Deriving Estimates of Inflation Expectations from the Prices of UK Government Bonds

by

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Erratum

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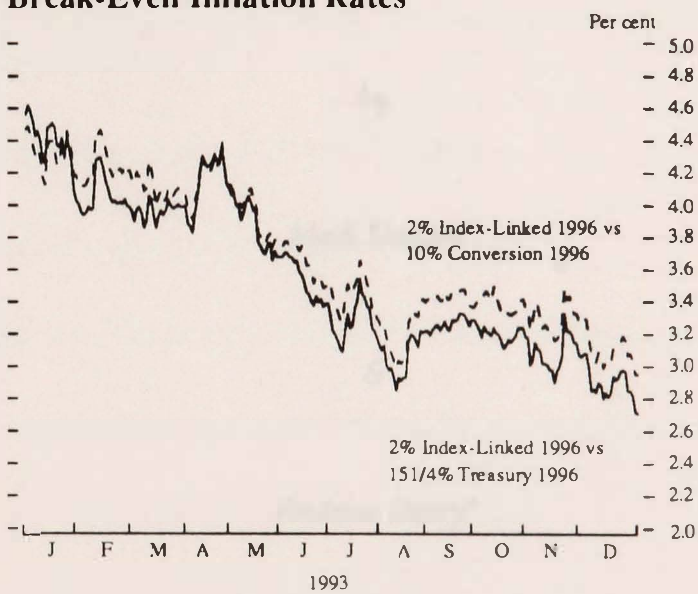
The following chart should replace Figure 4.5 on page 21.

Break-Even Inflation Rates



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Contents

Abstract

Bonds with payments linked to the Retail Price Index were first issued in the United Kingdom in 1981 and now account for a significant proportion of the total UK government bond market. This paper discusses various methods by which prices of these indexed bonds can be compared with prices of conventional bonds to infer market expectations of future inflation, and in particular outlines the approach currently used to produce the *inflation term structure* published in the Bank of England's *Inflation Report*. There are a number of estimation difficulties - both theoretical and practical - in deriving such term structures and therefore a number of caveats that should be borne in mind when interpreting such measures of expectations.

1 Introduction

There are several reasons why knowledge of inflation expectations is useful. By comparing market expectations of inflation with the government's inflation targets, it is possible to gauge financial markets' perceptions of the credibility of monetary policy.[1] Also, as market expectations of inflation encapsulate the markets' forecast of inflation they should contain information which can contribute to an individual institution's inflation forecasts. If business investment decisions are determined by prospective real returns, and we wish to forecast this investment, an estimate of inflation expectations is required. Finally, knowledge of the market's inflation expectations can assist decisions on whether to issue indexed or conventional debt.[2]

Traditionally, information on inflation expectations has been obtained by either survey or theory-based methods. However, these have shown themselves to be fairly unreliable - possibly because survey respondents have no incentive to answer accurately. More recently, efforts have been made to extract expectations from asset prices. For example, between 1985 and 1987 the New York Coffee, Sugar and Cocoa Exchange traded a futures contract on the US CPI-W[3] price index [see Petzel and Fabozzi (1988)]. Using the prices of these contracts, it was straightforward to obtain estimates of inflation expectations. No such contracts on the UK Retail Price Index (RPI) have ever been traded, so some alternative means of deriving inflation

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(1) Svensson (1993) discusses this in detail.

(2) One of the reasons given for the recent issue of the first ever Swedish indexed bond was that the Riksbank believed that market inflation expectations were above the likely outturn for inflation, making indexed bonds a cheaper source of funds than conventionals.

(3) CPI-W is the Consumer Price Index for urban wage earners and clerical workers.

expectations from the prices of financial assets must be considered for the United Kingdom.

If a financial market trades both indexed and non-indexed securities it is possible, subject to some assumptions about risk and liquidity premia, to derive estimates of inflation expectations from the prices of these instruments. In the United Kingdom, index-linked gilts, with cash flows linked to the RPI, were introduced in the 1981 Budget and such stocks now comprise 15% of the market. This has made it worthwhile to develop techniques to extract this information from UK Government bond prices. The principal aim of this paper is to review and critically assess several such techniques.

The paper is organised as follows. Section 2 briefly considers the survey and theory-based approaches for obtaining inflation expectations. In Section 3 we introduce the concept of index-linked bonds and explain in detail how the indexation works in the UK gilts market. In the next section, two simple techniques frequently used by the market to assess inflation expectations are examined. The first, which we call the *simple* measure, is effectively "a back of the envelope" calculation; the second, the so-called *break-even inflation rate,* is a more sophisticated adaptation of this calculation which involves an iterative procedure. Section 5 briefly sets out our preferred method, in the form of the inflation term structure. This uses an approach based on that of the break-even inflation rate but models both the nominal and the real interest rate term structures in their entirety rather than examining pairs of bonds of comparable maturity, and thereby generates a complete term structure of inflation expectations. Some of the complications that arise when attempting to estimate and interpret the inflation term structure are discussed in Section 6. Section 7 reports the results of an event study using this measure of inflation expectations, illustrating how market expectations were revised following sterling's withdrawal from the Exchange Rate Mechanism in September 1992. In Section 8 we provide a summary of the work so far.

2 Survey and theory-based methods for obtaining inflation expectations

2.1 Survey methods

There is a large literature on the quality and properties of measures of inflation expectations obtained from surveys. One of the best known of such surveys is the Livingston survey conducted in the United States, which records inflation expectations (ie inflation forecasts) of economists from a range of financial institutions. Figlewski and Wachtel (1981) report that the Livingston survey data do not conform to the rational expectations hypothesis. Pearce (1987) examines a different source of US expectations data, the surveys of money market participants conducted by Money Market Services (MMS): these poll securities dealers for their predictions of changes in economic variables to be announced in the near future including the CPI. The conclusion drawn by Pearce is that there is no evidence from the MMS data that expectations are not rational, and suggests that the data could be used to represent market expectations of inflation announcements. However, in addition to Figlewski and Wachtel (*op cit*) a number of other investigators have found that expectation measures from surveys are not rational (Evans and Gulamani 1984; Pesaran 1985).[4] Although results of this nature have led some researchers such as Mishkin (1983) to question the usefulness of survey data, even irrational expectations may be of interest if they affect the transmission of monetary policy into the real economy.

Part of the literature on surveys is concerned with techniques that can be used to quantify survey data of a qualitative nature. In the United Kingdom, Carlson and Parkin (1975) used the monthly Gallup Poll qualitative survey results on the expected direction of price (inflation) changes to construct a quantitative measure of the general

(4) In fact, empirical tests on survey data can only ever test the joint hypothesis of irrationality and/or a constrained information set.

public's expectations of price change. The study by Pesaran and Wright (1991) suggests several procedures for improving the usefulness of the information on industrialists' pricing intentions (to gauge expected increases in industrial prices) provided by the UK CBI[5] Industrial Trends Survey.

In addition to the rationality issue, there are several drawbacks to using data from surveys. For instance, surveys often take time to compile; so they may not give accurate estimates of current inflationary expectations. There is also a widely held view that, since respondents to surveys have no incentive to provide correct information, their responses may not be consistent with their normal "market" behaviour - where accurate assessments of future inflation could make the difference between a profit and a loss. Furthermore, surveys will generally weight responses equally rather than weighting by the relative importance of the respondent in the given market. Lastly, survey-based methods typically only give short-run expectations, as surveys are primarily concerned with the short-term outlook.

2.2 Theory-based methods

Alternatively, a theoretical model which makes some assumption about the expectations forming process can be used to produce inflation expectations. However, competing models can only be tested if we have an independent measure of inflation expectations. This means that it is not possible to use a theoretical approach without some other source of inflation expectations data. Figlewski and Wachtel (*op cit*), for instance, test models with rational, adaptive and regressive expectations processes using the Livingston survey data.

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(5) The CBI is the Confederation of British Industry.

3 Index-linked gilts and future inflation

United Kingdom Index-linked gilts are designed to give the investor a known real return independent of the inflation rate: both the coupon payments and the redemption payment are adjusted to keep pace with RPI inflation, thus preserving the real value of both income and capital. However, index-linked gilts (IGs) do not offer *complete* real value certainty, since there is an eight month lag in the indexation. The reason for this lag is the method employed in the United Kingdom to calculate accrued interest on gilts. For this calculation, the size of each dividend must be known six months before the payment is due; as this is linked to the RPI, a lag of at least six months must be used. However, even a six month lag is impractical since the RPI figures themselves are only available with a lag, the figure for a particular month typically being available only in the second week of the next month. A further month's lag is allowed because coupon payments on different stocks are made at different times of the month. The overall effect of the indexation lag is to lead to imperfect inflation protection: once each payment is translated into a known nominal amount using the latest RPI, the investor will gain in real terms if inflation falls over the eight month period and lose if it rises.[6]

One consequence of the eight month lag is that, when computing the real redemption yield (or internal rate of return) on an index-linked gilt, some assumption must be made about future inflation in order to discount the cash flows. This is illustrated by the price/yield relationship, a simplified[7] version of which is shown below.

