Estimating market expectations of changes in Bank Rate

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- The Bank uses a variety of methods to extract information about market participants' expectations of the future path of Bank Rate.
- This article examines some techniques for estimating, using market prices, market expectations of the timing of future changes in Bank Rate and the probability of Bank Rate being changed within a given period of time.
- These techniques are useful because the expected timing of changes in Bank Rate cannot be directly inferred from the mean expected path of the level of Bank Rate.

Overview

The Bank of England's Monetary Policy Committee (MPC) sets its policy rate, Bank Rate, in order to influence market interest rates, the level of activity in the economy, and inflation. People's expectations of future Bank Rate can have an important influence on economic activity. The MPC is, therefore, naturally interested in understanding how market expectations of its future policy are evolving.

The Bank uses a combination of surveys and financial market prices — interest rates and options prices — to obtain information on market participants' expectations of the future *level* of Bank Rate. But it is also interested in expectations of the timing of future *changes* in Bank Rate. There is an important distinction between these two expectations. **While market interest rates and options prices provide direct information on the expected level of Bank Rate at a given point in time, they do not, by themselves, provide direct information on the *time* at which Bank Rate is expected to change. To estimate the latter requires some assumptions to be made about Bank Rate's possible future paths.**

One means of doing so, discussed in this article, is the Libor Market Model (LMM), a framework originally developed to price interest rate derivatives. The LMM can also be used to estimate the probability that market participants attach to a change in the level of Bank Rate occurring within a given period of time.

The techniques described here are not exhaustive, and none offers a definitive view of market participants' expectations of future Bank Rate or the timing of its changes. All estimates involve a significant degree of uncertainty and rely on assumptions. In particular, those derived from market prices are affected by credit, liquidity and term premia. The Bank therefore continues to monitor and develop a range of indicators to assess market expectations of monetary policy.

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The Bank of England's Monetary Policy Committee (MPC) sets Bank Rate, the interest rate paid on commercial banks' reserves at the Bank of England. Expectations of the future level of Bank Rate — including the timing and pace of its changes — affect a range of market interest rates that influence financial asset prices and the cost of credit in the wider economy. The MPC is, therefore, naturally interested in understanding how these expectations are evolving. Such expectations may also contain useful information about investors' perceptions of economic developments.

The Bank has for some time used surveys to gather information about expectations of the future *level* of Bank Rate. Such expectations can also be estimated from the interest rates on instruments traded in financial markets. And the prices of options contracts written on these instruments can give a guide to the weight that investors attach to different possible levels of interest rates around these central expectations.

Surveys and financial market prices can also be informative about market participants' expectations of the timing of future *changes* in Bank Rate. But there is an important distinction between deriving market expectations of future levels and future changes in Bank Rate. Market interest rates, in themselves, can provide information on the time at which the expected level of Bank Rate reaches a certain point; but deriving expectations about the time at which Bank Rate changes requires further assumptions about its possible paths.

This article begins by reviewing the means by which the Bank extracts information about expectations of the future level of Bank Rate. It then explains some techniques used to extract information about the timing of future changes in Bank Rate — including the Libor Market Model (LMM), a framework originally developed to price interest rate derivatives.

Estimating market expectations of the future level of Bank Rate

The Bank uses a combination of market-based and survey-based measures to estimate expectations of the future level of Bank Rate. This section discusses these measures.[1]

Market-based measures

The Bank uses forward interest rates — that is, the rates at which investors can agree today to borrow or lend over some period beginning in the future[2] — to assess market expectations of the future level of Bank Rate. These forward rates can be derived from a range of financial instruments such as bonds, futures and swaps.

