Potential Credit Exposure on Interest Rate Swaps

by

*Ian Bond**

*Gareth Murphy**

and

Gary Robinson#

*Bank of England, Threadneedle Street, London, EC2R 8AH.
#BZW. The work reported here was carried out while on the staff of the Bank of England.

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Contents

Abstract

We develop in this paper an analytical analogue to the Monte Carlo techniques previously used by banking supervisors to assess the potential credit exposure of interest rate swaps, which permits a more thorough examination of swap exposure. We do so by using the Cox, Ingersoll and Ross (1985) one-factor model of the yield curve to generate interest rate paths from which swap credit exposure paths can be determined.

Even with such a relatively simple interest rate process, we find that the patterns of credit exposure are more complex than the supervisors' previous techniques allow: they vary with the level of interest rates, the slope of the yield curve and the volatility of the short rate - all factors which are ignored in the supervisors' risk measurement methodology - and have a significantly non-linear relationship with swap maturity. We conclude that market traders and regulators need to be alert to these factors in determining the appropriate level of capital to hold as protection against counterparty default.

I Introduction

The proliferation of new financial instruments confronts financial institutions and their supervisors with the problem of assessing their riskiness. Capital charges for credit risk inherent in the more established OTC derivative instruments such as interest rate swaps and forex forwards are covered by supervisors' existing guidelines (as detailed, for example, in the Basle Accord [BIS (1988)] and elsewhere). These rules adopt a very simple approach, which requires firms to distinguish only between interest rate and currency derivatives and between short-term (under one year) and long-term (over one year) contracts. The capital charges themselves are based on 'typical' maturity and vintage profiles within each of these four categories, as would be represented by a well-balanced and mature portfolio of deals, and do not pretend to be an accurate measure of risk on a transaction by transaction basis. Recent proposals from the Basle supervisors' committee [BIS (1994)] extend the classification somewhat, both in terms of distinctions between instruments and of maturity categories, but preserve the general 'balanced portfolio' assumption. So even for a 'typical' portfolio for which this approach will provide a reliable <u>overall</u> assessment of risk, it is of little relevance for tackling issues such as the design of internal risk management systems or of pricing models - for which an understanding of the risk characteristics of each deal is essential.

The established approach to assessing risk or capital charges on OTC derivatives is to use Monte-Carlo simulations to model the path of an instrument's likely value over time. This entails simulating the path of underlying variables (for example, interest rates or exchange rates) many times, evaluating the instrument's value (and therefore the credit exposure to the 'in the money' counterparty) which each path implies, and then choosing a particular quantile of the resulting value distribution to represent the "risk" of the instrument. [See Hull (1989) and Hull (1993), Ch. 11 for brief accounts of this approach and Bank of England (1987) for a description of the simulations underlying the weightings in the Basle Accord.]

Numerical methods of this kind are essential for precise evaluation of the risk of products when the behaviour of the underlying variables is not analytically tractable. But they are not especially transparent and, in particular, do not reveal, other than by exhaustive and time-consuming experimentation, how an instrument's characteristics - coupon, maturity, cash flow profile - and the volatility of the underlying variables contribute to its riskiness. It is therefore difficult to assess whether simple rules of thumb for calculating risk-weights and capital requirements - such as those in the Basle Accord - are reliable simplifications, and the ranges of parameter values for which they are appropriate.

In order to answer these questions, we here measure credit risk for plain-vanilla interest rate swaps using a relatively simple model of the term structure which avoids numerical simulation. In essence, we compute the quantiles of the state variable underlying the value of the swap and determine critical values for the path of the value of the swap itself (given that there is a strict monotone relationship between the swap value and the state variable). Measures of exposure are calculated from the swap value quantile paths.

This approach has two principal advantages over Monte-Carlo simulation: it makes it easier and quicker to evaluate an instrument's riskiness; and it greatly facilitates exploration of the relationship between its riskiness, its contractual terms and the behaviour of underlying variables. We demonstrate this by evaluating the risk inherent in interest rate swaps using not only the naive (flat yield curve) model implicit in the Basle weightings but also using the term structure model proposed in Cox, Ingersoll and Ross (1985) (CIR hereafter), which allows us to assess the effects on risk of mean reversion and of changes in the slope of the yield curve. The principal disadvantage of the method arises from the assumptions of the CIR framework. The issue here is not so much the utility maximising representative investor in a continuous time general equilibrium economy - which naturally bears little relation to the real environment of setting interest rates - but, rather, the inability of the model to capture the varieties of interest rate term structure which are frequently observed. This is particularly true of humped yield curves which only seem possible for relatively unrealistic settings of the model parameters, eg very high volatility levels.

The paper is arranged as follows. In Section II we set out some basic definitions used in the rest of the paper. Section III briefly describes the standard Monte-Carlo method and its analytical analogue. Section IV applies this alternative procedure to the most widely-used OTC derivative, the interest rate swap, with interest rates determined by a CIR term structure model. Section V concludes.

II Exposure, Replacement Cost and Capital Risk Weights

Unmargined OTC derivatives give rise to *credit exposure* to the 'in-the-money' counterparty (that is, the counterparty for whom the mark-to-market value of the contract is positive). This mark-to-market profit represents a credit exposure because it is not already earned (or secured - as it would be, if fully collateralised by margin payments) but is today's best estimate of the profit which can be expected to accrue to the 'in-the-money' counterparty over the remaining life of the deal. It will of course do so only if both parties continue to meet their contractual obligations to each other or if the in-the-money counterparty is able to assign or otherwise transfer their interest in the contract at fair value to a third party. As a result, the profit is correctly viewed as a claim on the 'out-of-the-money' counterparty. If that party defaults, the profit is likely to be lost.

On any deal, exposure in this sense can be positive or negative. But negative values do not constitute *credit* exposure: the out-of-the-money party is a debtor, and his position does not change if the other party defaults because he would still be required to meet his contractual obligations. Different considerations arise if there are other deals between the parties to which rights of offset or other legally enforceable netting rules apply, but these are not considered further here.

If the credit exposure at time t (for $t \geq 0$, where $t=0$ is the present) is denoted by S_t then the *replacement cost* in the event of default by the counterparty is given by $K_t = \max\{S_t, 0\}$. Noting that the value of a derivative at some future date t can be decomposed into its current value, S_0, and the change in value between now and time t, $(S_t - S_0)$, we can rewrite replacement cost as:

$$K_t = \max\{S_0 + (S_t - S_0), 0\} \quad \textbf{(1)}$$

or

$$K_t = S_0 + \max\{S_t - S_0, - S_0\} \quad \textbf{(2)}$$

The latter therefore decomposes replacement cost at some future date into current mark-to-market value, which is known with certainty, and *potential future exposure*, which is random since it depends upon future realisations of value, S_t. Because OTC derivatives are not readily transferable, it is prudent for the parties to a deal to assume that their participation commits them to whatever credit exposures will arise during the deal's life. So in measuring credit risk, they should be concerned (as are supervisors) as much with potential future exposure as with the exposure implied by the deal's current mark-to-market value.

It is replacement cost, as just defined, with which we shall be concerned in the rest of this paper although, in our illustrative swap examples in Section IV, we assume that current mark-to-market value is zero so that potential future exposure simplifies to $\max\{S_t, 0\}$. The analogy between credit risk measurement and option pricing is evident since both are convex functions of the underlying variable(s). Whilst the approach in this paper explores credit risk using quantiles as the main tool of analysis, it could equally have been replicated using contingent claims techniques.

In assessing the potential credit exposure of a swap, we require a measure of risk which summarises all possible paths which potential credit exposure can take. If there were risk of default only at time τ, the $100\alpha\%$ quantile of K_τ (that is, the value which is exceeded only with probability α) would summarise the riskiness of the financial security for a given degree of confidence. But the risk of default is not concentrated at any point in time: it exists throughout the term of the contract. So we require a measure of risk which summarises the $\alpha\%$ quantiles of the credit exposure over the remaining life of the deal.