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(6) For basic information on how IGs work, including the eight-month lag, readers should refer to the Bank of England's recent publication "*British Government Securities: The Market in Gilt-Edged Securities*", in particular Chapter 3.

(7) The key simplification here being the "assumption" of a six-month (or 1 period) indexation lag, rather than the eight months present in practice.

For an *n*-period bond:

$$\text{Price} = \frac{(1+i)C}{(1+i_a)(1+r)} + \frac{(1+i)(1+i_a)C}{(1+i_a)^2(1+r)^2} + \ldots\ldots + \frac{(1+i)(1+i_a)^{n-1}(R+C)}{(1+i_a)^n(1+r)^n}$$

where:

C	=	net coupon = gross coupon x (1-t), where t is the income tax rate[8]
R	=	redemption payment
r	=	net real yield
i	=	known RPI inflation used to set the next coupon payment
i_a	=	<u>assumed</u> average future inflation rate over the life of the bond

After simplification this reduces to:

$$\text{Price} = \frac{(1+i)C}{(1+i_a)(1+r)} + \frac{(1+i)C}{\left((1+i_a)(1+r)\right)^2} + \ldots\ldots + \frac{(1+i)(R+C)}{\left((1+i_a)(1+r)\right)^n}$$

clearly demonstrating the need to provide some assumption about future inflation.[9] This factor is important when estimating a real yield curve (for a more detailed explanation of this relationship, see Section 5). Figure 3.1 illustrates the relationship between the assumed

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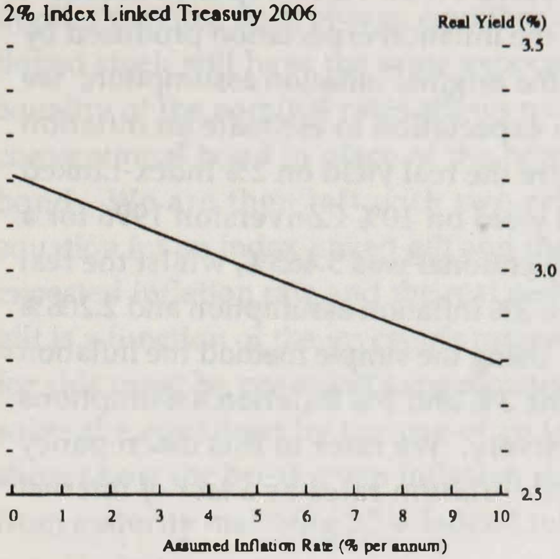
(8) In the United Kingdom, the <u>traders</u> in gilts are taxed on trading profits (a combination of capital gain and income) with relief on the associated expenses. Ordinary domestic investors - although exempt from capital gains tax - are liable to income tax on the dividends they receive.

(9) When publishing yields on index-linked bonds the current market convention is to assume 3% or 5% average inflation over the remainder of the bond's life. Although we could just assume that the average future inflation rate equals today's inflation figure, this is likely to lead to unrealistic yield estimates if current inflation is either high or low relative to the average.

inflation rate and the real yield on 2% Index-Linked Treasury 2006, for a date in November 1993.

Figure 3.1
Sensitivity of real yields to the assumption of constant inflation



4 Simple measures of inflation expectations

In this section we review two simple methods for deriving inflation expectations from gilt prices.

4.1 The "Simple" approach

We refer to the simplest method used to derive inflation expectations as the "simple" approach. Here, average expected inflation is calculated using the *Fisher identity*[10] as the straight difference of the real yield on an index-linked gilt (at some assumed average inflation rate) from the nominal yield on a conventional gilt of similar (preferably identical) maturity. For example, by subtracting the real yield on a five year

(10) The Fisher identity states that the nominal yield on a bond can be decomposed into components of real yield and expected inflation. In its simplest form: $y = r+i$.

index-linked stock from the nominal yield on a five year conventional this method gives an estimate of average expected inflation over the next five years. This measure is often misinterpreted as an expectation of inflation in five years' time, rather than the average rate over the next five years. Since the real yield on an IG is dependent on an assumed average rate of inflation the inflation expectation produced by this method depends directly on the original inflation assumption; we are effectively using an inflation expectation to estimate an inflation expectation. For example, compare the real yield on 2% Index-Linked Treasury 1996 with the nominal yield on 10% Conversion 1996 for a recent date. The yield on the conventional was 5.465%, whilst the real yield on the IG was 2.619% using a 3% inflation assumption and 2.208% using a 5% inflation assumption. Using the simple method the inflation expectations corresponding to the 3% and 5% inflation assumptions were 2.846% and 3.257% respectively. We refer to this discrepancy between assumed and estimated inflation rates as a *lack of internal consistency*.

4.2 Break-Even inflation rates

The method of calculating "break-even" inflation rates is more sophisticated, and eliminates this problem of consistency. Again, average inflation expectations are estimated by comparing the return on a conventional gilt with that on an index-linked stock of similar maturity, but this time by application of the compound form of the Fisher identity. This simply decomposes the nominal rate of return on a bond into components of real yield and inflation, using the usual rulxes of compound interest:[11]

$$\left(1 + \frac{y}{2}\right) = (1 + i)^{\frac{1}{2}} \left(1 + \frac{r}{2}\right)$$

(11) For the UK market the convention is to use semi-annual compounding, since coupon payments on gilts are (in general) made semi-annually.

where y is the nominal yield, r is the real yield and i is the inflation rate.

Implicit in the break-even methodology is the assumption that investors are risk-neutral (ie that they will require no (inflation) risk premium for holding either index-linked or conventional gilts).[12] This implies that in a no-arbitrage equilibrium a conventional and an index-linked stock will have the same expected nominal rate of return. This equality of the nominal rates allows us to use the nominal yield on the conventional bond in place of the nominal yield on the index-linked bond. We are then left with two relationships - the price/yield equation for an index-linked gilt and the Fisher identity, to solve for the expected inflation rate and the real yield. In addition, as the price of a gilt is a function of the investor's marginal income tax rate, an estimate for this must be provided exogenously. It is then straightforward to solve the equations by the use of an iterative procedure. Figure 4.1 shows how the break-even inflation rate (for 0% income tax) derived from maturity matching 2.5% Index-Linked Treasury 2003[13] has varied over the last year.

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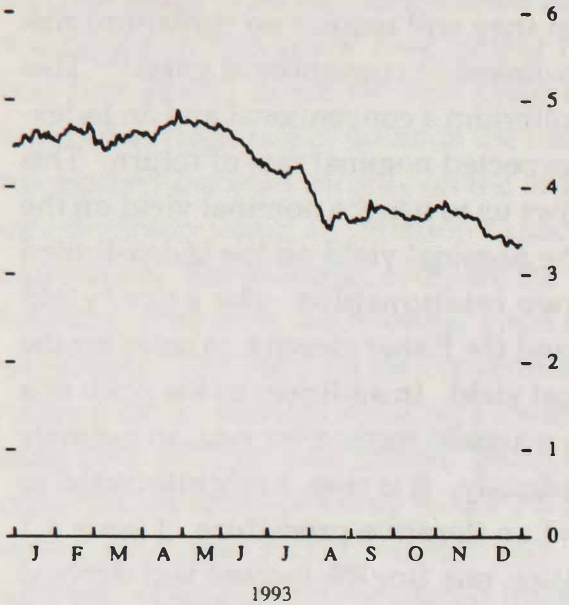
(12) Any liquidity effects that might be present are also ignored.

(13) In this particular example the conventional selected was 10% Treasury 2003 for the first half of the period and 8% Treasury 2003 for the second half.

Figure 4.1
Break-Even Inflation Rates
(Maturity Matched)



Arak & Kreicher (1985) adopt essentially the same methodology, but as a simplification they estimate and use a linear approximation to the price/yield equation. All their analysis is based on the 1996 index-linked bond and assumes 0% tax. Although a risk premium of zero is assumed, they do consider the likely effect that a non-zero risk premium would have on their results. Unfortunately their results are distorted, since their calculations incorrectly assume that the base date for indexation of coupon and principal on IGs is the stock issue date, rather than eight months prior to the issue date.

4.3 Duration-matched break-even inflation rates

One variation of the break-even methodology involves matching the conventional and index-linked stocks by *duration* rather than by maturity. Duration provides a means of normalising for the different risk characteristics of high and low coupon bonds. It is defined to be the weighted average maturity of a bond's cash flows, where the weights are the present values of each of the payments as a proportion of the total present value of all cash flows. Algebraically, (Macaulay) duration is defined by:

$$\text{Duration (years)} = \frac{\Sigma PVCF_i \times t_i}{\Sigma PVCF_i} \qquad (1)$$

where: t_i = time to the i^{th} cash flow in years

CF_i = i^{th} cash flow

y = gross redemption yield (expressed as a decimal)

$PVCF_i$ = present value of the i^{th} cash flow, mathematically:

$$PVCF_i = \frac{CF_i}{(1+y)^{t_i}}$$

From Macaulay duration it is straightforward to calculate the more widely used *modified duration.*[14]

$$\text{Modified Duration} = \frac{\text{Duration}}{(1+y)}$$

Modified duration can be used as a measure of the sensitivity of a bond's price to changes in yield. The mathematical relationship is:[15]

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(14) This division simply adjusts Macaulay's duration for the non-continuous compounding of interest (see Douglas page 60).

