Divisia Indices for Money:

An Appraisal of Theory and Practice

by

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Abstract

Increasing interest has been shown in recent years in index number measures of money which weight the different components within each monetary aggregate. This paper presents an assessment of Divisia measures of money including an appraisal of the theoretical arguments for the Divisia approach to monetary aggregation. It also describes the construction of a Divisia index for the United Kingdom and discusses the potential relevance of Divisia for the assessment of monetary conditions.

Divisia indices for money: an appraisal of theory and practice

1 Introduction

Existing monetary aggregates can be thought of as defining the quantity of money as the weighted sum of the total value of a set of monetary assets, with weights of unity if an asset is included in the definition or zero if it is excluded. Such simple-sum aggregation does not distinguish between the 'moneyness' of the components of the aggregate: thus notes and coin in circulation are treated in exactly the same way as interest-bearing deposits and substitution of one for the other has no effect on the aggregate. But the former are pure media of exchange and are non-interest-bearing, whereas the latter are primarily a store of value, earning an explicit rate of return; so such switching may in fact be significant. In addition, not only do components of monetary aggregates differ in terms of their use for transactions purposes, but these differences change over time. The assumption that all components are perfect substitutes is therefore not an accurate representation of fact.(1)

During the last ten years there has been increasing interest in index number measures of money which weight the different components within each monetary aggregate. The majority of authors favouring this approach have applied the Divisia weighting scheme.(2) Barnett, Fisher and Serletis (1992) provide an excellent survey of the theory underlying monetary aggregation and the rationale for the Divisia index.

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(1) Simple sum measures of money may, however, be more appropriate for other reasons, such as to ascertain the size of the banking industry, where the relevant aggregate is the size of balance sheets.

(2) The Divisia index was originally proposed by Francois Divisia in 1925 and was first used to analyse monetary data by William Barnett at the Federal Reserve Board. In the United Kingdom, its advocates have included Batchelor (1988a; b), Belongia and Chrystal (1991), Ford et al (1992) and Spencer (1989, 1992).

The Divisia approach attempts to allow for the varying transactions properties of the components of a monetary aggregate by giving them different weights. If these weights reflect the differences in transactions services provided by various monetary assets, then the resulting Divisia index should be more closely related to total spending in the economy than are conventional monetary aggregates. If this theoretical case for weighted aggregates is accepted, then the usual choice between various monetary indicators (ie M0, M4, etc) should be expressed instead in terms of weighted narrow and broad aggregates. However, the issue of how best to measure 'money' is logically separate from the question of whether changes in money are related to movements in other macroeconomic variables. Although policymakers may be interested primarily in monetary indicators which exhibit stable relationships with intermediate or final target variables, the principal attraction of Divisia is its consistency with the microeconomic theory of consumer behaviour.

This paper presents a detailed appraisal of Divisia. It assesses the theoretical relevance of the Divisia approach for monetary aggregation and identifies factors which in practice might make the Divisia index a misleading measure of monetary services.(3) The paper is organised as follows: Section 2 outlines the basics of indexation and examines the rationale for choosing the Divisia index; Section 3 sets out the criteria for an admissible aggregate; Section 4 examines what Divisia actually measures, in particular whether it is possible to isolate the transactions services of monetary assets from the other functions of money; Section 5 considers the problems encountered in constructing the weights; Section 6 presents a preliminary Divisia index for the United Kingdom, describing its construction and presenting alternative indices based on different assumptions concerning the data used; Section 7 sets out the results of preliminary econometric tests; and Section 8 concludes with a discussion of the implications for policy formulation.

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(3) Monetary services can be defined generally in relation to the functions of money, ie store of value, unit of account, medium of exchange, etc. Where monetary assets differ only in terms of their use as media of exchange, these differences can be regarded as differences in transactions services provided by the assets. This issue is discussed further in Section 4.

2 Statistical index numbers and the rationale for Divisia

The microeconomic theory of aggregation outlines conditions under which a group of assets can be aggregated together and treated as a single good. An 'exact' aggregate is defined as a group of assets for which a consumer's demand can be solved in two stages. The first stage is the decision over the total quantity of the aggregate to be held, while the second stage is a separate decision regarding the quantities held of each of the assets that comprise the aggregate. Only aggregates that satisfy this condition will display well-defined demand functions. In order to construct such economic aggregates, the relationship between the component assets - the extent to which the assets are substitutes - has to be ascertained. This relationship - more generally known as the aggregator function - is typically a utility function (in the case of consumers), or a cost function (for producers). The parameters of the aggregator function, which need to be estimated, will reflect the degree to which assets can be substituted. Since the resulting aggregate will then depend on both the choice of estimator and the specification of the aggregator function, this procedure is unlikely to prove popular for official measures of money. However, simple sum aggregation implicitly entails exactly this choice, albeit by default, since the parameters are not estimated but are set equal to unity.

Whereas an economic quantity aggregate depends only on the component quantities and unknown parameters, a statistical index does not depend on any unknown parameters but on the component quantities and their prices. Thus a statistical quantity index avoids the need to estimate parameters of the aggregator function by making the quantity index dependent on the component prices. There are a variety of statistical index numbers and until recently there were no clear criteria for selecting the most appropriate for monetary aggregates.

The link between economic quantity aggregates (from the microeconomic theory of aggregation) and statistical index numbers was provided by Diewert (1976), who defined a class of 'superlative' index numbers which approximate arbitrary 'exact' aggregator functions. He defined an index number to be exact if it equals the aggregator function whenever the data are consistent with maximising behaviour. Hulten (1973) showed that in continuous time the Divisia index would satisfy this criterion for any weakly separable aggregator function. In the case of discrete time these exact index numbers do not exist, but Diewert has shown that the Tornquist-Theil Divisia index (ie the discrete time version) could provide a second-order approximation to any linearly homogeneous aggregator function.

The second development, due to Barnett (1978, 1980), was to derive the 'user' cost of monetary services, analogous to the user cost of durable goods. These user costs could then be used as the prices that are required in order to calculate statistical index numbers. Previously it was not clear what the price of a monetary asset should be specified as, and since all monetary assets were denominated in the same currency, they were considered to be very similar assets.

2.1 A Divisia index for money

A Divisia index for money weights each of its components according to the extent to which they provide monetary services. Providing there exists a well-defined aggregate and the aggregator function is linearly homogeneous then the Divisia index exhibits several desirable characteristics, the most important of which is that it is consistent with the original optimisation problem faced by the representative consumer. The index is a weighted sum of its components' growth rates, where the weight for each component is the expenditure on that component as a proportion of the total expenditure on the aggregate as a whole. This result is derived from the optimisation problem of a representative consumer and thus has a theoretical underpinning in microeconomics. Appendix A shows how the continuous time Divisia index can be derived from the consumers' utility function.

The discrete time approximation is given by:

$$ln\, D_t - ln\, D_{(t-1)} = \Sigma_i\, n_{it}\, (ln\, M_{it} - ln\, M_{i(t-1)}) \qquad \textbf{(1)}$$

where: D_t = the Divisia measure at time t

M_{it} = holding of asset i at time t

n_{it} $= 1/2\,(s_{it} + s_{i(t-1)})$

s_{it} $= (p_{it}\,M_{it}) \,/\, \Sigma_i(p_{it}\,M_{it})$

p_{it} = price of asset i at time t (defined below)

When considering monetary assets, the relevant price is the asset's user cost. This is the interest foregone through holding a monetary asset rather than a financial asset which offers a higher return but provides no monetary services. The relevant user cost formula can be derived rigorously from an intertemporal maximisation problem [Barnett (1978)]. However, this much simpler foregone interest argument is sufficient to provide the intuition behind the relationship. Appendix B outlines a formal derivation of user costs based on Donovan (1978).

The user cost (price) of each asset is given by:

$$p_i = (R - r_i) \tag{2}$$

where: R = return on 'benchmark asset'

r_i = return on monetary asset i

p_i = price or user cost of monetary asset i(4)

Although most studies have used the Divisia index, Rotemberg, Driscoll and Poterba (1991) have proposed an alternative weighted aggregate which is closely related to Divisia. Their 'Currency-equivalent' aggregate (CE) is defined as the total stock of currency required to provide the same amount of transactions services as is provided by all monetary assets. The index is a simple time-varying weighted average of the stock of all monetary assets, where the weights are the ratio of each asset's user cost to a benchmark return,

(4) In Section 6 these user costs are adjusted for taxes.

ie $CE_t = \Sigma[(p_{it})/R_t]\ M_{it}$. This index is a weighted average of the levels of monetary assets, whereas Divisia is a weighted average of growth rates.

There are a number of differences between this index and Divisia. The major advantage of the CE index arises when new assets are added to the aggregate. Since the Divisia index is based on changes in the logarithm of the component assets, by definition when a new asset is added its rate of change will be infinity. The CE index is also intuitively more appealing since, being expressed in levels, it is simpler to interpret than the chain-linked Divisia index. The CE index is, however, derived from more restrictive assumptions regarding the flow of monetary services. Many of the other characteristics of CE and Divisia are very similar. Thus, although the rest of this paper addresses Divisia, the issues we discuss are in general applicable to the CE index. In Section 6.3 we construct a CE index for the United Kingdom.

3 Determining the components of a monetary aggregate - weak separability

Following from the discussion on aggregation (Section 2), a group of goods can be considered as an aggregate - as if it was an elementary good - if the decision regarding the level of the aggregate to be held is independent of its composition. This requires weak separability of the group of assets in the utility function. In the absence of such separability, changes in the relative prices of its components which left the aggregate's overall price index unchanged would imply different levels of demand for the aggregate as a whole. In this case, a stable microeconomic demand function for the aggregate could not exist. Thus weak separability is a necessary condition for any collection of assets to be considered as an admissible monetary aggregate, including the conventional aggregates.

Early studies investigated which assets should be grouped together to form a monetary aggregate by estimating the elasticities of substitution between different assets [Chetty (1969)]. But this approach does not provide evidence for the separability of a particular group and furthermore it is likely that such parametric tests are sensitive to specification error in the functional form. Even if the correct specification for any period could be found, it is unlikely that the elasticities of substitution will remain constant over time.

A non-parametric test for the weak separability of assets has been developed by Varian (1982). The selection of assets for aggregation is based on whether the observed holdings of assets at prevailing prices (user costs) appear consistent with utility maximisation. Varian applies the Generalised Axiom of Revealed Preference (GARP) which is a sufficient condition for utility maximisation. If a finite set of price and consumption data is consistent with GARP, then there exists a utility function that could have generated that behaviour. It allows for all behaviour consistent with optimisation by a representative consumer, including multi-valued demand functions where a flat part of the indifference curve leads to different levels of demand being consistent with the same prices. If the observed consumption pattern within a subset of all goods satisfies GARP, given the total expenditure on that subset, then that subset forms a weakly separable group.

There are, however, some serious disadvantages with this approach. It treats observed holdings as equilibrium values, making no allowance for measurement errors and adjustment costs. Thus assets may be ruled out when the failure of the test is due to adjustment costs, rather than their absence from the weakly separable group.(5) When there are adjustment costs, the appropriate prices should incorporate these costs since asset holdings and portfolio reallocations will be based on the 'effective' user costs, rather than the user costs derived from the explicit own rates of return. More generally, the Varian test is not appropriate for dynamic models. Also, it does not allow for stochastic shocks which could lead to observed holdings of monetary assets being away from their equilibrium values. As a result it gives a binary yes/no result rather than the confidence intervals generally associated with statistical tests.

In principle, any good can provide 'transactions services'. One of the main advantages of Divisia is that it imposes no arbitrary limit on this theoretical continuum of available 'monetary goods'. The yes/no result of the Varian test seems inconsistent with the existence of a continuous spectrum of monetary assets. However, Varian's approach tests the validity of a priori restrictions. If we measure the transactions services provided by *all* goods using the Divisia approach, we have imposed no such restrictions - the Varian test is then simply irrelevant. Logically, the group of 'all goods' must form a weakly separable subset of all goods: a test of separability is superfluous. Obviously, the transactions services provided by some goods are negligible and they would be assigned Divisia weights close to zero. To make calculation of the Divisia index tractable, zero weight restrictions would be imposed *a priori* on such goods. If these restrictions can be tested to ensure the remaining goods form a weakly separable group, and thus an admissible aggregate, there would be no logical or theoretical contradiction between Divisia and Varian's separability

(5) In general any costs (actual or perceived) which result in a gradual adjustment of stocks to their desired levels could lead to a failure of the admissibility test.

test. In practice, however, these separability tests cannot deal with adjustment costs, or any other source of inertia in portfolio adjustment.

Separability does not, however, provide an objective criterion for deciding what is money and what is not. The restrictions imposed are entirely subjective and separability is simply a technical issue regarding the consistency of such restrictions with aggregation theory. For example, the chosen separable group of 'monetary assets' may itself be a subset of a larger separable group; there is no reason for the smaller aggregate to be 'money' and the larger not. Indeed, the Divisia approach does not impose a definition of 'money'; it is the monetary services provided that are important.

Existing UK Divisia indices largely ignore the separability issue [Spencer (1989, 1992), Batchelor (1988a; b)]. Work which has used the Varian approach [Belongia & Chrystal (1991)] has concentrated on testing the separability of subsets of the components of M4. This assumes implicitly that M4 itself consists of a weakly separable group of component assets.

4 What does Divisia actually measure?

4.1 Monetary and transactions services

It is far from clear what 'monetary services' are. Textbook monetary theory suggests that money performs three functions: unit of account, store of value and medium of exchange. Proponents of Divisia indices are concerned only with the last of these. With regard to the unit of account function, the choice of numeraire in an economy at equilibrium is arbitrary. All capital-certain nominal assets denominated in the same currency provide equally good stores of value.(6) Since Divisia indices are normally restricted in their coverage to such assets, store of value services are controlled for in their construction. These services therefore do not need to be allowed for in the weighting procedure. Advocates argue that Divisia needs to allow only for the varying transactions services offered by monetary assets if the objective is to arrive at a more refined measure of money for which demand is likely to be stable with respect to the usual determinants.(7)

Monetary assets, however, display a range of characteristics, only some of which relate to their use as a medium of exchange. Many bank accounts offer investment advice, longer branch opening hours and easier overdraft facilities. Not all of these features provide for transactions, but they are 'monetary services' to the extent that their availability is contingent on holding monetary assets. Thus the Divisia measure is of wider scope than solely transactions services. Since these characteristics are not uniform across different categories, differences ought ideally to be allowed for in the weighting procedure.

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(6) This argument assumes the assets under consideration all have zero default risk.

(7) Since this demand function is derived from the representative agent's optimisation problem, only if certain restrictions are satisfied (identical preferences, endowments, etc) will there exist a stable macroeconomic demand function. We note this point to emphasise that the theoretically appealing framework of Divisia may not yield a stable macroeconomic monetary aggregate. This argument, however, is equally relevant to simple sum aggregates.

In principle, each characteristic of a bank or building society account could be assigned an implicit price and a pure transactions index calculated using the transactions elements. In practice, the multiplicity of accounts and characteristics would make the problem intractable. Non-price competition and product discrimination among banks and building societies have vastly expanded the range of accounts available. Furthermore, this process would require a subjective judgment about what to include, undermining some of the original attractiveness of the Divisia approach. To the extent that the Divisia index does not measure only transactions services, it may not have a stable relationship with macroeconomic variables. However, since the non-price characteristics of monetary assets are likely to be relatively stable on the whole, changing only slowly through time when viewed in the aggregate, failure to allow for them may not be a serious omission in the present context.

4.2 Pricing of transactions services

Divisia defines transactions services implicitly, using the observed interest rate to compute a user cost for the services provided by each asset. Monetary assets are viewed as durable goods which render their holders a variety of services. The interest rates are assumed to be at perfectly competitive full equilibrium levels; they act as a 'summary statistic' containing all the available information regarding how the market values the services provided by monetary assets. At full competitive equilibrium, the observed interest rates reflect fully the shadow price of the services provided by the asset.

If the banking industry is imperfectly competitive there may be significant non-price competition. The explicit own returns on assets will not then reflect the true returns to asset holders. A more serious issue is the failure of equilibrium market interest rates to capture the full shadow price in the presence of externalities. Given the 'social' nature of a medium of exchange, this is likely to be a recurring problem. For example, the transactions services provided to an individual by a bank current account depend on how many other people and institutions have such accounts. The more people with accounts, the wider the acceptability of bank cheques. Yet the increased benefits of a bank account to an individual arising from such externalities do not increase the bank's costs and thus, even in equilibrium, will not be reflected in market interest rates. Money performs some intrinsically social functions, a point made frequently in the literature on the origin of money.