Few financial instruments reference Bank Rate directly. But forward rates can be derived from overnight index swaps (OIS) and instruments referencing Libor (the London interbank offered rate). OIS are contracts involving payments based on the average overnight interest rate that prevails over their lifetime. For sterling contracts, the relevant overnight interest rate is the sterling overnight index average (SONIA).[3] And Libor is a quoted measure of the interest rates at which banks judge they can borrow from other banks.[4]

Forward rates derived from these market interest rates do not correspond perfectly to market participants' expectations of Bank Rate. This is because they include varying levels of credit, liquidity and term premia, which compensate investors for, respectively, the risk of counterparty default; the cost of taking or closing a position in the contract in the future; and the risk associated with borrowing or lending over a long period, compared to undertaking a series of (otherwise identical) shorter-term transactions.[5]

Since SONIA is an overnight rate, it contains a smaller credit premium than the rates underlying forward Libor rates, which are generally of a longer maturity. For this reason, forward OIS rates are the Bank's preferred means of inferring market expectations of the level of Bank Rate. Market intelligence suggests that participants in interest rate markets also use the forward OIS curve as a summary measure of market expectations of future monetary policy. Over the past three years, SONIA has tended to trade a few basis points below Bank Rate. This 'wedge' has to be considered when interpreting the forward OIS curve as a measure of Bank Rate expectations, and is discussed in more detail in the box on page 275.

Survey-based measures

The Bank also uses surveys to gather information on Bank Rate expectations. One such survey is the Reuters survey of economists, which has been running since the late 1990s. While the precise format has varied, in recent years the survey has asked economists for their expectations of future Bank Rate both after the next MPC meeting, and at the end of every quarter thereafter, out to a horizon of around 18 months. Other surveys of monetary policy expectations include the Bank's quarterly survey of external forecasters (SEF) — which is carried out in advance of the *Inflation Report* — and surveys carried out by Bloomberg and HM Treasury.

Survey measures have the advantage of not being affected by credit, liquidity and term premia, or the SONIA-Bank Rate

(1) For more detail, see Joyce and Meldrum (2008).
(2) For example, the one-year interest rate, five years forward, is the rate at which investors can currently agree to borrow or lend for a one-year period starting in five years' time.
(3) SONIA is compiled by the Wholesale Markets Brokers' Association. For further details, see www.wmba.org.uk.
(4) Libor is compiled by the ICE Benchmark Administrator. For further details, see www.theice.com/iba/libor.
(5) A number of recent studies estimate the level of term premia incorporated in market interest rates; for a summary, see Guimaraes (2014).

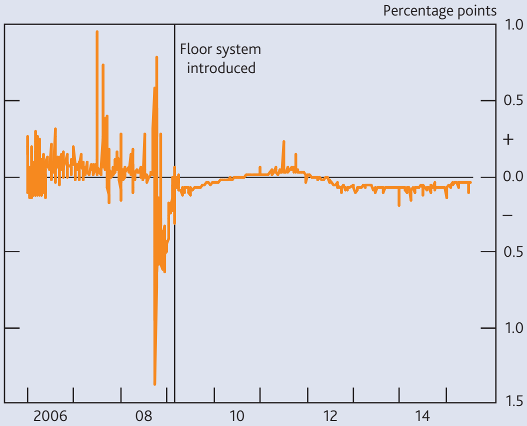
The wedge between SONIA and Bank Rate

Alongside the MPC decision on 5 March 2009, the Bank implemented a 'floor' system for implementing monetary policy. Under this framework, all reserves balances are remunerated at Bank Rate. This should, in theory, keep overnight interest rates close to Bank Rate: were market interest rates to fall below Bank Rate, banks could earn a risk-free profit by borrowing reserves in the market and depositing them with the Bank, where they earn Bank Rate. All else being equal, this should drive overnight interest rates back towards Bank Rate.

Since the introduction of the floor system, overnight rates have typically traded close to Bank Rate, with volatility at historically low levels. But since mid-2012, overnight interest rates, as measured by SONIA, have traded at an average of around 7 basis points below Bank Rate, although this 'wedge' has narrowed to around 4 basis points in recent months (**Chart A**). Market intelligence contacts report that banks have been relatively unwilling to borrow cash offered by non-banks (without reserves accounts) even at rates below Bank Rate, for two reasons:[(1)]

- First, there has been a fall in banks' demand for short-term borrowing. This reflects several factors, including banks' ongoing efforts to reduce their reliance on short-term wholesale funding.
- Second, banks have become less willing to borrow to arbitrage overnight rates against their reserves accounts. Such borrowing increases their balance sheet size and leverage. As a result, banks may have increased the returns they require to justify a given quantity of borrowing.