(15) See Appendix A for the derivation of this equation.

Percentage change
in the bond price = - 100 x modified duration x change in the yield

For example, a bond with a modified duration of 10 years will experience a 5% rise in price if there is a fall in yields (interest rates)[16] of 50 basis points:

Percentage change
in the bond price = - 100 x 10 x (-0.50/100) = 5

There are two important points to be noted from this formula. First, modified duration acts as a multiplier: the larger the modified duration, the greater the price impact for a given change in interest rates. In addition, for a bond of a given duration, large shifts in yield lead to a large percentage change in price.[17]

Given that duration measures a bond's price responsiveness to changes in nominal interest rates, and hence changes in expected inflation and the real interest rate, it might seem more reasonable to expect that gilts of equal duration (rather than of equal maturity) will have the same expected rate of return. The major drawback to duration matching is that the two bonds selected can be substantially mis-matched in terms of maturity, leading to difficulties when interpreting the results. For instance, using data from November 1993, 2.5% Index-Linked Treasury 2003 duration matches with 8.5% Treasury 2007, producing a maturity difference of four years. Should the resultant break-even inflation rate be treated as the average expected inflation rate over a ten year or a fourteen year horizon?

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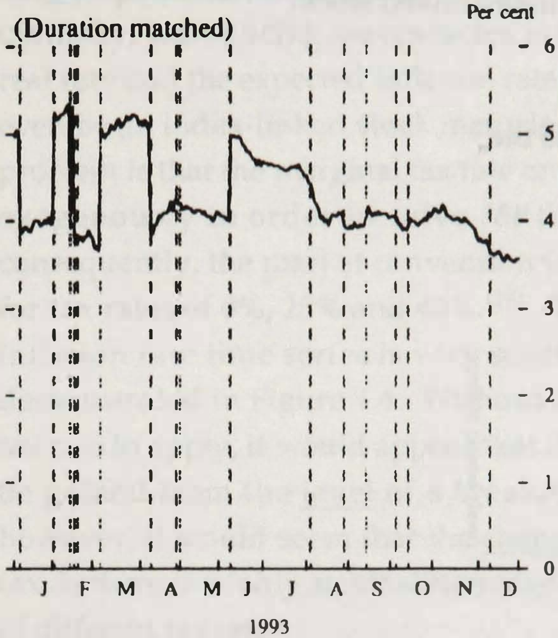
(16) This assumes parallel shifts in the yield curve.

(17) For a more detailed discussion of duration refer to either Belchamber (1988) or Douglas (1990).

Figure 4.2 below illustrates the break-even inflation rate series (for 0% tax) obtained by duration matching 2.5% Index-Linked Treasury 2003.(18) It is interesting to compare this chart with that obtained by maturity-matching the 2003 stock (Figure 4.1).

Figure 4.2
Break-Even Inflation Rates
(Duration matched)



4.4 Complications with duration-matching

In calculating the duration of a bond, we need to compute the present value of its future cash flows. Clearly, for an index-linked gilt this will mean making some assumption about the future inflation rate. Thus, when estimating break-even inflation rates by duration matching, the duration on the index-linked stock, as well as its real yield, will need to

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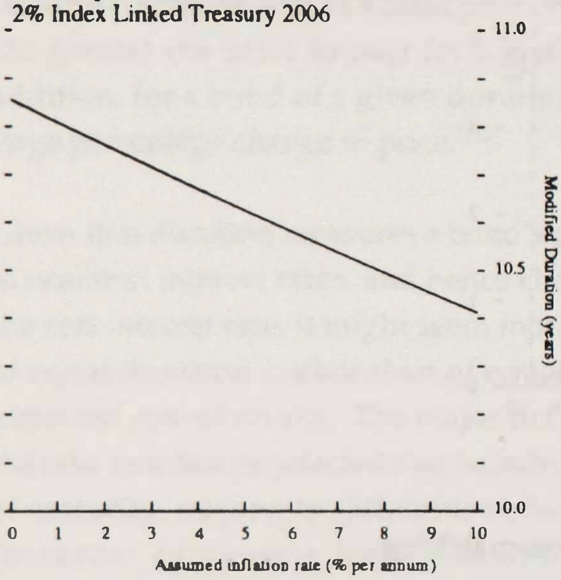
(18) In order to match bonds as precisely as possible, the conventional chosen to match 2 1/2% Index-Linked Treasury 2003 varied over the sample period - the switches are denoted by the vertical lines in Figure 4.2. Such switching <u>may</u> result in jumps in the break-even inflation rate time series, as is clearly the case in this example. This gives an indication of how sensitive break-even inflation rates are to the characteristics (eg coupon, liquidity etc) of the bonds being matched.

be re-evaluated for each iteration of the inflation rate. Figure 4.3 below shows how responsive an index-linked bond's duration is to the inflation assumption - in this particular example, a 1% point change in the inflation assumption induces a change of about 0.05 years in the modified duration. Given this inelastic response, it is generally unlikely that the process of iteration will result in a switch of conventional selected for matching against the index-linked stock.

Figure 4.3

Sensitivity of modified duration to the assumption of constant inflation

2% Index Linked Treasury 2006



Bootle (1991) expresses reservations about matching conventional and index-linked gilts by duration - a point that is re-iterated in Woodward (1990). They argue that normalising for interest rate risk by duration matching is "ill-conceived" in this context, since investors in conventionals face nominal interest rate risk (the risk of a change in real rates or inflation expectations) whilst IG investors are subject to real rate risk - only a change in the underlying level of real yields affecting both types of stock in the same way. Duration matching should only be applied to instruments facing the same type of risk.

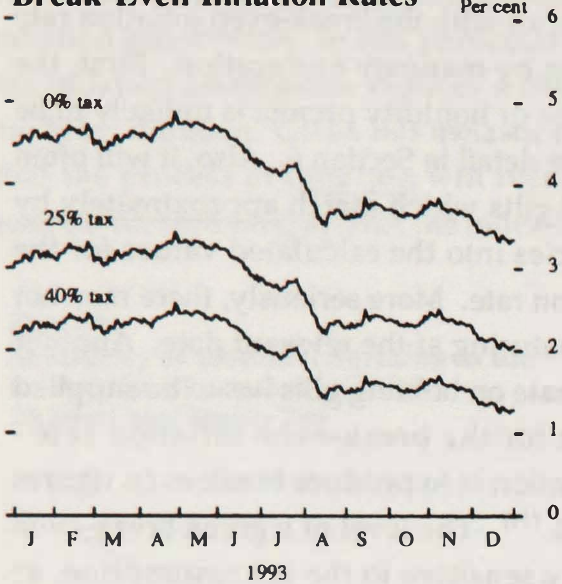
4.5 General problems with break-even inflation rates

There are clearly several deficiencies with the break-even inflation rate methodology, whether matching by maturity or duration. First, the assumption that gilts carry no risk or liquidity premia is unlikely to be realistic; this is discussed in more detail in Section 6. Also, it will often only be possible to find pairs of gilts which match approximately by maturity, introducing inaccuracies into the calculated values for the real rate and the expected inflation rate. More seriously, there may not even be an index-linked stock maturing at the relevant date. Another problem is that the marginal tax rate on holding gilts has to be supplied exogenously in order to solve for the break-even inflation rate - consequently, the market convention is to produce break-even figures for tax rates of 0%, 25% and 40%.(19) The level of a given break-even inflation rate time series is very sensitive to the tax assumption, as demonstrated in Figure 4.4. Without a clear view on the appropriate tax rate to apply, it would appear that little useful information can thus be gained from the level of a break-even rate series. Empirically, however, it would seem that the changes in the series vary little with tax, ie there is a fairly stable differential between break-even time series of different tax rates.

(19) For example, S G Warburg publish break-even inflation rates daily, for tax rates of 25% and 40%, and maturities of two, four and seven years.

Figure 4.4

Break-Even Inflation Rates

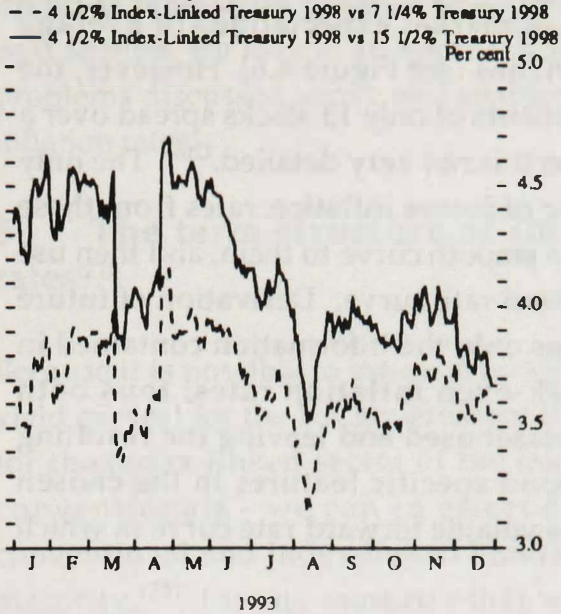


The fact that a break-even inflation rate is derived from only two gilt prices means that it is particularly vulnerable to distortions produced by the specific stocks selected. For instance, when matching stocks by maturity there may be two conventionals of roughly equal maturity but widely differing coupons; the difference in the break-even rates that this would produce is likely to be significant, as demonstrated by Figure 4.5. Also, by concentrating on only two stocks this approach ignores any inflation information contained in other bonds.

