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An investigation of the effect of funding on the slope of the yield curve

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Abstract

Market practititioners often have a firm view that funding operations have clearly observable effects on the slope of the yield curve. The standard theory of the expectations model of the yield curve, however, suggests that the sole determinant of the slope of the yield curve is expectations of future short-rates and so funding policy , ceteris paribus, should have no effect. This paper develops a high frequency set of data for the UK yield curve. It then uses principal components to decompose the yield curve into a set of factors which represent the level of returns, the slope of the curve and higher order effects. We then concentrate on the determinents of the second principal component as a measure of the slope and use a GARCH-M model to investigate the effects of funding on this variable. We find strongly significant effects from the stock of government bonds of varying maturity bands on the slope of the yield curve. This supports the practitioners view and argues that factors such as market segmentation are more important than simple theories might suggest.

Introduction

The determination of the yield curve is of interest from a number of viewpoints. Macroeconomists and policy makers for example are interested in the yield curve because they often believe that direct influence over interest rates can only be exercised at the short-term end of the spectrum while long rates have the most powerful effects on the economy (through investment, for example). Market traders and portfolio managers believe the determination of the yield curve to be a central question due to the link between yields and asset prices and the profit opportunities which may be created by a better understanding of yield curve changes. A central bank may be interested for both of these reasons, but may also interested in the question of the behaviour of monetary aggregates, which may be affected by the slope of the yield curve. A further consideration may be the cost of servicing the national debt and whether changing the structure of the debt can reduce servicing costs.

Earlier studies in the Bank [most notably Taylor (1990)], have outlined the basic theories of yield curve determination and have suggested that there is some evidence that the expectations theory of the yield curve is not a complete explanation. This paper is intended to pursue this line of empirical research further by developing a more comprehensive data set than has hitherto been used and by focusing more tightly on the specific question, 'does the composition of the stock of government bonds affect the slope of the yield curve?'

Below we present a brief description of the various theories of yield curve determination, a more detailed exposition may be found in Melino (1988) or Shiller and McCulloch (1987).

The dominant theory for many years within the academic literature has been the expectations theory, stemming from early work by Keynes (1930), Fisher (1930), Lutz (1940) and Hicks (1946) . This simply formalises the notion that, if agents are risk neutral profit maximizers with uniform expectations, a perfectly efficient market will set a long rate as a simple function of the expected future short rates so that expected profits will be equalised between holdings of either a long bond or a sequence of short bonds.

We may define the yield to maturity for a bond with n periods to maturity ($R^n{}_t$) as the solution to the following formula,

$$P_t^n = \sum_{i=t}^{t+n} \frac{C_i}{1+R_t^n} + \frac{V}{1+R_t^n} \qquad (1)$$

Where P^n is the price of the bond, C_i is the coupon paid on the bond in period i and V is the par value. This formula simply makes the yield to maturity the rate which equates the present value of the bond and its current price. So if a bond with n periods to maturity gives an overall return of $(1+R_t^n)^n$ then the expectations model would suggest that this should be equal to the expected product of short rates over the next n periods. That is,

$$(1+R_t^n)^n = (1+R_t^1)(1+E_t(R_{t+1}^1))\dots(1+E_t(R_{t+n}^1)) \tag{2}$$

where $E_t(R^1{}_{t+i})$ is the expectation formed at period t of the one period yield in period $t+i$. If this relationship does not hold it suggests that some market participants could expect to make profits simply by arbitraging between bonds of differing maturity. The consequence of this model is rather strong in that it suggests that the only factor which affects the slope of the yield curve is expectations and hence funding, or any other form of market intervention, can have no direct effect.

Another long standing model is the liquidity preference model, Hicks (1946). This may be thought of as a minor generalisation of the pure expectations model to allow for the possibility that agents are risk averse. So when an agent buys a six month bond he is uncertain of its value three months into the future while a sequence of two three month bonds would give complete capital certainty at three and six months (although the income over the whole period would then be uncertain). It is then argued that an agent will require a premium to give up the capital certainty of the three month bond in favour of holding the six-month bond and thus the yield curve would be expected to slope upwards. One way to motivate this model is in terms of the term premium; if we define the holding period return to be,

$$H_t^n = \frac{P_{t+1} - P_t + C_t}{P_t} \tag{3}$$

That is the total return earned by the bond between period t and $t+1$ expressed as a rate, then we may define the term premium as,

$$T_t^n = E(H_t^n) - R_t^1 \tag{4}$$

So, the term premium is the difference between the expected gain to holding a bond with n years to maturity over the next period and the gain to holding a one period bond over that period. Under the full expectations model the term premia should be zero, whereas the liquidity preference model suggests that the term premia should be an increasing function of n. The liquidity preference model is, however, still just a generalisation of the expectations model and thus there is no scope for other factors such as funding to enter the determination of the yield curve.

The only theory which offers a real scope for the positive manipulation of the yield curve by market participants is the market segmentation (or preferred habitat) model. This model sees the market for government debt as being made up of quite separate areas of operation with little or no overlap in market operators between different areas of the market. Under this condition it is possible that profit opportunities between different sectors could go unexploited and so there would be a real scope for effects from funding and other government policies on the yield curve.

Empirical work [Kessel (1965), Shiller, Campbell and Schoenholtz (1983), Fama (1984, 1984b), Pesando (1978, 1983), Mankiw (1986), Modigliani and Shiller (1973), Sargent (1979), Hansen and Sargent (1981), Cox et al (1981, 1985), Shiller (1981), Macdonald and Speight (1988) and Mills (1991)] has established that in its simplest form the expectations model is not completely adequate. It is generally agreed that the term premium increases with maturity for example. The notion that the market segmentation model may hold so that the outstanding amount of government bonds may actually affect the slope of the yield curve is, however, not widely accepted. Market participants often believe that the yield curve changes as they operate in the market, but academic studies have found it hard to substantiate this belief.

> *'Despite the fact that in a world of disparate expectations and risk aversion debt management operations ought to affect the yield curve, we could find no evidence from our studies on UK data that it has done so.'*
>
> [Goodhart and Gowland (1978)].

A good statement of the current academic view may be found in the conclusion of Shiller and McCulloch's (1987) survey paper

> *'Empirical work on the term structure has produced consensus on little more than that the rational expectations model, while containing an element of truth, can be rejected. There is no consensus on why term premia vary.'*

In this study we attempt to answer this question by taking a rather different line of empirical research to that normally followed:

- First we derive a high frequency (daily) data base on the yield curve of government bonds and the outstanding stock of bonds with various periods to maturity over the period 2nd January 1979 to 21 August 1990. The use of daily data is very important as one possible explanation of the lack of firm results in previous studies is that market segmentation effects may be important in the very short run but monthly or quarterly data may obscure these effects. This data set is new to the Bank and we will spend some time describing both the method of its construction and the data set itself in this paper. Using very high frequency data for estimation has a number of advantages. It obviously makes estimates much more reliable in a general sense simply because of the increase in degrees of freedom, but in this case there is another very strong reason for using a very high frequency data set. If we begin, as we do, from the premise that to characterise the yield curve we need more than the simple expectations theory, then we have to accept that some of the variables which may be important to test more elaborate models are simply not available. In particular, figures of the outstanding amount of non-government bonds are very hard to obtain as are stocks of foreign government and non-government bonds. However, at a daily level it would be very surprising if there was a strong correlation between government bonds for which we have measures, and the other factors for which we do not. On the not unreasonable assumption that at this frequency the correlation is low (or zero) then omitting such factors from our analysis should not induce biased coefficients but simply more complex dynamics. Another factor in choosing high frequency data is to deal with the criticisms that government bond stocks may be affected by changes in the yield curve and so estimation results may be biased. This may be true at the quarterly level but it is less obvious to propose that bond stocks respond at daily frequency to yield curve changes. Hence using high frequency data simplifies issues of causality.

- The second important aspect of this study is the use of principal components to produce a direct measure of the slope of the yield curve [a technique proposed by Steeley(1989)]. Thus we are able to focus directly on the question—does government funding policy affect the *slope* of the yield curve? This is important because changes in the slope of the yield curve account for only a very small part of the variation in yields so the chances of finding this effect must be much reduced.

- The final important feature concerns the time-varying nature of volatility in yield curve behaviour. This is now a widely accepted property of financial data in general. Here we adopt an estimation strategy which is specifically designed to deal with this feature, the details of which are given below.

The plan of this paper is then to begin in section 1 by describing the data set in terms of its construction. In section 2 we will apply a decomposition of the set of derived bond yield data using principal components to allow the estimation of a series which represents, as purely as possible, the slope of the yield curve. In section 3 we model this estimate of the slope of the yield curve, testing for stationarity, for a time varying covariance structure, and building a model which incorporates all these properties which allows a formal test of the effect of funding on the slope of the yield curve. Finally section 4 will draw some conclusions.

Section 1. The data

We have two rather different data sets, the yield to maturity on government bonds and the outstanding stock of government bonds. We will describe each in turn.