The fundamental problem may be due to the fact that the theory of the demand for money is not well founded. Including money in utility functions has often been questioned. Economists have generally had to resort to overlapping generations models or cash-in-advance type constraints to motivate the demand for money. In the absence of a coherent theory it is difficult to ascertain the appropriate prices. In some recent work Kiyotaki and Wright (1989,1991) have modelled the existence of money by relating it to its acceptability as a medium of exchange. They show that decisions by agents regarding whether to hold money depend on the probability of it being accepted as a medium of exchange and therefore their willingness to incur the costs of holding money can be related to the nature of monetary equilibrium. The relation between their notion of costs and the user costs in Divisia is not clear. What their work does suggest, however, is that actual user costs may be more complicated than the simple difference between the return on two assets.

Thus in practice Divisia measures all those services whose cost is reflected in the asset's equilibrium interest rate. Without modelling explicitly the supply side of the services market, these services cannot be identified precisely. Divisia simply assumes that differences in interest rates measure differences in transactions services provided. However, many of the services offered carry explicit charges which are part of the price of transactions services afforded by bank accounts. When, for example, banks increase the interest they pay on current accounts, and at the same time introduce charges for services provided, the Divisia user costs may indicate a significant but spurious change in transactions services provided. In principle the user costs should be calculated on the basis of net returns (explicit return less an imputed charge). Typically, however, the charges depend on account usage and the true user costs will not then be independent of quantities held and turnover. A rough attempt to explore the significance of the associated measurement problems is reported in Section 6.1.

4.3 Linear homogeneity

A related issue concerns the technical requirement that the transactions services function be homogeneous of degree one in asset holdings. The attractiveness of the Divisia approach is founded on its derivation from the first order conditions of an optimisation problem and its consequent consistency with microeconomic theory. To complete the derivation, linear homogeneity is required so that the Euler relationship can be employed. This relates the total value of transactions services to the partial derivative of each asset in the aggregator function and thus to equilibrium prices. Intuitively, linear homogeneity ensures that the aggregate will grow at the same rate as its components and that the Divisia weights will sum to unity (this is demonstrated formally in Appendix A).

The assumption of linear homogeneity seems plausible for a measure of transactions services - doubling all money holdings would double the transactions services available. However, the broader measure of services that Divisia actually provides is unlikely to satisfy this restriction. Investment advice and access to overdraft facilities are generally contingent on having a bank account rather than the amount held in it. Doubling bank deposits would not necessarily double the advice the bank provided. In cases where linear homogeneity is violated (for example, when services come in discrete 'lumps' rather than continuously, as with the example of investment advice), Divisia gives a poor measure since it relates the growth rate of asset holdings to that of total services, not allowing for the discontinuities that are likely to characterise this relationship.

5 Problems in constructing the weights

5.1 Financial innovation and the effects of gradual adjustment

Financial innovation has posed considerable problems in the empirical estimation of money demand equations. Proponents of Divisia have argued that financial innovation, particularly the introduction of interest-bearing accounts, is one of the main advantages of assigning different weights to the individual components of monetary aggregates. It is helpful to consider two forms of financial innovation - product innovation and technological innovation.

Product innovation is where banks use existing technology to introduce new types of accounts - the innovation is essentially to offer the existing characteristics of financial assets in different combinations. This type of product innovation, which has been extensive in the UK, is to a large extent the result of increased competition in the financial services industry. The Divisia index should in principle be able to account for this since the trade-off between the consumption of commodities and the consumption of transactions services will not be affected. Asset holders will reallocate their money holdings without altering the aggregate consumption of transactions services. In general the Divisia index should also capture the effects of financial deregulation, which allows banks to offer a wider product range.

The effects of technological innovations may change the parameters of the aggregator function, in which case the trade-off between the consumption of commodities and the consumption of transactions services will change. An example of this would be new technology, such as the introduction of ATMs and the wider use of credit cards, which increase the transactions services provided by existing asset holdings without increasing the user cost. Even when the representative agent is fully optimising in the face of given interest rates, the observed user costs and asset holdings may not reflect the direct impact of technological improvements. To the extent that such innovations are not reflected in *equilibrium* interest rates, the Divisia index will mismeasure the growth of transactions services (see Appendix A for a formal derivation of the deviation between the growth of transactions services and the Divisia index).

Koenig and Fomby (1990) and Ford, Peng and Mullineux (1992) suggest modifications of the Divisia index when there is non-neutral technical change. Koenig and Fomby recognise the need for parametric estimates of the aggregator function as the correct procedure, although as we have noted earlier this implies that the measure of money is subject to the choice of the specification of the functional form and the estimation procedure used. Ford et al *(op cit)* introduce a series of adjustments to the Divisia measure to account for technological change. They assume that interest rates fail to respond to innovations and impose 'refinements' upon the Divisia index. In the short run this is likely to be appropriate because of adjustment lags, but when the full competitive equilibrium required by Divisia obtains, observed interest rate differentials may fully reflect the costs and benefits of innovation. If they do, such adjustments are superfluous.

On the demand side, the Divisia approach treats observed holdings of assets as equilibrium values at the observed prices. Even if there are no supply side adjustments going on, that is to say, even if the observed interest rates can be assumed to be equilibrium rates, the assumption that asset holdings are at their desired values is inconsistent with the extensive evidence from both the theoretical and the empirical literature on the demand for money. There are likely to be adjustment costs, information asymmetries and so on, all of which imply that agents adjust their holdings of monetary assets gradually in response to changes in either the general level of interest rates or changes in relative interest rates between different types of bank and building society accounts. A recent example is the gradual decline of non-interest bearing accounts.

Spencer (1992) addresses this issue directly by smoothing the interest rate user costs before using them to construct the expenditure shares or growth weights. However, although the user costs are smoothed lagged interest rates, portfolio equilibrium is still assumed. For some assets this may not be plausible. It suggests, for example, that individuals fail to adjust out of non-interest-bearing current accounts into interest-bearing current accounts because of incorrect perceptions about the interest paid on interest-bearing accounts. This may be related to the general problem of using explicit interest rates to calculate user costs. In a banking system which is not perfectly competitive there will be some non-price competition, which implies that interest rates do not fully reflect total returns. It may therefore not be appropriate to group cash and non-interest-bearing accounts in one category. One implication of this procedure is that any subsequent estimated demand for money function will also have to be based on the same lags.

5.2 Choice of the benchmark asset

To assign user costs to each asset, an asset which does not yield any transactions services has to be selected against which the opportunity cost of these services can be measured. The user cost of a monetary asset is then simply the difference between the return on the non-monetary asset less its own rate of return.

In principle the non-monetary asset has to be capital certain in order to make it comparable to other monetary assets, and not to offer any transactions services. Assets which offer some transactions services should themselves be included in the Divisia aggregate. This implies that assets for which there are active secondary markets cannot be considered, since the existence of a secondary market would enable holdings of this asset to be converted readily into (more liquid) assets that could be used for transactions. There are not many assets which satisfy these two criteria. Some of the earlier work on Divisia [Spencer (1989) and Batchelor (1988a; b)] used the local authority deposit rate as the benchmark return. Another possible candidate could be National Savings certificates, although their holding period is typically longer than most monetary assets and the amount that can be held in this form is limited. It is however difficult to explain why some studies use corporate bond yields [eg Belongia and Chalfont (1989) and some of the earlier US studies by Barnett and others], since these include significant default premia and are traded in an active secondary market.

The benchmark asset will not in general be the same asset in different periods since money holders will in principle select the highest yielding non-monetary asset. Although advocates of the Divisia approach recognise this, in practice most studies use one single benchmark rate since, as noted earlier, there are few non-monetary assets that are capital certain and for which there is no active secondary market. More recent work, however, has adopted the approach of utilising the maximum available rate from a given data set of prices as the benchmark.

5.3 Negative user costs - the downward sloping yield curve

Even if a number of non-monetary assets fulfilled the above criteria for a benchmark asset there still remains the more serious problem of negative weights on some monetary assets in periods when markets expect interest rates in the future to be lower than current interest rates, ie when the yield curve is downward sloping. This problem arises primarily because yields to maturity are used instead of expected holding period returns, which are by definition not observable.(8) The theory underlying the Divisia approach suggests that agents decide on their distribution of monetary assets on the basis of expected (holding period) returns, and in principle should be allocating their portfolio between different assets according to instantaneous expected returns.

For any given holding period the problem of negative weights will not arise: if interest rates are expected to fall, all expected holding period returns will fall accordingly. The problem, however, is how to measure expected returns. Normally one could use the term structure of interest rates to work out what implied forward rates are, and by the expectations hypothesis of the term structure these can be treated as expected future interest rates. However, this would imply that there is no term premium, which is one characteristic which Divisia is attempting to capture. For example, the difference between holding

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(8) It is also possible that some assets have other characteristics. For example, they may be more acceptable as collateral and as a result have lower yields than assets which provide transactions services. Rotemberg et al *(op cit)* cite this argument for not using Treasury bills in the United States as the benchmark asset.

four successive seven-day accounts and a one-month account. From implied forward rates the expected holding period returns on these two assets would be equal, suggesting that they are not different in terms of their 'moneyness'. Therefore, given that the term structure cannot be used to calculate expected returns, an alternative way of measuring expectations would have to be found. Modelling the term premium on a range of very short-term monetary assets has to our knowledge not been attempted and is unlikely to be straightforward.

Advocates of the Divisia approach have argued that a moving average of lagged interest rates should be used to overcome this problem. This would be feasible - the necessary interest rate data on most of the relevant monetary assets should be available - but the user costs would not be the current opportunity costs. At times when interest rates change there would be significant distortions to the Divisia aggregate, especially because the true weights on cash relative to interest bearing assets would be significantly different from the historic (lagged) weights.

6 A Divisia index for the United Kingdom

The Divisia indices presented in this paper are constructed from the components of the M4 broad money aggregate and the following interest rates shown in Table 1.(9)

(9) Appendix C sets out, in detail, the data utilised and their sources and Appendix D graphs the levels of the components.

Table 1: M4 components and interest rates used to construct Divisia

COMPONENT		INTEREST RATE
Notes and coin in circulation with the public		zero
Non-interest-bearing UK private sector sterling sight deposits		zero
Interest-bearing UK private sector bank sight deposits		
o/w	*Persons*	*Clearing bank instant access account rate (gross rate)*
	Corporates	*Overnight London interbank deposit rate*
Interest-bearing UK private sector bank time deposits		
o/w	*Persons*	*Clearing bank interest-bearing personal account rate (gross rate)*
	Corporates	*Three-month London interbank deposit rate minus 0.5%*
Building society deposits		
o/w	*Persons*	*Building society savings account rate (gross rate)*
	Corporates	*Three-month London interbank deposit rate*
(Benchmark asset)		Three-month local authority deposit rate

The quarterly index covers the period 1977 Q1 to 1992 Q4, providing sixty-four observations. All series are seasonally unadjusted - the index is then itself seasonally adjusted.

6.1 User costs

Obtaining the correct specification for user costs is important not only for the time path of Divisia, but also for the empirical tests to be carried out. The user costs are constructed as in equation (2) above, with $(R-r_i)$ adjusted for taxes, giving $(R-r_i)(1-w)$, where w is the tax rate for asset i.(10)

As noted earlier, when calculating user costs and weighting component assets the instantaneous expected holding period return rather than the interest to maturity should be used. The transactions services derived from each asset will then reflect the difference between the holding period return on the benchmark asset and the holding period return on monetary assets. However, since the instantaneous holding period return is unobservable, we use the returns to maturity. Rates for maturities of less than three months are averaged over the quarter.

There are, of course, practical problems in assigning interest rates to particular classes of deposit. In particular, a detailed breakdown of deposits and their respective returns is not readily available, nor is it clear what the most appropriate single interest rate is for calculating the user costs applicable to each category of deposit. The following illustrates a number of the practical difficulties in calculating user costs.

(a) The importance of the benchmark interest rate

Since the same benchmark rate is used for computing each component weight, then the higher is this rate compared with other interest rates, the more equal all the relative weights become.

Our preferred index uses the three-month local authority deposit rate as the benchmark interest rate. These deposits are non-marketable and non-chequable

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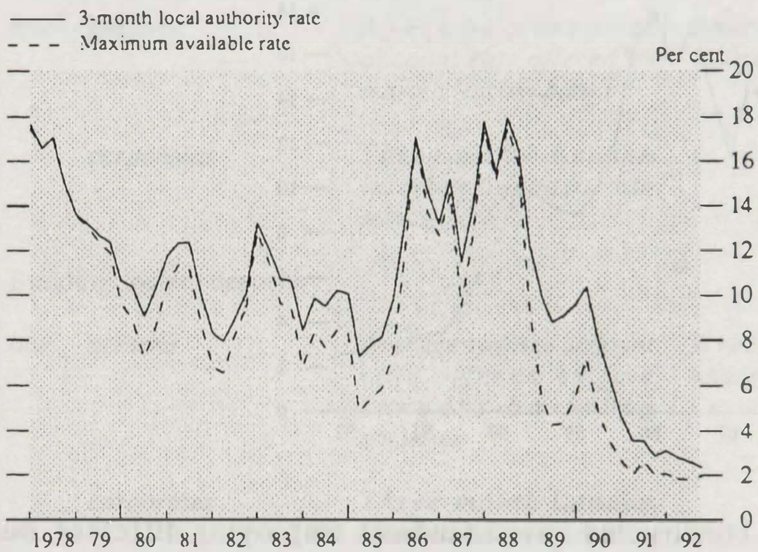
(10) The composite tax rate is used for interest-bearing retail deposits and the corporate tax rate for interest-bearing corporate deposits.

and so cannot be used for making transactions. Reflecting this, yields on this asset normally exceed those on other deposits, making the opportunity costs of these other deposits positive. However, there were periods in our sample when the returns offered on building society retail deposits and by banks and building societies on corporate deposits were higher than those available from local authorities, thus leading to negative weights. One simple solution to this problem is arbitrarily to add a constant to the benchmark rate. A constant of two percentage points is necessary to obtain positive weights throughout. This might be rationalised on the basis that local authority deposits have some residual liquidity, or a lower risk premium reflected in relatively lower interest rates.

Chart 1

Twelve-month growth rates of Divisia indices using different benchmarks



It is recognised that the benchmark asset will not in general be the same asset in each period since money holders will, in principle, select as their reference asset the one with the highest yield and this should be reflected in the calculation of user costs. A number of recent compilers of Divisia indices have adopted this approach and we have therefore constructed an alternative index where the benchmark rate of return is the maximum rate of return from the three-month local authority rate and from the interest rates on the different monetary assets within the index (see Chart 1).

A number of problems arise when using a maximum rate benchmark. In particular, the benchmark asset should not provide transactions services, so an asset included as money in one period should not be used as the benchmark in other periods.

(b) Rates of return

Chart 2

Twelve-month growth rates of Divisia indices using different rates of return

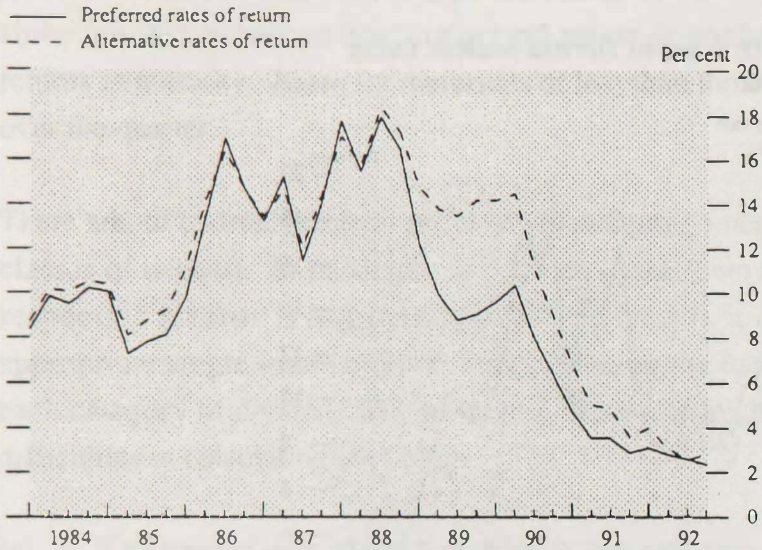


Chart 2 shows two constructed Divisia indices employing different, but arguably equally appropriate, interest rate series for corporate holdings of bank interest-bearing sight deposits and persons' bank interest-bearing time and building society deposits. The solid line represents the index analysed further in this paper, while the dashed line represents an index employing alternative rates of return, as set out in Table 2.