Figure 4.5

Break-Even Inflation Rates

(Stock sensitivity)

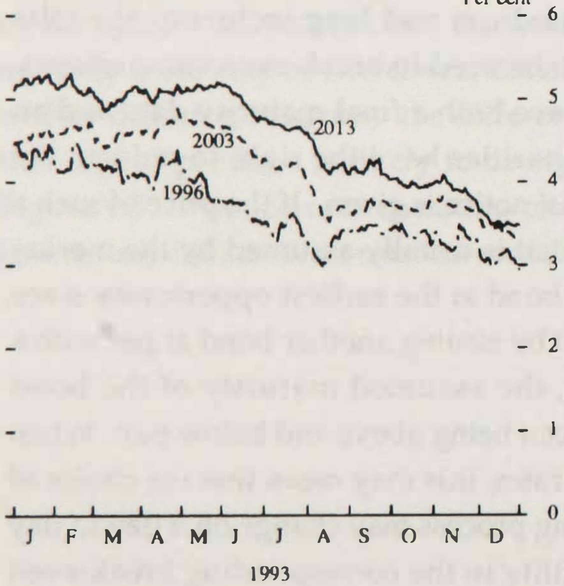


Given the limited number of conventional bonds at crucial points of the yield curve, especially in the medium and long sectors of the gilts market, double-dated bonds can be used in break-even comparisons. Double-dated or callable gilts have both a final maturity date and an earlier date after which the authorities have the right to redeem the stock, provided that three months' notice is given. If the price of such a bond is above par (ie face value), it is usually assumed by the market that the authorities will call the bond at the earliest opportunity since they would be able to refinance it by issuing another bond at par with a lower coupon. Consequently, the assumed maturity of the bond changes as its price moves between being above and below par. When calculating break-even inflation rates, this may mean that the choice of conventional used in the matching process may change on a day to day basis, causing misleading volatility in the corresponding break-even rate time series. Also, since the price of a callable bond will reflect a premium for the call option, it will have a higher apparent yield than an equivalent conventional bond.

By calculating a break-even inflation rate for each index-linked gilt, it is possible to build up a picture of market expectations of average inflation over different time horizons (see Figure 4.6). However, the index-linked market currently consists of only 13 stocks spread over a maturity range of 2 to 37 years so it is not very detailed.[20] The only way to back out a term structure of future inflation rates from these break-even rates would be to fit a smooth curve to them, and then use this to estimate an implied forward rate curve. Derivation of future inflation rates in such a way uses only the information contained in bonds used to create the break-even inflation rates, thus both unnecessarily restricting the dataset used and leaving the resulting estimates dependent on any bond specific features in the chosen dataset. This is likely to lead to a volatile forward rate curve in which we could place little confidence.

Figure 4.6

Break-Even Inflation Rates



(20) In fact, three index-linked stocks are beyond the maturity of the longest conventional, so we are restricted to only 10 break-even observations.

We conclude that, although we can learn little about the level of inflation expectations from break-even inflation rates, they can be useful as time-series to show how expectations have changed. In the next section, we look at an approach which eliminates several of the problems discussed above and enables us to derive implied forward inflation rates.

5 The term structure of implied forward inflation rates[21]

Because it is possible to estimate two interest rate term structures (or yield curves) for the UK government bond market - a real yield curve for the index-linked sector of the market and a nominal curve for conventionals - we can in effect create pairs of hypothetical conventional and index-linked bonds that are matched *perfectly* by maturity,[22] for any maturity that we desire.[23] The break-even approach can then be applied to these pairs to give continuous curves for both average and (more importantly) forward inflation expectations. As most[24] bonds are used in the estimation of the yield curves, this approach has the additional advantage of using almost all

(21) For convenience we refer to the implied forward inflation rate curve (both here and in the *Inflation Report*) as the inflation term structure (ITS).

(22) This approach could be adapted to duration match the bonds but this would be subject to the same caveats already discussed in the previous section.

(23) It is, however, unwise to extrapolate beyond the maturity of the longest actual bond of either type.

(24) Only irredeemables, tranches, convertibles (with outstanding conversion options), stocks with a small amount outstanding, and some low coupon stocks are currently excluded when estimating the yield curves.

the information available from the gilt market embodying inflation expectations.[25] It also ensures that the rates derived should be free of any stock-specific distortions (as discussed in Section 4) and can allow for tax effects.

Woodward (1990) adopts a similar approach to extract inflation expectations. However, he generates a par yield curve for conventional bonds[26] rather than a spot[27] curve. As Schaefer (1981) points out, the error introduced when yields are used as proxies for spot rates can be significant in the case of a sloping term structure. Woodward also does not attempt to estimate a real yield curve, but instead uses yields from the 14 observable index-linked bonds. This then limits the inflation term structure to a discrete curve as opposed to the continuous term structure produced by our method. One final difference between the approaches is in the treatment of taxation. Woodward estimates a single "effective" tax rate for conventional bonds separately from the nominal yield curve - simplistic analysis suggesting that the tax rate is not a function of maturity. He also decides that this effective tax rate would also be appropriate for the index-linked bonds. Neither assumption is correct, as we shall see.

5.1 The term structure of nominal interest rates

The choice of yield curve model can have a significant effect on the resulting inflation term structure, so in developing the inflation term

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(25) Not only are we using more conventional bonds in our estimation but also all of the IGs: when computing break-even inflation rates the three longest-dated IGs had to be excluded as there were no conventionals of suitable maturities to match against; however, when estimating the term structure of real interest rates these observations can be included.

(26) Based on the Bradley-Crane yield curve model (McEnally 1983).

(27) A t-period spot rate or zero-coupon yield represents the yield to maturity on a (hypothetical) t-period pure discount bond. The spot curve is the construct that economists are usually referring to when talking about the term structure of interest rates.

structure we have tested versions based on three different yield curve approaches - the current Bank of England yield curve model (Mastronikola 1991), the McCulloch yield curve (McCulloch 1975) and Schaefer's adaptation of the McCulloch approach (Schaefer 1981). A detailed comparison of these three methodologies appears in Deacon and Derry (1994).

There are two fundamental differences between the Bank and McCulloch methodologies. The first is that whilst McCulloch estimates a discount function by constraining the price of each bond to equal the sum of the discounted values of all cash flows on the bond, the Bank method estimates a par yield curve by fitting a yield surface to redemption yields on stocks as a function of time to maturity and coupon. As there are standard mathematical relationships between the discount function, par yield curve, spot curve and implied forward rate curve it is possible to derive comparable (but not necessarily identical) yield curves from the two approaches. Both McCulloch (1975) and the Bank approach use cubic spline functions in their estimation; however, in McCulloch it is the discount function that is specified by the spline function, whereas in the Bank model it is the par yield curve.

The second difference lies in the treatment of taxation of coupon income. McCulloch (1975) includes a single parameter for the effective tax rate; this produces a single tax rate for the whole yield curve.[28] The Bank method attempts to model investors' preferences by allowing the tax rate to vary with maturity and coupon. In contrast, Schaefer argues that a different yield curve should be produced for each tax rate faced by investors - the yield curve for a particular tax rate being estimated only from bonds which are "efficient" for investors subject to that tax rate [see Deacon and Derry (*op cit*)].

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(28) The McCulloch approach could however be modified to allow for a more complicated tax treatment.

5.2 The term structure of real interest rates

Using index-linked bond prices it is possible to estimate a term structure of *real* interest rates. The estimation of such a curve provides the 'real' counterpart to the nominal curves detailed in Deacon and Derry (*op cit*). In particular, it enables us to derive a real forward rate curve. In practice, however, there are two factors which complicate such estimation: there is the eight month lag in indexation, and there are far fewer index-linked gilts in issue. The first means that, without some measure of expected inflation, real bond yields and hence term structures derived from real yields are dependent to some degree upon the assumed rate of future inflation.(29) The second problem is more practical: there are currently only 13 index-linked bonds in issue and the Bank yield curve model (for example) estimates twelve parameters. Using the Bank model as it stands to estimate a real yield curve from index-linked bonds would therefore give an exact fit to observed yields and as such is impractical.

Despite these problems, both the McCulloch model and the Bank model can be amended to produce a real curve dependent upon an assumed rate of inflation. Schaefer's model is more difficult to apply since the lack of data will in general severely reduce the number of efficient bonds, making precise estimation difficult. The dataset needs to be expanded to include all index-linked bonds, in which case Schaefer's approach becomes equivalent to McCulloch's model.

