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# Implied exchange rate correlations and market perceptions of European Monetary Union

By Creon Butler and Neil Cooper, of the Bank's Monetary Instruments and Markets Division.

*A number of 'EMU calculators' have been developed to assess market expectations of the likelihood of particular countries joining European Monetary Union (EMU). Most of these techniques attempt to infer this information from interest rate differentials. Typically, they also require assumptions about the level of interest rates that would hold should a country not join EMU. This article discusses an alternative measure of EMU convergence—the expected correlation between currencies implicit in foreign exchange options prices. It shows how implied correlations may be calculated, and how they may be used to gauge expectations of EMU participation by continental European countries and to interpret sterling's movements since mid 1996.*

## Introduction

There has been increasing interest in techniques to gauge financial market expectations about the likelihood of European Monetary Union (EMU) going ahead and the probabilities attached to participation by certain countries. A number of 'EMU calculators' that attempt to assess EMU convergence have been developed.<sup>(1)</sup> Most of these techniques rely on interest rate differentials in the swap market. But some strong assumptions are also needed to interpret the results. After briefly reviewing some of the drawbacks of relying solely on interest rate differentials, this article presents an alternative indicator of EMU sentiment—the expected future correlation between currencies implied by foreign exchange options prices. These implied correlations provide information on the market's perceived likelihood of two countries joining EMU, since a necessary condition for them both to participate is that their exchange rates should be perfectly correlated beyond the date they join.

A simple approach to assessing market expectations of EMU relies on forward interest rate curves derived from government bond prices.<sup>(2)</sup> These enable an estimate to be made of the short-term interest rates expected to hold beyond 1 January 1999 in Germany—assumed to be a core member of EMU—and a second country of interest.<sup>(3)</sup> If the market were sure that the second country would join in the first wave of EMU, the expected short-term interest rates in the two countries after 1 January 1999 would be identical and the forward rate estimates of these expectations would be very close. But if there is uncertainty, it is argued that the expected interest rates in the second country would be above those in Germany. Moreover, the more doubtful the participation of the second country, the wider the divergence is likely to be. Though useful, this analysis is dependent on some key assumptions: (i) that Germany will definitely join EMU; and (ii) that the monetary policy of the second

country will be less credible outside EMU than in. But expectations of the second country's interest rates after 1999 could be very close to Germany's, even if it were not expected to join EMU. Put another way, convergence in expected short-term interest rates is a necessary, but not sufficient condition for the perception that both countries will join EMU.

The EMU calculators take this analysis further. They first estimate what interest rate spreads between countries would be if EMU were not in prospect, using either a time-series forecast or a full macroeconomic forecasting model. By comparing these estimates with the spreads that actually hold, and those that would hold if EMU participation were a certainty, they calculate the probability that individual countries will join EMU. But the results of this approach are inevitably dependent on the model for predicting interest rate spreads in the 'no-EMU' world.

Using implied (expected) exchange rate correlations as a gauge of EMU expectations does not generally require such detailed assumptions or forecasts about alternative scenarios. The key assumption is that there is a link between the probability the market attaches to two currencies joining EMU in 1999 and the implied correlation between their exchange rates *vis-à-vis* the dollar. The more likely they are to join, the closer to one the implied correlation will be, and *vice versa*. This is reasonable since if the two currencies do join, then the actual correlation coefficient must equal one (ie perfect correlation) from 1 January 1999 onwards; and there are few scenarios other than EMU that would produce an expectation of very high correlation between the two currencies.

The remainder of the article is structured as follows. The second section describes the technique for deriving implied

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(1) See, for example, J P Morgan (1997) and Goldman Sachs (1996).

(2) See Cooper and Steeley (1996a) and (1996b).

(3) On the assumption that interest rate risk premia are similar and/or small.

## Deriving implied correlations

From the price of an option on an underlying currency, we can derive information on the market's uncertainty about the future value of the currency. This is done by inverting a variant of the Black-Scholes formula with a given option price, to calculate the 'implied volatility'.

Since an option on a currency is quoted in terms of an exchange rate (ie the level of that currency *vis-à-vis* another currency), we can go a step further and derive an implied correlation—the market's expected future correlation between the exchange rates of any two currencies, using a third as a numeraire. This is a unique feature of currency options.

For example let  $S_1 = \$/¥$ , the dollar/yen exchange rate,  $S_2 = \$/DM$ , the dollar/Mark exchange rate; and  $S_3 = DM/¥ = S_1/S_2$ , the Mark/yen exchange rate. Then the proportional change in the exchange rate  $S_i$ ,  $r_i$ , is approximated by:

$$r_i = \ln \left( \frac{S_{i,t+1}}{S_{i,t}} \right) \quad (1)$$

Let the time interval be small—a day or less. From the definition of the Mark/yen cross-rate,  $S_3$ , it follows that the proportional change in the period is given by:

$$r_3 = \ln \left( \frac{S_{1,t+1}/S_{2,t+1}}{S_{1,t}/S_{2,t}} \right) \quad (2)$$

exchange rate correlations. The third discusses the information that they give about the perceived likelihood of certain continental European countries joining EMU. In the fourth, we look at sterling's behaviour since August 1996 in the light of this technique. The final section extends the technique to derive the expected future path of the short-term correlation between two exchange rates. This is then used to provide a further insight into the factors expected to influence sterling in the future.

### Derivation of implied exchange rate correlations

The Black-Scholes pricing formula shows how the fair market value of a call or put option on an equity will depend on the degree of uncertainty about the future value of the underlying asset, plus a number of other known factors. Extensions to the formula have also been developed to price options on a currency or interest rate. This means that we can take the price at which an option is traded in the market and, using the Black-Scholes formula, derive the expected volatility implied by the price. This is known as 'implied volatility'.

