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Speech

Real interest rates and risk

Speech given by

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1. Introduction

Today I want to delve deeply into the history of short term real interest rates and equity prices in the UK. My main motivation for doing so is to try and gain some insight into what drives the equilibrium level of real interest rates. This is key in understanding where interest rates might end up in the coming years and why, and will help us gauge to what extent current levels of interest rates are unusual or even “unprecedented”. Adding a second asset price, equities, to the analysis gives us an important extra dimension along which to verify our story: a simultaneous explanation of both real rates and equities is more convincing than an explanation of real rates alone.

As I have discussed in previous speeches, in the pre-crisis decade many central bankers and central bank observers thought of the equilibrium level of real interest rates as being broadly constant. The level in the UK was thought to be around 3% real, or 5% nominal (3% real rate + 2% inflation target), give or take a percentage point. Once short-term shocks to the economy dissipated, this was the level that interest rates were expected to return to.

Based on my long run analysis I will argue that it is misleading to think of the medium term equilibrium real rate as a constant.[1] Real interest rates move around a lot, and those changes can be persistent. Changes have been driven by both nominal factors, e.g. what the central bank targets and how credible it is, as well as real factors, e.g. the structure of the economy and the financial system.

There is therefore no “normal” level of interest rates. In history, we have had high interest rate regimes and low interest rate regimes, each of which lasted many years. Interest rates right now are neither at “emergency levels” nor at “unprecedented levels”. We are currently in a low interest rate regime, and we have been in such regimes before. Many find it disconcerting that it is so different from the high interest rate regime of the 1980s, but that regime was itself in many ways more unusual than the current regime.

I will provide some theoretical arguments on what determines the equilibrium real rate, and I will provide some data and some model calibrations that suggest the theory matches the data rather well. This will be a two-step story. The first step is to understand that the equilibrium real rate depends on how risky the economy is, and that this riskiness changes over time. The second step, somewhat more speculative, is to try to understand why the riskiness of the economy may change.

Crucially, the riskiness of the economy is not just determined by how volatile it is, but also by the extent to which booms and busts are asymmetric (the skew), and by the likelihood of outturns that are very far from the mean (kurtosis or fat tails). It turns out that the asymmetry of booms and busts, as well as the fatness of the tails, have changed significantly over the past few centuries: in the past century, business cycles have

[1] Galesi, Nuno & Thomas (2017) discuss the various similar but slightly different concepts of equilibrium real rates and neutral rates.

become more asymmetric and more fat-tailed, relative to the gold standard era of the 18th and 19th century. And these changes can explain why equilibrium interest rates have shifted down significantly relative to the gold standard era, and why the equity risk premium has moved up.

For the second step of the story, namely why the asymmetry and fatness of the tails have changed in the way the data show they have, I offer only a tentative answer. I suspect the increase in the importance of private sector debt and financial intermediation plays an important role, which in turn was facilitated by moving off the gold standard. An economy where debt and financial intermediation play a more important role can allocate resources more efficiently and achieve a higher growth rate, but also becomes more fragile. Small set-backs can have amplified downside effects and even lead to a financial crisis, while there is less scope for a large boom in an economy which already runs fairly efficiently.

Lastly, I examine sub-periods of high and low real interest rates during the past century. I show that periods of low real rates coincided with high risk events: wars, breaks in the monetary regime, financial crises or some combination of these. And low real interest rate regimes have coincided with falls in the ratio of private credit to GDP, i.e. the private sector was deleveraging. I note that the deleveraging trend since 2010 seems to have come to an end, or at least a pause. That may be telling us that equilibrium real rates are now rising slightly, although other drivers of low real rates do not show any signs of turning yet.

2. Long run history of short rates

In this section I want to look at the properties of short-term real interest rates and equity returns over a very long period of time. The UK is a unique country to study because of its long history of having a central bank, its long history of having well developed capital markets, and good availability of financial and macroeconomic data going back several centuries. I use the annual average of short term market rates[2] as the short-term nominal interest rate, and subtract inflation to obtain a real interest rate.[3]

Figure 1 shows the real short term rate and excess returns on equities since 1800. The next two charts in Figure 1 show the component parts of the real interest rate: the short-term nominal rate and the rate of inflation. As is immediately obvious, real short-term interest rates fluctuate in a wide range. There are peaks of nearly 20% and troughs of nearly -20%. The chart also shows clearly that the current level of real short-term interest rates is *not unprecented*. We have been here before, and even lower: for brief periods in the 19th century, and for several longer spells in the 20th century.

[2] For very long run studies it is important to use a market rate, such as a commercial bill rate or interbank rate, rather than the policy rate. Since the implementation of monetary policy has changed over time, this changes the relationship between the policy rate and short-term market rates, i.e. the same policy rate can imply a different stance of policy over time, even in nominal terms.
[3] The stylised facts highlighted here are robust to using either the GDP deflator or CPI, and using ex ante as well as ex- post real rates, proxied by different filters of inflation to capture inflation expectations. The charts and table show ex-post real rates using the GDP price deflator.

The rate of inflation behaved markedly differently in the early period of the sample, before WWI, compared with the later period of the sample. Inflation moved from being volatile but zero on average, with very little persistence, to being positive on average, with higher persistence. The most obvious explanation for this is the change in monetary regime.

Some version of a de-facto gold standard[4] prevailed until WWI,[5] while in most of the 20th century monetary policy instead targeted the exchange rate, monetary aggregates and the rate of inflation.

Monetary policy in a gold standard regime is like a rigid price level target: a short bout of inflation is soon followed by a short bout of deflation, returning prices to their earlier level. Over long horizons, inflation is expected to be zero.

Other monetary frameworks end up implicitly or explicitly targeting a rate of inflation[6] rather than the price level, and there is no mechanism that pushes the price level down quickly after a boost, or up quickly after a fall.

But the change in monetary regime seems to have done more than just change the behaviour of inflation. Real interest rates were, on average, much higher during the gold standard than during the non-gold standard era. Conversely the return on equities was lower and much more stable during the gold standard period than during the non-gold standard period.

I will examine in some detail what can explain this level shift in real interest rates and equity returns between the two eras. Thereafter, I will later take a more detailed look at the last century of data, focusing on the sub-periods of low real rates.

3. Real Interest Rates in Theory

To try and explain the different real interest rate regimes of the past, we need a theory of what drives real interest rates.

Let me dispense immediately with a fairly deeply rooted, but wrong, notion in modern macroeconomics, namely that real interest rates are primarily driven by the growth rate of the economy. This relationship does

[4] April and Lewis (1991) describe how, though Britain was under the bimetallic standard in the 18th century, Sir Isaac Newton, then Master of the Mint, set the relative price of silver and gold at an unfavourable rate for silver such that market forces drove Britain to an effective gold standard. This was more-or-less the case until the Coinage Act of 1816, declaring the Gold Standard in Britain. Britain remained almost alone in the Gold Standard until 1870s, when the US, Germany and Britain permitted full and automatic convertibility between gold and currency, ultimately giving way to the international gold standard.
[5] The UK returned to the gold standard from 1925 to 1931.
[6] A fixed exchange rate regime targets the exchange rate, of course. But one could argue that, if real exchange rates are broadly constant over long horizons, such a policy implicitly targets the rate of inflation of the country against whose currency we are fixing our exchange rate. Targeting monetary aggregates is an indirect way to target the rate of inflation.

not hold in the data, and it does not even hold in theory! Yet the idea persists, because of commonly adopted – but misleading – practices in solving macro models.

What follows is a brief tour of the finance theory of real interest rates. I realise it is a little terse for a central bank speech, but I want to illustrate that the misunderstanding about the strong link between real interest rates and growth does not come from bad theory, it comes from inappropriate simplification of good theory. This is actually a bit of economics and finance theory that works well, but only when applied correctly.

Consider the deep foundation of most economics models, namely that people act to maximise their expected welfare, subject to a budget constraint and possibly other constraints. This is a key feature of Keynesian, New Keynesian, Classical, Neo-classical, Behavioural schools of economic thought. You can add or remove constraints, you can change how expectations are formed, but some version of this idea will be at the heart of all these models.

