Granger causality in the presence of structural changes

Marco Bianchi

Bank of England, Threadneedle Street, London EC2R 8AH.

The views in this paper are those of the author and not necessarily those of the Bank of England. The paper is a revised version of Chapter 5 of the author's Ph.D. dissertation at the Université Catholique de Louvain, entitled "Time Series Modelling in the Presence of Structural Change". The author thanks his thesis advisors Prof. Wolfgang Härdle and Prof. Henri Sneessens for help and encouragement, and Fernando Barran, Marga Peeters and Danny Quah for comments and suggestions. Financial support from the European Community (Human Capital Mobility grant) during the years 1993–94 is gratefully acknowledged.

Issued by the Monetary Analysis Division, Bank of England, London, EC2R 8AH to which requests for individual copies should be addressed: envelopes should be marked for the attention of the Publications Group. (Telephone: 0171–601-4030).

Bank of England 1995
ISSN 0142–6753

Contents

Abstract

In this paper, we focus on Granger causality tests in the presence of regime shifts or structural breaks. We show that when the assumption of parameter constancy is violated, due to the occurrence of structural breaks, Granger causality tests can provide misleading inference about the underlying relationship of causality. We consider a Bayesian model for the detection of structural breaks which can make Granger causality tests 'robust' to the presence of structural instabilities in the sample. An application of the method to the Canadian series of GNP and M1 is presented.

1 Introduction

One of the basic assumptions for the use of Granger causality tests in econometrics is the stationarity of a vector autoregressive (VAR) time series representation. By stationarity, the absence of trends, seasonal components and structural instabilities in the sample period is generally meant. We look in this paper at the performance of the tests when the hypothesis of parameter constancy (that is absence of structural instabilities) is violated.

In a simulation study, Lütkepohl (1989) has demonstrated that Granger causality tests may provide quite incorrect inference about causality relations in the presence of structural changes. As long as Granger causality tests are tests for 'zero restrictions' on parameter estimates obtained by fitting a *linear* regression model to the data, the tests provide useful tools of analysis under the assumption of parameter constancy; in the presence of structural breaks, however, the linearity assumption in the regression equations of the VAR system can lead to the estimation of a mispecified model. This may strongly affect the outcome of Granger causality tests, leading to erroneous conclusions about causality.

A main argument of the paper is that the pitfalls of Granger causality tests may be avoided *if the number and the dating of the breaks would be known*, as the tests could be safely applied in those subsamples where no structural instabilities are detected. The problem of making Granger causality tests robust to structural breaks may be seen therefore equivalent to the problem of how to detect the breaks statistically.

A popular approach for the detection of structural breaks is to rely on predictions tests, as described in Lütkepohl (1989, 1991). A test statistic can be derived by comparing postsample predictions, obtained by estimating the model up to a certain date, with actually observed values after that date, over a fixed forecast horizon. Although prediction tests can work well in applications, they fail to address the key question of making inference about the *dating* of the structural change; indeed, these tests require that a date is fixed *a priori* as a candidate for a structural break. To have a list of dates for structural breaks which are 'data driven', that is not selected on the basis of prior information, we consider a Bayesian model for the detection of structural changes. The model, which is an adaptation of the approach introduced by Kashiwagi (1991), uses switching regressions to derive posterior probabilities for different dates to be structural changes in the sample.

The remainder of the paper is organised as follows. In Section 2 we simulate a bivariate VAR system in which the first endogeneous variable does not Granger-cause the second, but the second does Granger-cause the first. We allow both for a fixed and an abrupt change of the constant terms in the VAR representation and evaluate how the true causal structure emerges in the two different cases. We confirm the results obtained by Lütkepohl (1989) that structural instabilities have a substantial impact on Granger causality tests. However, we also make clear why the ability to detect the dating of the structural breaks is important for a correct inference on causality relationships. In Section 3 we focus on the problem of detecting the dates of the breaks. We consider various procedures such as recursive Granger causality tests, piecewise linear scatterplot smoothing, and Bayesian detection of structural breaks. In Section 4 we present an application to the Canadian series of income and money, quarterly data from 1955:I to 1977:IV, analysed by Hsiao (1979) and Lütkepohl (1989). Section 5 summarizes and concludes.

2 A Simulation Study

Following Lütkepohl (1989), we consider the design in which y_t and x_t are generated by the model

$$y_t = \mu_1 + \alpha_{11} y_{t-1} + \alpha_{12} x_{t-1} + v_{1t}, \quad (1)$$

$$x_t = \mu_2 + \alpha_{21} y_{t-1} + \alpha_{22} x_{t-1} + v_{2t}, \quad (2)$$

with $t = 0, 1, \ldots, 100$, $\mu_1 = \mu_2 = 0$, $\alpha_{11} = \alpha_{22} = 0.5$, $\alpha_{21} = 0.0$, and $\alpha_{12} = \pm 0.5$. The equation errors v_{1t} and v_{2t} are independent standard normal variates. We notice that x is not Granger caused (NGC) by y in the above design, but y is Granger caused by x. The parameter α_{12} governs the relation between the series. When $\alpha_{12} > 0$, an upward sloping OLS regression line well fits the scatterplot of the two variables. The opposite is true when $\alpha_{12} < 0$.

Replicating the design 100 times, with initial values $x_0 = y_0 = 0$, we estimate the VAR(1) model given by equations (1)–(2). We define pairwise Granger causality tests as usual by the null hypotheses

$$H_{01}: \quad \alpha_{21} = 0, \quad \Longleftrightarrow \quad x_t \; NGC \; by \; y_t, \tag{3}$$

$$H_{02}: \quad \alpha_{12} = 0, \quad \Longleftrightarrow \quad y_t \; NGC \; by \; x_t. \tag{4}$$

At each replication, we compute the p-values associated with the t-test statistics of the null hypotheses H_{01} and H_{02}. We also run a second experiment, in which we allow for an upward shift in the mean of the series, occurring at $t = 50$, from $\mu_i = 0$ to $\mu_i' = 1$, $i = 1, 2$. A sample realisation of the pairs x_t, y_t, obtained in the four different cases, is shown in Figure 1. The results of the Monte Carlo experiment are summarised in Table 1.

