Assessing the stability of narrow money demand in the United Kingdom

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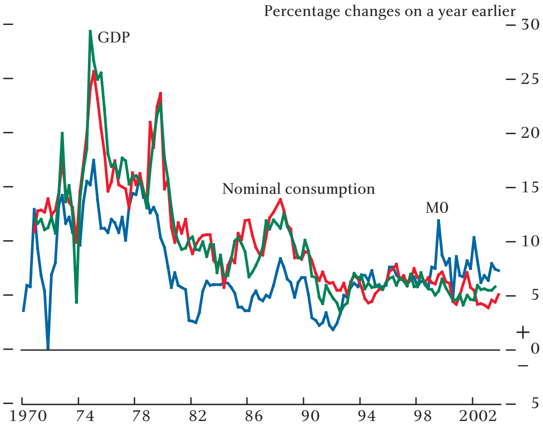
It is widely accepted that the introduction of cash-saving technologies, such as credit and debit cards, and the growing network of automated teller machines (ATMs) contributed to a prolonged upward shift in narrow money velocity towards the end of the 20th century.(2) *This article considers whether this upward shift might plausibly have come to an end. First, it presents data on four distinct manifestations of financial innovation, and asks whether the pace of change in each might have slowed. Second, it uses time-series data stretching back more than 100 years to present estimates of the demand for narrow money during different time periods. It finds tentative evidence that, since the early 1990s, narrow money velocity has been a broadly stable function of the short-term rate of interest.*

Introduction

Monetary policy makers take a keen interest in the monetary aggregates. In comparison with most other economic statistics, the monetary aggregates are more timely and less prone to revision. They are also based on a complete population—in this case banks and building societies operating in the United Kingdom—rather than a population sample. These are some of the reasons why the Monetary Policy Committee (MPC) looks at the monetary aggregates, alongside many other pieces of data, when assembling its projections for nominal demand, and hence for inflation.(3) The M0 aggregate, which comprises notes and coin in circulation, and bankers' operational balances held at the Bank of England, is one of the narrowest measures of money. Because it pays no interest, it is a relatively unattractive form of wealth. Consequently, the quantity of M0 might be more closely related to current nominal expenditure, on at least certain classes of goods and services, than broader measures of the money supply, such as M4.

Chart 1 confirms that, during the past 30 years or so, there has been a reasonably close relationship between the rate of growth of M0 and the rate of growth of nominal expenditure. All three series in the chart have followed a broadly similar pattern over the economic cycle. It is nonetheless evident that, during the late 1970s and for most of the 1980s, M0 grew markedly less quickly than nominal expenditure. That means that the velocity of circulation of narrow money, defined as the ratio of nominal expenditure to M0, was rising. Since the late 1990s, the picture has changed somewhat, with M0 tending to grow more quickly than nominal expenditure, and velocity tending to fall.

Chart 1
Narrow money and nominal expenditure

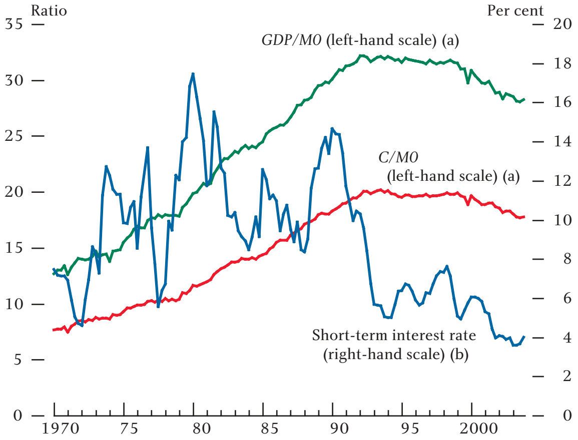


Sources: Bank of England and ONS.

In order to use data on the rate of growth of narrow money to draw inferences about the rate of growth of nominal expenditure, some view about the path of narrow money velocity is required. Chart 2 compares

(1) We are grateful to Zvi Eckstein for many useful discussions. He also provided us with the initial motivation for this project. We would also like to thank the Association for Payment Clearing Services (APACS), which provided us with all the means of payment data presented in this article.
(2) In this article, narrow money velocity is defined as nominal expenditure divided by M0.
(3) See Hauser and Brigden (2002) for a more detailed account of how the MPC makes use of the monetary aggregates.

Chart 2
Narrow money velocity and the short-term interest rate



Sources: Bank of England and ONS.

(a) Quarterly expenditure data are annualised.
(b) Discount rate on three-month eligible bills.

both the GDP and the consumption velocity of M0 with the discount rate on three-month eligible bills since 1970. A key identifying feature of cash, as a form of wealth, is that it pays no interest. Consequently, the opportunity cost of holding cash is the rate of interest that could be earned on an alternative asset. To the extent that the discount rate on three-month eligible bills is a suitable proxy for this rate of interest, then one should expect to see a positive relationship between the three series in Chart 2. But there appears to be no stable relationship of any kind. The most striking aspect of Chart 2 is the marked upward trend in narrow money velocity during the 1970s and the 1980s. At this time, the short-term interest rate was both unusually high and unusually volatile. Nonetheless, it reached a peak in 1980, and thereafter began to drift back down.