The implication is that, in equilibrium, we can find the theoretical price of any asset as the discounted value of future payoffs. For an infinitely lived claim on a dividend stream $\{d_{t+1}, d_{t+2},\ldots\}$ (e.g. equities) and discount factors $\{M_{t,t+1}, M_{t,t+2},\ldots\}$:

$$P_t = E_t\left[\sum_{i=1}^{\infty} M_{t,t+i} d_{t+i}\right]$$

The asset of interest to us is a risk-free one-period bond, whose price will be (the inverse of) the short-term real interest rate. With some manipulation (see appendix), it can be shown that the real interest rate is (the inverse) of the expected value of the discount factors. For example, the one-period risk free rate is the expected value of the discount factor for one period ahead:

$$\frac{1}{1+r_t^f} = E_t\left[M_{t,t+1}\right]$$

Finance people call it the stochastic discount factor. Macroeconomists call it the intertemporal marginal rate of substitution. Two ugly labels for the same intuitive concept: the real interest rate depends on how people value the future relative to today.

Now, to make a link back to the macro-economy, we need to make some assumptions about what drives the stochastic discount factor, or intertemporal marginal rate of substitution. An often-used assumption is that household welfare is well-described by a time-separable utility function (you can add up welfare across periods independently) for a representative agent (you can ignore differences between people).

Using these assumptions we can link the stochastic discount factor to aggregate macroeconomic variables, namely consumption. For example[7]

$$E_t[M_{t,t+1}] = E_t\left[e^{-\delta}\left(\frac{c_{t+1}}{c_t}\right)^{-\gamma}\right]$$

The representative agent step is already somewhat controversial. If heterogeneity across households matters, e.g. young vs old, rich vs poor, patient vs impatient, the stochastic discount factor will not just depend on aggregate macroeconomic variables, but on their entire cross-sectional distribution (of age, of wealth, of patience).[8] But let's just see how far we can get with simple aggregates.

To make it easier still for economists to solve their models, this highly non-linear equation is then linearised, and this is where the real trouble starts. Once linearised, what we are left with is a tight relationship between the real interest rates and growth, and nothing else.

$$E_t[M_{t,t+1}] \approx e^{-\delta} - \gamma E_t\left[\frac{c_{t+1} - c_t}{c_t}\right]$$

But this linearisation is hugely problematic. It kills off, mechanically, anything that might have been interesting about risk. Recall that we were trying to find what the theory says about the *risk-free* real interest rate. But in the linearised world, there is no risk-free real interest rate, as distinct from other interest rates. There is only THE real rate of return, and it is the same on short-term bonds, long-term bonds, equities, housing, and business capital.

How do we make progress then? We do not linearise.

A simple way forward, commonly used, is to assume a lognormal distribution of consumption, which makes the non-linear model have a simple solution.[9] Under such assumptions, the real interest rate depends positively on growth, but negatively on the volatility of growth. Empirically, Hartzmark (2016) shows that this volatility term is much more important than the growth term in explaining real interest rates over time. So the extended theory that includes the volatility of growth fits the data much better.[10]

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[7] The delta parameters is a pure time-discount preference (to capture the fact we are more or less impatient) and gamma controls the level of risk aversion (how much we dislike uncertainty).

[8] In fact there is a lot of evidence that the differences matter – see Constantinides & Ghosh (2017) for a recent example of the role of distributional characteristics in explaining asset prices and further references. The importance of heterogeneity in explaining real interest rates was also highlighted in my earlier speech (Vlieghe (2016a)) that examined the importance of the debt, demographics and the distribution of income, which would not matter or would not exist in a representative agent framework.

[9] There is a long tradition of using lognormal assumption in economics and asset pricing (see Hansen and Singleton (1983), Cochrane (2005), and Hartzmark (2016) for a detailed discussion.

[10] Hartzmark (2016) shows this for the US and a panel of countries. Only for the UK does he not find significant results. We have found that this is entirely due to the Gold Standard period, in particular when the UK rejoined the Gold Standard in 1925 at the pre-WWI level which was too high and led to persistent deflation and high real rates in a period of high volatility. See Guimarães & Vlieghe (2017) for details.

But we want to go one step further, along the lines of Martin (2013). We do not want to restrict ourselves to lognormal distributions. We want to allow for shocks that are asymmetric, or skewed. And we want to allow for a world where extreme events occur more frequently than in a normally distributed world, i.e. the distribution has fat tails. The technical term for this is excess kurtosis.

The details are in the appendix, but the key insight is that in a world that is asymmetric and fat-tailed, the entire distribution of consumption matters: the mean, the volatility, the skew (i.e. the asymmetry), the kurtosis (i.e. the fatness of the tails) and even higher moments as well.[11]

The intuition is straightforward. Households especially value assets that pay off when they need the income the most: when the level of consumption is low. In a riskier world of higher volatility, there are more states of the world where consumption is very low. On average, households will therefore pay more for a safe asset, which pays the same return in all states. This mechanism depresses the risk-free rate when the volatility of consumption is higher.

The reasoning carries over to higher moments of the distribution. If the shocks to consumption are negatively skewed, i.e. there is a higher chance of very bad outturns than very good outturns, households will again value a safe asset more highly, depressing the risk-free real interest rate. If the shocks to consumption are fat-tailed, so that extreme outcomes happen more frequently than a lognormal distribution would imply, again that leads to a lower risk-free real interest rate.

We now have a theory that says that the short-term risk-free interest rate depends (negatively) on the riskiness of the economy, as well as (positively) on growth. And we can be quite precise in saying that a riskier economy is one that has one or more of the following: a higher volatility, a more negative skew, fatter tails.

So far we have used this framework to analyse the risk-free short term interest rate. We can also use this framework to analyse the rate of return on equities. Equity returns are typically high when the economy is doing well, and low when the economy does badly. That is, equities pay off least when households need it most. To compensate for this risky feature of equities, average equity returns tend to be higher than returns on risk-free bonds. The difference between equity returns and risk-free returns is called the equity risk premium. The theory says that, the riskier the economy (higher volatility, more negative skew, fatter fails), the higher the equity risk premium (see Martin (2013)).

4. Applying the theory: gold standard vs non-gold standard

[11] The importance of skewness in asset pricing was recognised early on; see Krauss & Litzenberger (1976) for an extension of CAPM, and references therein for even earlier research on importance of skewness for choice under uncertainty. The importance of fat tails also has a long history in finance research and has recently been (re)popularized by Barro (2006, 2009) reviving the idea of rare disasters of Rietz (1988). Broadbent (2014) and Daly (2016) recently used the rare disaster framework.

Armed with our theoretical framework that risk-free rates depend negatively on risk, and that the equity risk premium depends positively on risk, we return to the data.

Table 1 summarises various properties of the data, split into the gold standard and non- gold standard regimes.[12]

The point I want to draw your attention to first is that real interest rates were, on average, 3.5% in the gold standard regime, much higher than the average of 1.6% in the non-gold standard regime (from which we also exclude the two world wars).

Now note the growth rates of consumption[13] in the economy: 0.8% before, 2% after. Clearly, an interest rate theory based on growth alone fails spectacularly here because it goes the wrong way. This failure of the notion that interest rates are determined by growth has also been documented by Hamilton et al. (2016) and Hartzmark (2016), who show that the correlation between real interest rates and growth over time is usually insignificant and often has the wrong sign.

But that is just a property of the overly simplified theory. The full theory of interest rates says that it should depend on risk, as measured by volatility, asymmetry and fat tails, as well as on average growth.

The volatility of consumption growth rose only slightly from the gold standard period to the non-gold standard period, to 2.3% from 1.7%. But the asymmetry and the fatness of the tails moved quite drastically. Consumption growth became strongly negatively skewed in the later period, with a skew of -1.3, from a near-zero skew of -0.2 in the earlier period. And consumption growth became much more fat-tailed, with a kurtosis of 8 in the later period, up from kurtosis of 2.4.[14]

To illustrate this same point, figure 2 shows a graph of the distribution of growth in the two periods: in the later period (in red), the distribution shifted to the right (higher average growth), but became more asymmetric (a higher chance of very low outcomes) and more fat-tailed.