Table 2: Alternative rates of return for constructing Divisia

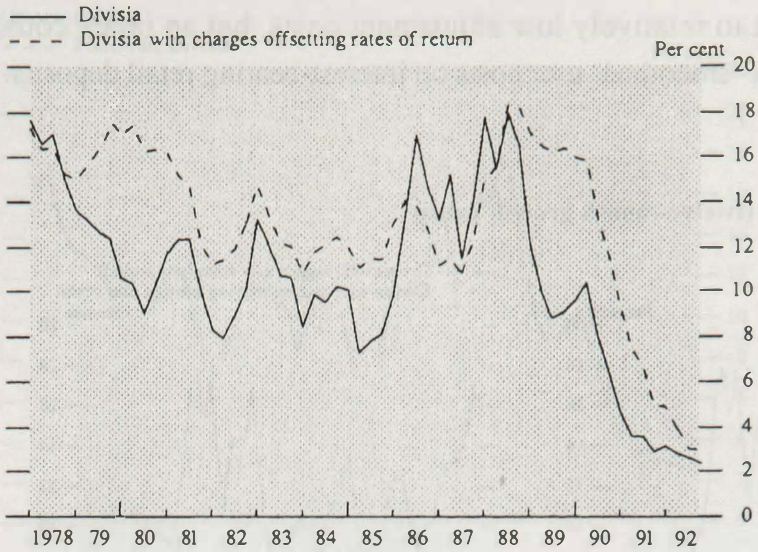
Component		Preferred Divisia rates of return	Alternative rates of return (where different)
Bank interest-bearing sight deposits			
o/w	*persons*	*Average current account rate offered by major clearing banks on deposits of £500*	
	corporates	*Overnight London interbank deposit rate*	*Base rate minus 3%*
Bank interest-bearing time deposits			
o/w	*persons*	*Average personal account rate offered by major clearing banks*	*Average rate on seven-day notice deposit accounts*
	corporates	*Three-month London interbank deposit rate minus 0.5%*	
Building society deposits			
o/w	*persons*	*Average savings account rate offered by five largest building societies*	*Average building society share rate*
	corporates	*Three-month London interbank deposit rate*	

The divergence between the two indices can be explained by changes in the respective user costs and growth weights of their components. The interest rate data for corporate sight and persons' time bank deposits used in constructing the preferred index are higher than those utilised in the alternative index, resulting in lower growth weights, but the retail building society rates are, on occasion, lower than those in the alternative index. One such period was from mid-1988 to 1991, with the largest differential emerging during 1989, which explains, in part, the stronger twelve-month growth rate of the alternative index during this period.

In addition to the question of the appropriate interest rates, there is the problem noted earlier of whether the appropriate rates of return, and hence the user costs, should take account of bank and building society charges. Each characteristic of a bank or building society account should, in principle, be assigned an implicit price. However, in the absence of adequate disaggregated information it is impossible to calculate a Divisia index which reflects accurately the impact of charges. Chart 3 plots a Divisia index where it is assumed that interest rates on interest-bearing retail components are fully offset by charges so that the rate of return is actually zero - in effect the user costs for retail interest-bearing deposits are calculated as the benchmark rate. The result is that the growth rate of the index is less trended and overall higher than the original index over the period in question.

Chart 3

Twelve-month growth rates of Divisia and of Divisia with charges offsetting rates of return



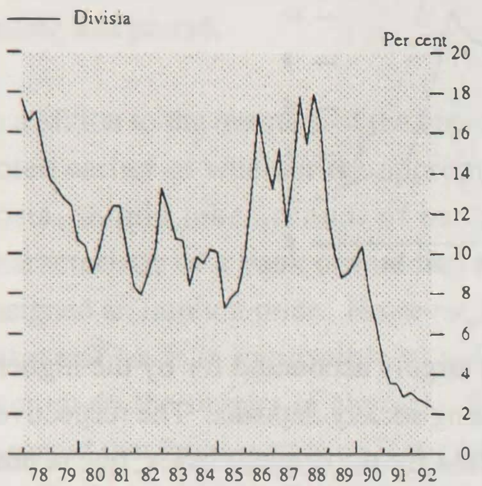
This increase in the annual growth rate is largely accounted for by the higher weights calculated for retail bank and building society deposits. The respective weights are more stable than those calculated for the original index, one result of which is that the strong growth from 1985 to mid-1986 is not fully replicated in the alternative index.

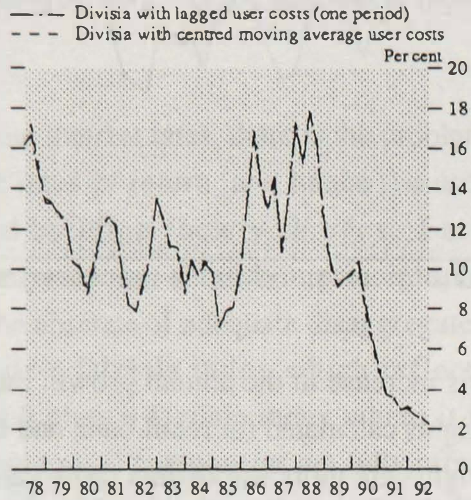
A further problem occurs if there are substantial costs of portfolio adjustment, or imperfect information regarding interest rates, as current user costs may then no longer be equated with marginal transactions services and Divisia will not provide an accurate measure of transactions services.

Various suggestions have been made to deal with this. One possible remedy is to use lagged interest rates to recalculate user costs. These can then be thought of as the perceived costs of holding monetary assets, or alternatively as the effective prices for asset holders who are subject to adjustment costs. An alternative is to use centred moving averages of user costs, on the argument that if individuals do not adjust their portfolios continuously, then their decisions will be based on present and expected values of this variable. The resulting user costs will move more smoothly than those calculated only from current returns, with the result that the weights assigned to asset growth rates will, other things being equal, also move more smoothly over time. It seems plausible that notes and coin, non-interest-bearing deposits and corporate deposits are subject to relatively low adjustment costs, but an index could incorporate lagged or 'smoothed' user costs on interest-bearing retail deposits.

Chart 4

Smoothing techniques (twelve-month growth rates)





These smoothing methods are rather ad hoc, but there is no way of determining the 'correct' values of the smoothing parameters, so their choice is inevitably arbitrary. As shown in Chart 4, such smoothing techniques make little difference to the outcome - a conclusion reached by other studies.

Chart 5

Twelve-month growth rates of Divisia and fixed rate Divisia

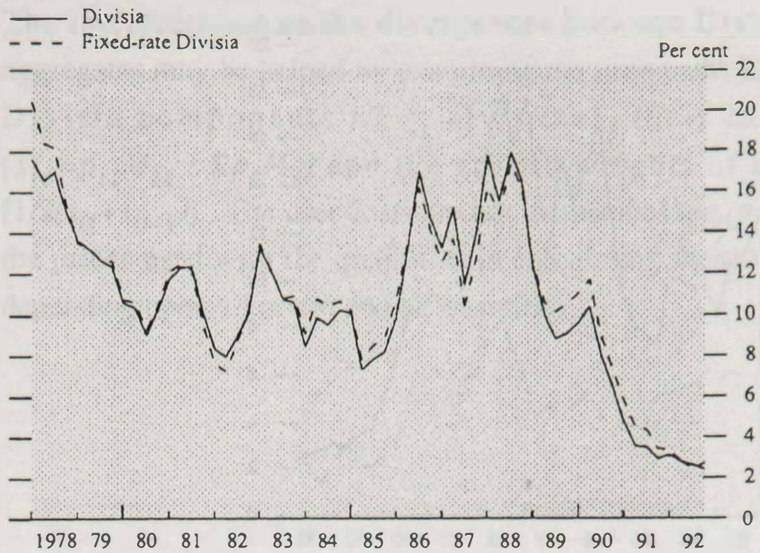


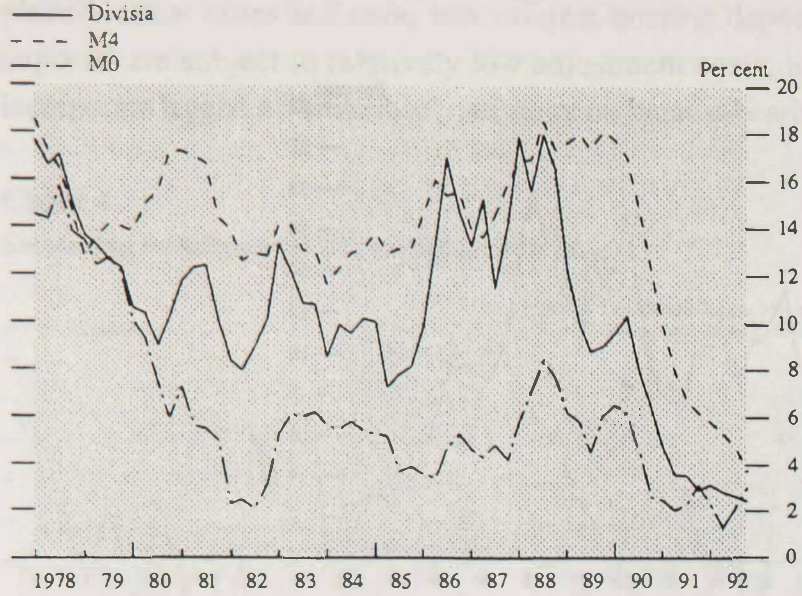
Chart 5 shows a Divisia index constructed with fixed user costs (the dashed line). The fixed user costs are the average user cost for each asset over the sample period. The outcome is quite similar to the index calculated with varying user costs and further supports the assertion that smoothing techniques make little difference.

6.2 The quarterly Divisia index

The twelve-month growth rate of the preferred quarterly Divisia index is compared with the existing simple sum aggregates in Chart 6.

Chart 6

Twelve-month growth rates of Divisia, M0, and M4



The growth paths of Divisia and of the simple sum aggregates diverge quite considerably. In the late 1970s both Divisia and M0 appeared to grow at almost identical rates. From 1980, however, M0 growth rates were much lower, whereas the growth of Divisia climbed to 12.5% by 1981 Q3, compared with 5.5% for M0 and 16.7% for M4. As would be expected given its construction, growth rates of the Divisia index usually lie somewhere between those of M0 and M4.

Of particular interest is the pick-up in Divisia growth from 1985 Q2 until 1986 Q3, after which it remained strong until the end of 1988. From this point until 1992 Q4, however, Divisia growth declined significantly, to below 3.0%. Although M4 also exhibits a sharp deceleration in its growth from 1990, the fall is not as severe.

In general, we would expect the behaviour of a broad money Divisia aggregate to be more like that of a narrow simple sum aggregate than the corresponding broad simple sum aggregate. This is because, as the general level of interest rates rise, funds will be shifted to less liquid assets. More weight will therefore be given to the more liquid, low yielding assets, whose growth rates would have been reduced.

The interpretation of the divergences between Divisia and simple sum aggregates may be helped by examining the time paths of the user costs of the Divisia components $(R-r_i$ after tax), their expenditure shares $(s_{it}=p_{it}M_{it} / \Sigma p_t M_t)$ and the growth weights of Divisia aggregates $[1/2(s_{it}+s_{it-1})]$. The user costs are not themselves the growth weights, but are the prices used with the quantities in calculating these weights, each weight depending upon all prices and all quantities.