Much has been written about the upward trend in narrow money velocity during the 1970s and the 1980s. The analysis has often focused on the impact of financial innovations that have allowed people to economise on their cash holdings: see for example Trundle (1982) or Westaway and Walton (1991). These cash-saving innovations included the introduction of ATMs, as well as a number of alternatives to cash as a means of payment, such as plastic cards and electronic funds transfer at the point of sale (EFTPOS) technology. More recently, a number of authors have considered why the upward trend in velocity might have apparently ended so abruptly during the 1990s. One example, from an earlier edition of this *Bulletin*, is Janssen (1996). He surveyed a number of direct measures of cash-saving innovations, and concluded that the pace at which these were being introduced had slowed.

The second section of this article revisits some of the direct measures of cash-saving innovations discussed in Janssen (1996). The third section provides some econometric evidence on the relationship between velocity and the short-term rate of interest. Using a set of annual data that stretches back more than 100 years, we find evidence that a gradual upward shift in velocity occurred during the middle of the period shown in Chart 2. During the period from 1870 to 1980, there appeared to have been a stable long-run relationship between velocity and the short-term rate of interest. That relationship seemed to break down between 1981 and 1992, but might have reasserted itself more recently.

Cash-saving financial innovations and narrow money velocity

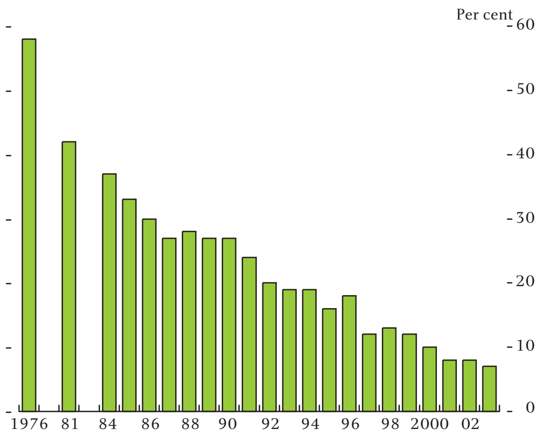
Estimates suggest that the greater part of M0 (around 80%) is held in the form of notes and coin by the household sector. A small amount (around 20%) is held in the form of notes and coin by monetary financial institutions (MFIs) and private non-financial corporations (PNFCs). This will include cash held in tills at bank and building society branches, and at retail outlets. The amount of notes and coin held by other financial corporations (OFCs) and bankers' operational balances at the Bank of England are both negligible. Since most cash is held by private individuals, our analysis concentrates on trends in household sector expenditure, and in the payments industry that serves it.

Imagine that, for a price, new technology could be installed that would allow a smaller amount of cash balances to be held for a given amount of expenditure. Then the incentive to invest in this new technology would depend not just on the opportunity cost of holding cash today, but on the expected opportunity cost of holding cash in the future. To the extent that the unprecedented pickup in inflation during the 1970s caused individuals to revise upwards their expectations for inflation and nominal interest rates, then one might imagine that the incentive to develop cash-saving innovations was considerable at this time. Over the past ten years or so, expectations of inflation and nominal interest rates have moderated significantly. So it is likely that the incentives to invest in cash-saving technology have lessened. However, the capital stock is still in place: ATMs and electronic funds transfer technologies decay only slowly over time. There are no comparable disincentives to encourage the removal of this new technology. So a gradual upward shift in velocity, which might now have come to an end, but is unlikely to be reversed, seems intuitively plausible. In this section, we review a number of direct measures of financial innovation.

Payment of wages in cash

One noticeable change has been a gradual movement away from payment of wages in cash. Chart 3 shows that, by 2003, only 7% of employees were paid in cash, down from more than 50% in the late 1970s. The high rates of interest during the late 1970s and the early 1980s would have made it very costly for firms to stockpile sufficient cash once a month, or in some cases once a week, to pay all of their workers. This, together with the fact that these high rates of interest were expected to persist, probably caused firms to seek other means of paying wages, such as through cheques or direct money transfer. Moreover, the statutory right of manual workers to demand payment in cash, enshrined in the Truck Acts, was removed in 1986. With so few employees now paid in cash, the scope for further upward shifts in narrow money velocity through an extension of payment by cheque, or by direct money transfer, is limited.

Chart 3
Percentage of employees paid in cash(a)



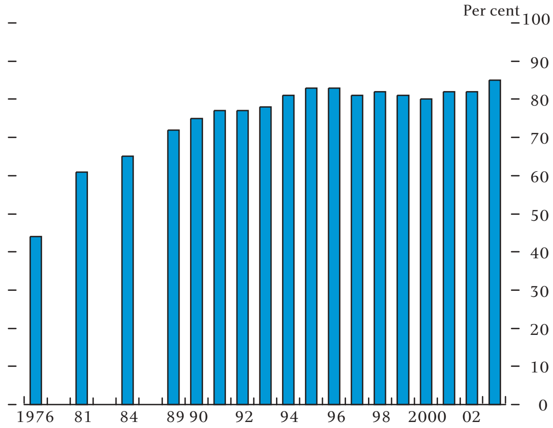
Source: APACS.

(a) Data for years not shown are not available.