So in the 20th century, economic dynamics became more negatively skewed, with more frequent outturns very far from average. These are risk characteristics that theory indeed predicts should lead to a lower real interest rate, but also to a higher equity risk premium. In the gold standard period, real equity returns were 3.6%, roughly equal to the risk-free rate, so the equity risk premium was zero. In the non-gold standard period, real equity returns were 6.9% with a real risk-free rate of 1.6%, so an average equity risk premium of 5.2%.

[12] We use the sample for which consumption is available (from 1830). In the appendix we show that the aspects of the data highlighted, our stylised facts, are robust to different choices of sample that roughly correspond to before and after WWI. Importantly, they are not affected by the inclusion or not of the two world wars.
[13] We focus on consumption because this is what our simple theory is based on. But the same patterns hold for GDP.
[14] We report the excess kurtosis (in excess of 3, which is the kurtosis of the normal distribution).

Note that the theory does not just explain the *direction* of the changes in risk-free rates and equity returns, it can match the *magnitude* as well. Using a quantitative model[15] outlined in the appendix, we feed in a series of parameters that determine the volatility, skew and kurtosis of consumption growth to match the gold standard era data. For plausible parameters the model generates a 3% risk-free real rate and near 0% equity risk premium in this period, as shown in Table 2. We then change the parameters to match the post-WWI mean, volatility, skew and kurtosis of consumption growth and see how large a change in the risk-free rate and equity risk premium the model predicts. It turns out we can account for almost all of the drop in the risk-free rate and almost all of the rise in the equity risk premium, again shown in Table 2.[16] Our model predicts a post-gold standard real rate of 1.8%, and an equity risk premium of 6.7%.[17]

5. Multiple regimes in the 20th century

So far we have explained the large change in gold standard era vs non-gold standard era interest rates and equity returns. But the non-gold standard era is not well-described as a single low-interest rate regime. The period contains multiple regimes, to which I now turn.

I apply a regime-switching framework to the data.[18] This is a statistical model that tries to identify regimes of high, medium and low interest rates, and allows for different levels of volatility.

Our regime classification method identifies four low interest rate regimes in the post-WWI period: the late 1910s, the 1930-40s, the 1970s and the current post-financial crisis period, as shown in figure 3.
The first three of these periods followed a change in the monetary regime.[19] After the monetary regime change, the late 1910s and 1970s were associated with a period of very high inflation (see figure 1). During 1930-40s inflation was less high, but nevertheless higher than today or the gold standard period, and very volatile.

In the 1910s and 1970s nominal interest rates were relatively high, while during the 1930-40s nominal rates were closer to zero, as they have been in the most recent decade.

[15] The model follows the approach of Martin (2013) and Backus, Chernov and Martin (2011)

[16] Two things are worth noting about the calibration. First, this is not a story about the occurrence of extreme disasters for the non-Gold Standard period. The magnitude of jumps in our calibration are an order of magnitude lower than in Barro (2006) and match the observed distribution of consumption closely, thus not subject to the criticism of Julliard & Ghosh (2012), Chen et al. (2017), Backus et al. (2011) and Martin (2013) that the extreme disasters are impossible to estimate with the data, or the results of Muir (2017) that suggests disasters cannot explain variation in risk premia. Second, in the calibration we target a range for each variable given by the estimates of mean and median (to account for the effect of large outlier in small samples) across a number of sample periods, which are shown in the appendix. See further discussion of the calibration in the appendix.

[17] The whole sample correlation between growth and real rates is near zero. Once we account for a level shift, such as from higher risk seen in the data, there is a low frequency correlation between real rates and growth rates in the post-WWI period. Moving sample estimates of skewness and volatility also correspond to low frequency movements in real rates as predicted by theory.

[18] The class of models we use, Markov Switching Regimes, were introduced by James Hamilton (1989) exactly to model time series over long spans when breaks in the behaviour of macroeconomic and financial time series data can occur because of "events such as wars, financial panics or significant changes in government policies" (Hamilton (1994)), exactly our case.

[19] The gold standard was effectively abandoned in 1914, resumed in 1925, abandoned again in 1931, before the UK joined the Bretton Woods system of fixed exchange rates in 1944, which collapsed in 1971. After 1971, the UK experimented with targeting monetary aggregates, targeting the exchange rate, before settling on targeting inflation since 1992. See Sayers (1976) and Howson (1993) for a fuller description of the various monetary regimes.

We can therefore say that the current low interest rate regime is unlike the others in that it did not follow a monetary regime change and has been characterised by low and stable inflation by historical standards, but otherwise most similar to the 1930-40s in that both featured near-zero nominal interest rates. As I mentioned in an earlier speech (Vlieghe (2016a)), in the 1930s some economists also worried about debt deleveraging and demographics, in a close parallel to today.

To fully account for each of the shorter low interest rate regimes, ideally we would like a measure of the expected volatility, expected skew and expected kurtosis of GDP year by year, but we do not have sufficient data or sufficiently powerful statistical techniques to identify these expectations. Such techniques only lend themselves to analysing the big break in real interest rates between the gold standard and non-gold standard period.

But we can still use the general insight that risk-free rates are pushed down by the riskiness of the economy. Each of the four low interest rate regimes in the post WWI period featured either a monetary regime change, a war, a financial crisis[20] or some combination thereof, all of which plausibly involve a material increase in risk, and often lower growth as well.

One further episode that I would like to highlight is the 1980s, as it is insufficiently appreciated to what extent this period was unusual, in real interest rate history terms.[21]

The loss of a monetary anchor after the collapse of the Bretton Woods system in 1971, pent up inflation pressure with inadequate monetary tightening, and a spike in the price of oil in 1973 led to persistently high inflation in the 1970s, which was largely unanticipated, meaning real rates in the 1970s were lower than if inflation had been as expected. In the 1980s, control over inflation was regained as nominal interest rates moved sharply higher. Inflation expectations, and inflation itself, moved back down only slowly. The slow adjustment of inflation expectations meant that real interest rates were required to be unusually high for an unusually long period[22], to limit growth and allow spare capacity to drag down inflation and inflation expectations to desired lower levels.[23] Real interest rates in the 1980s were higher and more persistent than at any time since the UK's attempt to resume its gold peg in the 1925-1931 period.

Real interest rates only started coming down in the late 1980s when inflation credibility of central banks became more firmly established, a process that continued into the 1990s, helped by the creation of inflation

[20] The global financial crisis that started in 1929, the UK banking crisis of the early 1970s, and the global financial crisis of 2008-09.
[21] Eichengreen (2015) had already noted that the low real rates of today might well be characterised as a reversion to the pre-1980s mean. Recently, Jorda et al. (2017) also make the point that the 1980s are the exceptional period when analysing rates for a panel of countries from 1870 to 2016. Using nearly 800 years of data for the UK makes the point even starker: a period of persistently high and rising inflation culminating in the early 80s had never been observed.
[22] In addition, there were a number of financial deregulation measures during that period that arguably loosened credit availability, adding further upward pressure to real interest rates.
[23] This was not just a UK phenomenon; there was a similar pattern in the US. Volcker expressed frequent frustration with how long it took before inflation expectations came back down (Silber (2012)). See Goodfriend and King (2005).

targeting regimes in the UK and in many other countries. Crucially, this was an adjustment of real interest rates back to modest levels from unusually elevated levels of the early 1980s (see also Jorda et al. (2017) for a similar discussion for other countries). Using the early 1980s as a starting point for establishing what is "normal" in real interest rate history is therefore unwise.

6. What determines the risk-regimes?

I have shown that finance theory can account for the high real interest rates of the gold standard and the low real interest rates of the non-gold standard era, based on the difference in the behaviour of the macroeconomy in the two periods.

The theoretical explanation is an increase in the riskiness of the economy, with a more negative skew and fatter tails of growth outturns playing an important role.

Somewhat speculatively, I would like to dig one level deeper still, and ask what fundamental drivers can explain the fact that growth has become more negatively skewed and tails have become fatter?