The yield data

Data construction

Gross prices on British Government stocks were collected for each working day between 2nd January 1979 and 21 August 1990 from 'Mullens' Blues' which are now published by Warburg Securities. The yield to maturity for each stock was calculated. For those bonds with dual redemption dates which were still traded after the 21 August 1990 it was assumed that they would be redeemed at their last redemption date. From this data set a more homogeneous set of gilts were derived by excluding index linked, undated and conversion stocks. Partly-paid stocks were also excluded until the first coupon payment after they became fully paid. To minimise possible tax effects any bond with a coupon rate well below the general yield was also excluded from the sample. After these exclusions 96 gilts remained of which 22 existed for the entire sample period. The longest maturity was the Exchequer 12% 2017 whilst the shortest maturity was the Treasury 9½% 1980.

The estimated yields to redemption and the maturity for each bond were used to calculate a yield curve for each working day using a cubic spline function. This was achieved using a version of the Bank of England's yield curve program [see Bank of England (1990)] in which the tax rates had been constrained to be equivalent for each of the gilts. The special tax calculations which are available within the yield curve program were unnecessary as those bonds which were affected were removed from our sample.

Data description

Having derived the spline function for the yield curve for each day over our sample (1979–90) it is then possible to read off a consistent set of rates for any maturity band. These are the yields to maturity for an n-year bond. We have done this for maturities of 2, 4 ,6, 8, 10, 12, 14, 16, 18, 20, 22 and 24 years. The time series for these yields are shown in Figures 1–12, while figure 13 shows the spread between the 2 year yield and the 24 year yield. In very broad terms we could characterise the period from 1979 to around 1985 (observation 1500) as being one of a fairly flat yield curve, 1985–87 was a period of moderate downward slope 1987–89 was one of a moderate upward slope and the period since 1989 as a fairly strong downward slope in the yield curve. Without prejudging the complex questions to be addressed later it is perhaps worth noting that this pattern accords fairly well with the expectations model 1985–87 could well be viewed as a period when expectations were looking forward to lower short-term interest rates and so we would have expected a falling yield curve. 1987–89 was a period of increasing concern about inflation rates and hence future interest rates and so we might expect the yield curve to rise and finally the most recent period would be seen as one where short term rates were above their expected long term level and so again we would expect the yield curve to be downward sloping.

The outstanding debt data

Data construction

Two sources of data were used to construct the outstanding amount of government bonds; one source for bonds without a redemption date and another for all other bonds. In the case of all bonds with an announced redemption date we obtained details of government transactions on any bonds which were outstanding over our data periods. This consists of the original quantity issued, the exact date of issue, the date and quantity of subsequent government trading in the gilt and the final redemption date (there were 725 market operations which affected the quantity of bonds over our data period). For each day of the sample period it was then possible to construct the total *face* value (not the market value) of bonds outstanding which would mature within any given period of that date. The chosen categories were bonds with less than one year to maturity, bonds with one–five years to maturity, bonds with five–ten years to maturity, bonds with ten–fifteen years to maturity and bonds with more than fifteen years to maturity. The data for irredeemable gilts were calculated in a slightly different way. In this case we had quantity data for total undated bonds on 5 July 1990 and a record of exact transactions (again amounts and dates) going back into the past. The construction of the outstanding debt series then consisted of deriving the time series from this transaction series.

Data description

The data for the amount of government bonds at varying maturities is shown in Figures 14–19. The most striking feature of these figures is simply how different they are from each other—the composition of debt has been varying quite markedly over the period. Bonds with less than ten years to maturity have clearly been rising quite rapidly, bonds between ten and fifteen years to maturity rose during the early part of the 1980s in line with shorter-term debt but the rise stopped in about 1985 and since 1988 a fairly steady reduction in debt has taken place. But perhaps the most striking feature is the behaviour of outstanding debt with more than fifteen years to maturity which has shown very marked fluctuations, rising rapidly in the 1979–81 period then falling sharply until 1985, rising again until 1988 and finally falling dramatically until the end of the period. The undated gilts fall consistently over the period and their actual quantity is much less than the other stocks.

Section 2: A decomposition of the yield data

Introduction

In this section we will take the yield to maturity data shown in Figures 1–13 and decompose it using principal components. A common practice is simply to take the spread between a short rate and a long rate as a measure of the slope of the yield curve. This practice builds a number of assumptions into the data; two actual data points are chosen, and this choice may affect the results. Also it is not clear how important this spread is in determining the yield at different maturities. Finally, it is not clear how this measure will respond to twists and changes in the shape of the yield curve. Principal components analysis circumvents having to choose just two arbitrary points and it also allows a systematic investigation of how important the slope (and other measures of the shape of the yield curve) are in determining yield levels.

We begin by giving a brief intuitive account of principal components, we then describe the application of this technique to our data, while a more rigorous account is presented in an appendix.

Principal components

As with many statistical techniques principal components can be derived in a number of ways depending on the motivation and framework used. One of the most common approaches is to motivate it as a way of dealing with multicollinearity. Taking an extreme assumption, if there are two variables which are perfectly collinear then obviously they cannot both be included in a regression model. Principal components will take these two variables and construct a variable which contains all the information in the two collinear variables. In the case of perfect collinearity it will need to construct only one variable which will contain all the explanatory power of the two collinear variables. In the case of less than perfect collinearity it is always possible to get the explanatory power of the original two variables with two new variables. But because principal components puts the variability of the two variables into the first principal component most of the explanatory power of the original two variables will be 'concentrated' in this one variable.

In a more formal way consider k variables $X_1 \ldots X_k$ and define a linear function of these variables as

$$l_1 = a_1X_1 + a_2X_2 + \ldots + a_kX_k$$

Now choose the a's so that the variance of l_1 is maximized subject to the condition that $\Sigma a_i^2 = 1$ (a normalization condition which simply prevents giving l an infinite variance by making the a's infinitely large); l_1 is then the first principal component. It will contain the largest proportion of explanatory power possible by combining all the X's together in one variable; in other words, it is the linear function of the X's which has the highest possible variance.

Now consider another linear function:

$$l_2 = b_1X_1 + b_2X_2 + \ldots + b_kX_k$$

and this time chose the b's to maximize the variance of l_2 subject to the condition that l_1 and l_2 are uncorrelated and the normalization condition $\Sigma b_i^2 = 1$. l_2 will then be the second principal component. This procedure can be repeated up to k times, giving $l_1 \ldots l_k$ mutually orthogonal variables which will be the full set of principal components. Furthermore it is possible to show that,

$$\Sigma\mathrm{var}(l_i) = \Sigma\mathrm{var}(X_i)$$

ie the total variability of the two sets of series will be the same. However, the l_i variables are uncorrelated with each other as opposed to the X_i variables which may potentially be highly correlated.

In the case of the yield curve we have a set of time series observations of points on the yield curve but these observations will tend to be highly collinear as the whole yield curve can be expected to broadly rise and fall with the overall level of interest rates. Our interest is largely focused on questions about changes in the shape of the yield curve rather than the more straightforward question of the determination of its overall level. But an investigation of these effects is rendered extremely difficulty by the collinearity between points on the yield curve. That is to say, we can explain ten-year rates very well by movements in five-year rates but what we are interested in is the relatively small changes which occur in the relationship between ten and five year rates. Principal components can be used in this context to isolate the different types of movement which take place in interest rates. The first principal component should capture all the movements which the term structure has in common across all maturities. We would expect this to explain most of the variation in all the rates. The higher principal components would then be associated with the various twists in the yield curve.

The number of points from the yield curve we feed into the principal component procedure will obviously affect the answer we get out. If we simply put two series in (a short maturity and a long one) then we could only derive two principal components, the first would have to be the level of interest rates while the second would have to be the slope of the yield curve. As we increase the number of series we input we can begin to allow for more complex twisting in the shape of the yield curve. Experimentation over the number of input series will allow us to judge how sensitive the results are to this choice.

Decomposing the yield curve

One of the major advantages of using principal components is that we can select a varying number of points on the yield curve and apply the technique to assess how sensitive it is to this choice. We have chosen to investigate 12 points (every two years from two to twenty four years), 6 points and three points. This means that for the first case up to 12 principal components can be generated, for the second up to six and for the third up to three. A number of features are of interest in these results; we can look at the cumulative R^2 to see how much explanatory power lies in each of the principal components. We can look to see how similar the estimate of the principal components are when different numbers of points are used and finally we can look at the 'factor loadings' which show how each component affects each point on the original yield curve.

