Divisia measures of money

Interest has increased in recent years in index-number measures of money which weight the different components within each monetary aggregate. Proponents of these index-number measures argue that they should be a more helpful indicator of monetary conditions than the current aggregates which are constructed on a simple-sum basis. This article[1] assesses Divisia measures of money, including the theoretical arguments for the Divisia approach to monetary aggregation.[2] It also describes the construction of a Divisia index for the United Kingdom and the indicator properties of such an index.

Introduction

Since 1976 the framework for monetary policy in the United Kingdom has included published targets (more recently monitoring ranges) for the growth of the money supply. This approach was adopted in part on the assumption that there existed a reasonably stable—or at least predictable—relationship between the growth of the chosen target monetary aggregate and nominal income. A further important consideration was to provide the financial markets, and the public more generally, with a clear signal of the commitment of the authorities to a counterinflationary policy, and thereby to influence expectations of future inflation.[3]

Experience with setting targets for broad monetary aggregates was not wholly encouraging. These targets were overshot during most of the first half of the 1980s, largely because of financial liberalisation and increased competition among banks and between banks and building societies; and no broad money targets have been published since 1986.[4] Even so, the importance of M4 as an indicator of monetary conditions has continued to be emphasised; and in October 1992 the Chancellor introduced a 4%–8% monitoring range for M4 (revised in the Spring 1993 Budget to 3%–9% per financial year, to run over the full term of the current Parliament). Meanwhile, target ranges[5] for a narrow measure of money, M0, have been published as part of the Medium Term Financial Strategy since 1984, and M0 has proved to be a useful indicator of monetary conditions.

But the performance of any monetary aggregate against its target (or monitoring) range has never been viewed by the authorities as the only guide to the conduct of monetary policy. A wide range of indicators embracing all the available relevant information is taken into account in forming a judgment on the appropriate stance of monetary policy. It is within this framework that this article discusses Divisia money, and in particular whether Divisia is a potentially useful additional indicator of monetary conditions.

Previous work on Divisia money by the Bank has included a discussion paper on composite monetary indicators[6] and, more recently, a statistical discussion paper on monetary aggregates,[7] in response to which the Bank received a number of comments on Divisia money.

Why Divisia?

The existing monetary aggregates, such as M0 and M4, are constructed simply by adding together their components. These simple-sum aggregates are based on the implicit assumption that their components are to all intents and purposes exactly the same. In other words, that they are perfect substitutes for one another. So, notes and coin in circulation are treated within M4 as perfect substitutes for interest-bearing deposits. But cash is primarily used as a medium of exchange (ie for transactions purposes), whereas many interest-bearing deposits are held for savings purposes. Treating all components as perfect substitutes for one another may therefore be misleading.

Because of this potentially misleading assumption behind simple-sum monetary aggregates, there has been increasing interest over the last ten years or so in measures of money which weight the different components within each monetary aggregate. In theory, such an aggregate would be

(1) Prepared by Paul Fisher, Suzanne Hudson and Mahmood Pradhan in the Bank's Economics Division. This article is based on a more detailed and technical working paper: Fisher, P G, Hudson, S L and Pradhan, M, 'Divisia indices for money: an appraisal of theory and practice' ***Bank of England Working Paper No 9***, May 1993.

(2) The Divisia index was originally proposed by Francois Divisia in 1925 and was used to analyse monetary data by William Barnett at the Federal Reserve Board in the United States. See Barnett, W A (1980), 'Economic monetary aggregation: an application of aggregation and index number theory', *Journal of Econometrics*, vol 14, pages 11–48. For a comprehensive survey of the theoretical rationale underlying the Divisia approach see Barnett, W A, Fisher, D and Serletis, A (1992), 'Consumer theory and the demand for money', *Journal of Economic Literature*, vol XXX (December), pages 2,086–119.

(3) The early years of monetary targeting in the United Kingdom are described in Fforde, J S, 'Setting monetary objectives', *Bank of England Quarterly Bulletin*, June 1983.

(4) The behaviour of broad money during the first half of the 1980s is discussed in the 1986 Loughborough University Banking Centre Annual Lecture in Finance, given by the Governor on 'Financial change and broad money', published in the *Bank of England Quarterly Bulletin*, December 1986.

(5) Redefined as a monitoring range in the 1993 Budget.

(6) Mills, T C (1983), 'Composite monetary indicators for the United Kingdom; construction and empirical analysis'. ***Bank of England Discussion Paper. Technical Series No 3.***

(7) 'Monetary aggregates in a changing environment: a statistical discussion paper', ***Bank of England Discussion Paper No 47***, March 1990.

expected to produce a more helpful measure of monetary conditions.

The majority of authors favouring this approach have applied the Divisia index[1] which attempts to allow for the varying transactions properties of different monetary assets (ie the ease with which they can be used for expenditure) by giving them different weights. If these weights reflect differences in the transactions services provided by various monetary assets, then the resulting Divisia index will provide a measure of the total quantity of money held in the economy for transactions purposes as opposed to savings purposes. In principle, such an index should be more closely related than conventional monetary aggregates to total expenditure in the economy.

A Divisia index for money is intended to weight each of its component assets according to the extent to which they provide transactions services. Assets which are more costly to hold, because they bear a lower rate of interest, are deemed to provide more transactions services. The price of the 'moneyness' of each asset is the 'user cost'; in other words, the interest return forgone through holding each monetary asset rather than a financial asset which offers a higher return but provides no transactions services. Where monetary assets differ only in terms of their use as media of exchange these differences are regarded as indicating the transactions services provided by the assets. The Divisia index measures these transactions services in terms of their rate of change, as a weighted average of the growth rates of the component assets.

The Divisia index

The rationale for grouping assets together in an index is normally that they share certain characteristics which lead consumers to regard them as close substitutes. Economists define a valid aggregate—assets that can be grouped together—as one where consumer demand for the aggregate as a whole does not depend on the quantities of each of the constituent assets held.[2]

In order to construct such aggregates, it is necessary to discover the extent to which the component assets are substitutes for one another. To determine this, economists typically try to estimate the elasticities of substitution. The resulting aggregate will differ, however, according to the estimation technique that has been chosen, and in practice the extent to which assets are substitutes may change over time.[3] Simple-sum aggregation has a similar drawback—all assets are treated as perfect substitutes.

Another way of determining the substitutability of the components of a monetary aggregate is to examine the holdings of the various monetary assets and their relative rates of return. Those holdings which appear preferable, in the sense that their returns dominate the returns from other assets, should be included in a monetary aggregate.[4] But this procedure does not allow for *gradual* adjustments away from assets with lower rates of return. For example, holdings of non-interest-bearing sight deposits have persisted long after the introduction of interest-bearing current accounts. Such behaviour would appear irrational according to the consistency criterion underlying this approach to aggregation.

Because of drawbacks inherent in each of these approaches, a purely statistical index which includes both quantities and their prices becomes an attractive option. There are a variety of statistical index numbers which could be constructed. But of these the Divisia index (see the box on page 160 for a detailed definition) seems the most suitable, since it provides a close approximation to the underlying relationship between component assets that motivates the demand for transactions servives.[5]

An alternative to the Divisia index is the 'Currency-equivalent' aggregate (CE), which is a simple time-varying weighted average of the stock of all monetary assets, where the weights are each asset's user cost as a proportion of a benchmark return, ie $CE = \sum[p_{it} / R_t]M_{it}$.[6] Because this index is expressed in levels rather than as a rate of change it is intuitively more appealing.[7] But in practice the *CE* index is much more volatile because, unlike the Divisia index, the weights are not chain linked. As a result, in constructing the index the weights have to be averaged over fairly long periods. For comparison, Chart 12 shows the *CE* index relative to the Divisia index.

Drawbacks of Divisia

As with the existing aggregates, the Divisia index has a number of shortcomings:

(a) Monetary and transactions services

It is far from clear precisely what Divisia is measuring when it measures 'transactions services'. Money performs three functions: a unit of account, a store of value and a medium

(1) There are a number of recent UK studies using the Divisia approach, including Batchelor, R A (1988), 'The monetary services index', *Economic Affairs*, vol 8; Belongia, M T and Chrystal, K A (1991), 'An admissable monetary aggregate for the United Kingdom', *The Review of Economics and Statistics*, vol 73, pages 497–503; Ford, J L, Peng, W S and Mullineux, A W (1992), 'Financial innovation and Divisia monetary aggregates', *Oxford Bulletin of Economics and Statistics*, vol 54 (1), February; and Spencer, P D (1992), 'UK monetary aggregates: in search of a better yardstick', *Kleinwort Benson Research—UK Economic Study*, October.