The World Wars were obviously two significant events that were both negative and far from the average, but I have shown that the change in the measured riskiness of the economy took place even in a sample where the wars are excluded. A good candidate explanation, in my view, is the rising importance of debt and financial intermediation.

Empirically, Jorda et al. (2013) have shown that booms associated with a more rapid private sector debt expansion are more frequently followed by financial crises, which tend to involve larger contractions in GDP than normal recessions. An increased frequency of very bad outturns is exactly what would generate a more negative skew and fatter tails in growth.[24]

The theoretical mechanism driving the link between debt and a more negative skew and fatter tails has been present in macroeconomic models of debt and leverage for many years. The basic insight from models where debt matters[25] is that increased debt and a larger financial sector brings the economy closer to an efficient allocation of resources, but also makes the economy more vulnerable to shocks. Higher leverage and a larger financial intermediation sector mean that a downturn can become amplified as reduced borrower net worth (final borrowers or financial intermediaries) causes a contraction in asset prices, which reduces net worth further, leading to a downward spiral of falling asset prices, falling borrowing capacity, falling lending capacity, falling spending. The mechanism is less powerful on the upside: if the economy is already close to an efficient allocation of resources, borrowing and lending constraints do not bite, and the

[24] More recently, Jensen et al. (2017) document that US growth has become more negatively skewed in the US over the period where household and business leverage has risen.
[25] There is both a literature on finance and volatility (Kiyotaki and Moore (1997) and many others), as well as finance and growth (King and Levine (1993), Levine (1997) and references therein).

scope for a large further boom is limited. The asymmetry and fat tails of growth outturns therefore follow logically from that framework.[26]

Figure 4 and 5 show that the ratio of credit[27] to household income was low and stable during the gold standard period, but became larger and more volatile in the post-gold standard period, consistent with these theories of the impact of credit and financial intermediation.[28]

The fact that the rise in the importance of credit and financial intermediation occurred at the start of the 20th century is probably not a coincidence. The end of the gold standard plausibly contributed to this process, as it allowed for the supply of money and credit to be decoupled from a fixed gold supply, bringing huge benefits in terms of growth and more efficient allocation of resources, but costs in terms of fragility.

Specifically, figure 5 shows that, within the generally low real interest rate regime of the non-gold standard era, regimes of persistent negative real interest rates coincided with contractions in the ratio of credit to income, in other words deleveraging.

This deleveraging mechanism was one of the three factors (debt, demographics, distribution of income) that I highlighted earlier as a key driver for low real interest rates.[29]

7. Policy outlook

The link from low equilibrium rates to deleveraging brings me to the current policy outlook. Figure 6 is a more up-to-date and comprehensive measure of the ratio of credit to GDP.[30] It shows that the period of deleveraging that started in 2010 might be coming to an end. For most of the subdued economic recovery since 2010, credit growth has been below income growth. Households and firms were repairing their balance sheets. As I have discussed previously (Vlieghe (2016a)), this was a welcome process of repair, but one that should be expected to weigh on growth. The desire to reduce the debt burden translates into reduced spending by those sectors with the highest debt.

But over the past year or so that process of balance sheet repair seems to have come to an end, or at least to a pause. That potentially says something important about the equilibrium real interest rate. Households and firms, by their actions, are showing us that they have reached their desired debt to income ratios for

[26] See Brunnermeier and Sannikov (2014) for a full nonlinear solution of the model where debt matters, with a detailed discussion of the distributional impact of financial intermediation.
[27] This particular chart shows only mortgage credit due to data limitations. There is also a long history of a measure of total credit available, which has similar dynamics, but it includes interbank credit as well.
[28] This explanation is only partial at best. There is a large literature documenting the link between development and the volatility of growth. See Koren and Tenreyro (2013), and references therein, who show that emerging economies tend to have growth rates that are more volatile and negatively skewed, despite generally lower levels of financial depth.
[29] Vlieghe (2016a)
[30] We use the Financial Policy Committee's Counter Cyclical Buffer data as it is higher frequency, more timely and contains total households and PNFC's liabilities

now, and do not wish to deleverage further. That also means that the deleveraging process is no longer weighing on demand as much as it was previously. In turn, that suggests that the equilibrium real interest rate might be rising slightly from the very low levels in the post-crisis period.

Deleveraging is not the only factor driving the equilibrium interest rate. Other medium-term dynamics, such as demographics and the income distribution, do not show any signs of turning. And, as I have argued in this speech, even if the equilibrium interest rate is moving up, it is only moving up within a framework of a lower average real interest rate in the post gold standard era. The high real interest rates of the 1980s remain an outlier, the drivers of which are not applicable at all today.

While these medium-term considerations form an important backdrop to my thinking about Bank Rate now and in the medium term, they are not enough to guide actual meeting-by-meeting interest rate decisions. Those decisions require significant further analysis of recent developments in the UK economy: growth, slack, inflation.

Until recently, I thought the appropriate response of monetary policy was to be patient, given modest growth and subdued underlying inflationary pressure. But the evolution of the data is increasingly suggesting that we are approaching the moment when Bank Rate may need to rise.

In recent months, I have been struck particularly by the following developments:

First, despite a clear weakening of GDP growth in the first half of this year, the amount of economic slack continues to be eroded. Employment growth has re-accelerated after slowing late last year, and the unemployment rate keeps making new lows, reaching 4.3% on the most recent data, down from 4.9% a year ago. Wage growth is not as weak as it was earlier in the year: over the past 5 months, annualised growth in private sector pay[31] has averaged just over 3%. And some pay-related surveys also suggest a modest rise in wage pressure in recent months. If these near-term labour market trends continue, I would expect this to lead to somewhat more upward pressure on medium-term inflation.

Second, consumption growth generally held up better than I expected over the past year. Consumption did slow earlier this year, partly in response to weaker real income growth, as wage growth has not kept up with the exchange-rate-driven, temporary, rise in inflation. Consumption growth in Q2 was particularly weak. But there are some early signs of stronger consumption growth in Q3. Whether that improvement will last is still an open question. Rising real wage growth, both due to improving nominal wage growth and the expected easing back of inflation, would support consumption growth further out. An easing of the rate of growth of consumer credit following some tentative signs of a modest tightening in credit conditions is likely to act as a slight drag.

[31] Private sector AWE excluding bonuses.

The wider economic backdrop over the past year has been one of improving global growth, and in particular an improving outlook for Eurozone growth, which generally benefits UK external demand.

But, acting in the other direction, is the continued uncertainty about the UK's future trading relations with the EU and the rest of the world. That uncertainty is likely to be dampening investment growth at the moment as firms put some projects on hold pending further clarity.

There remains a risk that, at some stage, the uncertainty surrounding the Brexit process has a larger impact on the economy than we have seen so far. If that happens, monetary policy would respond appropriately. But for now, it seems the net effect of the many underlying forces acting on the UK economy is that slack is continually being eroded and wage pressure is gently building.

If these data trends of reducing slack, rising pay pressure, strengthening household spending and robust global growth continue, the appropriate time for a rise in Bank Rate might be as early as in the coming months.