The estimates of the first, second and third principal components from the 12 point case are presented in Figures 20, 21 and 22. The estimates for the first two principal components from the six and three point cases were virtually identical to these and so we will not present them, but the third principal component varied markedly between the three cases as one might expect. The first principal component very clearly reproduces the pattern of the level of interest rates (compare Figure 20 with Figure 10, the 20-year yield), and it is interesting to note that the principal component is actually smoother than any of the yield observations. So by extracting an overall measure of the level of yield rates we are able to filter out much of the noise which seems to affect individual bonds. The second principal component is also very similar in overall movement to the spread between short and long rates (Figure 13), so it is clearly capturing the basic notion of the 'slope' of the yield curve, although again it is clearly filtering out some of the noise associated with choosing individual points. We will not attempt any firm interpretation of the third component except to say that it seems to represent a tendency for a 'kink' to develop in the yield curve around 16 years maturity (this is shown in the factor loading tables below).

Now consider the cumulative R^2 for the three cases. Table 1 shows this.

Table 1
Cumulative R^2 for 12, 6 and 3 points from the yield curve

Principal component no.	12 point case	6 point case	3 point case
1	0.85	0.86	0.88
2	0.90	0.93	0.97
3	0.93	0.95	1.0
4	0.94	0.97	—
5	0.96	0.99	—
6	0.97	1.0	—

The interpretation of this table is that the general level of interest rates (the first principal component) explains over 85% of the variation in all individual rates. In the

12 component case the second principal component (the 'slope') explains about another 5% while in the 6 component case it explains about another 7% and in the three component case it explains a further 9% of the variation in the individual rates. Higher components each explain only a very small part (1 or 2%) of interest rate variation. This justifies focusing primarily on the two first components as the major explanation of yield rate variation.

Table 2 gives the factor loadings for the 12 component case. The factor loading show the correlation between each principal component and the original data.

Table 2.
The factor loadings for the 12 component case

Maturity of bond (years)	first component	second component	third component	fourth component
2	0.82	0.49	0.03	0.16
4	0.91	0.31	0.01	-0.09
6	0.95	0.11	0.04	-0.02
8	0.93	0.06	0.17	0.10
10	0.97	-0.07	0.12	-0.02
12	0.96	0.05	0.03	-0.13
14	0.93	0.01	-0.13	-0.28
16	0.88	-0.04	-0.44	0.06
18	0.92	-0.18	-0.14	0.19
20	0.91	-0.24	0.29	0.00
22	0.92	-0.29	0.03	0.03
24	0.95	-0.17	-0.03	0.02

The factor loadings on the first principal component are interesting in that they are all around 0.9 except for the loading on the very short term yield. The interpretation of this is that for bonds of four years maturity and beyond a change in the general level of interest rates is reflected fairly uniformly amongst all rates but the short-term rate does not follow this simple relationship. Instead the second component loading factor shows that there is a high loading on the 2-year maturity yield relative to all the other maturities, so a large part of changes in the slope are reflected purely at the shortest end of the yield curve. Another point to note about the loading factors for the second component is the sign pattern—the effect of maturities of two to eight years is clearly positive, maturities from ten to sixteen years are very small and for practical purposes zero while maturities from eighteen to twenty-four years are clearly negative. This means that, for example, when the yield curve tilts so as to slope downwards more steeply we expect short-term rates to rise a lot, there will be a general increase in rates up to eight years in maturity, we expect no change in rates between ten and sixteen years maturity and we expect rates beyond eighteen years to fall. In other words *the yield curve generally tends to pivot around a point in the ten to sixteen year maturity band.* The higher principal components loading factors do not seem to be of great interest. The set of factors for the third component seems to indicate the presence of an occasional kink which develops around sixteen years maturity and most of the loadings for the fourth component are small. We do not report the loadings for the other components.

To confirm the strong link between the spread between short two-year rate and long-term 24-year yields and the second principal component (SPC) we performed the following simple regression of the second principal component on the spread.

$$\text{spread} = \underset{(70.5)}{0.711} + \underset{(139.9)}{1.410}\,\text{SPC} \qquad R^2{=}0.87$$

where spread = two-year rate minus 24-year rate.

This indeed confirms that there is a very close correlation between the second component and the spread.

The next section will investigate the time series properties of the second principal component and it will introduce the data on the stock of bonds into a formal model of slope determination.

Section 3. Testing for the effects of changing the debt structure

Introduction

This section will analyze the second principal component (SPC) derived above as a measure of the slope of the yield curve, by examining the univariate properties of the series and investigating the effects of changing the stock of government debt on the slope of the yield curve. Much recent work on financial data has stressed the presence of time varying volatility, as periods of high volatility seem to come in waves with intermittent periods of relative calm. It is important to recognise this property of financial data when carrying out formal tests as to ignore these effects when they are in fact present can completely invalidate the test procedure. We will proceed by first estimating a univariate model of the SPC and then estimating a model which allows for this changing volatility. This second model will demonstrate the important presence of time varying volatility and so the formal tests of the effects of bond stocks on the SPC can then be conducted within this framework. The specific model we use to deal with the time varying volatility is a member of the class of ARCH models. The next section will outline these models before giving the formal tests.

ARCH models

Engle, Lilien and Robins (1987) suggest an extension of Engle's (1982) ARCH model whereby the conditional first moment of a time series itself becomes a function of the conditional second moment, which follows an ARCH process:

$$y_t = \alpha' x_t + \delta h_t^2 + \varepsilon_t \tag{5}$$

$$\varepsilon_t \mid \Omega_{t-1} \sim N(0, h_t^2)$$

$$h_t^2 = \gamma_0 + \sum_{i=1}^{n} \gamma_i \varepsilon_{t-i}^2 \tag{6}$$

where x_t is a vector of conditioning variables, weakly exogenous for all parameters of (5) and (6). Engle, Lilien and Robins (1987) term this kind of model ARCH-in-mean or ARCH-M. Note the h_t^2 is the conditional variance of ε_t formed at period t based on the set (α-field) of all information up to period t-1.

A further extension of the ARCH formulation, which imposes smoother behaviour on the conditional second moments, has been suggested by Bollerslev (1986). In Bollerslev's GARCH formulation, the conditional second moments are functions of their own lagged values as well as the squares and cross-products of lagged forecast errors. Bollerslev did not consider the GARCH-M extension although this is a fairly obvious one which was subsequently used in Bollerslev, Engle and Wooldridge (1988). Thus, for example, the GARCH-M (n, p) formulation of the above model would consist of (5) and

$$h_t^2 = A_0 + \sum_{i=1}^{n} A_i \varepsilon_{t-i}^2 + \sum_{i=1}^{p} B_i h_{t-i}^2 \tag{7}$$

where the B_i and A_i are coefficients.

Stacking all of the parameters of the system into a single vector

$$\mu = (\alpha, \delta, A_0, A_1, \ldots, A_n, B_1, \ldots, B_p) \tag{8}$$

and applying Schweppe's (1965) prediction error decomposition form of the likelihood function, the log-likelihood for a sample of T observations (conditional on initial values) is proportional to

$$L(\mu) = \sum_{t=1}^{T} (-\log \mid h_t^2 \mid - \frac{\varepsilon_t^2}{h_t^2}) \tag{9}$$

(where we have assumed normality of the forecast errors).

Although the analytic derivatives of (9) can be computed [see Engle, Lilien and Robins (1987)] variable-metric algorithms which employ numerical derivatives are simpler to use and easily allow changes in specification. Under suitable regularity conditions [Crowder (1976)], maximization of (9) will yield maximum-likelihood estimates with the usual properties.

Testing for the effects of bonds on the slope of the yield curve

We begin by presenting conventional tests of stationarity for the first and second principal components. These tests are of considerable relevance to the policy question as, in this context, we can associate the notion of stationarity with that of persistence. If a shock hits the level of the yield curve and the level is a non-stationary process we will, in general, expect that shock to persist indefinitely (that is to say the level will be permanently higher). If the level of the yield curve is a stationary process then the shock will be expected to persist for only a short time and the overall effect will be reversed. We may asses this question of non-stationarity in a number of ways, by using formal tests for stationarity (the augmented Dickey-Fuller test will be used) as well as informal descriptive statistics (the autocorrelation and partial autocorrelation function).

For completeness we begin by considering the first principal component.

Time-series properties of the first principal component

Augmented Dickey-Fuller test = -1.67

1	2	3	4	5	6
Autocorrelation function.					
0.998	0.997	0.995	0.993	0.991	0.989
Partial autocorrelation function.					
0.998	-0.060	0.018	-0.038	-0.004	-0.004
Autocorrelation function for squares.					
0.995	0.990	0.984	0.978	0.972	0.965
Partial autocorrelation function for squares.					
0.995	-0.084	-0.006	-0.053	0.004	-0.067

Note: The Dickey-Fuller test was augmented by four lags, this was sufficient to whiten the error process, although the broad conclusions were not affected by using other dynamic structures.

The Dickey-Fuller test does not allow us to reject the hypothesis that the first principal component is non-stationary and all the evidence from the descriptive statistics strongly supports the view that the first principal component (the level of rates) is non-stationary and shocks would be expected to persist.

We now turn to the second principal component.

