# **G7 yield curves**

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In November 1994, the Bank of England adopted a new method for estimating yield curves from the gilt-edged market. The curves are used for measuring expectations of future interest rates and inflation. Recently the Bank used the same method to estimate the yield curves of the other G7 countries' government debt. This article describes these yield curves and explains how the estimation method was adapted to each particular market.

# Introduction

Yield curves obtained from government bond prices may contain valuable information about market expectations of future interest rates. For instance, measurements from yield curves may indicate whether interest rates are expected to rise, fall or to stay the same.<sup>(1)</sup> Yield curves can also indicate by how much market participants expect interest rates to change. Such information is useful when judging the form and timing of monetary policy interventions.

Moreover, comparing the yield curves of different countries' government debt can indicate whether interest rate differentials between countries are expected to rise or fall. It also provides a measure of how monetary policy varies between countries and can provide information on whether these variations are expected to persist.

This article continues a series on the estimation and interpretation of yield curves. Previous articles<sup>(2)</sup> in the series outlined the Bank of England's approach to estimating the UK yield curve in both the conventional and index-linked bond markets, and considered both the interpretation and the forecasting power of the expectations derived from those yield curves. The Bank has recently used the same technique<sup>(3)</sup> to estimate nominal yield curves in the government bond markets for each of the other G7 countries. This article describes the resulting estimated yield curves. It begins by highlighting the different factors, including institutional characteristics, that have to be considered when estimating yield curves. A section for each country describes the domestic bond market and estimated yield curves for that market.

# **Estimating yield curves**

The zero-coupon yield curve, estimated from the prices of coupon-bearing bonds, represents the term (or maturity) structure of spot interest rates in that bond market. Spot interest rates are the rates at which the individual cash flows arising from a coupon-bearing bond-the coupon and redemption payments-are discounted to determine the price of the bond today. Related to these spot interest rates are a set of implicit forward interest rates. The spot rate for payments arising at date t+1 represents the average rate of return between today and date t+1, while the forward rate associated with date t+1 represents the one-period rate of return implicit in the difference between the *t*-maturity spot rate and the (t+1) maturity spot rate.<sup>(4)</sup>

In principle, recovering the underlying term structure of spot interest rates from coupon bond prices is straightforward. The set of spot interest rates define a set of discount factors—the value today of £1 to be paid at date t. The price of a coupon-bearing bond is equal to the sum of each cash flow arising from that bond multiplied by the discount factor applicable to the date of that cash flow. If a group of bond prices are written in this way, a huge system of linear simultaneous equations is generated, where the only unknowns are the discount factors common to all the bonds in the group. The values of the discount factors can be found by solving the equation system using matrix algebra. The spot and forward interest rates can then be recovered from the discount factors.

This procedure has two limitations. First, a necessary condition for solving this equation system is that the number of bonds in the group exceeds the number of payment dates. Second, the method produces discount factors only for dates when coupons are paid: it is unable to 'fill in' the rest of the curve. For these reasons, sophisticated mathematical techniques are introduced to estimate the complete discount function-the set of discount factors at all maturities.

The intended use of the yield curves is central to the decision on the type of estimation technique to be used. For example, there is less need to know the precise shape of the yield curve for macroeconomic policy, than for pricing interest rate instruments. A method that captures the

The most widely used measure is the set of implied forward interest rates. It is recognised, however, that risk premia and the convexity of yield curves can introduce a wedge between forward interest rates and market expectations of future interest rates. Deacon, M P and Derry, A J (1994a); Bank of England (1994); King, M A (1995); Breedon, F J (1995). Some of the estimated forward rate curves have appeared in previous editions of the *Quarterly Bulletin* and *Inflation Report*. The Bank estimates forward rates based on a six-month interval. Further details of the differences between spot and forward interest rates can be the restincted forward rates based on a six-month interval. found in Deacon, M P and Derry, A J (1994b).

fundamental features in a smooth curve is more desirable. Having tested different yield-curve estimation techniques, in November 1994,<sup>(1)</sup> the Bank adopted the method proposed by Lars Svensson<sup>(2)</sup> to estimate yield curves for monetary policy purposes. Further details are given in the May 1995 Quarterly Bulletin.<sup>(3)</sup>

#### Data considerations

For comparative purposes it is important to have a set of yield curves estimated using a common technique for each country. Although the estimation method accommodates differing institutional factors, such as the calculation of accrued interest, comparisons may be difficult where the number of bonds and the maturity ranges in the markets are very different. The overall definition, or quality, of the estimated curve will reflect both the availability of bonds in general and how they are spread along the maturity range. For example, while the UK market has conventional bonds distributed out to 25 years with relatively few gaps, other countries such as Germany and Italy have relatively few bonds beyond ten years. Differences between the United Kingdom and Germany can be seen in Charts 1 and 2. Although a paucity of long bonds will tend to reduce definition at the long end of the yield curve, the Svensson technique has the advantage of ensuring that the estimated curves will settle down to a fixed level at long maturities.