We recall that according to our design H_{01} is true and H_{02} is false, that is correct inference is made upon causality if H_{01} is not frequently rejected, but H_{02} is frequently rejected. It is interesting to see from Table 1 that in the absence of regime shifts, this is actually the case: H_{01} is rejected in a very small percentage of cases, and H_{02} is rejected in 100% of the cases, regardless whether α_{12} is positive or negative. In the presence of a single regime shift in both series, rather different results are obtained. H_{01} is now rejected too frequently, especially when α_{12} is negative.

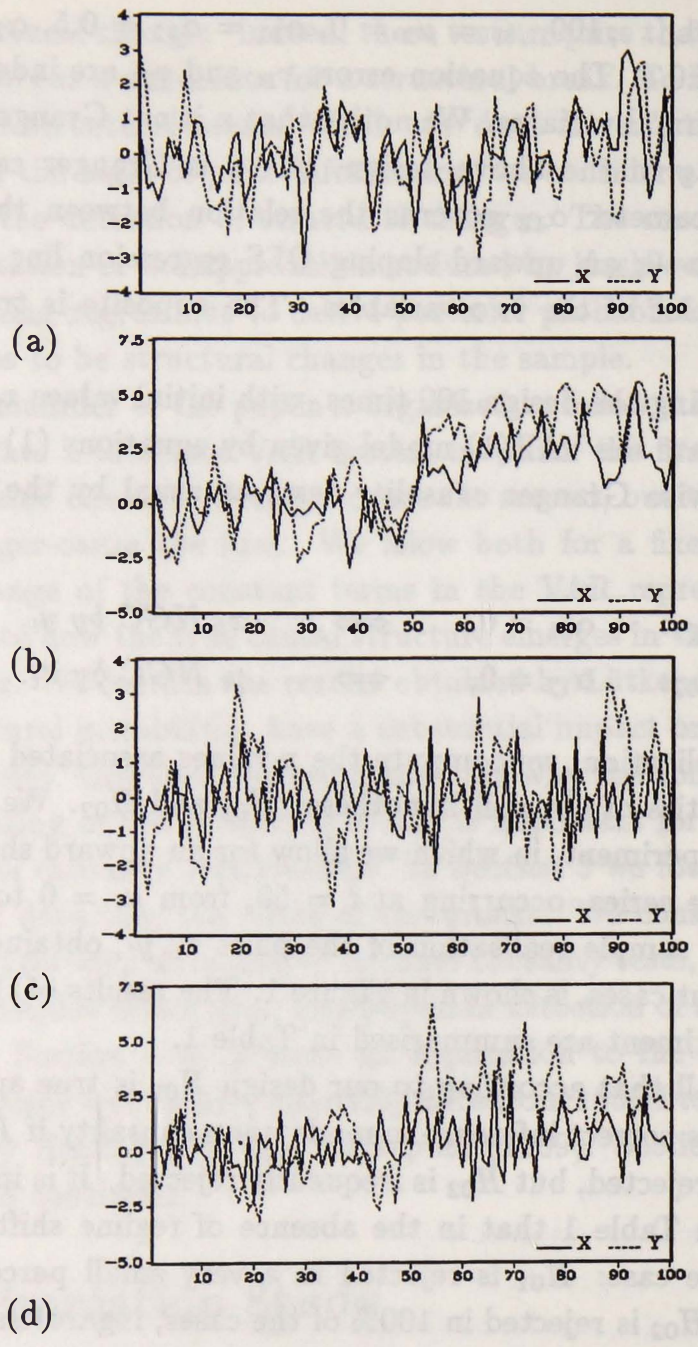


Figure 1: Plots of one realisation of the simulated series x_t (solid line) and y_t (dashed line) in the Monte Carlo experiment. (a) no regime shift, $\alpha_{12} > 0$; (b) regime shift, $\alpha_{12} > 0$; (c) no regime shift, $\alpha_{12} < 0$; (d) regime shift, $\alpha_{12} < 0$.

Table 1: Frequency of rejection of H_{0i}.

	No regime shift		Regime shift	
	H_{01}	H_{02}	H_{01}	H_{02}
$\alpha_{12} > 0$	4%	100%	**59%**	100%
$\alpha_{12} < 0$	5%	100%	**92%**	100%

The numbers reported in the table are the frequency of rejection of H_{0i}, $i = 1, 2$, at 5% critical level. Recall that H_{01} is true, and should therefore be rejected in 5% of the cases; H_{02}, on the other hand, is false, and should be rejected in 95% of the cases. Unsatisfactory outcomes in the table are in bold.

We also notice that H_{02} is rejected in 100% of the cases, that is correct inference is maintained concerning the direction of causality from x_t to y_t.

The above results are clearly consistent with those derived by Lütkepohl (1989). The natural question arising from the simulation exercise is though: Why does the simple presence of a shift in the mean of the series leads to incorrect inference upon causality? The scatterplots of the sample series displayed in Figure 2 provide some help to answer the question, by giving the intuition of why Granger causality tests can lead to misleading results in the presence of structural breaks.

Even though the scatterplots are on a contemporaneous correlations, they shed light on causality tests (i.e. on lagged conditional behaviour) as they make clear that in the context of the classical *linear* regression model the point estimates of the parameters representing the relationship between a pair of time series can be ***strongly biased*** in the presence of structural changes. We notice for example that when $\alpha_{12} < 0$, the scatterplot of x_t versus y_t is well fitted by a downward sloping regression line in the absence of regime shifts (Figure 2c). In the presence of a regime shift, however, the upward jump occurring in the mean of the series makes that two clusters of points are fitted, in a way that the OLS regression line becomes positively sloped (Figure 2d).

