The Statistical Distribution of Short-Term Libor Rates Under Two Monetary Regimes

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CONTENTS

Abstract

We present a statistical analysis of sterling libor interest rates in two monetary regimes: free-floating of sterling, prior to ERM entry, and the recent ERM regime. Our findings indicate that short-term interest rates follow a random walk with time-varying volatility, and increments drawn from a kurtotic distribution. Moreover, we find that the process followed by the short rate is sensitive to the regime: interest rate changes observed in the floating-rate regime are drawn from a distribution with much fatter tails (the Cauchy distribution) than for the ERM regime (t-distribution). This characterisation of the process followed by the short interest rate is inconsistent with existing pricing models for interest-rate-derivative securities, which assume either that short rates follow simple Geometric Brownian Motion, or that they are mean-reverting.

1. Introduction

Early pricing models for interest rate derivative securities [such as the Rendleman and Bartter (1980), Black and Scholes (1973) and Black (1976) models applied to bond options] and the calculation of capital charges [see Bank of England (1988)] assumed that short-term interest rates could be adequately described by a random walk (in discrete time), or Geometric Brownian Motion (GBM) (in continuous time). GBM is characterised by a *constant* drift rate (possibly zero) and a *constant* volatility. If the interest rate does follow GBM, then we would expect (from the Central Limit Theorem) the distribution of interest rate changes to be normal. However, as with other financial prices, we observe that the empirical distribution of interest rate changes is highly non-normal, with fat tails and high peaks relative to the normal distribution (see Figure 1). We can therefore infer that GBM is not a good model of the short interest rate process. As we shall see, this departure from GBM and associated normality is particularly evident for interest rate changes calculated over short intervals.

More recently developed derivative pricing models have proposed an underlying process for interest rate changes in which the drift rate of the process is not constant, but varies through time. It has been argued that, unlike the prices of securities such as currencies, we might expect the path of interest rates to be influenced by their current level. These models, therefore, do not have the Markov property of GBM, that the expected interest rate next period is uncorrelated with current and past interest rates. In particular, since interest rates cannot rise or fall without limit, the conditional probability of a rise/fall in interest rates will vary with their current level in such a way that if current rates are 'high', a fall will be more likely than a rise, while if current rates are 'low', a rise will be more likely than a fall. Such a 'mean-reverting' underlying process for interest rates would, *inter alia*, account for the lower volatility of long-term interest rates relative to short-term rates. Examples of pricing models which assume a mean-reverting underlying interest rate process, include the general equilibrium models of Cox, Ingersoll and Ross (1985) and Longstaff and Schwartz (1992), and the arbitrage-free models of Black, Derman and Toy (1990), Heath, Jarrow and Morton (1989) and Hull and White (1990).

Statistical modelling of asset prices and rates of return has concentrated almost exclusively on exchange rates, stock prices and commodity prices [see Mandelbrot (1965) and Fama (1965) for early research of this kind, and Bollerslev (1987), Hsieh (1988), Hall, Brorsen and Irwin (1989) and Fujihara and Park (1990) among recent contributions]. However, in a recent paper, Chan, Karolyi, Longstaff and Sanders (1992) test several short rate/ term structure models, using monthly US Treasury Bill data for the period 1964 to 1989, against the following 'general' alternative stochastic process for the short rate, *r*:

$$dr=(\alpha+\beta.r).dt+\sigma.r^{\gamma}.dz \quad \textbf{(1)}$$

where dz is a normally distributed Wiener process, and α,β,σ and γ are constant parameters. These authors find, *inter alia*, that models which assume that interest rate changes are homoskedastic perform poorly, and that mean-reversion, if present, is extremely weak. Our results are very similar. However, we note that the testing procedure employed by Chan *et al* is incomplete, in that the stationarity properties of the short rate used for testing are not established, and the assumption of normally distributed increments to the interest rate process is not tested. The present paper focuses on these two questions. We present an analysis of the empirical distributions of two short-term interest rates: 3-month sterling libor and 6-month sterling libor. The latter is used as a reference rate for swap pricing, and is therefore of special interest in the calculation of swap values and credit exposure from interest rate swaps.

Another focus of interest in the present study is the effect on the interest rate distribution and volatility of membership of a credible exchange rate target zone regime. In October 1990, sterling entered the European Exchange Rate Mechanism (ERM). The modelling exercise is carried out for pre-ERM and ERM sample periods, in recognition of the possibility that this change in regime caused a change in the process governing short-term interest rates. Ideally, we would have liked to analyse data from the period following sterling's recent withdrawal from the ERM. However, the latter time series is very short. Our findings indicate that the monetary regime is an important conditioning factor for the distribution of interest rates. In particular, membership of a credible exchange rate regime renders the interest rate distribution less kurtotic and more nearly normal than under a free-floating rate regime - a result which should impinge on the pricing of interest rate derivatives.

The rest of the paper is arranged as follows. In Section 2 we establish for each interest rate/sample period:

(a) *whether the interest rate follows a random walk;*

(b) *whether the variance of the random walk is constant or time-varying;*

(c) *the nature of the distribution of interest rate changes.*

Section 3 interprets these results, and the paper ends with a discussion of the implications of our findings for the pricing of interest rate derivative securities and the calculation of capital charges against interest rate derivatives.

2. Statistical Analysis

We analyse daily observations on the logarithm of 3 and 6-month sterling libor for two sample periods: a period of (relatively) free-floating of sterling, from *5 April 1988 to 5 October 1990*. (This period could not have been extended backwards without including the period in which the sterling-deutschmark rate was targeted, which may have implied a different process for interest rates); and the period of sterling membership of the ERM, here measured from *19 November 1990 to 17 July 1992*.

Histograms of the log-differences in 3-month and 6-month libor rates are shown in Figures 2(a to f) and 3(a to f) for differences of 1, 30 and 90 days, and for pre-ERM and ERM samples. Also shown are the closest fitting normal curves in each case. These distributions exhibit several striking features, which are shared by those of other asset prices and rates of return. First, they are typically non-normal, characterised by higher peaks than the normal curve and by extra weight in the tails of the distributions. The distributions are also unstable: as the differencing interval increases from 1 to 30 to 90 days, the distributions become progressively more normal, the high peaks and fat tails becoming less pronounced. A particularly interesting contrast is apparent between the distributions in the two sample periods. While the distributions of the ERM sample are quite unstable, tending quickly to normality as the differencing interval is increased, the pre-ERM sample distribution is quite stable: even the 90-day change in interest rates has a prominent peak and fatter tails than normal. This result is fairly extreme in comparison with the distributions of other rates of return, which are typically close to normality after quarterly aggregation.

Each of these results confirms that the simple GBM model of short-term interest rates is not supported by the data. This is especially true under the regime of freely floating exchange rates. We now proceed to establish the origins of this non-normality.

(a) Tests of the Random Walk Hypothesis

Table 1 shows test statistics [Augmented Dickey-Fuller (ADF) statistics (see Dickey and Fuller (1979)] for the hypothesis that the interest rate follows a random walk. These tests without exception fail to reject the random walk model of interest rates, for both rates and in both regimes. The results are *not* borderline, so that the low power of the Dickey-Fuller tests is not an issue. Moreover, the autocorrelation functions and spectra of the libor rates provide corroborative evidence, giving no indication of departures from the random walk. Because any significant mean-reversion in the series would have led to the rejection of the unit root hypothesis, these results imply, *inter alia*, that models which have the property of mean-reversion are inappropriate for pricing derivatives of libor short interest rates.