Rearranging the terms in brackets, we get:

$$r_3 = \ln \left( \frac{S_{1,t+1}/S_{1,t}}{S_{2,t+1}/S_{2,t}} \right) \quad (3)$$

$$= \ln \left( \frac{S_{1,t+1}}{S_{1,t}} \right) - \ln \left( \frac{S_{2,t+1}}{S_{2,t}} \right) \quad (4)$$

so that:

$$r_3 = r_1 - r_2 \quad (5)$$

It then follows that:

$$\text{var}(r_3) = \text{var}(r_1) + \text{var}(r_2) - 2\text{cov}(r_1, r_2) \quad (6)$$

Now take the implied volatility derived from an option on  $\$/¥$  as an estimate of the expected average standard deviation of movements of the US dollar expressed in terms of yen for the lifetime of the option (we will return to whether this is a reasonable assumption later). Let the implied volatilities for  $\$/¥$ ,  $\$/DM$  and  $DM/¥$  be termed  $\sigma_1$ ,  $\sigma_2$ , and  $\sigma_3$  respectively. Inserting these into equation (6), we get:

$$\sigma_3^2 = \sigma_1^2 + \sigma_2^2 - 2\text{cov}(r_1, r_2) \quad (7)$$

so

$$\sigma_3^2 = \sigma_1^2 + \sigma_2^2 - 2\sigma_1\sigma_2\rho_{12} \quad (8)$$

Because an exchange rate option gives us information on the market's uncertainty about the price of one currency in terms of another, with three currencies and options on each of the possible exchange rate pairings, we can derive an estimate of the market's expected future, or implied, correlation between any two of the exchange rates. To see the intuition behind this, let the three currencies be the US dollar, Japanese yen and Deutsche Mark (Mark). Suppose that the expected volatilities of the yen and of the Mark against the dollar are both very high, but that the volatility of the yen against the Mark is expected to be very low. This means that the market expects the dollar to drive most of the volatility between the yen and the dollar, and between the Mark and the dollar. It follows that the market will expect the dollar/yen exchange rate and the dollar/Mark exchange rate to be highly correlated. Another way to characterise the implied correlation is that it represents the degree of co-movement between two currencies using a third as numeraire. The box above provides a technical description of the method for deriving implied correlations.

What value is added by knowing the implied correlation for two exchange rates with a common numeraire if

and rearranging, we get the implied correlation between the Mark and the yen using the US dollar as numeraire:

$$\rho_{12} = \frac{\sigma_1^2 + \sigma_2^2 - \sigma_3^2}{2\sigma_1\sigma_2} \quad (9)$$

To calculate the implied correlation between \$/¥ and \$/DM on a particular date, we insert the observed implied volatilities for that date into equation (9).

We use data from over-the-counter (OTC) market makers,<sup>(1)</sup> rather than from the FX option exchanges such as Philadelphia and the Chicago Mercantile Exchange. This is first because liquidity is generally much higher on the OTC market than on the exchanges. Second, prices in the OTC market are quoted directly in terms of implied volatilities. This avoids the errors that may be introduced by slightly non-synchronous data, when trying to calculate implied volatilities using quotes on spot exchange rates and options prices. Finally, the OTC market trades options on a wider range of cross-rates than the exchanges. This is important, since to calculate the implied correlation we need implied volatilities for the three relevant currency pairings. Even where we use the dollar as the numeraire, this means that we need to use one cross-rate implied volatility, which may not be available from the exchanges.

A further feature of using OTC quotes is that they have constant maturities. Typically, one can observe quotes ranging from one week to one year on market makers' screens. This may be an advantage or a disadvantage, depending on the application. Having continuously

quoted data with the same time horizon makes it easier to generate meaningful time series. On the other hand, it makes it harder to see how expectations about a future fixed date have changed.

One possible objection to the use of implied volatilities is that the Black-Scholes pricing model assumes that the underlying asset price has constant volatility. Yet it is widely recognised that volatility changes over time. Does this affect the validity of our estimates of implied correlation?

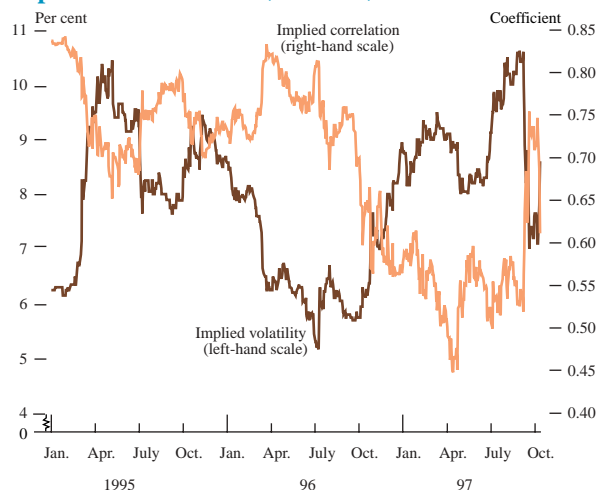
Fortunately, the method appears robust to the presence of variable exchange rate volatility. From a theoretical perspective, Feinstein (1989) investigated the true value of an at-the-money equity option in the presence of uncertain time-varying volatility. He showed that it is approximately equal to the Black-Scholes valuation, provided that the volatility estimate used in the Black-Scholes formula is the average expected volatility of the underlying stock for the remaining lifetime of the option.

Heynen, Kemna and Vorst (1994) extended this work to examine the relationship between implied volatilities derived using the Black-Scholes formula, and the true volatilities under three alternative stochastic models with uncertain time-varying volatility. From each of these models, they generated theoretical option prices. They then compared the Black-Scholes implied volatilities derived from these prices with the true average expected volatilities. For all three models the implied volatilities were very close to the average expected volatilities. This suggests that our use of Black-Scholes volatility is acceptable.