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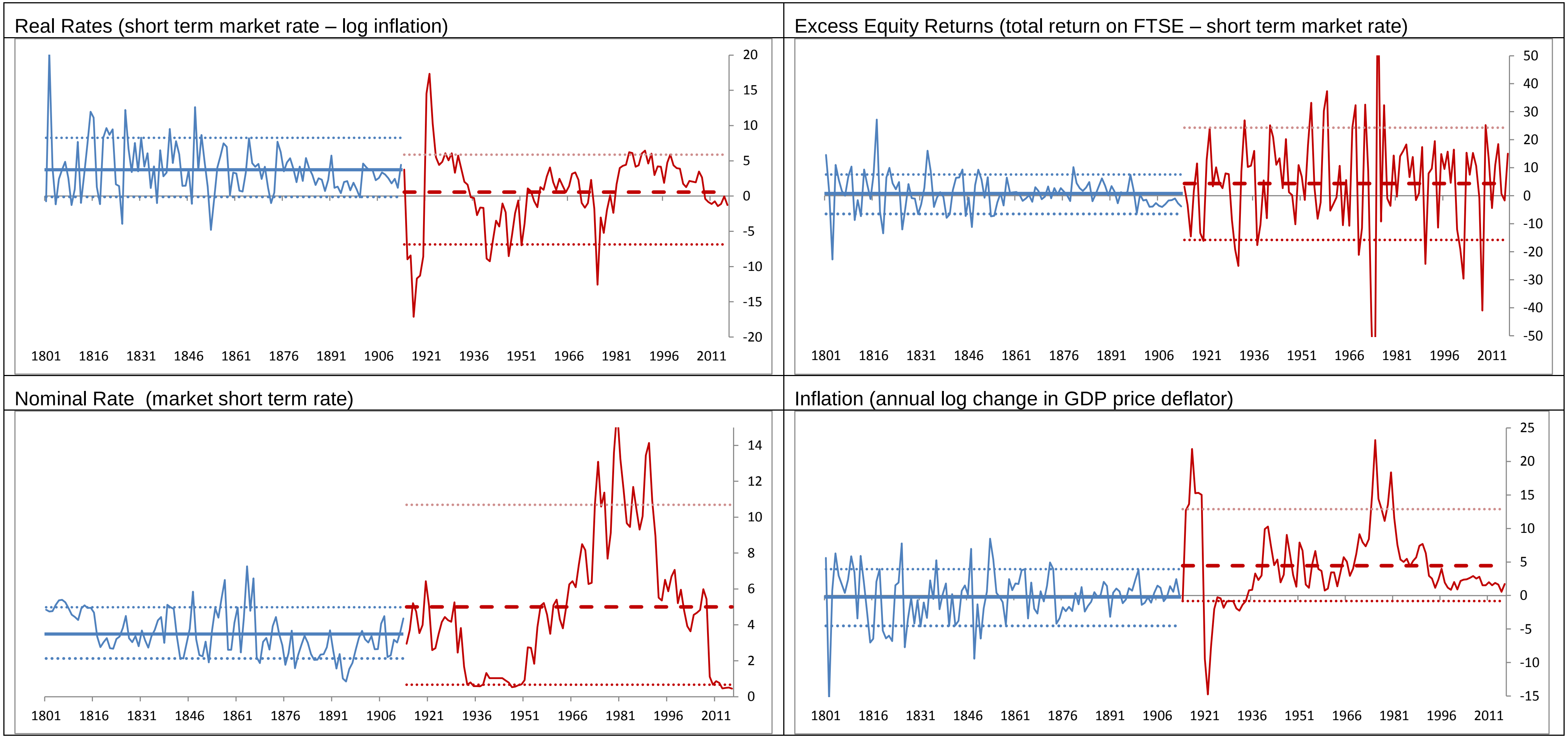
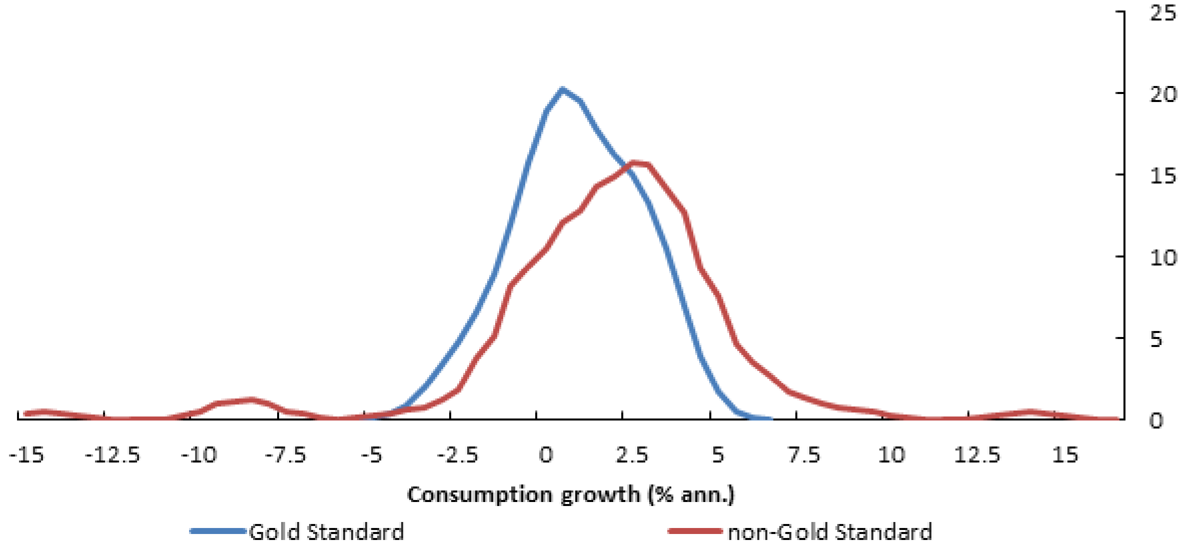
Figure 1: Long run view of asset prices and inflation



Source: Nominal interest rates and inflation are from the Bank of England's '[A millennium of macroeconomic data]' (the dataset was originally called the 'Three centuries of macroeconomic data), total equity return data is from Global Financial Data.

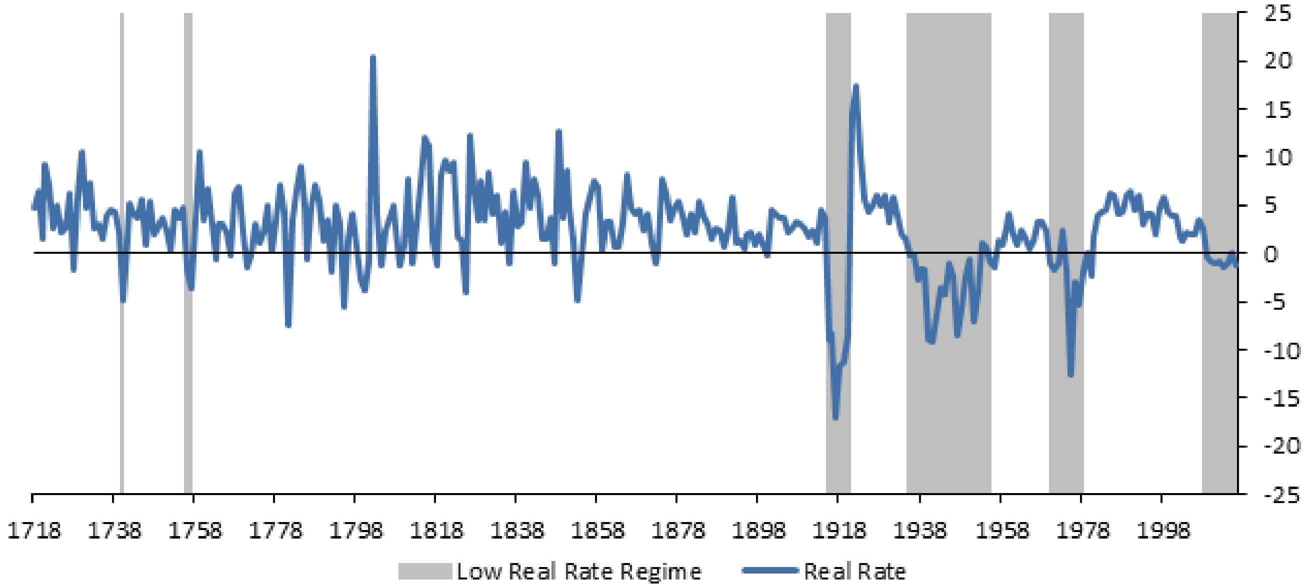
Figure 2: Distribution of UK consumption growth



Notes: The figure shows the kernel density estimates for the UK consumption per capita growth. The Gold Standard sample contains the years 1831-1913 and 1925-1931, while the non-Gold Standard sample contains the years 1914-1924 and 1932-2016.