Chart 7

User costs

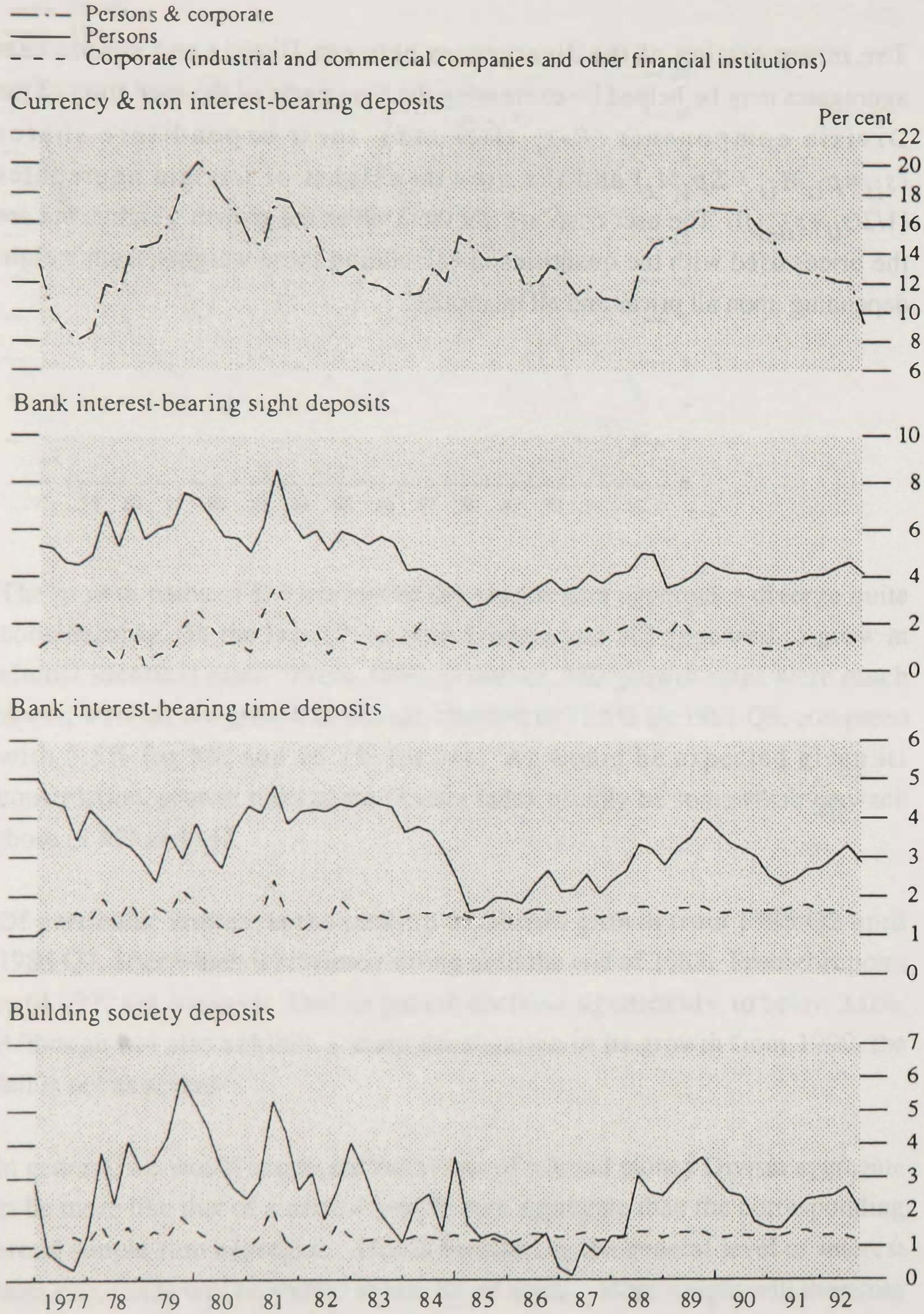


Chart 8

Expenditure Shares

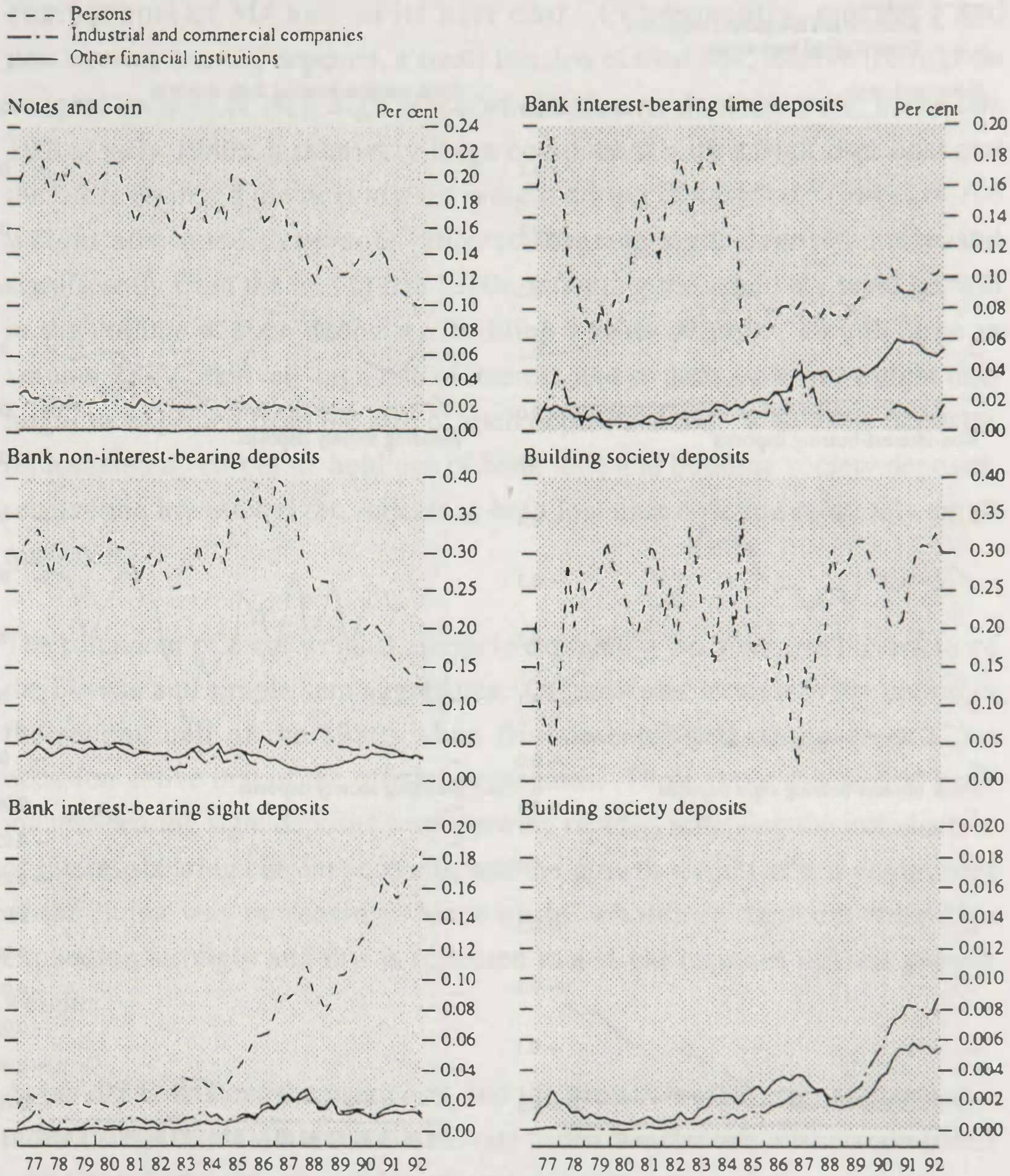
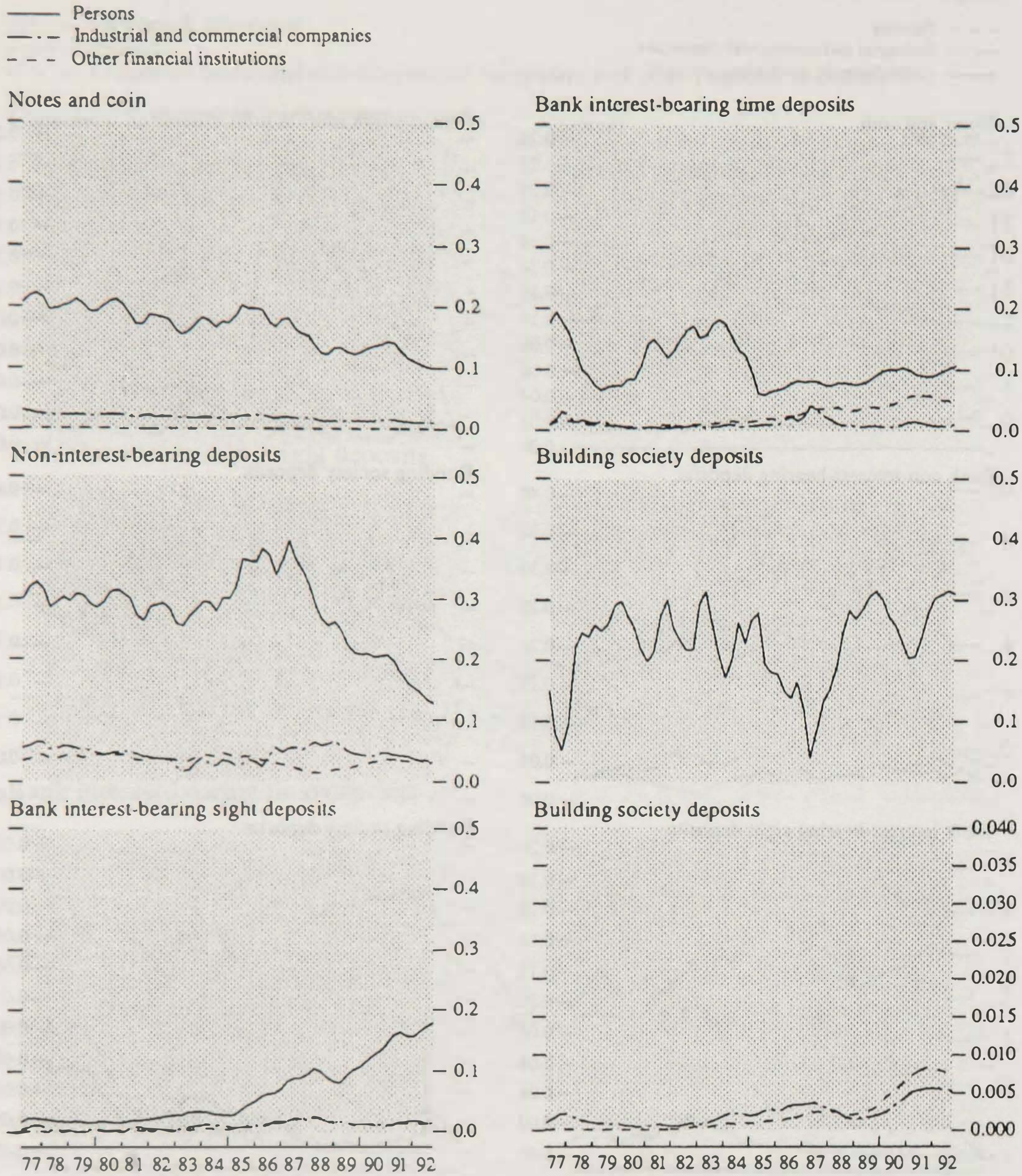


Chart 9

Growth weights



The growth weight of each component depends on its size relative to other components of M4 and on its user cost. Consequently, currency and non-interest-bearing deposits, a small fraction of total M4, receive the highest weights because of their high user cost. Bank time deposits were, in the late 1970s/early 1980s, a relatively large component with a high user cost and therefore received the next highest weight during that period. However, the weight attributed to persons' interest-bearing sight deposits increased significantly from the mid to late 1980s, reflecting the relatively rapid growth in the volume of such deposits. Building society deposits, though large in volume, pay a relatively high rate of interest, and so attract a lower weight than might be expected from the size of such deposits alone. Wholesale deposits, represented by corporate holdings of bank time and building society deposits, receive the lowest weight, reflecting both low user cost and relatively small quantities.

The behaviour of these weights assists in explaining the divergent behaviour of the Divisia and simple sum aggregates. Of particular interest is the period in the second half of the 1980s when Divisia exhibited strong growth, on occasion above that of the official aggregates. Throughout this period bank interest-bearing sight deposits were growing rapidly, reflecting the introduction of interest-bearing current accounts, and the growth weight of this component of the index was increasing. Meanwhile, wholesale deposits were also expanding strongly and this is reflected in a slight increase in their growth weights.

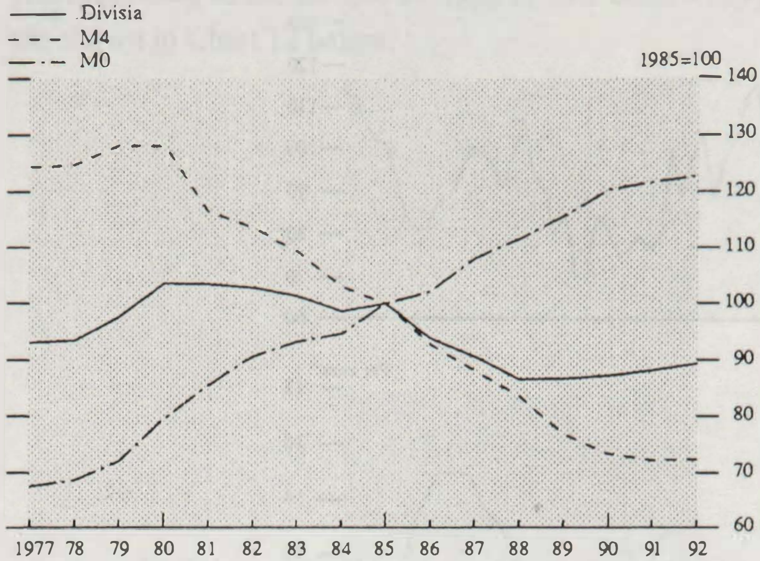
Appendix E sets out the user costs and the growth weights of an alternative index using the maximum available rate as the benchmark. When compared with Chart 9, the growth weights of interest-bearing deposits now appear more volatile. One outcome of utilising such a benchmark is that interest rates for monetary assets operate as the benchmark when they exceed the local authority deposit rate, with the effect that their respective user costs and growth weights are zero. This is clearly illustrated in Appendix E with the retail building society and corporate deposits periodically experiencing zero user costs and growth weights when their respective rates took the role of the benchmark.

Of particular interest is the collapse in the building society deposit growth weight during 1986 and 1987. The financial services industry was extremely competitive during this period and in order to attract new business the building societies offered very.competitive rates. It should be recognised, however, that the average deposit with the building societies did not earn this return; only the marginal deposit. This provides a good illustration of one of the practical difficulties with Divisia - a detailed breakdown of deposits and their respective returns is not available.

The velocities of Divisia and its simple sum counterparts are shown in Chart 10. Although declining for most of the 1980s, Divisia velocity appeared to stabilise and then subsequently increased, albeit slowly, from 1988. Divisia velocity has been more stable than that of M4 or M0.

Chart 10

Velocity of Divisia, M0 and M4



Corresponding to the Divisia quantity index is a price index - the price dual. The Divisia index of prices is obtained by cumulating over time a weighted sum of the rates of change of the component prices, where the weights are the current shares of the component assets in the total current expenditure on all assets in the index:

$$ln\,P_t - ln\,P_{t-1} = \Sigma n_{it}(ln\,p_{it} - ln\,p_{it-1}) \qquad (3)$$

where: $n_{it} = p_{it}M_{it} / \Sigma p_{it}M_{it}$

In our econometric work on Divisia we use the level of this price dual instead of the level of nominal interest rates. The user cost formula used in this case is that derived in Appendix B equation (**B11**), dividing through by P_{nt} to obtain a *real* price dual, ρ_t.

Chart 11

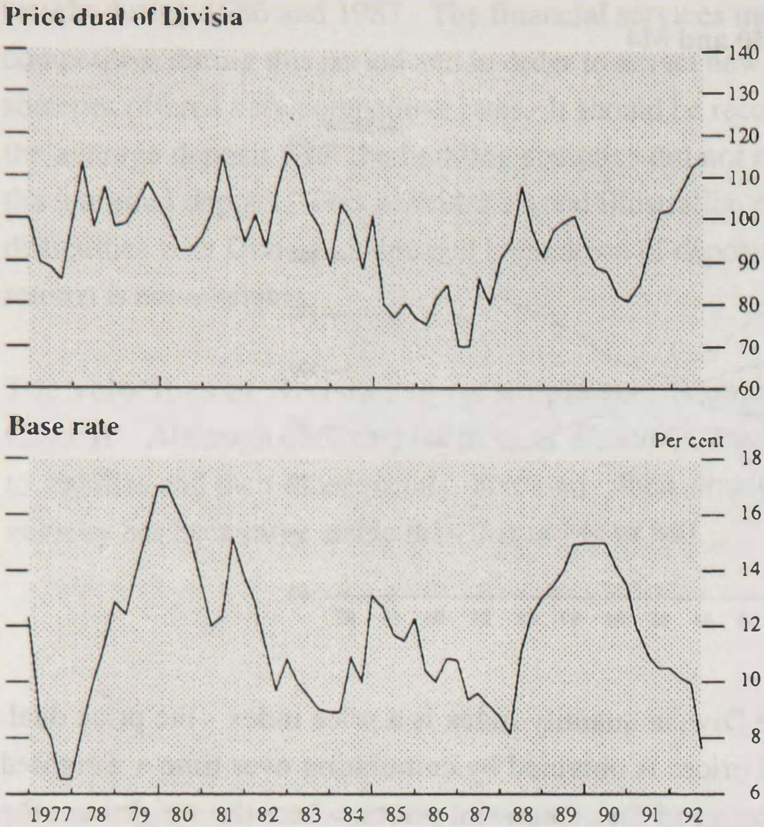


Chart 11 plots the Divisia price dual and the base rate. As the price dual is based on interest differentials it is not surprising that its historical behaviour bears little resemblance to the level of the base rate.

6.3 Currency equivalent aggregate

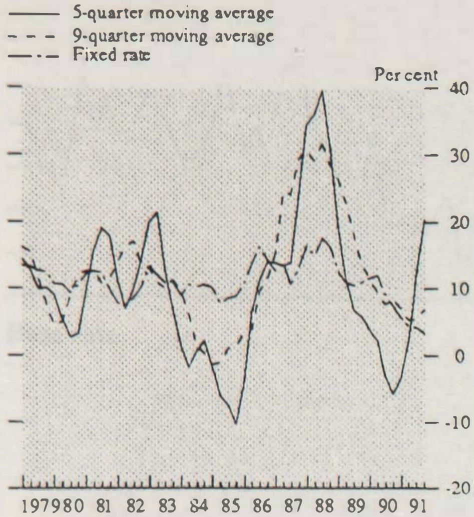
The currency equivalent aggregate proposed by Rotemberg, Driscoll and Poterba *(op cit)* takes the form:

$$CE = \Sigma \frac{R - r_i}{R} M_i \tag{4}$$

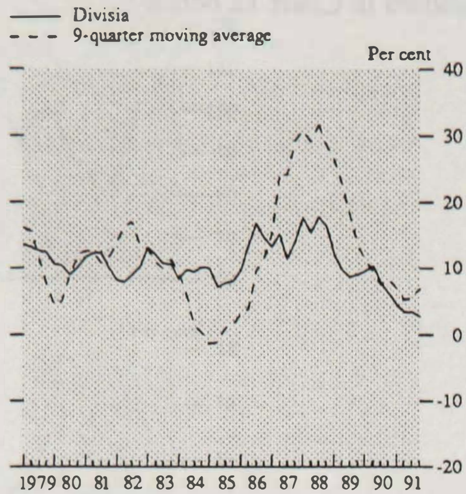
As with Rotemberg et al we calculate the aggregate with various centred moving averages of user costs and an aggregate which uses fixed weights corresponding to the sample average of user costs - the growth rates of which are shown in Chart 12 below.

Chart 12

Twelve-month growth rates of currency equivalent aggregate



Twelve-month growth rates of currency equivalent aggregate and of Divisia



The second graph shows the annual growth rate of Divisia compared with the currency equivalent aggregate. Because the currency equivalent aggregate's short-term fluctuations are sensitive to high frequency interest rate changes, its annual growth rate, even with a nine-quarter centred moving average, exhibits greater variation than that of Divisia.

6.4 Sectoral Divisia

In addition to an aggregated Divisia index, our data set allows us to investigate the historical behaviour of corporate [encompassing both Industrial and Commercial Companies (ICCs) and Other Financial Institutions (OFIs)] and personal sector Divisia indices and their money demand behaviour. As with our preferred aggregated index the sectoral indices are constructed with the three-month local authority deposit rate operating as the benchmark.(11)

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(11) A corporate index was also constructed utilising the three-month Treasury bill rate operating as the benchmark - it made little difference to the resultant index.