As a result of the bias in the point estimates of the model, the t-statistics (or F statistics) on the significance of the lags in the VAR model, that is Granger causality tests, are unreliable. The failure of Granger causality tests simply reflects a problem of *functional form mispecification* in the VAR estimation.

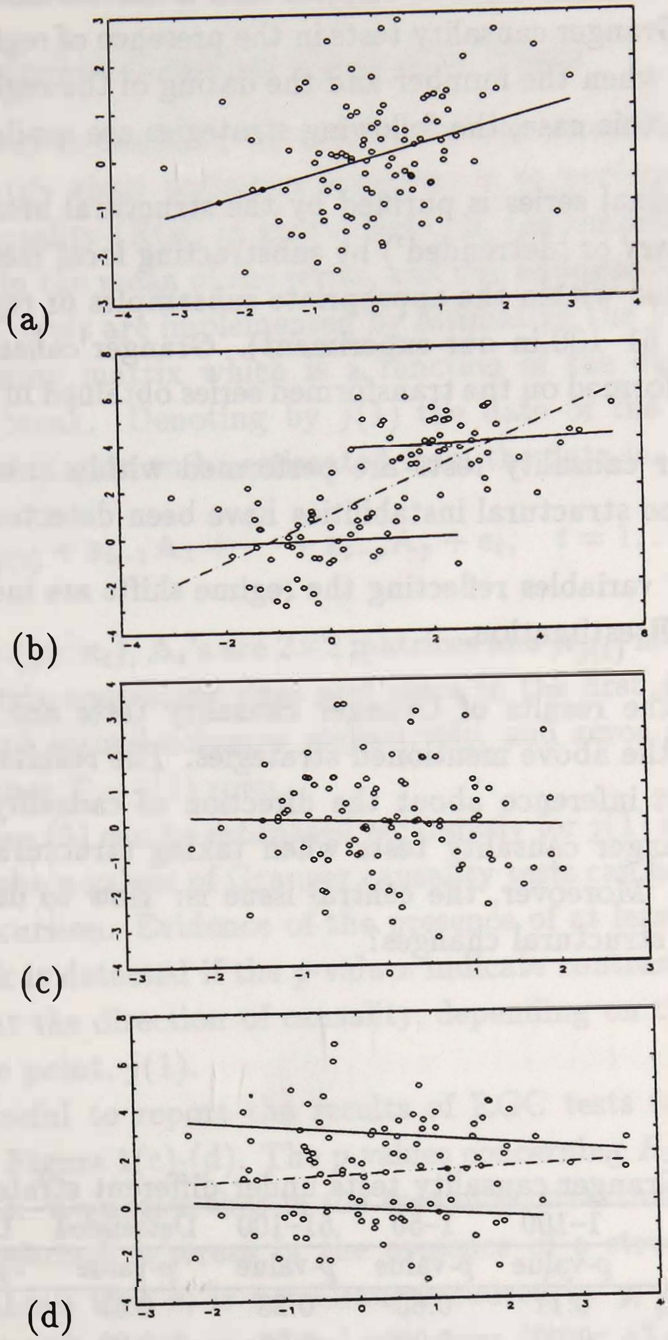


Figure 2: Scatterplots of the series plotted in Figure 1. (a) no regime shift, $\alpha_{12} > 0$; (b) regime shift, $\alpha_{12} > 0$; (c) no regime shift, $\alpha_{12} < 0$; (d) regime shift, $\alpha_{12} < 0$.

The above considerations suggest that a correct inference on the basis of Granger causality tests in the presence of regime shifts can be made when the number and the dating of the regime shifts in known. In this case, the following strategies are available:

- the original series is purified by the structural break (made stationary or 'detrended') by substracting local mean values computed within the appropriate subsamples or regimes (1–50 and 51–100 in our experiment). Granger causality tests are performed on the transformed series obtained in this way;
- Granger causality tests are performed within sub–samples where no structural instabilities have been detected;
- dummy variables reflecting the regime shifts are included in the VAR estimation.

In Table 2, the results of Granger causality tests are reported according to the above mentioned strategies. The results indicate that a correct inference about the direction of causality can be made by Granger causality tests when taking structural breaks into account. Moreover, the central issue is: How to detect the dating of the structural changes?

Table 2: Granger causality tests under different strategies.

Samples:	1–100	1–50	51–100	Detrended	Dummy
$\alpha_{12} > 0$	p-value	p-value	p-value	p-value	p-value
x_t *NGC by* y_t	0.11	0.60	0.28	0.63	0.82
y_t *NGC by* x_t	0.00	0.00	0.00	0.00	0.00
$\alpha_{12} < 0$	p-value	p-value	p-value	p-value	p-value
x_t *NGC by* y_t	0.02	0.98	0.42	0.40	0.64
y_t *NGC by* x_t	0.00	0.00	0.02	0.00	0.00

3 Detection of Structural Breaks

3.1 Recursive Granger Causality Tests

A natural way to check for the presence of structural changes (but not to identify their dating or location) is to perform recursive Granger causality (RGC in short) tests. In the simplest case of a single shift in the mean of the series, and two endogenous variables y_t, x_t, RGC tests are implemented by estimating the VAR system with a dummy matrix which is a function of the dating of the structural break. Denoting by $j(1)$ the date of the break, the VAR representation to be estimated from the data is

$$\mathbf{z}_t = \boldsymbol{\mu}_{j(1)} + \mathbf{z}_{t-1}\mathbf{A}_1 + \cdots + \mathbf{z}_{t-p}\mathbf{A}_p + \boldsymbol{\epsilon}_t, \quad t = 1, \ldots, T \tag{5}$$

where $\mathbf{z}_t = (\mathbf{y}_t \vdots \mathbf{x}_t)$, $\mathbf{A}_i$'s are 2×2 matrices and $\boldsymbol{\mu}_{j(1)}$ is a $(T-p) \times 2$ design matrix containing ones and zeros in the first $j(1)$ rows of the first and second columns respectively, and zeros and ones in the remaining $T - j(1)$ rows.

