Monetary policy and uncertainty

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This article describes various types of uncertainty that policy-makers may face. It summarises analysis, including recent work by Bank staff, that shows how different forms of uncertainty could lead to different policy responses.

Introduction

Monetary policy-makers take decisions in an uncertain world. This has been long recognised by policy-makers and is reflected in the *Inflation Report* fan charts, for example, which display the many uncertainties surrounding the inflation and growth projections. But academic studies often assume that policy-makers act as if certain when determining policy. This rests on the notion that policy-makers' uncertainty about the economy is only of one particular form.

Recent research has begun to explore the implications for monetary policy of a wider range of uncertainties facing policy-makers (for example, Sack (1998), Sargent (1998) and Aoki (1999)). One strand of this, on which Bank of England staff have worked, has been the analysis of whether uncertainty about the relationship between economic variables in the economy (for example, between nominal interest rates and the demand for money) could entail a slower, or smoother, policy response to shocks to the economy than otherwise. This analysis, which follows a proposition first put forward by Brainard (1967), is based on the premise that uncertainty about the relationship between the official interest rate and the rest of the economy (a form of 'parameter uncertainty') creates a trade-off for policy-makers: the parameter uncertainty may mean that movements in the official interest rate themselves increase uncertainty about the future path of the economy. This could lead policy-makers to use their policy instruments more cautiously, even if this is likely to result in a worse outcome on average, in order to reduce the chance of missing the target significantly.

The next section of this article describes the relationship between monetary policy and uncertainty. It discusses in detail the parameter-uncertainty effect identified by Brainard, and reviews other forms of uncertainty. The third section summarises the results from two empirical studies carried out at the Bank, which, by focusing only on the parameter-uncertainty effect identified by Brainard, explore in a preliminary way the quantitative importance of uncertainty for the United Kingdom.

Uncertainty and monetary policy: theoretical considerations

A standard approach to analysing monetary policy is to specify an objective for policy-makers, and a model of the economy, and then to determine how monetary policy should be operated in response to disturbances or 'shocks' to the economy.

How uncertainty is supposed to affect monetary policy will depend on how the model is specified. Researchers have generally specified models in which uncertainty is independent of the policy-maker's behaviour. In these models, the only uncertainty is whether the economy will deviate from the path policy-makers expect on account of what are known as 'additive shocks'.⁽¹⁾ As Theil (1958) showed, the best that policy-makers could do in this case would be to ignore the effects of uncertainty upon the economy. This is known as 'certainty-equivalence'.

But this restrictive approach does not take account of many of the uncertainties faced by policy-makers. As outlined below, a number of recent papers have explored the implications for policy of allowing more general treatments of uncertainty.

The certainty-equivalence approach and the recent generalisations can be illustrated using a simple model of inflation targeting, based on Svensson (1996). The core of the model is a simple two-equation system. The first equation (a form of Phillips curve) links inflation to the output gap, ie:

$$\pi_{t+1} = a\pi_t + y_{t+1}$$
(1)

where π_t is the inflation rate and y_t is the output gap.

The second equation (a form of IS curve) links the output gap to nominal interest rates. The output gap is inversely

Additive shocks cause a variable to deviate from the path implied by its identified determinants. For example, in the following equation, additive shocks (*e*) cause variable x (say, exports) to deviate from the level implied by its identified determinants—the previous-period outcomes for x and y (say, world demand) given the multipliers, α and β, that relate x to its determinants: x_{t+1} = αx_{t+1} + βy_t + e_{t+1}.

related to previous changes in the short-term nominal interest rate, i_t , and is subject to additive shocks, ε_{t+1} , which average zero, and have a variance of $\sigma_{\varepsilon}^{2:(1)}$

$$y_{t+1} = -bi_t + \varepsilon_{t+1} \tag{2}$$

Substituting (2) into (1) gives the following reduced form for inflation:

$$\pi_{t+1} = a\pi_t - bi_t + \varepsilon_{t+1} \tag{3}$$

Policy-makers set nominal interest rates, i_t , with the aim of meeting the inflation target. Specifically, it is assumed that their objective is to minimise the expected squared deviations of inflation from target (normalised to zero).⁽²⁾ This objective can be interpreted as saying that the policy-maker is concerned with minimising expected future deviations of inflation from target (the bias in future inflation), and uncertainty about future inflation (the variance of inflation).⁽³⁾ This concern about both bias and variance is vital to understanding Brainard's insight.