Chart A The SONIA-Bank Rate wedge[(a)(b)]



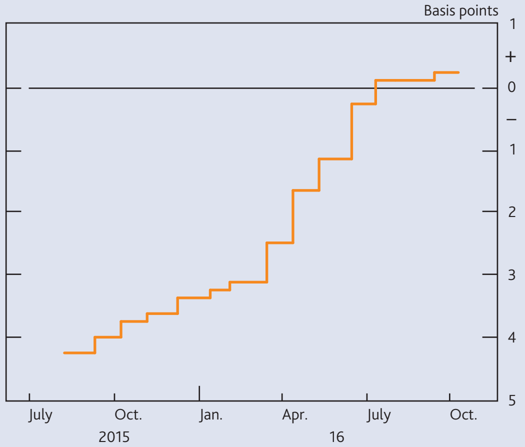
Sources: Bloomberg and Bank calculations.

(a) SONIA minus Bank Rate.
(b) Data to 30 July 2015.

This wedge complicates analysis of market expectations of Bank Rate and its future changes. Any estimate of these expectations must make an assumption about expectations of the level of the wedge in the future.

One way to gain an insight into the degree to which market participants expect the wedge to persist is from the prices of SONIA-Bank Rate basis swaps — contracts that involve payments linked to the average value of the difference between SONIA and Bank Rate that has prevailed over their life. The prices of these securities allow for the construction of a SONIA-Bank Rate forward curve, which, abstracting from credit, liquidity and term premia, gives an indication of the SONIA-Bank Rate wedge that market participants expect to apply at a given future time (**Chart B**). The SONIA-Bank Rate forward curve currently suggests that markets expect the wedge between the two rates to decrease over the next year.

Chart B MPC-dated forward SONIA-Bank Rate basis swap rates[(a)]



Source: Bloomberg.

(a) Data to 30 July 2015.

There are, however, reasons to be cautious of drawing a firm conclusion on investor expectations of the future value of the SONIA-Bank Rate wedge from swaps. Market intelligence suggests that the market for these swaps is relatively illiquid, with the volume of traded contracts far lower than that in other interest rate markets, including that for OIS. As such, their prices may embody liquidity premia to compensate investors for the ease with which they can take or close positions. This may drive the prices of these instruments away from a level commensurate with a pure read on investor expectations of the wedge.

(1) For further details see Jackson and Sim (2013).

wedge. But they are only carried out periodically, meaning that they are not available for day-to-day monitoring. They are also typically carried out over a period of several days, which potentially complicates their interpretation if, for example, the sample period coincides with economic news.

There is also the possibility that market prices and surveys capture different types of information. Survey respondents might report their *modal* expectations (the outcome for Bank Rate that they believe to be most likely), whereas market prices are indicative of market participants' *mean* expectations (the probability-weighted average outcome for Bank Rate). If the distribution of possible outcomes for Bank Rate is skewed, this may cause expectations obtained from surveys to differ from those derived from financial market prices, even if the modal expectation is the same in both cases.

Together, these considerations mean that the Bank uses surveys and market-based measures as complements, rather than relying on one or the other. The Bank also gathers market intelligence on monetary policy expectations through its regular dialogue with market participants.

Deriving the market's view of the distribution of possible future levels of Bank Rate

The measures of expectations discussed in the previous section provide the Bank with a useful summary of market views about the future level of Bank Rate. But the Bank is also interested in the perceived balance of risks around these central expectations. It therefore also monitors a range of survey-based and market-based measures of the *distribution* of possible future levels of Bank Rate.

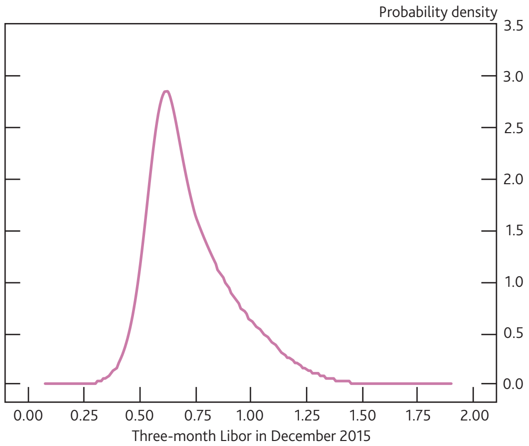
Since 2014, the Bank's SEF and the Reuters survey of economists have included questions about the probability respondents attach to different future levels of Bank Rate, in addition to the questions about their central projections.