However, we know that if interest rates follow a random walk with increments drawn from a normal distribution, then the distribution of daily changes in interest rates will itself be normal. Indeed, because of the stability under temporal aggregation of the normal distribution, the distribution of n-day changes in interest rates will be normally distributed, for all n. Yet we have observed two features of the data which fail to conform with the GBM model: the distribution of daily interest rate changes is highly non-normal, exhibiting a very high peak and fat tails (ie a high degree of kurtosis) relative to the normal distribution; and the distribution becomes more nearly normal as the differencing interval is increased. There are three possible explanations for these findings: *either* the variance of the increments to the interest rate process varies through time [this was shown by Diebold (1986) to generate both the features which characterise the libor interest rate distributions], *or* the increments to the random walk are drawn from a distribution with fatter tails than the normal, *or* both. We explore each of these possibilities in the following two subsections.

(b) Time-Varying Volatility

Figure 4 (a,b) shows the log changes in libor 3-month and 6-month rates over the entire sample period. Even a cursory glance at these data suggests that the volatility of interest rates varies through time. More specifically, there appear to be episodes which are extremely 'noisy' and episodes which are by comparison 'quiet'. During a noisy (quiet) period, a large (small) change on one day is likely to be followed by a large (small) change. Large changes follow large changes of either sign, and small changes follow small changes. This suggests that interest rate volatility may be autoregressive. This observation in other financial prices led to the development of AutoRegressive Conditional

Heteroskedasticity (so-called ARCH) models. [See Engle (1982).] It follows that an ARCH model may be an appropriate representation of the variation in libor interest rate volatility through time.

Consider the following random walk model, which nests the discrete time approximation of the GBM model of interest rates. The interest rate, r_t, drifts through time (measured in days) at a constant rate, μ, (possibly zero) with increments drawn from a normal distribution with variance, σ_t^2, which varies through time. Thus:

$$\Delta ln r_t = \mu + \epsilon_t \quad (2)$$

$$\text{where } \epsilon_t \mid \Omega_{t-1} \sim N(0, \sigma_t^2) \quad (3)$$

$$\sigma_t^2 = \alpha + \beta(L).\sigma_{t-1}^2 + \gamma(L).\epsilon_{t-1}^2 \;;\beta_0 = \gamma_0 = 0 \quad (4)$$

and Ω_t represents the information available up to, and including, period t.

The variance of the process is assumed to evolve autoregressively (though the lag polynomial β(L)) and to depend upon the squared residuals (through the lag polynomial γ(L)). That is, the process ϵ_t exhibits Generalised AutoRegressive Conditional Heteroskedasticity (GARCH) [see Bollerslev (1986)]. This model therefore accounts for the characteristic feature of financial markets, that volatile days tend to be followed by volatile days, quiet days by quiet days. This model reduces to the simple random walk approximation to the GBM model when β(L)=γ(L)=0, in which case variance is constant, $\sigma_t^2 = \alpha$, for all t.

Table 2 presents test statistics for the presence of GARCH processes in short-term interest rates. The presence of GARCH in each libor rate and for each sub-period is confirmed. This finding accounts (in part at least) for the kurtosis and instability of the distribution of daily interest rate changes, and echoes the results established for exchange rates [see, for example, Fujihara and Park (1990)]. We also note that there are much stronger ARCH effects in the pre-ERM sample than in the ERM sample, which largely explains the high degree of kurtosis present in the pre-ERM data.

(c) Conditional Distributions of Interest Rate Changes

GARCH models of the form described in (1), (2) and (3) were estimated for each libor rate and each sample. In each case, an ARCH model of low order adequately accounted for the heteroskedasticity present. The residuals from each of these regressions were scaled by the conditional standard deviation, $\sigma_t \mid \Omega_{t-1}$, to form the standardised error process ϵ_t/σ_t. Provided the error term ϵ_t is normally distributed, then the scaled residual ϵ_t/σ_t should be normally distributed. This was tested for in each case, but the scaled residuals were found to exhibit excess kurtosis relative to the normal distribution.

This result may be explained by the conditional distribution being other than normal - specifically, a distribution with a higher degree of kurtosis than the normal distribution. Bollerslev (1987) addressed a similar problem in the analysis of other financial prices, and used the student *t* distribution as a non-normal alternative. We therefore looked at the *t* distribution as a general alternative conditioning distribution of the interest rate process. The *t* distribution has a degree of kurtosis which varies directly with the number of degrees of freedom, ν: for low degrees of freedom, the distribution is highly kurtotic, with fat tails and a high peak; as the number of degrees of freedom rises, the distribution tends to normality.

We follow Bollerslev's approach, and write the model of the short-term interest rate as:

$$\Delta logr_t = b_0 + b_1.(L)\Delta logr_{t-1} + \epsilon_t \tag{5}$$

$$\epsilon_t \mid \Omega_{t-1} \sim f_\nu(\epsilon_t \mid \Omega_{t-1})$$

$$f_{\nu}(z) = \pi^{-1/2}\Gamma((\nu+1)/2)\,\Gamma(\nu/2)^{-1}\,((\nu-2)h_t)^{-1/2}$$

$$* (1 + z^2\, h_t^{-1}(\nu-2)^{-1})^{-(\nu+1)/2} \qquad \textbf{(6)}$$

$$h_t = a_0^2 + \sum_{i=1}^{q} a_i^2\, \epsilon^2_{t-i} \qquad \textbf{(7)}$$

This model was estimated with varying success. The estimation results for the best-fitting models are displayed in Table 3, while histograms of scaled model residuals and their closest-fitting distributions are shown in Figures 5 (a to d). For both 3-month and 6-month libor in the ERM period, the *t* model was successfully estimated with a low number of degrees of freedom (4.64 and 3.75 for the 3 month and 6 month rates respectively). Estimation of the 3 and 6-month rates for the pre-ERM sample proved to be problematic, because the distributions are so extremely kurtotic. Our results suggested that the number of degrees of freedom was less than 2. We therefore tried the Cauchy distribution, which is a *t* distribution with 1 degree of freedom.

That the pre-ERM data fit the Cauchy distribution so well [as can be judged from Figure 5(a, c)], is not entirely surprising. The Cauchy distribution is special, in that it is one of the Stable Paretian class of distributions. A distinguishing characteristic of these distributions is that they are stable under addition (including temporal aggregation) - that is, the distribution of a sum of Cauchy variates has the same overall shape (in particular, has the same relative weight in the tail) as the individual Cauchy variates which form the sum. We have seen from the preliminary data analysis, that the pre-ERM data are remarkably stable under temporal aggregation, suggesting that one of the stable Paretian distributions may have generated pre-ERM data.

3. Interpretation

Our principal findings are that:

(i) interest rates were less volatile during sterling's membership of the ERM than prior to sterling joining the ERM;

(ii) strong ARCH present in the pre-ERM sample is followed by significantly weaker ARCH in the ERM sample.

We now briefly discuss and interpret these results.