More widespread access to bank and building society current accounts

With fewer employees receiving wages in the form of cash then, almost by necessity, a greater proportion of adults now has access to bank and building society accounts. Chart 4 shows that the proportion of the UK adult population holding a current account at a bank or building society rose from below 50% in the mid-1970s to about 80% by the mid-1990s. It has remained broadly stable since that time. Individuals with current accounts have access to a range of non-cash means of payment, such as cheques, direct debits and standing orders. Consequently, they are no longer obliged to withdraw cash in order to purchase goods and services, or to settle debts more generally.

Chart 4
Percentage of adults with a current account(a)



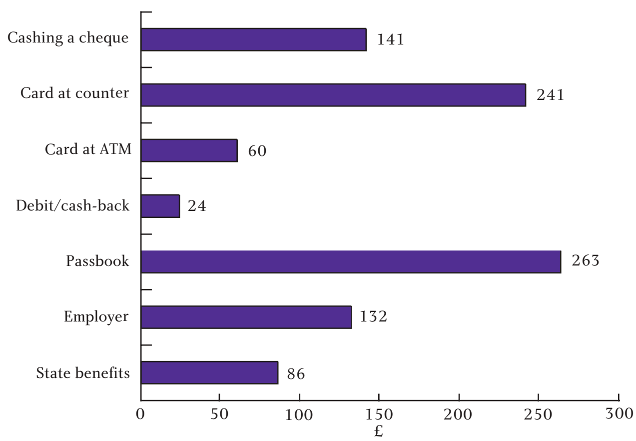
Source: APACS.

(a) Data for years not shown are not available.

Ability to make quick and easy cash withdrawals

Automated teller machines (ATMs) provide a convenient alternative to entering a bank or a building society branch in order to obtain cash 'across the counter'. Chart 5 shows that individual cash acquisitions from ATMs, or from cash-back transactions, are on average much smaller in value than any other method of obtaining cash. This is not surprising. These technologies allow individuals to obtain cash at little or

Chart 5
Average value of cash acquired on each occasion, by different methods



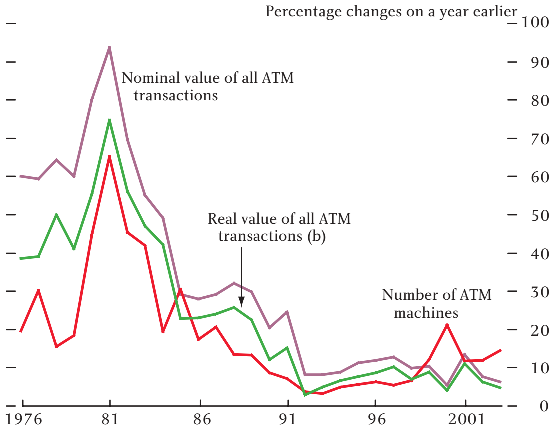
Source: APACS.

no cost, from a wide number of locations at any time of day or night. It is the fact that cash is available quickly and easily in this way that encourages individuals to make more frequent, smaller withdrawals, and consequently to hold smaller cash balances on average.

The first cash dispenser was introduced in 1967, and since that time the ATM network has grown rapidly. Machines are no longer situated only at banks and building societies, but are dispersed throughout the country: at supermarkets, convenience stores, petrol stations, and bars. These latter innovations—known as remote and independently operated ATMs—have kept the ATM network as a whole growing.

Chart 6 shows that the rate of growth of both the number of ATMs and the value of withdrawals made from them has slowed considerably from a peak in the early 1980s. The rate of growth of the value of cash withdrawals made from ATMs is now about the same as the rate of growth of M0 itself. And in each of the past two years, the average amount withdrawn from each ATM has fallen slightly, as the number of ATMs has continued to rise. It appears that the placing of new machines at more convenient locations is, to a degree, taking custom away from the existing network, rather than encouraging greater overall use.

Chart 6
Growth of the ATM network(a)



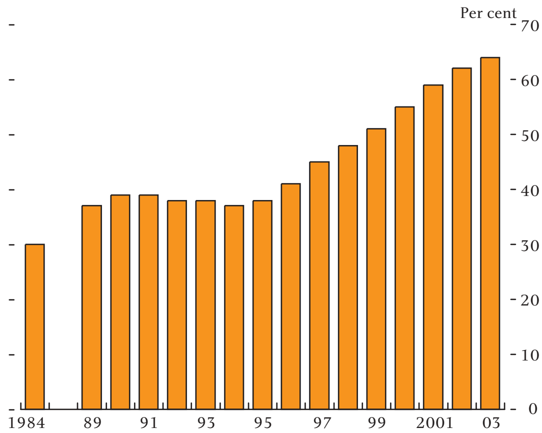
Source: APACS.

(a) Independently operated ATMs were first included in 2000, inflating the annual growth rate for that year.
(b) The real value of ATM transactions was constructed by deflating nominal values by the consumption deflator.

Introduction of alternative means of payment

Cheques have been in existence for many years. But plastic cards, and in particular debit cards, are a much more recent alternative to cash. Increased ownership and usage of plastic cards has been one of the most noticeable trends in payment systems over recent years. The proportions of adults holding credit or charge cards and of adults holding debit cards have both risen particularly strongly (see Charts 7 and 8).