(2) Formally, the group of assets that are aggregated needs to be 'weakly separable' from other assets. In the absence of such separability, changes in the relative prices of the component assets which left the aggregate's price index unchanged would imply different levels of demand for the aggregate as a whole.

(3) See Chetty, V K (1969), 'On measuring the nearness of near-moneys', *American Economic Review*, vol 5, pages 115–45, for an early application of this approach.

(4) See Varian, H R (1982), 'The nonparametric approach to demand analysis', *Econometrica*, pages 945–74.

(5) See Diewert, W E (1976), 'Exact and superlative index numbers', *Journal of Econometrics*, vol 4, pages 115–45.

(6) See Rotemberg, J J, Driscoll, J C and Poterba, J M (1991), 'Money, output and prices: evidence from a new aggregate', *NBER Working Paper, No 3824*.

(7) The *CE* index is also able to cope better with the introduction of new assets to monetary aggregates than the Divisia index. Since the Divisia is based on rates of change, expressed in natural logarithms, in the period when a new asset is introduced, its rate of growth will be undefined. This is because its contribution to the index in the previous period is zero.

The Divisia index for money

The Divisia index is expressed as the rate of change of the weighted sum of the rates of change of the individual component assets. The growth rate of each component asset is assigned a weight which reflects its user cost. Formally, the user cost of monetary asset i, is given by p_i,

$$p_i = (R - r_i)$$

where R is the return on the alternative benchmark asset (which does not provide transactions services) and r_i is the return on monetary asset i.(1)

The actual weight of asset i, (the total cost of holding asset i) is calculated as its user cost times the quantity of asset i held, expressed as a proportion of the total cost of all monetary assets held. Thus the weight applied to each asset is its share of total expenditure on transactions services.

The Divisia index of money is then given by,(2)

$$\ln D_t - \ln D_{t-1} = \sum_i n_{it} (\ln M_{it} - \ln M_{it-1})$$

Where,
ln denotes the natural logarithm of a variable
D_t is the Divisia measure at time t
M_{it} is the holding of monetary asset i at time t
$n_{it} = {}^1/_2 (s_{it} + s_{it-1})$

$$s_{it} = (p_{it} M_{it}) / \sum_i p_{it} M_{it}$$

p_{it} is the user cost of asset i at time t

As this is a discrete time approximation, the actual weights (n_i) are the simple two-period moving averages of the expenditure shares. The level of Divisia money can be obtained by arbitrarily assigning a base value of 100 in any one period, from which percentage growth rates can be calculated. The index is based on the assumption that a doubling of all money holdings would double the transactions services available. This property of the index implies that the aggregate will grow at the same rate as its components and that the weights will sum to unity. Where user costs reflect more than simply the usefulness of assets in affecting transactions, this feature of the index is clearly restrictive, and the growth of transactions services may not be captured accurately by this measure.

(1) For a formal derivation of the user cost see Barnett, W A (1978), 'The user cost of money', *Economic Letters*, vol 1, pages 145–49.
(2) The index given here is the discrete time approximation (known as the Tornquist-Theil approximation) of the continuous time Divisia index.

of exchange. Proponents of Divisia indices are concerned only with the last of these. The choice of numeraire in an economy is arbitrary, and all capital-certain nominal assets denominated in the same currency provide equally good stores of value.(1) So because *all* monetary assets provide these two functions in equal measure, they do not affect the weighting procedure in the Divisia index and are thus irrelevant to the Divisia approach. As a result, it is argued that Divisia indices measure the 'transactions services' offered by monetary assets in their role as a medium of exchange, and are therefore *the* relevant aggregate for a stable demand-for-money equation.

But in practice Divisia measures far more than just transactions services. Monetary assets have a range of characteristics, only some of which relate to their use as a medium of exchange. For example, many bank accounts offer investment advice, easier overdraft facilities and other financial services such as insurance and pensions advice. Not all of these features enhance the transactions properties of an asset, but they are 'monetary' services to the extent that they are only available if you hold the relevant monetary assets.

To avoid this difficulty, each characteristic of a bank account could, in principle, be assigned an implicit price and a pure transactions index could be calculated using the transactions elements. But in practice the multiplicity of accounts and characteristics would make the problem intractable. Non-price competition and product discrimination among banks has vastly expanded the range of accounts available. And if it were decided to choose only a sample of products, the need for subjective judgments about which to include would undermine some of the original attractiveness of the Divisia approach. Some transactions services, for example, are provided by means—such as unused credit card facilities—which do not depend on holdings of monetary assets. So to the extent that the Divisia index constructed from available data cannot provide a true measure of transactions services, it is less likely to have a stable relationship with macroeconomic variables.

(b) Measuring transactions services

The Divisia index defines transactions services implicitly, by using the observed interest rate to compute a user cost for the services provided. These interest rates are assumed to be at competitive levels: they act as a 'summary statistic' containing all the available information about how the market values the services provided by monetary assets. In a competitive financial services industry, the observed interest rates would fully reflect the shadow price of the transactions services provided by the asset. But in practice the degree of competition may change over time. For example, the financial system became more competitive throughout the 1980s, and banks and building societies offered accounts which offered both increasingly easy

(1) This argument assumes the assets under consideration all have zero default risk.

access (shorter periods of notice for withdrawals) and increasing interest rates relative to wholesale market rates.

Market interest rates may also fail to capture the full shadow price in the presence of externalities. Given the 'social' nature of a medium of exchange, this is likely to be a recurring problem. For example, the transactions services provided to an individual by a bank current account depend on how many other people and institutions have such accounts. The more accounts, the wider the acceptability of bank cheques. Yet the increased benefits of a bank account to an individual arising from their widespread use may not be reflected in market interest rates.

Thus in practice the transactions services that Divisia measures are all those services whose cost is reflected in the asset's interest rate. These services cannot be identified precisely without explicitly modelling the supply side of the transactions services market. So Divisia simply assumes that differences in interest rates measure differences in services provided. But this can be misleading. Many of the services offered could be explicitly charged for. These charges are part of the price of monetary services provided by bank accounts. When, for example, banks started offering interest on current accounts, and at the same time instituted charges for various services provided, the Divisia user costs of these accounts fell, indicating a change in transactions services provided. This apparent change is misleading, although the current user costs may capture more accurately the level of transactions services provided by these accounts.

The broader measure of transactions services that Divisia actually provides is also unlikely to satisfy the restriction that doubling all money holdings would double the transactions services available. Investment advice and other financial services generally depend on having a bank account rather than on the amount held in it. Doubling bank deposits would not necessarily double the advice a bank provided. Moreover, new technology, such as the introduction of ATMs and the wider use of credit cards, increases the transactions services provided by existing asset holdings without increasing the user cost to the agent. To the extent that such innovations are not reflected in interest rates, the Divisia index will mismeasure transactions services. This implies that the Divisia approach may not in practice be able to overcome the problems posed for monetary aggregates by financial innovation.(1)

(c) Portfolio adjustment

The Divisia approach assumes that asset holdings are at their desired values and makes no allowance for adjustment costs and/or measurement errors. This appears to be inconsistent with the extensive evidence from both the theoretical and empirical literature on the demand for money. When there are adjustment costs they should be reflected in the appropriate prices, since asset holdings and portfolio reallocations will be based on the 'effective' user costs, rather than the user costs based on the explicit own rates of return. In practice, agents adjust their holdings of monetary assets over time in response to changes in relative interest rates between different types of deposits. A recent example is the gradual decline in non-interest-bearing deposits.

(d) Choice of the benchmark asset

In order to assign user costs to each asset, an asset which does not yield any transactions services has to be selected against which the opportunity cost of these services can be measured. The user cost of a monetary asset is then simply the return on the non-monetary asset less the monetary asset's own rate of return.

In principle, the non-monetary asset has to be capital-certain in order to make it comparable to other monetary assets. But it must not offer any transactions services, since assets which offer some transactions services should themselves be included in the Divisia aggregate. This implies that assets for which there are active secondary markets should not be considered, because a secondary market would enable holdings to be readily converted into more liquid assets that could be used for transactions.

There are not many assets which satisfy these two criteria. Most of the recent UK work on Divisia uses the local authority deposit rate as the benchmark return. Another possible candidate could be National Savings certificates, although their holding period is far longer than that of most monetary assets, and the amount that can be held in this form is usually limited. The benchmark asset need not be the same asset in different periods if relative yields on alternative benchmark assets change over time.