The only uncertainty in this version of the model arises from the additive disturbance entering the IS curve (2). Policy-makers are assumed to know with certainty: (i) the parameter values linking variables in the economy; (ii) the state of the economy (so that the output gap and inflation are measured with certainty); and (iii) most basically, the functional form of the economy (ie how inflation and the output gap are actually related). An optimal rule can then be determined, which, in this hypothetical world, would enable policy-makers to minimise expected deviations of inflation from target:

$$i_t = \frac{a}{b}\pi_t \tag{4}$$

The rule is certainty-equivalent: the same interest rate rule would be optimal in a world with no uncertainty about additive shocks. If policy-makers followed this rule, they would completely offset the effects of shocks to inflation, so that the expectation at time t of next period's inflation would always be equal to target. Hence, although policy-makers cannot prevent temporary deviations of inflation from target, they can ensure that the effects of such shocks do not persist. The reason why the model implies that policy-makers can control inflation so accurately is because assumptions (i) to (iii) imply that they can unambiguously identify a shock to inflation from inflation outturns, and know by exactly how much they need to move the instrument to offset the effects of the shock on subsequent inflation.

Of course, in the real world, policy-makers cannot identify shocks or best responses so clearly. So we need to consider how the optimal rule varies as conditions (i) to (iii) are relaxed.

(i) Parameter uncertainty

Parameter uncertainty arises where policy-makers are unsure how changes in one variable will affect another. In (2), for example, if policy-makers did not know the value of the parameter b, they would be unsure how changes in interest rates would feed through to the output gap and hence inflation.

To understand how policy-makers might best act in this situation, we need to make an assumption about precisely what policy-makers are unsure about. The first, and most influential, analysis of parameter uncertainty was provided by Brainard (1967). He assumed that policy-makers were uncertain about the actual value of parameters in the model, but knew the distribution from which they were drawn.

The effect of this form of parameter uncertainty can be illustrated in the model developed above. Assume that the policy-makers know that the parameters a and b in the reduced-form equation for inflation (3) are drawn from independent normal distributions, with means \overline{a} and \overline{b} , and variances σ_a^2 and σ_b^2 respectively. In this case, the optimal rule becomes:

$$i_t = \left[\frac{\overline{a}\overline{b}}{\overline{b}^2 + \sigma_b^2}\right] \pi_t$$
(5)

This seems more complicated than (4). But in fact it is closely related, and reflects Brainard's insight that the optimal policy response should be modified to take account of any uncertainty about parameters in the transmission mechanism: as uncertainty increases about how inflation will respond to changes in the monetary policy instrument (ie as σ_h^2 becomes larger), so the interest rate response to inflation deviations from target becomes smaller, with the result that inflation is not returned to target straight away. This is what Blinder (1998) called 'Brainard conservatism'.(4)

The trade-off between returning inflation to target and increasing uncertainty about inflation depends on the size of the variance σ_b^2 of the policy multiplier *b* relative to its average level \overline{b} . This policy multiplier measures how effectively interest rates reduce the bias in inflation, while its variance σ_h^2 , measures how much uncertainty is injected by the policy-maker. The ratio of the standard deviation to the mean, $\frac{\sigma_b}{\overline{b}}$, is known as the 'coefficient of variation' and summarises this trade-off. For example, a large coefficient of variation means that for a small reduction in the inflation

A more sophisticated model would relate the output gap to real interest rates, and real interest rates to nominal rates, via the Fisher identity. Martin and Salmon (1999) present a version of the model that makes this distinction.
 For simplicity, this objective implies that policy-makers have no regard for the variability of output. This is not the case in practice, as is recognised in the third section.
 This is a consequence of Jensen's inequality, which arises in many applications, and implies that the expected squared value of a variable equals the square of the bias plus the variance: E(π²) = (E(π)²) + var(π).
 Brainard himself noted examples where the conservatism principle may fail to hold, even in a model as simple as the one presented here. This may occur if parameters co-vary in a certain way.