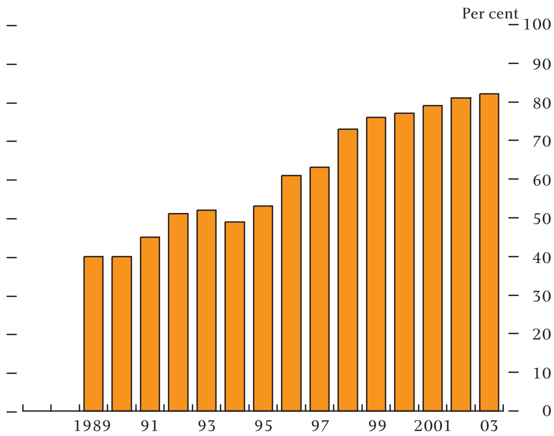
Chart 7
Percentage of adults holding a credit or charge card(a)



Source: APACS.

(a) Data for years not shown are not available.

Chart 8
Percentage of adults holding a debit card(a)



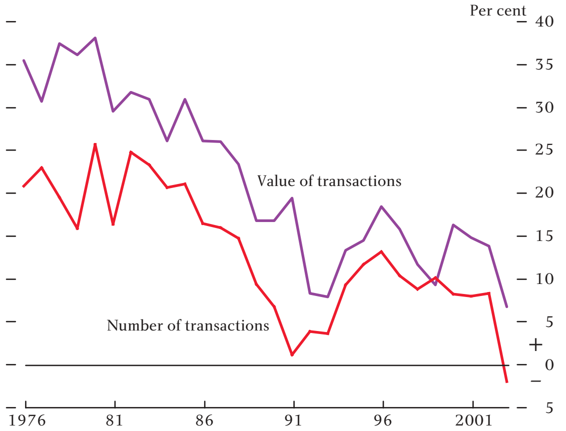
Source: APACS.

(a) Data for years not shown are not available.

Data on the number of cards in circulation are not necessarily informative about card usage, which is what matters if we are interested in factors affecting narrow money velocity. A large number of credit card accounts are inactive (as many as 35% of Visa and MasterCard accounts are inactive, according to data from the British Bankers' Association). Chart 9 shows that, while expenditure on credit cards is still growing strongly, growth rates are well below the peaks seen in the 1970s and the 1980s.

The fact that the value of credit card transactions is growing faster than consumption tells us that the

Chart 9
Growth of credit card transactions(a)

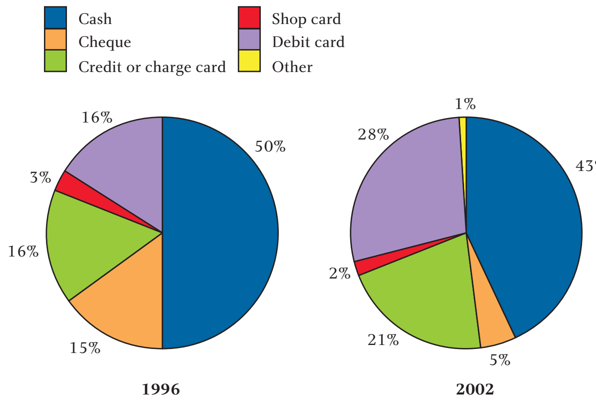


Sources: British Bankers' Association and Committee of London and Scottish Bankers.

(a) There is a break in both series in 1991 (prior to this, building society and travel/entertainment cards were excluded).

proportion of transactions completed by credit card must be rising. This is confirmed by Chart 10, which focuses on transactions above £1 in value at retail outlets. Between 1996 and 2002, the share of payments made by any form of plastic card rose by 16 percentage points, from 35% to 51%. But this was not entirely at the expense of payments by cash. The share of payments made by cheque fell by 10 percentage points, from 15% to just 5%, while the share of payments made by cash fell by 7 percentage points, from 50% to 43%.

Chart 10
Method used to make payments above £1 in value at retail outlets



Source: APACS Consumer Payments Survey.

In summary, it is widely accepted that a number of cash-saving innovations permitted a substantial rise in narrow money velocity, over and above that which would have been expected given fluctuations in the opportunity cost of holding cash. These innovations are thought to have been particularly prevalent during the 1980s. The main aim of this section has been to consider whether, for the time being at least, the increase in velocity has come to an end. We started by arguing that there were good theoretical reasons for believing that it had. The substantial rise in expected future nominal rates of interest during the 1970s, brought about by an unprecedented pickup in inflation, created a big incentive to invest in cash-saving technology. Although expected future nominal rates of interest, as measured by yields on long-dated government bonds, have fallen back sharply over the past ten years or so, the relevant capital, such as the ATM network, remains in place and will not decay rapidly.

We examined evidence on four different manifestations of financial innovation. Three out of the four were supportive of the hypothesis that the period of innovation was approaching an end. In particular, the proportion of employees paid in cash is now so low that it cannot fall much further. The proportion of adults with access to a current account has been broadly constant since the mid-1990s. The number of ATMs continues to rise. Nevertheless, the value of ATM withdrawals is now growing in line with M0, and the average value of withdrawals per ATM actually fell in 2002 and in 2003. Conversely, the proportion of transactions completed by credit and debit cards is still rising. This appears to be partly at the expense of cash, but largely at the expense of cheques.