5.3 Simplifying the Bank's model

A simplified version of the Bank model perhaps represents the most straightforward way of estimating a real yield curve. The Bank model is stripped down to ignore all tax effects and instead just fits the yield to maturity structure. In addition, the number of knot points defining

(29) This dependence can be important when calculating the yields on index-linked gilts approaching maturity, but becomes less important the longer the maturity of the bond.

the cubic spline is reduced from six to three (at present), reflecting the relative lack of data. Currently, therefore, the simplified model estimates three parameters (defining a spline consisting of two cubic functions) using 13 data points.

The procedure is as follows: gross real redemption yields (assuming a fixed rate of inflation) are calculated from the market prices of index-linked bonds. The initial values of the three parameters representing the real yields at each of the knot points define an initial estimate of the real yield curve. A non-linear technique is then used to estimate the values of the parameters that minimise the sum of squared residuals between the observed and fitted real yields. This fitted curve is interpreted as the real par yield curve, from which the term structure of real interest rates and the implied real forward rate curve can be calculated using the relationships detailed in Deacon and Derry (*op cit*).

The main drawback with this approach is the lack of parameters to account for taxation effects. Not only is it impractical to apply the full Bank model, but it may not even be appropriate given the difference in nature between the index-linked and conventional markets. Compared to the conventional market, the variation of coupons on index-linked bonds is not large and so tax rules are unlikely to affect prices of indexed bonds with the same maturity but different coupons to the extent observed in the conventional market. However, indexed bonds with different maturities may be attractive to different categories of investor. Anecdotal evidence suggests that high rate income tax payers, who are attracted to IGs because of the advantageous ratio of capital to income, prefer short-dated securities for reasons of liquidity and reduced price volatility. In contrast, long-dated index-linked gilts are favoured by pension funds, which are exempt from income tax. Currently, we implicitly assume that the marginal investor at all maturities in the IG market faces a 0% income tax rate and so any distortions induced by this assumption are likely to be at the short maturities. Research continues in this area.

5.4 Fitting a discount function to indexed bonds

The method first proposed by McCulloch (1971) can be adapted in a reasonably straightforward manner to produce a real yield curve. The tax treatment suggested in his second paper (1975) can also be included, but for the reasons outlined above may not be appropriate for the index-linked market. However, even if this is indeed the case, estimation of such a parameter at least allows to some extent for any coupon effect that may exist in the gilt market; so even if a precise meaning cannot be attached to the estimated parameter, it may be still be desirable to include it in the model.

It can be shown (see Appendix B for a full derivation) that when using McCulloch's methodology to fit a discount function using indexed bonds equation (20) in Deacon and Derry (*op cit*) need only be amended by a scaling factor (Λ_i) that is known for each gilt once an assumption has been made about future inflation expectations. So if $\delta_r(m)$ is the real discount function, defined by

$$\delta_r(m) = 1 + \sum_{j=1}^{k} a_j f_j(m)$$

where $f_j(m)$ is the j^{th} basis function, we estimate the coefficients a_j from:

$$y_i = \sum_{j=1}^{k} a_j x_{ij}$$

where: $$y_i = P_i - \Lambda_i C_i m_i - \Lambda_i R_i$$

$$x_{ij} = \Lambda_i C_i \int_0^{m_i} f_j(\mu)\, d\mu + \Lambda_i R_i f_j(m_i)$$

$$u = (1 + \pi^e)^{-1/2}$$

$$\Lambda_i = \begin{cases} [u^{tdj}]_i \cdot \dfrac{RPID_j}{RPIB_i} & \text{if } RPID_j \text{ is known} \\ [u^{tdl-L/6}]_i \cdot \dfrac{RPIL}{RIPB_i} & \text{otherwise} \end{cases}$$

Here P_i, C_i, m_i and R_i are the price, coupon, maturity and real redemption payment of the i^{th} indexed bond. Λ_i is the scaling factor on the i^{th} bond, and depends on the ratio of the RPI defining each dividend to the gilt's "base" RPI level. If the RPI setting a given dividend is unknown then we must estimate it using the latest available RPI figure in conjunction with some assumption about π^e the path of future inflation.

As with the Bank model, the number of estimating functions is reduced to reflect the lower number of data points available for use in the estimation.

5.5 The term structure of inflation expectations

From term structure models for nominal and real spot interest rates we can construct respective curves for nominal and real implied forward rates [see Deacon and Derry (*op cit*) for a full derivation]. The term structure of implied forward inflation rates can then be obtained from these using an interpretation of the Fisher relationship discussed in Section 4:

$$\left(1 + \frac{y}{2}\right) = (1 + i)^{\frac{1}{2}} \left(1 + \frac{r}{2}\right) \qquad (2)$$

where y is the implied nominal forward rate, r the implied real forward rate and i the implied forward inflation rate.

Like the term structure of real spot rates, the real implied forward rate curve will depend on an inflation assumption. In order to make this assumption consistent with the inflation term structure (ITS) that we calculate, a procedure has been developed which iterates on the assumed inflation rate. In simple terms, for each iteration the real yield curve is re-estimated[30] until consistency between the assumed ITS and the estimated ITS is achieved. Initially, real yields are calculated for all index-linked gilts[31] assuming a flat (eg 5%) inflation rate. The real yield curve is then estimated, from which the real forward rate curve is obtained. By applying equation (2), the initial estimate of the inflation term structure is produced. This forward inflation curve is then converted into an average inflation curve, using the equation:

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(30) Only the real yield curve is re-estimated as part of the iteration, not the nominal curve - which is estimated only once.

(31) Index-linked gilts with less than one year to maturity are excluded from the ITS estimation, since as they approach maturity they effectively trade as conventionals.

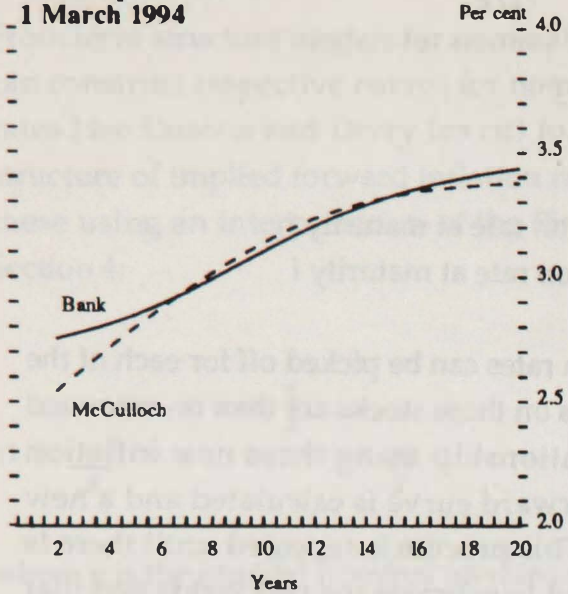
$$is_k = \prod_{j=1}^{k} (1 + if_j)^{-1/k} - 1$$

where: if_i = forward inflation rate at maturity i
is_i = average inflation rate at maturity i

From this average curve, inflation rates can be picked off for each of the index-linked gilts. The real yields on these stocks are then re-estimated from the bond price/yield relationship using these new inflation assumptions. Again the real forward curve is calculated and a new estimate of the ITS produced. This process is repeated until there is consistency between the ITS used to estimate the real yields and that produced by equation (2); typically this requires about 10 iterations.

Figure 5.1 compares real yield curves for the Bank and McCulloch methodologies for a recent date. In each case, real yields are evaluated using inflation assumptions derived from the respective average inflation curve.

Figure 5.1
Real Spot Curve
1 March 1994



Figures 5.2 and 5.3 illustrate inflation term structures ie forward inflation rates for the Bank, McCulloch and Schaefer[32] approaches. Although there is some difference between the curves they are all of broadly the same shape.

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(32) As mentioned above, Schaefer's approach is only used to estimate the nominal curve. To generate the ITS this curve (assuming an income tax rate of 0%) is compared with a McCulloch real yield curve since Schaefer's approach cannot be applied to the IG market due to the lack of index-linked bonds.

Figure 5.2
Inflation Term Structure
1 March 1994

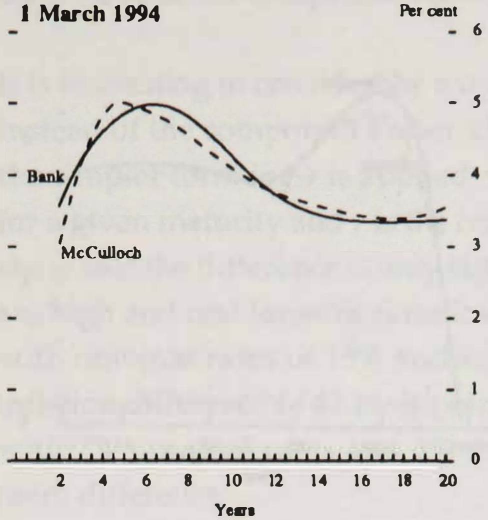
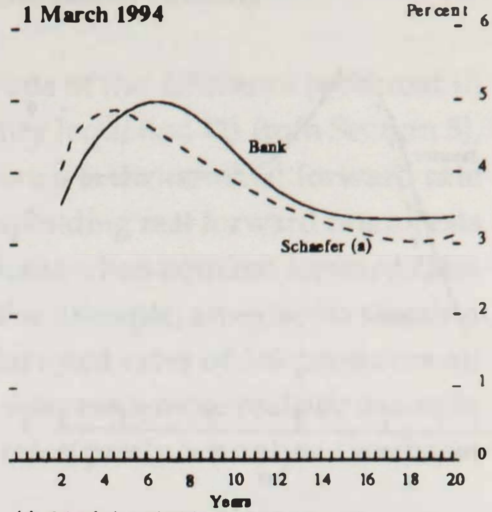


Figure 5.3
Inflation Term Structure
1 March 1994



(a) Schaefer's methodology estimated for a 0% tax payer (including near efficient bonds)

6 Estimation problems with the inflation term structure

The following section examines some of the complications that arise in estimating the inflation term structure.