Chart 13

Twelve-month growth rates of aggregate Divisia, personal sector Divisia and corporate sector Divisia

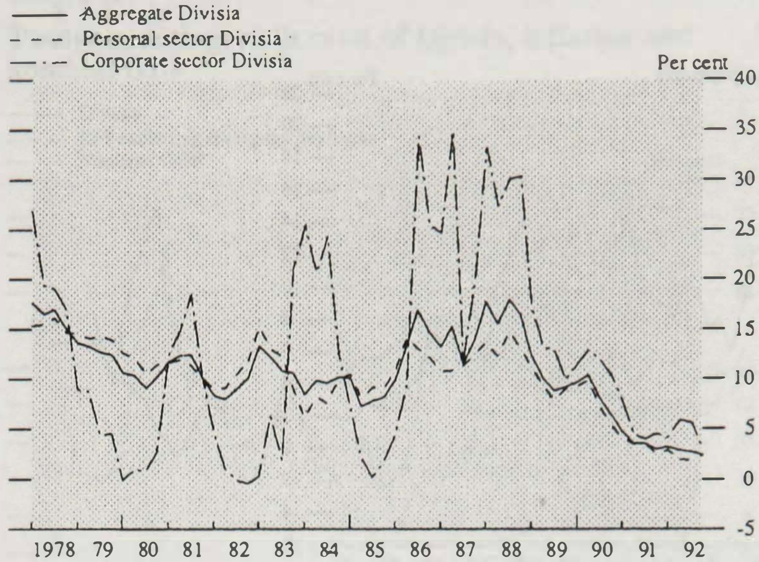
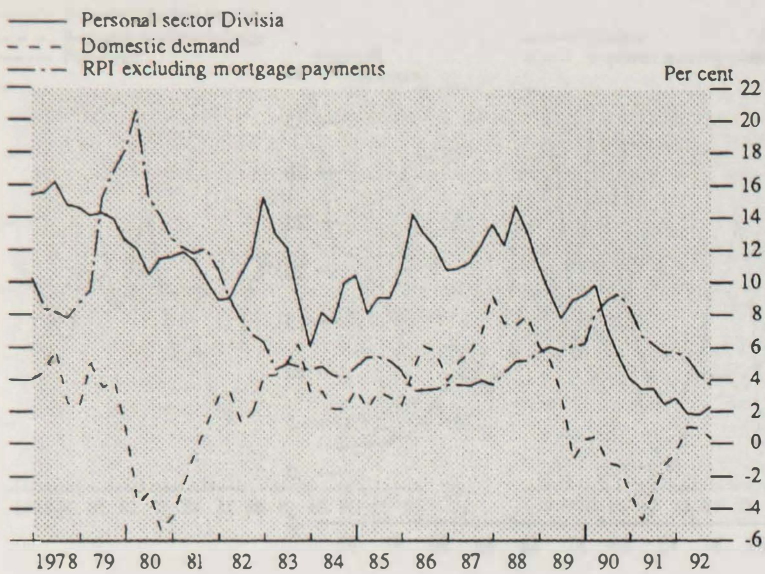


Chart 13 plots the annual growth rate of the aggregate Divisia against the personal and corporate sector Divisia. The historical behaviour of the personal sector Divisia is similar to the aggregate index. This is not unexpected as persons are dominant holders of M4 deposits. Corporate sector Divisia, on the other hand, exhibits a more volatile growth path than the aggregate index. This can be explained by both ICCs' and OFIs' relatively small, and somewhat variable, holdings of M4 deposits.

Chart 14

Twelve-month growth rates of personal sector Divisia, inflation and domestic demand



Twelve-month growth rates of corporate sector Divisia, inflation and nominal GDP

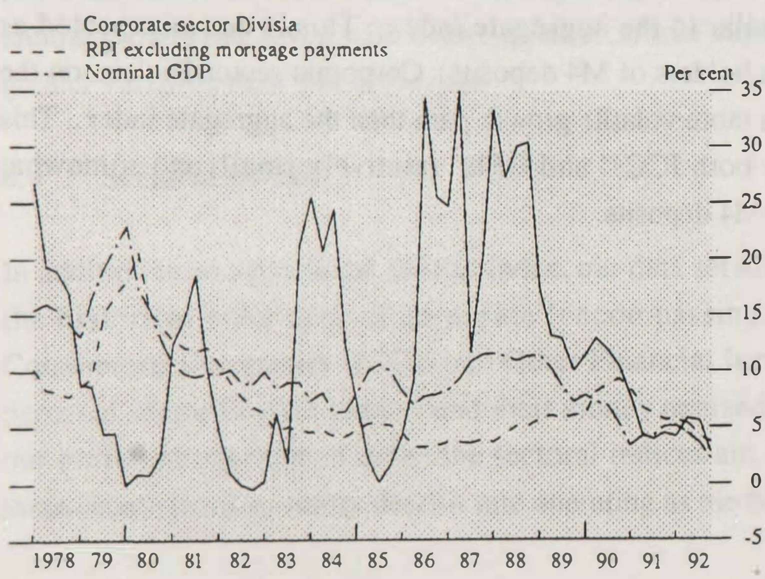
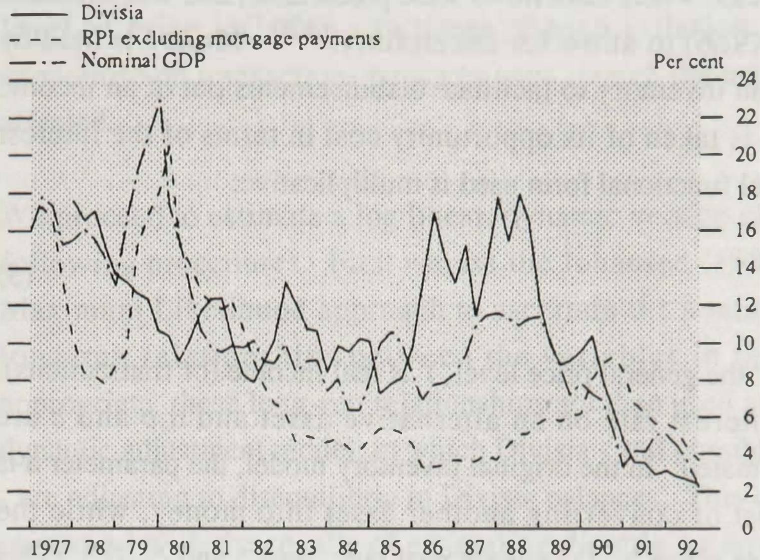


Chart 15

Twelve-month growth rates of Divisia, inflation and nominal GDP



Although no strong relationship is immediately apparent between Divisia and nominal GDP, Divisia does appear to lead on a number of turning points (see Chart 15): in 1977/78 Divisia money growth rose strongly and appeared to lead the subsequent upturn seen in nominal GDP during 1979; and the strong growth in Divisia seen over the period from mid-1985-88 is followed by the upturn in GDP growth in 1986-89, with the subsequent deceleration in Divisia growth replicated with a lag of a year by GDP. No such association, however, is apparent between Divisia and inflation.

For an economic variable to be useful as an indicator, it must be systematically correlated with current or future movements in final objectives. To be useful as an intermediate target it is also necessary to be able to account for its own variation. We therefore evaluate Divisia on two counts. First, we attempt to establish stable econometric relationships for the behaviour of both the aggregate and sectoral Divisia indices. Second, we investigate the informational content of Divisia using bivariate causality tests.

Since Divisia is constructed as a monetary aggregate based largely on transactions services, it seems sensible to apply the theory of the transactions demand for money. This approach was developed by Baumol (1952) and Tobin (1956) for the case where cash flows were predictable, and was extended by Miller and Orr (1966) to allow for uncertainty.(12) Money is held by economic agents as an inventory to facilitate disbursements out of an income stream, but account is taken of its opportunity cost in terms of the interest foregone. The general functional form used is multiplicative:

$$M/P = a\,Y^b\,R^c \tag{5}$$

where M is money, P the general price level, Y is real income (or transactions), R is the nominal interest rate on an alternative asset and a,b and c are parameters to be estimated. In the original inventory model, the parameter a is the transactions cost of converting another asset into money, while the elasticities b and c are given by the square root law: b=0.5 and c=-0.5. This particular model may be too stringent in its assumptions and it is normal practice to satisfy the less restrictive conditions $0.5 \leq b \leq 1.0$ and $c \leq 0$.

Since some of the components of Divisia are interest-bearing we replace the level of the nominal interest rate in the standard equation with a user cost measure which is based on interest rate differentials. The measure used here is the *real* price dual of Divisia.(13)

In estimation, real income is often replaced by expenditure or output as being more closely related to the volume of transactions. The appropriate measure will also differ across sectors. We have chosen to use total domestic demand

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(12) For more recent surveys of the literature on the demand for money see Judd and Scadding (1982).

(13) The level of the price dual weights together interest rate differentials and multiplies by the general price level (see Appendix B, equation B11). To obtain the real price dual we therefore divide by the general price level.

for the personal sector and GDP for the corporate sector, with their relevant deflators as price indices. All data are seasonally adjusted and logged.

Equation (5) represents a static equilibrium relationship. In the short-run or dynamic equilibrium we also allow holdings of Divisia to be affected by the level of price inflation - in times of high inflation agents will tend to economise on transactions balances even though the user cost is not directly affected.

We proceed to estimate a log linear, dynamic version of equation (5) by the following procedures. First we use the Johansen (1988) Full Information Maximum Likelihood approach to ascertain the number and nature of the long-run relationships between the variables in the data set. Where appropriate, these long-run relationships are then used as the foundation of a dynamic adjustment model, in which Divisia - and possibly the other variables - are adjusting to disequilibria in Divisia balances. This estimation strategy is compared with the results of estimating directly an error correction model based on OLS - which can be viewed as a test of the restrictions imposed by the Johansen procedure.

The data sample is restricted by the availability of the Divisia index to be 1977 Q1-92 Q4. Initial investigation shows that over this sample, aggregate real Divisia (M/P, M^p/P, M^c/P for aggregate, personal and corporate), domestic demand at constant prices (DD), GDP and the price deflators (P^d,P^g for demand and GDP) are all on the borderline between $I(1)$ and $I(2)$ processes, while the user cost indices (ρ) are borderline $I(0)/I(1)$ (borderline in the sense of conflicting results from different tests for non-stationarity and test values close to the 5% significance level). Given the well-known small sample problems of such tests we use our judgment to treat all the series as $I(1)$. The similarity of the time series properties holds out some hope of cointegration.

7.1 The personal sector

For the personal sector an expenditure measure is most likely to represent accurately the volume of transactions. After some experimentation with consumption, we preferred total domestic demand as the scale variable. Estimates of the long-run relationships are inconclusive - there could be between zero and two cointegrating vectors according to the choice of test and significance level (details of test statistics are reported in Appendix F). If there are no cointegrating vectors then we will not be able to obtain a satisfactory explanation for the level of Divisia. If there is more than one vector then we may need to apply identification conditions to extract that combination which is relevant to a behavioural model of the demand for Divisia balances. In this instance the first vector is clearly suitable for a Divisia equation on theoretical grounds and we feel able to ignore the possibility of a second, marginal relationship.

Long-run relationship:

$$ln(M^p/P^d) = 0.93\, ln(DD) - 0.22\, ln(\rho) \qquad (6)$$

Notes: 3 lags in the VAR, additional $I(0)$ variables $\Delta ln(P^d)$. Sample 1977 Q4-92 Q4. Tests of elasticity on DD, H_0:b=1 $\chi^2(1)=0.2$; H_0:b=0.5 $\chi^2(1)=0.9$.

Tests show that the activity elasticity could be imposed at either unity or one-half (in which case the user cost elasticity varies between -0.18 and -0.50). We proceed with the unrestricted estimate.

Dynamic relationship:

(a) Based on Johansen estimate of the long run:

$$\Delta ln(M^p/P^d)_t = \underset{(-3.7)}{-0.49} + \underset{(2.0)}{0.21}\, \Delta ln(M^p/P^d)_{t-1} + \underset{(3.0)}{0.32}\, \Delta ln(DD_t)$$

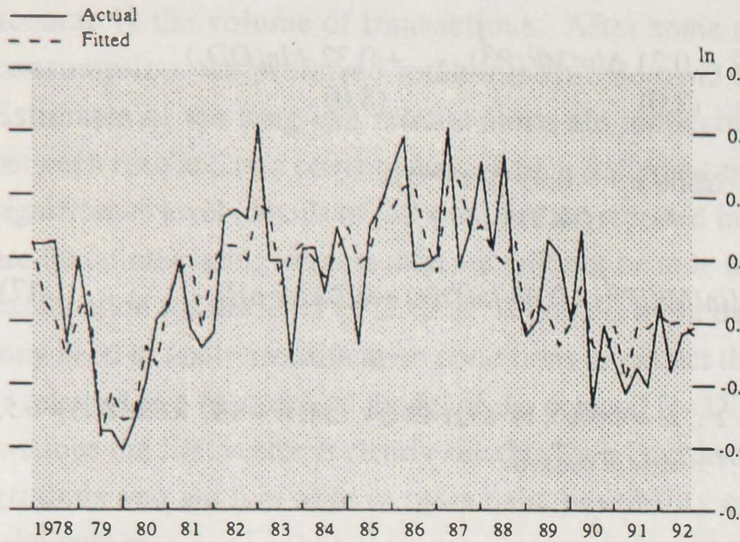
$$\underset{(-1.2)}{-0.007}\, \Delta\Delta ln(\rho_t) \underset{(-4.3)}{-0.49}\, \Delta ln(P^d{}_t)$$

$$\underset{(-3.7)}{-0.11}\, [ln(M^p/P^d) - 0.93\, ln(DD) + 0.22\, ln(\rho)]_{t-1} \tag{7}$$

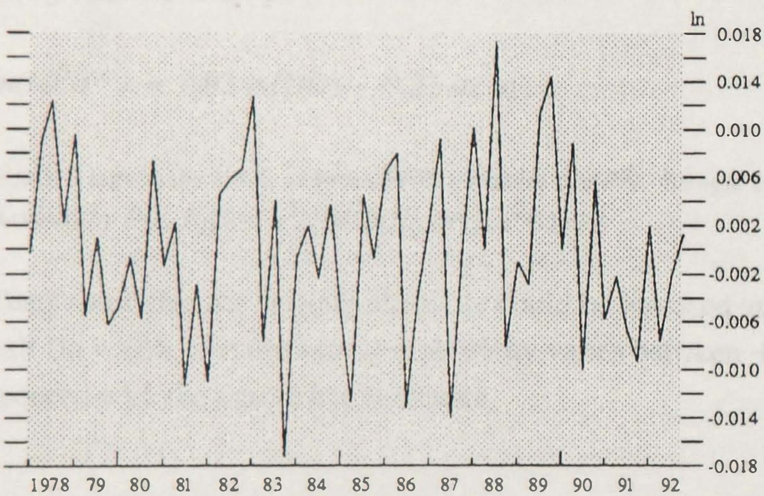
Notes: $Rbar^2 = 0.58$, $DW = 2.1$, $se = 0.0080$, 1978 Q1-92 Q4, LM(4) = 6.9, RESET(1) = 0.5, NORM(2) = 0.7, HET(1) = 1.7, t-ratios in brackets.

Chart 16

Personal sector Divisia: actual and fitted values from equation 7



Personal sector Divisia: residuals from equation 7



The equation is parsimonious, reasonably stable under recursive estimation and passes all mis-specification diagnostics at a 5% probability value. The equation explains 60% of the quarter-to-quarter variation in the dependent variable and the residual standard error is 0.8%. All variables enter contemporaneously although the user cost term becomes double-differenced and has a 95% confidence interval which includes zero. Plots of actual and fitted, and the estimated residuals are shown in Chart 16.

(b) Based on OLS:

$$\Delta ln(M^p/P^d)_t = \underset{(-1.2)}{-0.59} + \underset{(2.0)}{0.22}\,\Delta ln(M^p/P^d)_{t-1} + \underset{(2.8)}{0.33}\,\Delta ln(DD_t)$$

$$\underset{(-1.0)}{-0.007}\,\Delta\Delta ln(\rho_t) \underset{(-3.6)}{-0.48}\,\Delta ln(P^d{}_t)$$

$$\underset{(-2.0)}{-0.12}\,[ln(M^p/P^d)_{t-1} \underset{(6.8)}{-0.96}\,ln(DD_{t-1}) + \underset{(1.8)}{0.20}\,ln(\rho_{t-1})] \qquad (8)$$

Notes: $Rbar^2 = 0.56$, DW = 2.1, se = 0.0082, 1978 Q1-92 Q4, LM(4) = 7.1, RESET(1) = 0.6, NORM(2) = 0.8, HET(1) = 1.8, t-ratios in brackets.

Direct estimation by OLS shows minimal differences - the long-run elasticity on activity increases slightly and the overall fit worsens marginally (two fewer degrees of freedom). A similar equation can be found if the long-run activity elasticity is restricted to unity. The t-ratios reported on the long-run coefficients are calculated so as to preserve valid inference on the $I(1)$ variables. Interestingly, these show that the long-run user cost term is relatively imprecisely estimated.

In summary, the dynamic and long-run equations for personal sector Divisia both seem to work reasonably well, despite the relatively short sample available. One possible cause for concern is the relative imprecision of the user cost terms, although their coefficients are correctly signed and of reasonable magnitude. The imprecision probably reflects the presence of considerable noise in what is a relatively volatile series.