If K_t^α denotes the $\alpha\%$ quantile of K_t at time t, then the most conservative summary measure of risk for the given degree of confidence is $\max\{K_t^\alpha, 0 \le t \le T\}$: we can be $\alpha\%$ confident that the loss will not exceed that value even if default occurs at the worst possible time. But if default is equally likely at *any* time, this summary measure is unsatisfactory because it takes no account of the (lower) levels of loss which we can similarly be $\alpha\%$ confident will not be exceeded at other times. A natural measure to reflect losses in all relevant time periods, and on which we will therefore focus in the rest of the paper, is the average $\alpha\%$ quantile path over an interval τ which is measured as

$$\bar{K}^\alpha = \tau^{-1} \int_0^\tau K_t^\alpha \, dt \qquad \textbf{(3)}$$

These risk measurement quantiles depend on the parameters describing the probability distributions of the underlying variables (drift, volatility etc) and the structure of the derivative (maturity, coupon etc). We will illustrate the way in which the average $\alpha\%$ quantile replacement cost changes for different values of these parameters, standardised by expressing the cost as a percentage of the contract's face value or notional principal.

We are aware that this measure of overall riskiness is not as intuitive as perhaps is the *expected replacement cost*, where the expectation is taken jointly over time $[0,T]$ and the support of the underlying asset $[0,\infty]$, which is a variation on the contingent claim credit risk measurement technique mentioned in Hull (1989). Implicitly the averaging above assumes a uniform probability of default (conditional on the event of assuming default occurring) over the life of the instrument. However, joint averaging cannot be easily rationalised in response to questions like "will this measure cover risk 90% of the time, if an individual capital requirement is set for the swap". Our average quantile measures will answer questions like this assuming that little is known about the time of default, ie a uniform distribution.

III The Monte-Carlo method and its analytical analogue

Let S_t be the value of a pay-floating/receive-fixed swap at time t, which depends upon the time t interest rate r_t(1) and time thus: $S_t=f(r_t,t)$. Suppose for the moment that r_t is log-normally distributed conditional on an initial interest rate r_0, ie

$$log(r_t)-log(r_0)-(\mu-\sigma^2/2)t \sim N(0,\sigma^2 t) \qquad \textbf{(4)}$$

where μ is the drift of r_t and σ is its volatility. This time-dependent family of distributions for $(S_t,t\geq 0)$ is equivalent to S_t being a geometric Brownian motion (GBM). If we define $l_t \equiv log(r_t)-log(r_0)-(\mu-\sigma^2/2)t$, then we have $S_t=f(r_0.exp(l_t+(\mu-\sigma^2/2)t), t)$. Thus we may

(1) Implicit in the price formula is an interest rate term structure which is generated by a single stochastic factor, eg a flat yield curve or the term structure model of Cox, Ingersoll and Ross (1985). If the yield curve is flat, then the instantaneous (short) interest rate suffices to specify the whole yield curve.

write S_t as a function of $l_t \sim N(0,\sigma^2 t)$ and time t which aids the construction of quantile paths.

The Monte-Carlo simulation approach to measuring S_t, the riskiness of the swap, is to simulate the path of r_t many times over the time interval $[0,T]$. Given the distributional assumptions for r_t (which specify how r_t can change over time), these paths will generate corresponding paths for the swap exposure S_t on $[0,T]$. At any given time t between 0 and T, there will be a distribution of simulated values for S_t. The $\alpha\%$ quantile of this sample of simulated values of S_t is an estimate of the population $\alpha\%$ quantile of S_t. Having obtained the $\alpha\%$ quantile, there are several ways in which this information may be summarised. For example, capital requirements have been calculated on the basis of averages derived from these sample paths. Our proposed analytical method is to select a path for the random variable r_t (in the case of an interest rate swap, the single underlying interest rate factor which is assumed to drive the yield curve) which corresponds to the $\alpha\%$ quantile of its distribution. This path for interest rates can then be substituted into the derivative pricing formula to obtain the $\alpha\%$ quantile of the derivative price where there is a monotone relationship between S_t and r_t. For example, if:

$$S_t=f(r_0\exp(l_t+(\mu-\sigma^2/2)t),t) \tag{5}$$

where $l_t \sim N(0,\sigma^2 t)$, then the average $\alpha\%$ quantile for S_t over the interval $[0,\tau]$ is given by:

$$\bar{K}_t = (1/\tau)\int_0^\tau (S_t-S_0)\,dt \tag{6}$$

where l_t is set equal to $q(\alpha)\sigma\sqrt{t}$ where $q(\alpha)$ is the number of standard deviations appropriate to the $\alpha\%$ quantile of the standard normal distribution. We compute this quantity by approximating it with the average half-yearly exposure.

This measure of risk depends on (i) μ, the drift rate of r_t; (ii) σ, the volatility of r_t; (iii) τ, the length of the time interval over which the average is taken; (iv) $q(\alpha)$, and (v) the terms of the financial instrument, which define its cost given the value of the underlying stochastic variable. It provides an analytical framework in which to compute a measure of risk which is much quicker and more accurate than the measure provided by Monte-Carlo simulation.

To illustrate the use of this analytical method, suppose - as in the simulations underlying the Basle requirements - that the yield curve is always flat and subject only to parallel shifts. This allows us to concentrate on a single interest rate - the instantaneous short rate, r_t, appropriate for all terms to maturity. The value of the swap just after payment date t is given by:

$$S_t = ((r_0 - r_t)/n) \sum_{s=1}^{(T-t)} (1+r_t/n)^{-s} \quad (t \geq 1/n) \tag{7}$$

where T is the time when the last payment is made, n is the number of payments per year and r_0 is the fixed coupon. This formula can be rewritten as:

$$S_t = (r_0/r_t - 1)(1-(1+r_t/n)^{-n(T-t)}) \tag{8}$$

The swap is then evaluated at the $\alpha\%$ quantile of the interest rate, r_t, and a measure of average potential exposure over the life of the swap is calculated, as described in formula (6) [see **Appendix (a)** for details of calculations]. This measure of risk is evaluated for a variety of numerical values of the initial short rate and its volatility σ. The *ceteris paribus* effects of each of these factors on potential credit exposure is assessed by keeping constant all but the parameter of interest. The results of these experiments are displayed in Figures 2 to 5. In all the graphs, the units on the vertical axis are percentages of notional principal. A list of all the parameters underlying the plots in this paper is given in Appendix (d).

Figure 1

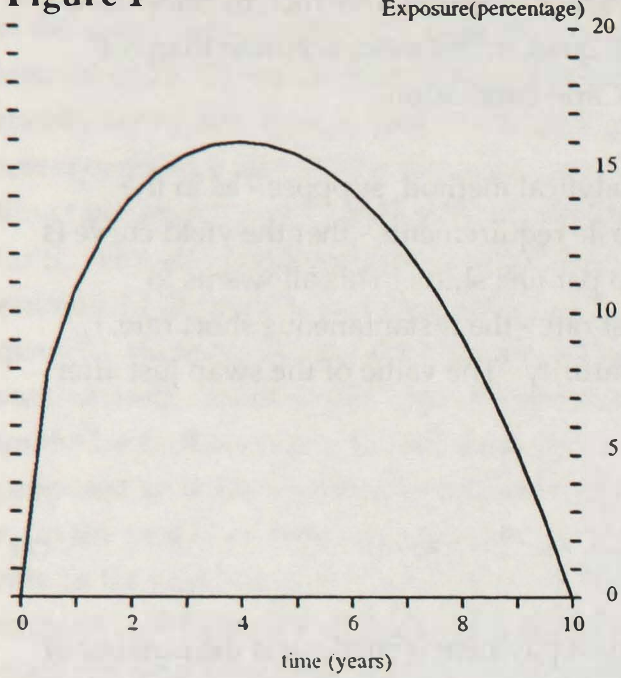

Figure 1 illustrates the 95% quantile path over its full life for the replacement cost of a 10 year, 6% coupon swap with semi-annual exchange of floating rate and fixed rate payments. This displays the familiar "hump" shape, which reflects the changing balance between the effects of falling interest rates (which increase the credit exposure faced by the receiver of the fixed coupon flows) and the falling number of remaining payments (which decrease credit exposure as the swap approaches maturity).