Equation (5) can be estimated recursively for $j(1) = 1, \ldots, T - p - 1$, and the p-values of Granger causality tests can be calculated at each recursion. Evidence of the presence of at least one structural break is detected if the p-values indicate contrasting conclusions about the direction of causality, depending on the dating of the change point, $j(1)$.

It is useful to report the results of RGC tests for the series plotted in Figure 1(c)-(d). The p-values concerning H_{01} are shown in Figure 3. We notice that in the case of no structural break H_{01} is never rejected, whereas in the presence of a structural break the hypothesis that x_t is not Granger caused by y_t is incorrectly rejected at 10% significance level whenever $j(1) < 43$ or $j(1) > 58$. Because inference about the direction of causality changes with the location of the dummy variable, this presents evidence for the occurrence of a structural break.

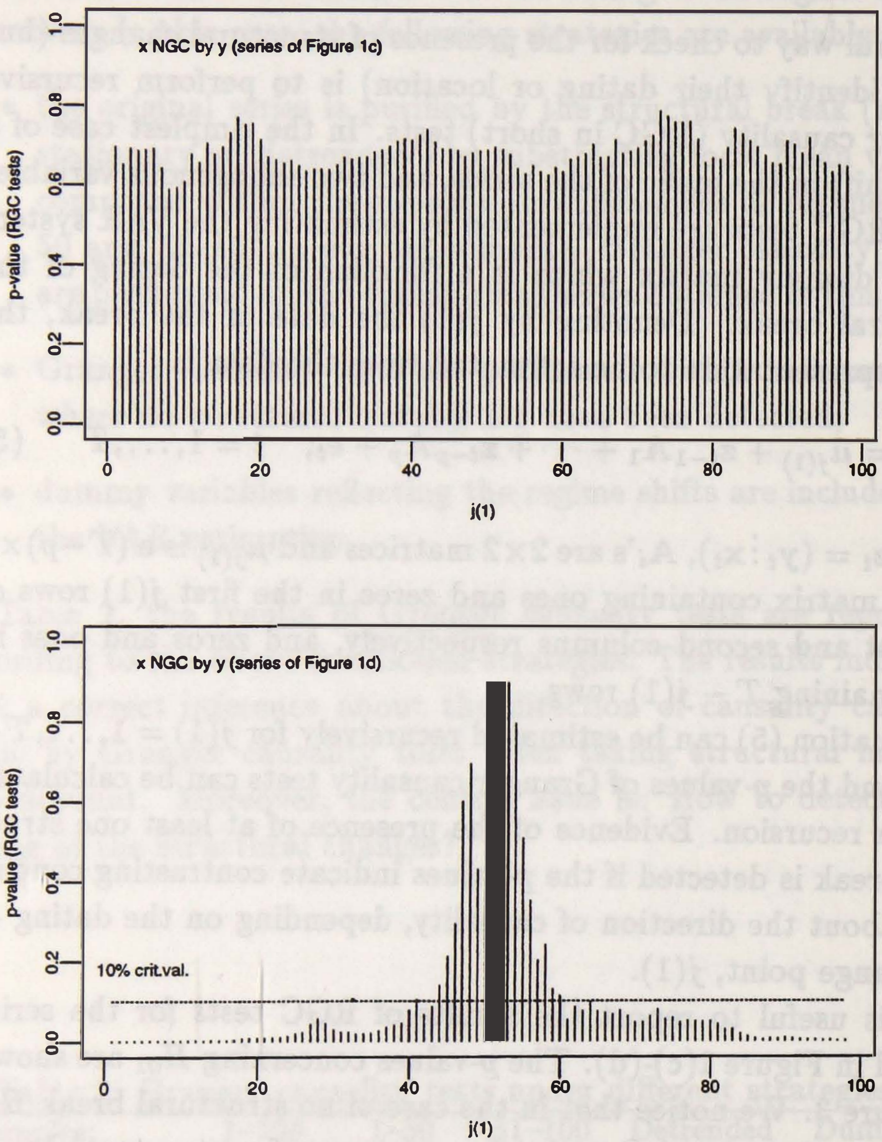


Figure 3: p-values of the null hypothesis H_{01}, obtained by Granger causality tests when estimating a VAR(1) model with a dummy variable shifting over time. Note: the results refer to the series reported in Figure 1(c)-(d).

3.2 Scatterplot Smoothing by Switching Regressions

Representing hypotheses tests within the classical linear regression model, Granger causality tests suffer from the drawbacks typical of the linear regression model, in particular the fact that OLS estimates are not robust with respect to outliers or groups in the data. The problem of the simple linear regression model is that an attempt is made to fit the scatterplot of two series by a simple straight line.

The linear regression model may be made more flexible by allowing for changes in the relationship between variables, by means of the switching regression model

$$y_t = \alpha_i + \beta_i x_t + \epsilon_t, \qquad j(i)+1 \leq t \leq j(i+1), \qquad i = 0, \ldots, n, \quad (6)$$

with $j(0) = 0$, $j(n+1) = T$ and $\epsilon_t \sim \mathrm{NID}(0, \sigma^2)$. The idea is that a classical linear model with given parameters holds within a regime, whereas the same linear model, but with different parameters, holds in different regimes. The switching regression model represents a useful tool for smoothing the scatterplot of two series by means of a *piecewise linear* fit, in order to avoid functional form mispecification leading to biased parameter estimates in the model.