A substantial research effort has explored the question of what should be the effect on exchange rate and interest rate volatility of membership of a credible exchange rate target zone regime (ie one in which effective intervention at the limits of the zone is expected). Early expositions [for example, Svensson (1991) and Krugman (1991)] showed that under rational expectations, membership of a credible regime should cause an exchange rate to be less variable than in a floating-rate regime. [A related study of the sterling-deutschmark exchange rate and membership of the ERM (Pesaran and Robinson (1993)) found that membership was indeed associated with weakened ARCH and lower exchange rate volatility.] This property results from the fact that, because intervention at the limits of the zone is expected by market participants, this expectation will constrain the dynamics of the exchange rate within the target zone - in particular, the exchange rate cannot respond as fully to changes in fundamentals as in a free-floating regime. However, for a given flow of news on exchange rate fundamentals and the associated fundamentals volatility, lower volatility in the exchange rate implies greater volatility in interest rates.

This prediction of greater interest rate volatility in target zone regimes was tested and contradicted in subsequent empirical studies of ERM interest rates and exchange rates. For example, Flood, Rose and Mathieson (1990) found that, while for some currencies a trade-off was found between exchange rate and interest rate volatility, other currencies exhibited a positive relationship. This mis-match between theory and evidence was addressed in models developed by Bertola and Caballero (1990) and Bertola and Svensson (1991). These studies demonstrated that a risk of realignment in exchange rate parities renders ambiguous the relationship between interest rate and exchange rate volatility, so that both a trade-off and a positive association between these volatilities is consistent with rational expectations. Our finding that membership of the ERM reduced libor volatility is therefore consistent with what might be expected for a target zone regime for which there is a perceived risk of realignment.

The presence of ARCH in speculative prices and rates of return has been rationalised in terms of the nature of the information flow reaching the market. If the flow of news is heterogeneous through time, with news clustering around particular calendar days, then this may generate heteroskedasticity in the price change measured in calendar time; alternatively, financial markets may be unable to assess immediately the value of news, so that its effect on rates may under or overshoot [see Fujihara and Park (1990)]. Moreover, this rationalisation can be extended: entry to the ERM would have changed the *significance* of news items. In particular, many news items, such as trade figures which would have significance for the conduct of interest rate policy in a free-floating regime, would cease to have significance in a target zone regime where interest rate policy is directed at maintaining the exchange rate within the target zone. This reformulation of policy may have reduced significantly the flow of news items which could impinge upon interest rates, and weakened ARCH in the ERM regime interest rate data.

4. Conclusions - Implications for Derivative Pricing and the Calculation of Capital Weights

We have presented a statistical analysis of the process followed by two key sterling interest rates through two distinct monetary regimes: the period prior to sterling's entry into the ERM, and that of sterling's membership of the ERM. We have found that these short-term rates exhibit excess kurtosis relative to the normal distribution, and that there are two sources of this kurtosis: the volatility of increments in interest rates is time-varying (well represented by a low order ARCH process) and the increments are drawn from conditional distributions which are themselves kurtotic (well represented by *t* distributions with a low number of degrees of freedom).

This characterisation of the distribution of short-term interest rates is inconsistent with derivative pricing models which assume that the short rate follows simple GBM, and with those models which incorporate mean-reversion in interest rates, which is found to be absent in the rates studied here. There are no interest-rate-derivative pricing models which incorporate both non-normality of conditional distribution and heteroskedasticity, although pricing models which incorporate a diffusion limit of the GARCH model have been developed [for example, Longstaff and Schwartz (1992)]. This suggests a useful line of research.

Interestingly, our results suggest rather different conclusions for bond and option pricing (and the calculation of capital risk-weights), according to the period (ie regime) under study and the length of exposure. Although the distributions of libor rates are significantly non-normal in both sample periods, under the ERM regime non-normality was rather less of a problem than it appears to have been prior to ERM membership, suggesting that bond and option pricing models which assume GBM will price risk more accurately in a fixed exchange rate regime than in a floating rate regime. Similarly, risk-weights and capital charges which assume GBM will give less protection than they are designed to provide in a floating rate regime, relative to a fixed rate regime.

TABLE 1: TESTS FOR UNIT ROOTS IN SHORT-TERM INTEREST RATES

Test statistics for the presence of unit roots in short-term libor rates. Each of the results strongly indicates the presence of unit roots, so that we cannot reject the random walk hypothesis. This also implies the rejection of mean-reversion.

Period		3-month libor	6-month libor
PRE-ERM	DF	-1.80	-1.76
	ADF(5)	-1.59	-1.58
	ADF(10)	-1.57	-1.69
ERM	DF	-1.41	-1.63
	ADF(5)	-1.54	-1.93
	ADF(10)	-1.36	-1.67

[Dickey-Fuller (DF), and Augmented Dickey-Fuller (ADF) statistics, critical value = -3.42]

TABLE 2: ARCH IN RESIDUAL TESTS

Test statistics for the presence of AutoRegressive Conditional Heteroskedasticity (ARCH). The results indicate that ARCH is present in both 3-month and 6-month libor rates and for each sample period, although the effect is stronger in the floating rate regime than in the ERM regime. The presence of ARCH effects partly accounts for the kurtosis which the libor rate distributions exhibit.

Period		3-month libor	6-month libor
PRE-ERM	LM(1)	24.99	19.99
	LM(4)	26.07	24.45
ERM	LM(1)	3.02	3.82
	LM(4)	9.95	10.98

[Lagrange Multiplier (LM) tests for 1st and 4th order ARCH, distributed χ^2_1 and χ^2_4 with critical values 3.84 and 9.49 respectively at the 95% confidence level.]

TABLE 3: DIAGNOSTIC STATISTICS

This table gives the regression results from estimation of the model:

$$\Delta lnr_t = a_0 + a\,(L).\Delta lnr_{t-1} + \epsilon_t$$

$$\epsilon_t \mid \Omega_{t-1} \sim g(0, \sigma_t^2 \mid \Omega_{t-1})$$

$$\sigma_t^2 = \alpha + \beta(L).\sigma_{t-1}^2 + \gamma(L).\epsilon_{t-1}^2$$

This specification allows for the presence of (G)ARCH effects (time-varying conditional volatility) and for a conditioning t-distribution. ARCH is found to give a parsimonious description of the heteroskedasticity present. For the ERM regime, a conditioning t-distribution with low degrees of freedom is found to account for the remaining excess kurtosis, while for the pre-ERM floating regime the Cauchy distribution (t-distribution with 1 degree of freedom) is appropriate.

Dependent Variable	LIBOR 3-month		LIBOR 6-month	
Sample period	pre-ERM	ERM	pre-ERM	ERM
Conditional Distribution	Cauchy	t(4.64)	Cauchy	t(3.75)
a_0	-0.0014	-0.052	-0.219	-0.031
	(0.113)	(1.759)	(2.009)	(1.085)
α	0.0203	0.408	0.0237	0.387
	(7.003)	(6.094)	(2.894)	(4.720)
$\gamma(1)$	0.129	0.109	0.0739	0.218
	(4.498)	(1.316)	(2.768)	(1.75)
$\gamma(2)$		0.107	0.0576	0.218
		(1.295)	(2.486)	(1.79)
$\gamma(3)$			0.0197	
			(1.529)	
$\gamma(4)$			0.0196	
			(1.608)	

t statistics are in parentheses.