(e) Interpreting changes in the Divisia index

A practical objection to the use of Divisia indices has been the problem of interpretation. When interest rates change, the weights on the component assets will change even before any portfolio changes have occurred. So, if interest rates rise, the user cost of cash will rise and it will attract a higher weight. Once the portfolio shifts have occurred, less cash will be held and its weight will then be lower. Because of this lag, current weights will not be equilibrium weights—unless, of course, the portfolio shifts are instantaneous. The immediate effect on the Divisia index of a rise in the general level of interest rates would then be different from the long-run effects, after portfolio shifts have taken place. Whether the initial effect is to reduce or to increase the growth rate of Divisia relative to the long run depends on the initial relative growth rates of interest-bearing and non-interest-bearing deposits. When cash holdings are growing more rapidly than interest-bearing deposits, an increase in interest rates will raise the weight on cash and reduce that on interest-bearing deposits, thereby leading to an increase in the Divisia growth rate. As a result, it is

(1) Two recent studies have attempted to identify the extent of the divergence between Divisia and 'true' growth of transactions services. See Koenig, E F and Fomby, T B (1990). 'A new monetary aggregate'. *Federal Reserve Bank of Dallas Economic Review* (May), pages 1–15, and Ford *et al. op cit.*

possible that for short periods of time the Divisia index could be a misleading indicator of the monetary stance.

Summary

This discussion of some of the problems in implementing the Divisia approach suggests that user costs may reflect more than just the transactions services of assets. Moreover, interest rate data on financial assets are not sufficiently detailed to provide accurate measures of transactions services. These arguments do not, however, invalidate the Divisia approach. Conventional simple-sum aggregates, which have not been assessed in this article, may suffer from more serious drawbacks. The central case for Divisia indices is straightforward—components of monetary aggregates are not perfect substitutes. Therefore, differential weighting, even with the imperfections noted here, is likely to yield a more accurate measure of transactions services.

A Divisia index for the United Kingdom

It is possible to calculate a number of Divisia indices on the basis of different assumptions about the specifications of user costs and the extent to which asset holdings are disaggregated. The Divisia index presented in this article is constructed from the components of the M4 broad money aggregate and the following interest rates:(1)

Component	Interest rate
Notes and coin in circulation with the public	zero
Non-interest-bearing UK private sector sterling sight deposits	zero
Interest-bearing UK private sector bank sight deposits	
of which:	
Persons	Clearing bank instant access account rate (gross rate)
Corporates	Overnight London interbank deposit rate
Interest-bearing UK private sector bank time deposits	
of which:	
Persons	Clearing bank interest-bearing personal account rate (gross rate)
Corporates	Three-month London interbank deposit rate minus 0.5%
Building society deposits	
of which:	
Persons	Building society savings account rate (gross rate)
Corporates	Three-month London interbank deposit rate
(Benchmark asset)	Three-month local authority deposit rate

The quarterly index covers the period 1977 Q1 to 1992 Q4, providing sixty-four observations. All the component series are seasonally unadjusted—the index is then itself seasonally adjusted.

User costs

Obtaining the correct specification for user costs is important not only for the time path of Divisia, but also for the empirical tests to be carried out. The user costs are constructed as shown in the box on page 242, adjusted for taxation.(2)

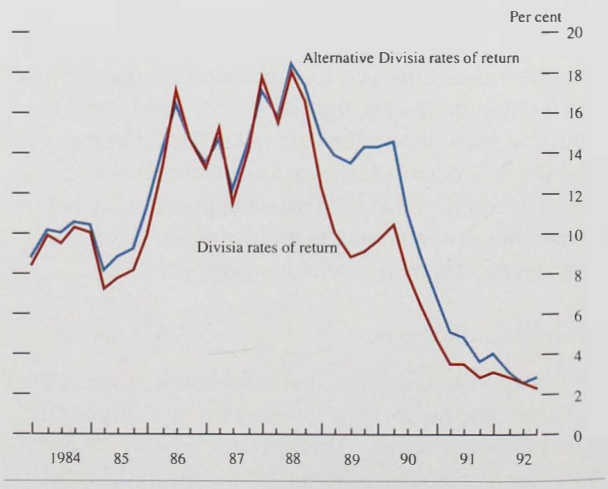
In calculating the user costs and weighting the component assets, the instantaneous expected holding period return should ideally be used rather than the interest to maturity. The difference between the holding period return on the benchmark asset and the holding period return on monetary assets will then reflect the differences in transactions services derived from each asset. But, because the instantaneous holding period return is unobservable, we use the returns to maturity. Rates for maturities of less than three months are averaged over the quarter.

There are, of course, practical problems in assigning interest rates to particular types of deposit. In particular, a detailed breakdown of deposits and their respective returns is not readily available. Nor is it clear what the most appropriate single interest rate is for calculating the user costs for each category of deposit. The following illustrates a number of the practical difficulties in obtaining a correct specification for user costs.

(a) Rates of return

Chart 1 shows two Divisia indices. Each employs different, but arguably both appropriate, interest rate series for corporate holdings of bank interest-bearing sight deposits and for persons' bank interest-bearing time and building

Chart 1
Twelve-month growth rates of Divisia indices using different rates of return



society deposits. The red line represents the index analysed further in this article. The blue line represents an index employing the alternative rates of return, as set out below:

(1) Appendix A sets out the data and their sources.
(2) The composite tax rate is used for interest-bearing retail deposits and the corporate tax rate for deposits held by the corporate sector.

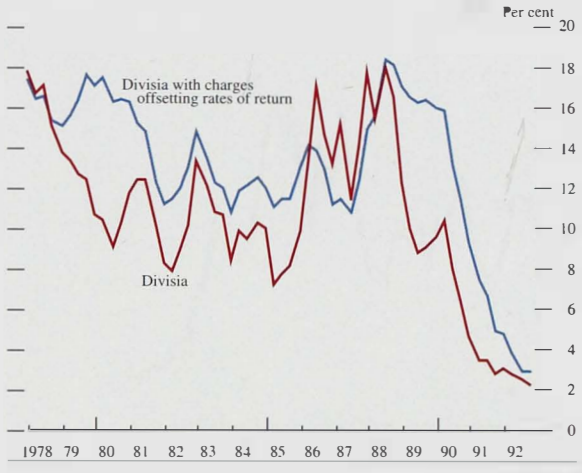
Component	Divisia rates of return	Alternative rates of return (where different)
Bank interest-bearing sight deposits		
of which:		
Persons	Average current account rate offered by major clearing banks on deposits of £500	
Corporates	Overnight London interbank deposit rate	Base rate minus 3%
Bank interest-bearing time deposits		
of which:		
Persons	Average personal account rate offered by major clearing banks	Average rate on seven-day notice deposit accounts
Corporates	Three-month London interbank deposit rate minus 0.5%	
Building society deposits		
of which:		
Persons	Average savings account rate offered by five largest building societies	Average building society share rate
Corporates	Three-month London interbank deposit rate	

The divergence between the two indices can be explained by changes in the respective user costs and growth weights of the components. The interest rate data for corporate sight and for persons' time bank deposits which are used in the preferred index are higher than those used in the alternative index, and result in lower growth weights. On occasion, the building society rates are lower than those in the alternative index. One such period was from mid-1988 to 1991, with the largest differential emerging during 1989. This partly explains the stronger twelve-month growth rate of the alternative index during this period.

There is also a further question about whether the appropriate rate of return, and hence the user costs, should take account of bank and building society charges. As discussed above, each characteristic of a bank account should, in principle, be given an implicit price. But not all charges are likely to be relevant and, in the absence of adequate disaggregation, the resulting index might well be distorted. Chart 2 shows a Divisia index which assumes that interest rates on interest-bearing retail components are fully offset by charges—in effect, the user costs for retail interest-bearing deposits are calculated as the benchmark rate. This makes the growth rate of the index less trended and higher overall than the original index over the period in question.

The generally higher annual growth rate of the alternative index is largely accounted for by the higher weights calculated for retail bank and building society deposits. These weights are more stable than those calculated for the primary index. One result of this is that the strong growth from 1985 to mid-1986 is not fully replicated in the alternative index.

