The impact of government spending on demand pressure

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When assessing the outlook for inflation, the growth of real GDP is commonly used as an indicator of changes in current demand pressures. But as GDP includes the output of the government sector, this approach can in some circumstances be misleading. Government output is not necessarily an informative guide to the impact of government spending on the balance of demand and supply pressures in the marketed sector of the economy. Instead, it may be more informative to consider the quantity of resources that the government absorbs — that is, how much private sector output it buys and how much labour it hires — rather than the quantity of output it produces.

Introduction

In the National Accounts, gross domestic product (GDP) includes the output of both the private sector and the government. When assessing the outlook for inflation, the growth of real GDP is commonly used as an indicator of changes in current demand pressures. But this can give a misleading impression of the impact of the government sector.

There are well-documented difficulties of accurately measuring the government output component of real GDP, which was the subject of a recent review led by Sir Tony Atkinson (2005). But even if government output were measured perfectly, GDP would not necessarily provide a useful guide to inflationary pressures in the economy. That is because, as this article explains, the government’s demand for inputs (consisting of private sector goods and services and labour) tends to be a better indicator of the impact of the public sector on inflation than the outputs that the government produces (eg health and education services). So a different measure of aggregate activity may be a more informative indicator of inflationary pressures than GDP.(1)

The structure of the article is as follows. First, we discuss how the government’s demand for private sector resources can have an impact on inflationary pressure. Within this, we briefly consider the measurement of government output and productivity in the National Accounts, and explore the channels by which government output might in theory influence the inflation outlook. Second, we consider the implications for macroeconomic analysis. We look at what this means for the role of government output and GDP in macroeconomic models. We then look at alternative methods of constructing measures of aggregate activity, which may be more informative about the outlook for inflation than those based on GDP.

The role of government spending in the determination of inflation

The prices that make up the consumer prices index (CPI) basket and other price indices are almost entirely set by private sector firms. So the prospects for CPI inflation are likely to depend on the balance of demand and supply pressures exerted within the marketed sector of the economy. The output of this sector is produced almost entirely by private sector employees, working for private sector firms, largely using private sector capital. In the rest of this article we will refer to this concept loosely as ‘private sector’ output. But perhaps a more precise description would be ‘marketed’ or ‘business sector’ output.(2)

There are two main channels by which government behaviour has an effect on activity and prices in the marketed sector. First, the government purchases output directly from the private sector (typically called ‘procurement’). Second, the government’s demand for labour competes with the demand for labour from

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(1) For an earlier discussion of this issue, see Bank of England (2004).
(2) For example, some elements of the private sector, such as charities, produce non-marketed services. Similarly, in practice, the government sector does produce some output that can be interpreted as ‘marketed’ (eg the provision of sports facilities or passports, on which revenue is earned).
private sector firms, and so will affect wage costs in the private sector.\(^{(1)}\)

So it is the government's demand for inputs (eg purchases of schoolbooks and employment of teachers) that is of primary relevance for CPI inflation, rather than the output those inputs are used to produce (eg education services). The government's demand for inputs adds directly to the demand for private sector output, and absorbs scarce resources (in the form of labour inputs) that could have been used to produce private sector output. In doing so it affects the balance of demand and supply of marketed output in the economy as a whole. By contrast, under assumptions that we discuss below, it is not clear that how much output the government actually produces, given its purchases of inputs (ie how productive the government is), is directly relevant for private sector activity or firms' pricing decisions in the short to medium term.

This raises the issue of whether GDP is the most informative guide to demand pressures in the economy. GDP includes the government's output. In the current System of National Accounts,\(^{(2)}\) the government sector is assumed to consume all of the non-marketed output it produces (which is typically provided free at the point of delivery to citizens). This consumption is treated as a component of final demand in GDP — called 'General Government final consumption'. So, in principle at least, the volume of real government consumption is estimated by measuring the volume of government outputs provided free at the point of delivery to the private sector, rather than by the real inputs the government uses.