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Figure 1: **Theoretical and Empirical Distributions of Interest Rate Changes**

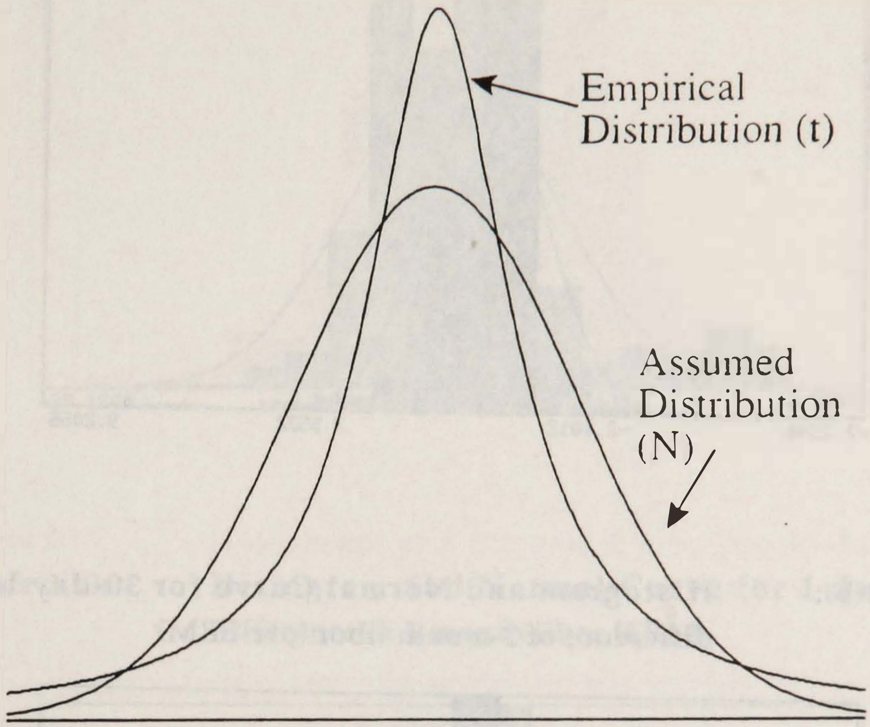


Figure 2(a): Histogram and Normal Curve for 1 day log difference of 3-month libor (pre-ERM)

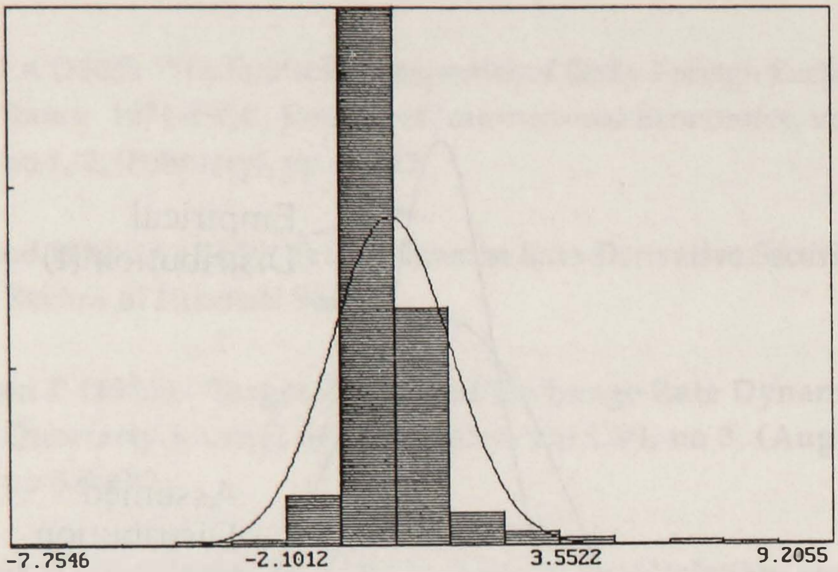


Figure 2(b): Histogram and Normal Curve for 30-day log difference of 3-month libor (pre-ERM)

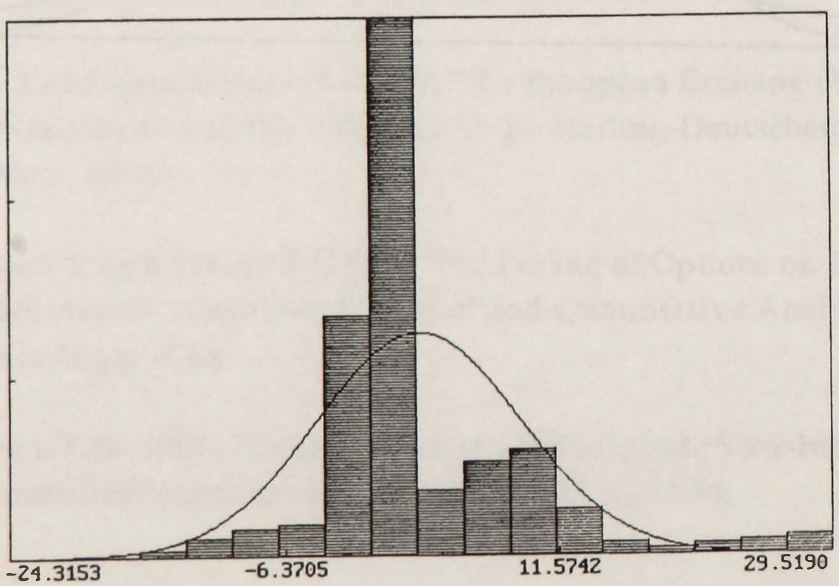


Figure 2(c): Histogram and Normal Curve for 90-day log difference of 3-month libor (pre-ERM)

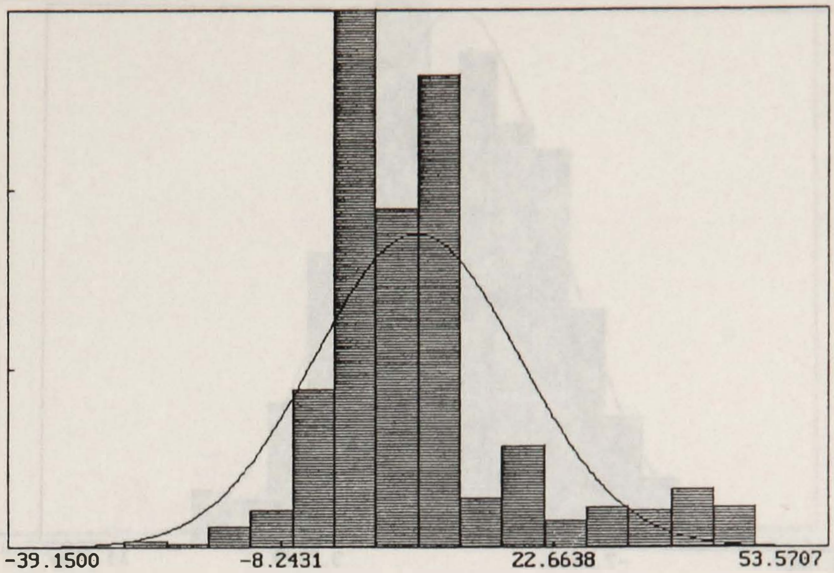


Figure 2(d): Histogram and Normal Curve for 1 day log difference of 3-month libor (ERM)