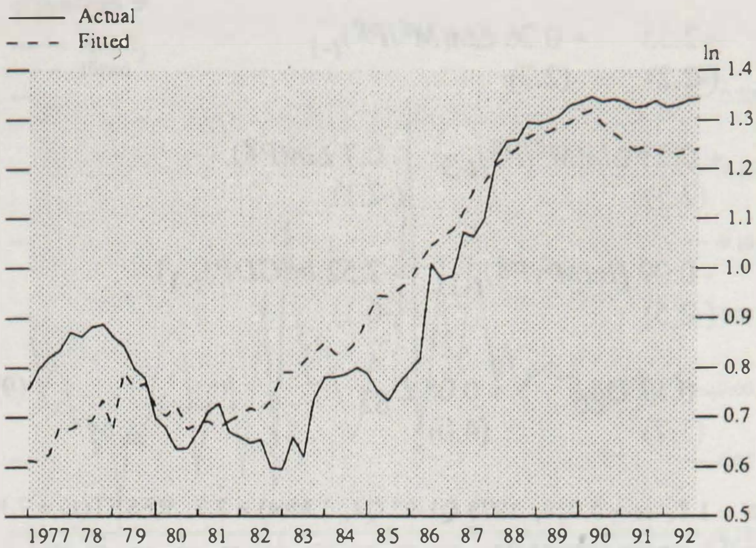
7.2 The corporate sector

The corporate sector has proved much more difficult to model than the personal sector. We might expect corporate transactions balances (at least for industrial and commercial companies) to be held on account of production costs (mostly wage and raw material costs but also land rents and the cost of capital), which in turn are related to total output. Attempting to strip out corporate income or expenditure from the national accounts is not straightforward, especially for financial institutions, and hence we choose GDP as the activity variable.

The principal problem is a failure to find any sensible cointegrating relationships. The (real) corporate Divisia index does not cointegrate with output, but when the user cost is included it has the wrong sign and/or an implausibly large coefficient [partly arising from the fact that the sectoral user cost is probably $I(0)$]. The nature of the problem is illustrated graphically in Chart 17 which reports the fitted value from regressing the (log) level of real corporate Divisia on GDP alone (the elasticity is 2.3). The Chart shows that GDP cannot account for the degree of variation in Divisia and the timing of peaks and troughs is not close. When the user cost series is entered, it cannot account for the remaining variation. Furthermore, the first difference (in logs) of the corporate Divisia series has a standard deviation nearly four times as large as the personal sector series, and its higher moments indicate severe non-normality.

Chart 17

Regression of real corporate sector Divisia on GDP



Disaggregating Industrial and Commercial Companies (ICCs) and Other Financial Institutions (OFIs) might be expected to help. In practice neither of the components is any more amenable to explanation. The extra degree of variation in the corporate sector means that we cannot expect to hide the problem by subsuming it within the total - hoping for the personal sector to dominate. The best equation that we have found is based on an unrestricted error correction model, with no user cost terms included and two ad hoc dummy variables for 1983 (1, -1 in Q3, Q4) and 1986 (1, -1 in Q3, Q4).

Dynamic equation (OLS):

$$\Delta ln(M^c/P^g)_t = \underset{(-2.2)}{-2.55} \; \underset{(2.7)}{+ 0.26}\, \Delta ln(M^c/P^g)_{t-1}$$

$$\underset{(2.7)}{+ 0.23}\, \Delta ln(M^c/P^g)_{t-3} \; \underset{(-2.1)}{- 0.7}\, \Delta ln(P^g)$$

$$\underset{(-2.5)}{- 0.09}\, [ln(M^c/P^g)_{t-1} \; \underset{(-6.1)}{- 2.52}\, ln(GDP)_{t-1}]$$

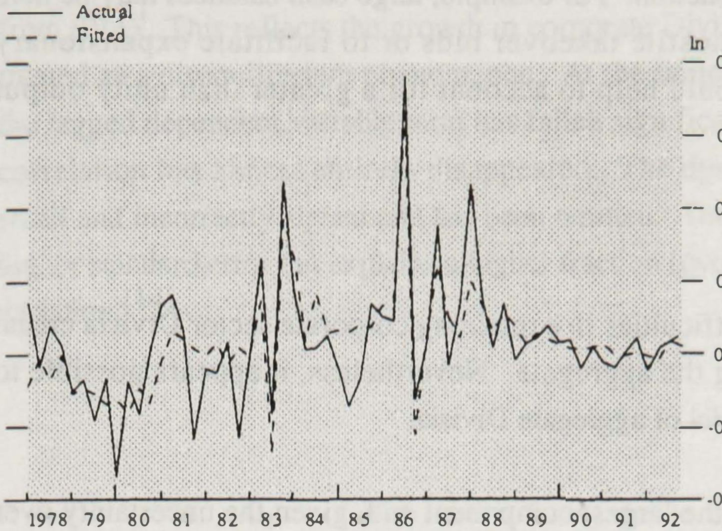
$$\underset{(5.7)}{+ 0.12}\, D_{86} \; \underset{(4.0)}{+ 0.08}\, D_{83} \qquad \textbf{(9)}$$

Notes: $Rbar^2 = 0.59$, DW = 1.8, se = 0.028, 1978 Q1-92 Q4, LM(4) = 2.7, RESET(1) = 7.1, NORM(2) = 0.8, HET(1) = 0.1, t-ratios in brackets.

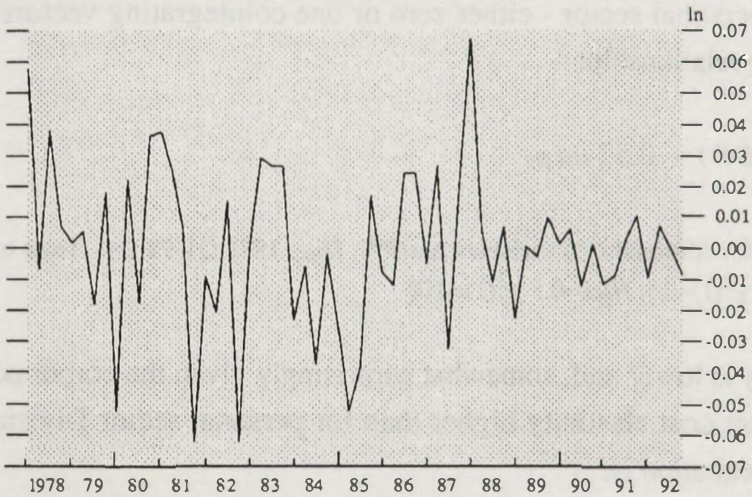
The OLS estimation contradicts the Johansen results in that the error correction term is significant - which implicitly indicates cointegration. Two dummy variables need to be included but the equation still fails badly on the RESET test for functional form (regression of the residuals on the square of the fitted values: test value 7.1, 5% critical value 3.84). The equation standard error is relatively high at 2.8%. Although over half of the quarterly variation is explained, this drops to a quarter if the dummy variables are excluded. The user cost terms, if entered, are incorrectly signed. Actual and fitted values are shown in Chart 18.

Chart 18

Corporate sector Divisia: actual and fitted values from equation 9



Corporate sector Divisia: residuals from equation 9



Why should it be so much more difficult to model corporate sector holdings of Divisia? The corporate sector generally has wider access to capital markets than the personal sector - in terms of both liabilities and assets. Assets which might be regarded as illiquid by the personal sector - equities, government stock, foreign currency balances - may be highly liquid to the corporate sector. Hence the restriction of transactions balances to be a function of M4 components alone is less likely to be valid. Alternatively, the corporate sector may need to hold liquid assets for purposes other than transactions, or for particular types of transaction. For example, large cash balances may be held as a reserve to fight hostile takeover bids or to facilitate expansionary acquisitions. This would help to account for a greater than unity output elasticity.

7.3 Aggregate Divisia

As noted above, the difficulties in explaining corporate sector Divisia create problems in modelling the aggregate. Nevertheless, it appears possible to obtain a reasonable model of aggregate Divisia.

The personal sector is the largest component and, given the uncertainty over the appropriate corporate sector activity variable, we use domestic demand as the scale variable. The cointegration analysis gives slightly more conclusive results than for the personal sector - either zero or one cointegrating vectors. We use the following relationship:

$$ln(M/P^d) = 0.72\ ln(DD) - 0.52\ ln(\rho)$$

Notes: 4 lags in the VAR, additional $I(0)$ variables $\Delta ln(P^d)$, D_{86}, 1978 Q1-92 Q4. Tests of elasticity on DD, $H_0{:}b=1$ $\chi^2(1)=0.5$; $H_0{:}b=0.5$ $\chi^2(1)=0.08$.

The activity elasticity is lower and, somewhat surprisingly given the corporate sector results, the user cost elasticity higher than for personal sector Divisia alone. The dynamic equation is:

$$\begin{aligned}\Delta ln(M/P^d)_t = &\ \underset{(-3.0)}{-\,0.05} + \underset{(2.1)}{0.18}\ \Delta ln(M/P^d)_{t-1} + \underset{(3.6)}{0.39}\ \Delta ln(DD_t) - \underset{(-5.2)}{0.64}\ \Delta ln(P^d{}_t) \\ &\ \underset{(-3.7)}{-\,0.045}\ [(ln(M/P^d) - 0.72\ ln(DD) + 0.52\ ln(\rho)]_{t-1} + \underset{(3.0)}{0.02}\ D_{86} \qquad \textbf{(10)}\end{aligned}$$

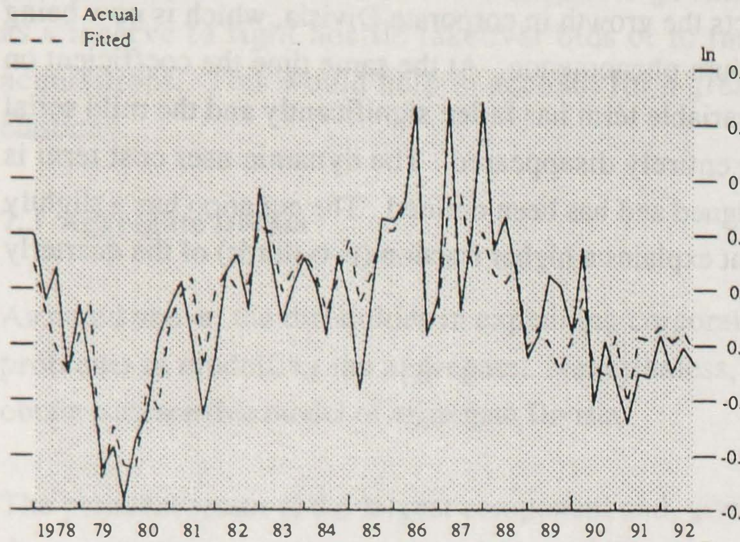
Notes: $Rbar^2 = 0.68$, DW = 1.9, se = 0.0086, 1978 Q1-92 Q4, LM(4) = 2.2, RESET(1) = 0.2, NORM(2) = 2.2, HET(1) = 1.2, t-ratios in brackets.

This equation differs from the personal sector equation in several ways. Most important is that the error correction coefficient is half the magnitude (0.045 from 0.11). This reflects the growth in corporate Divisia, which is now being treated as a disequilibrium phenomenon. At the same time the coefficient on the lagged dependent variable term has fallen significantly and the mild serial correlation has almost entirely disappeared. The dynamic user cost term is small and incorrectly signed and has been omitted. The equation has a slightly higher standard error but explains a higher fraction (two-thirds) of the quarterly variation.(14)

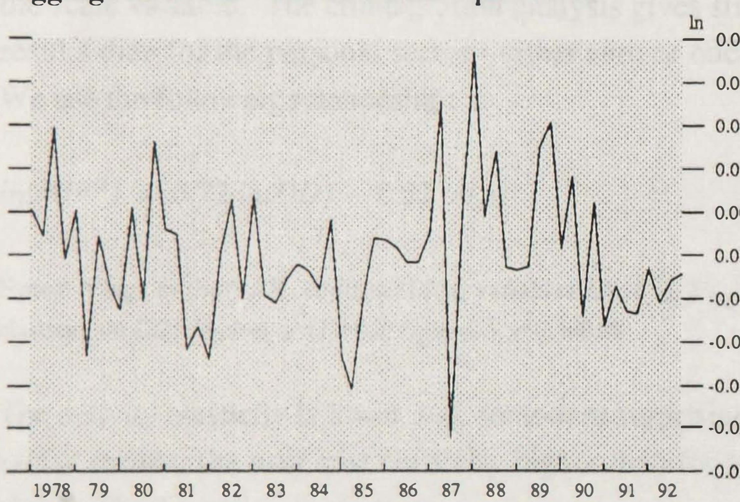
(14) An OLS based estimate of the error-correction model gives similar coefficient estimates - but all the levels terms are not well determined. This equation is not reported.

Chart 19

Aggregate Divisia: actual and fitted values from equation 10



Aggregate Divisia: residuals from equation 10



On some diagnostics, and on the basis of the long-run user cost elasticity, this equation is more attractive than the personal sector equation. Plots of actual and fitted, and of residuals, are given in Chart 19. The equation is reasonably stable and thus meets our first criterion for an intermediate target. But despite this conclusion a standard error of 0.9% allows considerable scope for unexplained variation on a quarter-to-quarter basis.

7.4 Causality tests

To be useful as an indicator, Divisia should contain information on final policy objectives. We evaluate the informational content by means of bivariate autoregressions using the price level and (nominal) output as the objectives. For prices we use the RPI excluding the effects of mortgage interest payments, the community charge and indirect taxes. For output we use nominal GDP.

Tests such as these are weak - there is no behavioural content to the equation specification and it is therefore quite likely that such simple autoregressions are not stable over time. However, indicator variables do tend to be examined in a bivariate context and the strength of such correlations is therefore of some interest.

The tests are performed in two stages. First we attempt to establish an unrestricted cointegrating vector using the Johansen approach. Second, we estimate an unrestricted autoregression for the objective variable, specified in the first difference of logs with five lags of the dependent variable and an equal number of lags in the indicator variable, together with the cointegrating term at lag one and an intercept. The tests are:

(I) the exclusion of the cointegrating term,

(II) the exclusion of lags in the indicator, conditional on excluding the cointegrating term, and

(III) the inclusion of a contemporaneous term in the indicator variable.

Test (I) establishes causality from the level of the indicator to that of the objective. If the cointegrating term cannot be excluded, this test automatically implies dynamic as well as levels effects - there is no need to test additionally for short-run causality. Test (II) is the traditional causality test based on a differenced equation - which is valid only if the cointegrating term is insignificant. This test turns out to be largely superfluous but we report the results to demonstrate that incorrect omission of the levels terms could alter the conclusions. Test (III) establishes contemporaneous correlation, but the causality here could run in either direction.

The behavioural equations have already demonstrated causality from inflation and activity to Divisia. We do not need to test this further. Unfortunately we cannot specify a simple behavioural equation for domestic demand or GDP which would include monetary aggregates. Instead the causality tests can be thought of as a partial reduced form approach.

The tests are performed with some additional variables to ensure that the autoregressions represent data consistent models. For GDP we included a (1,-1) dummy for 1979 Q1 and Q2 to account for a road haulage dispute which caused a switch in recorded net trade. For prices we include three quarterly dummies because the RPI is not seasonally adjusted and contains mild seasonality (much of which is removed because we strip out the effects of indirect taxes which are uprated in the budget quarter). These additional variables were included as additional $I(0)$ terms in the Johansen procedure as appropriate.

The regressions are all based on the same sample 1978 Q4-92 Q4 (the exact test values are very sample specific but the qualitative nature of the results should be more robust). Both Chi-square (asymptotic) and F tests (small sample) are reported. Degrees of freedom for the tests are as shown in brackets. Rejection of the null hypothesis of no causality at 5% is indicated in Table 3 by §.

Similar tests are performed for aggregate Divisia, its personal and corporate components, and for M4, M4 lending and M0. The results are reasonably encouraging. Although the strength of the cointegration tests vary, they are sufficient to conclude that there is a cointegrating relation in each case. There is causality in levels for each of the monetary aggregates to both GDP and prices, with the exception that M0 does not appear to cause nominal GDP. The levels term is also correctly signed (negative) in every case. There is a general lack of contemporaneous correlation - again with the sole exception of M0 and nominal GDP (but causality here could run in either direction). If (incorrectly) the levels effects were to be ignored then it would be much less clear whether there was indeed any causality from the money aggregates to either GDP or prices. Of the small sample F tests only M0→Prices is significant for test (II).

On these grounds there appears to be useful information in all the monetary aggregates considered. M0 is possibly the most useful forward indicator in this statistical sense for retail prices and as a contemporaneous indicator of GDP. At the same time M0 does not seem to contain much longer-term information on nominal GDP. Divisia, particularly the personal sector index, appears to be slightly more robust across the different tests and different objective variables.