As a result, GDP may not always be the best indicator of demand pressures in the marketed sector of the economy. This does not imply that including government output in ONS-measured GDP is in any sense 'wrong' — simply that it is aiming to capture a different concept. As discussed in detail in Atkinson (2005), several purposes are served by measures of national income, and no single number can serve all of these purposes.\(^{(3)}\) In particular, GDP is often used as a broad measure of welfare, and it is clear that the contribution of the government's output should be reflected in such a measure. In a later section, we discuss an alternative indicator of aggregate activity that better reflects the two components of the government's demand for resources than does GDP.

So the issue here is not one of measurement: even if government output were perfectly measured, it may not necessarily provide an appropriate guide to the impact of government spending on demand pressures in the economy. It is worth noting in passing, though, that measuring the volume of government output is not a straightforward task; the box on page 142 discusses some of the recent developments in this area.

To make clear the distinction between the role of government inputs and outputs in determining demand pressures, and why movements in GDP can be a misleading guide, it may be helpful to look at an example. Take a situation in which the National Health Service becomes more efficient, such that more operations are performed, to the same quality, for a given number of doctors and nurses employed, and surgical instruments and other equipment bought from the private sector. The ONS would (rightly) measure this as increased real government consumption and GDP. This might, in the first instance, appear to be an increase in the volume of resources being consumed in the economy. But we know there is no extra demand pressure on the marketed sector of the economy, because the health service is not acquiring any more private sector goods and services, or hiring any more labour to produce this extra output.

Contrast this with the case where the increase in operations is largely the result of more procurement from the private sector (eg an increase in hospital equipment). Again GDP would increase, reflecting the increase in private sector output. But in this case the impact of the rise in procurement spending is likely to be more similar to that of an increase in private consumption expenditure. The resulting increase in private sector output is likely to require an increase in factor inputs hired by the private sector and, depending on the monetary policy response, may lead to higher wage costs and prices.

There are, of course, some channels through which changes in government output and productivity might potentially have an impact on the balance between demand and supply in the marketed sector. We consider

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\(^{(1)}\) This assumes that the two labour markets are not completely segmented, so that some workers could in principle work in either sector.


\(^{(3)}\) Hicks (1940) makes the same point: 'There may be more than one money value of the social income, each corresponding to a different purpose of calculation'. Of the many papers since that have discussed the different purposes to which National Accounts data can be used, see in particular: Okun (1971), Nordhaus and Tobin (1972), Weitzman (1976), Mamalakis (1996), and Asheim and Weitzman (2001).
Measuring government output in the United Kingdom: recent developments

In the United Kingdom, the ONS — which is at the forefront of international best practice in this area — currently measures about two thirds of government output by directly estimating the change in the volume of services provided. For instance, health output is partly measured by counting the number of patient consultations in the NHS, and education output partly by counting the number of pupils taught. An associated implied price can then be imputed by dividing nominal spending by this estimated volume of output.\(^{(1)}\)

The outputs of the government sector are particularly difficult to measure accurately. The Atkinson Review was set up to consider these problems, and produced its final report in January 2005 (Atkinson (2005)). A particular problem is allowing adequately for quality improvements. For instance, if the NHS spends money improving the clinical effect of an operation, the measured output (one operation) would not change. The Department of Health has in fact commissioned research from the University of York and the National Institute for Economic and Social Research into developing measures of clinical outcomes. Under the current system, though, to the extent that resources are put into improving the quality as well as the quantity of public services, we might expect that the level and growth of measured government outputs are biased downwards.

The remaining one third of real government consumption is assumed to grow at the same rate as the real value of the inputs used to produce it. This is used mainly for services such as defence, which are consumed collectively, and for which the benefits are particularly difficult to quantify.

The problem of measuring the government’s output may have been particularly important in recent years. An indicator of this is that since 1997 Q1, nominal government consumption — current spending on labour and procurement for services such as education, health, defence, law and order and local government, which comprises about a fifth of GDP — has risen by 71%. Over the same period, the ONS measure of real government consumption has risen by just 23%. So the implied price deflator has risen by 39%; by contrast, the CPI has risen by just 11% over that period, and the GDP deflator (which includes the government consumption deflator) has risen by only 20% (see Charts A and B).