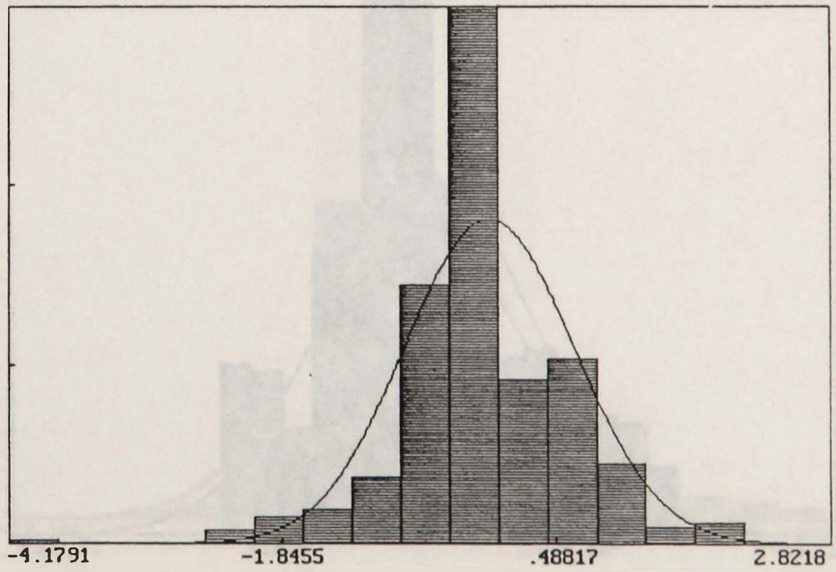


Figure 2(e): Histogram and Normal Curve for 30-day log difference of 3-month libor (ERM)

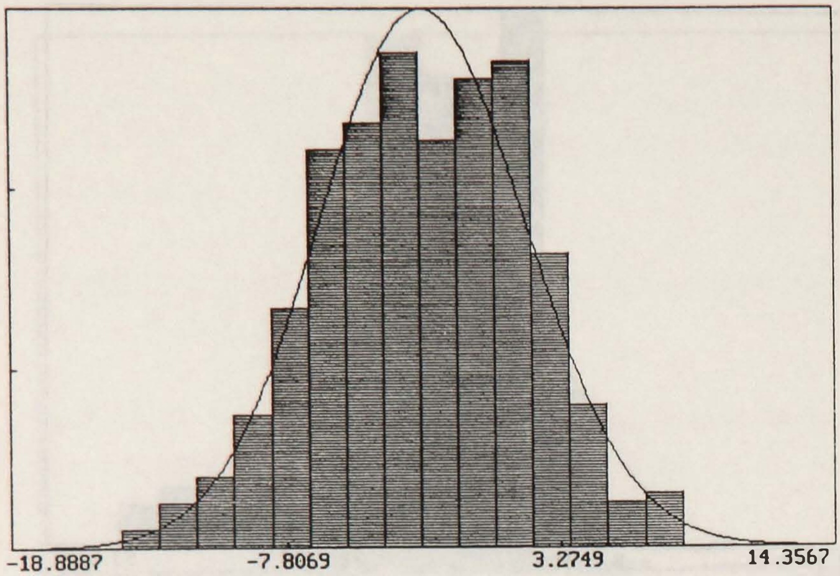


Figure 2(f): Histogram and Normal Curve for 90-day log difference of 3-month libor (ERM)

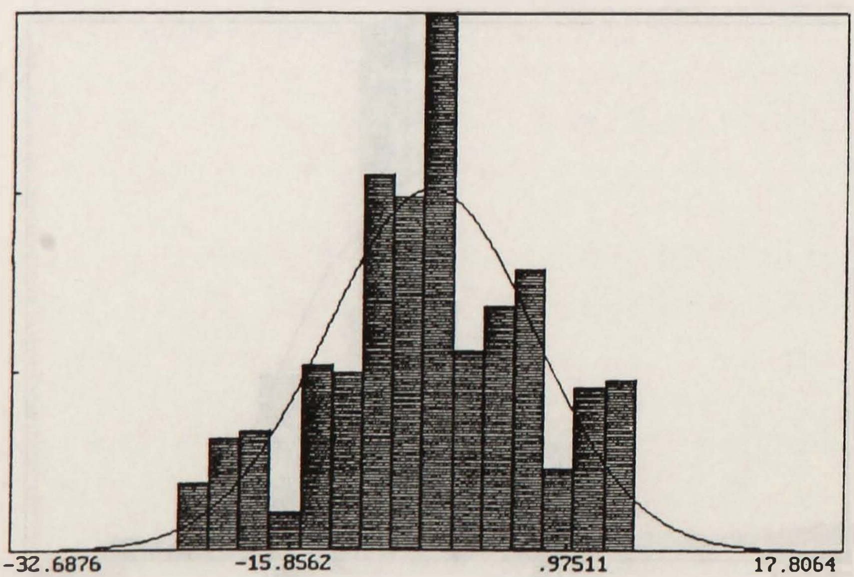


Figure 3(a): Histogram and Normal Curve for 1 day log difference of 6-month libor (pre-ERM)

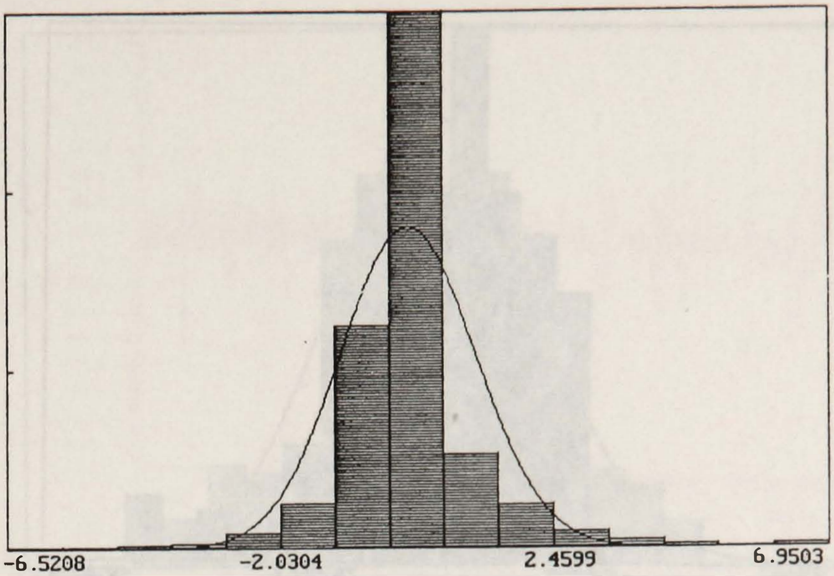


Figure 3(b): Histogram and Normal Curve for 30-day log difference of 6-month libor (pre-ERM)

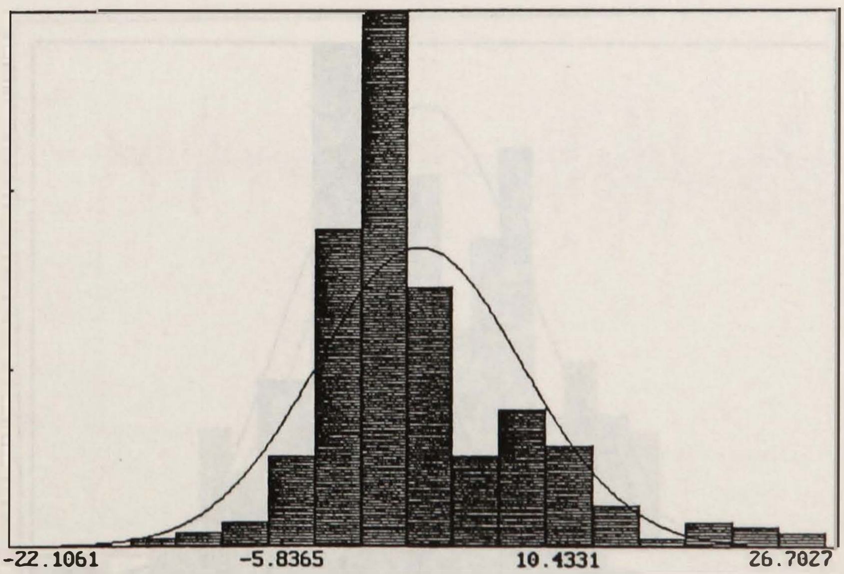


Figure 3(c): Histogram and Normal Curve for 90-day log difference of 6-month libor (pre-ERM)