In this section, we have necessarily focused on those types of financial innovation for which data are readily available. Other changes in payments technology might have affected narrow money velocity in the past, and may continue to do so in the future. A recent article in this *Bulletin* by Allen (2003) discussed so-called 'e-payments'—the settlement of debt using mobile phone or internet technology. And further processes may be at work, other than financial innovation, that will have a bearing on velocity. These include changes in the size of the hidden economy, which by its nature is cash intensive.

Modelling the demand for narrow money

In an environment where the prices of most goods and services are rising, and the returns to holding financial assets are on average positive, cash is a dominated asset, in the sense that it would appear preferable always to hold wealth in the form of some alternative asset, such as a savings account that pays a positive rate of interest. Why then do people in practice choose to hold non interest-bearing forms of money, such as M0? Over time, researchers have proposed a variety of solutions to this puzzle. In this article, we focus on just one solution, put forward by Sidrauski (1967). He argued that cash balances provide a flow of services, essentially by facilitating economic transactions. Individuals are able to obtain economic benefits simply by holding cash for a period of time. Models that adopt this approach are known as 'money in the utility function' (MIU) models.

In a technical appendix to this article, we describe an MIU model in some detail. We show that, when an individual maximises his or her welfare by choosing each period how much to consume, how much to hold in the form of an interest-bearing asset, such as a bank deposit, and how much to hold in the form of non interest-bearing narrow money, the demand for narrow money takes the following form:

$$\frac{m_t}{c_t} = \left(\frac{\delta}{1-\delta}\right)\left[\frac{1+R_t}{R_t}\right]^{\sigma} \quad \textbf{(1)}$$

where c_t is real consumption expenditure at time t, m_t is real money balances at time t and R_t is the nominal interest rate payable at time t on the interest-bearing asset. δ and σ are parameters of the model.(1) Equation **(1)** says that the ratio of real money balances to real consumption, which is the same as the ratio of nominal money balances to nominal consumption or the inverse of velocity, depends on R_t. Of particular interest to policymakers is the interest elasticity of money demand (or ε_t). This measures the percentage change in money demand that would occur following a given percentage change in the nominal rate of interest. After differentiating **(1)** with respect to R_t, multiplying by the ratio of R_t to m_t/c_t and rearranging, the following expression for ε_t can be obtained:

$$\varepsilon_t = -\sigma\left(\frac{1}{1+R_t}\right) \quad \textbf{(2)}$$

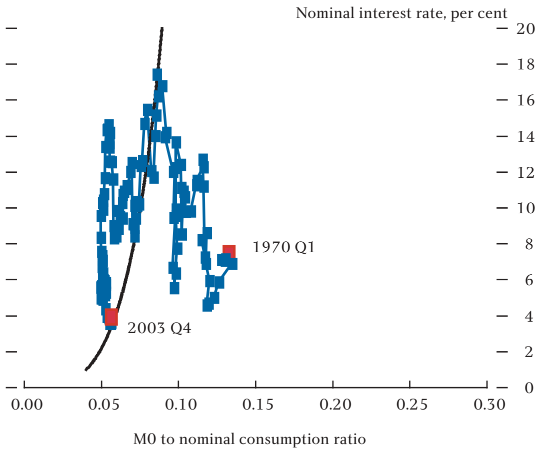
We begin by estimating equation **(1)** over the period 1970 Q1 to 2002 Q4.(2) The results are shown in the first row of Table A. The central estimate of σ for the period 1970 Q1 to 2002 Q4 is negative. Using equation **(2)**, it is clear that the interest elasticity of money demand would be positive if σ were negative. In other words, the demand for real money balances would

Table A
Parameter estimates of the narrow money demand equation over three different sample periods

	σ		δ	
Sample period	Central estimate	95% confidence interval (a)	Central estimate	95% confidence interval (a)
1970 Q1–2002 Q4	-0.283	(-0.593, -0.028)	0.129	(0.064, 0.242)
1870–1980	0.198	(0.118, 0.277)	0.067	(0.051, 0.086)
1992 Q4–2002 Q4	0.178	(0.150, 0.207)	0.030	(0.027, 0.032)

(a) We interpret **(1)** as a description of long-run equilibrium, rather than a condition that should hold period by period. This means that we should expect to find serially correlated error terms, which indeed we do. Consequently the standard errors have been corrected using the procedure of Newey and West (1987).

Chart 11
M0 to nominal consumption ratio since 1970 Q1



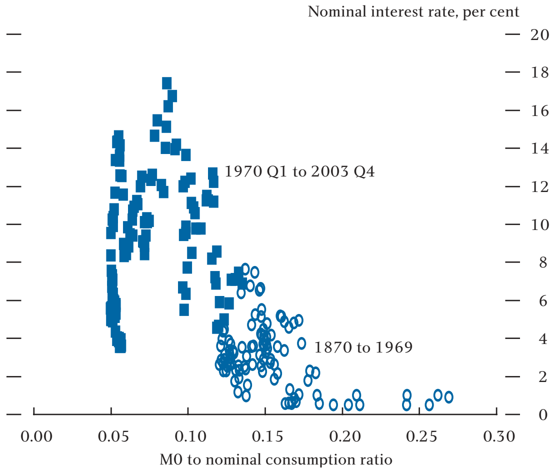
Sources: Bank of England and ONS.

