# Technical appendix

Reliable partners – speech by Ben Broadbent

30 March 2022

This Appendix provides further details of the simulations reported in Charts 4, 5 and 6.

We use a simple New Keynesian model, in which each time period, $t$, is one quarter of a year:

$$\begin{aligned} x_t = & \quad \delta \mathbb{E}_t x_{t+1} - \sigma\delta(i_t - \mathbb{E}_t \pi_{t+1} - r_t^*) \\ \pi_t - \pi^* = & \quad \beta \mathbb{E}_t(\pi_{t+1} - \pi^*) + \kappa x_t + u_t \end{aligned}$$

where $x$ is the output gap, $\pi$ denotes inflation and $\pi^*$ denotes the inflation target. Following Haberis, Harrison, and Waldron (2019)[1], we set $\sigma = 1$ and $\kappa = 0.025$. We set $\beta = 0.995$.

Textbook New Keynesian models (for example Woodford 2003[2]; Galí 2008[3]) typically assume that $\delta = 1$. This in turn implies that the effect on the output gap of a change in the real interest rate in the very distant future is identical to the effect of a change in the short-term real interest rate. That property of the model gives rise to the so-called 'forward guidance puzzle' (Del Negro, Giannoni, and Patterson 2015[4]; Gabaix 2020[5]). A variety of small changes to the underlying textbook model give rise to an equation for aggregate demand with $\delta < 1$ so that changes in the real interest rate in the distant future have strictly less effect on spending today than changes in real interest rates in the near term. Our implementation follows Rannenberg (2021)[6] and we set $\delta = 0.9$, midway between the values of 0.85 and 0.96 used by Gabaix (2020) and Rannenberg (2021) respectively.

The model is driven by two exogenous shocks, which follow simple stochastic processes:

$$\begin{aligned} r_t^* - R^* = & \quad \rho_{r^*}(r_{t-1}^* - R^*) + s_{r^*}\varepsilon_t^{r^*} \\ u_t = & \quad s_u \varepsilon_t^u \end{aligned}$$

where $\varepsilon_t^{r^*}, \varepsilon_t^u \sim \mathbb{N}(0,1)$. We set $\rho_{r^*} = 0.85$ following Haberis, Harrison, and Waldron (2019) and choose $s_{r^*}$ and $s_u$ so that the model implied variances of inflation, output gap and the short-term interest rate are similar to those in the data.[7] The values chosen are $s_{r^*} = 0.4, s_u = 0.15$.

The loss function used to guide monetary policy and evaluate outcomes is

$$\mathcal{V}_t = \mathbb{E}_t \sum_{\tau=t}^{\infty} \beta^\tau \left((\pi_\tau - \pi^*)^2 + \lambda x_\tau^2\right) = \mathbb{E}_t \sum_{\tau=t}^{\infty} \beta^\tau \mathcal{L}_t \qquad (1)$$

and $\mathcal{L}$ denotes the 'period loss' and we set $\lambda = 0.25$. This is somewhat higher than the values often implied by approximations to the utility of representative households in a textbook model (for example Woodford 2003; Galí 2008) but closer to the values often used by policymakers in indicative simulations (for example, Yellen 2012[8]; Carney 2017[9]).

---

[1] Haberis, Harrison and Waldron (2019), 'Uncertain Policy Promises.' European Economic Review.
[2] Woodford (2003), 'Interest and Prices: Foundations of a Theory of Monetary Policy'.
[3] Gali (2008), 'Monetary Policy, Inflation, and the Business Cycle'.
[4] Del Negro, Giannoni, and Patterson (2015), 'The Forward Guidance Puzzle'. Federal Reserve Bank of New York Staff Report No. 574.
[5] Gabaix (2020), 'A Behavioral New Keynesian Model'. American Economic Review.
[6] Rannenberg (2021), 'State-Dependent Fiscal Multipliers with Preferences over Safe Assets.' Journal of Monetary Economics.
[7] Under a simple Taylor rule for monetary policy and ignoring the zero lower bound. The resulting standard deviations for the output gap, annual inflation and the annualised policy rate are (approximately) 1.5, 1 and 2 respectively.
[8] Yellen (2012), 'Perspectives on Monetary Policy'.
[9] Carney (2017), 'Lambda'.

Our baseline assumptions for monetary policy assume that the policymaker sets the nominal interest rate to minimize (1), subject to a lower bound on the nominal interest rate:

$$i_t \geq b$$

and the lower bound is imposed in the simulations using the algorithms described in Harrison and Waldron (2021).[10]

The average distance of the interest rate from the lower bound is given by $R^* + \pi^* - b$. We simulate the model for alternative assumptions about the average amount of policy space and the conduct of monetary policy. In each case, the model is simulated for 256,000 periods and the average period loss, $\mathcal{L}$, is computed. The charts in the main text plot the square root of this loss, normalised to 1 for the case of commitment policy with 5% average policy space.

The model is simulated under four alternative assumptions about the conduct of monetary policy:

**Discretionary policy**: the policymaker sets the interest rate in a time-consistent manner to minimize (1). Time consistent policy means that the policymaker sets policy optimally today recognising that future policymakers will act in the same way. So today's policymaker cannot influence future policy actions and therefore cannot make credible commitments about the future path of the interest rate.

**Commitment policy**: the policymaker sets the interest rate according to a policy rule that minimises (1) under commitment. The commitment policy allows the policymaker to influence output and inflation in the near term by making credible promises about the behaviour of interest rates in the future.

**Imperfectly credible commitment policy**: the policymaker sets the interest rate according to a policy rule that minimises (1) under commitment but recognising that private agents believe that the policymaker will switch to the discretionary policy with a constant (5%) probability each period. Even though the policymaker never switches from the commitment policy, the fact that agents doubt the credibility of the policymaker's commitments implies that credible promises about the behaviour of interest rates in the future have less traction over output and inflation in the near term.

**Unresponsive policy**: every $N$ periods the policymaker announces a path for the interest rate over the next $K$ periods. The policymaker then follows the plan (for the $N-1$ subsequent periods) and then announces a new path and the process repeats. Each announced path is consistent with the path for the interest rate under optimal commitment, conditional on *no future shocks arriving*. Because shocks subsequently *do* arrive, the policymaker is therefore unresponsive to the economic news (interest rates are set according to the pre-announced plan). For simplicity we consider the case in which $N = 2$ and $K = 4$.

[10] Harrison and Waldron (2021), 'Optimal Policy with Occasionally Binding Constraints: Piecewise Linear Solution Methods'. Bank of England Staff Working Paper No. 911.