Figure 2 gives the average replacement cost of swaps of differing original maturities (where the average is calculated over the life of the swap) and also the corresponding maximum replacement cost (as defined in Section II above). Comparison of the two charts shows that average replacement cost is equal to (approximately) three-quarters of the maximum replacement cost.

Figure 2

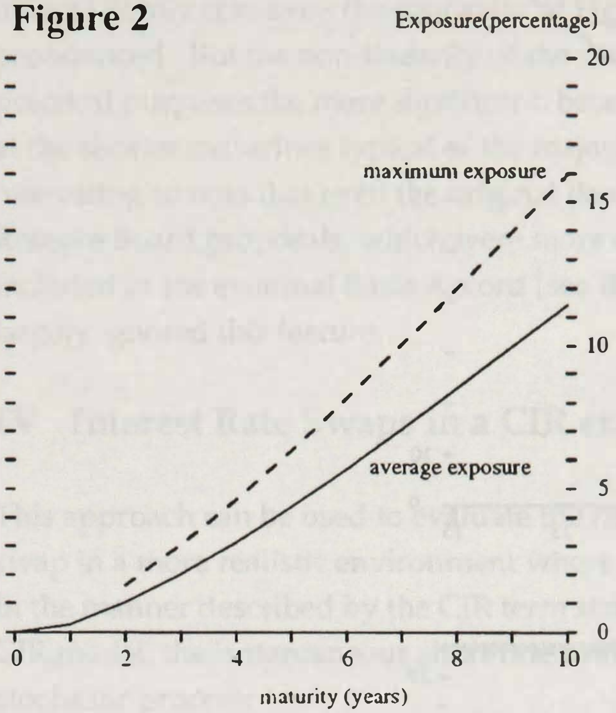

Figure 2 also shows clearly that replacement cost increases more than proportionately with maturity: a 2-year swap has average replacement cost of about 1% of notional principal, whereas the corresponding figures for 5-year and 10-year swaps are about 4.5% and 10% respectively.

Figure 3

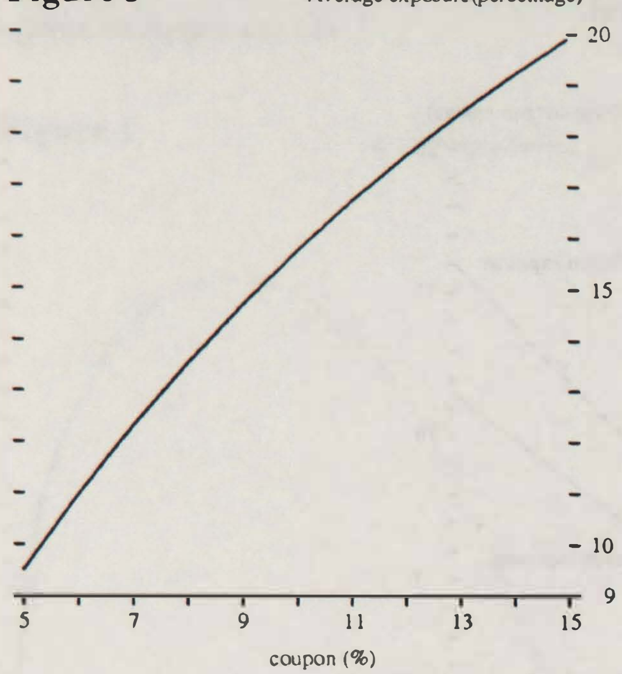

Figure 4

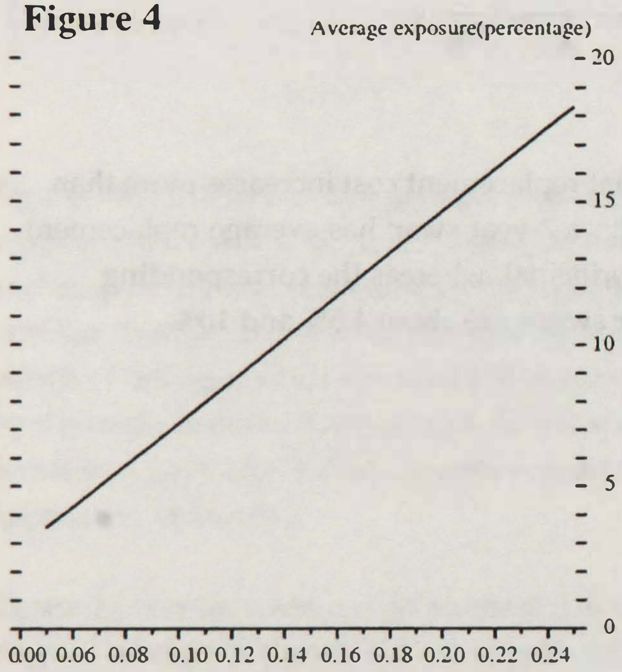

Figure 3 illustrates the effect of changes in coupon on average replacement cost and Figure 4 the effect of interest rate volatility. Though the plots in these latter diagrams appear to be linear, they are in fact slightly concave - the concavity of Figure 3 being more pronounced. But the non-linearity of the maturity-related curve is for practical purposes the more significant, because it is most pronounced at the shorter maturities typical of the majority of transactions. It is interesting to note that even the original Bank of England/Federal Reserve Board proposals, which were more detailed than those included in the eventual Basle Accord [see Bank of England (1987)], largely ignored this feature.

IV Interest Rate Swaps in a CIR environment

This approach can be used to evaluate the riskiness of an interest rate swap in a more realistic environment where short interest rates evolve in the manner described by the CIR term structure model. Under the CIR model, the instantaneous short rate evolves according to the stochastic process:

$$dr_t = \kappa(\theta - r_t)dt + \sigma\sqrt{r_t}dZ_t, \qquad (9)$$

where $\kappa(\theta - r_t)$ is the drift of the short rate, $\sigma\sqrt{r_t}$ is the instantaneous standard deviation and Z_t is a Wiener process.(2) This implies that r_t is distributed as a non-central chi-square distribution [(see **Appendix (b)**].

This model has a number of attractive properties, including:

(i) the error-correction term, $\kappa.(\theta - r).dt$, ensures that the short rate gravitates towards the steady-state level, θ, at a rate κ, ie short

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(2) The short rate drifts towards θ (the mean of the short rate in the infinite future). The speed with which it drifts towards θ is given by the mean reversion parameter κ. The volatility of the short rate is determined by σ and the square root of the short rate at that point in time. Heuristically, $dZ_t \sim N(0,dt)$.

rates mean-revert around θ. This determines the slope of the yield curve generated by the CIR model and allows us to analyse swap exposure when the short-end of the yield curve tends towards θ. The long-run short rate θ is not to be confused with the yield on a perpetuity which equals $2\kappa/(\kappa+\lambda+\gamma)$ where $\gamma=\sqrt{((\kappa+\lambda)^2+2\sigma^2)}$;

(ii) the second term ensures that the short rate cannot become negative: the volatility of interest rate changes is proportional to the square root of the interest rate;

(iii) the CIR interest rate process allows the derivation of closed-form price formulae for zero coupon bonds (and hence the yield curve and implied forward rates) and therefore for other interest rate derivatives.

