Bank of England

Annex 1: A formal model of the expectations augmented Phillips Curve

Annex 1 of the Quarterly Bulletin article 'Inflation targetting considerations for central banks in Africa'

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The basic macroeconomic model used to illustrate the advantages of inflation targeting (IT) is based on the expectations-augmented Phillips Curve (EAPC). The EAPC can be set out in many different formulations, with more or less economic underpinning, based on differing assumptions about the macroeconomy, expectations formation, and the policy framework. The following simple illustration is designed to demonstrate the main results given in Section 2 of the paper.

A1.1: The model

Equation (1) shows inflation (Π_t) depending on (a) the deviation of (current) real demand (Y_t) from supply (Y^*) – the output gap – and (b) the deviation of expected inflation ($E[\Pi_{t+1}]$) from its target, Π^*, plus other shocks, μ_t.

$$\Pi_t = \boldsymbol{\alpha} . (Y_t - Y^*) + \boldsymbol{\beta} . E[\Pi_{t+1}] + (1- \boldsymbol{\beta}) . \Pi^* + \mu_t \qquad (1)$$

The coefficient $\boldsymbol{\beta}$ lies between zero and one and reflects the (lack of) credibility of the target. We can define full monetary credibility to be when agents set wages and prices assuming that the target will always be hit, ignoring any (small) expected fluctuations. In that case $\boldsymbol{\beta}$ would be zero. In that instance, equation (1) resembles the original Phillips Curve, showing a correlation between inflation and some measure of excess demand (unemployment is sometimes used, rather than an output gap).

With no credibility, wage and price-setters ignore the target and adjust wages and prices in line with any expected changes in inflation, then $\boldsymbol{\beta} = 1$. For any value of $\boldsymbol{\beta}$ that is non-zero, the short-run correlation between excess demand and inflation shifts with inflation expectations.

Note that, by construction this model is inherently subject to the Lucas[1] critique: $\boldsymbol{\beta}$ is determined by the credibility of the policy regime.

The output gap ($Y_t - Y^*$), is shown in Equation (2), depending on the extent that the real interest rate (r_t) deviates from its equilibrium level (r^*). Of course, other factors are also important but for simplicity those factors are treated as exogenous shocks.

$$Y_t - Y^* = \boldsymbol{\theta} . (r^* - r_t) + \varepsilon_t \qquad (2)$$

Demand shocks are represented by ε_t which will have the effect of pushing both inflation and output in the same direction. A supply shock – defined as impacting on output with the opposite sign to its impact on inflation – can be constructed as a combination of shocks to both equations eg, $\mu_t = -\Psi \varepsilon_t$. Many shocks will contain elements of demand and supply. For

1 For example, Lucas (1976).

a shock to be recognised as a net supply shock in practice would require $\mu_t > -\alpha.\ \varepsilon_t$. This can make recognition of shocks in practice quite problematic.

The real interest rate (r_t) is conventionally defined as the difference between the nominal policy rate (i_t) and expected inflation: ($E[\Pi_{t+1}]$):

$$r_t = i_t - E[\Pi_{t+1}] \tag{3}$$

To complete the model, and turn it from being static into genuinely dynamic, one needs to specify an expectations formation process and add a policy function for setting the nominal interest rate.

Inflation expectations can be backward or forward looking and they can be quick or slow to adapt. Initially we set expected inflation equal to its previous (lagged) value, bearing in mind that there is a measurement lag.

$$E[\Pi_{t+1}] = \Pi_{t-1} \tag{4}$$

Monetary policy works in this model directly through changing the policy rate (i_t) to affect real interest rates temporarily and hence real demand (Y_t), plus indirectly through the credibility of the target ($\boldsymbol{\beta}$).

If actual demand (and output) is above potential output ($Y_t > Y^*$) then to reduce inflationary pressure the real interest rate (r_t) needs to be increased in order to reduce demand and hence inflation.

A1.2: The policy reaction function

A minimal necessary condition for any stable policy reaction function is that the policy rate (i_t) needs to be increased more than any variation in inflation expectations ($E[\Pi_{t+1}]$) otherwise the resultant change in real interest rates will have the wrong sign. There are many examples where this is breached in practice – one can observe the policy rate ‘chasing’ the inflation rate upwards, never moving enough to bring inflation down.