Bonds with 'special' features are not included in the estimation process, since these features will tend to distort the prices of these bonds. These bonds include callable and convertible bonds whose prices reflect the embedded options. This has the most noticeable impact in the US Treasury market where there are no non-callable securities with maturities between 10 and 18 years; see Chart 3. All outstanding Japanese government bonds are callable. Since none has ever been called and the market does not appear to expect any to be called, the bonds are

# Chart 2 German redemption yields











treated as conventional for the purpose of estimating a yield curve.

For all countries apart from the United Kingdom, the data were collected from a wire-service feed, not directly from the exchanges. Difficulties were experienced in collecting a complete set of price data for periods in 1993 and 1994 for most of the countries. This means that during these periods, definition of the estimated curves is reduced, because some of the bonds normally included in the estimation process were absent temporarily from the data set. This explains some of the volatility observed in the estimated yield curves during this time.

## Tax effects

The tax treatment of coupon-bearing bonds is one of several factors to be considered when estimating yield curves. If coupon income and capital gains on the value of a bond are subject to different tax rates, then some bonds will contain a

<sup>(1)</sup> (2)

See Bank of England (1994). Svensson, LE O (1994). This technique is an extension of a technique proposed in Nelson, C F and Siegel, A F (1987). A simplified version of the Nelson and Siegel method is used to fit the UK real yield curve. Other techniques have been developed by, for example, Steeley (1991), and Fisher, Nychka and Zervos (1995).

<sup>(3)</sup> See Breedon, F J (1995)

price premium over others. Those investors facing relatively high marginal income tax rates would tend to favour bonds whose payment stream was skewed towards capital gains rather than income, such as low-coupon stocks. If such investors form a large proportion of the participants in the market, low-coupon stocks could carry a price premium. If this premium is not recognised during yield curve estimation, it could cause the estimated curve to be biased downwards.

Two approaches have been developed to adjust the measured yield curve for these tax effects. The first approach<sup>(1)</sup> recognises that yield curves are tax-specific, so that investors in different tax brackets face different after-tax returns, and would therefore optimally choose to invest in different bonds. Yield curves are then estimated using only those bonds that are 'optimal' for any given tax bracket. The second approach controls more generally for the bias in the yield curve caused by the differential tax treatment of coupon income and capital gains. This approach is characterised by two methods. The first method<sup>(2)</sup> estimates the yield curve subject to the coupon payments being down-weighted by a fixed proportion, which is estimated along with the yield curve. This fixed proportion is called the 'effective tax rate'. The second method, used by the Bank,(3) recognises that taxes will affect the yield of some bonds more than others and introduces three further parameters to model this relationship and the tax adjustment more accurately.

In all of the G7 countries, at least some participants face different tax treatment of capital gains and coupon income in the domestic bond markets. The bond markets in the United Kingdom, Germany and Japan, for example, illustrate this issue. In the United Kingdom before April 1996, a substantial proportion of investors in gilt-edged securities were subject to income tax on coupons received but were exempt from tax on capital gains. During and after April 1996, the tax treatment of gilt-edged securities changed with some classes of investors being liable to capital gains tax on the price change in a given period, in addition to their existing liability for income tax on coupon interest.<sup>(4)</sup> Other classes of market participants, such as private investors, continue to be exempt from capital gains tax on gilts, while some participants, such as pension funds, continue to be exempt from all tax in respect of gilts. In Germany, coupon income is subject to a flat-rate withholding tax, while capital gains on bonds held in excess of six months are free of this tax. For bonds held for less than six months, capital gains are taxed at the personal (progressive) rate of income tax if, in combination with coupon income, certain thresholds are exceeded. In Japan, there is a particular preference for low-coupon stocks. This may in part reflect the tax treatment of capital gains, but the market's preference for current-coupon stocks is generally thought to be the main explanation. For all G7 countries, the Bank's

tax model is used to adjust for the effects of tax-related premia.

### Other factors

The Bank's yield-curve estimation procedure requires the calculation of redemption yields. This is the rate of return offered on a bond if the bond is purchased at the current market price and held until redemption. Redemption yields are calculated from gross bond prices, that is, a price that includes accrued interest. Accrued interest (AI) is calculated as.