It is also possible to derive market perceptions of the distribution of possible future levels of Bank Rate from interest rate options. Options are contracts giving the holder the right (but not the obligation) to buy or sell an asset on (or before) a specified future date at a specified price (the 'strike' price). An option's price therefore reflects the weight that investors attach to the possibility of the price of the underlying asset reaching the strike price. And if option prices for a range of strike prices are available, it is possible to infer information about the weight that investors attach to the underlying asset price reaching these different levels.

As explained in the previous section, forward OIS rates are the Bank's preferred means of inferring market expectations of the level of Bank Rate. But options on OIS rates are not widely traded. Instead, a guide to the market-implied distribution of future levels of Bank Rate can be obtained from options referencing Libor, which are more frequently traded.

One way to display this information is in the form of a risk-neutral probability density function (PDF).(1)(2) **Chart 1** shows a recent example of a risk-neutral PDF of three-month Libor. Possible levels of the interest rate are measured horizontally and probability is measured vertically, so that the area under the line corresponds to the probability a risk-neutral investor might attach to three-month Libor lying within the corresponding range. The risk-neutral PDF has been positively skewed recently, which may reflect market views that interest rates are close to their effective lower bound.

Chart 1 Option-implied risk-neutral probability density function of Libor, as of 30 July 2015



Sources: Bloomberg and Bank calculations.

Estimating market expectations of future changes in Bank Rate

The previous sections discussed some ways of extracting information about market participants' expectations of the *level* of Bank Rate at a given point in time, along with the weight they attach to outcomes around that central expectation. But the Bank is also interested in expectations of the *timing of changes* in Bank Rate. This section begins by describing one simple proxy for measuring these expectations, and explains its limitations. It then introduces the LMM, a technique that can be used to estimate market expectations of the future path of Bank Rate, including the expected timing of its changes.

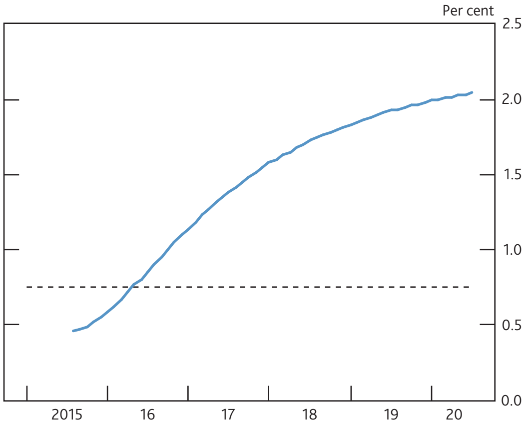
(1) See Breeden and Litzenberger (1978) and Clews, Panigirtzoglou and Proudman (2000). Estimated PDFs can be obtained from www.bankofengland.co.uk/statistics/Pages/impliedpdfs/.
(2) The probabilities shown in **Chart 1** are those that would be perceived by a 'risk-neutral' investor who was indifferent between a pay-off with certainty and a gamble with the same expected pay-off. In the likely case that investors are in fact risk-averse, the actual probabilities they attach to different levels of Libor may differ to those suggested by the risk-neutral PDF.

The techniques considered here can be applied when Bank Rate is at any level, and can be used to estimate expectations of both increases and decreases in Bank Rate. The remainder of the article applies the techniques to the current conjuncture, in which Bank Rate is at 0.5% and the upward-sloping forward OIS curve indicates that markets generally expect the next change in Bank Rate to be an increase.

A simple proxy based on OIS rates

Under the assumption that market participants expect the MPC to change Bank Rate in increments of 25 basis points, one simple proxy for when markets expect Bank Rate to change is the date at which forward OIS rates reach a level 25 basis points above or below the current level of Bank Rate. For example, **Chart 2** shows a recent forward OIS curve. This simple proxy would suggest that markets expected Bank Rate to change at the date when the curve reaches 0.75%. Market intelligence suggests that participants in interest rate markets typically use variants of this proxy to estimate market expectations of the timing of changes in Bank Rate.