In fact, this difference is now somewhat less acute than it was prior to the revisions in the Blue Book 2004. The ONS has been developing more sophisticated measures of government output, as part of the Atkinson Review. One result of this work was that a new measure of health output was introduced in the June 2004 Blue Book. The new measure (see Pritchard (2004)) includes services that were excluded from the old one, such as NHS Direct and Walk-in Centres. It also involves looking at output data in more detail — the new measure distinguishes between 1,700 different activity types, compared with 16 types covered in the old one. The new measure raised the growth rate of health output in most years since 1996, and added about 0.5 percentage points a year to the growth of real government consumption since 1999, but did not affect nominal consumption. So the difference between real and nominal government consumption growth is now considerably less marked. But real government consumption is still not fully adjusted for quality, so it would be reasonable to conclude that its growth is still somewhat understated. The ONS continues to develop new measures to address this issue.

\(^{(1)}\) See Pritchard (2003) for detailed explanation of current ONS practice.
two of these channels below, and an appendix provides a technical discussion. The issue is whether these effects are likely to be sufficiently large to have a material impact on demand and supply within the usual timescale for monetary policy making of two to three years.

(i) Effect of public consumption on private consumption

If government outputs affect private consumption decisions, then they could matter for overall demand pressure. An obvious example, at a disaggregated level of spending, is that households might respond to increased government provision of healthcare by reducing their own spending on private healthcare.\(^1\) But, the key issue for analysing inflationary pressures is whether an increase in government output has an effect on aggregate private consumption and saving behaviour. For instance, even if households responded to the government’s increased provision of public healthcare by decreasing their consumption of private health services, they may increase their consumption of other consumer goods, with little impact on aggregate private consumer spending.

Whether government output and private consumption are complements or substitutes will determine the response of private consumption to an increase in government output. And whether the increase is permanent or temporary will also matter. Naturally the strength of any effect is an empirical question. Unfortunately, the evidence is somewhat inconclusive, but the effects found so far are relatively small.\(^2\) So it seems a reasonable central assumption that this effect is not material, at least over the 2–3 year period relevant for monetary policy.

(ii) Effect of public capital or output on private productivity

Government capital often raises private sector productivity.\(^3\) If the government did not provide transport infrastructure, for instance, the returns to private capital would decrease enormously. The same also applies to certain other kinds of government output — for instance, education increases human capital and the provision of health services leads to healthier, more productive workers.\(^4\)

The issue from the point of view of monetary policy, though, is quite a narrow one: do movements in public capital materially affect potential private supply within a 2–3 year forecast horizon? It is reasonable to conclude that they do not over this time period, although they may well do so over a longer period.

One key reason for this is that the proportionate change in public capital is likely to be relatively small over a 2–3 year horizon. Even a large increase in government investment (say 10% per annum) would tend to have a relatively small impact (of around 1%) on the public sector capital stock over a 2–3 year period. So even if public sector capital were a perfect substitute for private capital, the implied proportionate increase in the total capital stock available to the private sector would be small (around 0.25%).

The basic insight from the discussion above is that inflationary pressures depend on the government’s demand for inputs of private sector output and labour, rather than its output of services. In the rest of the article, we describe how this insight can inform the ways in which we should analyse the economy, and how we can construct more useful indicators of aggregate demand pressure than those based on GDP.

Implications for macroeconomic analysis

Most standard macroeconomic models are based on profit-maximising behaviour by private sector firms, which are assumed to sell their goods and services in ( imperfectly) competitive markets at market-determined prices. As noted earlier, this suggests that the measure of demand relevant to firms is a measure of the volume of expenditure on ‘marketed’ or ‘business sector’ output rather than GDP. This implies combining both private sector and government expenditure on private sector goods and services and then removing imports from overseas to get expenditure on the ‘value added’ of the domestic private sector.\(^5\)