The CIR model is of course unrealistic in a number of respects: it was derived for an economy without any medium of exchange, so that the term structures derived are effectively *real* interest rate term structures; it implies that the rate on perpetuities is constant, which may be true for real interest rates [see Brown and Schaefer (1991)] but not for nominal interest rates; and empirical evidence for the United States suggests that volatility is proportional to a higher power of the interest rate than its square root [see Chan et al (1992)]. Forthcoming work by one of the authors suggests UK 3-month and 6-month LIBOR data may be consistent with the CIR short-rate process, though the estimated equations have poor predictive power, ie R^2 of less than 0.01. It is nevertheless a significant improvement on the assumption of a flat yield curve, being both analytically tractable and capable of generating yield curves which are upward sloping, downward sloping and humped. The value (at time t) of a pure discount bond $P(r,T\text{-}t)$ (which matures at time T) given the current value of the short rate, (denoted by r instead of r_t), is of the form $P(r,T\text{-}t) = A(T\text{-}t)e^{-rB(T-t)}$. The functions A and B depend on κ,θ,σ,T,t and the market price of short rate risk (λ) [their functional forms are set out in the Appendix (b)]. Thus the continuously compounded yield-to-maturity, $y(r,T-t)$, of a pure discount bond, issued at time t to mature at time T, is given by:[3]

$$y(r,T-t) = (rB(T-t) - \log A(T-t))/(T-t) \qquad \textbf{(10)}$$

The above equation defines the interest rate term structure in a CIR economy.

The value of a pay-floating/receive-fixed interest rate swap with half-yearly payments, a coupon C_0 and maturity at time T is given as

$$S_t = \Sigma_{i=1}^{2(T-t)} \{(1+C_0/2)\, P(r,i/2) - P(r,(i-1)/2)\} \qquad \textbf{(11)}$$

Generally, the coupon (fixed side) of a swap is set so that the net discounted flows equal zero. In effect, this means that the coupon is a par yield corresponding to the maturity of the swap. Equation **(11)** can be solved (with S_t=0, where the swap is initially 'at-the-money' at t=0), to give the coupon as

$$C_0 = 2\left[\frac{\sum_{i=1}^{2T} P(r,(i-1)/2)}{\sum_{i=1}^{2T} P(r,i/2)} - 1\right]$$

Clearly, the swap value S_t is a function of the current value of the short rate r which has a non-central chi-square distribution. By rewriting equation (10), one can check that S_t is a monotone decreasing function of r (see Appendix (c) for details). This monotonicity allows us to establish quantiles for the value of a swap. For example, if we find the 5% quantile of the short rate, $r^{0.05}$, then $S_t(r^{0.05})$ will give the 95% quantile for the value of the swap (which

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(3) We recognise that this solution provides yields which assume continuous compounding; however, for our purposes, this approach is computationally faster and will not change any of the individual results.

was previously denoted by $K_t^{0.95}$). We will focus, in particular, on the α% swap quantile paths $\{S_t(r_t^{1-\alpha}): 0 \le t \le T\}$.

Capital requirements must protect against interest rate developments during the life of the swap. In this section we make a (stylised) distinction between two sets of factors which determine the capital requirement:

(a) structural factors which affect interest rate developments and determine the parameters r_0 (the initial short rate), θ, κ, λ and σ underlying the model. For example, the mean reversion parameter may be a consequence of the monetary policy of the time which determines the speed with which short-term interest rates adjust and the amplitude of the interest rate cycle. These parameters affect (i) the initial level of the short-end of the yield curve, (ii) the slope of the yield curve and (iii) the direction of yield curve movements over time and (iv) the speed with which the yield curve shifts and they are fixed over the life of the swap;

(b) random movements in the short rate which cause unexpected shifts in the yield curve over time.

A more sophisticated model might allow the parameters in (a) to be random variables, thus allowing for a degree of structural change over time which may be consistent with observed interest rate cycles. Longstaff and Schwartz (1993) allow the volatility parameter to be generated by a mean-reverting square root process. The only random variable in this model is the short rate. The results of the following section explore the effect on swap exposure for different settings of the factors in (a). In the case of (b), the effect of random movements in the short rate enters the capital requirement through the short-rate quantile path which ensures that the capital requirement protects against a fixed percentage of random (interest rate) events.

1: The effect of a sloped yield curve on swap exposure

Given a short rate r_0 at time 0, the 100(1-α)% quantile ($r_t^{1-\alpha}$) bounds the short rate (at time t) from above 100(1-α)% of the time. The quantile $K_t^{\alpha} = S_t(r_t^{\ 1-\alpha})$ sets a capital requirement which protects against 100α% of the possible swaps values at time t. We plot the 100α% quantile path of the swap over its lifetime. These plots are the main tool employed to analyse the riskiness of swaps in this paper.

We investigate the 5% and 95% swap quantile paths under three different interest rate scenarios:

Scenario 1:	Initial short rate 6%
	Long-run short rate, θ=6%
Scenario 2:	Initial short rate 6%
	Long-run short rate, θ=3%
Scenario 3:	Initial short rate 6%
	Long-run short rate, θ=9%

Figure 5

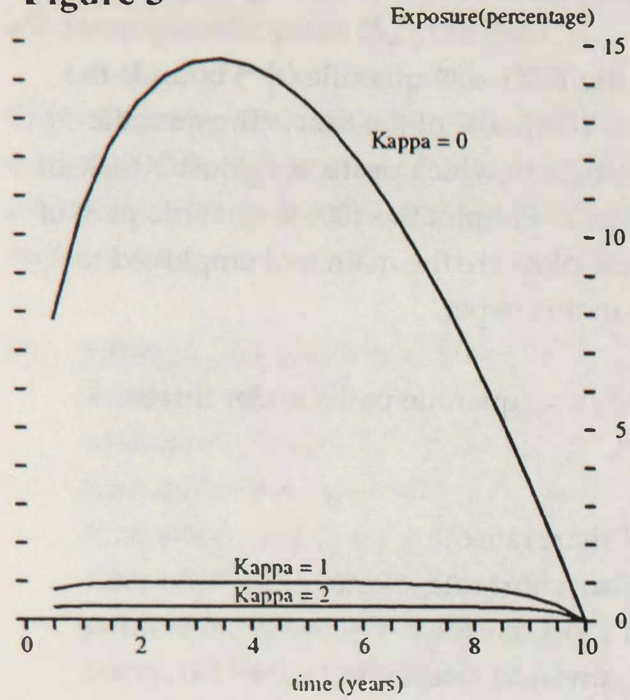

Figure 5 shows the 95% quantile exposure for a 10-year swap under scenario 1 where the mean reversion parameter κ=0,1 and 2. Zero mean reversion (κ=0) generates a 95% quantile exposure profile which resembles that of Figure 1. In comparing the GBM model with the CIR model it is helpful - for the purposes of establishing a benchmark for comparison - to identify the case of no mean reversion (κ=0) in the CIR model with an interest rate process generated by GBM.

Observe that the effect of mean reversion is to reduce the exposure of the swaps dramatically. This is the case even when the yield curve is almost flat, ie the $\kappa(\theta - r_t)$ is small because r_0 equals θ, but becomes more pronounced when the short rate is reverting to a point which is different from the initial starting rate - the maximum quantile exposures (over the life of a 10-year swap) for κ=1 and 2 are less than 2% of notional principal for scenarios 2 and 3.[4] In this respect, capital

(4) In the cases of scenarios 2 and 3, κ=1 and 2 correspond to the cases where (in a deterministic world, σ=0) the short-rate changes after one year equal 1.9% and 2.6% (percentage points), respectively.

requirements calculated on the basis of the CIR model will be smaller than those of the GBM model. Section 3 considers the effect of mean reversion in more detail.

Figure 5 shows the 95% quantile exposure profiles for a (receive-fixed/pay-floating) 10-year swap when the yield curve is flat and unchanging - scenario 1. In fact, the 5% quantile path looks (almost)(5) identical for this swap except that the (credit) exposure is negative. Thus there is a close symmetry between the 5% and 95% quantiles of the receive-fixed/pay-floating swap when the yield curve is flat. **Thus when the yield curve is flat (and unchanging) over the life of the swap, neither side of the swap will be predominantly in (or out of) the money.**

When θ (the long-run short rate) is different from the initial short rate in the presence of mean reversion ($\kappa \neq 0$), the yield curve will have an upward or downward slope. Figures 6(a,b,c) and 7(a,b,c) show the effect of sloped yield curves under scenarios 2 and 3. Plots of the quantile exposure profiles over the lives of swaps of maturity 2.5, 5, 7.5 and 10 years are drawn. The plots have been drawn so that the swap value is (almost) always positive by taking the appropriate side of the swap in each case.