Chart 2 Forward OIS curve, as of 30 July 2015(a)



Sources: Bloomberg and Bank calculations.

(a) Instantaneous forward interest rates derived from the Bank's OIS curves.

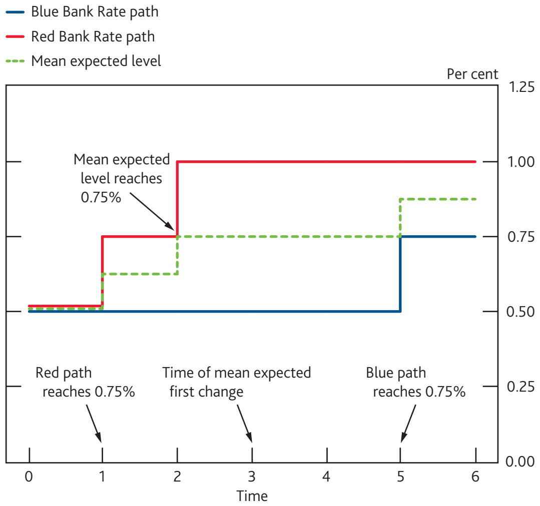
This proxy can be adjusted to control for any difference between SONIA and Bank Rate. If market participants expect a difference to persist, then it might be appropriate to compare forward OIS rates to SONIA, rather than Bank Rate. In practice, it is difficult to determine market expectations of the future difference with confidence (see the box on page 275). For simplicity, therefore, the remainder of the article assumes that market participants do not expect a significant difference to persist. The remainder of the article also abstracts from credit, liquidity and term premia.

The distinction between the expected level of Bank Rate and the expected timing of its changes

The date at which forward OIS rates reach 25 basis points above or below the current level of Bank Rate may be a reasonable measure of the *date at which the expected level* of Bank Rate reaches that level. But that date may differ to the *date at which Bank Rate is expected to change* to that level.

This distinction is illustrated in **Figure 1**, which shows two purely hypothetical future paths of Bank Rate, illustrated by the red and blue lines. Market participants are assumed to place a 50% probability on each. In the scenario shown by the red line, Bank Rate increases first in period 1 and quickly increases again in period 2. In the scenario shown by the blue line, Bank Rate remains unchanged for longer, before increasing once in period 5. The green line shows the mean expected level of Bank Rate at each point in time: since a 50% probability is attached to each of the two paths, this is the simple average of the two. This line can be thought of as a hypothetical forward OIS curve.

Figure 1 Two hypothetical future paths of Bank Rate(a)



(a) These paths of Bank Rate are entirely hypothetical and bear no relation to MPC policy.

There are two points to take from this example:

- The **mean expected level** of Bank Rate — as measured by the green line — increases to 0.75% in period 2, because in that period, one of the possible paths is at 0.5% and the other is at 1%.
- But the **mean expected time** of Bank Rate's first increase is later, in period 3. This is because one of the possible paths first increases in period 1 and the other does so in period 5; the mean time is halfway between these dates.

This simple example shows that differences in the expected level of Bank Rate do not necessarily correspond to the expected timing of its changes. This means that the simple proxy based on OIS rates may not give a clear read on market expectations of the timing of changes in Bank Rate.(1)

In order to estimate expectations of the timing of changes in Bank Rate, it is therefore necessary to make some assumptions about its possible future paths. One means of doing so is to use the LMM,(2) a framework originally developed to price interest rate derivatives.(3)

The Libor Market Model

The LMM uses information from current forward interest rates and options prices to identify a statistical distribution governing the future path of interest rates. The analysis in this article is based on an LMM calibrated so that the distribution of the forward rate at each maturity has a mean consistent with the current forward OIS curve, and volatility consistent with the prices of options on Libor.(4) More details are provided in the annex at the end of this article.

While the option-implied PDFs discussed above provide an indication of market participants' view of the distribution of future Bank Rate at a given point in time, they do not provide any information on how markets expect Bank Rate to move between these points in time. The advantage of the LMM is that it models changes in interest rates as well as their level, which means it can be used to estimate a wider range of distributions and statistics, under the assumptions set out in the annex.