Time-series properties of the second principal component

Augmented Dickey-Fuller test = -4.00

1	2	3	4	5	6
Autocorrelation function.					
0.991	0.981	0.970	0.960	0.949	0.939
Partial autocorrelation function.					
0.991	-0.035	-0.022	-0.015	-0.005	-0.008
Autocorrelation function for squares.					
0.973	0.944	0.916	0.890	0.865	0.838
Partial autocorrelation function for squares.					
0.973	-0.040	-0.006	0.022	-0.001	-0.053

In this case we can decisively reject the hypothesis of non-stationarity in the Dickey-Fuller test and the descriptive statistics give a clear picture (bearing in mind that this is daily data) of a stationary process. So we can reach a fairly robust conclusion that the second principal component is a stationary time series. This means that over time any shock which hits the slope of the yield curve will tend to reverse itself. In view of the work presented below it is also worth noting that the partial autocorrelation function for the SPC suggests a fairly low order autoregressive process would be adequate as a time-series representation of this series. Also the partial autocorrelation function for the squares of the series suggests that a low order GARCH model of the variance process would also probably be appropriate.

Before testing directly for the effects of bond stocks on the SPC it is important to establish a reasonably adequate-time series model, so next we report a univariate model of the SPC estimated both by OLS and as a GARCH-M(1,1) model over the full daily data set. The regressions reported in the following tables use the second principal component using three points from the yield curve. The conclusions are unaltered if the second principal component using twelve points from the yield curve are used.

A time-series model of the SPC

	OLS	GARCH-M
constant	0.00063 (0.1)	-0.004 (1.6)
SPC_{t-1}	0.846 (46.6)	0.671 (82.9)
SPC_{t-2}	0.093 (3.9)	0.186 (8.4)
SPC_{t-3}	0.033 (1.8)	0.097 (4.9)
α	—	-0.765 (1.7)
A_0	—	0.002 (3.6)
A_1	—	0.185 (3.5)
B_1	—	0.454 (71.5)
COR1	-0.0001	0.056
COR2	-0.0007	0.042
COR4	-0.0077	0.035
COR8	0.0161	0.046

(t-statistics in parenthesis. CORi is the *i*th point on the correlogram).

In terms of serial correlation both models are reasonably satisfactory, the GARCH-M model does not have a highly significant role for the effect of volatility on the SPC, so it seems that periods of high volatility do not systematically change the shape of the yield curve. There are, however, very strong signs of ARCH effects in the residuals as shown by the highly significant coefficients A_1 and B_1. This suggests that proper tests of the market segmentation hypothesis should be conducted within a GARCH-M framework.

Having established this basic framework we can proceed to introduce a variable to capture the effects of changes in the composition of government bonds. Given the profile of bond stocks outlined in Figures 14 to 19 a sensible split seems to exist between bonds with less than ten years to maturity, which have generally been trending upwards, and bonds of over ten years to maturity which have been falling in recent years. So as the simplest measure of bond movements we use the proportion of bonds with a maturity of over ten years (PB10), this proportion is shown in Figure 23. This variable may then be entered into the model outlined above as a test of the market segmentation hypothesis, and under the expectations hypothesis it should have a zero coefficient. According to the market segmentation hypothesis we would expect an increase in this proportion to lower the price of long bonds thus raising their yield. As SPC is essentially the short yield minus the long yield, we would expect the SPC to go down giving a negative relationship between PB10 and SPC.

Testing the market segmentation hypothesis

	OLS	GARCH-M	
constant	0.09 (2.8)	0.164	(9.5)
SPC_{t-1}	0.84 (46.4)	0.80	(34.9)
SPC_{t-2}	0.09 (3.9)	0.15	(14.0)
SPC_{t-3}	0.03 (1.7)	-0.03	(1.8)
PB10	-0.19 (2.9)	-0.377	(10.7)
α	—	-0.59	(0.74)
A_0	—	0.0003	(2.9)
A_1	—	0.03	(2.8)
B_1	—	0.46	(64.9)
COR1	0.0000	0.03	
COR2	-0.0003	0.04	
COR4	-0.0053	0.07	
COR8	0.0178	0.07	

(t-statistics in parenthesis. CORi is the *i*th point on the correlogram)

We report here both the OLS and the GARCH-M results even though the OLS assumptions are violated mainly for completeness and to demonstrate that the test conclusions are not reliant solely on using this sophisticated estimation methodology. In both estimation exercises PB10 is negative and significant. The conclusion is therefore clearly that we can reject the expectations hypothesis in favour of the market segmentation view of the determination of the yield. Changing the composition of government debt does appear to change the shape of the yield curve.

Some subsidiary hypothesis

We will now consider a number of alternative hypotheses which might reconcile our findings with the simple expectations theory. The first of these is the idea, put forward by Goodhart and Gowland (1978), that the expectations theory may hold in the longer run but that market segmentation may exist for a short period. The results presented above suggest a long-run effect from the stock of bonds but in fairness this is presupposed by our choice of functional form. That is, by using the level of the percentage of government bonds we impose the result that, if this variable is significant, it will have a long-run effect. We can test this by including both the change in the variable as well as the level, so if there is only a temporary effect only the difference term will be significant but if the effect is permanent the level term will also be significant.

Testing for dynamic effects

	OLS	GARCH-M
constant	0.089 (2.8)	0.151 (8.7)
SPC_{t-1}	0.843 (46.4)	0.702 (90.9)
SPC_{t-2}	0.092 (3.9)	0.188 (8.8)
SPC_{t-3}	0.031 (1.7)	0.052 (4.6)
PB10	-0.185 (2.8)	-0.322 (9.2)
ΔPB10	-0.293 (0.1)	0.026 (0.02)
α	—	-0.812 (1.4)
A_0	—	0.002 (3.0)
A_1	—	0.162 (2.9)
B_1	—	0.453 (64.8)
COR1	0.0000	0.054
COR2	-0.0003	0.042
COR4	-0.0053	0.051
COR8	0.0178	0.059

(t-statistics in parenthesis, CORi is the *i*th point on the correlogram)

This result is quite striking in that the inclusion of the change in bond stocks is clearly insignificant in both the GARCH and OLS estimates. The conclusion then seems to be quite strong, the effect of bond stocks on the yield curve is permanent rather than transitory. Of course we must make a slight caveat to this, as with daily data it is very hard to be confident about very long run properties. It is always possible that the effect of stocks fades out over a period of years, and we cannot rule out this possibility. But the evidence we report points strongly towards a long lasting effect.

Another area in which our choice of model structure has been arbitrary has been the specific way we have proxied bond stocks. The choice of using the proportion of bonds with over 10 years to maturity was made partly on account of how bond stocks had changed over the sample and partly with a view to other principal component evidence which suggested that the yield curve had tended to pivot at a maturity around ten to sixteen years. But this is clearly something that warrants further investigation. So we produced a disaggregation of our single bond measure by defining the proportion of bonds with maturities between five and ten years (PB5-10), the proportion of bonds with between ten and fifteen years to maturity (PB10-15) and the proportion of bonds with over fifteen years to maturity (PB15+). The following table gives the results of investigating this disaggregation.

Testing more disaggregated bond stocks

	OLS	GARCH-M
constant	0.240 (1.6)	0.197 (2.5)
SPC_{t-1}	0.842 (46.3)	0.811 (94.1)
SPC_{t-2}	0.092 (3.9)	0.128 (5.2)
SPC_{t-3}	0.030 (1.65)	-0.023 (1.1)
PB5-10	-0.168 (0.8)	0.140 (1.2)
PB10-15	-0.554 (2.1)	-0.818 (5.5)
PB15+	-0.345 (1.6)	-0.283 (2.6)
α	—	-0.453 (0.9)
A_0	—	0.001 (2.6)
A_1	—	0.062 (2.6)
B_1	—	0.465 (57.4)
COR1	0.0000	0.04
COR2	-0.0002	0.05
COR4	-0.0047	0.08
COR8	0.0182	0.08

(t-statistics in parenthesis, CORi is the *i*th point on the correlogram)

In both sets of estimates the proportion of bonds between five and ten years is insignificant, in the OLS results it is negative while the GARCH results actually find an insignificant but positive effect. This suggests that the choice of a break between under and over ten years to maturity was a correct one. The category between ten and fifteen years to maturity has a more powerful and significant effect than the very long maturity group, although both are correctly signed. This may reflect the fact that the change in this group has been numerically much larger than in any of the others. On the whole we would judge that these results conform well with our initial aggregation choices.

Section 4. Conclusions

In this paper we have addressed the very specific question of whether the composition of government debt affects the slope of the yield curve. We have generated a high quality data set consisting of something over three thousand daily observations of both the yield curve, and the stock and composition of government debt. Using principal component analysis we have derived a measure of the slope of the yield curve which is thus separately identified from the general level of interest rates. We then model this slope measure using both standard OLS techniques and more appropriate GARCH estimation.