 $AI = (d/n) \ge C$ 

where:

d is the number of days between the previous coupon date and the settlement date,

*n* is the assumed number of days in a year, and

C is the coupon rate.

The different rules regarding the values of d and n used in the calculation of accrued interest are summarised in the table.

### Different rules used in calculating accrued interest by country

Country	Settlement date	d	п
United Kingdom	Next business day	Actual (a)	365
Germany	(b)	(c)	360
France	Next business day (d)	Actual	365
United States	Next business day	Actual	(e)
Japan	(f)	Actual	365
Italy	+3 business days	(c)	360
Canada	+5 business days (g)	Actual	365

d is the number of days between the previous coupon date and the settlement date. n is the assumed number of days in a year.

(a) This is the actual number of days between the last coupon date and the settlement date.
 (b) Exchange-traded bonds settle at +2 business days, OTC-traded bonds settle at

(b) Exchange-traded bonds settle at +2 business days, OTC-traded bonds settle at +3 business days.
(c) If the last coupon date is dd/mm/yy and the settlement date is DD/MM/YY, then d = Min(DD, 30) - Min(dd, 30) + 30 (MM-mm) + 360 (YY-yy).
(d) Some bonds have a 3-day settlement period.
(e) The number of days in a year is assumed to be double the number of days between the semi-annual coupon dates, and can range from 362-68.
(f) Settlement dates are published in advance of the corresponding trade date and can be variable.
(g) This recently changed to +3 business days.

Bonds in the United Kingdom trade ex-dividend; that is, bonds are purchased in a certain period without the right to the forthcoming coupon. In the ex-dividend period, accrued interest is calculated using the settlement date and the next coupon date, and will be negative.<sup>(5)</sup> Furthermore, for newly issued bonds, the first coupon may be adjusted if the issue date did not fall on one of its semi-annual coupon dates. This added technicality is overcome by excluding from the estimation process all bonds that have yet to receive their first coupon. This 'infant-bond rule' rarely excludes more than six bonds from the estimation procedure.

The yield-curve estimation technique used by the Bank introduces two further tax parameters. If bonds trade ex-dividend, then the preferential tax treatment of capital gains can induce a further price premium. Purchasing bonds

Schaefer, S M (1981). McCulloch, J H (1975). See Mastronikola, K (1991).

However, capital losses on gilts may be offset against capital gains arising elsewhere Further details are given in, for example, Fage, P (1986).

ex-dividend will remove the liability to income tax on the next coupon payment, making them particularly attractive to high-rate taxpayers. One parameter permits the accrued interest calculation to be weighted differently according to whether bonds trade cum or ex-dividend. The other parameter accounts for the FOTRA (Free Of Tax for Residents Abroad) status of some gilts, which could impart a price premium to those bonds.

## The UK yield curve

There are currently 72 conventional gilt-edged securities, with coupons ranging from 2.5% to 15.5%. These include a group of callable bonds, a group of undated bonds and a convertible bond.<sup>(1)</sup> The yield curve is estimated using around 43 single-dated conventional bonds and some undated bonds to assist in the definition of the long end of the curve. The United Kingdom also has a market in inflation index-linked bonds. Comparisons of conventional and index-linked yields can be used to generate measures of inflation expectations.<sup>(2)</sup> The estimated spot and forward curves for the UK nominal government bond market on 8 March 1996 are shown in Chart 4.<sup>(3)</sup>





Time-series plots of two, five and ten-year spot and forward rates over the period from January 1992 to March 1996 appear in Charts 5 and 6. Interpreting the behaviour of spot and forward curves over time, and comparing with our knowledge of economic events, can help to validate the yield-curve estimation technique. For instance, periods of relatively high volatility may be attributable to sudden changes in market conditions, such as the departure of sterling from the ERM. But they may also reflect other factors, such as data limitations, that may distort the shape of the estimated curve. The charts show that the UK spot-rate curve has been upward sloping since October 1992. Prior to this, although the curve was inverted, it was

## Chart 5 Two, five and ten-year spot rates estimated from UK government bonds



Chart 6

Two, five and ten-year implied forward rates estimated from UK government bonds



relatively flat. This suggests that interest rates in the near and longer term were expected to remain at broadly the same level. Since then, movements in the curve suggest that there have been three turning points; for example, the increase in yields around February 1994 coincided with the general rise in bond yields following the Federal Reserve's tightening of monetary policy.<sup>(4)</sup> In general, the spread between the five and ten-year spot rates has been more variable than the spread between the two and five-year rates. This may suggest that changes in expectations of long-term interest rates have been more variable than changes in expectations of short-term interest rates.