Once calibrated, the LMM can be used to simulate possible paths of interest rates, the distribution of which is consistent with current forward rates and options prices. Two examples of the simulated paths of forward rates are shown by the green and purple lines in **Figure 2**. These simulated paths can then be used to make inferences about market expectations of Bank Rate.

For example, the mean expected time at which Bank Rate changes to a given level corresponds to the average time at which the simulated paths of interest rates reach that level. The two paths in **Figure 2** reach 0.75% at t_1 and t_2. In this simple example, placing equal weight on each path, the expected time at which Bank Rate changes to 0.75% would therefore be the average of t_1 and t_2.

By simulating many such paths of forward interest rates, the LMM can be used to estimate a PDF showing the weight a risk-neutral market participant might place on Bank Rate changing at different future times. A recent example of such a PDF is illustrated in **Chart 3**. Whereas the horizontal axis in **Chart 1** shows possible levels of interest rates at a given time, this axis in **Chart 3** shows possible dates of changes in Bank Rate. It is important to remember that these results abstract from all risk premia, as well as any expected difference between SONIA and Bank Rate. In practice, credit, liquidity and term premia, as well as any expected wedge between SONIA and Bank Rate, may lead market participants' true expectations to differ from the estimates presented in this article.

Figure 2 Two examples of simulated paths of forward interest rates

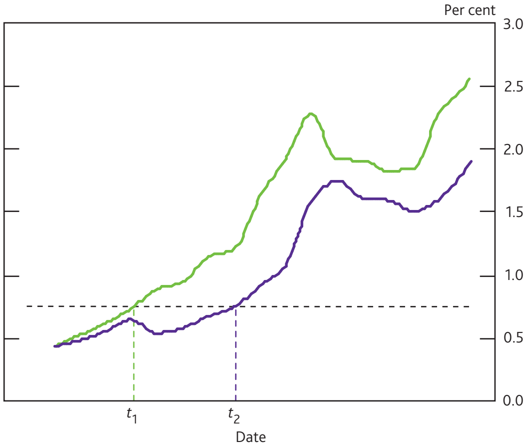
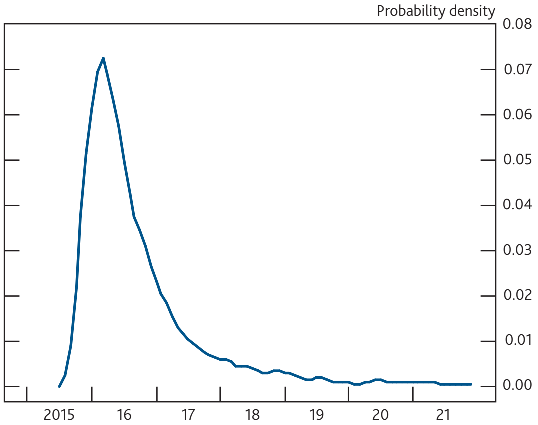


Chart 3 Estimated risk-neutral probability density function of the time of the next increase in Bank Rate, as of 30 July 2015(a)



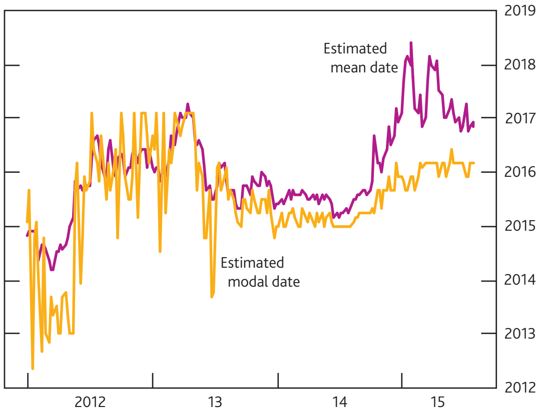
Sources: Bloomberg and Bank calculations.

(a) Estimated using the LMM.