Table 3: Causality test results

Causality from Divisia (personal sector)

Cointegrating Relation	→GDP $lnNGDP = 0.85\ lnD^p$		→Prices $lnP = 0.36\ lnD^p$	
Test	χ^2	F	χ^2	F
I	§16.1 [1]	§14.4 [1,44]	§11.0 [1]	§8.9 [1,43]
II	§11.3 [5]	2.0 [5,45]	§12.9 [5]	2.2 [5,43]
III	1.2 [1]	0.9 [1,43]	0.0 [1]	0.0 [1,41]

Causality from Divisia (corporate sector)

Cointegrating Relation	→GDP $lnNGDP = 0.51\ lnD^c$		→Prices $lnP = 0.38\ lnD^c$	
Test	χ^2	F	χ^2	F
I	§6.6 [1]	§5.4 [1,44]	20.2 [1]	17.8 [1,43]
II	5.2 [5]	0.9 [5,45]	3.0 [5]	0.5 [5,43]
III	0.1 [1]	0.1 [1,43]	0.0 [1]	0.0 [1,41]

Causality from Divisia (aggregate)

Cointegrating Relation	→GDP $lnNGDP = 0.79\ lnD$		→Prices $lnP = 0.41\ lnD$	
Test	χ^2	F	χ^2	F
I	§11.2 [1]	§9.6 [1,44]	§16.9 [1]	§14.5 [1,42]
II	8.6 [5]	1.5 [5,45]	§12.0 [5]	2.0 [5,43]
III	1.8 [1]	1.4 [1,43]	0.1 [1]	0.1 [1,41]

Causality from M4

Cointegrating Relation	→GDP $lnNGDP = 0.62\ lnM4$		→Prices $lnP = 0.41\ lnM4$	
Test	χ^2	F	χ^2	F
I	§16.9 [1]	§15.2 [1,44]	§19.7 [1]	§17.4 [1,42]
II	2.4 [5]	0.4 [5,45]	8.6 [5]	1.4 [5,43]
III	3.3 [1]	2.6 [1,43]	0.0 [1]	0.0 [1,41]

Causality from M4 lending

Cointegrating Relation	→GDP $lnNGDP = 0.48\ lnM4L$		→Prices $lnP = 0.29\ lnM4L$	
Test	χ^2	F	χ^2	F
I	§18.1 [1]	§16.4 [1,44]	§15.8 [1]	§13.4 [1,42]
II	§12.6 [5]	2.2 [5,45]	2.8 [5]	0.4 [5,43]
III	0.0 [1]	0.0 [1,43]	2.2 [1]	1.6 [1,41]

Causality from M0

Cointegrating Relation	→GDP $lnNGDP = 1.7\ lnM0$		→Prices $lnP = 1.02\ lnM0$	
Test	χ^2	F	χ^2	F
I	0.4 [1]	0.3 [1,44]	§7.0 [1]	§5.5 [1,42]
II	11.0 [5]	1.9 [5,45]	§21.4 [5]	§3.9 [5,43]
III	§7.5 [1]	§6.1 [1,43]	0.1 [1]	0.1 [1,41]

8 Conclusion

In principle, a Divisia measure of money has considerable attractions as a measure of transactions services, weighting each type of deposit according to the transactions services it offers. Such a measure might have a closer relationship with total expenditure in the economy than do the conventional monetary aggregates. This, however, does not imply that Divisia will necessarily be useful in predicting inflation. If monetary aggregates are generally not good leading indicators then Divisia may also be disappointing in this respect.

As discussed in this paper, there are both theoretical and practical difficulties in constructing an index which measures the transactions services provided by different types of monetary asset. It would, however, be wrong to conclude from this that a Divisia index would be inferior to the conventional monetary aggregates. First, these theoretical and practical difficulties may not be severe; and second, some of these difficulties apply at least equally to the conventional monetary aggregates. So even a Divisia index which captures transactions services only imperfectly may nevertheless provide a better measure of money than other monetary aggregates.

This paper has presented a Divisia index for the United Kingdom and has illustrated the impact of alternative - and possibly no less valid - solutions to some of the practical and theoretical difficulties which arise in the construction of such an index. It is difficult to judge the significance of the differences among the various indices which are presented, but it may be observed that these differences are much smaller than the differences between the path of a Divisia index and the path of any of the conventional monetary aggregates.

A Divisia measure of money appears to have some leading indicator properties for predicting both nominal output and inflation. These results do not suggest that Divisia is unambiguously superior to other monetary aggregates as a leading indicator, although a case can clearly be made for including Divisia in the range of indicators analysed by the authorities when forming their judgments on monetary conditions.

Appendix A - Linear homogeneity

When the aggregator function is linearly homogeneous the Divisia index will exactly reflect the growth of transactions (or, more generally, monetary) services in the economy. In this appendix, we first show how the Divisia index is derived from the transactions services function, and then consider the divergence between transactions services and the Divisia index when linear homogeneity is relaxed.

Consider a simple case of only two monetary assets, cash (C) and one interest bearing asset (I). Transactions services M will be a function of holdings of these two assets.

$$M = f(C, I) \tag{A1}$$

Differentiating (A1) and dividing by M yields

$$\frac{\dot{M}}{M} = \frac{dM}{M} = f_C \frac{dC}{M} + f_I \frac{dI}{M} \tag{A2}$$

where f_C and f_I are the partial derivative of f with respect to C and I.

Multiplying the first term in (A2) by C/C and the second term by I/I gives

$$\frac{\dot{M}}{M} = f_C \frac{C}{M} \cdot \frac{\dot{C}}{C} + f_I \frac{I}{M} \frac{\dot{I}}{I} \tag{A3}$$

Euler's Law states that if f is linearly homogeneous then,

$$f_C C + f_I I = f(C, I) = M \tag{A4}$$

Substituting the expression for M from (A4) in (A3) yields

$$\frac{\dot{M}}{M} = \frac{f_C C}{f_C C + f_I I}\left[\frac{\dot{C}}{C}\right] + \frac{f_I I}{f_C C + f_I I}\left[\frac{\dot{I}}{I}\right] \quad \textbf{(A5)}$$

In equilibrium consumers will equate their marginal utilities to the prices of assets, such that the marginal rate of substitution between any two assets will equal the ratio of the prices of the two assets, ie $f_C/f_I = P_C/P_I$, where P_C and P_I are the user costs as derived in Appendix B. Substituting these prices in place of the partial derivatives yields

$$\frac{\dot{M}}{M} = s_C\left[\frac{\dot{C}}{C}\right] + s_I\left[\frac{\dot{I}}{I}\right] \quad \textbf{(A6)}$$

where

$$s_C = \frac{P_C C}{P_C C + P_I I}$$

$$s_I = \frac{P_I I}{P_C C + P_I I}$$

In continuous time $\dot{M}/M \equiv \dot{D}/D$ where D is the Divisia measure. The formula for the Divisia index in the text [equation (1)] is the discrete time approximation.

Suppose now the transactions services function is given by

$$M = f(\alpha C, \beta I) \quad \textbf{(A7)}$$

where α and β are time varying parameters reflecting the transactions technology (eg ATMs, direct debit facilities, etc) which are not fully reflected in the own rates of return used to derive user costs.

Following the same procedure outlined above, it can be shown that

$$\frac{\dot{M}}{M} = S_C \left[\frac{\dot{C}}{C} \right] + S_I \left[\frac{\dot{I}}{I} \right] S_C \left[\frac{\dot{\alpha}}{\alpha} \right] + S_I \left[\frac{\dot{\beta}}{\beta} \right] \qquad \textbf{(A8)}$$

The last two terms in (A8) capture the extent of the departure of the Divisia measure from the 'true' growth of transactions services in the economy. In order to measure this divergence, some functional form for the payments technology (ie α and β) has to be specified and estimated.

Appendix B - Derivation of user cost

In order to derive the user cost of a monetary asset, one can begin with the analogous case for a durable good. Denoting the one period rental price of a durable good as P_{nt}, its current price as P^*_{nt} and the depreciation rate as δ_n, the user cost is given by

$$P_{nt} = P^*_{nt} - \frac{(1-\delta_n)\, P^{*e}_{t+1}}{(1+R_t)} \tag{B1}$$

In equation (B1), P^{*e}_{t+1} is the expected resale price in the next period and R_t is the one period nominal rate of return on bonds which do not provide any transactions services. P_{nt} can be thought of as the holding period return from t to $t+1$.

For the user costs of monetary assets we consider first the case when there is no inflation $(P^{*e}_{t+1} = P_{nt})$. In general P_n can be considered as the price index of goods and services, instead of durable goods only.

The real value of an individual's cash holdings (C) is then equal to C_t/P_{nt}. Measuring the cost of holding cash in terms of real goods, the user cost of cash, by analogy to (B1), is given by

$$P_{ct} = P_{nt} - \frac{P_{nt}}{1+R_t} \tag{B2}$$

where it is assumed that $\delta_n = 0$. P_{ct} is the rental price for non-interest-bearing monetary assets. For an interest-bearing asset M_i, the own rate of return would be included, such that

$$P_{it} = P_{nt} - \frac{(1+r_{it})\, P_{nt}}{1+R_t} \tag{B3}$$

where r_{it} is the return on asset i.

When inflation is positive the nominal quantity of cash will be the same next period, but the real value will be lower, given by C_t/P^{*e}_{nt+1}. The depreciation rate of real balances δ_t can be solved from

$$\frac{C_t}{P^{*e}_{nt+1}} = (1-\delta_t)\,\frac{C_t}{P_{nt}} \tag{B4}$$

$$\therefore\ \delta_t = \frac{P^{*e}_{nt+1} - P_{nt}}{P^{*e}_{nt+1}} \tag{B5}$$

The user cost of non-interest-bearing money is:

$$P_{ct} = P_{nt} - \frac{(1-\delta_t)\, P^{*e}_{nt+1}}{(1+R_t)} \tag{B6}$$

Substituting (B5) in (B6) yields the user cost of cash as

$$P_{ct} = P_{nt} - \frac{P_{nt}}{(1+R_t)} \tag{B7}$$

This is identical to (B2), which is the user cost of cash in the absence of inflation.

For an interest-bearing asset M_i, the depreciation rate of real balances can be solved in a similar manner, but taking account of interest earnings. Therefore,

$$\frac{M_i(1+r_{it})}{P^{*e}_{nt+1}} = (1-\delta_t)\frac{M_{it}}{P_{nt}} \tag{B8}$$

which implies,

$$\delta_t = \frac{P^e_{nt+1} - P^*_{nt}(1+r_{it})}{P^e_{nt+1}} \tag{B9}$$

Substituting (B9) in (B8) gives the user cost of interest-bearing asset M_i as

$$p_{it} = P_{nt} - \frac{P_{nt}(1+r_{it})}{(1+R_t)} \tag{B10}$$

which is again identical to (B3), the user cost in the absence of inflation.

Re-arranging (B10), yields

$$p_{it} = \frac{P_{nt}(R_t - r_{it})}{(1+R_t)} \tag{B11}$$

This simplifies to R_t - r_{it} as shown in equation (2) in the text when calculating the Divisia weights s_{it}. That inflation does not appear to affect the user cost of money may appear surprising. This is because we have assumed R_t is constant. When R_t moves with inflation, the price (user cost) of real goods will fall relative to the price of money.

Appendix C - Data utilised in the construction of Divisia indices

Notes and coin - published level data non-seasonally adjusted (nsa).

Non-interest-bearing bank deposits - ICCs' holdings provided by the Bank's Financial Statistics Division. This series was then subtracted from the known total with the residual divided between Persons and OFIs on an estimated basis.

Interest-bearing sight bank deposits - ICCs' holdings provided by the Bank's Financial Statistics Division. This series was then subtracted from the known total with the residual divided between Persons and OFIs on an estimated basis.

Interest-bearing time bank deposits - break-adjusted sectoral flow data (nsa) provided by the Bank's Financial Statistics Division which are subsequently calculated to levels. Building society holdings of bank certificates of deposit and of bank deposits were deducted from OFIs' holdings of bank time deposits.

Building society deposits - break-adjusted sectoral flow data (nsa) provided by the Bank's Financial Statistics Division which are subsequently calculated to levels.

TESSAs(15) - persons' bank time and building society retail deposits are adjusted for the introduction of TESSAs by subtracting the published levels of TESSAs (nsa) from the components. This is a reasonable calculation to make as TESSAs are not held for transactions purposes and as such should not be incorporated in the Divisia indices.

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(15) Tax Exempt Special Savings Accounts.

Our indices are adjusted statistically for Abbey National's flotation in 1989 by incorporating the relevant break-adjusted flow data for bank sight and time deposits and building society deposits.

Bank current account (gross rate) - up to 1984 the series is a rate provided by a single bank which offered interest-bearing sight deposits. Thereafter, it is an average of the rates offered by the major clearing banks on deposits of £500.

Clearing banks interest-bearing personal account (gross rate) - pre-1984 series is interest payable on seven-day notice deposit accounts with the clearing banks; thereafter it is an average of the rates payable on two or more similar accounts with tiered interest rates according to the size of balance held. We take the rate payable on the median tier at any one time (currently £10,000; it has risen over time).

Building society deposit (gross rate) - pre-1984 series utilises the average building society share rate, as published by the Building Societies Commission (this provides a net figure; the gross rate is derived by including the composite tax rate). Thereafter it is an average of the savings account gross rates offered by the largest five building societies.

London interbank overnight deposit rate - observed rate at about 10.30am; as published in Financial Statistics.

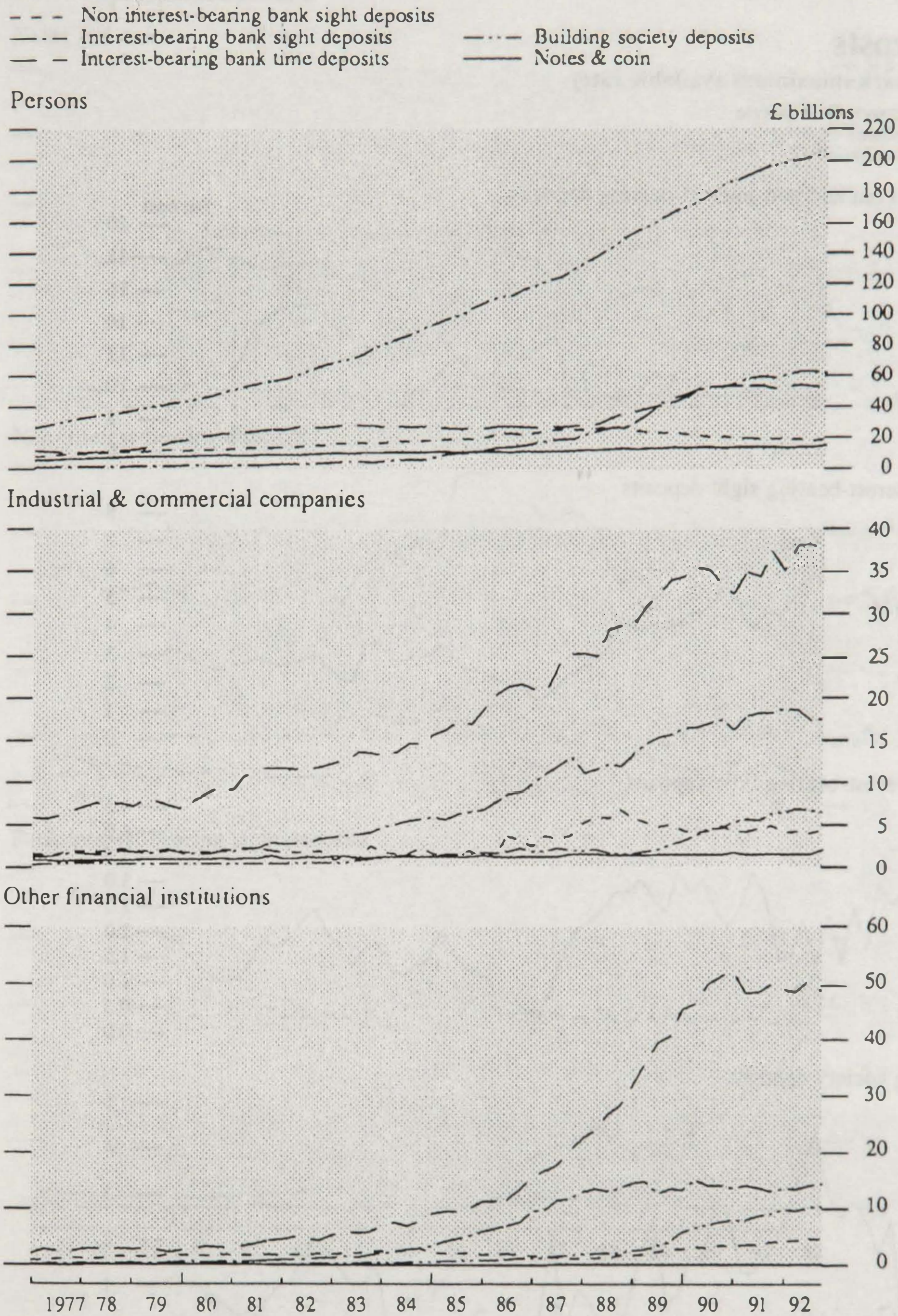
London interbank three-month deposit rate - as published in Financial Statistics.