6.1 Inflation iteration

One of the main complexities in deriving an inflation term structure from a given yield curve model is in ensuring internal consistency, the necessity of which was discussed in Section 5. Figures 6.1 - 6.3 compare internally consistent inflation term structures for several dates with the corresponding simple (ie non-iterated) term structures.

Figure 6.1
ITS: Iterative vs Simple approach
7 May 1993

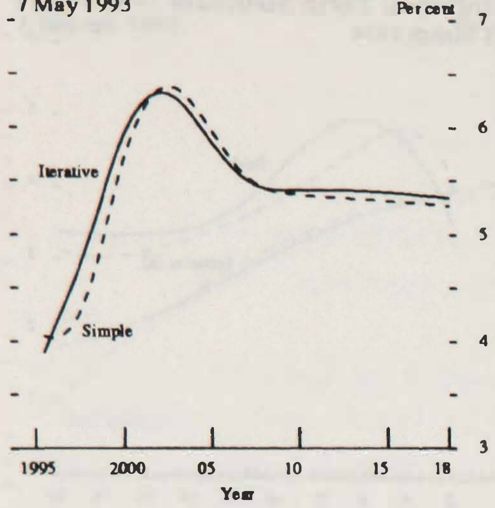


Figure 6.2
ITS: Iterative vs Simple approach
2 August 1993

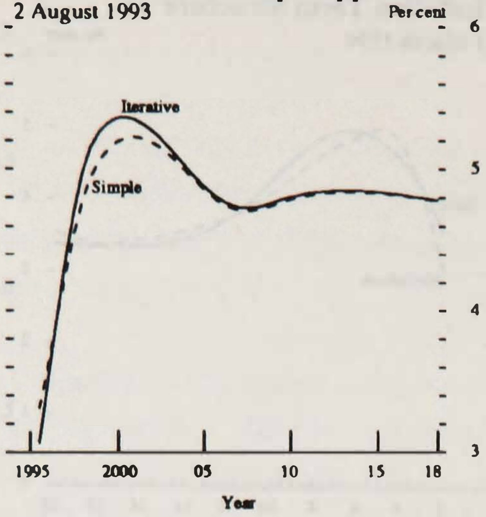
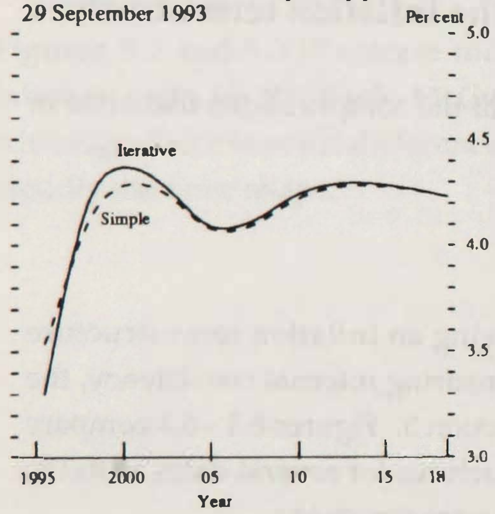


Figure 6.3
ITS: Iterative vs Simple approach
29 September 1993



Clearly the most significant change brought about by iterating is in the short-to-medium term. For 7 May 1993, the difference is as high as 47 basis points. The shapes of the term structures are largely unaltered, though the peaks have shifted marginally.

6.2 Use of the compound form of the Fisher identity

It is interesting to consider the magnitude of the difference produced if, instead of the compound Fisher identity [equation (2) from Section 5], the simpler form $i=y-r$ is applied, where y is the nominal forward rate for a given maturity and r is the corresponding real forward rate. Tests show that the difference is only significant when nominal forward rates are high and real forward rates low. For example, an extreme scenario with nominal rates of 15% and real forward rates of 1% produces an inflation difference of 42 basis points, whereas a more realistic example with 7% nominal rates and 3% real rates produces only a two basis point difference.

6.3 Choice of curve fitting approach

When deriving models for the term structure of nominal and real interest rates, it is desirable that these are smooth and continuous. Given the constraint that there are only a finite number of observations (ie bonds trading in the market) from which to estimate the models, some form of curve fitting technique must be applied. For our research on inflation expectations, it is particularly important that the forward rate curves are stable. Allowing the yield curve to be too flexible can result in "humps" in the corresponding forward rate curve which may not reflect any underlying events in the market. Curve fitting techniques are discussed in some depth by Deacon and Derry (*op cit*).

6.4 Asymptotically flat forward rate curves

One of the economic priors that it may be desirable to impose on a model of the inflation term structure is that it is flat for long maturities. This can be achieved by restricting both real and nominal forward curves to be either parallel (ie equal gradient) or flat at long maturities.

We have adopted the latter approach, as it seems more realistic (particularly if one thinks of the plausibility of the long run real rate) and is simpler to implement. This is achieved in the Bank model by constraining the cubic spline to flatten at the long end. Although McCulloch's cubic spline will not in general produce asymptotically flat forward rate curves, it can be constrained to do so by application of a technique due to Vasicek and Fong (1982). The more recent parsimonious term structure model of Nelson and Siegel (1987) was specifically designed to produce forward rate curves with horizontal asymptotes. A full exposition of the Vasicek and Fong adjustment and the Nelson and Siegel model can be found in Deacon and Derry (*op cit*).

6.5 Tax effects

The diversity of coupons in the conventional gilt market coupled with the differential taxation of income and capital gain in the UK results in a noticeable coupon effect, and a number of models have been proposed to remove such distortions [a summary of methodologies is provided in Deacon and Derry (*op cit*)]. As previously noted, the coupon effect is less prominent in the IG market, since there is less diversity in coupons, but anecdotal evidence suggests tax segmentation by maturity. It is important that any such effects are adequately accounted for when modelling the real yield curve, but this task is complicated by the small number of index-linked gilts issued. Our current methodology ignores such effects, so clearly needs to be improved.

Estimation of the inflation term structure relies on a no-arbitrage condition, and it is therefore necessary to produce nominal and real curves for the same category of investor. Our current methodology involves modelling both nominal and real curves from the perspective of a zero rate tax payer, the nominal curve by modelling the tax effect on the prices of those bonds not naturally held by such investors and thereby adjusting them to be comparable with the remainder, whilst the prices of IGs are explicitly assumed to be set by 0% taxpayers. This enables us to apply the no-arbitrage condition, since we are comparing the IG and conventional market from the perspective of the same (possibly hypothetical) investor paying no income or capital gains tax.[33]

6.6 Jensen's inequality and risk premia

In order to interpret our results as "true" market expectations of inflation we must assume that the forward interest rates that we calculate are expected future short rates. However, in practice there are several factors which are likely to lead to a difference between the two. These factors can be grouped into three categories: risk premia, liquidity premia and Jensen's inequality.[34]

There are three principal sources of risk for holders of bonds: the risk of default, the risk of changes in inflation, and the risk of changes in the spot interest rate. Since the UK Government is unlikely to default, we can safely ignore the first of these. *Inflation risk* is incurred by holders of bonds with variable real returns i.e without a guaranteed real return. The inflation risk premium on index-linked bonds is likely to be small since they offer a high degree of real value certainty,[35] whereas for conventionals it may be significant as all payments are fixed in nominal, but not in real, terms. The *interest rate* (or *price risk) premium* represents the compensation a bond holder requires for likely variability in the value of the bond through time. As the prices of high duration bonds will generally be more sensitive to a change in interest

(33) These estimates can clearly be scaled in the usual way for investors facing other tax treatments.

(34) There may also be some effect on both index-linked and conventional yield curves at specific maturities where new stock has just been issued or is about to be issued.

(35) If there was no eight-month lag in the indexation, IGs would offer *complete* real value certainty.

rates than low duration bonds (assuming parallel shifts of the yield curve), the price risk premium included in their return will be higher.