rise following an increase in the rate of interest. This makes little economic sense. Chart 11 plots those combinations of the nominal interest rate and the M0 to nominal consumption ratio that have occurred since 1970 Q1, together with the line of best fit implied by the parameter estimates in the first row of Table A. This confirms that, since 1970 Q1, the demand for money curve has had a perverse upward slope. But it also shows that any relationship between nominal interest rates and the M0 to nominal consumption ratio during this period has been very weak. The line of best fit may be upward sloping, but many of the points lie some distance from it. Starting from 1970 Q1, the points in Chart 11 have a general tendency to drift to the left. In other words, the M0 to nominal consumption ratio has generally been falling, and velocity has generally been rising, irrespective of fluctuations in the rate of interest. The results in the first row of Table A merely provide statistical confirmation of what we should already have come to suspect. The gradual introduction of cash-saving financial innovations, perhaps starting in the late 1970s, but then continuing through the 1980s, means that conventional money demand equations, when fitted to data from this period, do not work well.

(1) More details are provided in the technical appendix.
(2) Although equation **(1)** is non-linear in $(1+R_t)/R_t$, it becomes linear after taking natural logarithms. Since R_t and m_t/c_t are both endogenous, we then apply the dynamic ordinary least squares technique of Stock and Watson (1993).

Chart 12 reproduces, as blue squares, the quarterly data from 1970 Q1 to 2003 Q4 shown in Chart 11. But, in addition, it includes annual data covering the period from 1870 to 1969, represented by blue circles. Arguably, these data cast a somewhat different light on the experience of the past three decades. Many of the points in Chart 12, including all of the blue circles, and a large number of the blue squares that relate to the 1970s, appear to fit quite neatly on the kind of downward-sloping money demand curve predicted by equation **(1)**. There is undoubtedly a further cluster of blue squares that do not fit on this, or indeed any, downward-sloping money demand curve. But there is possibly a third set of blue squares, lying in the bottom left-hand corner of Chart 12, and relating to the most recent past, that lie on a second downward-sloping money demand curve, positioned further to the left than the first.

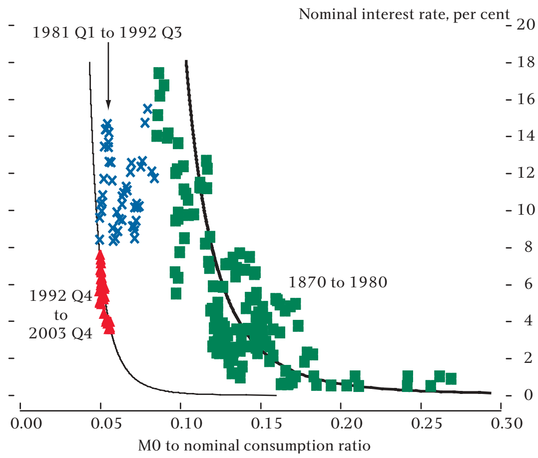
Chart 12
M0 to nominal consumption ratio since 1870



Sources: Bank of England; Mitchell, B R (1988), *British historical statistics*, Cambridge University Press; and ONS.

This scenario, of a prolonged leftward shift in the demand for money curve, would fit the hypothesis advanced earlier, that the introduction of cash-saving payments technology, most likely brought about by the high inflation and high nominal interest rate era of the 1970s, caused a prolonged upward shift in velocity. This shift took place over a number of years, and might now have come to an end. In order to investigate this hypothesis more formally, and because we wish to remain agnostic about the precise timing of the velocity shift, we adopted the following approach. First, we estimated equation **(1)** using annual data from 1870. The sample was gradually extended forwards in time until the estimates for σ and δ became unstable. This seemed to occur after 1980. Second, we estimated equation **(1)** using quarterly data up until 2002 Q4. The sample was gradually extended backwards in time until the estimates of σ and δ became unstable. This seemed to occur before 1992 Q4. In this way we identified: an early period, running from 1870 to 1980, represented by green squares in Chart 13; a middle period running from 1981 Q1 to 1992 Q3, represented by blue crosses in Chart 13; and a late period running from 1992 Q4 to 2003 Q4, represented by red triangles in Chart 13. The parameter estimates we obtained for the early and the late periods are shown in the second and third rows of Table A. The associated lines of best fit are plotted in Chart 13.

Chart 13
M0 to nominal consumption ratio since 1870



Sources: Bank of England; Mitchell, B R (1988), *British historical statistics*, Cambridge University Press; and ONS.

The central estimate of σ is positive, for both the early and the late periods, giving a more conventional downward slope to the money demand curves. Moreover, the two central estimates of σ are reasonably close, at 0.198 for the early period and 0.178 for the late period. The 95% confidence intervals around the central estimates of σ for the early and late periods overlap, suggesting that one might not reject the null hypothesis that they are in fact the same.(1) It would appear that, during the period from 1992 Q4 to 2002 Q4, the demand for narrow money responded to changes in the rate of interest in more or less the same way as it had done during the period from 1870 to 1980. Conversely, the two central estimates of δ, at 0.067 for the early period and 0.030 for the late period, look rather different. The 95% confidence intervals around these