Chart 2
Twelve-month growth rates of Divisia and of Divisia with charges offsetting rates of return



A further problem occurs if there are substantial costs of portfolio adjustment, or imperfect information about interest rates. If this is so, then current user costs may no longer be equated with marginal transactions services and Divisia will not provide an entirely accurate measure of transactions services. One possible remedy is to use lagged interest rates to calculate user costs, on the basis that these lagged rates reflect the perceived costs of holding monetary assets or, alternatively, the effective prices for asset holders who are subject to adjustment costs. Other suggestions have focused on various smoothing techniques. For example, centred moving averages of the user costs could be used, on the argument that if individuals do not adjust their portfolios continuously, then their decisions will be based on present and expected values of this variable. The resulting user costs will move more smoothly than those calculated only from current returns, with the result that the weights assigned to asset growth rates will, other things being equal, also move more smoothly over time. It seems plausible that notes and coin, non-interest-bearing deposits and wholesale deposits are subject to relatively low adjustment costs, but an index could incorporate lagged or 'smoothed' user costs on interest-bearing retail deposits.

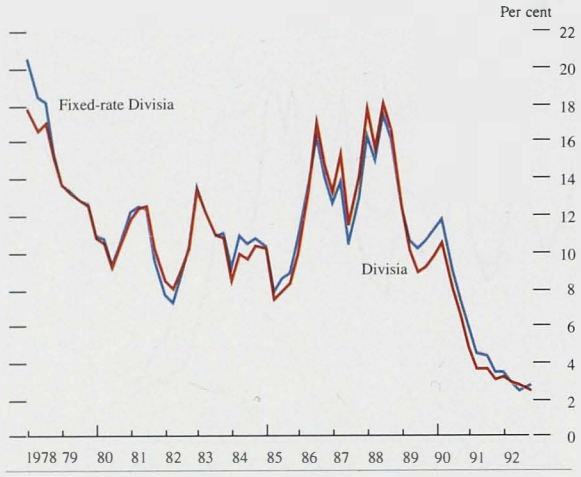
These smoothing methods are rather ad hoc. But there is no way of determining the 'correct' values of the smoothing parameters, so their choice is inevitably arbitrary. In practice, such smoothing techniques make little difference to the outcome—a conclusion reached by other studies.

For illustrative purposes, Chart 3 shows a Divisia index constructed with fixed user costs compared with the original Divisia index. The user costs are fixed as the overall sample averages. The outcome, as shown, is quite similar to the index calculated with varying user costs and further supports the view that smoothing techniques make little difference.

(b) The benchmark interest rate

The index presented here uses the three-month local authority deposit rate as the benchmark interest rate. These deposits are non-marketable and non-chequable and so

Chart 3
Twelve-month growth rates of Divisia and fixed-rate Divisia



cannot be used for making transactions. Reflecting this, yields on these deposits normally exceed those on other deposits, making the opportunity cost on monetary assets positive.

The sample includes periods, however, when the returns offered on building society retail deposits and by banks and building societies on corporate deposits were higher than the returns on local authority deposits, which would generate negative weights for these components. One simple solution to the problem of negative weights is arbitrarily to add a constant to the benchmark rate. Using the data as set out in Appendix A, a constant of two percentage points is necessary to obtain positive weights throughout.

An alternative to the single-rate benchmark is to use the maximum available rate as the benchmark. Chart 4 compares the quarterly index incorporating the three-month local authority deposit rate plus a constant as the benchmark with an index constructed using the maximum available rate from the component assets as the benchmark in each quarter.

Chart 4
Twelve-month growth rates of Divisia indices using different benchmarks

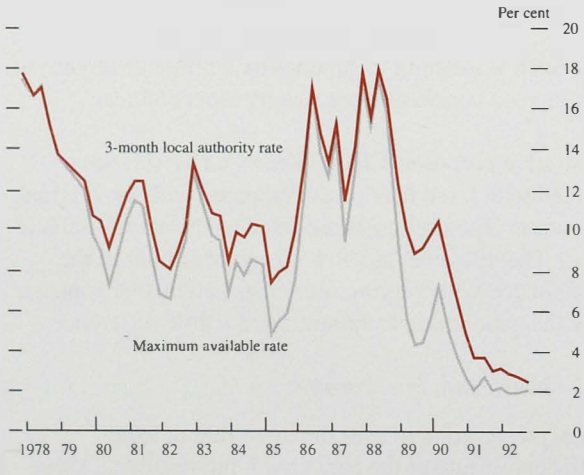
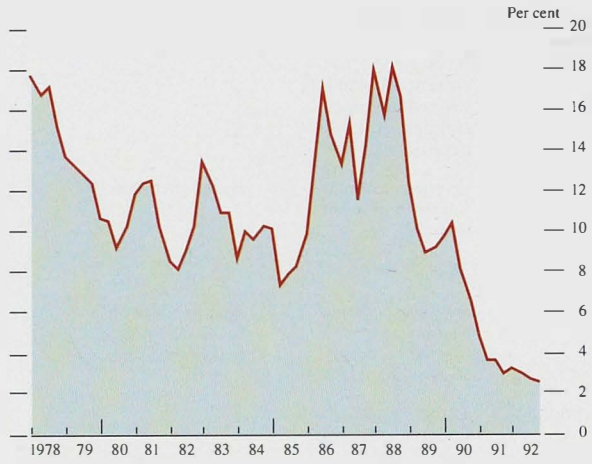
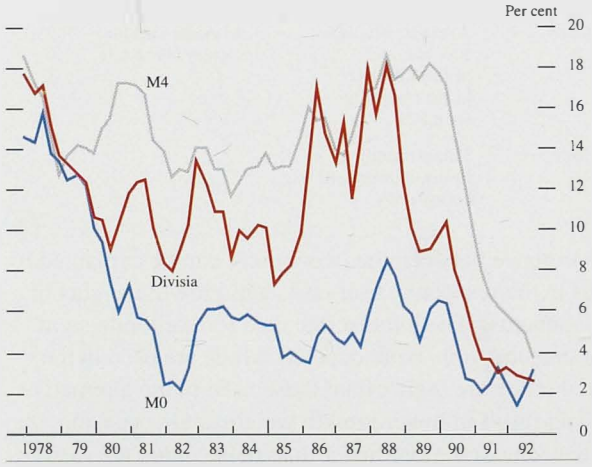


Chart 5
Twelve-month growth rate of Divisia



Twelve-month growth rates of Divisia, M0 and M4



A number of problems arise when using a maximum-rate benchmark. In principle the benchmark asset should not provide monetary services and, as such, an asset that is included as money in a previous time period should not later be used as the benchmark. One outcome of using the maximum available rate from the component assets is that interest rates for monetary assets operate periodically as the benchmark when they exceed the local authority deposit rate, with the effect that their respective user costs and growth weights are zero.(1) In the period in question, the building society and wholesale deposits operate, on occasion, as the benchmark asset.

The quarterly Divisia index

The twelve-month growth rate of the preferred quarterly Divisia index is illustrated above and compared with the conventional simple-sum monetary aggregates.

The growth paths of Divisia and of the simple-sum aggregates diverge quite considerably over the sample period. In the late 1970s Divisia and M0 appeared to grow at almost identical rates. From 1980, however, M0 growth rates continued to fall, whereas the growth of Divisia had climbed to 12.5% by the third quarter of 1981, compared with 5.5% for M0 and 16.7% for M4.

(1) Strictly speaking the local authority deposit would then have to be included in the aggregate and therefore treated as money.

Of particular interest is the pick-up in Divisia growth from 1985 Q2 until 1986 Q3 , after which it remained strong until the end of 1988. Between end-1988 and 1992 Q4 , however, Divisia growth fell sharply to below 3.0%. Although M4 growth also fell sharply from 1990, its fall was not so severe.