bias, the policy-maker injects a lot of variance into future inflation, and implies that (5) would result in a different policy response from (4).(1)

Given that policy-makers are assumed in this particular model to be able to control inflation speedily and accurately, why would they not offset inflationary shocks completely and immediately? Recall the assumption that policy-makers wish to minimise the expected bias and variance of inflation. If the parameters of the model are known with certainty, the variance of inflation is independent of monetary policy-the only source of uncertainty is the additive disturbance (ε_{t+1}), which is outside the policy-makers' control. So interest rates can be set with the aim of putting expected inflation back on target, or completely eliminating the bias in inflation, as in (4). But once account is taken of the uncertainty about the parameters of the model, the variance of inflation also depends on the level of interest rates, and so the policy-makers' actions affect uncertainty about future inflation. In this model, large movements in interest rates in response to shocks increase the variance of inflation. So the quicker a policy-maker attempts to return inflation to target, the higher will be the probability of missing the target by a long way. This conflict between the bias and uncertainty about future inflation underlies Brainard's prescription that the optimal policy response cannot ignore uncertainty about parameters in the transmission of monetary policy.

If policy-makers choose to follow (5), because of parameter uncertainty, a more sustained interest rate reaction will be required than if policy were set according to the certainty-equivalent rule, (4). This is because (5) does not offset the entire inflation shock when it occurs, so that policy in the next period will have to react to the residual consequences of the initial shock to inflation. Under (4), the whole of the shock to inflation is offset immediately. Spreading out a policy response has been labelled 'gradualism'.(2)

Several studies have sought to examine the quantitative importance of the Brainard effect. In particular, Sack (1998) has examined how sensitive the optimal monetary policy rule in the United States is to this form of uncertainty. He finds that an optimal rule that assumes no parameter uncertainty leads to a larger, or more 'aggressive', policy response to shocks than has been observed in the past couple of decades. The optimal rule allowing for uncertainty implies a somewhat smaller, or more 'conservative', response to shocks. The first of the Bank studies described in the third section applies Sack's method to the United Kingdom.

(ii) Knowledge of the current economic state

The assumption that the true state of the economy can be measured ignores a potentially serious form of uncertainty for the policy-maker: measurement error. This can arise

because many important variables like GDP are only available with a lag, and are subject to revision; moreover, some variables, such as the output gap, cannot be directly measured at all.

Chow (1977) showed that if the measurement error can be treated as another additive error, it should not affect policy. In the simple model described above, this will be the case if it is assumed that the output gap is measured with error. Instead of (2), policy-makers would then use (2'), where \hat{y}_t is their measure of the output gap and η_t^y is the difference between the actual output gap and that estimate:

$$\hat{y}_{t+1} = -bi_t + \varepsilon_{t+1} + \eta_{t+1}^{y}$$
(2')

The policy-maker would not be able to distinguish between the contributions of the additive error (ε_{t+1}) and the measurement error (η_{t+1}^y) to their estimate of the output gap. Nevertheless, the optimal rule given (1) and (2') would remain (4).

But this will not always be so. In particular, in models where there are different kinds of additive shock, which require a different policy response, measurement error can make it harder for policy-makers to identify what additive shocks have occurred. In these circumstances, the optimal policy response may be more 'conservative' than when there is no measurement error.

For example, policy-makers whose objective was to prevent large fluctuations of inflation from target and fluctuations in the output gap would typically raise interest rates in response to a demand shock that increased inflation and output. Conversely, they might cut interest rates in response to a supply shock that raised inflation but lowered output. But if output could initially only be measured with uncertainty, it might not be clear whether a measured increase in output and inflation reflected a supply or demand shock, or simply measurement error. Then to assume, for example, that the measured rise in output is wholly accounted for by a demand shock, and to raise the interest rate, could be a significant policy mistake. The best that policy-makers could do would be to respond to their estimate of the state of the economy, which would reflect each of the possibilities that there had been a demand or supply shock, and that the observed change in output was entirely due to measurement error. The optimal policy response would depend on the severity of measurement error in the economy, and the weight placed on each possibility. Aoki (1999) shows how this strategy could lead to a more conservative (ie lesser) response than if there were no measurement error. The intuition is that policy-makers benefit by not 'putting all their eggs in one basket' when interpreting the data.