# The German yield curve

The market in conventional, coupon-paying bonds comprises four groups of instruments, distinguished by their initial time to maturity. Bundesanleihen have an initial maturity of between 10 and 30 years. Bundesobligationen

depicted in the charts. See Ganley, J and Noblet, G (1995) (4)

This also includes a single floating-rate bond which pays a variable coupon on a quarterly basis. See Breedon, F J (1995). Estimation difficulties mean that we do not place reliance on the fitted curve for maturities of less than two years. Consequently this segment is not (2) (3)

have an initial maturity of five years. Bundesschatzanweisungen have an initial maturity of four years and are currently the shortest-maturity debt instrument. In addition, there are a group of special issues: Bundespost, Bundesbahn and Fonds Deutsche Einheit bonds, with initial maturities of 5–15 years. About 120 bonds are used to estimate the yield curve. The estimated spot and forward interest rate curves on 8 March 1996 are shown in Chart 7.





As with the United Kingdom, the German yield curve has been upward sloping since the beginning of 1993: see Chart 8. Prior to this, the curve was initially inverted and

## Chart 8





stable before becoming relatively flat while shifting down. Since March 1993, changes in German yields have been broadly similar to the United Kingdom, although the two to ten-year spread has widened rather than narrowed. The high volatility, particularly in the forward rates (see Chart 9), around the end of 1993 and the beginning of 1994 most probably reflects missing price data in our data set rather than any particular economic event, although 1994

# Chart 9

# Two, five and ten-year implied forward rates estimated from German government bonds



represented a period of higher volatility in major bond markets (see the reference in footnote 4 on page 202).

# The French yield curve

French government debt is classified by initial time to maturity. BTFs (Bons du Trésor à taux Fixe et intérêt précompté) are zero-coupon bonds with residual maturities of one year or less. BTANs (Bons du Trésor à taux fixe et intérêts ANnuels) are coupon-bearing bonds with either a two or five-year initial maturity. OATs (Obligations Assimilables du Trésor) are longer-term, coupon-bearing bonds with an initial maturity of up to 30 years. Only the coupon-bearing bonds are used in the yield-curve estimation procedure. While the zero-coupon bonds could be included to improve the curve definition at the short end, their exclusion permits an independent check to be made on the short end of the yield curve. About 30 bonds are typically used to estimate the yield curve and the estimated spot and forward rate curves appear in Chart 10. The relatively low number of issues in France reflects the issuance strategy. As





well as creating new issues, the French government issues a considerable portion of new debt in the form of tranches of existing stocks, a process known as *Assimilation*. The relatively small number of distinct securities are not however concentrated at particular maturities, although some gaps begin to appear at longer maturities. None of these gaps would be filled by including the omitted zero-coupon bonds.

Movements in French spot and forward rates, plotted in Charts 11 and 12, show a similar pattern to movements in both the UK and German curves. The two to ten-year spread widened in 1995, by roughly the same amount as the German curve. Implied forward rates appeared relatively volatile during 1993, although as explained above, this may reflect a reduction in available data in this period.

## Chart 11

# Two, five and ten-year spot rates estimated from French government bonds



## Chart 12





# The US yield curve

The market in long-term US Treasury securities comprises Treasury Notes, which have an initial maturity of two to ten years, and Treasury Bonds, which have an initial maturity of more than ten years. There are around 170 Treasury Notes and Bonds. The most recently issued US Treasury securities are known as On-the-Run bonds. These benchmark securities tend to be the most heavily traded securities in the market and are thought to enjoy a liquidity premium. Since the Bank's yield curve estimation technique does not include bonds until they pass their first coupon date, some of the On-the-Run securities are excluded. Chart 3 shows the redemption yields for the Treasury securities used in the estimation (around 155 securities). While the US Treasury market is distinguished by the large number of its marketable securities, it can be seen that there are no non-callable issues in the ten to eighteen-year maturity range. Nevertheless, the spot and forward rates for these maturities were not found to be unusually volatile. Chart 13 shows the estimated spot and forward-rate curves for 8 March 1996.

## Chart 13 US spot and forward rate curves 8 March 1996



Unlike the yield curves considered so far, the US curve has remained upward sloping for the entire sample period. The two to ten-year spread has narrowed over the sample period



## Chart 15 Two, five and ten-year implied forward rates estimated from US government bonds



as the curve has flattened out. The movements of the estimated spot and forward curves are shown in Charts 14 and 15.