Source: Bank of England's 'A millennium of macroeconomic data' database, Bank estimated kernel densities

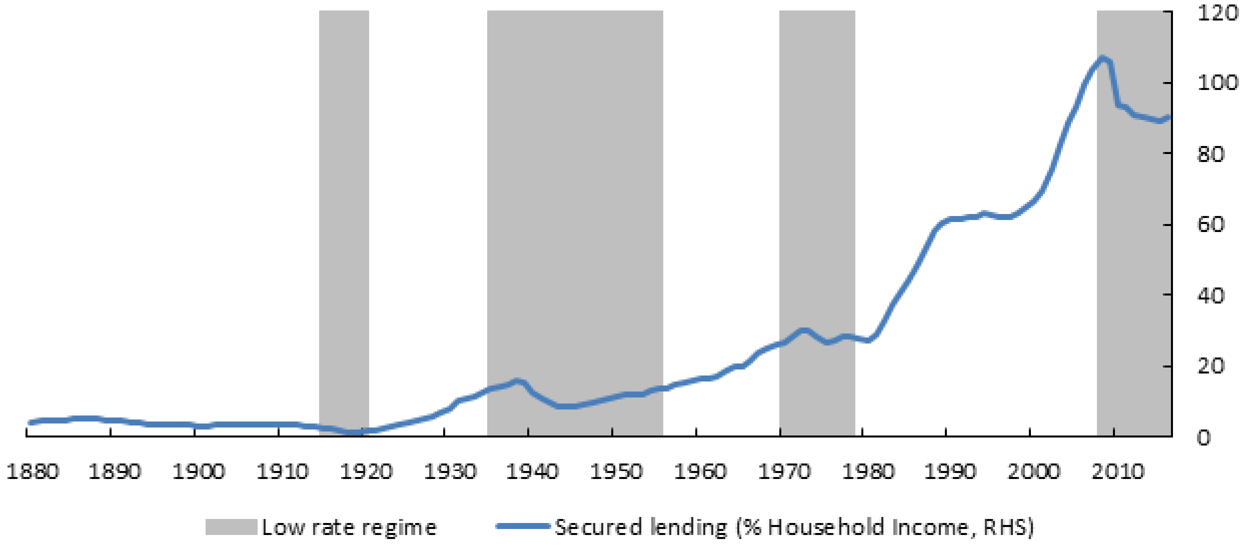
Figure 3: Low rate regimes in UK real rates (full sample estimation of regimes)



Notes: The low real rate regime is defined by the Markov Switching Model in Section 5.

Source: Bank of England's 'A millennium of macroeconomic data' database

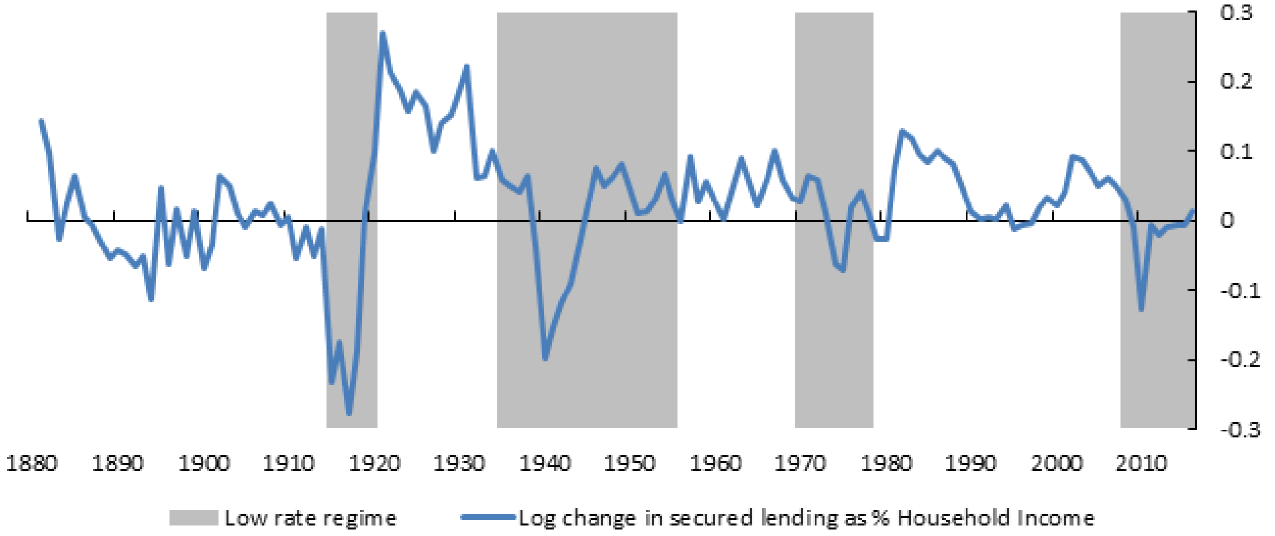
Figure 4: Credit-to-household income ratio with real low rate regimes



Notes: The low real rate regime is defined by the Markov Switching Model in Section 5. Secured lending measure excludes impact of securitizations

Source: Bank of England's 'A millennium of macroeconomic data' database

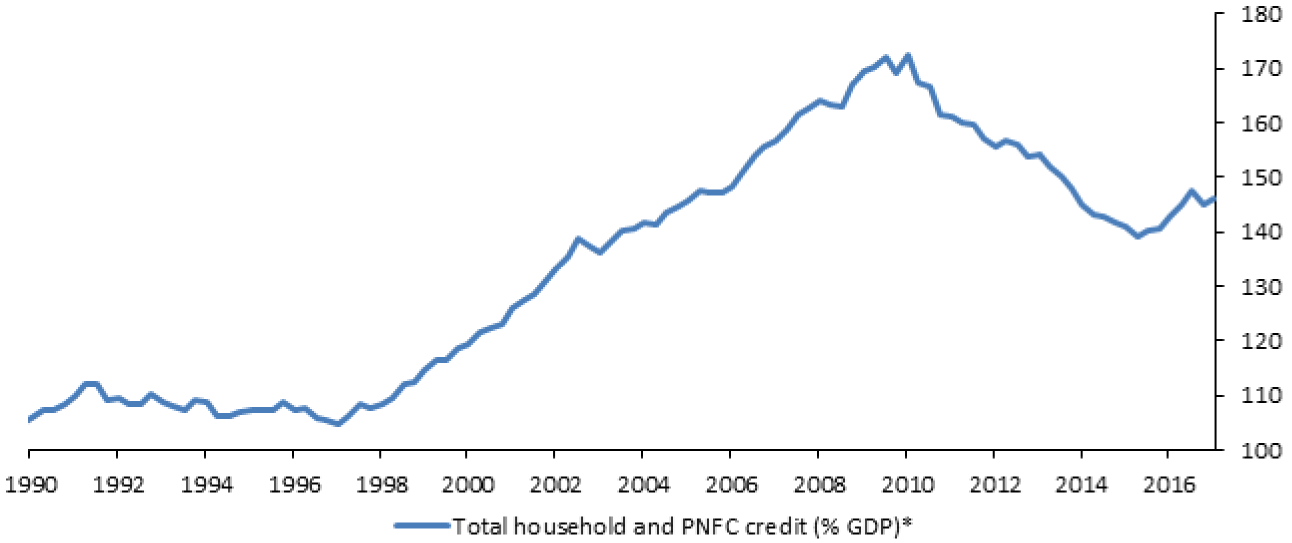
Figure 5: Growth rate of credit-to-household income ratio with real low rate regimes



Notes: The low real rate regime is defined by the Markov Switching Model in Section 5. Secured lending measure excludes impact of securitizations

Source: Bank of England's 'A millennium of macroeconomic data' database

Figure 6: Total Credit-to-GDP (households and credit to non-financial sectors)



Note: *All liabilities of the household and not-for-profit sector (ex. unfunded pensions, derivatives), and private non-financial corporations' (PNFCs') loans and debt securities