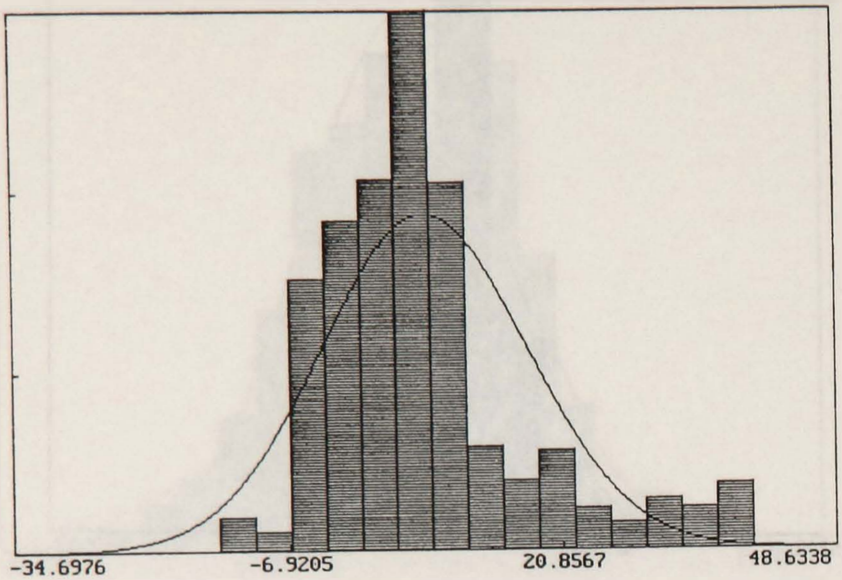


Figure 3(d): Histogram and Normal Curve for 1 day log difference of 6-month libor (ERM)

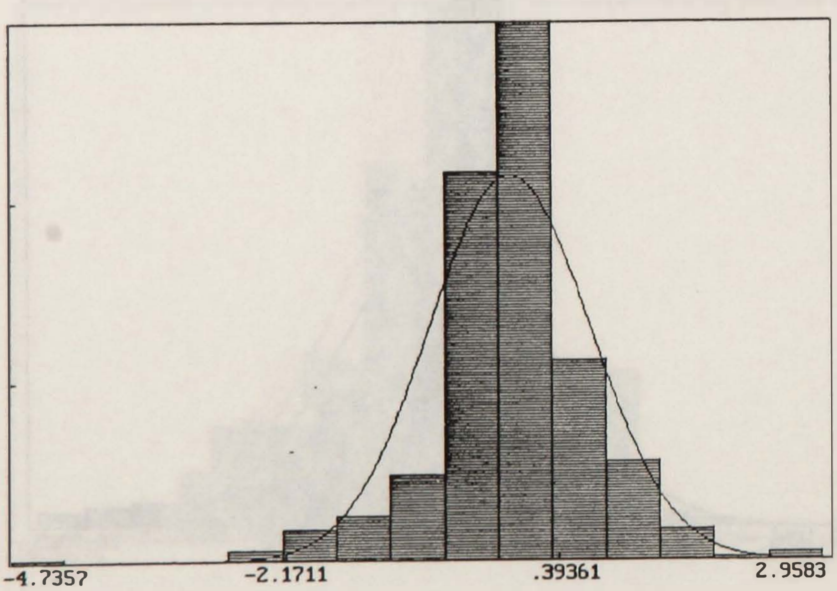


Figure 3(e): Histogram and Normal Curve for 30-day log difference of 6-month libor (ERM)

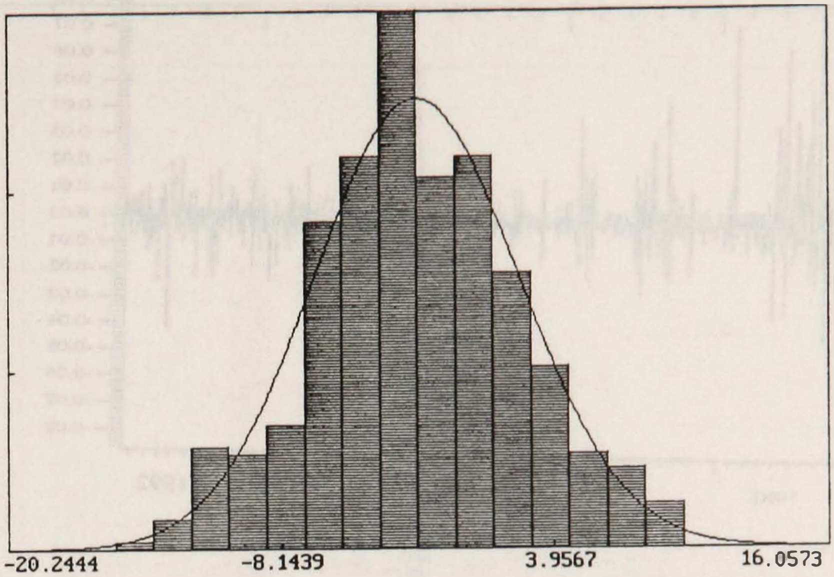


Figure 3(f): Histogram and Normal Curve for 90-day log difference of 6-month libor (ERM)