Figure 6(a) shows the 95% quantile path of the (receive-fixed/pay-floating) swaps where the yield curve is downward sloping over the life of the swap. Figure 6(b) shows the 5% quantile path of the swaps.

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(5) There is a slight difference in exposure due to the fact that each tail corresponds to interest rates at different levels above and below 6%. The 5% quantile of a receive-fixed/pay-floating swap corresponds to a higher level of interest rates (above 6%) than the 95% quantile swap exposure where interest rates are lower.

Figure 6a

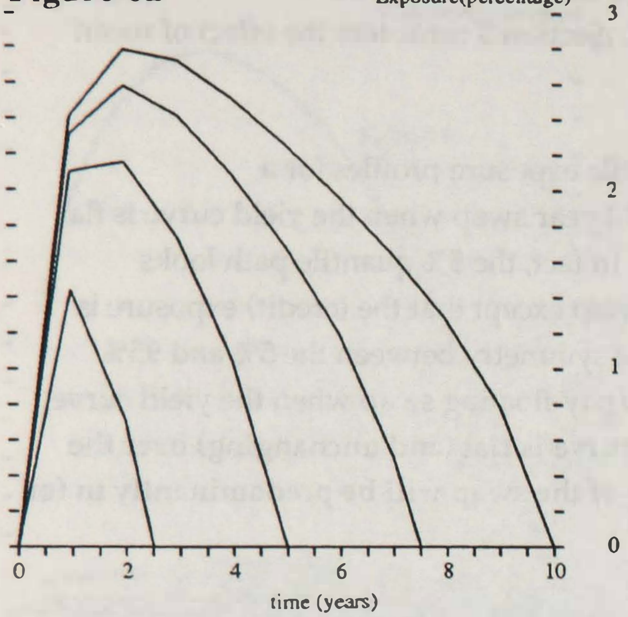
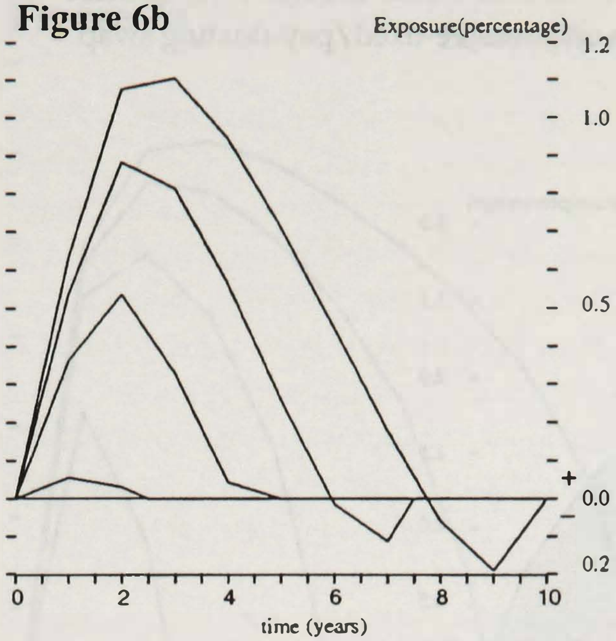


Figure 6(a) shows the 95% quantile path for receive-fixed/pay-floating swaps of different maturities. The curves have inverted-U shapes where credit exposure increases for (roughly) the first 35% of the swap's life and declines thereafter. Bearing in mind that the short rate declines monotonically along its quantile path, giving rise to a series of downward sloping yield curves which are shifting downwards over time, the decline in swap credit exposure is the result of (i) the effect of fewer remaining payments as maturity approaches (which reduce credit exposure) dominating (ii) the downward yield curve shifts (which increase credit exposure).

Figure 6b



In the cases of swaps of maturity greater than five years, the credit exposures in figure 6(b) change sign as the swap approaches maturity. In scenario 2, the initial yield curve is downward sloping and implies a 10-year coupon of 3.31% on the fixed side, ie 1.655% of notional principal is paid every half year. Since the short rate evolves along its 95% path, it remains above its asymptotic mean of 3%. Therefore, all the yield curves generated along this 95% path are downward sloping. In particular, the half-yearly implied forward rate curves (which are expected future payments on the floating side) are (a) downward sloping and (b) particularly high at the short end because of the 95% quantile short rate path. The short end of the forward rate curve will lie above the half-yearly coupon rate of 1.655%. As the swap approaches maturity, the half-yearly investment rates at the long end drop out of the swap value formula, leaving only the higher rates at the short end. For a receive-fixed/pay-floating swap, the initial positive exposure is due to the greater number of payments (at the long end) which lie below 1.655%. However, as the forward rate curve evolves over time, these rates at the long end drop out and only the rates at the short end are left. As these lie above 1.655% (due to the particularly high short rate quantile path which determines the short end of these curves) therefore the exposure on a receive-fixed/pay-floating swap will be negative.

Figure 6c

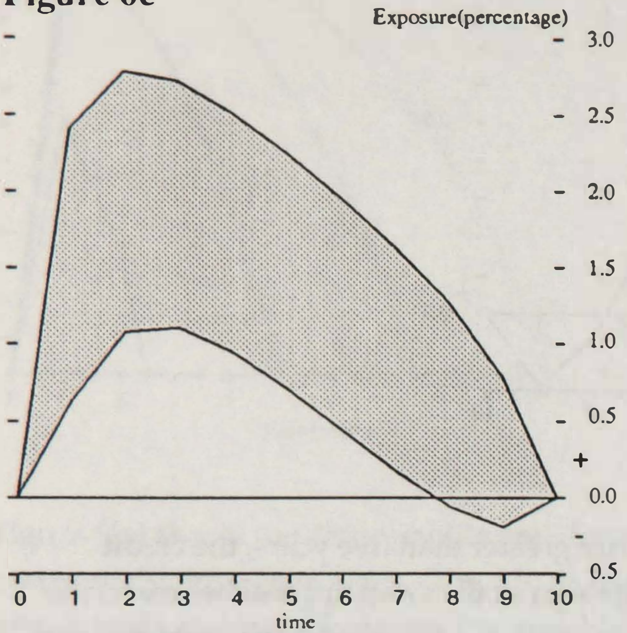
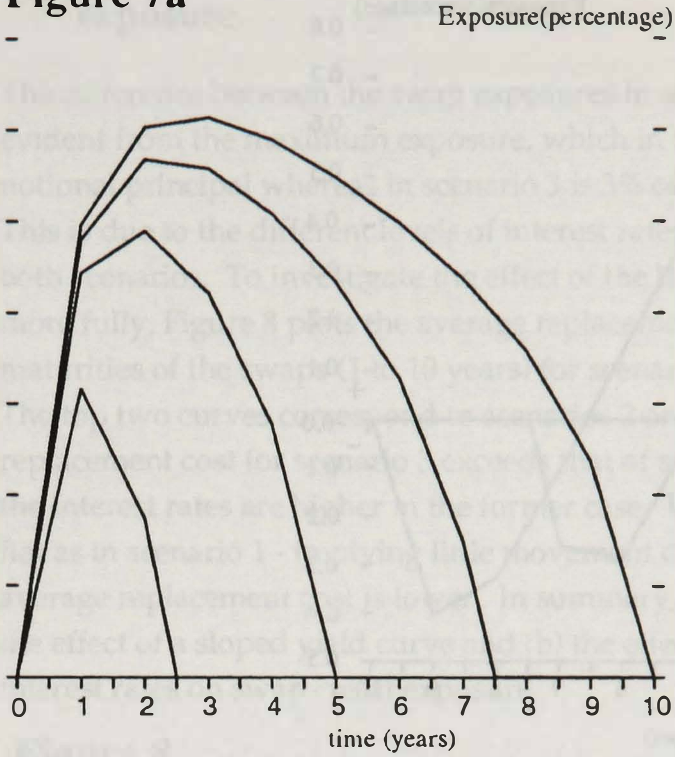


Figure 6(c) shows the 90% confidence interval for exposure over the life of a 10-year swap. The conclusion which emerges from Figures 6(a,b,c) is that **when the yield curve is downward sloping and shifting downwards, the credit exposure on a receive-fixed/pay-floating swap is mostly positive.** Furthermore, the greater the maturity, then larger is the probability that the value of the swap will be negative at some stage in its lifetime.