Recent studies, for example by Smets (1998) and Rudebusch (1998) have analysed whether measurement error of the output gap (Smets) and of output and inflation

Conversely, as the variance of b tends to zero, (5) collapses to (4). A third consideration is whether or not the gradual policy response implied by (5) will result in policy-makers cumulatively moving interest rates by more, in order to offset the shock. This will be determined by the persistence of the additive shock. See Martin (1999) for more details.

(Rudebusch) might significantly affect the optimal response to developments in the US economy. Both studies assume that policy is set according to a form of the 'Taylor rule', which relates interest rates to developments in output and inflation.⁽¹⁾ They first calculate the optimal form for the Taylor rule, conditional on a model that assumes no measurement error, and then recalculate this rule allowing for measurement error. Both studies show that the optimal policy response could decline markedly if measurement error is significant.

(iii) Model uncertainty

The analysis described above still assumes that the policy-makers know precisely how uncertain they are: to calculate rule (5), policy-makers must know the variance of the uncertain parameters and additive disturbances to the economy. Similarly, the studies by Aoki, Rudebusch and Smets assume that the variance of any measurement error is known.

A more realistic assumption may be that uncertainty is more pervasive than this. In particular, and fundamentally, policy-makers are uncertain about the basic form of the 'true' model of the economy. This would be the case, for example, if it was unclear which variables to omit or include in the model.

Theorists have considered how policy should be set, given such 'model uncertainty'. One idea is that policy rules could be designed that perform well across a range of plausible models of the economy. Such 'robust' policy rules would not, by definition, perform as well as an optimal rule designed for a particular model. But they would be designed to perform quite well both with this model and a range of similar models, whereas the optimal rule might perform poorly with other models.

This analysis is still evolving, and a consensus on how to identify robust rules has yet to emerge. McCallum (1988 and subsequent papers) investigated whether particular policy rules performed credibly across a small range of models, on the basis of qualitative criteria. More recent contributions (for example, Sargent (1998) and Onatski and Stock (1999)) have used formal mathematical criteria to investigate robustness. Sargent takes an open-economy variant of the model set out above (due to Ball (1998)) and calculates a policy rule robust to small mis-specifications around this model.⁽²⁾ Unlike the Brainard conservatism result for parameter uncertainty, Sargent finds that the robust rule for Ball's model may

be more aggressive than the certainty-equivalent optimal rule in the model.⁽³⁾ For the simple model presented above in (1) to (3), it is possible to show that a 'Sargent robust rule' coincides with the certainty-equivalent optimal rule.(4)

To summarise, this section has discussed a range of theoretical approaches to studying the uncertainty faced by monetary policy-makers. The most common approach to uncertainty, where only additive uncertainty is considered, is theoretically attractive and implies certainty-equivalent policy responses. But this approach is restrictive: once more uncertainties are allowed for, it becomes apparent that the dictum that policy-makers can act as if the world is certain is unlikely to be appropriate. Beyond this, it is hard to draw general lessons. It seems likely that parameter uncertainty and measurement error are both likely to reduce the size of the optimal response to shocks, but more generic model uncertainty could imply a need for a more aggressive policy response.

Parameter uncertainty in the United Kingdom

This section focuses on a particular form of uncertainty, namely parameter uncertainty as defined by Brainard. It summarises two empirical studies by Bank staff. The first paper (Martin and Salmon (1999)) takes a vector autoregressive (VAR) model of the economy, and calculates optimal rules under alternative assumptions about the presence of parameter uncertainty. The second (Hall, Salmon, Yates and Batini (1999)) analyses the effectiveness of different degrees of policy responsiveness (as in the first paper, under alternative assumptions about the presence of parameter uncertainty) on the assumption that policy is set according to a simple policy rule.(5)

(i) Policy rules with and without parameter uncertainty

The approach of the Martin and Salmon study mirrors that described in the previous section: a model is specified, objectives for policy-makers are hypothesised, and optimal rules are calculated. The model is solved under alternative assumptions about parameter uncertainty, and the optimal rules are compared.

