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# Band-pass filtering, cointegration, and business cycle analysis

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In recent years, band-pass filtering—the non-structural, frequency-domain based decomposition of economic time series into trend and cyclical components—has become more and more popular among macroeconomists, as a way of capturing and describing business cycle stylised facts. Compared with the Hodrick-Prescott filter, the band-pass filter offers the advantage of allowing the researcher to target a specific frequency band, thus extracting from the series of interest all the components associated with that band, while essentially discarding all the others. The growing interest of the macroeconomics profession in band-pass filtering techniques is demonstrated, first by the number of recent papers on business cycle stylised facts that make use of the band-pass filter, second, by the inclusion in the recent *Handbook of Macroeconomics* of a chapter on US post-World War II business cycle stylised facts entirely based on band-pass filtering, and third, by the continuing attempts to develop new and better approximations to the ideal band-pass filter.

This paper critically assesses the practice of band-pass filtering, making two main points. First, it is shown that, depending on the stochastic properties of the filtered process, the band-pass filtered cyclical component could be entirely authentic, partly or mostly spurious, or even entirely spurious. While, in general, there does not exist any universally valid measure of authenticity for band-pass filtered cyclical components, it is shown that for unobserved components (UCARIMA) processes there does indeed exist such a natural measure, based on the integral of the spectral density of the band-pass filtered process. Taking a simple sticky-price DSGE model as the data-generation process, it is shown that: (a) under a number of circumstances, band-pass filtered output may provide a surprisingly bad proxy for the structural output gap; and (b) as a technique for extracting a proxy for the output gap, band-pass filtering suffers from

the distinct disadvantage that, as a simple consequence of the Lucas critique, the accuracy of the approximation is not invariant to the monetary rule followed by the policy-maker, and in fact crucially depends on it.

Second, taking some alternative macroeconomic models as data-generation processes, it is shown that band-pass filtering: (1) may markedly distort key business cycle stylised facts, as captured by the cross-correlations and the cross-spectral statistics (gain, phase angle, and coherence) between the cyclical components of the variables of interest and the cyclical component of GDP; and (2) may well create entirely spurious stylised facts. For example: (a) the Phillips correlation between inflation and the cyclical component of economic activity will in general appear weaker than it is in reality; (b) both money supply and productivity may appear procyclical even when they follow random walks by construction; (c) the real wage may appear procyclical when in fact it is countercyclical. These results are not peculiar to a particular class of model, but instead illustrate a general problem: the presence of stochastic trends, and possibly of cointegrating relationships among macroeconomic variables, may significantly alter the business cycle stylised facts as captured by the band-pass filter. Again, the degree of authenticity of business cycle stylised facts uncovered via band-pass filtering crucially depends on the monetary rule followed by the policy-maker.

The general conclusion emerging from the paper is that, far from being the neutral, atheoretical, and objective approach to the study of business cycle stylised facts that it is often claimed to be, band-pass filtering may markedly distort those very same stylised facts in unpredictable ways, simply because such distortions crucially depend on the unknown true structure of the economy that the researcher is investigating.