Liquidity is important in two respects: first, prices of bonds that are identical in all respects other than their liquidity may differ. This is particularly true if one of the bonds is perceived by the markets as a "benchmark". How such effects are treated depends to an extent on how they are viewed; if a bond's price is relatively high because comparable bonds are illiquid, the liquid bond's price is likely to be more representative of market interest rates. However, if this bond is being used primarily as a hedge instrument then its price is likely to reflect factors other than solely the term structure of interest rates. The second liquidity effect important in the estimation of the ITS is the relative liquidity of conventional and index-linked gilts. The index-linked market is less liquid than the conventional market, and therefore any comparison between the two is likely to require an additional factor to allow for the difference in liquidity. Our methodology currently assumes that all liquidity effects are negligible, though this may be unrealistic.

There are several competing hypotheses for the economic relationship between expected future rates and bond prices. If the underlying process corresponds to either the yield to maturity or the local expectations hypotheses, implied expected future rates will not correspond to actual expected future rates but will actually be less, to an extent dependent upon the volatility in future rates. Essentially this arises from the difference between $E[(1+r)^{-1}]$ and $(1+E[r])^{-1}$, where $E(.)$ is the expected value function and r is a future interest rate - this is an example of *Jensen's inequality*. More precisely, Jensen's inequality states that, for a strictly convex function, the expectation of the function of a random variable will be greater than the function of the expectation of the variable ie.

$$E[g(x)] > g(E[x])$$

For a detailed exposition on the effect of Jensen's inequality when estimating expected interest rates, see Anderson and Barr (1994).

When deriving estimates of inflation expectations, we must take into account both risk premia and Jensen's inequality. The nominal inflation risk premium, nominal interest rate risk premium and the effect of Jensen's inequality on real rates will all tend to bias our estimates of inflation expectations upwards. Factors tending to cause an understatement of inflation expectations will be the real inflation risk premium (likely to be small), the real interest rate risk premium and the Jensen effect on nominal rates.

Preliminary investigations into Jensen's effect suggest that it is unlikely to be significant if the short-term interest rate follows a strongly mean - reverting process (see Anderson and Barr *(op cit)*). Although it is not clear at this stage what the magnitude of the net effect of risk premia and Jensen's inequality is likely to be, we believe that the inflation term structure overstates "true" inflation expectations. Further work is in hand to model these effects more accurately.

Most studies of inflation expectations make simplistic assumptions about the effect of risk premia and Jensen's inequality. For instance, Arak and Kreicher (*op cit*) assume risk premia of zero, though they admit that this assumption is unlikely to be realistic. Woodward (*op cit*), and Robertson and Symons (*op cit*) also use zero risk premia, though the former does examine how his results would differ if the risk premia were non-zero.

7 An event study of inflation expectations

Given the method for deriving inflation expectations outlined in the preceding sections, it is useful to examine how these expectations responded to a recent monetary event. For this we have chosen sterling's exit from the Exchange Rate Mechanism (ERM) on 16 September 1992 (Black Wednesday).

On suspension of sterling's membership of the ERM gilt prices reacted sharply, largely reflecting changes in investor's expectations - particularly of future interest rates. These are likely to have changed significantly, since UK interest rates would no longer be as closely tied to the interest rates of other ERM member countries, hence giving the government more flexibility over setting rates.

Figure 7.1 illustrates the impact that sterling's suspension had on inflation expectations(36), whilst Figure 7.2 shows how the nominal and real forward interest rate components responded.

Figure 7.1
Inflation Expectations: Before and after Sterling's exit from the ERM (16/9/92)

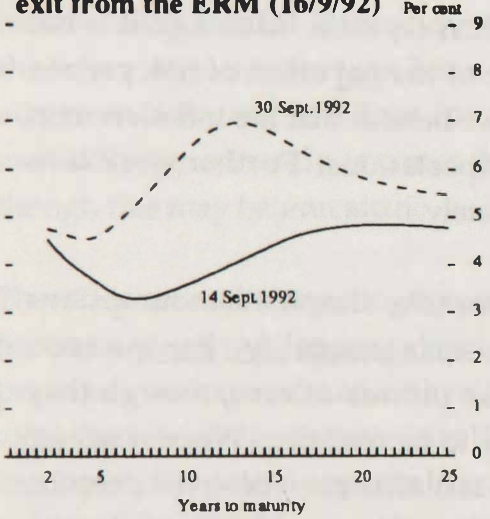
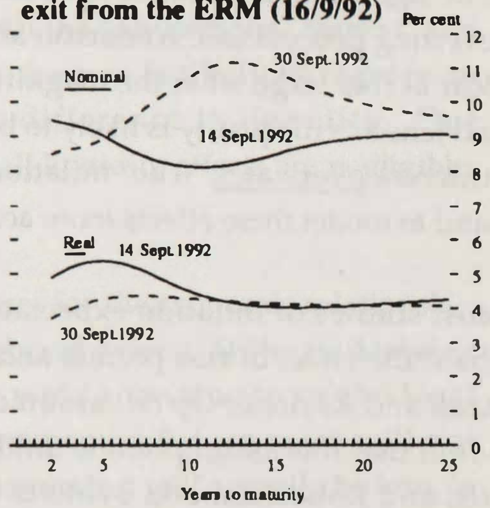


Figure 7.2
Implied forward interest rates: Before and after Sterling's exit from the ERM (16/9/92)



The charts suggest that the effect of sterling's exit from the ERM on expected nominal interest rates was largely due to a change in inflation expectations (or a change in the risk premia on conventional bonds), expected real rates only responding up to the 10 year horizon.

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(36) In this comparison we have assumed a net risk premia/Jensen effect of zero. In reality this will probably not have been the case, so some of the change in figures 7.1 and 7.2 will probably be due to a shift in these factors.

Investors expected lower nominal interest rates in the short term but over the longer term they expected rates to rise by about 50 basis points. Clearly the rise in inflation expectations was significant, especially over the medium to long term.

8 Summary

This paper has described a number of techniques that can be employed to extract information on inflation expectations from the prices of UK Government bonds. The simple chart overleaf helps to illustrate how these different measures of inflation expectations are linked.

Our preferred method (and that which is used to produce the inflation term structure published in the Bank of England's *Inflation Report*) is a natural refinement of techniques researched by Arak and Kreicher (*op cit*), and Woodward (*op cit*). The most significant development is in the use of term structure models for both the conventional and index-linked sectors of the gilt market. Given some assumptions about risk premia and the bias due to Jensen's inequality,[37] this enables us to produce a term structure for inflation expectations which is continuous and uses almost all the information available in the market. The use of term structure models also makes it possible to extract forward as well as average estimates for inflation expectations - something that was not possible in earlier studies. An improved treatment of taxation is inherent in the nominal term structure model employed.

One area in which further work is required is in assessing the likely magnitude of the bias due to Jensen's inequality and risk premia. Until these effects are quantified and taken into account in our computations, caveats must remain on any results produced. That said, our view is that the net effect of risk premia and Jensen's effect is to overstate estimates of inflation expectations.

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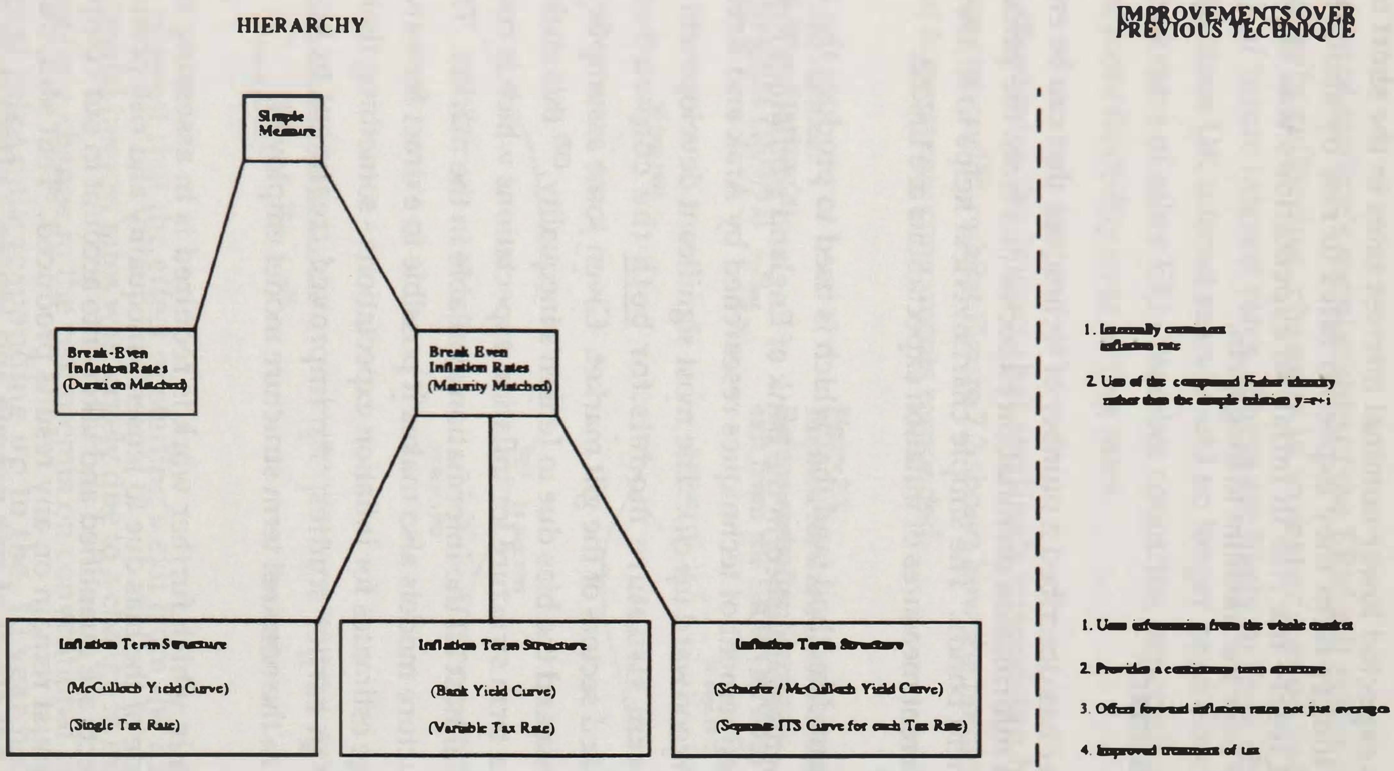
(37) At present we assume that the net effect is zero.