(1) The data used in this article comes from Citibank FX Options, London.

one already knows the implied volatility for the two currencies in which one is interested? Chart 1 shows

**Chart 1**  
Twelve-month implied volatility for £/DM and implied correlation of \$/£ with \$/DM

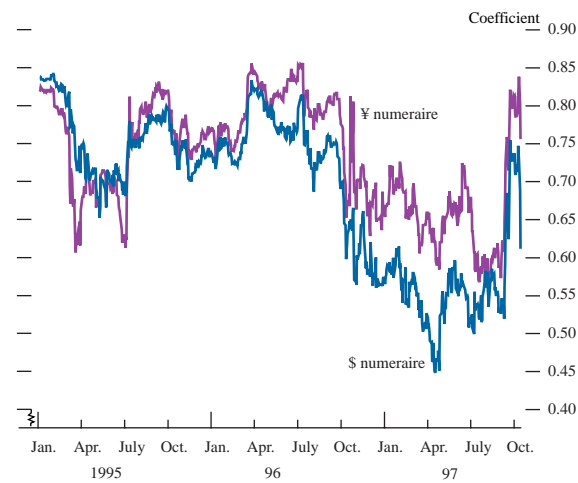


that, as one would expect from the above intuition, the implied volatility for £/DM moves inversely with the implied correlation between the \$/£ and \$/DM. The inverse relationship is fairly close, though there are clearly times when it breaks down, such as in spring 1995, which was a period of generally high exchange rate volatility. This illustrates one advantage of implied correlations: they adjust for general shifts in uncertainty affecting all countries.<sup>(1)</sup>

One concern is that the choice of numeraire may affect the results. To check this, we compared the implied correlation between sterling and the Mark using two different numeraires—the dollar and the yen. The results are shown in Chart 2. It can be seen that the choice of numeraire has not altered the general pattern of movement, but it does affect the absolute level of the implied correlation. This suggests that the choice of numeraire may at times be important. We use the US dollar as the benchmark currency where possible in the analysis that follows.

(1) A second advantage is that the correlation coefficient—a number between -1 and +1, where a figure close to +1 represents a very high degree of co-movement—is more readily understood than the implied volatility measure, which can in theory take any positive value.

**Chart 2**  
**Twelve-month implied correlations of \$/£**  
**with \$/DM and £/¥ with DM/¥**



### Implied exchange rate correlations for continental Europe

Limits on the liquid maturities available in the market for over-the-counter (OTC) exchange rate options mean that twelve months is the longest time horizon up to which we can calculate implied exchange rate correlations. So at the time of writing this article, the furthest time horizon is October 1998. This goes beyond the date when the initial members of EMU and their bilateral conversion rates are due to be announced, but stops short of the start date for EMU itself. Nonetheless, we can already learn quite a lot about market perceptions of EMU from the implied correlations we do have. In this section we use implied correlations to investigate changing market perceptions of the likelihood of different continental European countries participating in EMU. In the following section, we look at the information provided on market perceptions of sterling.

If EMU is expected to proceed on schedule, the market will expect the dollar exchange rates of any two participants to be perfectly correlated from 1 January 1999 onwards. But the market is also likely to expect two currencies that it thinks will participate at the start of EMU to be closely correlated between now and 1 January 1999. The current forward exchange rate between one currency and the other will be the market's best guess (or average view) of the level at which the exchange rate will in due course be irrevocably locked. Where short-term interest rates in the two countries are the same, the forward rate will be the same as the current spot rate. So in the absence of any news, the market would expect the dollar exchange rates for the two countries to be highly correlated in the run-up to EMU. If there is an interest rate differential between the two countries, it means that their exchange rate is expected to move to a new level in the period to 1 January 1999. But given that daily interest rate differentials are typically small and stable relative to daily exchange rate fluctuations, this movement on its own will have at most a small impact on the expected future correlation between the two countries' dollar exchange rates. Only if there is significant

uncertainty about whether one or other of the countries will join, or about the exchange rate at which they will join, will the market look forward to news that could lead to uncorrelated movements in the dollar exchange rates for the two currencies. Otherwise, future news will be expected to affect both currencies similarly.

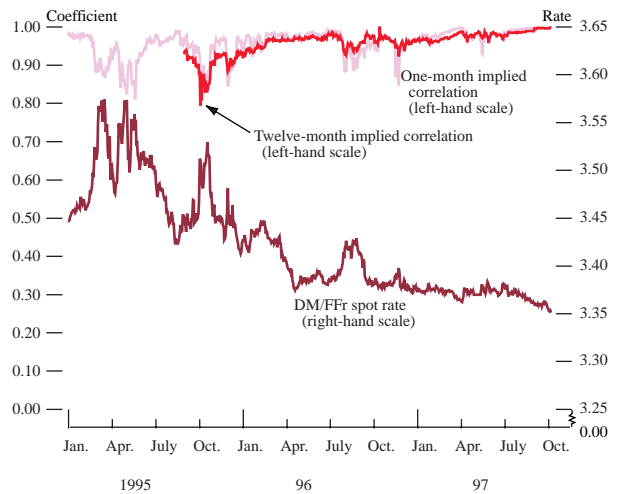
Equally, if we observe a very high twelve-month implied correlation between two potential participants in EMU, this should generally be a good indicator that both currencies are expected to join. But there are two possible exceptions to this conclusion. The first is the scenario where there is considerable uncertainty about whether one of the countries will join, but it is all focused in the period between October 1998 and 1 January 1999, for which we have no data. This seems very unlikely in practice, since the initial participation in EMU is due to be announced in spring 1998. Second, certain currencies that do not participate in EMU as from 1 January 1999 may join a new exchange rate mechanism ('ERM2'). If the fluctuation bands were very tight, it would be impossible *ex ante* to distinguish this scenario from that of EMU participation. But it seems unlikely that the market would perceive a country to be able to maintain a tight ERM band after 1 January 1999, and yet not expect it to qualify for participation in EMU at the outset, though some countries that are perceived as eligible to participate in EMU, but that have decided not to join on 1 January 1999, might be an exception.