The divergences between the growth paths of Divisia and simple-sum monetary aggregates may be explained by examining the time paths of the Divisia components, their user costs, their expenditure shares and the weights attached to the growth of the Divisia components. These are shown in Charts 6–9. The user costs are not themselves the

Chart 6
Components of Divisia (levels, unadjusted)

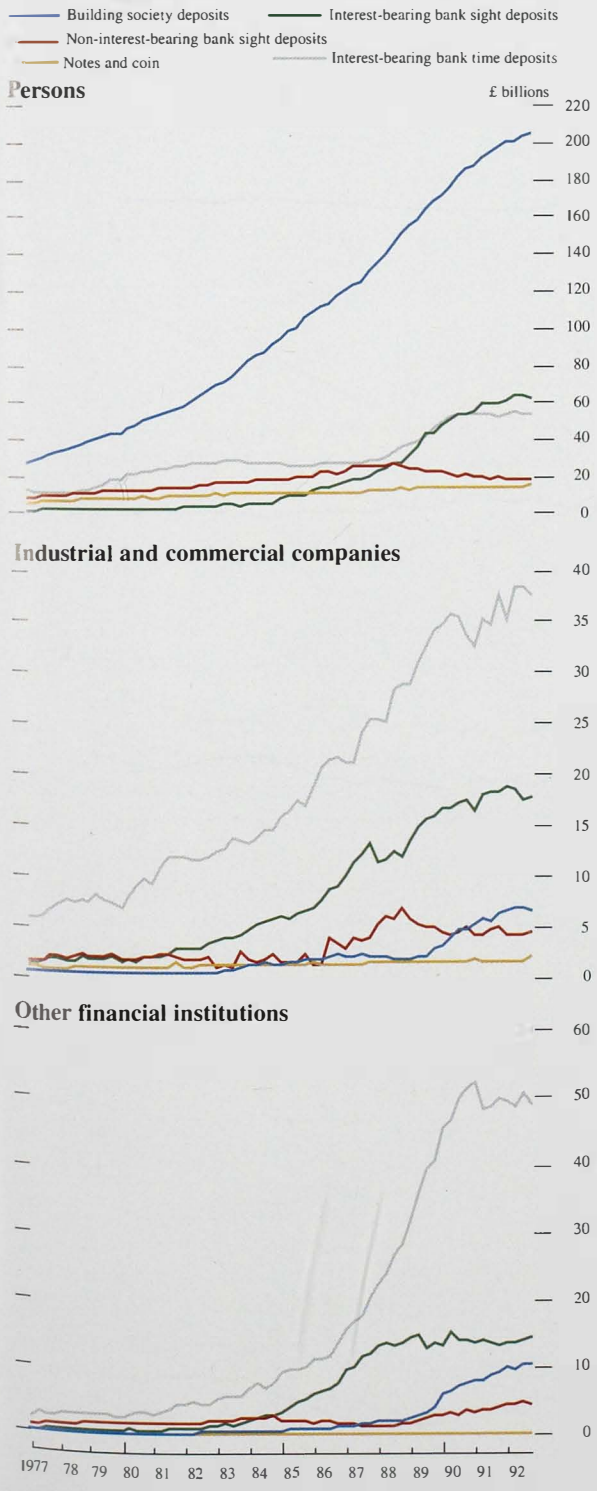
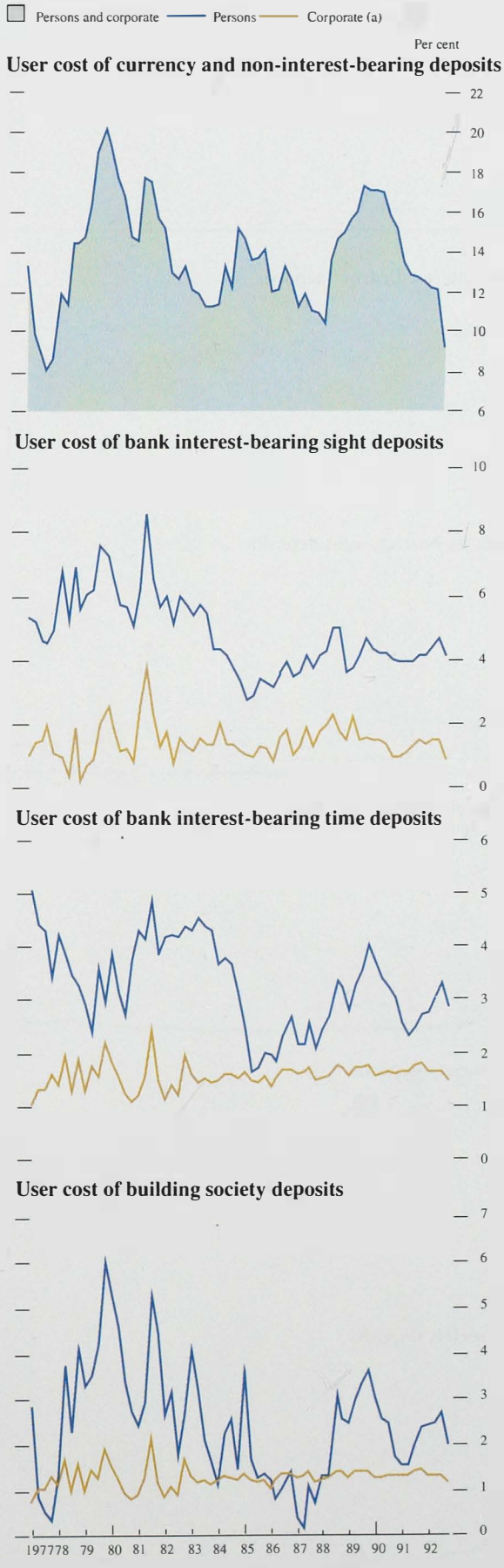


Chart 7
User costs



(a) Industrial and commercial companies and other financial institutions.