Equations (1)–(3) can be reduced by substitution to:

$$\Pi_t = \boldsymbol{\alpha}.\ \boldsymbol{\theta}.\ (r^* - r_t) + \boldsymbol{\beta}\ .\ E[\Pi_{t+1}] + (1-\boldsymbol{\beta})\ .\ \Pi^* + \mu_t + \boldsymbol{\alpha}\ \varepsilon_t \quad \text{and hence}$$

$$\Pi_t = \boldsymbol{\alpha}.\ \boldsymbol{\theta}.\ r^* - \boldsymbol{\alpha}.\ \boldsymbol{\theta}.\ i_t + (\boldsymbol{\alpha}.\ \boldsymbol{\theta} + \boldsymbol{\beta})\ .\ E[\Pi_{t+1}] + (1-\boldsymbol{\beta})\ .\ \Pi^* + \mu_t + \boldsymbol{\alpha}\ \varepsilon_t \tag{5}$$

If expectations do not change, then to keep inflation constant, (5) tells us that nominal interest rates need to react enough to offset the shock. Let Δ be the change operator, then eliminating all constants in (5) and re-arranging yields:

$$\Delta\, i_t = (\mu_t + \boldsymbol{\alpha}\, \varepsilon_t) / \boldsymbol{\alpha}.\, \boldsymbol{\theta}. \qquad (6a)$$

So, the change in nominal interest rates for a demand shock ε_t depends both on the size of the shock and the strength of $\boldsymbol{\theta}$ ie, the necessary change in nominal interest rates in response to a demand shock depends on the impact of real interest rates on output. The size of $\boldsymbol{\alpha}$ (the impact of excess demand on inflation) is also important but only if there is an element of a supply shock μ_t, otherwise it cancels out.

If there is a change in inflation expectations, then nominal interest rates need to react according to the following rule:

$$\Delta\, i_t = (\mu_t + \boldsymbol{\alpha}\, \varepsilon_t) / \boldsymbol{\alpha}.\, \boldsymbol{\theta} + (1 + \boldsymbol{\beta} / \boldsymbol{\alpha}.\, \boldsymbol{\theta})\, \Delta\, E[\Pi_{t+1}] \qquad (6b)$$

Assuming positive values of $\boldsymbol{\alpha}$, $\boldsymbol{\theta}$, $\boldsymbol{\beta}$ the reaction coefficient is a number greater than 1 and hence interest rates need to be changed by an additional amount when inflation expectations react. Here we can see that the higher $\boldsymbol{\beta}$, the lower credibility, and the greater must be the change in nominal interest rates for a given change in expectations. Meanwhile the greater the effect of interest rates on excess demand (of $\boldsymbol{\theta}$), and the greater the impact of excess demand on inflation the lower the policy change needs to be.

We note that this model does not determine a unique equilibrium level of inflation – rather that is simply assumed to be the target level. Nor is this formulation based on any particular microeconomic theory of price or wage setting behaviour. It is merely a pedagogic device to capture the workings of an IT framework.

In this model, inflation can only be stable when output is at its equilibrium Y* which is assumed to be driven solely by exogenous factors and hence invariant to interest rates, real or nominal. The model assumes that there is no long-run effect of monetary policy on real output: the influence of nominal interest rates on the level of output, via shifts in the real interest rate, can only be transitory.

In a conventional version of the model which allows for output growth, equilibrium growth would be driven fundamentally by a combination of technology growth (driving investment and hence productivity) and the growth of the working population.

In general, we observe that volatility in both inflation and output can be damaging for output growth and welfare, and that the more stable are both factors, the higher the average growth rate is likely to be because agents can concentrate on making long-term productive investment decisions.

A1.3: The Taylor Rule

A common monetary policy response function posited is the Taylor rule,[2] in which interest rates are adjusted in response to the output gap $(Y_t - Y^*)$ and the deviation of actual inflation from target. In practise the actual inflation rate is only observed with a lag, so we might use $(\Pi_{t-1} - \Pi^*)$.

$$i_t = \Pi_{t-1} + \mu.(Y_t - Y^*) + (1-\mu).(\Pi_{t-1} - \Pi^*) + r^* \quad (7a)$$

$$i_t - \Pi_{t-1} = \mu.(Y_t - Y^*) + (1-\mu).(\Pi_{t-1} - \Pi^*) + r^* \quad (7b)$$

The Taylor rule is often simplified by assuming a fixed value for the equilibrium real interest rate and for the inflation target (both are sometimes set to have the value 2 and equal weights, such as $\mu = 0.5$.

Equation (7b) tells us that the nominal interest rate should be set so that the current estimate of the real interest rate $(i_t - \Pi_{t-1})$ is equal to the equilibrium real interest rate plus/minus a term that reflects the size of the output gap and the deviation of the inflation rate relative to target. This is consistent with the basic properties derived from the EAPC model.

The Taylor rule generates some nice properties in these simple models. With the right choice of parameters (reflecting the degree of credibility and the impact of interest rates on output) it would be sufficient to bring inflation back to target after a simple demand shock. It has also been shown to be a reasonably good approximation to how central banks set interest rates in practice eg, the US Federal Reserve.[3]

In high income countries, θ is often estimated to be weak – possibly due to the decline of manufacturing industry and falling investment, making the economy potentially less sensitive to interest rates. The impact of establishing credibility may then be more powerful than the transmission mechanism of monetary policy through the real economy.

2 Taylor (1993) and Taylor (1999).

3 Technically the decision is made by the FOMC.