As described in the introduction, implied exchange rate correlations and the spread between implied forward interest rates provide alternative measures of market expectations about EMU. One advantage of implied correlations is that aside from EMU, there are relatively few scenarios that could lead to the expectation that two currencies will be very highly correlated during a given period of time—the narrow band ERM2 discussed above is perhaps the most likely. By contrast, there are a number of economic scenarios other than EMU that could lead to two countries having similar expected future short-term interest rates for a period. Another advantage of implied correlations is that they are not susceptible to the estimation errors involved in fitting yield curves and deriving implied forward interest rates. They also avoid the problem introduced by market expectations that longer-term government debt from different countries participating in EMU will carry different credit or liquidity spreads. This could result in two countries having different implied forward interest rates, even though both were thought certain to participate. On the other hand, implied forward interest rate spreads have the important advantage at present that we can estimate them for the period after EMU is due to begin.

Chart 3 shows the implied correlation between the dollar/Mark exchange rate and the dollar/French franc exchange rate up to a one-month and twelve-month horizon since the start of 1995 and mid 1995 respectively. The implied correlations have both remained high throughout the period, but in the past six months have become more stable, with a rising trend towards almost one. This suggests that

the market has had a reasonably high degree of confidence during the period that both the Mark and the French franc will participate in EMU, and that it has become even more confident in the past six months.<sup>(1)</sup> The spread between ten-year implied forward rates in Germany and France provides a much noisier series, from which one cannot so readily read a trend.

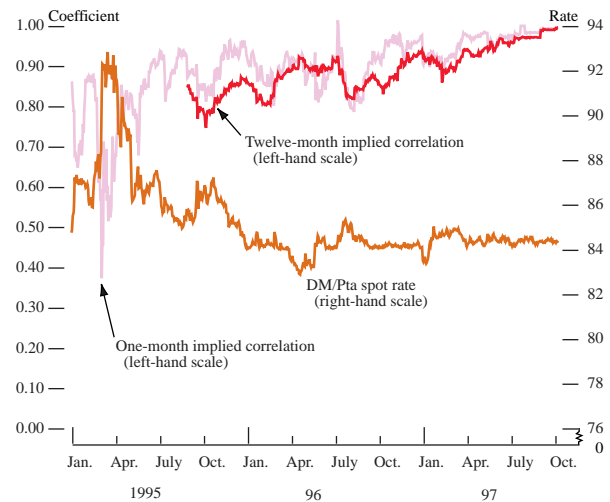
**Chart 3**  
One-month and twelve-month implied correlations of \$/DM with \$/FFr, and DM/FFr exchange rate



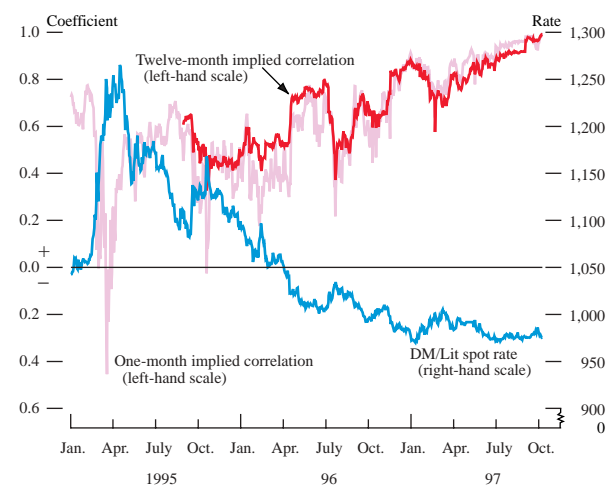
Charts 4 and 5 show implied correlations between the Spanish peseta and the Mark, and between the Italian lira and the Mark, respectively. At the start of 1996, the implied twelve-month correlation between the peseta and the Mark was 0.87, whereas for the lira and the Mark, the figure was 0.44. In both cases the implied correlations have been rising since then, but more markedly in the case of the lira. Given the independent evidence from the French franc/Mark comparison that a high probability has been assigned during the whole period to the Mark participating in EMU, this suggests that the market has attached a rising probability to these two countries joining. But the shift in perception has been greater for Italy than for Spain. Both charts show a step up in the twelve-month implied correlation to nearly one on 16 September, shortly after an ECOFIN meeting that confirmed that the initial participants in EMU and their bilateral parities would be announced in spring 1998.

Chart 6 shows the spread between the implied forward interest rates in Germany and Italy ten years ahead. The general picture of a narrowing spread since the start of 1996 tells a similar story to the implied correlations. But the month-to-month changes are sometimes different. For example, the measures in both charts suggest there was a sudden but temporary fall in the perceived probability of Italy joining Germany in EMU in August 1996, but this appears sharper when judged by the implied correlation measure than when judged by the forward interest rate spread measure.

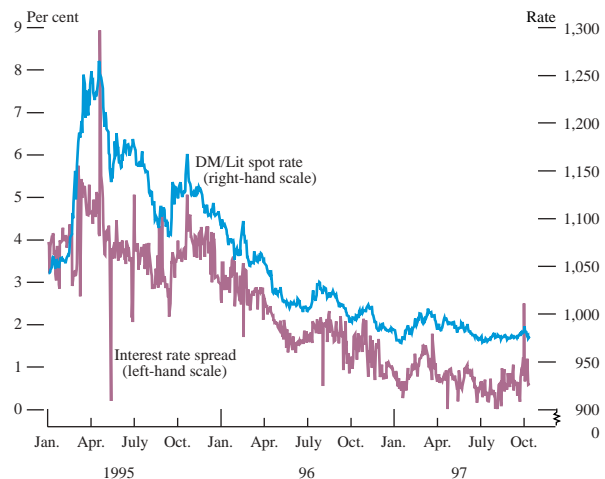
**Chart 4**  
One-month and twelve-month implied correlations of \$/DM with \$/Pta, and DM/Pta exchange rate



**Chart 5**  
One-month and twelve-month implied correlations of \$/DM with \$/Lit, and DM/Lit exchange rate



**Chart 6**  
DM/Lit exchange rate vs spread between ten-year forward interest rates in Italy and Germany



(1) In this article we do not try to model what the implied correlation between the two exchange rates would have been in the absence of EMU. But such an extension should be possible, using similar techniques to those already employed with yield spreads. In some cases, this might provide a more precise estimate of the probability of two countries joining EMU. But in the case of the Mark and French franc, the implied correlation is so high that any extension of this kind would produce a probability very close to one.