In the context of switching regressions, a natural method for detecting the structural changes in the relation between two variables is to look for the piecewise linear fit minimising the residual sum of squares. For the series x_t, y_t plotted in Figure 1(d), for example, fitting equation (6) with $n = 1$ and $j(1) = 3, \ldots, T-3$ we obtain the results reported in Figure 4. Not surprisingly, the smallest residual sum of squares is obtained in correspondence of $j(1) = 50$, that is the true change point is actually detected.

3.3 Bayesian Detection of Structural Changes

On the basis of the switching regression model of equation (6), a Bayesian approach may be used to derive posterior probabilities

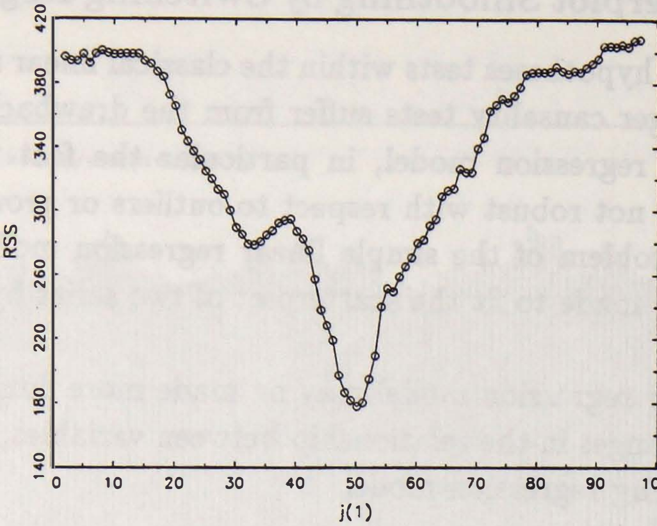


Figure 4: Smoothing the scatterplot of Figure 2(d) by the switching regression model of equation (6) with $n = 1$ change point placed at $j(1) = 3, \ldots, 97$. The best fitting (smallest residual sum of squares) is obtained for $j(1) = 50$, which is the true location of the structural change in the relationship between the two variables.

for different dates to be structural changes in the relationship between two variables. Within a segmented trend framework, that is in the particular case that t (time) replaces x_t as a predictor in equation (6), such an approach has been proposed by Kashiwagi (1991).

Given a pair of time series $\mathbf{z}_t = (x_t, y_t)$, prior probabilities are assigned to the event that there are n structural breaks in the relationship between the series and to the conditional event (conditional to n) that the breaks are located at $j(1)$, $j(2)$, ..., $j(n)$. Posterior probabilities are derived by combining the sample information obtained when fitting the switching regression model with *flat* priors, through the Bayes theorem. Formally, the events are defined as

J : joint event that the scatterplot of y_t versus x_t contains (i) n change points, (ii) at $j(1), j(2), \ldots, j(n)$;

b : event that there are n change points in the scatterplot, i.e. n structural changes in the relationship between the two variables;

c : conditional event that, given the scatterplot of y_t versus x_t contains n change points, these are located at $j(1), j(2), \ldots, j(n)$.

As in Kashiwagi (1991), we denote by $p(N = n|\mathbf{z})$ the posterior probability that there are n breaks, for $n = 0, 1, \ldots, \bar{n}$, where $\bar{n}$ is the maximum number of breaks allowed for, which cannot exceed $\bar{T} \equiv T - 1$. We also denote by $p(J_t|\mathbf{z})$ the posterior probability that a structural break actually occurs at time t.

Since flat prior probabilities are assumed in the model, any number of change points, from 0 to $\bar{n}$, has the same probability to occur; moreover, change points may occur at all possible combinations of the $\bar{T}$ dates with equal probability. Denoting $\omega(b)$ and $\omega(c)$ the priors, we have therefore

$$\omega(b) = \frac{1}{n+1}, \tag{7}$$

$$\omega(c) = \frac{1}{C_n^{\bar{T}}} = \frac{n!(\bar{T}-n)!}{\bar{T}!}, \tag{8}$$

where $C_n^{\bar{T}}$ stands for the combination of $\bar{T}$ elements taken n by n.

It is pointed out by Kashiwagi (1991) that in a Bayesian context the integrated likelihood of J is approximated by the exponential of the so called predictive log–likelihood

$$f(\mathbf{z}|J) = \int \ldots \int f(\mathbf{z}|J, \boldsymbol{\theta})\omega(\boldsymbol{\theta})\, d\boldsymbol{\theta} \approx \exp\{PL\}, \tag{9}$$

where $\boldsymbol{\theta}$ is the parameter vector and the predictive log–likelihood, PL, is an information criterion (that is a quantity representing a trade–off between goodness of fit and number of parameters in the model) defined as (see Kashiwagi, 1991, for details)

$$PL = -\frac{T}{2} - \frac{T}{2}\ln(2\pi\hat{\sigma}^2) - \frac{T(k+1)}{T-k-2}. \tag{10}$$