(1) The forward OIS curve plots the means of a set of probability distributions over possible interest rates at given points in time. On the other hand, the expected date of the next change in Bank Rate is the mean of a probability distribution over dates at which a given rate is reached. The relationship between these distributions will generally not be simple.
(2) Other models that have been used for similar purposes include those in Andreasen and Meldrum (2015) and Bauer and Rudebusch (2014).
(3) See Brace, Gatarek and Musiela (1997), Miltersen, Sandmann and Sondermann (1997) and Jamshidian (1997).
(4) When using the LMM to make inferences about Bank Rate, it would be preferable to use the prices of options on Bank Rate or OIS. Since these are not widely traded, options on Libor are used here. This should be a reasonable approximation to the extent that the volatility of OIS rates is similar to the volatility of Libor.

Chart 4 compares the mean and mode of the estimated distribution of the expected timing of the first rise in Bank Rate. This provides an indication of how the location and shape of the distribution shown in **Chart 3** have changed over time. For most of the sample, the mean date has been later than the modal (most likely) date. This is because the estimated distribution has tended to be positively skewed; that is, market participants appear to place more weight on the first increase in Bank Rate occurring after the estimated modal date than before it. The gap between the mean and mode has also varied over time. In particular, this gap widened in late 2014 and early 2015, as market interest rates fell and the estimated mean expected time of the first increase in Bank Rate moved later.

Chart 4 Estimated mean and modal expected dates of the next increase in Bank Rate[(a)]

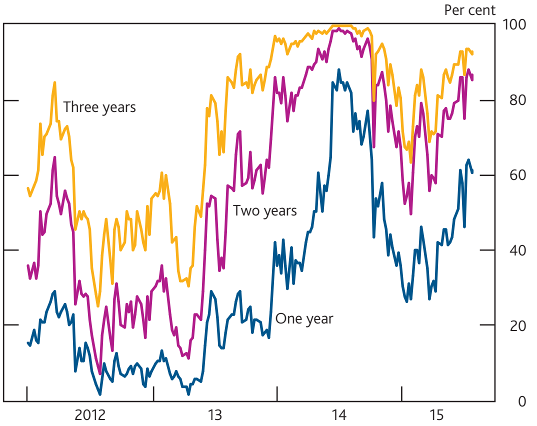


Sources: Bloomberg and Bank calculations.

(a) Estimated using the LMM. Data to 30 July 2015.

The LMM can also be used to estimate the weight that a risk-neutral market participant might attach to Bank Rate changing within a given period of time. **Chart 5** shows how such estimates have varied over time.

Chart 5 Estimated risk-neutral probability of a Bank Rate increase within one, two and three years[(a)]



Sources: Bloomberg and Bank calculations.

(a) Estimated using the LMM. Data to 30 July 2015.

Conclusion

This article has discussed a range of market-based and survey-based tools that can be used to estimate market expectations of Bank Rate. Some of these methodologies are used to estimate market expectations of the level of Bank Rate at a given point in time, and some for measuring market expectations of the timing of its changes.

None of the methods discussed offers a definitive view of expectations of the future path of Bank Rate. The estimates derived from all of them are subject to uncertainty and rely on assumptions. In particular, those derived from market prices are affected by credit, liquidity and term premia, as well as any difference between SONIA and Bank Rate. The Bank therefore continues to monitor and develop a range of indicators to assess market expectations of future monetary policy.

Annex
Outline of the Libor Market Model

Full technical details of the Libor Market Model (LMM) are described in Brace, Gatarek and Musiela (1997). This annex provides an outline based on the implementation and notation in Brigo and Mercurio (2006).

The LMM specifies a set of equations governing changes in each of a set of forward interest rates. The parameters of these equations are then chosen to minimise the sum of squared differences between observed implied interest rate volatilities and those given by the model. The remainder of this annex discusses this procedure in more detail.

Evolution of forward interest rates

The LMM models a set of forward interest rates of maturities, $T_1, T_2, \ldots, T_M$.

Each forward interest rate is assumed to follow a 'diffusion process', whereby the rate of increase in the interest rate in any small space of time dt is the sum of a deterministic rate of increase (or drift) and a normally distributed random fluctuation:

$$\frac{dF_i(t)}{F_i} = \mu_i dt + \sigma_i(t)dW_i \text{ for } i = 1, \ldots, M$$

where $F_i(t)$ denotes the forward interest rate applying between times T_{i-1} and T_i viewed at time t;
μ_i denotes the deterministic rate of drift;
σ_i denotes the volatility of the return on the forward rate $F_i(t)$; and
dW_i denotes normally distributed random fluctuations with mean zero and variance dt.