Chart 8
Expenditure shares

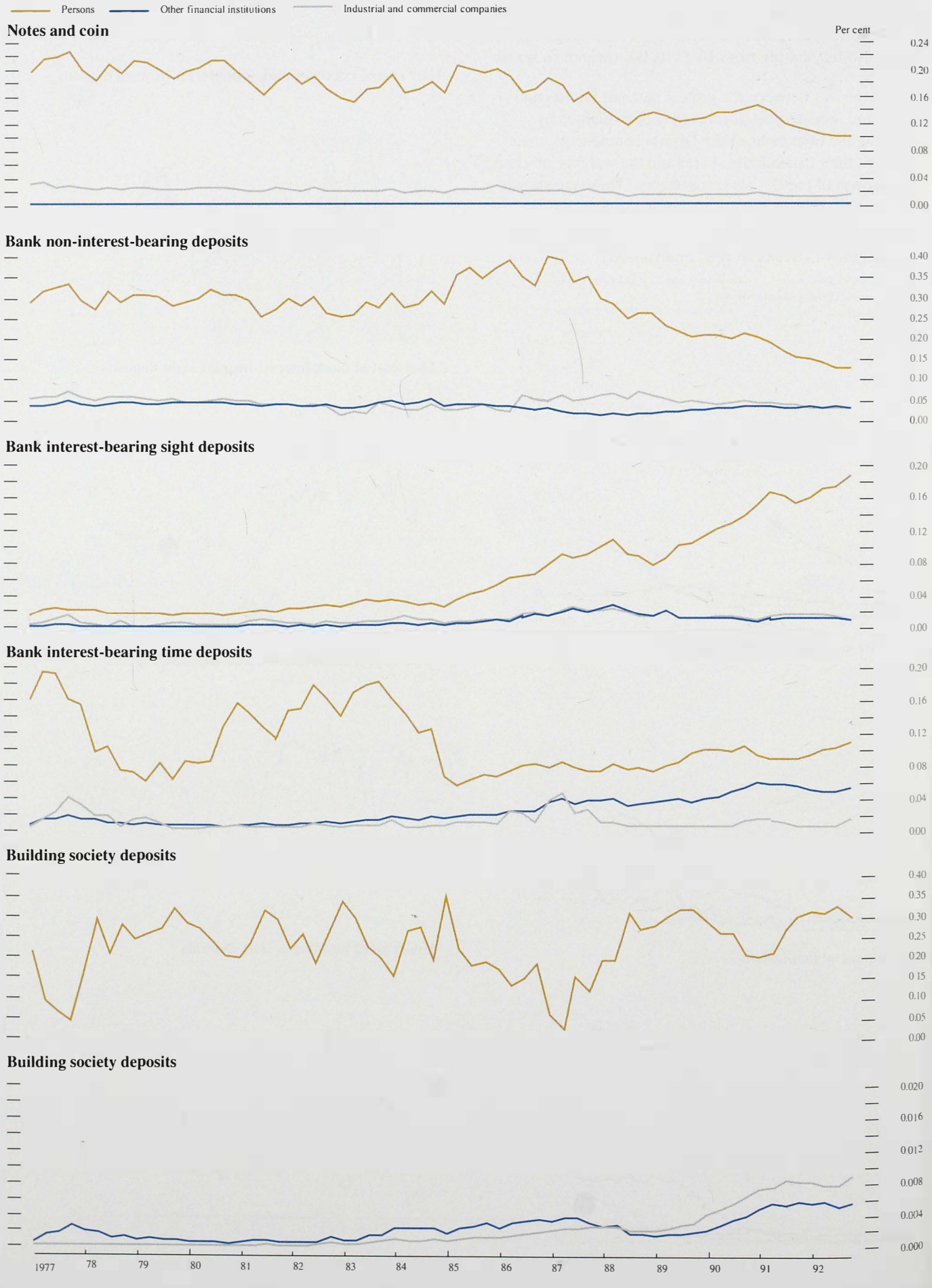


Chart 9
Growth rates

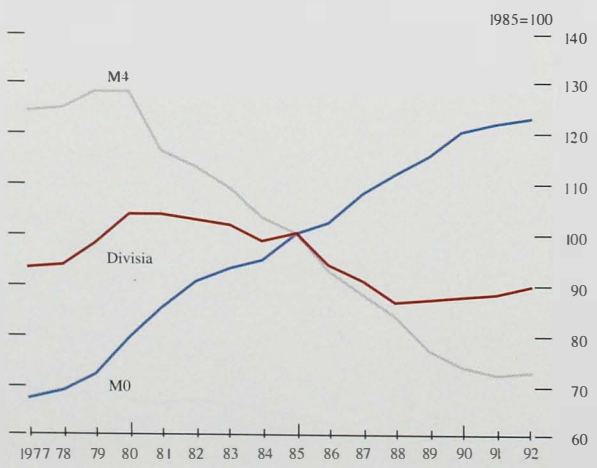
weights, but are the prices used with the quantities in calculating these weights, each weight depending upon all prices and all quantities.

The weight each component receives depends on its size relative to the other components of M4 and on its user cost. Because of their high user costs, currency and non-interest-bearing deposits—a small fraction of total M4—receive the highest weights. Bank time deposits were, in the late 1970s and early 1980s, a relatively large component of the money supply with high user costs and therefore carried a relatively high weight over this period. The weight attributed to interest-bearing sight deposits increased significantly from the mid to late 1980s owing to the sharp increase in the quantity of such deposits, despite the decline in their user costs as banks began to pay interest on current accounts. Building society deposits, though large, pay a relatively high rate of interest, and so attract a lower weight than might be expected from the size of such deposits alone. Wholesale deposits, represented by corporate holdings of bank time and building society deposits, receive the lowest weight, reflecting both low user costs and relatively small quantities.

The behaviour of these share weights helps to explain why Divisia and simple-sum monetary aggregates diverge. The second half of the 1980s when Divisia exhibited strong growth, on occasion above that for the official aggregates, is particularly interesting. Throughout this period bank interest-bearing sight deposits were growing rapidly, following the introduction of interest-bearing current accounts, and the growth weight on this component of the index was increasing. Meanwhile, wholesale deposits were also expanding strongly and this is reflected in a slight increase in their growth weights.

Chart 10 shows Divisia's velocity alongside that of M4 and M0. The velocity of M0 has risen relatively steadily (M0 has risen less than money GDP). M4 velocity has been more variable, rising in the late 1970s and falling since 1980, largely reflecting the deceleration in velocity of bank

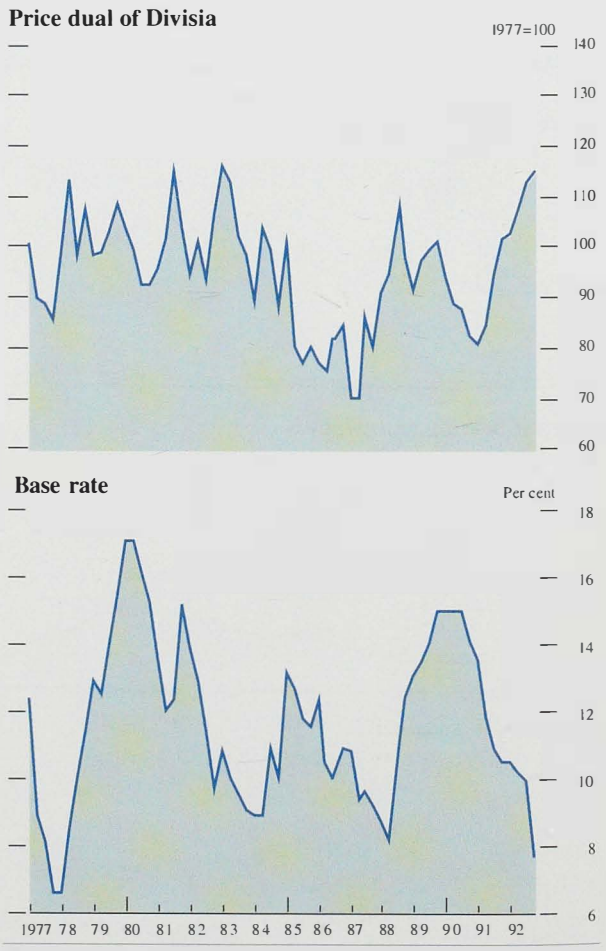
Chart 10
Velocity of Divisia, M0 and M4



interest-bearing sight and time deposits. Divisia velocity, however, appeared to stabilise in the late 1980s and then increased slowly from 1988. Divisia velocity has been more stable than that of M4.

Corresponding to the Divisia quantity index is a price index —the price dual.[1] The Divisia index of prices (see Chart 11) is obtained by cumulating over time a weighted sum of the rates of change of the component prices, where the weights are the current shares of the component assets in the total current holdings of all assets in the index. As the price dual is based on interest differentials it is not surprising that its historical behaviour bears little resemblance to the level of the base rate. The price dual is employed later when testing for indicator properties of Divisia.

Chart 11
Divisia price index

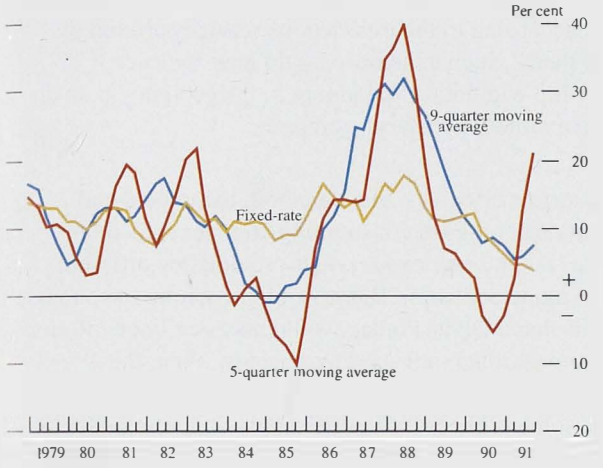


As mentioned above, an alternative to Divisia is the 'Currency-equivalent' aggregate. As with Rotemberg *et al (op cit)* this is calculated using various centred moving averages of user costs and an aggregate which uses fixed weights corresponding to the sample average of user costs—the growth rates of which are shown in Chart 12 below.

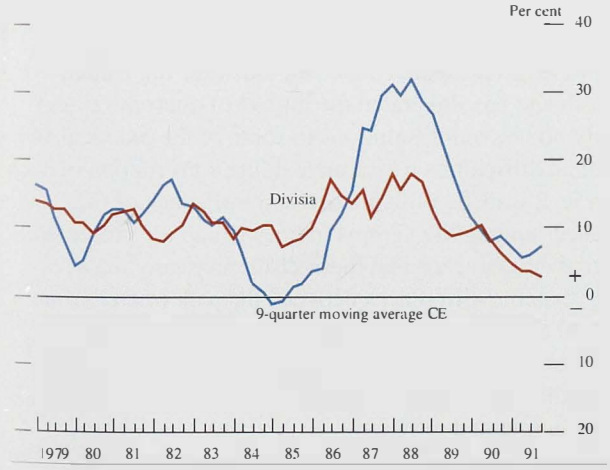
Because the Currency-equivalent aggregate's short-term fluctuations are sensitive to high frequency interest rate changes, its annual growth rate, even with a nine-quarter

(1) This is calculated as $Ln\ P_t - Ln\ P_{t-1} = \Sigma n_{it}(Ln\ p_{it} - Ln\ p_{it-1})$ where: n_{it} = the arithmetic average of the expenditure shares $(p_{it}m_{it}/\Sigma p_{it}m_{it})$.