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1. If, as in this example, an increase in government output lowers the marginal utility of private consumption, then the two are said to be ‘substitutes’. If, instead, it increases the marginal utility of private consumption, the two are said to be ‘complements’. In either case, the two are said to be ‘non-separable’ in utility. See the Appendix for an example of a utility function that embodies this non-separability, and how this affects private sector behaviour in a simple model of the economy.
2. In particular, there is no consensus as to whether aggregate private and government consumption (or indeed their subcomponents) are substitutes or complements. And in most cases the estimate of the elasticity of substitution between the two aggregates is quite small. See for instance Aschauer (1985), Graham (1995), Karras (1994), Ni (1995), Kuehlwein (1998), Rouakes and Rebwi (2005), Cardina, Kohaya and Ruge-Murcia (2005), or Okolo (2003).
4. See the Appendix for an example of how this works in a simple model of the economy.
5. In practice this aggregate should be constructed by weighting the different components of demand together using chain-linking techniques, but we use an additive description here for simplicity of exposition. Also the aggregate should be adjusted for indirect taxes (net of subsidies) on products. Note also that government procurement includes imported goods and services.
\[ Y_p = C + I + P + X - M \]

where \( Y_p \) is expenditure on private sector output, \( C \) is private consumption spending, \( I \) is total (ie private and public) investment spending, \( P \) is real government procurement of private (consumption) goods and services, \( X \) is real spending on exports and \( M \) is real expenditure on imports.

We can compare this with the equivalent relationship for GDP:

\[ GDP = C + I + G + X - M \]

where \( G \) is real total government final consumption spending. As noted earlier, \( G \) represents the government's real consumption of its own non-market output of goods and services.

The difference between the government's final consumption spending, \( G \), and the government's procurement of private goods and services, \( P \), is implicitly the government's output in 'value added' terms \( (Y_g) \). The government's value added is essentially the quantity of government output after accounting for the contribution of inputs of private sector goods and services used in their production. So the difference between GDP and private sector output is the exclusion of the government's value added output from GDP.

\[ GDP - Y_p = G - P = Y_g \]

As noted earlier, there is an additional channel through which government spending affects private sector prices. The government’s labour demand competes with the private sector for scarce labour resources, and so affects private sector pricing behaviour via the supply side, through its impact on private sector wages and costs. For example, in a simple model of the labour market where there is a supply of labour \( L_s \), then wages must move to ensure that the aggregate demand and supply for labour are consistent:

\[ L_p + L_g = L_s \]

where \( L_g \) is the government's labour demand and \( L_p \) is the private sector's labour demand (which will depend on the level of private sector output \( Y_p \)). An increase in government employment will use resources that otherwise could be used to produce private sector goods and services, and this is likely to imply higher wage costs for private sector firms. Provided the private and public sector labour markets are not entirely segmented, and labour can move between sectors, then in general the government’s demand for labour is likely to influence private sector wages.

So macroeconomic analysis should try to capture two key channels via which government spending can affect the demand for private sector output and prices:

(i) The 'direct' effect resulting from the government's procurement of private sector goods and services.

(ii) The 'indirect' effect resulting from the government's purchases of factor inputs (largely labour) which affects private sector prices largely through the impact on private sector (wage) costs.

This can be done straightforwardly in a macroeconomic model by making private sector production the underlying concept of output on the supply side and articulating the government's role in the labour market. The box on page 145 discusses the details of how this is implemented in the Bank's quarterly forecasting model BEQM. Other macroeconomic models also articulate these channels, though to varying degrees.

More generally, we may want to analyse inflationary pressures in the economy without the use of an explicit model. The analysis above suggests the use of a broader set of activity indicators. In particular, the two important indicators identified above are: private sector output — because this directly reflects the demand pressures on the marketed sector of the economy and ultimately pins down its demand for labour — and government employment which, together with the private sector's demand for labour, affects the overall tightness of the labour market. How difficult is it to construct or obtain these indicators in practice?

Estimates of private sector output can be estimated from both the output and expenditure sides of the National Accounts. For example, on the output side of the accounts, we can aggregate together all of the industry components of GDP, except for those that can be

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(1) Note this includes government purchases of capital goods from the private sector.