Benchmark rate - the index uses the three-month local authority deposit rate as the benchmark rate (as published in Financial Statistics).

All interest rates are average rates over the quarter.

Appendix D - Components of Divisia

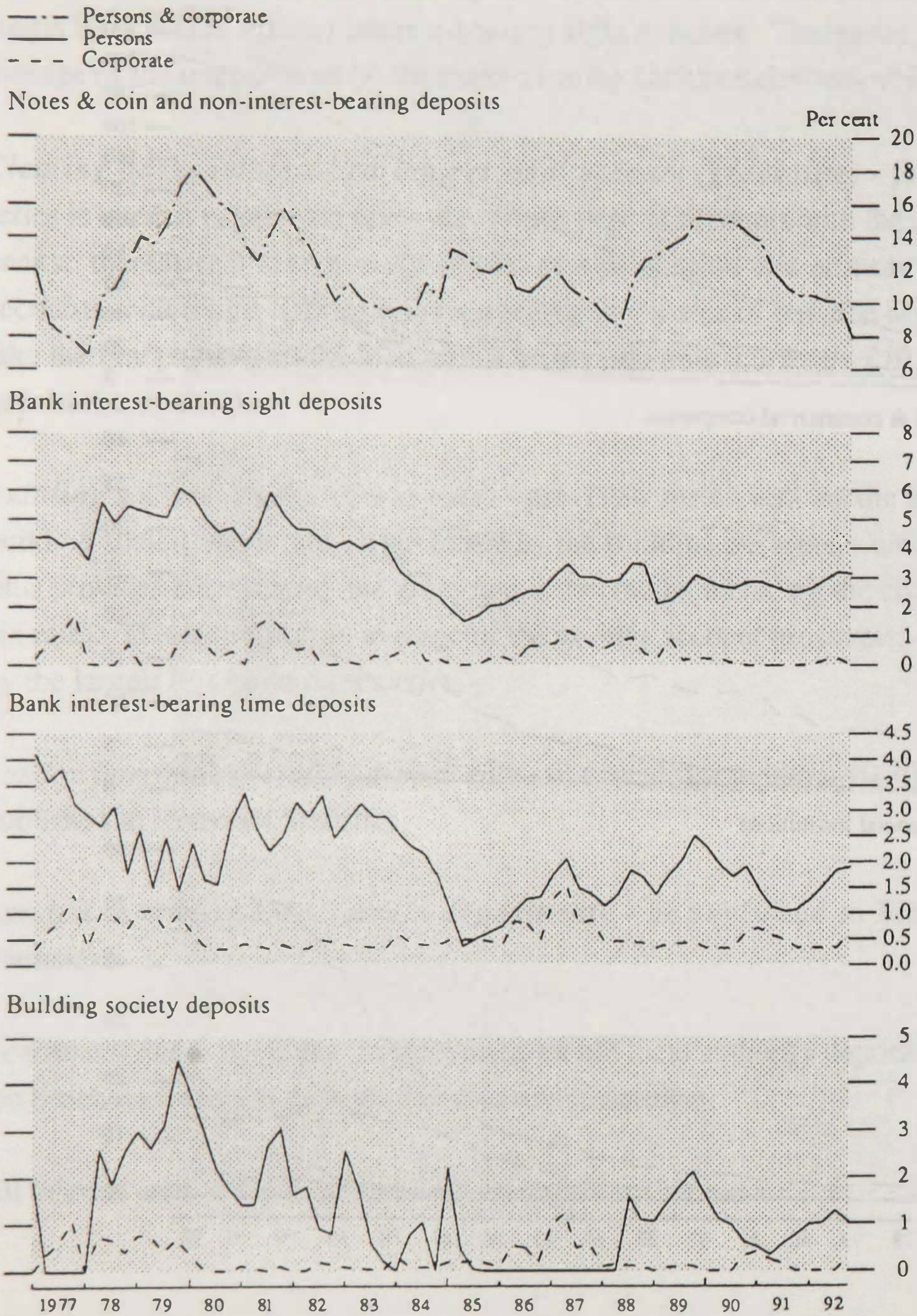
Components of Divisia (levels, ua)



Appendix E - User costs and growth weights with maximum available rate as benchmark

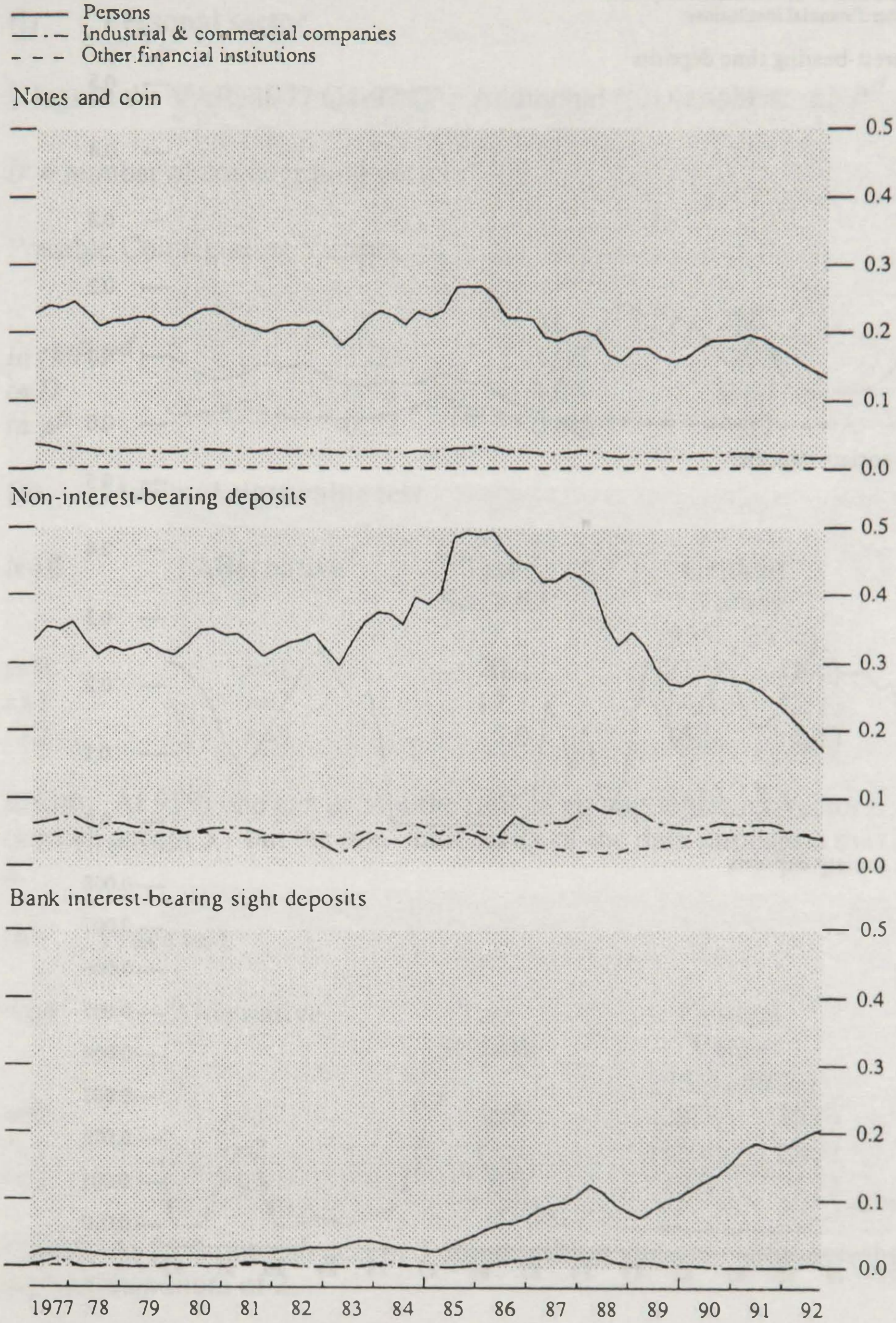
User costs

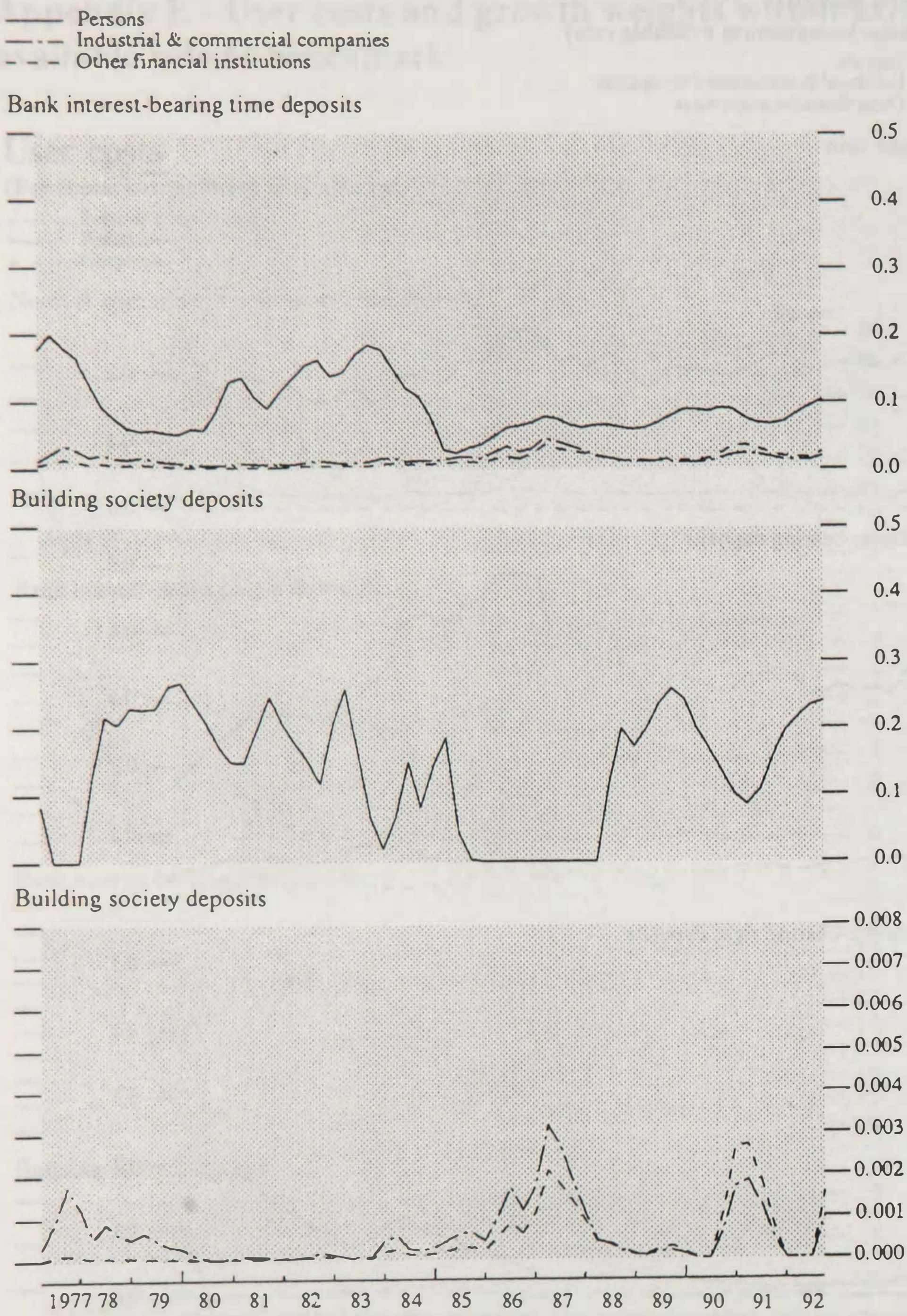
(Benchmark=maximum available rate)



Growth weights

(Benchmark=maximum available rate)




Persons
Industrial & commercial companies
Other financial institutions
Bank interest-bearing time deposits
0.5
0.4
0.3
0.2
0.1
0.0
Building society deposits
0.5
0.4
0.3
0.2
0.1
0.0
Building society deposits
0.008
0.007
0.006
0.005
0.004
0.003
0.002
0.001
0.000
1977 78 79 80 81 82 83 84 85 86 87 88 89 90 91 92

Appendix F - Johansen tests for cointegration

(i) Personal sector

3 lags in the VAR, 1977 Q4-92 Q4. Additional $I(0)$ variables: ΔlnP^d

(r = number of cointegrating vectors)

Possible Cointegrating Vectors

	I	II	III
$ln\,(M^p/P^d)$	-1.00	-1.00	-1.00
$ln\,D$	0.93	1.26	0.56
$ln\,\rho^p$	-0.22	-0.0017	1.22

(a) Maximal eigenvalue test

Null	Alternative	Test Statistic	Critical Values (95%, 90%)
r=0	r=1	20.2	(21.0, 18.6)
$r\leq1$	r=2	12.7	(14.1, 12.1)
$r\leq2$	r=3	5.0	(3.8, 2.7)

Result: At 95% one can accept the null of no cointegrating vectors. At 90% one can accept 3 - but the non-stationarity of the data imposes a maximum of 2.

(b) Trace test

Null	Alternative	Test Statistic	Critical Values (95%, 90%)
r=0	r=1	38.0	(29.7, 26.8)
$r\leq1$	r=2	17.7	(15.4, 13.3)
$r\leq2$	r=3	5.0	(3.8, 2.7)

Result: At 95% one can accept 3 cointegrating vectors - although there is an implicit maximum of 2.

Overall conclusion: One sensible cointegrating vector. Possibly a second which can be ignored as weak.

(ii) Corporate sector

4 lags in the VAR, 1978 Q1-92 Q4. Additional $I(0)$ variables, D_{86}, ΔlnP^{g}.

(r = number of cointegrating vectors)

Possible Cointegrating Vectors

	I	II	III
$ln\,(M^{c}/P^{g})$	-1.00	-1.00	-1.00
$ln\,GDP$	3.42	1.83	3.94
$ln\,\rho^{c}$	6.43	-0.31	0.66

(a) Maximal eigenvalue test

Null	Alternative	Test Statistic	Critical Values (95%, 90%)
$r=0$	$r=1$	19.2	(21.0, 18.6)
$r\leq1$	$r=2$	5.3	(14.1, 12.1)
$r\leq2$	$r=3$	3.5	(3.8, 2.7)

Result: At 95% one can accept the null of no cointegrating vectors. At 90% there may be one.

(b) Trace test

Null	Alternative	Test Statistics	Critical Values (95%, 90%)
$r=0$	$r=1$	28.0	(29.7, 26.8)
$r\leq1$	$r=2$	8.8	(15.4, 13.3)
$r\leq2$	$r=3$	3.5	(3.8, 2.7)

Result: At 95% one can accept the null of no cointegrating vectors. At 90% there may be one.

Overall conclusion: No cointegrating vectors.

(iii) Aggregate index

4 lags in the VAR, 1978 Q1-92 Q4, Additional $I(0)$ variable: ΔlnP^d, $D86$.

(r = number of cointegrating vectors)

Possible cointegrating vectors

	I	II	III
$ln\,(M/P^d)$	-1.00	-1.00	-1.00
$ln\,D$	0.72	1.41	1.51
$ln\,\rho$	-0.52	0.34	-0.22

(a) Maximal eigenvalue test

Null	Alternative	Test Statistic	Critical Values (95%, 90%)
r=0	r=1	19.6	(21.0, 18.6)
r≤1	r=2	12.0	(14.1, 12.1)
r≤2	r=3	3.4	(3.8, 2.7)

Result: At 90% one can accept a single cointegrating vector, at 95% no cointegrating vectors.

(b) Trace test

Null	Alternative	Test Statistic	Critical Values (95%, 90%)
r=0	r=1	35.0	(29.7, 26.8)
r≤1	r=2	15.4	(15.4, 13.3)
r≤2	r=3	3.4	(3.8, 2.7)

Result: At 95% one can accept a single cointegrating vector, at 90% one can accept two cointegrating vectors.

Overall conclusion: one cointegrating vector.

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