## Implied exchange rate correlations for sterling

A number of factors have contributed to sterling's appreciation and subsequent fall in the past 15 months. These include news about the relative stance of monetary and fiscal policy in the United Kingdom and other countries; the impact of changes in the oil price; and shifts in the demand and supply curves for UK goods and services. At the same time, market comment in the period suggests that another important additional factor has been shifts in international investors' portfolio preferences for sterling-denominated assets, as a result of changing perceptions about EMU. A range of possible rationalisations for such preference shifts has been put forward. We analyse these below, and see how the predictions compare with the evidence from implied exchange rate correlations and other financial market information.<sup>(1)</sup> This approach cannot prove that a particular scenario underlies sterling's behaviour during the period, but it does help to distinguish those scenarios that are consistent with the way other financial assets have been priced, and those that are not. We mainly focus on the twelve-month implied correlation in this section. In the next section we turn to implied correlations up to shorter time horizons.

### *Scenario (i): portfolio diversification*

In this scenario, investors become increasingly confident that EMU will proceed on schedule. Those holding what they expect will shortly become euro-denominated assets face a reduction in the extent to which their wealth is diversified against demand shocks. This creates an incentive for them to transfer some of their wealth into assets denominated in currencies that are expected to remain outside the euro area. The demand for diversification may be enhanced by the growing belief that the euro will have a broad initial membership, since more investors will then be affected by the arrival of EMU. As the market becomes more confident that sterling will not join EMU at an early date, it begins to strengthen as a result of this demand. Moreover, while the likelihood of the United Kingdom's participation in EMU is changing, the impact on sterling's value of increased demand for diversification is likely to be proportionately greater than for other currencies that are also potential homes for these funds, but which have never been candidates for membership, such as the US dollar.<sup>(2)</sup>

### *Scenario (ii): weak euro*

In this scenario, investors also come to expect EMU to proceed on schedule with a broad initial membership. But this leads them to expect that the initial monetary policy stance of the European Central Bank (ECB) will be excessively lax, reflecting the average of the historic behaviour of the different participating states. Sterling strengthens on this concern, the more so the

more confident the market is that it will remain outside the euro.

### *Scenario (iii): euro uncertainty*

In this scenario, the market's increasing belief that EMU will proceed with broad initial membership leads it to become more uncertain about what kind of monetary policy will operate in the euro-zone countries after monetary union. On the one hand, the market perceives the risk of an excessively lax monetary policy as discussed above, but it also sees a risk that the ECB might be forced to adopt a very tight monetary stance to establish its credibility. This uncertainty in turn creates uncertainty about the levels of euro interest rates and the euro exchange rate that will hold after 1 January 1999. As long as sterling is expected to remain outside the euro, the effect is to increase investor preferences for sterling assets.

### *Scenario (iv): pre-EMU uncertainty*

This final scenario differs from the first three insofar as the market becomes increasingly uncertain in the run-up to 1 January 1999 about whether EMU will actually happen and who will join. As a result of this uncertainty, investors have an increased preference for sterling assets, and this is stronger the more confident they are that sterling will remain outside the euro.

Scenarios (ii), (iii) and (iv) could be characterised as safe-haven stories. Other possible scenarios include those in which the market is increasingly confident that sterling will stay outside the euro, but does expect it to participate in ERM2, a successor to the ERM. But market comment along these lines has been patchy, making it unlikely that this scenario has been sustained for any length of time. Moreover, unless the market expects sterling to participate in an ERM2 with very tight effective margins of variation, it may make little difference to the scenarios already presented.

Any of the four scenarios listed would explain how changing investor perspectives on EMU could have contributed to sterling's appreciation since August 1996. But are they consistent with what we observe from implied correlations?

The evidence presented in the third section on implied correlations between continental European currencies does not appear to be consistent with scenario (iv). Increasing uncertainty about whether EMU will go ahead and who will join means that future news should be expected to cause divergent movements (lower correlation) between the dollar/Mark and dollar/French franc exchange rates, or between the dollar/Mark and dollar/lira exchange rates. But in practice, all the currency pairs examined now have very high absolute expected future correlations. On the other hand, the evidence in the third section is consistent with

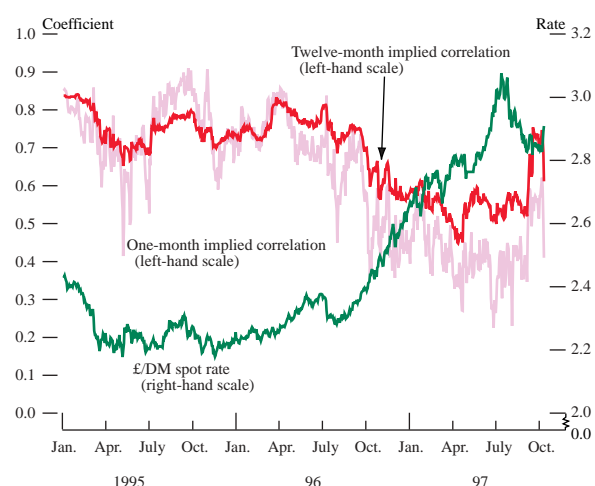
(1) All of these scenarios assume that the market can be modelled as a representative agent. In a model with heterogeneous beliefs, another class of outcomes would be possible.

(2) In contrast with the argument made in this scenario, Alogoskoufis *et al* (1997) have argued that the creation of the euro could lead to portfolio inflows to the euro zone, as the euro begins to share the role of international reserve currency and medium of exchange with the US dollar.

scenarios (i) to (iii) insofar as the rising implied correlations for the peseta and the Mark, and the lira and the Mark, are consistent with a growing market expectation that EMU will have a broad initial membership.