(2) See Pain and Westaway (1996) and Finn (1998) for examples which make these distinctions clear.
The role of government output in the Bank of England’s Quarterly Model (BEQM)

The treatment of government demand in the Bank’s Quarterly Model (BEQM) largely follows the description in this article. The determination of inflation in BEQM is based on the production and pricing of private goods and services. So rather than using the ONS estimates of GDP, the primary concept of output in the model is a measure of private sector output, which excludes the government’s value-added. The measure of private sector output in BEQM also excludes actual and imputed rentals on dwellings as this type of output is assumed not to require (non-housing) capital and labour inputs to produce. So the relationship between GDP and BEQM’s measure of private sector output is given by:

\[ GDP = Y_p + Y_g + Y_{rents} \]

where \( Y_p \) is private sector value-added, \( Y_g \) is government value-added and \( Y_{rents} \) is the consumption of actual and imputed rentals on dwellings. On the supply side, BEQM uses a constant elasticity of substitution production function to relate private sector output to private sector inputs of labour and capital.

In the labour market, the government is assumed to compete with the private sector for scarce labour resources, and so affects private sector pricing behaviour through its impact on private sector wages. In BEQM, private sector wages are determined by a wage-bargaining process. The government affects this in two ways. First, its labour demand affects the number of unemployed workers, which influences the nature of the private sector wage bargain and hence the pressure on wages. Second, the government wage affects the options of private sector workers. So a higher government wage means private sector workers may push for higher wages, because the alternative of working in the government sector is more rewarding than it was. The government’s labour demand and wage in BEQM are determined in a relatively straightforward way. Its demand for labour depends on a simple rule governing the government’s wage bill. And there is a simple mark-up relationship between private and public sector wages, although for most shocks this mark-up is usually assumed to stay constant so that public and private sector wages are assumed to move together.

attributed to the government sector. From the expenditure side, an estimate of private sector output requires data on government procurement in nominal terms. And to construct a chained volume measure, an estimate of the deflator for government procurement is required. An important issue here for the construction of either measure is how the government and the private sector are defined. For example, should public corporations or non-profit institutions such as charities and universities be included in the definition of the government sector? The Bank of England has recently undertaken a joint project with the ONS to develop a measure of output for the marketed or business sector of the economy. So far, the project has developed some preliminary estimates based on the output side of the accounts. This measure is described in detail on the ONS website. The project will also examine whether a consistent estimate of marketed output can be derived from the expenditure side of the National Accounts.

For public sector employment, the ONS has recently published estimates on a quarterly basis, although at present these are only available up to 2004 Q1. These estimates are based on a survey of government departments and other public sector institutions. Estimates of government sector employment can also be derived from the Labour Force Survey (LFS) of households. But these estimates may be less precise to the extent that respondents may not know or misreport in which sector they are working. The workforce jobs (WFJ) survey can also be used to

(1) The exact approach that we have taken is described in more detail in Harrison et al (2005).
(2) Although private sector output is the main activity variable used in the production of the MPC’s inflation forecast, the Inflation Report includes a forecast for GDP rather than private sector output. This is because GDP is a more familiar concept and allows greater comparability with external forecasters.

(2) See Hicks and Lindsay (2005).
Constructing an alternative indicator of aggregate demand

There are a number of issues to deal with when constructing an aggregate demand indicator that captures the pressure on resources arising from government spending. In particular, how do we combine the government’s procurement of private sector output and the government’s demand for labour to construct a measure of the government’s total demand for resources? The two components are measured in different units, so it would not be meaningful simply to add them together. One way we can convert them into the same units is to work out how many private sector goods the government’s labour force could have produced, if they had instead worked in the private sector. This is done by multiplying the number of workers employed by the government by a measure of private sector labour productivity. This gives a measure of the ‘opportunity cost’ of workers employed by the government, in terms of private output foregone. So the government’s total demand for resources (GDFR), in terms of private sector output units, can be expressed as:

\[
GDFR = \text{Real volume of procurement of private sector goods and services} + \text{Opportunity cost of government employment}
\]