Figure 7a



In terms of shape, Figures 7(a,b) are the reverse of Figure 6 since the former correspond to a pay-floating/receive-fixed swap while the latter correspond to a pay-fixed/receive-floating swap, therefore similar explanations apply as above. **Thus when the yield curve is upward sloping and is shifting upwards, the receive-floating/pay-fixed swap has (mostly) positive credit exposure.**

Figure 7b

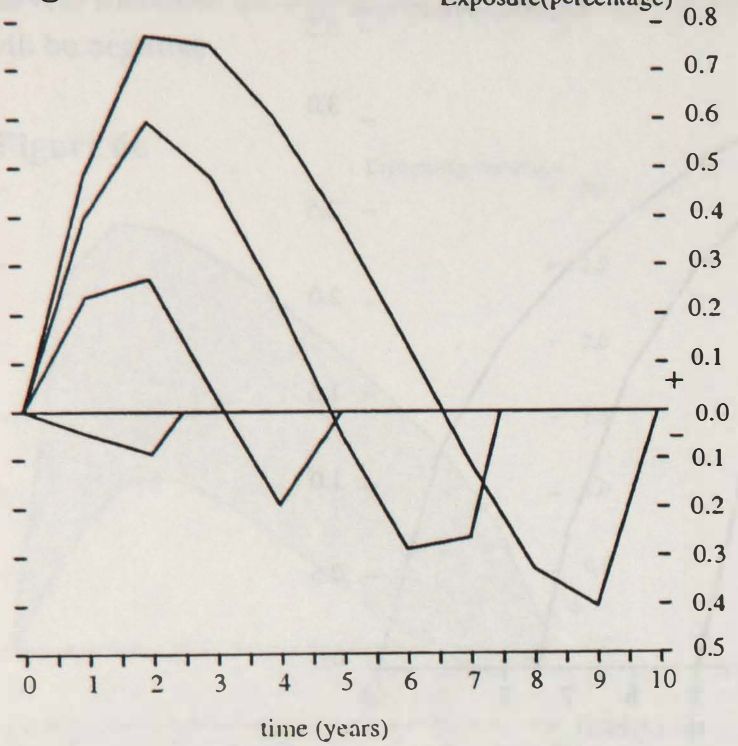
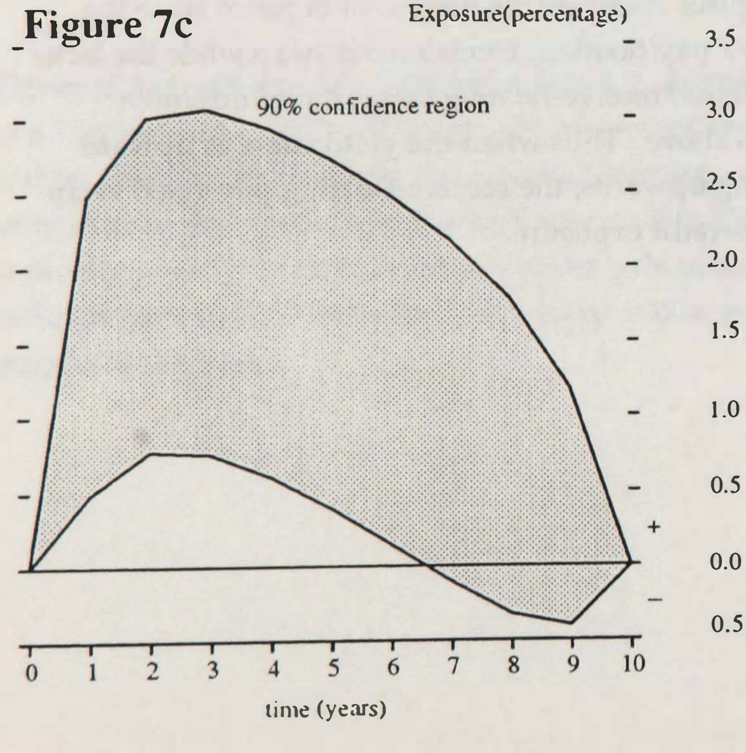


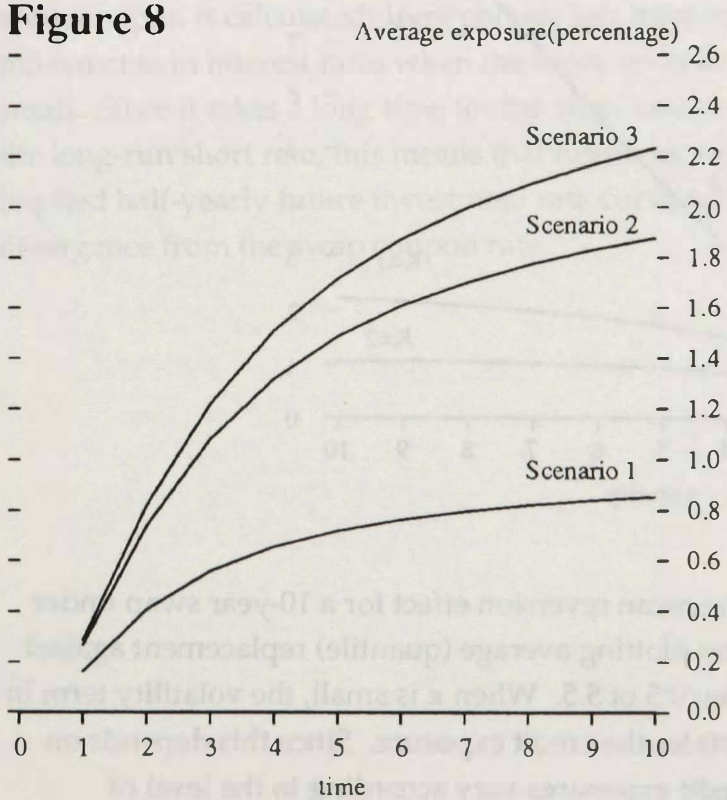
Figure 7c



2: The effect of the level of interest rates on swap exposure

The difference between the swap exposures in scenarios 2 and 3 is only evident from the maximum exposure, which in scenario 2 is 2.5% of notional principal whereas in scenario 3 is 3% of notional principal. This is due to the different levels of interest rates being considered in both scenarios. To investigate the effect of the level of interest rates more fully, Figure 8 plots the average replacement cost[6] against the maturities of the swaps (1 to 10 years) for scenarios 1,2 and 3 with κ=1. The top two curves correspond to scenarios 2 and 3. Average replacement cost for scenario 3 exceeds that of scenario 2 due to the fact the interest rates are higher in the former case. When the yield curve is flat as in scenario 1 - implying little movement due to mean reversion - average replacement cost is lower. In summary, Figure 8 illustrates (a) the effect of a sloped yield curve and (b) the effect of a higher level of interest rates on swap credit exposure.

Figure 8



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(6) The average replacement cost of a swap is defined to be the average of the 100α% quantile replacement cost at each payment date in the life of the swap, ie $n^{-1} \sum_{t=1}^{n} max\{K_t^{\alpha},0\}$ if there are no payment dates.

3: Effect of κ on swap exposure

If we compare Figure 1 and Figure 8 - in both graphs average replacement costs are plotted against the maturities of the swaps - we see that, in the case of the GBM model, the graph is convex whereas in the latter case (with mean reversion, κ,=1) average quantile replacement cost rises along a concave path with maturity. Figure 9 illustrates this point for the CIR model for the cases κ=0,1 and 2 by plotting average (quantile) replacement cost against maturity under scenario 3.