Chart 12
Currency equivalent aggregate (twelve-month growth rate)



Twelve-month growth rates of Currency-equivalent aggregate and of Divisia



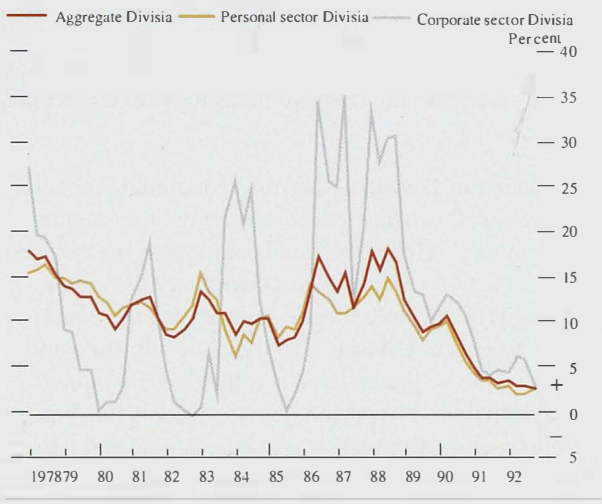
centred moving average, exhibits greater variation than that of Divisia.

Sectoral Divisia

In addition to an aggregated Divisia index, the data set allows the investigation of the historical behaviour of corporate (encompassing both Industrial and Commercial Companies (ICCs) and Other Financial Institutions (OFIs)) and personal sector Divisia indices and their money demand behaviour. As with the preferred aggregated index the sectoral indices are constructed with the three-month local authority deposit rate as the benchmark.(1)

Chart 13 plots the annual growth rate of the aggregate Divisia against the personal and corporate sector Divisia. The historical behaviour of the personal sector Divisia is similar to the aggregate. This is not unexpected as persons are dominant holders of M4 deposits. Corporate sector Divisia, on the other hand, exhibits a more volatile growth path than the aggregate index. This can be explained by the somewhat variable holdings of M4 deposits by ICCs and OFIs.

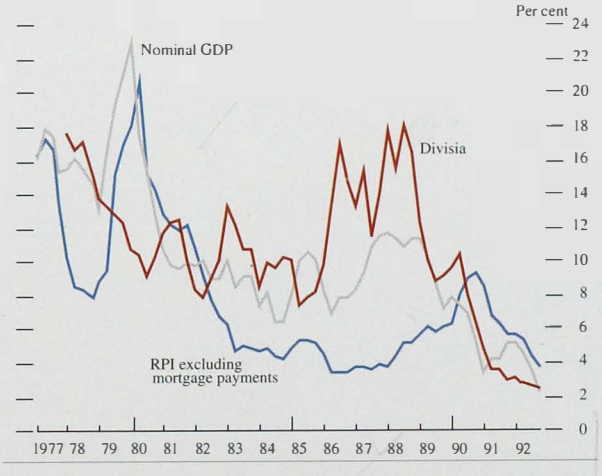
Chart 13
Twelve-month growth rates of aggregate Divisia, personal sector Divisia, and corporate sector Divisia



An econometric evaluation of Divisia

Although no particularly strong relationship between Divisia and nominal GDP is immediately apparent from Chart 14, Divisia does appear to lead nominal income on a number of turning points: Divisia money growth rose strongly in 1977–78 and appeared to lead the subsequent upturn seen in nominal GDP during 1979; the strong growth in Divisia between 1985 and 1987 was followed by an upturn in GDP growth in 1986–89; and the subsequent deceleration in Divisia growth also appeared to lead the decline in nominal GDP growth. No such association, however, is immediately apparent between Divisia and inflation.

Chart 14
Twelve-month growth rates of Divisia, inflation and nominal GDP



As discussed in the introduction, the use of broad money aggregates as intermediate targets or even as monetary indicators has been hampered by the relative unpredictability of their relationships with nominal income and inflation. It has often been difficult to understand why monetary aggregates behave in a particular way and even harder to

(1) A corporate index was also constructed using the three-month Treasury bill operating as the benchmark—it makes little difference to the index.

extract what these movements mean for the macroeconomic outlook. The Divisia indices are therefore evaluated on two counts. First, econometric relationships are estimated which explain the demand for Divisia balances—the results of this exercise are reported in Appendix B. Second, simple causality tests are used to assess the informational content of Divisia with respect to future movements in nominal income and inflation.(1)

The construction of Divisia allows the econometric models to be based on well-established theories of the transactions demand for money. Hence the trend component is explained by prices, real activity and user costs, with an assumption that all changes in the price level eventually lead to equal percentage changes in Divisia. The dynamic adjustment to trend is allowed to be gradual and is influenced by output growth and inflation. For personal sector Divisia this type of model works well—the equation is stable, simple and passes standard mis-specification tests. For the corporate sector the results are less convincing. *Real* Divisia growth in this sector has been much faster than can satisfactorily be explained by real activity and user costs; there have also been some episodes of extreme quarter-on-quarter variation. Despite this, a model of the aggregate index, similar to the personal sector equation, appears to satisfy the statistical criteria of stability and predictability.

The results of causality tests show that Divisia and measures of broad money such as M4 and M4 lending are all useful as medium-term indicators of inflation and nominal output. Divisia is perhaps slightly more robust. M0 is probably the best short-term indicator for inflation and is useful as a contemporaneous indicator of nominal income; but it seems to contain significantly less information on medium-term activity.

Conclusion

A Divisia measure of money has theoretical attractions as a measure of transactions services, weighting each type of deposit according to the transactions services offered by each of them. Such a measure might have a closer relationship with total expenditure in the economy than do the conventional monetary aggregates.

As this article discusses, there are both theoretical and practical difficulties in constructing an index which measures the transactions services provided by different types of monetary asset. But it would be wrong to conclude from this that a Divisia index would necessarily be inferior to the conventional monetary aggregates. First, these theoretical and practical difficulties may not be severe; and second, some of these difficulties apply at least equally to the conventional monetary aggregates. So even a Divisia index which captures transactions services only imperfectly may nevertheless provide a better measure of money than other monetary aggregates.

This article has presented a Divisia index for the United Kingdom and has illustrated the impact of alternative, and possibly no less valid, solutions to some of the practical and theoretical difficulties which arise in the construction of such an index. It is difficult to judge the significance of the differences among the various indices which are presented, but it may be observed that these differences are much smaller than the differences between the path of a Divisia index and the path of any of the conventional monetary aggregates. A Divisia measure of money appears to have some leading indicator properties for predicting both nominal output and inflation.

(1) The detailed results are reported in **Bank of England Working Paper No 9**, May 1993.

Appendix A

Data used in the construction of the Divisia indices

Notes and coin: published level data non-seasonally adjusted (nsa).

Non-interest-bearing bank deposits: ICCs' holdings provided by the Bank's Financial Statistics Division. This series was then subtracted from the known total with the residual divided between Persons and OFIs on an estimated basis.

Interest-bearing sight bank deposits: ICCs' holdings provided by the Bank's Financial Statistics Division. This series was then subtracted from the known total with the residual divided between Persons and OFIs on an estimated basis.

Interest-bearing time bank deposits: break-adjusted sectoral flow data (nsa) provided by the Bank's Financial Statistics Division which are subsequently calculated to levels. Building society holdings of bank Certificates of Deposit and of bank deposits have been deducted from OFIs' holdings of bank time deposits.

Building society deposits: break-adjusted sectoral flow data (nsa) provided by the Bank's Financial Statistics Division which are subsequently calculated to levels.

TESSAs:[1] persons' bank time and building society retail deposits are adjusted for the introduction of TESSAs by subtracting the published levels of TESSAs (nsa) from the components. This is a reasonable calculation to make as TESSAs are not held for transactions purposes and as such should not be incorporated in the Divisia indices.

The indices are statistically adjusted for Abbey National's flotation in 1988 by incorporating the relevant break-adjusted flow data for bank sight and time deposits and building society deposits.

Bank current account (gross rate): up to 1984 the series is a rate provided by a single bank which offered interest-bearing sight deposits. Thereafter, it is an average of the rates offered by the major clearing banks on deposits of £500.

Clearing banks interest-bearing personal account (gross rate): pre-1984 the series is interest payable on seven-day notice deposit accounts with the clearing banks; thereafter it is an average of the rates payable on two or more similar accounts with tiered interest rates according to the size of balance held. We take the rate payable on the median tier at any one time (currently £10,000; it has risen over time).

Building society deposit (gross rate): pre-1984 the series uses the average building society share rate, as published by the Building Societies Commission (this provides a net figure; the gross rate is derived by including the composite tax rate). Thereafter it is an average of the largest five building society savings account gross rates.