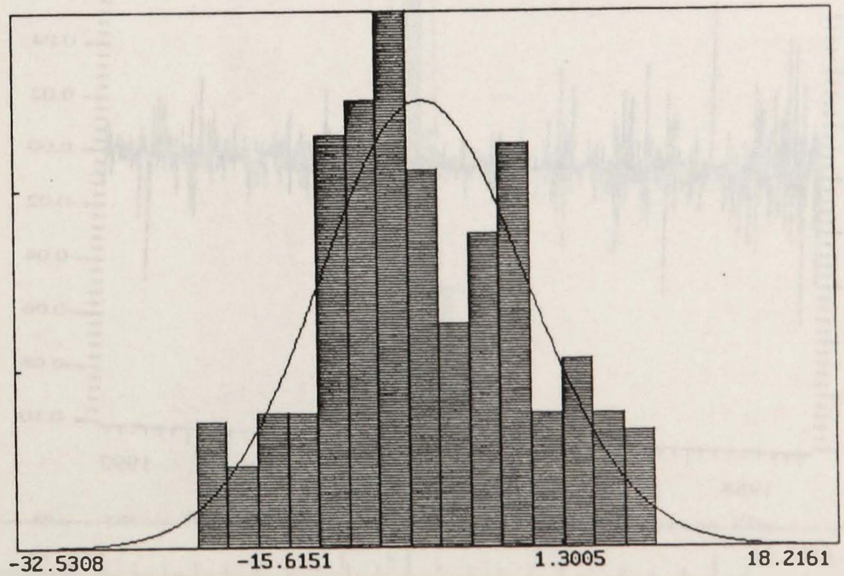


Figure 4(a): 3-month libor - log changes

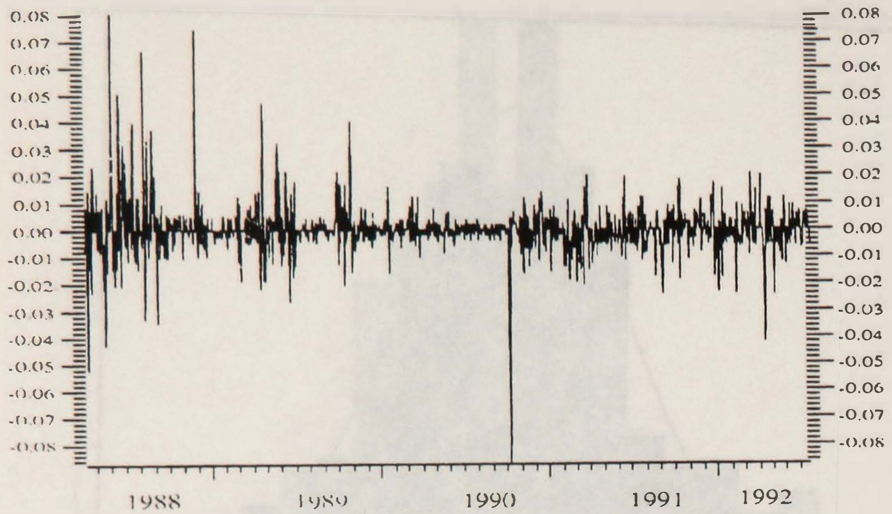


Figure 4(b): 6-month libor - log changes

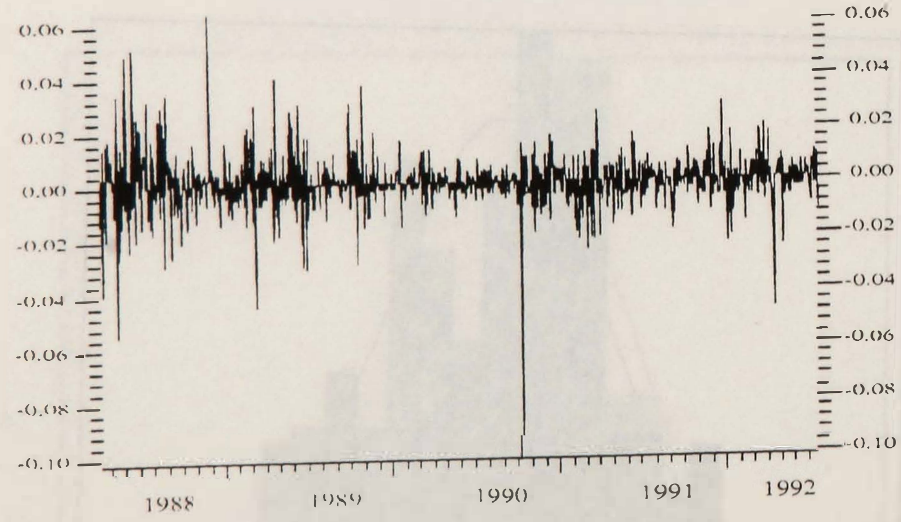


Figure 5(a): Histogram of 3-month libor (pre-ERM) and Cauchy Distribution

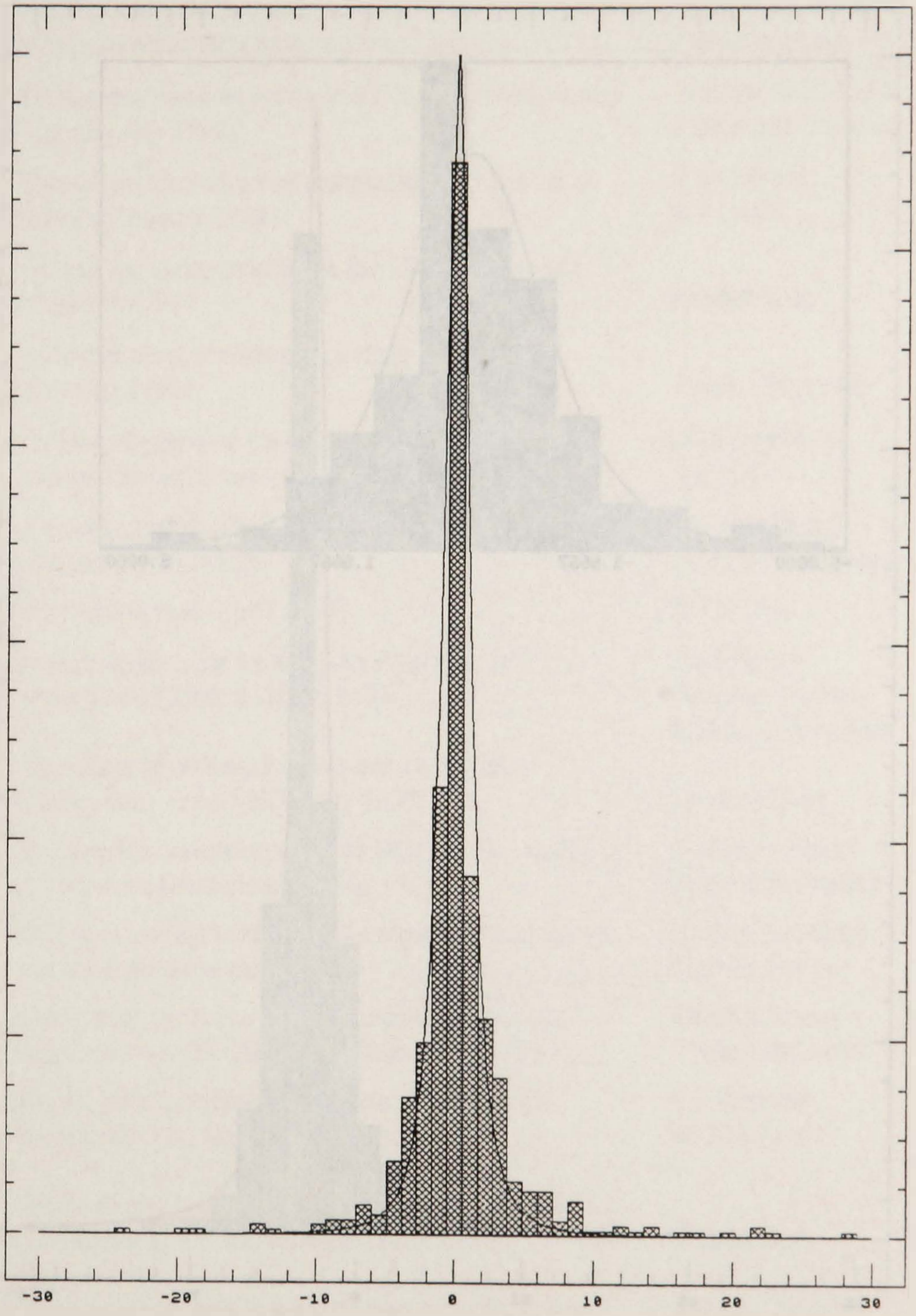


Figure 5(b): Histogram of 3-month libor (ERM) and the Closest Fitting t-Distribution

