A new yield curve model

The Bank has developed a new model of prices and yields in the gilt-edged market to replace one used since 1973. This note briefly describes the new model.

There are at present about one hundred British government stocks, or gilts. The yields on these stocks appear to be related to their different coupons and maturities, but the diversity of stocks and yields makes it difficult to say at any time what the interest rate is at a specified maturity. Even allowing in some way for coupon differences, it might still be necessary to interpolate between observed maturities in order to infer the interest rate at some intermediate maturity, when, for instance, considering the terms of issue of a new stock.

For many years the Bank has used a model of prices and yields in the gilt-edged market for these purposes. The main application of the model is to generate estimates of the so-called par yield at any given maturity. The par yield is the coupon which a hypothetical stock would have to bear in order for its price to equal its face value, that is to be priced at par. The set of par yields at every maturity forms the par yield curve. With the aid of this curve the Bank advises on the rates of interest to be charged by central government for lending to public corporations and local authorities. Par yields are published regularly in Table 9.1 of the statistical annex.

The construction and subsequent amendments of the model used by the Bank since 1973, originally developed by J P Burman and W R White, have been described in a number of *Bulletin* articles.[1] Burman and White based their work on certain strong assumptions regarding the nature and behaviour of market participants: they supposed investors to be either tax-paying or non-tax-paying, to operate within a preferred habitat or segment of the market, and to have a planning horizon within this preferred habitat. Over time, it has become clear that this model, despite the plausibility of these assumptions, sometimes fails to fit the data sufficiently well, particularly when the yield curve is downward sloping. For this reason, a replacement model has been developed which fits the recorded data more closely.

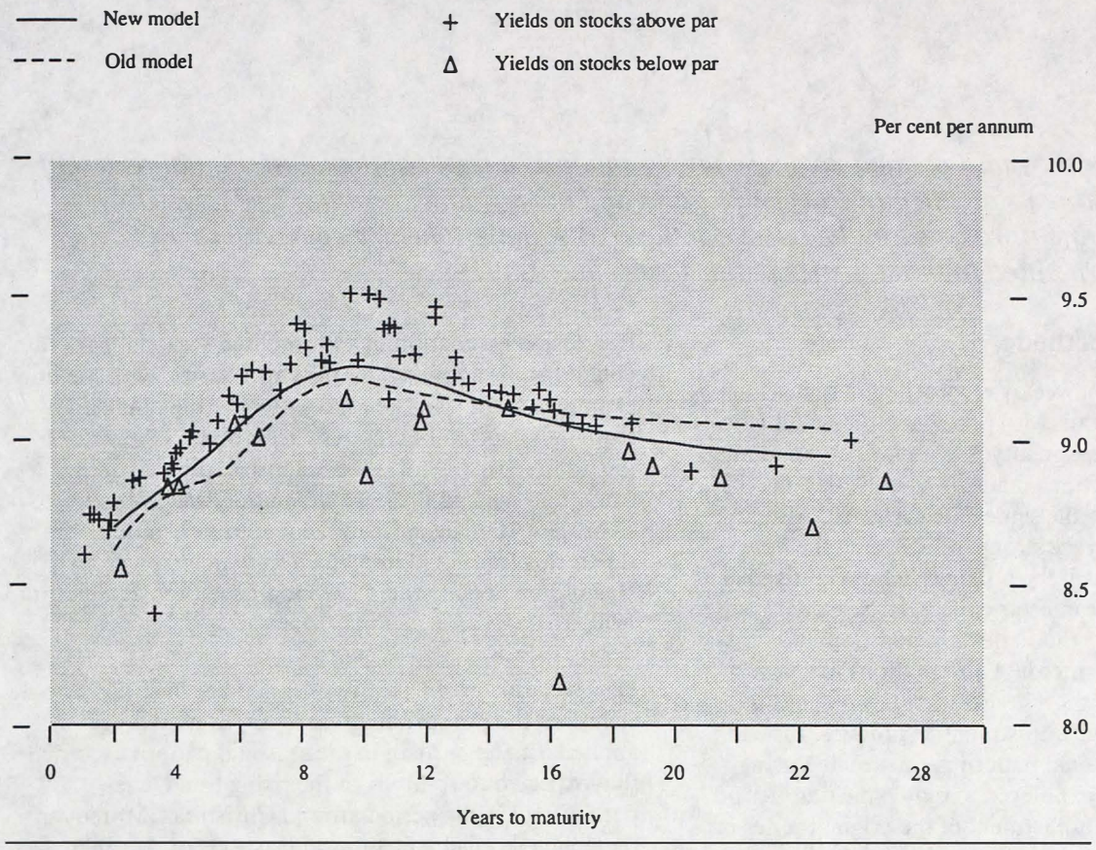
In developing the new model an alternative and simpler assumption is made that the yields on all stocks can be approximated by an equation which relates the yield on a hypothetical stock to its maturity and its coupon. In addition to the maturity and coupon the equation used in the new model includes twelve parameters, whose values are estimated in the course of fitting the model, using non-linear regression techniques, to the yields on actual stocks. Only conventional dated stocks with a significant amount in issue and having more than one year to maturity are used; index-linked stocks, irredeemable stocks and stocks with existing conversion options are therefore not included. Some of the improvement in the closeness of the fit over the old model can be attributed to the increase in the number of fitted parameters—twelve, instead of the eight required by the old model.

The equation used in the model describes a three-dimensional surface, a point on which has co-ordinates representing the yield, maturity and coupon of a hypothetical stock. This surface is built up in the following way. Since the par yield curve is the aspect of the model of most importance to the Bank, it is given an explicit functional form, being a modification of what is termed a cubic spline function. In deciding the exact form of the curve a balance must be struck between suppleness and rigidity: the curve must be able to take a wide variety of shapes without being so flexible that a single mispriced stock might significantly affect it. This curve forms the 'backbone' of the whole surface. The 'ribs' of the surface describe the price-coupon relationship of stocks when the term to maturity remains unchanged. If a stock has a coupon greater than the par yield (an above-par stock) then it is assumed that the price and coupon are linearly related. This linear relationship is given in terms of the effective tax rate facing investors in high coupon stocks. Where a stock has a coupon less than the par yield (a below-par stock) then the price-coupon relationship becomes more complex. This allows the model to take account of the observed preference which tax-payers with relatively high marginal tax rates on income have for stocks offering the prospect of capital gains. Finally, adjustments are made to allow for stocks which are free of tax to residents abroad and for any ex-dividend effect.

During the course of development the goodness of fit of the model was assessed using data for the period 1974–89. Thus the model has been tested on many different types of market condition. The accompanying chart shows the par yield curves from both the old and new models for 30 March 1988, on which date the distinction between the two models is particularly clear. It can be seen that the new curve separates the yields on stocks above par from

(1) See the December 1972 *Bulletin*, pages 467–86; the September 1973 *Bulletin*, pages 315–26; the June 1976 *Bulletin*, pages 212–15; and the June 1982 *Bulletin*, pages 226–31.

Par yield curves for 30 March 1988



those below par more distinctly than did the old curve, suggesting that the par yield curve is more accurately determined by the new model, at least on this particular date. Similar effects are observable on other dates.

A more detailed description in the new model will appear in a forthcoming discussion paper in the Bank of England technical paper series. It is intended that the Bank will commence use of the new model from 1 March, 1990.