For the switching regression model, $k = \dim(\boldsymbol{\theta})$ is the number of parameters estimated in equation (6), and $\hat{\sigma}^2$ the residual sum of squares of the regression divided by T. By Bayes theorem, posterior probabilities $p(N = n|\mathbf{z})$ are given by

$$p(N = n|\mathbf{z}) = \frac{p(\mathbf{z}|b)\omega(b)}{p(\mathbf{z})} = \frac{p(\mathbf{z}|b)\omega(b)}{\sum_{n=0}^{\bar{n}} p(\mathbf{z}|b)\omega(b)}, \tag{11}$$

with

$$p(\mathbf{z}|b) = \sum_{\Omega_n} f(\mathbf{z}|J)\omega(c), \tag{12}$$

where Ω_n is the set of all possible models with n change points. We note that $p(N = n|\mathbf{z})$ is a function of $p(\mathbf{z}|b)$, and that $p(\mathbf{z}|b)$ is derived from equations (8), (9), and (10). The posterior probabilities for different dates being change points are derived by

$$p(J_t|\mathbf{z}) = \sum_{n=0}^{\bar{n}} p(J_t|\mathbf{z}, b) \cdot p(N = n|\mathbf{z}), \tag{13}$$

with

$$p(J_t|\mathbf{z}, b) = \sum_{\Omega_{n,t}} \left(\frac{f(\mathbf{z}|J)\omega(c)}{p(\mathbf{z}|b)} \right), \tag{14}$$

where $\Omega_{n,t}$, for given n and t, is the set of all possible models with n change points and one change point located at time t.[1]

[1] We note here that $p(\mathbf{z}|b)$ is the sum of predictive likelihoods $p(c|\mathbf{z})\omega(c)$ obtained when estimating the switching regression model for *all possible combinations* of change points (Ω_n), for a given n. $p(J|\mathbf{z}, b)$, instead, is the sum of the predictive likelihoods obtained when estimating the switching regression model for *all combinations involving the same* t ($\Omega_{n,t}$), normalised by $p(\mathbf{z}|b)$. Details of the algorithm for the derivation of the posterior probabilities are given by Kashiwagi (1991) and Bianchi (1995), Chapter 2.

For the simulated series of Figure 1(d), the posterior probabilities $p(J_t|\mathbf{z})$ obtained by the Bayesian approach are reported in Figure 5. Observation number 50 is clearly detected as the date of the structural break.

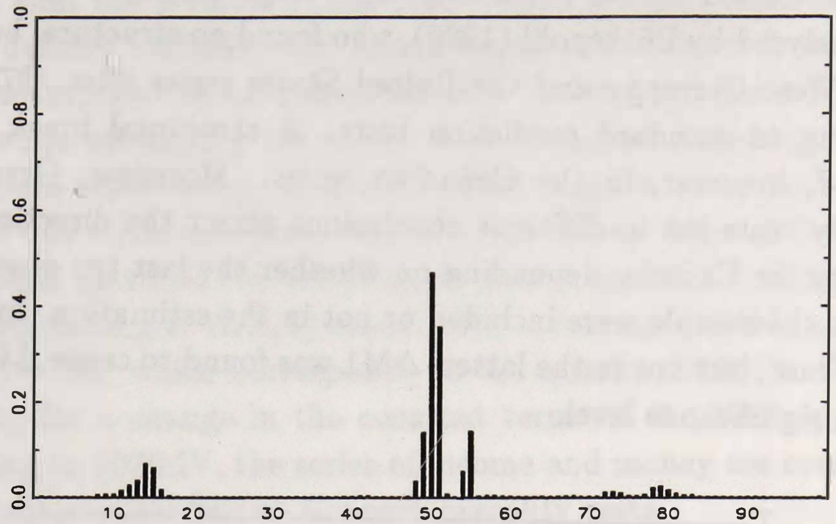


Figure 5: Plots of the posterior probabilities $p(J_t|\mathbf{z})$ obtained by the Bayesian model.

4 Canadian GNP and M1 Series

We present in this Section an application of the methods described in Section 3 to the series of gross national product (GNP) and money supply (M1) in Canada.

The series of income and money for Canada, West Germany and the United States (quarterly data from 1955:I to 1977:IV) were analysed by Lütkepohl (1989), who found no structural break in the West Germany and the United States series after 1973:II, according to standard prediction tests. A structural break was detected, however, in the Canadian series. Moreover, Granger causality tests led to different conclusions about the direction of causality for Canada, depending on whether the last ten observations in the sample were included or not in the estimation. In the former case, but not in the latter, ΔM1 was found to cause ΔGNP at 10% significance level.