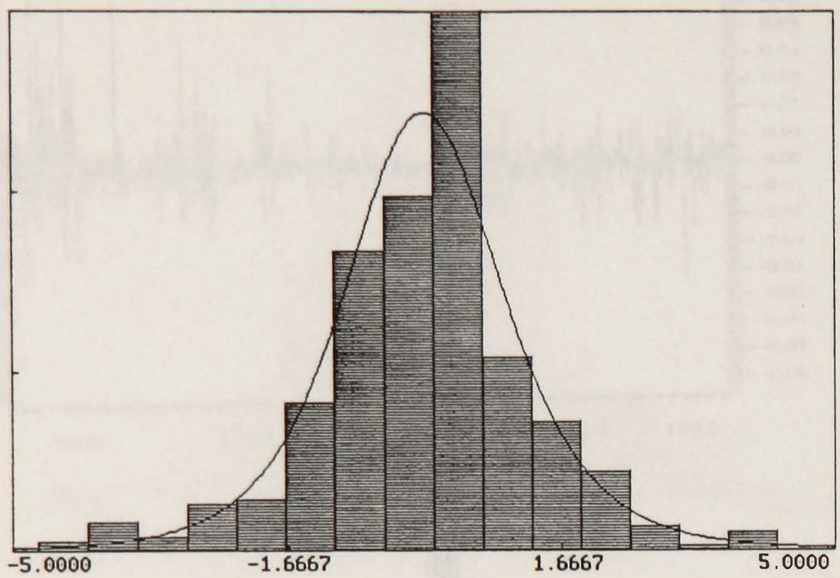


Figure 5(c): Histogram of 6-month libor (pre-ERM) and Cauchy Distribution

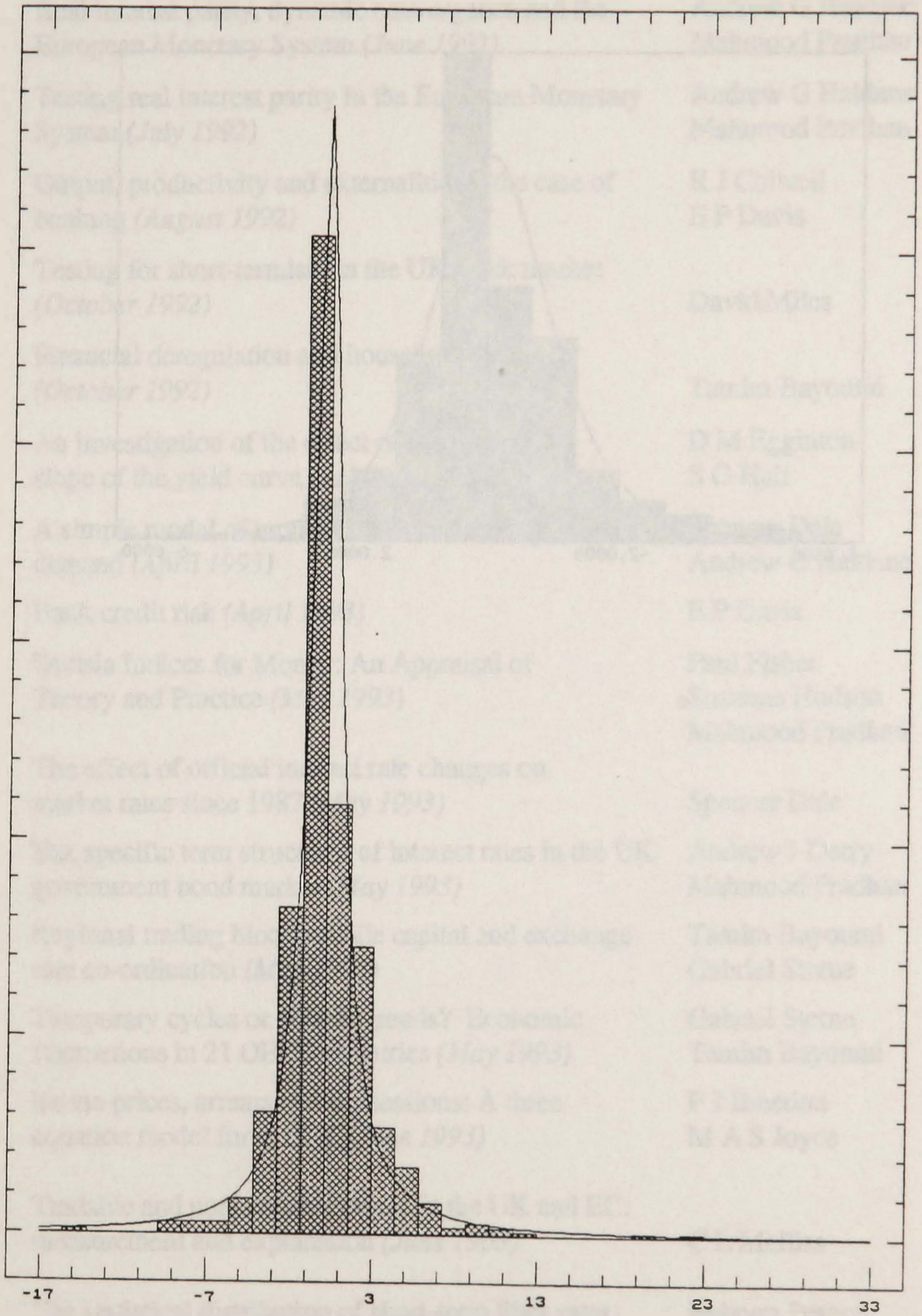
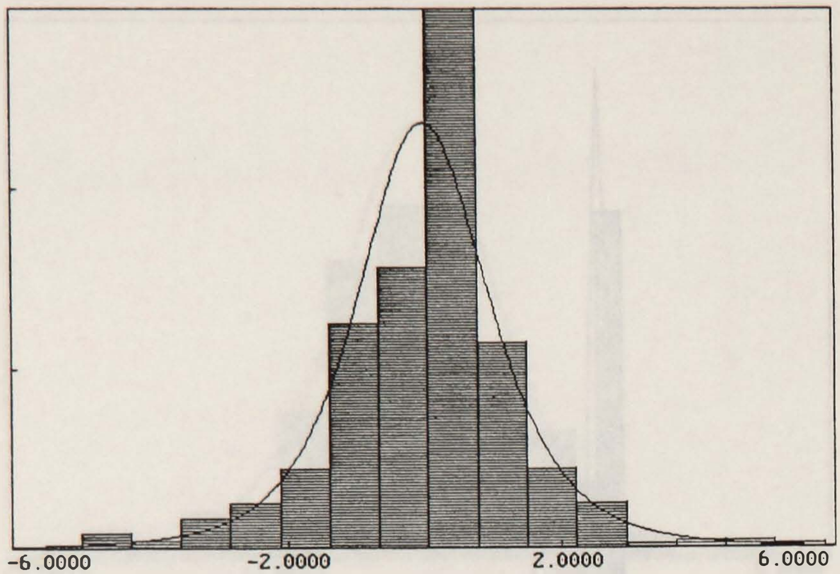


Figure 5(d): Histogram of 6-month libor (ERM) and Closest Fitting t-Distribution



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