Chart 7 shows the one-month and twelve-month implied correlations between sterling and the Mark, using the dollar as numeraire plotted against the £/DM exchange rate. The path of the twelve-month implied correlation falls into four distinct phases. In the period up to August 1996, it averaged close to 0.8. During this period the £/DM exchange rate was reasonably stable. Through the next nine months, the twelve-month implied correlation declined progressively, reaching 0.45 in May 1997. At the same time sterling rose sharply against the Mark, and stood 24% higher on 6 May 1997 than on 6 August 1996. During the next four months, the implied correlation fluctuated at around 0.55, as sterling first rose by another 9% against the Mark and then fell by about the same amount. Finally, in mid September this year, the twelve-month implied correlation rose sharply to 0.75, and remained there for a month before falling back to 0.6 at the time of writing (21 October). There was no sustained move in sterling in mid September, but the rate against the Mark rose sharply as the implied correlation fell on 20–21 October. During the period as a whole, the one-month implied correlation has been much more volatile than the twelve-month implied correlation, though since March 1996 it has fallen below the twelve-month implied correlation by an increasing margin. We discuss the possible interpretation of this in the next section.

**Chart 7**  
One-month and twelve-month implied correlations of \$/DM with \$/£, and £/DM exchange rate



Scenarios (i) to (iii) are consistent with the fall in implied correlation between the Mark and sterling between August 1996 and May 1997. In all three scenarios, sterling's attractiveness to investors increases because they become increasingly confident that it will not be affected by certain kinds of economic and political shock that are expected to affect the prospective members of EMU.

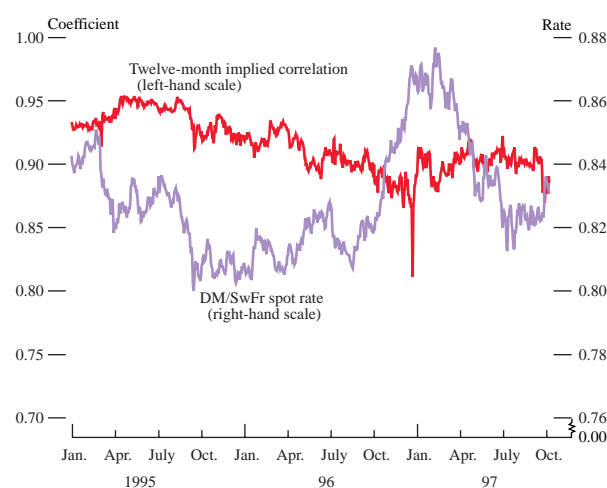
A similar analysis would suggest that, from May 1997 to August 1997, there was no change in how the market

expected future news to differentiate between sterling and the Mark, since during that period the twelve-month implied correlation between sterling and the Mark was broadly stable. But it is still possible that shifts in the level of concern about EMU of the kind outlined in scenarios (i) to (iii) contributed in part to the sharp appreciation and then depreciation in sterling during the period. This is because heightened EMU concerns may lead investors to value the already distinctive behaviour of sterling more highly, alongside other safe-haven currencies.

Finally, the sharp rise and then fall in the twelve-month implied correlation between sterling and the Mark between mid September and 21 October is consistent with a rise and then fall in the probability attached by the market to sterling participating in EMU on or fairly soon after 1 January 1999. This is because with heightened expectations of EMU participation, the market should expect the impact of future news—whether EMU-related or economic—on the Mark and sterling to be more similar. The fact that sterling's exchange rate against the Mark remained broadly unchanged during the initial rise in implied correlation in mid September suggests that other influences may have offset the effect of a declining EMU factor during that period.

To compare market perceptions of sterling with other possible diversification or short-term safe-haven currencies, Chart 8 shows the implied correlation between the Swiss franc and the Mark using the US dollar as numeraire, and Chart 9 shows the implied correlation between the US dollar and the Mark using the Japanese yen as numeraire. The expected correlation between the Swiss franc and the Mark has fallen slightly since the start of 1996, but is still nearly 0.9. This may reflect the interdependence between the Swiss and German economies, and may partly explain why the Swiss franc has not experienced trend appreciation in the past 18 months as the start date for EMU has approached. By contrast, the implied twelve-month correlation between the dollar and the Mark has remained roughly constant at around 0.55 since the start of 1996. This means that unlike

**Chart 8**  
Twelve-month implied correlation of \$/DM with \$/SwFr, and DM/SwFr exchange rate



## Forward implied correlations

In the previous box on deriving implied correlations, we showed how it was possible to extract market expectations of the average correlation between two exchange rates for a given horizon using volatility quotes with a matching maturity. Typically, we can observe these quotes for maturities of one week; one, two, three and six months; and one year. Market practitioners call this the term structure of implied volatility. From this term structure we can construct a term structure of implied correlation simply by using equation (9) at each maturity. But we may want to know what the short-term correlation is expected to be at some point in the future, rather than an average in the period from now to the future date. In other words, we are interested in 'forward' correlations. Here we describe a method for deriving forward correlation curves that give us this information.

Our approach works in two stages. First we calculate 'instantaneous' forward volatility curves for the three relevant currency pairings. These tell us what the instantaneous volatility of each pairing is expected to be at each point in the future up to one year. Then we use the implied correlation equation—equation (9)—in the box on page 414 at each maturity, using the forward volatilities as inputs. This gives us the instantaneous forward correlation curve. This curve should be interpreted as the market's expectation of the instantaneous correlation at each point in the future.

But how do we calculate the forward volatility curves? What is needed is a way to disentangle the implicit volatilities for each sub-period from the volatility quotes we observe. Since volatility changes with time, we need a model that incorporates uncertain time-varying

volatility. Here we follow an approach used by Campa and Chang (1995), based on the Hull and White (1987) stochastic volatility model.