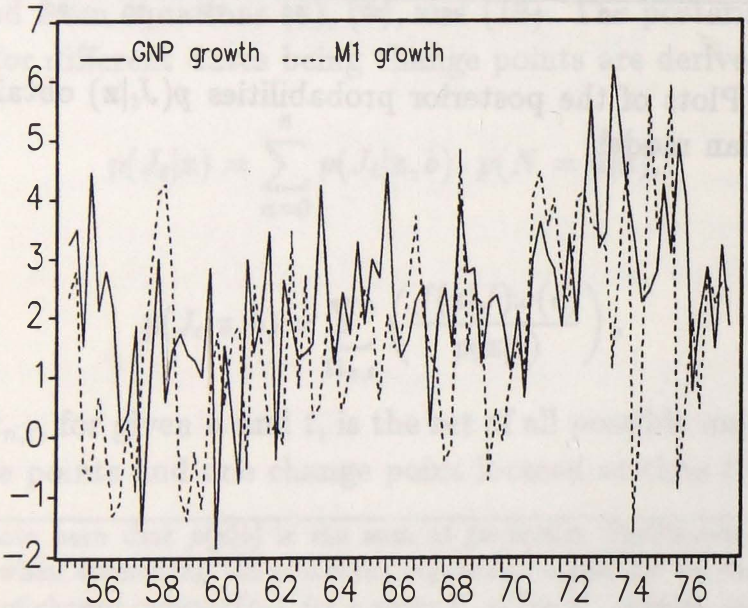


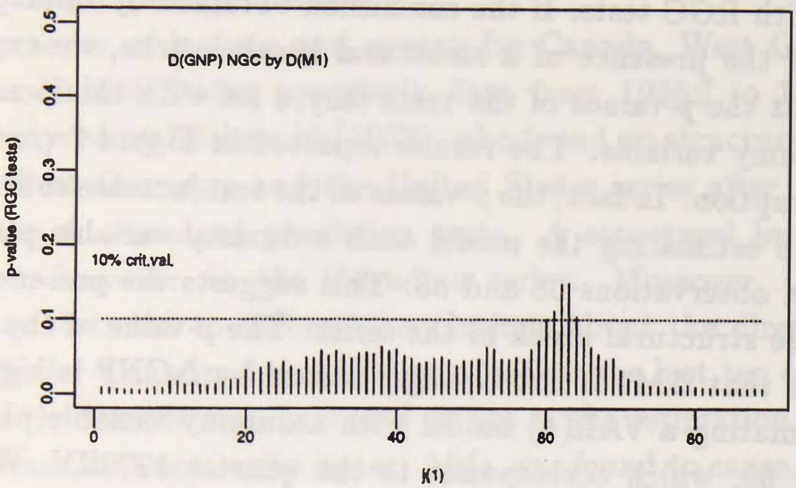
Figure 6: Growth rates of GNP (solid line) and M1 (dotted line): quarterly data from 1955:II to 1977:IV.

The annual growth rates (first differences of the series in logs) of the Canadian money supply and income series analysed by Hsiao (1979) and Lütkepohl (1989) are shown in Figure 6. In the following, the methods of Section 3 are employed on these series, starting with RGC tests. If the conclusion obtained by Lütkepohl concerning the presence of a structural break is true, we expect to find that the p-values of the tests vary a lot with the location of the dummy variable. The results reported in Figure 7 confirm this presumption. In fact, the p-values of the test increase substantially when estimating the model with a dummy variable placed in between observations 58 and 66. This suggests the presence of at least one structural break in the series. The p-value of the null hypothesis that ΔM1 is not Granger caused by ΔGNP is highest when estimating a VAR(1) model with a dummy variable placed at $j(1) = 63$, which corresponds to the quarter 1970:IV. When allowing for a change in the constant terms of the VAR system occurring in 1970:IV, the series of income and money are causally independent according to Granger causality tests.

Turning next to the switching regression model

$$\Delta\mathrm{GNP}_t = \alpha_i + \beta_i \cdot \Delta\mathrm{M1}_t + \epsilon_t, \tag{15}$$

where $j(i) + 1 \leq t \leq j(i+1)$ for $i = 0, \ldots, n$, we look for the best piecewise linear fit of the scatterplot of $\Delta\mathrm{GNP}_t$ versus $\Delta\mathrm{M1}_t$, for different numbers of break points, $n = 0, 1, \ldots, \bar{n}$, with $\bar{n} = 3$. We obtain the results reported in Table 3. The piecewise linear fits derived for $n = 0, 1, 2$ according to the best fitting change points of Table 3 are shown in Figure 8.



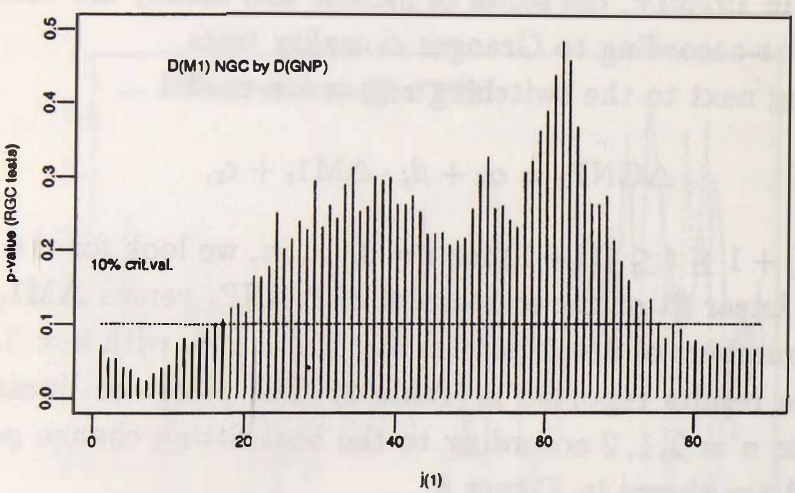


Figure 7: RCG tests for the series of income and money, based on the estimation of an integrated VAR(1) model with one dummy variable.

Table 3: RSS of switching regressions

	$j(1)$	$j(2)$	$j(3)$	RSS	R^2
$n = 0$	-	-	-	86.04	0.04
$n = 1$	63 (1970:IV)	-	-	71.52	0.21
$n = 2$	70 (1972:III)	77 (1974:II)	-	63.84	0.29
$n = 3$	7 (1956:IV)	24 (1961:I)	70 (1972:III)	57.34	0.36

Minimum values of RSS when fitting switching regressions with different number of change points. Note: results have been obtained for the series standardised (we subtracted the mean and divided for the sample standard deviation).

It is interesting to compare the way the scatterplot of ΔGNP_t versus ΔM1_t is smoothed by piecewise linear fits when assuming the occurrence of a different number of structural changes. If no break is assumed, a positive relation holds between income and money, represented by the upward sloped regression line of Figure 8(a). Higher growth rates in the money supply are associated with faster rates of economic activity. Assuming the presence of a structural break in 1970:IV, however, we obtain two regression lines, fitted respectively in the subsamples 1955:II–1970:IV and 1971:I–1977:IV. These lines are almost parallel to the x-axis, indicating no significant relationship between money and income growth rates. It is also suggested that, similarly to our simulated example, the positive relationship discovered between ΔGNP and ΔM1 when excluding the presence of a structural break can be the result of fitting two clusters of points in the scatterplot of the two variables, corresponding to observations generated by two different regimes. If two structural breaks are assumed, more structure appears in the scatterplot of ΔGNP_t versus ΔM1_t (see Figure 8c), with a negative relationship emerging in the subsample from 1972:III to 1977.IV; the main results, however, remain qualitatively unchanged from the case of a single structural break.