We find very significant effects from the composition of government debt on the slope of the yield curve. The effect seems to be long lasting, and it is not simply a transitory effect while expectations adjust over some short period. There seems to be a division at around ten years to maturity, so that increasing the stock of bonds over this maturity tends to make the yield curve slope upwards while increasing stock with a maturity of less than ten years tends to make it slope downwards.

Given our estimated model we can work backwards through the principal components to calculate the expected effect of a change in debt composition on the shape of the yield curve. Let us ask the question; *what would be the effect of raising the proportion of debt with more than ten years to maturity by ten percentage points (eg from 40% to 50%)?* The impact effect would be an increase in the long rate relative to the short rate by around 0.05 (eg from 1% to 1.05%) on the actual day of the effect. This would then be expected to grow over time as the market adjusted to produce a long run effect of almost 0.7 (eg from 1% to 1.7%).

These findings refute the simple expectations model of the yield curve in favour of a richer model which would undoubtedly fall within the general area covered by the market segmentation model. These results open up the possibility of policy makers actively manipulating the yield curve, something which is not possible if the expectations model holds in its simple form.

Figure 1
2-year ahead yield

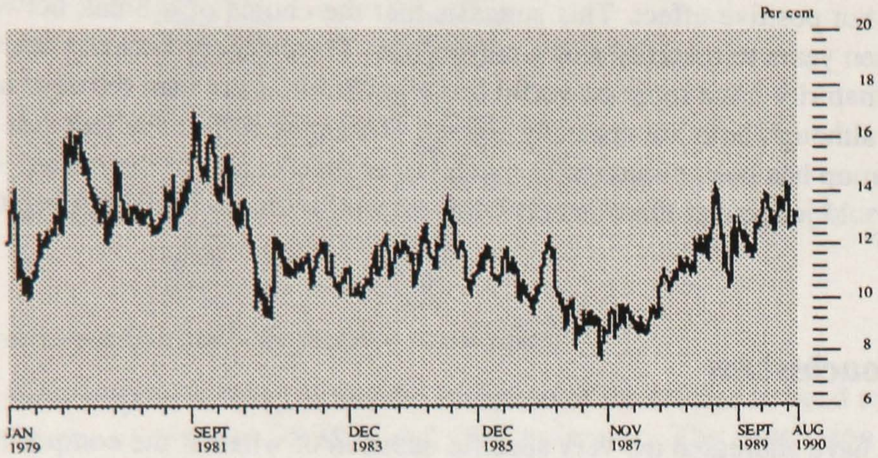


Figure 2
4-year ahead yield

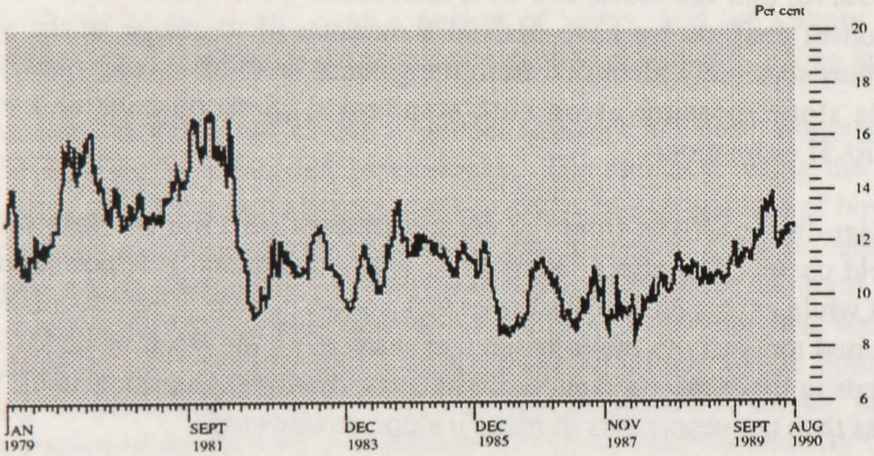


Figure 3
6-year ahead yield

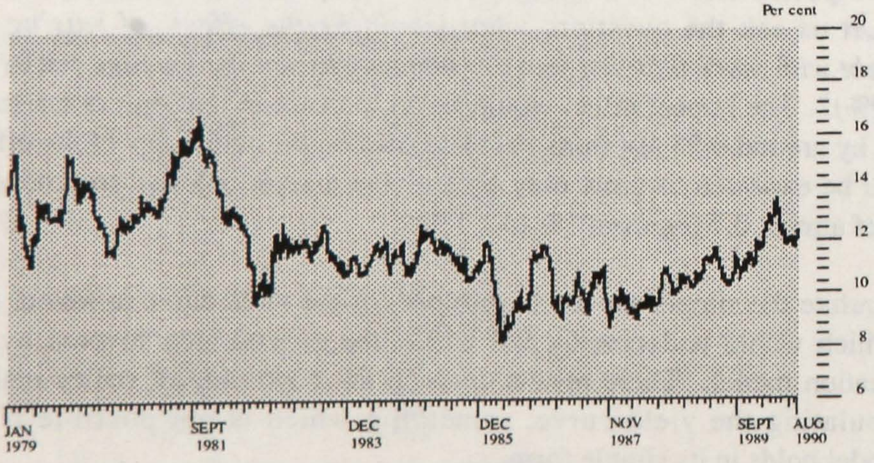


Figure 4
8-year ahead yield

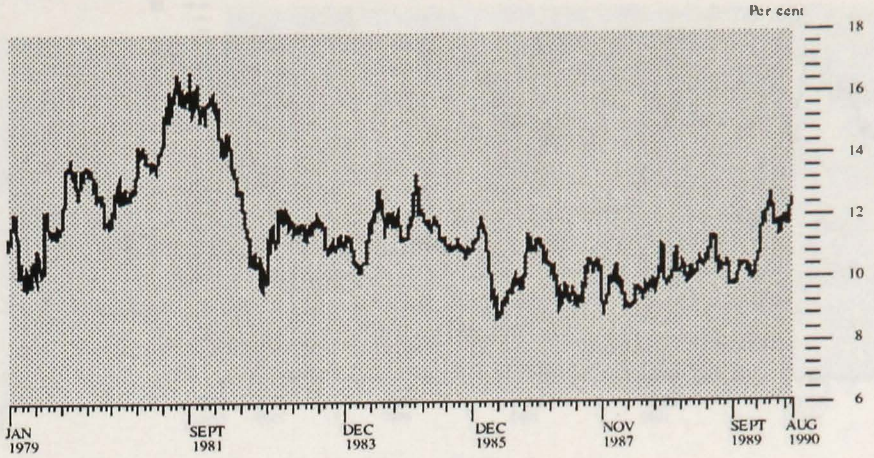


Figure 5
10-year ahead yield

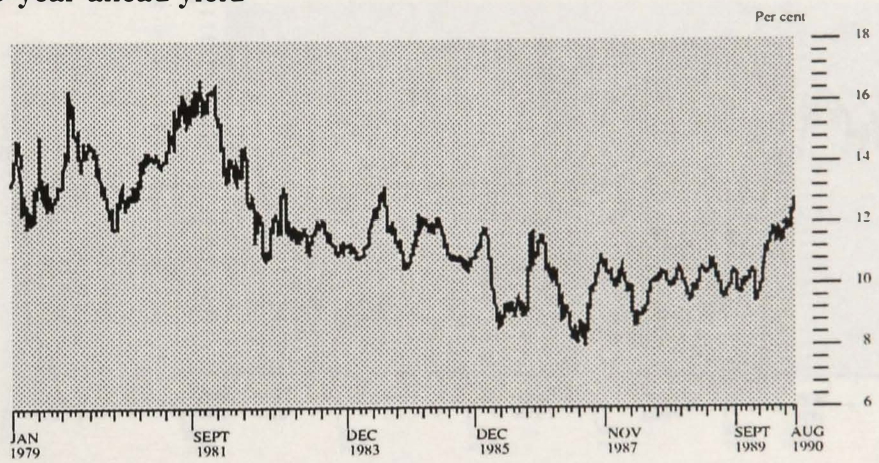


Figure 6
12-year ahead yield

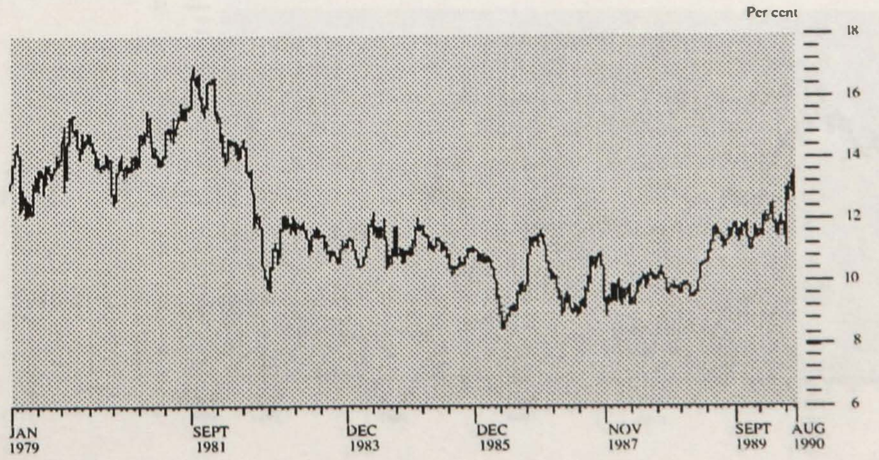


Figure 7
14-year ahead yield

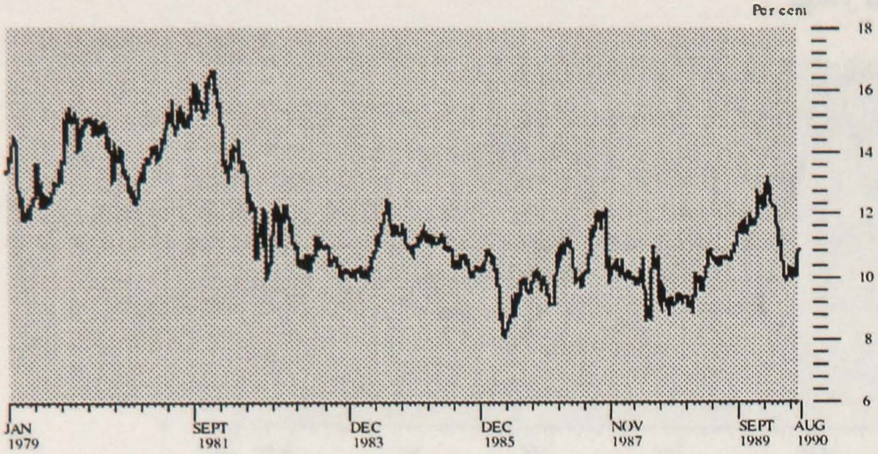


Figure 8
16-year ahead yield

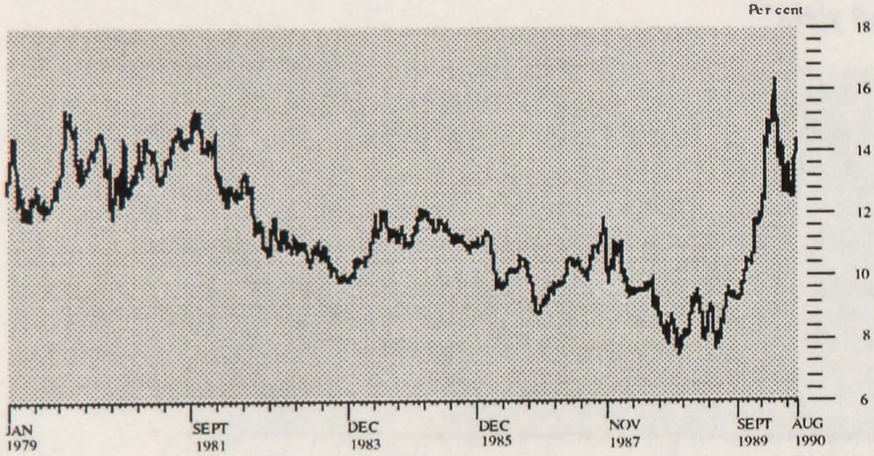


Figure 9
18-year ahead yield

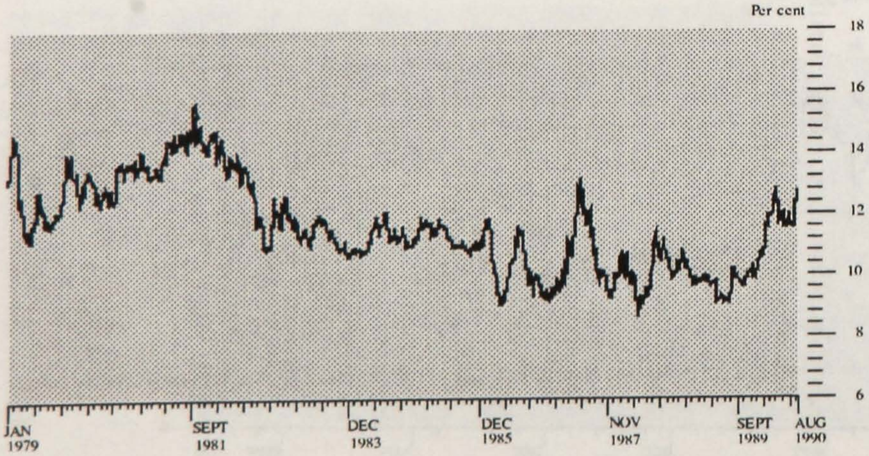


Figure 10
20-year ahead yield

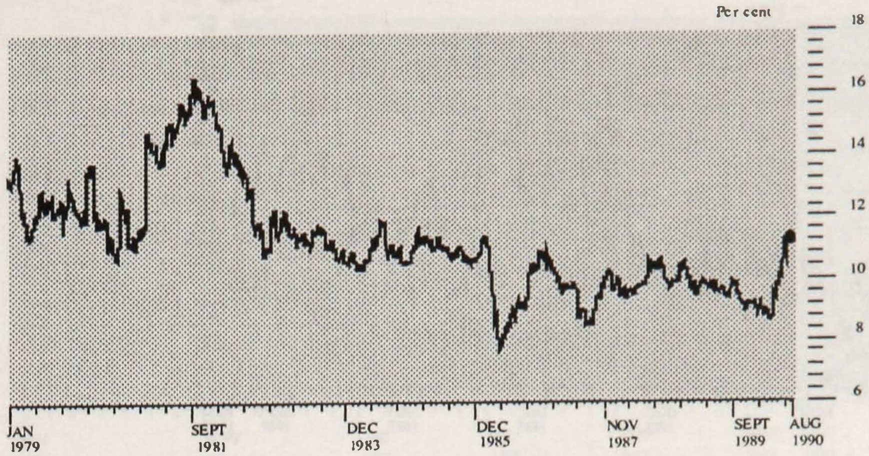


Figure 11
22-year ahead yield

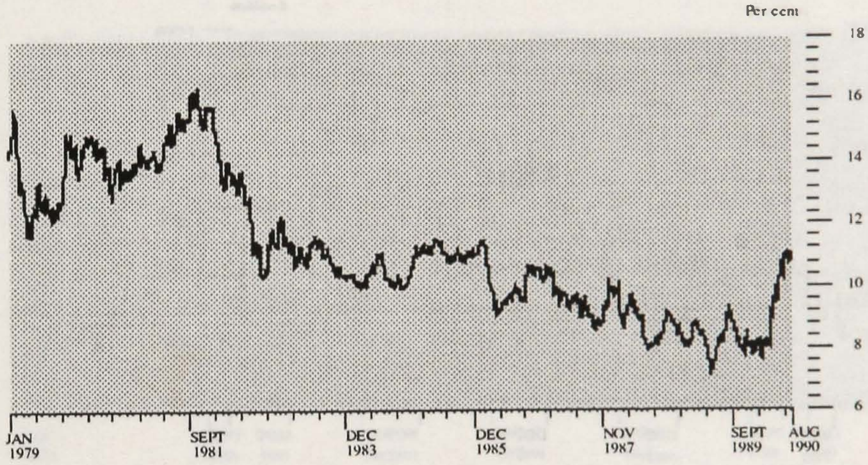


Figure 12
24-year ahead yield

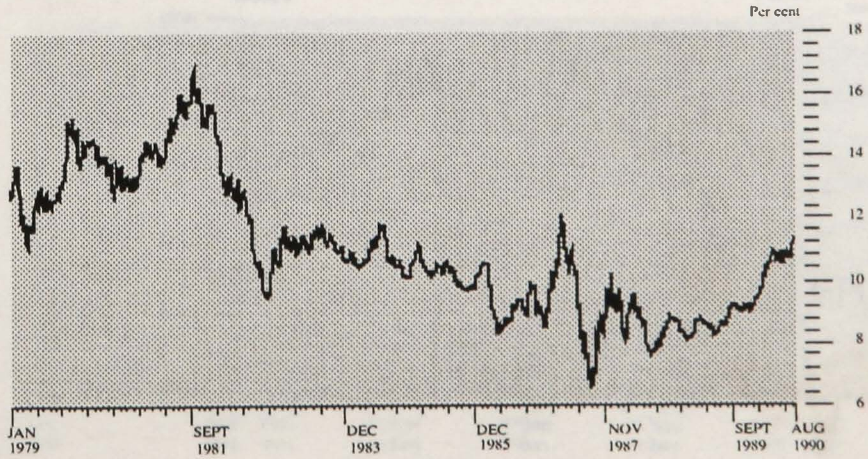


Figure 13
Difference between short and long rates