London interbank overnight deposit rate: observed rate at about 10.30am; as published in *Financial Statistics*.

London interbank three-month deposit rate: as published in *Financial Statistics*.

Benchmark rate: the index uses the three-month local authority deposit rate as the benchmark rate; as published in *Financial Statistics*.

All interest rates are average rates over the quarter.

(1) Tax Exempt Special Savings Account.

Appendix B

Econometric relationships

Since Divisia is constructed as a monetary aggregate based on transactions services, it can be modelled on the basis of the theory of the transactions demand for money. Money is held by economic agents as an inventory to facilitate disbursements out of an income stream and is sensitive to its opportunity cost in terms of the interest forgone. The general functional form used is multiplicative:

$$M/P = a\, Y^b\, R^c \qquad \textbf{(B.1)}$$

where M is money, P the general price level, Y is real income (or transactions), R is the nominal interest rate on an alternative asset ($-R$ gives the real differential rate of return on non-interest-bearing money) and a,b, and c are parameters to be estimated. In the original inventory model, the parameter a is the transactions cost of converting another asset into money and the elasticities b and c are given by the square root law: b=0.5 and c=-0.5. This particular model may be too stringent in its assumptions and it is normal practice to satisfy the less restrictive conditions $0.5 \leq b \leq 1.0$, and $c \leq 0$.

When modelling Divisia, some of its components are interest-bearing; the level of the nominal interest rate is therefore replaced with a user-cost measure which is based on interest rate differentials. The measure used here is the *real* price dual of Divisia (ρ).

The appropriate measure of transactions will differ across sectors. Total domestic demand has been chosen for the personal sector, GDP for the corporate sector and their relevant deflators as price indices. All data are seasonally adjusted and logged. Equation **(B.1)** represents a static equilibrium relationship. In the short run, holdings of Divisia will be affected by the level of price inflation—in times of high inflation agents will tend to economise on real transactions balances even though the user cost is not directly affected.

A log linear, dynamic version of equation **(B.1)** is estimated by the following methods. First, the Johansen full information[1] approach is used to ascertain the number and nature of the long-run relationships between the variables in the data set—we expect to find a relationship to match equation **(B.1)**. Where appropriate, these long-run relationships are then used as the foundation of a dynamic adjustment model, in which Divisia—and possibly the other variables—are seen to be adjusting to the deviation in Divisia balances from their desired long-run levels.

The available data sample is 1977 Q1–1992 Q4 (estimation sample 1978 Q1–1992 Q4 after allowing for lags). Initial investigation shows that over this sample, real Divisia (M/P, M^p for personal, M^c for corporate), domestic demand (DD), GDP and the price deflators (P^d,P^g for domestic demand and GDP) can be treated as difference stationary processes.

The personal sector

The long-run relationship is estimated to be:

$$ln(M^p/P^d) = 0.93\, ln(DD) - 0.22\, ln(\rho)$$

Tests show that the activity elasticity could be imposed at either unity or one half (in which case the user-cost elasticity varies between -0.18 and - 0.50). Proceeding with the unrestricted estimate suggests the following dynamic relationship (t-ratios in brackets):

$$\begin{aligned}\Delta ln(M^p/P^d)_t = & -0.49 + 0.21\, \Delta ln(M^p/P^d)_{t-1} \\ & \quad (-3.7) \quad (2.0) \\ & + 0.32\, \Delta ln(DD)_t - 0.007\, \Delta\Delta ln(\rho)_t \\ & \quad (3.0) \qquad\qquad (-1.2) \\ & - 0.49\, \Delta ln(P^d)_t - 0.11\, [ln(M^p/P^d) \\ & \quad (-4.3) \qquad\qquad (-3.7) \\ & - 0.93 ln(DD) + 0.22\, ln(\rho)]_{t-1}\end{aligned}$$

This equation is parsimonious, reasonably stable under recursive estimation and passes all mis-specification diagnostics at a 5% probability value. The equation explains 60% of the quarter-to-quarter variation in the dependent variable and the residual standard error is 0.8%. All variables enter contemporaneously although the user-cost term becomes double-differenced and has a 95% confidence interval which includes zero. Direct estimation by OLS shows minimal differences but reveals that the levels effect of the user cost is weakly determined.

The dynamic and long-run equations for personal sector Divisia both seem to work reasonably well, despite the relatively short sample available. One possible cause for concern is the relative imprecision of the user-cost terms, although their coefficients are correctly signed and of reasonable magnitude.

The corporate sector

The corporate sector has proved considerably more difficult to model than the personal sector. The main problem is a failure to find any sensible, statistically acceptable long-run relationships. GDP cannot account for the trend or the degree of variation in corporate Divisia and the respective timing of peaks and troughs are not close. When the

(1) Johansen, S (1988), 'Statisical analysis of co-integrating vectors', *Journal of Economic Dynamics and Control*, 12 (2/3), pages 231–54.

user-cost series is entered, it cannot account for the remaining variation. The best estimates are based on an unrestricted error correction model, with no user-cost terms included and two data-determined dummy variables for 1983 (1, -1 in Q3, Q4) and 1986 (1, -1 in Q3, Q4) (t-ratios in brackets):

$$\begin{aligned}\Delta ln(M^c/P^g)_t = &- 2.55 + 0.26\ \Delta ln(M^c/P^g)_{t-1} \\ &(-2.2) \quad (2.7) \\ &+ 0.23\ \Delta ln(M^c/P^g)_{t-3} - 0.7\ \Delta ln(P^g)_t \\ &(2.7) \qquad\qquad (-2.1) \\ &- 0.09\ [ln(M^c/P^g)_{t-1} - 2.52\ ln(GDP)_{t-1}] \\ &(-2.5) \qquad\qquad (-6.1) \\ &+ 0.12\ D_{86} + 0.08\ D_{83} \\ &(5.7) \qquad (4.0)\end{aligned}$$

The equation fails badly on a test for functional form but is otherwise robust. The residual standard error is 2.8%. Although over half of the quarterly variation is explained, this drops to a quarter if the dummy variables are excluded. The user-cost terms, if entered, are incorrectly signed.

These difficulties in modelling corporate sector holdings of Divisia may reflect in part the wider access of the corporate sector to capital markets than the personal sector—in terms of both liabilities and assets. Assets which would be regarded as illiquid by the personal sector—equities, government stock, foreign currency balances—may be highly liquid to the corporate sector. Hence the restriction of transactions balances to be a function of M4 components is less likely to be valid. Also, the corporate sector may need to hold liquid assets for purposes other than transactions, or for particular types of transaction. For example, large cash/bank balances may be held to facilitate (or defend against) expansionary acquisitions. This might account for a greater than unit output elasticity during the 1980s.

Aggregate Divisia

The difficulties in explaining corporate sector Divisia create problems in modelling the aggregate index. Nevertheless, it appears possible to obtain a statistically acceptable model of aggregate Divisia. The personal sector is the largest component and, given some uncertainty over the appropriate corporate sector activity variable, domestic demand can represent the scale variable. The long-run relationship is found to be:

$$ln(M/P^d) = 0.72\ ln(DD) - 0.52\ ln(\rho)$$

Based on this, the dynamic equation is (t-ratios in brackets):

$$\begin{aligned}\Delta ln(M/P^d)_t = &- 0.05 + 0.18\ \Delta ln(M/P^d)_{t-1} \\ &(-3.0) \quad (2.1) \\ &+ 0.39\ \Delta ln(DD)_t - 0.64\ \Delta ln(P^d)_t \\ &(3.6) \qquad\qquad (-5.2) \\ &- 0.045\ [ln(M/P^d) - 0.72\ ln(DD) \\ &(-3.7) \\ &+ 0.52\ ln(\rho)]_{t-1} + 0.02\ D_{86} \\ &\qquad\qquad (3.0)\end{aligned}$$

This equation differs from the personal sector equation in several ways. Most important is that the coefficient on the term representing deviations from static equilibrium is half the magnitude (0.045 from 0.11). This reflects the fact that the previously unexplained growth in corporate Divisia is now being treated as a dynamic adjustment phenomenon. The coefficient on the dynamic user-cost term is small and incorrectly signed and has been omitted. The equation has a slightly higher standard error of 0.85% but explains a higher fraction (two thirds) of the quarterly variation. The equation is reasonably stable and thus meets the main criterion. In other respects, the equations are simple and are consistent with the theoretical model of transactions demand. Despite this, the standard error of 0.85% allows considerable scope for unexplained variation on a quarter-to-quarter basis.