Source: Financial Policy Committee's Counter Cyclical Buffer core indicators

Table 1: Descriptive data summary

	Nominal Interest rate (1)	Equity Returns (annual) (2)	Excess Equity Returns (2)-(1)	Inflation (GDP deflator) (3)	Real Interest Rate (1)-(3)	Real Equity Return (2)-(3)	Real per-capita Consumption growth (4)	Real per-capita GDP growth (4)
	Gold Standard - consumption sample (1830-1913, 1925-1931)							
Mean	3.3	3.4	0.1	-0.1	3.5	3.6	0.8	0.9
Median	3.1	3.2	0.4	-0.2	3.3	3.5	0.8	1.3
Volatility	1.2	5.5	5.7	2.7	2.7	6.6	1.7	2.5
Skew	0.7	-1.2	-1.0	0.1	0.3	-0.5	-0.2	-0.2
Kurtosis	3.6	8.2	7.3	4.7	4.3	4.9	2.4	2.8
AR(1)	0.5	0.5	0.5	0.1	0.2	0.4	-0.1	0.0
	Non Gold Standard or World War sample (1921-1925, 1932-1938, 1948-2016)							
Mean	5.6	10.8	5.2	4.0	1.6	6.9	2.0	2.1
Median	5.2	11.8	8.3	2.9	1.7	10.5	2.3	2.3
Volatility	4.0	20.6	20.8	5.3	4.2	21.0	2.3	2.4
Skew	0.5	-0.3	-0.7	0.3	0.4	-1.0	-1.3	-2.2
Kurtosis	2.5	7.8	8.4	6.5	6.6	7.6	8.0	11.5
AR(1)	0.9	-0.1	-0.1	0.8	0.7	0.0	0.3	0.1

Note: The appendix shows that the precise choice of sample dates for the GS and non-GS periods is not important for the stylised facts we aim to match to theory - the level of real rate and equity premia and the growth rate, volatility, skewness and kurtosis of (consumption) growth rates.

Table 2: Model calibration results

	GS/pre-WWI		post-GS/WW	
	Data	Model	Data	Model
real rate	3.0,3.4	3.0	1.1,1.7	1.8
equity premium	0.4,0.6	0.3	7.6,8.6	6.7
cons growth	0.8	1.2	2.0,2.3	2.2
cons vol	1.5,1.8	1.5	2.3,3.1	5.5
cons skew	-0.2,0.2	-0.0	-2.1,-1.3	-1.2
cons kurtosis	2.3,2.4	2.5	8.0,13.9	9.4

Note: Model calibration only changes the parameters for the consumption process (mean and volatility of Gaussian term and arrival intensity of jumps and mean and volatility of jump sizes), the preference parameters (risk aversion of 6, time discount of 0.025 and intertemporal elasticity of substitution of 2) are kept fixed for both periods.

Table A1 – Robustness of sample moments to cut-off dates

		Period I (GS/pre-WWI)				Period II (post GS/WW)			
		1718-1913,1925-1931	1830-1913,1925-1931	1870-1913,1925-1931	1718-1913	1921-1925,1932-2106	1921-1925,1932-1938, 1948-2016	1921-2016	1932-2016
		GS full sample	GS cons sample	GS global sample	pre-WWI	not-GS	not-GS or WWs	post-WWI	post-GS
real rate	mean	3.5	3.5	3.1	3.4	1.0	1.6	1.3	0.4
	median	3.4	3.3	3.0	3.3	1.3	1.7	1.7	1.1
equity premium	mean	0.5	0.1	-0.2	0.8	5.7	5.2	5.0	5.4
	median	0.6	0.4	0.4	0.5	8.6	8.3	7.6	7.9
consumption growth	mean	0.8	0.8	0.8	0.8	1.8	2.0	1.8	1.9
	median	0.8	0.8	0.8	0.8	2.2	2.3	2.0	2.1
	volatility	1.7	1.7	1.5	1.8	3.1	2.3	3.0	2.9
	Skew	-0.2	-0.2	0.2	-0.1	-2.0	-1.3	-2.1	-2.0
	Kurtosis	2.4	2.4	2.4	2.3	12.2	8.0	12.8	13.9
GDP growth	mean	0.6	0.9	0.8	0.6	1.9	2.1	1.8	2.0
	median	0.7	1.3	1.3	0.7	2.3	2.3	2.3	2.3
	volatility	3.0	2.5	2.7	2.9	2.8	2.4	2.9	2.5
	Skew	0.1	-0.2	-0.1	0.1	-1.3	-2.2	-1.2	-0.5
	Kurtosis	2.9	2.8	2.9	2.9	7.3	11.5	6.2	4.3

Appendix

The fundamental asset pricing equation says that the price $(P_{i,t})$ of any traded asset i paying dividends $\{d_{i,t+s}\}_{s=1:\infty}$ is given by:

$$P_{i,t} = E_t \left[M_{t,t+1} \left(P_{i,t+1} + d_{i,t+1} \right) \right]$$

or

$$1 = E_t \left[M_{t,t+1} \left(1 + R_{i,t,t+1} \right) \right]$$

where $M_{t,t+1}$ is the stochastic discount factor (SDF). This is the usual Euler Equation that is part of any general equilibrium macro model. In the case of a real 1 period risk-free bond $(P_{t+1}^f = 1; d_{t+1} = 0)$ the Euler equation yields the 1-period risk-free (ex-ante) real rate $R_{t,t+1}^f$ (which is known at time t):

$$\frac{1}{1 + R_{t,t+1}^f} = \frac{P_t^f}{P_{t+1}^f} = E_t \left[M_{t,t+1} \right] \tag{1}$$

In a representative agent economy with time separable additive expected power utility $U\left(\{C_s\}_{s=t:\infty}\right) = \sum_{s=t:\infty} e^{-\delta s} u\left(C_s\right)$, with $u\left(C\right) = \frac{C^{1-\gamma}}{1-\gamma}$, the SDF is given by the intertemporal marginal rate of substitution

$$M_{t,t+k} = e^{-\delta k} \frac{u'\left(C_{t+k}\right)}{u'\left(C_t\right)} = e^{-\delta k} \left(\frac{C_{t+k}}{C_t} \right)^{-\gamma}$$

Both the representative agent and time separability have been relaxed by Martin (2013). In the calibration we will keep the representative agent but use non-separable Epstein-Zin results from Martin (2013). For expositional simplicity we show the real rate expression using the time separable power utility case. Following Martin (2013), to calculate the expectations in (1) we will use cumulant generating functions, for which it will be useful to introduce some notation.

Expansions using cumulants

Let $\varphi_{t,x}\left(\theta\right) \equiv \ln E_t \left[e^{\theta x} \right]$ be the cumulant generating function of x, and let $\kappa_{t,x}\left(n\right)$ be the n^{th} cumulant of x. The first cumulant of x is the expected value (μ_x), the second is the variance $\left(\sigma_x^2\right)$, the third is the centred third moment of x, or $skewness \times \sigma_x^3$, but higher order cumulants do not correspond to centred moments (see e.g. Shiryaev (1996) for the general link between cumulants and moments). Then if the n^{th} moment of x exists and $\varphi_{t,x}\left(\theta\right)$ exists in the neighborhood of θ then

$$\varphi_{t,x}\left(\theta\right) = \sum_{j=1:n} \frac{\theta^j}{j!} \kappa_{t,x}\left(n\right) + o\left(\left|\theta^n\right|\right)$$

If we define $c_{t+1} \equiv \ln\left(\frac{C_{t+1}}{C_t}\right)$, then from (1) the log real risk-free rate $r_t^f \equiv \ln\left(1 + R_{t,t+1}^f\right)$ can be expressed in terms of the log consumption cumulants:

$$\begin{aligned} r_t^f &= -\ln E_t \left[e^{-\delta} \left(\frac{C_{t+1}}{C_t} \right)^{-\gamma} \right] = \delta - \ln E_t \left[e^{-\gamma c_{t+1}} \right] \\ &= \delta - \sum_{j=1:\infty} \frac{\left(-\gamma\right)^j}{j!} \kappa_{t,c}\left(n\right) \end{aligned}$$

If real log consumption innovations are iid lognormally distributed $\ln\left(C_{t+1}/C_t\right) \sim iidN\left(\mu_c, \sigma_c^2\right)$ then

$$r_t^f = \delta + \gamma \mu_{t,c} - \frac{\gamma^2}{2} \sigma_{t,c}^2$$

where $\mu_{t,c} = E_t c_{t+1} = E_t \ln \frac{C_{t+1}}{C_t}$ and $\sigma_{t,c}^2 = Var_t\left(c_{t+1}\right)$ (note that $\ln E_t \frac{c_{t+1}}{c_t} = \mu_c + \frac{\sigma_c^2}{2}$). In the general case, skewness, kurtosis and higher moments also matter

$$\begin{aligned} r_t^f &= \delta - \ln E_t\left[e^{-\gamma \ln\left(\frac{c_{t+1}}{c_t}\right)}\right] = \delta - \sum_{j=1}^{\infty} \frac{(-\gamma)^j}{j!} \kappa_{t,c}(j) \\ &= \delta + \gamma \mu_{t,c} - \frac{\gamma^2}{2} \sigma_{t,c}^2 + \frac{\gamma^3}{3!} skew_{t,c} \sigma_{t,c}^3 - \frac{\gamma^4}{4!} kurt_{t,c} \sigma_{t,c}^4 + \text{higher order log consumption moments} \end{aligned}$$

So the real rate will be higher, the higher the time discount preference parameter, the higher the growth rate of consumption, the lower the variance and even higher moments, and the higher the skewness and odd higher moments of consumption.