MEASURES OF INFLATION EXPECTATIONS



Appendix A: Derivation of the duration/volatility relation

Using the notation of Section 4, from equation (1):

$$\textit{Modified duration (years)} = \frac{\Sigma PVCF_i \times t_i}{(1+y) \times \Sigma_i PVCF_i}$$

From the bond price/yield relationship $P = \Sigma_i PVCF_i$

$$= \Sigma_i \frac{CF_i}{(1+y)^{t_i}}$$

Therefore

$$\frac{dP}{dy} = -\Sigma_i \frac{CF_i \times t_i}{(1+y)^{t_i+1}} = -\Sigma_i \frac{CF_i}{(1+y)^{t_i}} \times \frac{t_i}{(1+y)}$$

$$= -\Sigma_i PVCF_i \times t_i \times \frac{1}{(1+y)}$$

$$\text{So} \quad MD = -\frac{1}{P}\frac{dP}{dy}$$

$$= -\frac{\text{percentage change in the bond price}}{100} \times \frac{1}{\text{change in yield}}$$

Appendix B: Derivation of the McCulloch real yield curve equations

In order to estimate a real yield curve we need to calculate a real redemption yield for each index-linked gilt. To do this we must estimate values for the outstanding dividends and the corresponding discount rates for each such bond. This appendix sets out how we produce these estimates for the net present value of each cash flow on a given index-linked stock, and how this enables us to produce a McCulloch real yield curve.

Calculation of the dividends

For an index-linked gilt the i^{th} remaining dividend payment[(38)] is half the coupon scaled by a factor:

$$\frac{RPID_i}{RPIB}$$

where: $RPIB$ = Base RPI level for the stock in question (the RPI level eight months before the stock was issued).

$RPID_i$ = RPI value defining the *ith* remaining dividend payment (the RPI level eight months before the date of the ith dividend).

For a given date it is likely that only $RPID_1$, the RPI defining the next dividend, is known with all subsequent dividends depending on yet unpublished RPI values. In order to provide values for these dividends in our yield calculations we must make use of the latest available RPI

(38) We ignore here the (obvious) adjustments required if the stock has not yet reached its first ever dividend payment, since in general this will not be simply C/2 (in real terms).

figure in conjunction with some assumption about the path of future RPI inflation.

By way of example, consider a hypothetical index-linked gilt of 4% coupon, issued in January 1991 and paying dividends in September and March of each year. Suppose also, that for a given settlement date in June 1994 that the latest known RPI is for May 1994.

The base RPI value (RPIB) for the bond is the RPI level for May 1990, and $RPID_1$ - the RPI defining the next dividend (due in September 1994) is defined by the RPI for January 1994. Thus the next dividend is simply half the coupon multiplied by the RPI scaling factor:

$$\frac{RPID_1}{RPIB} \times \frac{4.0}{2}$$

and is a known quantity for our given settlement date.

However the subsequent dividend, which is due in March 1995, will be defined by the RPI level for July 1994 which is as yet unknown. In order to estimate this dividend payment we use the latest known RPI (May 1994) and assume an inflation rate to bridge the two month gap between May and July 1994. For publication purposes current market convention is to assume an inflation rate of either 3% or 5% for this period.

Let: π^e = the assumed annual inflation rate,

tl = time (in half-years) from the settlement date to the date of the latest RPI,

tdi = time (in half-years) from the settlement date to the date of the RPI defining the ith remaining dividend,

$$u \quad = \quad (1+\pi^e)^{-1/2}.$$

If $RPID_2$ is not yet known then we estimate it as:

$$RPID_2 \quad = \quad RPIL \times u^{tl-td2}$$

$$= \quad RPIL \times u^{tl-(td1+1)}$$

$$= \quad RPIL \times u^{tl-td1} \times u^{-1}$$

$$= \quad RPIL \times u^{-L/6} \times u^{-1}$$

Here L is defined to be $6(td1-tl)$ and is the number of months from the date of the latest RPI value to the date of the RPI defining the next dividend. L is usually negative, but may be positive when the stock is XD. Clearly, if $RPID_2$ is known we do not need to make such assumptions.

It can be shown that in the general case where the ith dividend is unknown we can estimate the appropriate RPI level by:

$$RPID_i = RPIL \times u^{-L/6} \times u^{-(i-1)}$$

Calculation of the discount factors

Let: r = the real yield

w = the real discount factor = $(1+r/2)^{-1}$

Then, using our previous notation, the discount factor for the first dividend payment $FACTOR_1$ is given by:

$$FACTOR_1 = (wu)^{td1} = w^{td1} \times u^{td1}$$

The discount factor for the second dividend payment ($FACTOR_2$) is defined similarly to be:

$$FACTOR_2 = (wu)^{td2} = w^{td2} \times u^{td2}$$

$$= w^{td2} \times u^{(td1+1)}$$

$$= w^{td2}\, u^{td1}\, u$$

In general it can be shown that,

$$FACTOR_i = w^{tdi}\, u^{td1}\, u^{(i-l)}$$

Calculation of the cash flow net present values

It is now possible to calculate the net present value of each cash flow. The net present value of the ith remaining dividend payment is given by:

$$\frac{RPID_i}{RPIB} \times \frac{C}{2} \times FACTOR_i$$

In the case where $RPID_i$ is <u>known</u> this reduces to:

$$\left(\frac{RPID_i}{RPIB} \times u^{tdi}\right) \times \frac{C}{2} \times w^{tdi}$$

When the value of $RPID_i$ is <u>unknown</u> the relation becomes:

$$\frac{RPIL}{RPIB}\, u^{-L/6}\, u^{-(i-1)}\, \frac{C}{2} \times w^{tdi}\, u^{td1}\, u^{(i-1)}$$

$$= \left(\frac{RPIL}{RPIB}\, u^{td1-L/6}\right) \times \frac{C}{2} \times w^{tdi}$$

Now define a term Λ to be:

$$\Lambda = \begin{cases} [u^{tdi}]^{\frac{RPID_i}{RPIB}} & \text{if } RPID_i \text{ is known} \\ [u^{tdi - L/6}]^{\frac{RPIL}{RPIB}} & \text{otherwise} \end{cases}$$

Then the net present value of the *i*th dividend payment reduces to $\Lambda \times C/2 \times w^{tdi}$.

If we now define $\delta_r(m)$ to be the real discount function at maturity m then the above can be written as:

$$\Lambda \times \frac{C}{2} \times \delta_r(tdi) .$$

Similarly, for an indexed bond with N cash flows the net present value of the real redemption payment R can be written as:

$$\Lambda \times R \times \delta_r(tdN)$$

In the case of an N-period conventional bond of (nominal) coupon C the present value of each dividend takes the form:

$$\frac{C}{2} \times \delta_n(tdi)$$

and the present value of the (nominal) redemption payment R the form:

$$R \times \delta_n(tdN)$$

where $\delta_n(m)$ is the nominal discount function at time m.

McCulloch real yield curve equations

It follows that in order to adapt the McCulloch yield curve equations to generate a real yield curve, we simply scale each cash flow of <u>each bond</u> by the relevant Λ factor.

We can then define the regression equation for the McCulloch real yield curve by

$$y_i = \sum_{j=1}^{K} a_j x_{ij}$$

where: $y_i = p_i - \Lambda_i C_i m_i - \Lambda_i R_i$

$$x_{ij} = \Lambda_i C_i \int_o^{m_i} f_j(\mu)\, d\mu + \Lambda_i R_i f_j(m_i)$$

$$u = (1 + \pi^e)^{-1/2}$$

$$\Lambda_i = \begin{cases} \left[u^{tdj}\right]_i \cdot \dfrac{RPID_j}{RPIB_i} & \textbf{if } RPID_j \textbf{ is known} \\[2ex] \left[u^{tdl-L/6}\right]_i \cdot \dfrac{RPIL}{RPIB_i} & \textbf{otherwise} \end{cases}$$

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