The model is a VAR that relates developments in inflation, output, the exchange rate and the official interest rate to each other. It is estimated between 1980 Q2 and 1997 Q2.60 It is assumed that the primary objective of policy is to minimise expected squared deviations of RPIX inflation from target.(7)

⁽¹⁾(2)

See Stuart (1996) for a fuller description of the Taylor rule. Sargent's method is complex, but its essence is that model uncertainty can be introduced by assuming that the policy-maker does not know the properties of the shocks to the model. If these can take a variety of forms, then in effect the properties of the entire model are uncertain. As long as a limit is put on the possible types of behaviour that shocks can exhibit, it is possible to work out how a policy rule would perform in each shock 'world'. Sargent identifies the robust rule as the rule that results in the least-bad possible outcome. The particular assumption that Sargent makes is that the autocorrelation, or persistence, of additive disturbances to the model is uncertain. In this case, the main risk to policy is that shocks will affect inflation for longer than anticipated. It is possible, for a given baseline model and characterisation of uncertainty around it, that the robust rule implies more conservative responses than a certainty-equivalent rule. Onatski and Stock (1999) found examples of this in their paper, though the robust rules they identified were mainly aggressive, relative to a certainty-equivalent rule. Others have focused on parameter uncertainty in an attempt to understand the historical behaviour of official interest rates in the major industrialised countries. Charles Goodhart's 1998 Keynes lecture summarises this analysis, and shows how the optimal rules described here can be used in that context.

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The econometric estimates of the VAR can be used to compute optimal rules for nominal interest rates that relate interest rates to current and past outcomes of all of the model variables. The authors calculate two rules: one assuming that parameters are certain, and one allowing for parameter uncertainty. These rules are generalisations of (4) and (5). As Brainard showed, the coefficients in the optimal rule that allows for parameter uncertainty depend on the variances of the parameters in the VAR model. Brainard suggested that parameter uncertainty could arise either if the underlying model was uncertain, or if the 'true' model was deterministic but the policy-maker had to estimate it. Econometric estimation techniques such as ordinary least-squares (used here to estimate the VAR model) provide estimates not only of the parameters themselves, but also of the variances and covariances between the parameters. The authors follow the second interpretation of parameter uncertainty, and treat the econometric measures of parameter variance and covariance as measures of the policy-maker's uncertainty about parameters. The paper investigates the practical importance of the conservatism and gradualism effects upon the rule identified in the previous section.

Hypothetical paths for interest rates can be calculated for the sample period, on the assumption that policy was set according to each of the two rules, and that the economy was subject to the same set of shocks as historically occurred. These hypothetical paths suggest that the additive-uncertainty rule, calculated on the assumption that policy-makers act as if they know the true parameter values in the economy, would have resulted in more aggressive responses to shocks than the parameter-uncertainty rule, which assumes that policy-makers are uncertain about parameters. This can be seen from the table, which reports summary statistics on the volatility of interest rates implied by each rule. The maximum and minimum deviations and standard errors of interest rates from trend according to the additive-uncertainty rule are larger than those for the parameter-uncertainty rule.(1)

Deviations of interest rates from trend

	Additive-uncertainty rule	Parameter-uncertainty rule
Mean	-0.10	-0.06
Standard error	0.98	0.60
Minimum	-2.85	-1.66
Maximum	2.46	1.91

Impulse response functions, which show the optimal path for interest rates in response to a hypothetical shock, provide evidence on both the immediate response to a shock, and how interest rates evolve in subsequent periods.

The chart shows the impulse responses of rates under the optimal rules to an (additive) shock to output identified from the VAR model. The initial response of the additive-uncertainty rule is around 1.5 times larger than the

Interest rate responses to an output shock



initial parameter-uncertainty response, which is consistent with conservatism affecting the latter. And the parameter-uncertainty response is more drawn-out thereafter, which is consistent with the effect of gradualism. More detailed analysis of the cumulative total response to the shock in the quarters after it occurs suggests that gradualism is important for the first two or three quarters only. Thereafter, the cumulative response implied by each rule is similar.⁽²⁾

Interpreted narrowly, these results suggest that a policy-maker who took account of uncertainty about parameters would choose to act differently from a policy-maker who did not. In particular, the initial response to developments would be less, but two or three quarters after the shock, the cumulative policy responses of the two policy-makers would be similar. But the results rely on many auxiliary assumptions-for example, it is assumed that the economy can be accurately represented by a rather simple VAR model-and so provide only indicative evidence.