# The Japanese yield curve

There are currently around 120 Japanese government bond issues outstanding. Secondary market activity is typically concentrated into one liquid sector of the market at ten years. Within this sector, liquidity is concentrated in the ten-year benchmark bond and those bonds deliverable under the ten-year futures contract. The benchmark issue accounts for around 80% of trading volume and has not been excluded from the estimation process despite the existence of an associated price premium. The remaining relatively illiquid stocks are purchased and held for their long-term yield premium by insurance companies and trust banks. Some of the illiquid bonds may not trade on a particular day and when this occurs they are excluded from the estimation process, to prevent bias from 'stale' price information. Although there are

### Chart 16



potentially few gaps in the maturity spectrum up to 20 years, the number of bonds used to calculate the yield curve can differ substantially between consecutive business dates, due to lack of trading. As an example, Chart 16 shows the number of bonds used to calculate the yield curve for 1 and 2 May 1995. The chart shows that on 1 May, relatively few bonds at the long end of the curve were traded. This comparison is typical of the sample as a whole.

Chart 17 shows the estimated spot and forward rate curves for 8 March 1996. The time-series plots of spot and

## Chart 17

# Japanese spot and forward-rate curves 8 March 1996



forward rates, Charts 18 and 19, show that the Japanese yield curve has been upward sloping throughout the sample period.

## Chart 18 Two, five and ten-year spot rates estimated from Japanese government bonds



The two to ten-year spread in spot yields has remained broadly constant for most of the period shown, although in recent months it has been historically wide.

## Chart 19





## The Italian yield curve

The Italian Treasury issues various types of lire-denominated debt instruments. BOTs (Buoni Ordinari del Tesoro) are zero-coupon bonds issued with three, six and twelve-month initial maturities. BTPs (Buoni del Tesoro Poliennali) are conventional, coupon-paying bonds with an initial maturity range of between 3 and 30 years. CCTs (Certificati di Credito del Tesoro) are variable-rate instruments indexed to BOT yields. The initial maturity range is five to ten years. CTOs (Certificati del Tesoro con Opzione) are callable Treasury notes with an initial maturity range of three to ten years. The Italian yield curve is estimated using the conventional BTPs—around 40 bonds. Chart 20 shows the estimated spot and forward curve for 8 March 1996.





The relative scarcity of long-maturity bonds, and a gap between 10 and 28 years in the maturity spectrum, mean that it is particularly difficult to obtain satisfactory estimates of long-maturity spot and forward interest rates in the Italian government bond market. Prior to the Italian Treasury's introduction of a 30-year BTP at the end of 1993, the estimated yield curve beyond ten years was entirely determined by extrapolating the yield curve estimated over maturities shorter than ten years. As the curve was not being fitted to any data at longer maturities, the long end of the curve was highly sensitive to relatively small changes in redemption yields up to ten years out. This consideration is not peculiar to the Italian bond market; for example, as mentioned above, the German bond market also has few long-maturity bonds. However, the long end of the German yield curve is much less volatile than the Italian curve. While this may in part reflect lower volatility in interest rate expectations in Germany, it could also be a consequence of the smaller number of bonds in the Italian market. Also, the difficulties in obtaining price data were particularly severe for the Italian bond market.

The volatility induced in the yield curve by the limited data is evident in Charts 21 and 22 of the spot and forward rates. In general, the curves moved roughly in line with other European countries. The volatility around September 1992 may in part reflect the departure of the lira from the ERM.





#### Chart 22

## Two, five and ten-year implied forward rates estimated from Italian government bonds



## The Canadian yield curve

The Canadian government issues fixed-term, coupon-paying bonds, with initial maturities of 2–30 years. In general, around 80 bonds are used to estimate the Canadian yield curve. Chart 23 shows the estimated spot and forward rate curve for 8 March 1996. The distribution of Canadian debt shares similarities with the structure of the UK debt market, with relatively few gaps and a relatively well-defined long end. The Canadian government, like the United Kingdom's, issues index-linked bonds but, since there are only two outstanding index-linked stocks in Canada, it is not possible to estimate a real yield curve in the same way as for the United Kingdom.

## Chart 23





The time series plots of spot and forward rates, Charts 24 and 25, indicate that the Canadian yield curve moved broadly in line with the US yield curve over the sample period. But the extent to which the two to ten-year

# Chart 24

Two, five and ten-year spot rates estimated from Canadian government bonds



# Chart 25 Two, five and ten-year implied forward rates

estimated from Canadian government bonds



yield spread narrowed was less pronounced in the Canadian market.

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