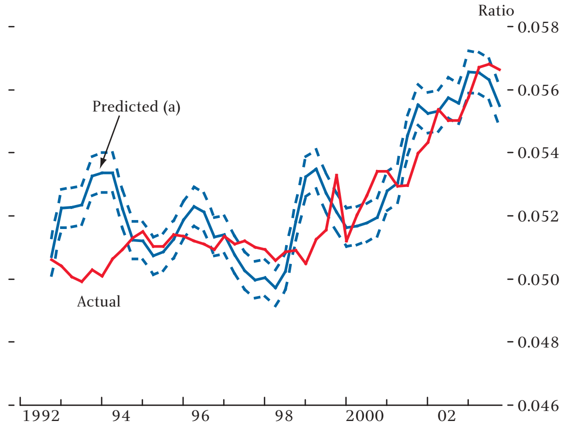
(1) This is a rather informal line of reasoning. Under the null hypothesis, the estimates of σ taken from the quarterly and the annual data sets would come from different distributions, and so would not be directly comparable.

central estimates do not overlap. One interpretation of the change in the central estimate of the δ parameter is that an individual would now require less than one half the quantity of narrow money balances to fund a given amount of consumption expenditure than had been necessary before the velocity shift.

It is relevant that the non-linear money demand equation obtained using the MIU model described in the technical appendix appears to cope well with the period of very low interest rates around the time of World War II. The five green squares in the bottom right-hand corner of Chart 13 cover the period from 1943 to 1947, when the eligible bill rate dropped below 1%. A feature of this specification is that the demand for money becomes infinite as the rate of interest approaches 0%. An alternative specification, which has often been estimated by researchers in the past, involves regressing the logarithm of the M0 to nominal consumption ratio on the level, rather than the logarithm, of the nominal interest rate. Known as the semi-logarithmic specification, this approach is associated with the work of Cagan (1956). Under the semi-logarithmic specification, the M0 to nominal consumption ratio has an upper bound and the line of best fit is forced to cut the horizontal axis. A more detailed consideration of alternative money demand equations is contained in Chadha, Haldane and Janssen (1998).

Can our econometric work shed any light on the recent behaviour of M0? In particular, can it account for the fact that, since the beginning of 1998, the M0 to nominal consumption ratio has risen, with M0 growing at an average annual rate of 7.1% and nominal consumption growing more slowly, at an average annual rate of 5.3%? Chart 14 compares the M0 to nominal consumption ratio predicted by the equation for the late period shown in the final row of Table A with the actual outturns. Owing to force of habit, individuals are likely to adjust their money balances only slowly over time. But our model makes no allowance for this. Instead, it assumes that individuals adjust their money balances immediately to the quantity implied by the prevailing level of nominal expenditure, and the prevailing nominal interest rate. Despite this potential shortcoming, the actual and predicted series shown in Chart 14 track each other quite closely. Between 1998 Q2, when the Bank of England repo rate reached a peak, and 2003 Q4, the discount rate on eligible bills fell from 7.5% to 3.9%. According to our equation, that implies that the ratio of

Chart 14
Actual and predicted values of m_t/c_t since 1992 Q4



Sources: Bank of England calculations and ONS.

(a) The predicted values for m_t/c_t should be regarded as estimates of the long-run equilibrium M0 to nominal consumption ratio. The dashed lines represent 95% confidence intervals around those estimates of long-run equilibrium.

M0 to nominal consumption ought to have risen by a little over 10%. As Chart 14 shows, this is more or less what happened. It seems probable that the approximate halving of nominal interest rates during that five and a half year period accounted for much of the pickup in the money to consumption ratio, or much of the slowdown in velocity.

Chart 14 uses data up until 2003 Q4. At that point in time, the M0 to nominal consumption ratio was a little above its estimated long-run equilibrium. Moreover, since the end of last year, short-term interest rates have tended to rise. Taken together, these two observations imply that the rate of growth of M0 could moderate during the second half of this year, without necessarily implying a slowdown in the rate of growth of nominal demand.

Summary

This article has used time-series data stretching back more than 100 years in an attempt to shed some light on the recent behaviour of M0. Modelling the demand for M0 in the United Kingdom is problematic, owing to a steady upward drift in velocity during the 1970s and the 1980s that cannot be explained by changes in the opportunity cost of holding cash, as measured by the short-term rate of interest. Rather than address this problem directly, we have avoided it. Casual inspection of the data suggested that, for much of the period since 1870, there appeared to have been a stable relationship between the M0 to nominal consumption ratio and the short-term rate of interest. It was clear that this relationship broke down towards the end of the 20th century, although it may have reasserted itself in recent years. After some experimentation, we divided our data set into three distinct time periods. During the middle period, which ran from 1981 Q1 to 1992 Q3, our money demand equation did not fit the data. By contrast, during the early and the late periods, our money demand equation fitted the data reasonably well. Moreover, although the quantity of M0 used to finance a given amount of nominal consumption had approximately halved between the early and the late periods, the response to changes in the short-term rate of interest rate was little changed. For the past ten years or so, the M0 to nominal consumption ratio appears to have been a reasonably stable function of the short-term rate of interest. We have asserted that this might plausibly be because the rate at which cash-saving financial innovations are being introduced has slowed somewhat, and we have provided some direct evidence on payment technologies to support this assertion.