(ii) Simple policy rules with and without parameter uncertainty

In a recent paper, Hall, Salmon, Yates and Batini (1999) analysed how additive and parameter uncertainty might affect policy, on the assumption that policy is set according to a simple rule. The paper makes use of an existing model, due to Haldane, McCallum and Salmon (1996), which was developed before the new monetary arrangements were put in place, and does not reflect current institutional structures. It therefore provides only indirect evidence for the United Kingdom.⁽³⁾ But it acts as a cross-check on the findings from the Martin and Salmon paper.

The paper addresses the following three questions.

First, does the optimal degree of feedback (the feedback in the simple rule that delivers the best

The *Working Paper* describes how each of the data in the VAR, including interest rates, are de-trended. The *Working Paper* analyses the impulse response functions to shocks to each variable in the VAR model. The results are broadly similar, though the other impulse responses are harder to interpret. See the forthcoming *Working Paper* for details of the modelling approach used. (1)(2)

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stabilisation properties) fall when additive uncertainty is introduced into the model of the economy?

- Second, does this optimal degree of feedback fall when parameter uncertainty is introduced into the model?
- Finally, how does uncertainty about particular parameters in the model influence the optimal degree of feedback in the policy rule? In other words, is uncertainty about some specific relationship in the economy more important for the operation of monetary policy than uncertainty about others (in terms of optimal degree of reaction to news)?

The final question is interesting because discussions of the merits of a gradualist monetary policy have often been couched with reference to uncertainties about particular aspects of the economy.⁽¹⁾

The paper shows that the optimal degree of feedback to developments in the economy is largely unchanged by the size of additive shocks to the economy. This is not surprising, and accords with the certainty-equivalence result described earlier.

If parameter uncertainty is introduced, the picture changes. In this case, the optimal degree of feedback varies inversely with the extent of uncertainty: optimal policy is more conservative in an uncertain world.

To address the issue of uncertainty about specific parameters in the model, the paper analyses the effect of assuming that there is uncertainty either about the policy multipliers in the model or about the relationship between the output gap and inflation. It shows that uncertainty about the policy multipliers has a more significant impact upon policy-makers' optimal degree of reactiveness than uncertainty about the output gap.

In the estimated model underlying the analysis, the coefficients of variation on the policy multipliers in the

model are greater than those for the output gap parameters. As discussed earlier, a higher coefficient of variation, other things being equal, implies that the variance costs of policy reaction will increase relative to gains from attempted stabilisation, such that it will become optimal to respond less actively to policy shocks. Once again, the empirical results accord with the theoretical predictions.

Summary

This article has reviewed how economic theory suggests that monetary policy-makers should take account of different types of uncertainty. This is an area where economic theory lags behind practice. Policy-makers have always had to make allowances for all the uncertainties that they perceive.

Theoretical analysis has tended to consider only very specific and tightly defined forms of uncertainty. A key result—that policy-makers should act as if certain—is applicable only when policy-makers have considerable information about the structure and state of the economy. The second section of this article showed how this certainty-equivalence result breaks down once it is assumed that policy-makers are unsure about the relationship between variables in the economy or, in some circumstances, on account of measurement error, about the current state of the economy. Such uncertainties by themselves are likely to result in smaller policy responses to economic developments. The results from the studies summarised in the third section provide some evidence of this effect.

But these studies take account only of the effect of parameter uncertainty. In practice, policy-makers' uncertainty is likely to be deep-seated, not least because they are unsure about the basic structure of the 'true' economy. Neither these studies, nor other empirical work, provide a unified analysis of the effects of all forms of uncertainty upon policy. In short, a consensus view has yet to emerge from the academic literature as to how policy-makers should deal with uncertainty.

(1) For instance, there has been much recent debate in the United Kingdom and in the United States about both the level of the NAIRU and the output gap and their relation to inflation. Wieland (1998) shows how this can lead to parameter uncertainty.

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