To find the nominal interest rate, and ex-post real rate implied by the nominal bond, we need to introduce the joint cumulant function. See Guimarães & Vlieghe (2017) for the analysis which can then also explain term spread as a function of inflation and consumption dynamics and inflation surprises (not discussed in this speech).

In the Epstein-Zin case Martin (2013a) shows the real rate is given by:

$$r_t^f = \delta - \varphi_{t,c}(-\gamma) + \left(1 - \frac{1}{\vartheta}\right) \varphi_{t,c}(1-\gamma)$$

where $\vartheta \equiv \frac{1-\gamma}{1-\frac{1}{\psi}}$ and ψ is the intertemporal elasticity of substitution, with the expression for risk premia unchanged. Table 2 uses Epstein-Zin preferences.

Heterogeneity introduces many more possible drivers of risk premia, which is potentially relevant for long term real rates. Heterogeneity provides a theoretical link to many of the possible low frequency drivers of persistent movements in real rates: debt deleveraging, distribution of income and demographics (3Ds). Martin (2013) shows that introducing heterogeneity in the model will lower the real rate and increase the risk premia relative to a similar representative agent economy. Important for calibration is the evidence on the difference in the dynamics of consumption of the wealthy equity holders vs aggregate consumption, particularly towards the end of the sample (see Parker & Vissing-Jorgensen (2009)), and role of financial intermediaries (see Adrian, Crump & Muir (2014), He, Kelly & Manela (2016) and Muir (2017)).

Empirical Model

We calibrate a jump-diffusion model as in Martin (2013) and Backus, Chernov & Martin (2011, BCM henceforth) to match the real rate; consumption growth moments and equity returns. The model for consumption growth $G_t = \ln \frac{C_t}{C_0}$ is

$$G_t = \mu_t t + \sigma_t B_t + \sum_{i=0}^{N(t)} Y_{t,i}$$

where B_t is a Brownian motion, $N(t)$ is a Poisson counting process and Y (the jumps) are iid random variables. If $Y \tilde{} N(-b_t, s_t^2), b > 0$ (disaster) then the CGF is given by

$$\varphi_{t,c}(\theta) = \mu_t \theta + \frac{1}{2} \sigma_t^2 \theta^2 + \omega_t \left(e^{-\theta b_t + \frac{1}{2}\theta^2 s_t^2} - 1\right)$$

which can be used to find the nth cumulant by taking derivative with respect to θ at $\theta = 0$:

$$\frac{\partial^n \varphi_{t,c}(\theta)}{\partial \theta^n}|_{\theta=0} = \kappa_{t,c}(n)$$

As shown by Martin (2013) in the power utility case and Epstein-Zin case the risk premia (expected excess return) on equities (defined as $\ln(1 + E[R_{t,t+1}])$) is given by:

$$\begin{aligned} r_t^f &= \delta - \varphi_{t,c}(-\gamma) \\ \widetilde{rp}_t &= \varphi_{t,c}(-\gamma) - \varphi_{t,c}(\lambda - \gamma) + \varphi_{t,c}(\lambda) \end{aligned}$$

where λ is the parameter linking dividends to consumption $D_t = C_t^\lambda$. We follow BCM and report risk premia as the expected log excess return ($E[\ln(1 + R_{t,t+1})]$) since this is what we calculate from the data in Table 1 (which because of Jensen's inequality is lower than $\widetilde{rp}_t$ - $E[\ln(1 + R_{t,t+1})] \leq \ln(1 + E[R_{t,t+1}])$ - and hence poses a tougher challenge to match the post-GS period equity premium - for some of the calibrations the difference can be significant):

$$rp_t = \varphi_{t,c}(-\gamma) - \varphi_{t,c}(\lambda - \gamma) + \lambda\mu_{t,c} < \widetilde{rp}_t$$

Calibration

Data in Table 1 and used for the calibration targets are from the Bank of England 'A millennium of macroeconomic data' dataset (available at http://www.bankofengland.co.uk/research/Documents/datasets) and total returns for FTSE are from Global Financial Data.

The Barro (2006) explanation of the equity premium relies on an extreme specification of disasters (roughly 1% chance of a 40% decline in consumption), which in practice is impossible to estimate from available data from one country (Martin (2013), Chen, Dou & Kogan (2016)) and has been criticised for being implausible and inconsistent with asset price data (Backus, Chernov & Martin (2011, BCM)), and US macro data (Julliard & Ghosh (2013)). In particular, Barro uses a large cross-section of countries and assumes they all share the same probability distribution of disasters (see also Nakamura et al (2015)). As shown by BCM the data in options implies a much milder size of jumps as well as more frequent jumps. BCM and Martin (2013) show that the real distinct characteristic of Barro's jump calibration is about the extreme tail events (e.g. 8 standard deviation events).

We show that it is possible to generate the level of real rate, equity premium and consumption distribution (growth, volatility, skewness and kurtosis) without relying on Barro's extreme disaster calibration. The Gold Standard does not require as much non-Gaussianity (as there is no equity risk premia, no skewness and low excess kurtosis in growth). For the post-GS sample jumps are necessary to generate the skewness/kurtosis in the data and the high equity premia with low real rates. But the jump distribution required to do so is "reasonable" in the sense that the probability of observing large falls in consumption roughly matches the data well and we can losely match the skewness and excess kurtosis in the data. In contrast, the Barro-type calibration would imply having seen more than two falls of more than 20% when the largest fall observed was 14%, with the model-implied skewness of -10 and kurtosis of nearly 100 (see Martin (2013) and BCM), an order of magnitude more than in the data.

Keeping the same preference parameters across regimes $\{\gamma, \delta, \varphi\} = \{6, 0.025, 2\}$, which are reasonable and close to those used in Martin (2013) and BCM, we get the numbers shown in Table 2. We are able to match real rates, equity premium, average growth rate and skewness and kurtosis of growth but with a higher volatility than observed in the data (5.5% vs 3%). The probabilities of large falls, though much less extreme than in Barro and more in line with BCM, are higher than the empirical counterparts (but not way off). To reduce the volatility of consumption to 4% while still matching the real rates and equity premium would require more extreme jumps, leading to skewness and kurtosis of -6 and 39 - three times as high as in the data, similar to Nakamura et al (2015), but not as extreme as original Barro-type calibration.

One way to reconcile the higher implied volatility than in the data is that it has been shown (Parker & Vissing-Jorgensen AER 2009) that the aggregate exposure of (top) stockholders has increased, which Malloy, Moskowitz & Vissing-Jorgensen JF2009 have shown is able to explain asset prices better with more reasonable measures of risk aversion (around 10, as opposed to 30-40 for aggregate consumption).

Besides the necessity for more negative jumps in the post war period, the preference parameters needed to match the data exactly would also be different. The time discount parameter would have to be lower during the post-GS period and the risk aversion higher. This could be consistent with the increase in financial intermediation: the Brunnermeier & Sannikov (AER 2014) model, and many of the models in the financial intermediation literature, the financial intermediary is more patient; and leverage can raise the effective risk aversion (can act as habit in that the subsistence level is the amount of debt that need repaying instead of zero) as well as heterogeneity.