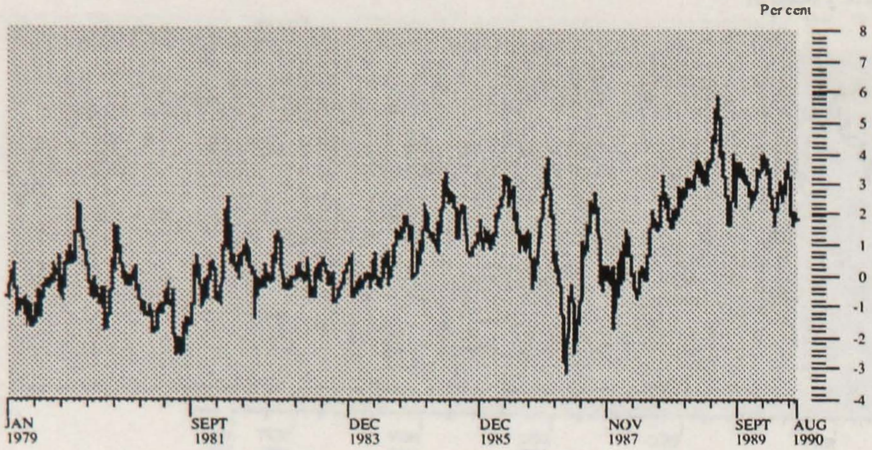


Figure 14
Bonds with less than 1 year to maturity

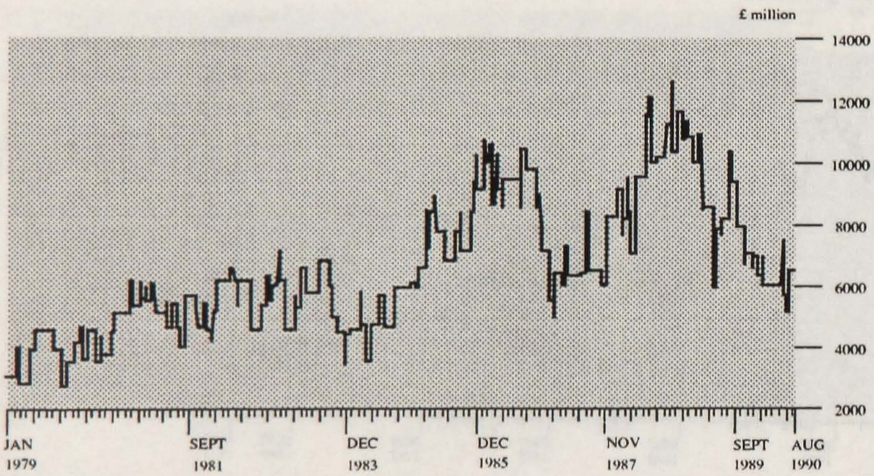


Figure 15
Bonds with between 1 and 5 years to maturity

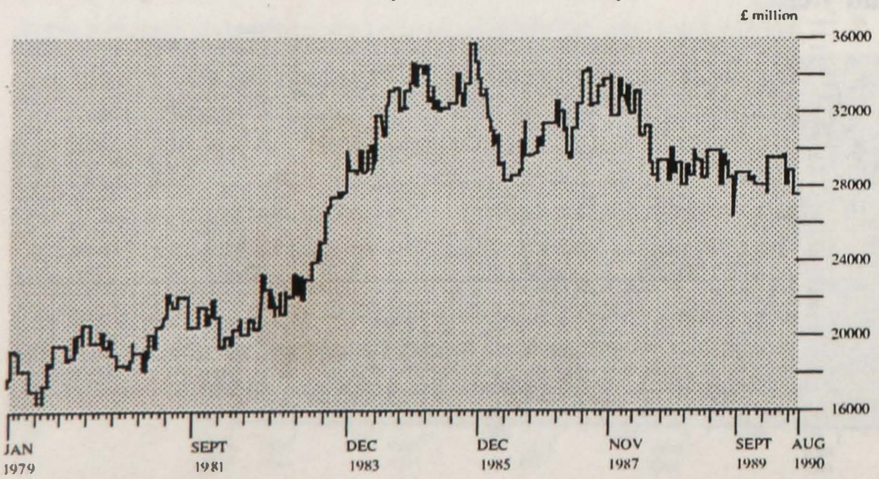


Figure 16
Bonds with between 5 and 10 years to maturity

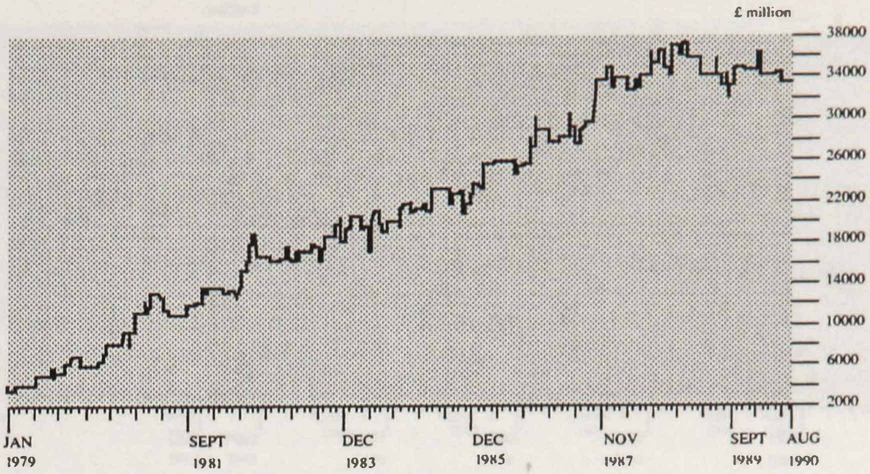


Figure 17
Bonds with between 10 and 15 years to maturity

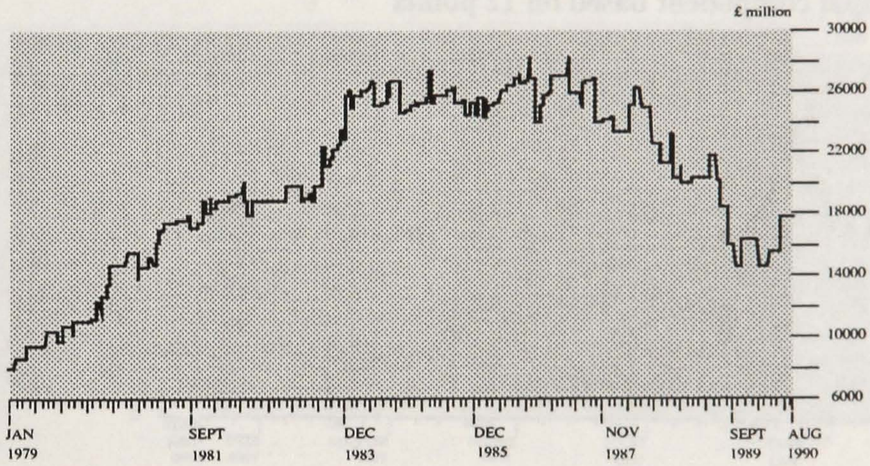


Figure 18
Bonds with more than 15 years to maturity

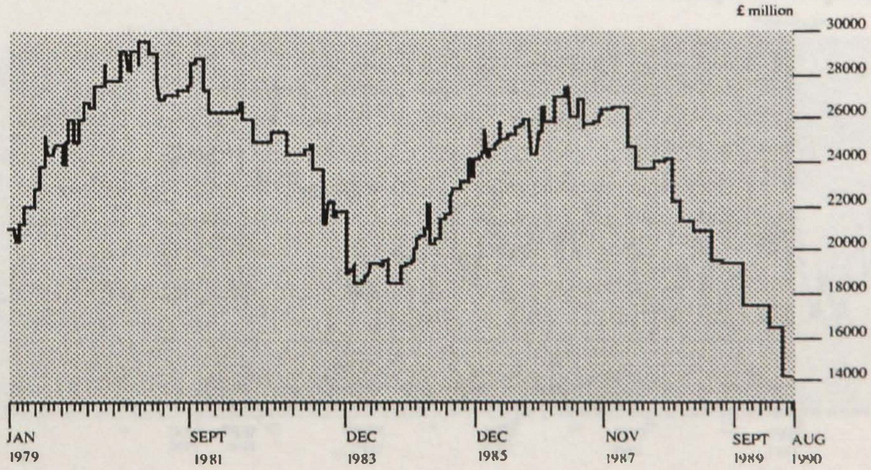


Figure 19
Stock of irredemable bonds

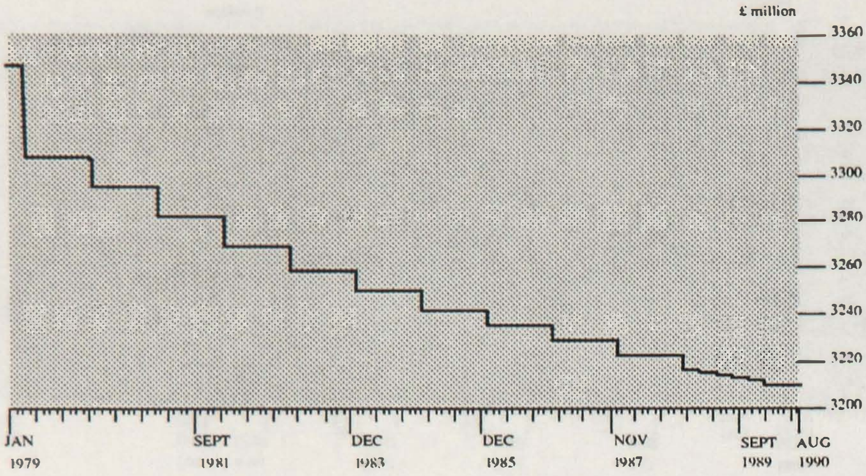


Figure 20
First principal component based on 12 points

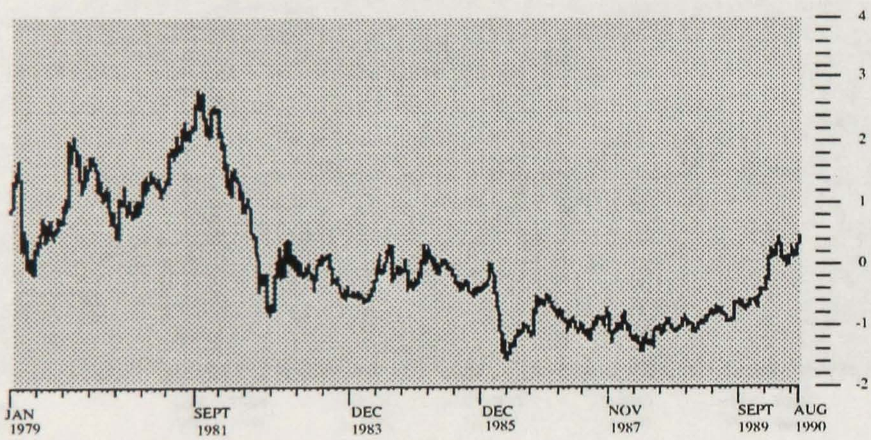


Figure 21
Second principal component based on 12 points

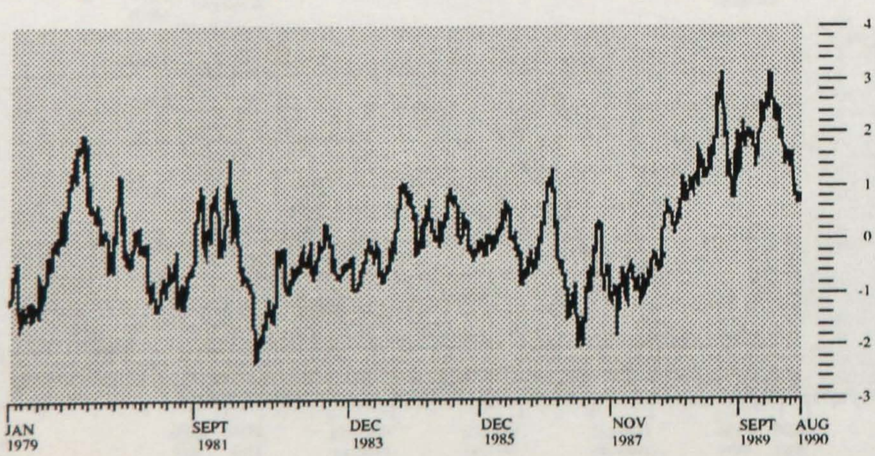


Figure 22
Third principal component based on 12 points

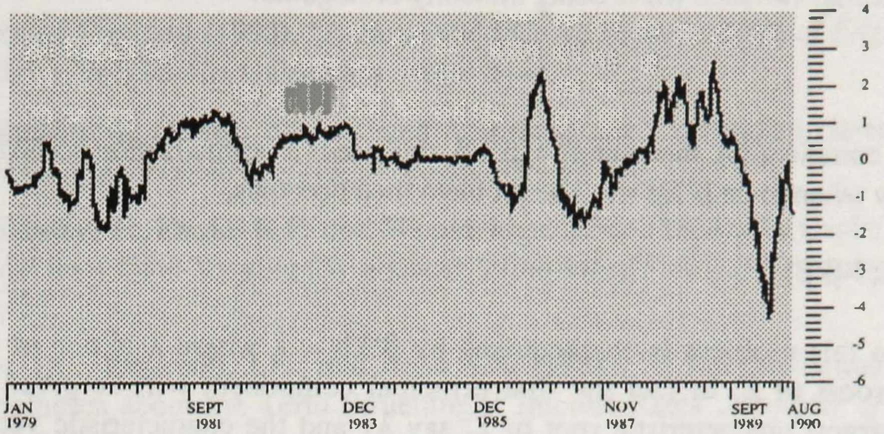


Figure 23
Proportion of gilts with a maturity of 10 years and over

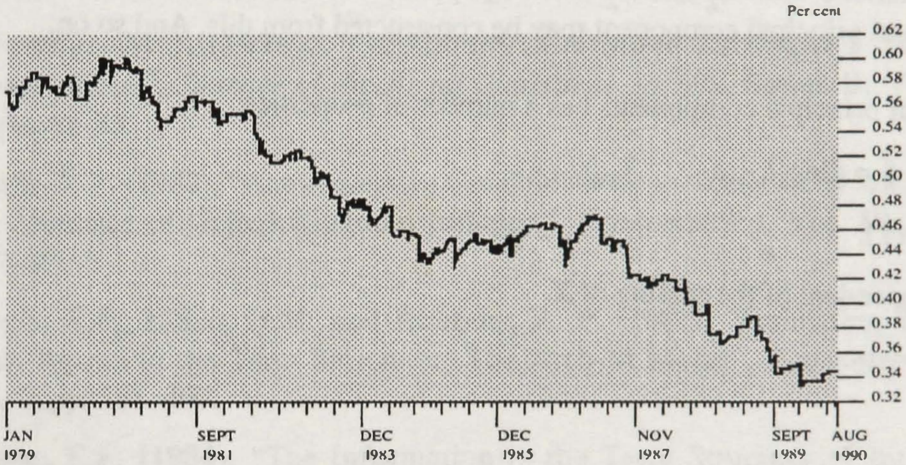
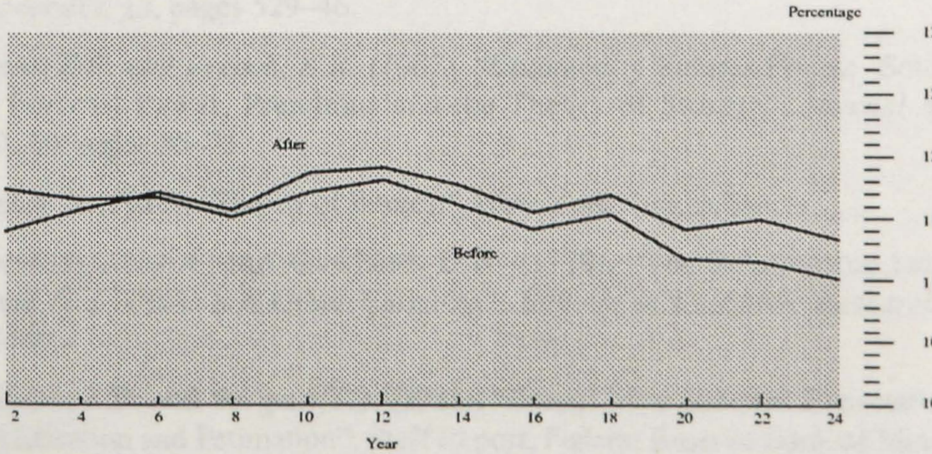


Figure 24
Yield curves before and after change in Gilts



APPENDIX. PRINCIPAL COMPONENTS

Let $X^{\sim}(\alpha,\Sigma)$ be an m dimensional vector, our objective is to define a set of variables which have maximal variance while being mutually orthogonal.

if $Y = \beta' X$

is such a linear combination, then var(Y) = $\beta'\Sigma\beta$, this needs a normalization constraint which is usually taken to be $\beta'\Sigma\beta = 1$. The problem then becomes,

MAX $\beta'\Sigma\beta + \lambda(1-\beta'\beta)$

The solution to this problem is characterised by $\beta'\Sigma\beta = \lambda$ where λ is one of the characteristic roots of Σ. In fact the first principal component will be given by choosing the largest characteristic root of Σ, say λ_1 and the characteristic vector associated with it will give β_1 from which the first principal component may be calculated. The second principal component will be given by choosing the second largest characteristic root, λ_2 and λ_2 will be the characteristic vector associated with it and the second principal component may be constructed from this. And so on.

The vector Y of principal components has a number of properties;

1 $Y \sim N(0,\S)$ $\quad \S = \mathrm{diag}(\lambda_1, \lambda_2, \ldots \lambda_m)$.

2 var(X) = var(Y).

3 Y is not independent of the scaling of X.

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