Figure 9

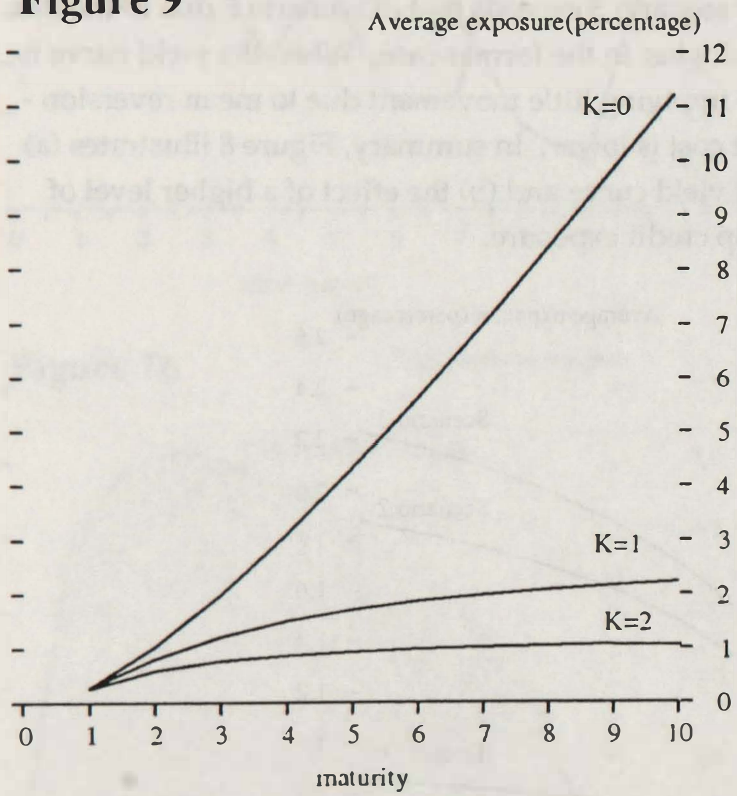
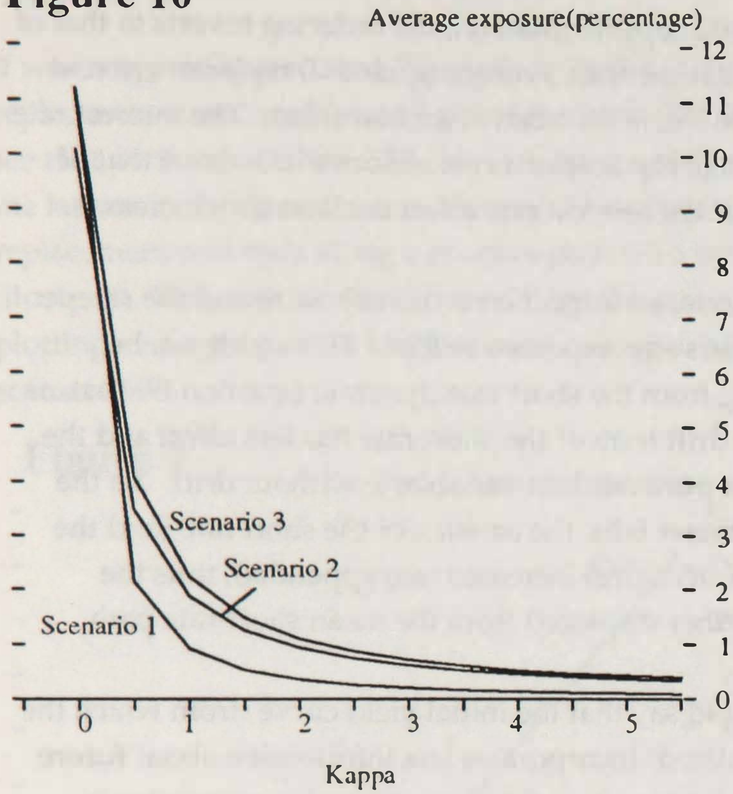


Figure 10 explores the mean reversion effect for a 10-year swap under scenarios 1, 2 and 3 by plotting average (quantile) replacement against κ where κ ranges from 0.5 ot 5.5. When κ is small, the volatility term in equation (9), $\sigma\sqrt{r}$, dictates the credit exposure. Since this depends on the short rate, the credit exposures vary according to the level of interest rates. For κ=0.2 (approximately), the ordering reverts to that of Figure 8. Scenario 1 has the least average (quantile) replacement cost due to the flat yield curve, ie no mean reversion effect. The interest rate effect places the average replacement cost of scenario 3 above that of scenario 2. The size of the interest rate effect declines as κ increases.

Overall, κ has a pronounced impact on swap exposure and the steeper the yield curve the less swap exposure will be. This result can be rationalised by noting from the short rate dynamic equation (9) that, as κ becomes small, the drift term of the short rate has less effect and the short rate resembles a pure random variable, ie without drift. As the mean reversion parameter falls, the *variance* of the short rate (and the yields along the term structure) increases (see appendix), thus the quantile paths are further displaced from the mean short-rate path.

Alternatively, one could say that the initial yield curve (from which the swap coupon is calculated) incorporates less information about future movements in interest rates when the mean reversion parameter is small. Since it takes a long time for the short rate to settle down around the long-run short rate, this means that future movements in the implied half-yearly future investment rate curve will show greater divergence from the swap coupon rate.

Figure 10



4: Effect of volatility on average replacement cost

The (local) volatility of the short rate in the CIR model is given by $\sigma\sqrt{r}$. By varying σ, we can investigate the effect of changes in short-rate volatility on swap exposure. A rise in σ has the effect of widening the tails of the distribution of the short rate, thereby shifting the quantile paths further away from the mean short rate path. The variance of the yield at time t given the short rate at time s ($s<t$) is an increasing function of σ. Thus if σ increases, the variability of the yield curve increases.

Figure 11

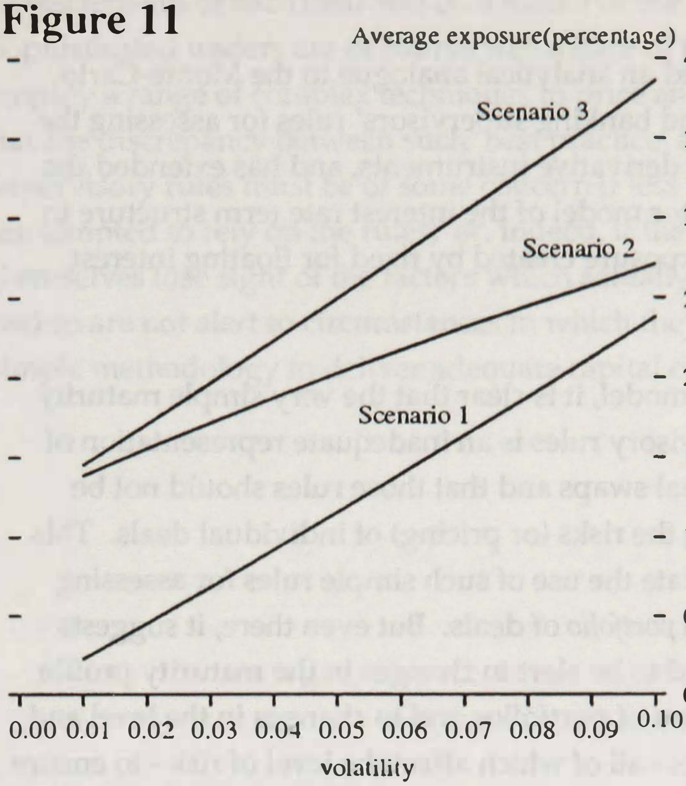


Figure 11 illustrates the effects of different levels of volatility under scenarios 1,2 and 3. In each graph, the average replacement cost of a 10-year swap (assuming an initial short rate of 3%, 6% or 9%) is plotted against the volatility parameter σ. Average replacement cost increases linearly with volatility in all cases. The ordering of the curves is consistent with Figures 9 and 8. The slope of the curves is positive: as the level of interest rate rises, volatility of the short rate ($\sigma\sqrt{r}$) rises. However, the slope of the curves is the most interesting feature. As volatility rises, the short-rate gap (θ-r) which determines the slope of the yield curve (along with κ) becomes less significant and the effect of the volatility term in equation (9) becomes more pronounced. For this reason, the curves corresponding to scenarios 1 and 3 are steeper than that of scenario 2 where interest rates eventually settle around 3%.