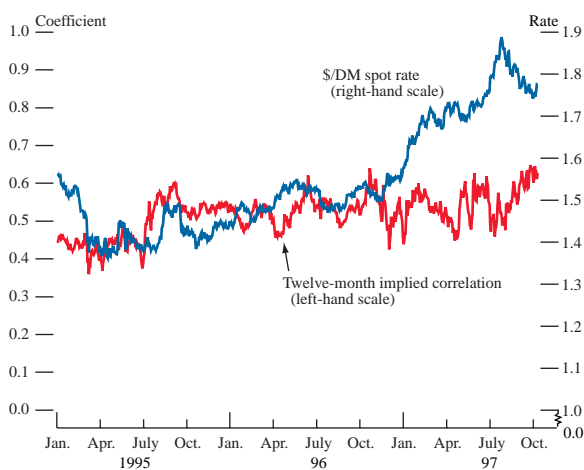
Campa and Chang derive the following linear relationship between per-period expected variances:

$$V_{0,km} = \left(\frac{1}{k}\right) E_0 \left[ \sum_{i=0}^{k-1} V_{im,(i+1)m} \right]$$

Here, future time is divided up into  $k$  sub-periods, each of length  $m$ . For example, if we are looking at expectations of volatilities in the next twelve months, and we divide that time up into twelve single-month periods, then  $k = 12$  sub-periods and  $m = 1/12$  of a year.  $V_{0,km}$  then represents the twelve-month squared volatility and the  $V_{im,(i+1)m}$  terms represent the individual future one-month squared volatilities. So in this example, the current twelve-month squared volatility equals the average of current and expected future one-month squared volatilities. Now, if we knew what the eleven and twelve-month volatility quotes were, we could infer the expected future (or forward) one-month volatility in eleven months' time simply by rearranging the equation and plugging in the appropriate values.

In practice, we only observe a limited number of maturities for implied volatilities in the OTC market. We do not see, for example, eleven-month volatility quotes. So to exploit this relationship, we first need to interpolate across the term structure of volatility. We do this by employing a cubic spline. Once we have a continuous term structure we extract a virtually instantaneous (rather than a one-month) forward curve by dividing up the

**Chart 9**  
\$/DM exchange rate and twelve-month implied correlation of \$/¥ with DM/¥



for sterling, there has been no change during this period in the US dollar's safe-haven or diversification characteristics

as regards the Mark. But it is also consistent with the dollar appreciating against the Mark in response to a general shift in investor preferences towards currencies offering diversification or safe-haven potential.

One cannot readily use the information from implied correlations to distinguish between scenarios (i) to (iii). But other market information can help. Chart 10 shows implied forward interest rates in Germany and the United States ten years ahead. Under scenario (ii), weak euro, long-term inflation expectations in Germany (representing the future euro zone) should rise relative to the world level (represented by the United States). But the chart shows that implied forward interest rates in Germany have fallen both in absolute terms and relative to those in the United States since the start of 1996. So it seems very unlikely that long-term inflation expectations for the euro zone have risen.

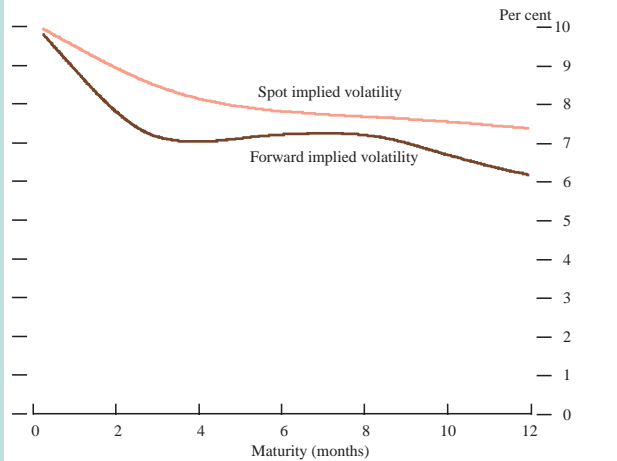
Under scenario (iii), euro uncertainty, one might also expect some rise in ten-year implied forward interest rates in



future into a very large number of sub-periods—a very high  $k$ —and employing this relationship recursively across maturities up to one year.

Chart A above plots both the interpolated term structure of volatility (the spot volatility curve) and the forward curve derived from it for £/DM on 2 October 1997. The difference between these curves is that the spot curve gives us the average expected volatility up to a point in the future, whereas the forward curve gives us the instantaneous volatility expected at a given point in the future. This difference is, at least in part, analogous to the difference between zero coupon and implied forward interest rates. And for the same reasons, we should look at the implied forward volatility curve if we want to examine expectations of future volatilities.

**Chart A**  
Term structure of £/DM volatility on 2 October 1997

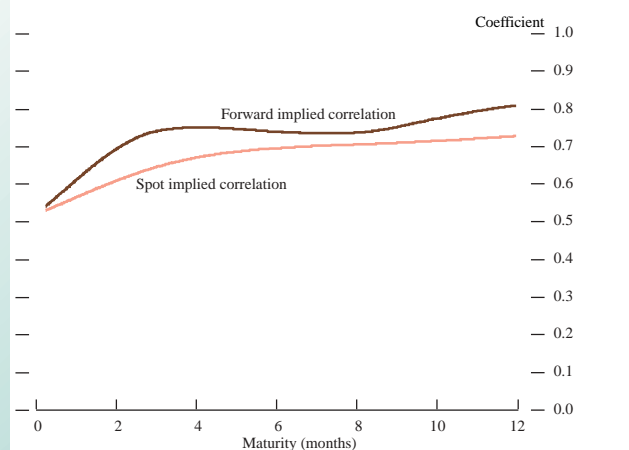


To generate the implied forward correlation curve, we first need to construct the implied forward volatility curves for the three appropriate currency pairings. For example, if we want to derive the implied forward

correlation of sterling and the Mark with the dollar as numeraire, we need the forward volatility curves for the £/US\$, US\$/DM, and £/DM. Once this is done, all that is needed to generate the forward correlation curve is to use equation (9) at each maturity, using the forward volatilities as inputs.

Chart B portrays the implied forward correlation for sterling and the Mark with the dollar as numeraire calculated on 2 October 1997. Of course, it is also possible to calculate a spot correlation curve by using the spot volatilities as inputs. This latter curve should be interpreted as the average expected correlation between the beginning of October and alternative points in the future. The forward correlation curve, on the other hand, tells us what the instantaneous correlation is expected to be at different horizons. It therefore gives us a more easily interpreted measure of how the market expects the correlation between two exchange rates to change over time.