The random fluctuations have correlation given by:

$$E\left(dW_i(t)dW_j(t)\right) = \rho_{i,j} \text{ for } i, j = 1, \ldots, M.$$

Given this specification, it can be shown that in order to ensure that the modelled forward interest rates do not permit investors the opportunity of arbitrage (that is, the opportunity to generate riskless profit by agreeing to borrow and lend at different forward interest rates simultaneously), the drifts μ_i must satisfy the relationship:

$$\mu_i(t) = \sigma_i(t)\sum_{j=q(t)}^{i} \frac{\tau_j \rho_{i,j} \sigma_j(t) F_j(t)}{1 + \tau_j F_j(t)}$$

where the function $q(t)$ indexes a set of times t where $T_{q(t)-1} < t < T_{q(t)}$. This implies that the drift of forward rate F_i is determined by its volatility σ_i and by past forward rates F_j and their volatilities σ_j.

Functions for volatility and correlation

Absent any other assumptions, calibrating the LMM to a large number of forward rates becomes very computationally burdensome. Fitting the model to n forward rates necessitates the calibration of $n \times n$ variance and covariance parameters.

In order to improve the model's tractability, functional forms are imposed that govern how the volatilities associated with each forward rate, and the correlations between them, vary over time. In the specification used here, these functions take the form

$$\sigma_i(t) = \left(a\left(T_i - t\right) + b\right)e^{-c\left(T_i - t\right)} + d$$

and

$$\rho_{i,j} = e^{-\beta|i-j|}$$

where a, b, c, d and β are constant parameters to be estimated.

This specification is used elsewhere in the recent literature.(1) It reduces the complexity of the model, while permitting sufficient flexibility for the modelled volatilities to match those observed in the market. Intuitively, it implies that the volatilities of forward rates of maturities close together are more highly correlated than those of forward rates of maturities far apart.

Calibration

Calibrating the model amounts to choosing the parameters a, b, c, d and β in order to minimise the difference between the implied volatilities at different horizons given by the model, and those observed in the market.

To do so, it is useful to express the implied volatilities given by the model as an analytical function of the parameters to be estimated. Here the analytical approximation developed by Rebonato (1999) is used:

$$\left(\upsilon_{p,q}^{LFM}\right)^2 = \sum_{i,j=p+1}^{q} \frac{w_i(0)F_i(0)w_j(0)F_j(0)}{S_{p,q}(0)^2} \int_0^{T_p} \rho_{i,j}\sigma_i(t)\sigma_j(t)dt$$

where $\left(\upsilon_{p,q}^{LFM}\right)^2$ is the (squared) implied volatility of the interest rate applying between times p and q, $S_{p,q}(0)$ is the forward swap rate between times p and q, and the weights $w_i(t)$ are given by the formula:

(1) See summary in Brigo and Mercurio (2006).

$$w_i(t)=\frac{\tau_i P(t,T_i)}{\sum_{k=p+1}^{q}\tau_k P(t,t_k)}$$

where $P(t, u)$ is the price of a bond at time t with tenor $u - t$, and τ_i is the fraction of time between T_{i-1} and T_i.

Numerical optimisation is then used to choose parameters that minimise the sum of squared differences between the predicted implied volatilities and observed implied volatilities; that is:

$$\left(\hat{a},\hat{b},\hat{c},\hat{d},\hat{\beta}\right)=argmin_{a,b,c,d,\beta}\sum\left(\upsilon_{p,q}^{LFM}-\sigma_{p,q}\right)^2.$$

The distinction between expected levels and expected changes

Once calibrated, the LMM can be used to estimate the time at which Bank Rate is first expected to increase to a given level x%, that is:

$$E\left[min\ \left\{t:B_t\geq x\%\right\}\right]$$

where B_t refers to Bank Rate in period t and E is the expectations operator.

This time may differ to the time at which forward overnight index swap rates first reach x%, which instead indicates the time at which the expected level of Bank Rate first reaches x% (abstracting from the effects of term premia and the SONIA-Bank Rate wedge). That is:

$$min\ \left\{t:E\left[B_t\right]\geq x\%\right\}.$$

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