The private sector’s demand for resources, expressed in the same units, is simply the sum\(^1\) of the private sector expenditure components. So the aggregate demand for resources (DFR) in the economy as a whole is given by:

\[
DFR = \text{Total private and government expenditure on private sector output (} Y_p \text{)} + \text{Opportunity cost of government employment}
\]

There are clearly a large number of issues in developing such a ‘demand for resources’ measure in practice. In particular, when estimating the opportunity cost of government labour we could make a number of assumptions about government workers’ hypothetical productivity in the private sector. We could simply assume they would have exactly the same average productivity as a private sector worker. But public sector workers typically work shorter hours than private sector workers. So should we assume that government employees work private sector hours or the average hours they currently work in the public sector, when estimating the opportunity cost? In addition, labour productivity depends critically on the capital-labour ratio. Should we assume that the public sector workers work at the existing private sector capital-labour ratio, or at the government’s, or at some other ratio? There are no ‘right’ answers to questions such as these. The main lesson is that we should not read too much into small differences between different estimates.

In Chart A we show an estimate of the growth rate of the aggregate demand for resources, along with the volume of GDP as estimated by the ONS. Chart B shows the contribution to the growth of the demand for resources by private sector output and the opportunity cost of government labour. To construct the opportunity cost of government labour we have taken the recently published ONS estimates of government employment up to 2004 Q1 and then extended those data to 2004 Q4 using figures from the LFS survey. We have then multiplied this series by average private sector output per head, using the measure of marketed or business sector output derived from the output side of the National Accounts discussed earlier. Those estimates suggest that the ONS measure of real GDP has recently been growing more slowly than the demand for resources (implied by the growth in private sector output and the opportunity cost of the government’s labour).

Although the measurement issues discussed above mean that this is not in any sense a definitive measure of the demand for resources, it nevertheless gives a rough idea of the extent of the differences between the two measures of demand.

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(1) Strictly the chain-weighted aggregate rather than the sum.
estimate employment in the public sector dominated industries.

Clearly we can look at these indicators separately along with a wide variety of other indicators of demand pressures in the economy, as is currently done in the Bank’s Inflation Report. But it may also be useful to create an aggregate demand indicator that combines the demand for private sector output and the government’s labour demand in a consistent way, and which reflects the two sectors’ aggregate demand for resources in the economy. This would then allow a comparison with GDP and estimate how far we might be wrong by using GDP as an indicator of aggregate demand pressures. The box on page 146 discusses such an estimate and shows that recent GDP growth may have significantly understated changes in demand pressure in the recent past.

**Conclusion**

This article has shown that published GDP may not be the most appropriate guide to demand pressures in the economy because it includes the output of the government. It is more important to consider the quantity of resources the government absorbs and how that affects the ability of the private sector to meet the demand for its goods and services. This has important implications for the macroeconomic models and corroborative indicators that are used to analyse inflationary pressures in the economy.
Appendix

Here we explore formally the interaction between private sector output and government output and productivity using a simple model.  We first derive expressions for private sector output under the following conditions: that government output is valued non-separably from private consumption; and that government output affects private sector productivity.  We contrast this with a situation when government output is valued separably from private consumption; and private sector productivity is unaffected by government output.

Consumers maximise a utility function $U$, in which they gain utility from private consumption $(c)$, services provided by government to consumers $(g_c)$ and leisure $(h)$.  In addition, the increment to utility from an additional unit of private consumption is dependent on the level of government output.

$$U = \ln(c + \phi g_c) + \theta \ln g_c + \ln h$$  \hspace{1cm} (1)

They maximise utility subject to a standard time constraint and budget constraint, where $l$ is hours spent labouring; the number of hours in the day has been normalised to 1; $w$ is the wage rate; and $\tau$ is a lump-sum tax levied by the government.

$$1 = l + h$$ \hspace{1cm} (2)

$$c = l.w - \tau$$ \hspace{1cm} (3)

Consumers maximise utility by choosing the hours that they work given the constraints, the wages they face, and taxes. Substituting the constraints into the utility function, we get:

$$U = \ln(lw - \tau + \phi g_c) + \theta \ln g_c + \ln(1 - l)$$ \hspace{1cm} (4)