V Conclusion

This paper has developed an analytical analogue to the Monte-Carlo methods which lie behind banking supervisors' rules for assessing the credit exposure on OTC derivative instruments, and has extended the analysis using a one-factor model of the interest rate term structure to examine the potential exposure created by fixed for floating interest rate swaps.

Even in the naive GBM model, it is clear that the very simple maturity distinction in the supervisory rules is an inadequate representation of the riskiness of individual swaps and that those rules should not be relied upon in assessing the risks (or pricing) of individual deals. This does not of itself invalidate the use of such simple rules for assessing the riskiness of a typical *portfolio* of deals. But even there, it suggests that the supervisors need to be alert to changes in the maturity profile and currency composition of portfolios and to changes in the level and volatility of interest rates - all of which affect the level of risk - to ensure that banks have adequate capital cover for this business. At a minimum then, periodic re-evaluation of the add-ons would seem to be required. And in dealing with 'untypical' portfolios - for example, those which would arise if a bank were significantly to alter the scale of its derivatives business - the internationally-agreed rules could mis-state the risks involved; this suggests that supervisors need the capacity to evaluate the risks of such portfolios more precisely than their rules of thumb allow, to assure themselves that risks are not materially undercapitalised.

Extending the analysis to a one factor model of the interest rate term structure, with the sloping yield curves and mean reversion of interest rates which are a more realistic representation of the interest rate environment in which swap traders operate, further emphasises the limitations of the existing capital rules and confirms that accurate pricing of such deals to reflect their capital usage cannot safely rely on simple rules of thumb but should be based explicitly on the characteristics of each deal and on a model of the interest rate process. Sophisticated traders are of course well aware of this and already employ a range of complex techniques to price and manage these risks. But the discrepancy between such 'best practice' approaches and the supervisory rules must be of some concern if less sophisticated firms are tempted to rely on the rules; or, indeed, if the supervisors themselves lose sight of the factors which actually determine the risks, and so are not alert to circumstances in which they cannot rely on the simple methodology to deliver adequate capital cover.

APPENDIX: Details of calculations

(a) Interest Rate Swap (flat yield curve)

Where the yield curve is flat for all states of the world, the value of a swap just after the payment at time t is given by:

$$S_t(r_t) = (r_0/r_t \;\; - 1)(1 - (1+r_t/n)^{-n(T-\tau)}),$$

where n is the number of payments per year, T is the maturity of the swap and r_0 is the coupon (ie the fixed side of the swap) which has been determined on the basis of the initial (flat) yield curve.

Since the value of the swap at time t, S_t, is a strict monotone function of r_t, it follows that quantiles of r_t imply quantiles of S_t when the value of the swap is evaluated at these quantile points. Since r_t is conditionally distributed as

$$r_t = r_0\, exp((\mu\text{-}\sigma^2/2)t + l_t),\ and\ l_t \sim N(0,\sigma^2 t),\ for\ t \geq 0,$$

then the 5% quantile path for r_t is given by

$$r_t = r_0\, exp((\mu - \sigma^2/r)t - 1.645\ \sigma\sqrt{t}).$$

Substituting this into $S_t(r_t)$ we obtain the 95% quantile path for the value of the "naive" swap.

The average replacement cost along the 95% quantile path is determined as a simple average of the swap exposures at each payment date over the life of the swap. This approximates to integrating under the graph of the replacement cost quantile path and dividing by the maturity - the approximation becomes more accurate as the payment frequency (n) increases.

(b) Interest Rate Swap (CIR yield curve)

Cox, Ingersoll and Ross (1985) specify a stochastic differential equation for the short rate which is given by

$$dr_t = \kappa(\theta - r_t)dt + \sigma\sqrt{r_t}\, dZ_t.$$

This implies that r_t is a non-central chi-squared distributed random variable (conditional on r_0, $0<t$), ie

$$r_t \sim \chi^2\,(2r_0;\, 2q+2,\, 2u)$$

where $c = 2\kappa\theta/\sigma^2-1$ and r_0 is the short rate at time 0. This implies that r_t has a conditional mean of

$$r_0 e^{-t} + \theta(1-e^{-t})$$

and a conditional variance of

$$\{\sigma^2(1-e^{-t})/\kappa\}\{r_0 e^{-t}+(\theta/2)(1-e^{-t})\}.$$

One factor models of the yield curve allow the notion of a quantile yield curve path. In the case of the CIR model, we determine the "5% quantile" yield curves and hence the 95% quantile replacement cost of the swap at each point in time during the life of the swap. The 5% quantile paths for the instantaneous short rate have been derived by approximating the non-central chi-square distribution. We use the central chi-square cumulent approximation to this distribution given in Kendall and Stuart (1961), ie

$$\chi^2(s;\gamma,\lambda)ds = \rho^{-1}\,\chi^2(t/\rho;\,\gamma^*,0)dt$$

where $\rho = 1 + \lambda/(\gamma+\lambda)$ and $\gamma^* = \gamma + \lambda^2/(\gamma+2\lambda)$.

Given an initial short rate, this allows the construction of a 5% quantile path for the short rate over the life of the swap. The "5% quantile" yield curves are those yield curves which are generated by short rates on 5% quantile path. The "95% quantile" path for the replacement cost of the swap is determined on the basis of this sequence of yield curves.

In the CIR model, the price of a pure discount bond at time t which pays £1 at time T is shown to be:

$$P(r, t, T) = A(t,T).e^{-B(t,T).r}$$

where:

$$A(t,T) \equiv \left[\frac{2\gamma \exp[(\kappa+\lambda+\gamma)(T-t)/2]}{(\kappa+\lambda+\gamma)(\exp[\gamma(T-t)]-1)+2\gamma}\right]^{2\kappa\theta/\sigma^2}$$

$$B(t,T) \equiv \left[\frac{2(\exp[\gamma(T-t)]-1)}{(\kappa+\lambda+\gamma)(\exp[\gamma(T-t)]-1)+2\gamma}\right]$$

$$\gamma \equiv [(\kappa+\lambda)^2 + 2\sigma^2]^{1/2}$$

(c) Monotonicity between the short rate and swap exposure

Consider equation **(11)** which gives the value of a receive-fixed/pay-floating swap at time t (where t is just after a coupon payment date).

$$S_t=\Sigma_{i=1}^{2(T-t)}\{(1+C_0/2)P(r,i/2)-P(r,(i-1)/2)\} \qquad \textbf{(11)}$$

This can be rewritten as:

$$S_t=\Sigma_{i=1}^{2(T-t)-1}\{(C_0/2)P(r,i/2)-P(r,(i-1)/2)\}+(1+C_0/2)P(r,T-t)-P(r,0)$$

Since (i) $dP(r,\tau)/dr=-B(\tau)A(\tau)exp(-rB(\tau))<0$ for $\tau>0$ and (ii) $dP(r,0)/dr=0$ and (iii) the terms in the summation will not be positive for reasonable parameter values, it follows that $dS_t/dr<0$.

(d) Parameters underlying the graphs

GBM model	μ	σ	r_0	maturity
Figure 1	0	0.15	6%	10 years
Figure 2	0	0.15	6%	x-axis
Figure 3	0	0.15	x-axis	10 years
Figure 4	0	x-axis	6%	10 years

CIR model	κ	σ	r_0	maturity	θ
Figure 5	0,1,2	0.04	6%	10 years	6%
Figure 6(a,b,c)	1	0.04	6%	2.5,5,7.5,10 years	3%,9%
Figure 7(a,b,c)	1	0.04	6%	2.5,5,7.5,10 years	3%,9%
Figure 8	1	0.04	6%	x-axis	3,6,9%
Figure 9	0,1,2	0.04	6%	x-axis	9%
Figure 10	x-axis	0.04	6%	10 years	3,6,9%
Figure 11	1	x-axis	6%	10 years	3,6,9%

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