**Chart B**  
Implied correlation of \$/£ with \$/DM on 2 October 1997



**Chart 10**  
Ten-year forward interest rates for the United States and Germany



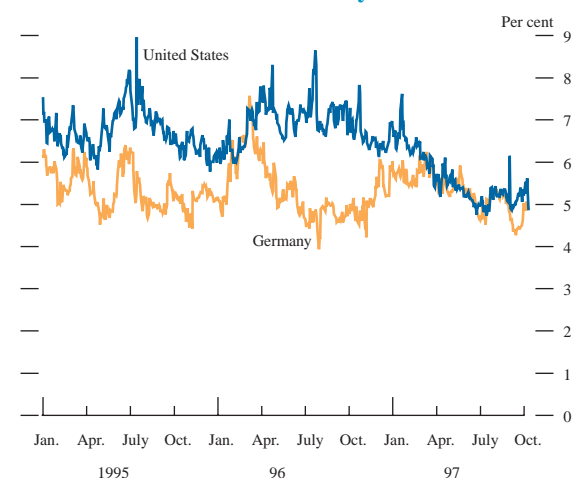
Germany, to reflect an increased inflation risk premium in euro interest rates. But the size of the change could well be small, and other factors will also be at work. So Chart 10 is less conclusive evidence against this scenario. But one would expect the uncertainty about the future monetary regime for the euro to show up in greater uncertainty about long-term interest rates in Germany. Yet Chart 11, which shows the implied volatility for German Bunds up to a short-term time horizon,<sup>(1)</sup> suggests that short-term uncertainty about German long-term interest rates has changed little since 1995 and is now at about the same level as in the summer of 1996. In the same period, uncertainty about US long-term rates has fallen somewhat relative to that for German long-term rates, but this does not suggest any significant relative increase in uncertainty about German long-term rates.

Taken together, the evidence from implied forward interest rates and implied volatilities argues against scenarios (ii)

(1) Varying between one and three months.

and (iii), leaving scenario (i) as the most likely of the four scenarios proposed above.

**Chart 11**  
Implied volatility of long-term interest rates in the United States and Germany<sup>(a)</sup>



(a) Derived from front contracts.

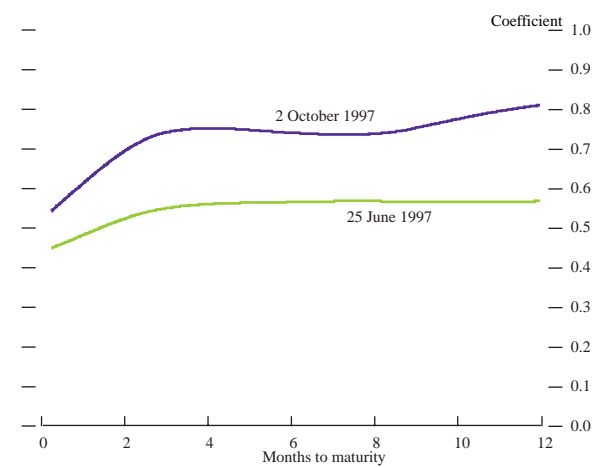
### Implied forward correlations

The discussion so far has focused on twelve-month implied correlations. These contain less noise than one-month correlations and also provide more information on market expectations about what will happen near to 1 January 1999. But it is clear from Chart 7 that the relationship between the one-month and twelve-month implied correlations for sterling and the Mark has changed significantly. At the start of 1996, the two measures were equal. But since then, the one-month implied correlation has tended to fall more sharply than the twelve-month implied correlation. What does this mean? Intuitively, the relationship between the two should tell us something about how the short-term implied correlation is expected to evolve. This is because the twelve-month implied correlation reflects the expected average of the one-month correlation during the twelve-month period.

The box on page 420 describes how one can estimate an implied forward correlation curve, showing the expected path of the very short-term correlation between two exchange rates. Chart 12 shows the shape of this curve for the dollar/sterling and dollar/Mark exchange rates on two dates: 25 June 1997 and 2 October 1997. On the first date, the curve has a slight upward slope for the first three months before flattening out. This suggests that future news is expected to have a more divergent impact on sterling and the Mark in the short term than in the longer term. On the second date, the curve has shifted up at all time horizons, but has also acquired a much steeper slope in the first three months.

A possible interpretation for the upward-sloping portion of the curve on both dates is that the market expected euro news to be more significant in the first two to three months than subsequently. In June, when the market

**Chart 12**  
Implied forward correlation curves of \$/£ with \$/DM on 25 June and 2 October 1997



appeared very confident that sterling would not join EMU at an early stage, the news expected in the near term could have been related to the likelihood of other EU members joining the euro. But on 2 October, in the period of heightened speculation that the United Kingdom might participate in EMU relatively early, the news might also have related to clarification on the United Kingdom's position. At the time, the two to three-month time horizon fitted the Maastricht Treaty requirement for the United Kingdom to decide by the end of 1997 whether or not to exercise its opt-out.

But the main advantage of the implied forward correlation curve as we get closer to 1 January 1999 is that it will soon enable us to estimate the market's expectation of the short-term correlation between two exchange rates for a period starting beyond 1 January 1999. This could then provide the most accurate reading available about which countries are expected to participate in EMU.

### Conclusion

In this paper we have described a technique for deriving the expected future (or implied) correlation between two exchange rates up to a twelve-month time horizon. We use this information to obtain a new perspective on market expectations about whether particular EU members will join EMU. This suggests that since the start of 1996, the market has become increasingly confident that EMU will proceed on a broad basis. We also use the technique, together with other information, to derive insights on how speculation about EMU may have contributed to the appreciation in sterling since August 1996. This suggests that a desire for diversification on the part of investors holding what they expect will shortly become euro-denominated assets is the most plausible of the various possible EMU scenarios which have been proposed. Though the technique cannot provide conclusive evidence that one particular scenario or sentiment underlies market expectations, implied correlations can help to pin down more accurately the underlying nature of these expectations.

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