We can write the consumers’ first-order condition:

$$w(1 - l) = c + \phi g_c$$ \hspace{1cm} (5)

This is the labour supply curve in this model.  We assume that we can aggregate up labour supply curves of individuals to get an aggregate labour supply curve.  Consumers, once they decide how much to work $(l)$, work some part of $l$ in the public sector $(l_g)$ and some part in the private sector $(l_p)$, the division being chosen to clear markets.  So the time constraint is actually:

$$1 = h + l_p + l_g$$ \hspace{1cm} (6)

Firms are perfectly competitive and maximise profits:

$$\pi = y - w l_p$$ \hspace{1cm} (7)
Where $\pi$ is profits, $y$ is the revenue from selling output, and $l_p$ is the amount of labour employed by private sector firms. They maximise profits subject to a constant returns to scale production function, in which government capital $g_k$ is assumed to contribute to firms’ productive capacity:

$$y = A_p g_k^\alpha l_p$$  \hspace{1cm} (8)

Firms’ labour demand comes from their first-order condition, which we get by substituting the production constraint (8) into the profit function (7), and differentiating with respect to $l_p$. Thus:

$$w = A_p g_k^\alpha$$  \hspace{1cm} (9)

Or, in words, wages equal the marginal product of labour. Rearranging the production function (8), private sector labour demand will be:

$$l_p = \frac{y}{A_p g_k^\alpha}$$  \hspace{1cm} (10)

The government runs a balanced budget, choosing a level of tax revenues $\tau$, taking the wage rate $w$ as given, and spending all of its taxes on wages:

$$\tau = w_l$$  \hspace{1cm} (11)

The government produces output $g$ using a linear technology:

$$g = A_g l_g$$  \hspace{1cm} (12)

It then divides up $g$ into a portion handed directly to consumers as services ($g_c$) and a portion given to firms as capital ($g_k$).

$$g = g_c + g_k = \psi g + (1 - \psi)g$$  \hspace{1cm} (13)

Substituting the expression for the wage rate from (9) into the government’s budget constraint (11) gives an expression for the government’s labour demand:

$$l_g = \frac{\tau}{A_p g_k^\alpha}$$  \hspace{1cm} (14)

Adding together the expressions for private (10) and public (14) labour demand, we get an expression for aggregate labour demand:

$$\frac{y}{A_p g_k^\alpha} + \frac{\tau}{A_p g_k^\alpha}$$  \hspace{1cm} (15)
To derive an expression for equilibrium output, we equate total labour demand to labour supply (combining equations (5), (9) and (15)), and equate the demand and supply for goods \( y = c \). The expression that results is:

\[
y = \frac{1}{2} \left[ A_k g_k^a - \tau - \phi g_c \right]
\]  

This tells us that output and consumption rises if government capital rises (since it increases firms’ capacity). By contrast, if the government increases taxes, then private output and consumption both fall. The effect of an increase in government services \( g_c \) depends on the sign of its coefficient \( \phi \): if \( \phi \) is positive, then an increase in government services decreases the marginal utility of private consumption (ie the two are substitutes), decreasing private output and consumption; and vice versa.

We can explain this more formally by writing down an equation (17) that shows how the marginal utility of private consumption varies with the quantity of government services provided. This is obtained by differentiating the utility function (1) first with respect to \( c \) and then with respect to \( g_c \). This shows that, if \( \phi \) is positive, then an increase in government services decreases the marginal utility of private consumption, and vice versa.

\[
U_{cs} = \frac{-\phi}{(c + \phi g_c)^2}
\]  

We turn to derive output under our contrasting set of assumptions. These are: that the marginal utility of private consumption is unaffected by the level of government output, so \( \phi = 0 \) in equation (4); and that government capital is unproductive, so \( \alpha = 0 \) in equation (8). In this case, our expression for output is:

\[
y = \frac{A_k - \tau}{2}
\]  

This tells us that output and consumption rises if private sector productivity rises, and falls if the government increases taxes. But it is unaffected by the level of government output or government productivity.
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