It is always easier to identify stable money demand functions with the benefit of hindsight. One can imagine many different scenarios, such as an increase in the use of plastic cards for small purchases, perhaps following the implementation of 'chip and PIN' technology, or a change in the size of the hidden economy, that would cause another shift in narrow money velocity. The challenge for monetary policy makers, when next faced with a set of narrow money data that appear inconsistent with projections for nominal expenditure, will be, as always, to determine whether the surprising data contain genuine news about economic activity, or whether they are merely a sign that the stable money demand function has broken down once again.

Technical appendix

This technical appendix shows how the money demand equation on page 136 of the main text can be derived from the optimising behaviour of a representative individual.

Let the representative individual solve the following problem:

$$\max E_t \sum_{j=0}^{\infty} \beta^j u\left(c_{t+j}, m_{t+j}\right) \quad \textbf{(A1)}$$

subject to:

$$M_t + D_t = Y_t - p_t c_t + M_{t-1} + (1 + R_{t-1})D_{t-1} \quad \textbf{(A2)}$$

In **(A1)** β is the subjective discount factor measuring the individual's impatience to consume, c_t is real consumption expenditure at time t and m_t is real money balances at time t. $u(c_t,m_t)$ expresses the individual's utility as a function of c_t and m_t. The fact that m_t is one of the arguments makes this a 'money in the utility function' (MIU) model. In **(A2)** M_t and D_t are nominal money balances and nominal bank deposits respectively at time t. Y_t is nominal labour income at time t and p_t is the price level at time t, so p_tc_t is nominal consumption expenditure at time t. R_t is the nominal interest rate on bank deposits at time t. **(A2)** is the nominal budget constraint. It says that the change in the stock of assets between time $t-1$ and time t is equal to savings during period t, where savings are defined as labour and interest income net of consumption expenditure.

The first stage in solving this problem is to rewrite **(A2)** in real terms. Dividing **(A2)** through by p_t we obtain:

$$m_t + d_t = y_t - c_t + m_{t-1}\left(\frac{p_{t-1}}{p_t}\right) + (1+R_{t-1})d_{t-1}\left(\frac{p_{t-1}}{p_t}\right) \quad \textbf{(A3)}$$

where lower-case letters are used to denote real variables. The Lagrangean for this problem can then be written as:

$$\begin{aligned} l = {} & E_t \sum_{j=0}^{\infty} \beta^j u(c_{t+j}, m_{t+j}) \\ & - E_t \sum_{j=0}^{\infty} \beta^j \lambda_{t+j}\left[m_{t+j} + d_{t+j} - y_{t+j} + c_{t+j} - m_{t+j-1}\left(\frac{p_{t+j-1}}{p_{t+j}}\right) - \left(1+R_{t+j-1}\right)d_{t+j-1}\left(\frac{p_{t+j-1}}{p_{t+j}}\right)\right] \end{aligned} \quad \textbf{(A4)}$$

Differentiating **(A4)** with respect to c_t, m_t and then d_t, gives three first-order conditions:

$$u_{c,t} = \lambda_t \quad \textbf{(A5)}$$

$$u_{m,t} = \lambda_t - \beta E_t \lambda_{t+1}\left(\frac{p_t}{p_{t+1}}\right) \quad \textbf{(A6)}$$

$$\lambda_t = \beta E_t\left[\lambda_{t+1}(1+R_t)\left(\frac{p_t}{p_{t+1}}\right)\right] \quad \textbf{(A7)}$$

After combining **(A5)**, **(A6)** and **(A7)**, the following expression for the ratio of the marginal utilities can be obtained:

$$\begin{aligned} \frac{u_{c,t}}{u_{m,t}} &= \frac{\lambda_t}{\lambda_t - \lambda_t\left(\dfrac{1}{1+R_t}\right)} \\ &= \frac{1+R_t}{R_t} \end{aligned} \quad \textbf{(A8)}$$

We assume a constant elasticity of substitution (CES) utility function of the following form:

$$u(c_t, m_t) = \left[(1-\delta)^{\rho+1} c_t^{-\rho} + \delta^{\rho+1} m_t^{-\rho} \right]^{\frac{-1}{\rho}} \quad \textbf{(A9)}$$

From **(A9)**, the marginal utilities with respect to c_t and m_t are given by:

$$u_{c,t} = \frac{-1}{\rho} \left[(1-\delta)^{\rho+1} c_t^{-\rho} + \delta^{\rho+1} m_t^{-\rho} \right]^{\frac{-1-\rho}{\rho}} (1-\delta)^{\rho+1} (-\rho) c_t^{-\rho-1} \quad \textbf{(A10)}$$

$$u_{m,t} = \frac{-1}{\rho} \left[(1-\delta)^{\rho+1} c_t^{-\rho} + \delta^{\rho+1} m_t^{-\rho} \right]^{\frac{-1-\rho}{\rho}} \delta^{\rho+1} (-\rho) m_t^{-\rho-1} \quad \textbf{(A11)}$$

Finally, the money demand equation is obtained by using **(A10)** and **(A11)** in **(A8)**:

$$\frac{m_t}{c_t} = \left(\frac{\delta}{1-\delta} \right) \left(\frac{1+R_t}{R_t} \right)^{\sigma} \quad \textbf{(A12)}$$

where $\sigma = (1/(1+\rho))$ is the constant elasticity of substitution between real consumption expenditure and real money balances. Equation **(A12)** is equation **(1)** in the main text.

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