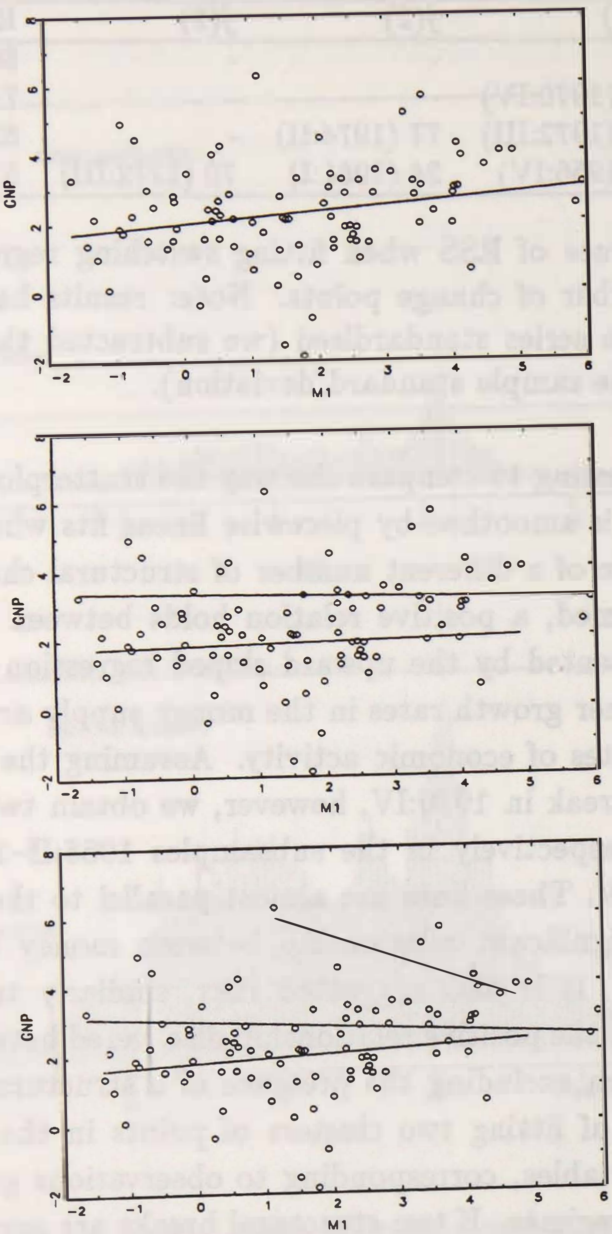


Figure 8: Piecewise linear fit obtained when assuming a different number of change points. (a) $n = 0$; (b) $n = 1$; (c) $n = 2$.

Posterior probabilities derived with the Bayesian approach confirm the presence of a single structural break (see Table 4) identified at observation number 63, i.e. 1970:IV (see Figure 9). Prediction tests for structural change also strongly indicate 1970:IV as a structural break. The results presented in Table 5, in particular, make a comparison between 1973:II, the date suspected of being a change point by Lütkepohl (1989) on the basis of prior information (the date before the first oil shock), and 1970:IV, the date suspected of being a change point by our data-driven analysis. The results validate that 1970:IV is more likely to be a structural change than 1973:II. A similar conclusion also appears from Figure 10, where a substantial upward jump in the value of the series is apparent after 1970:4.[2]

Finally, the results of Granger causality tests reported in Table 6 indicate that ΔM1 and $\Delta \mathrm{GNP}_t$ are causally independent when taking the break at 1970:IV into account. This is sharply in contrast with the conclusion that $\Delta \mathrm{M1}_t$ causes $\Delta \mathrm{GNP}_t$ at 1% significance level, which would be derived on the basis of Granger causality tests when estimating the model using all the available observations.

[2] In fact, the average growth rate of GNP in the subsample 1955:I–1970:IV is 1.85% versus a mean rate of 3.23% in the subsample 1971:I–1977:IV. Similarly, the growth rates of money supply in the two subsamples are 1.14% and 2.79% respectively.

Table 4: Post. probabilities

n	$p(N = n\|z)$
0	0.02
1	$\boxed{0.37}$
2	0.34
3	0.27

Bayesian posterior probabilities for the number of breaks, obtained estimating the switching regression model with a maximum number of $\bar{n} = 3$ change points.

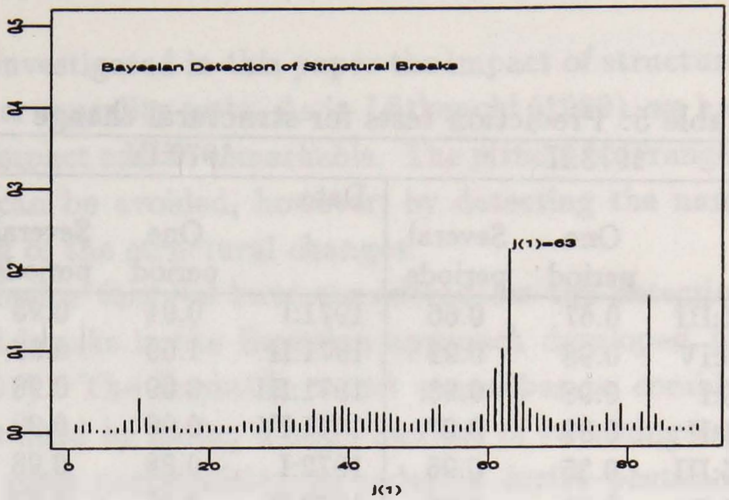


Figure 9: Bayesian posterior probabilities $p(J_t|\mathbf{z})$ obtained estimating the switching regression model, with a maximum number of $\bar{n} = 3$ change points.

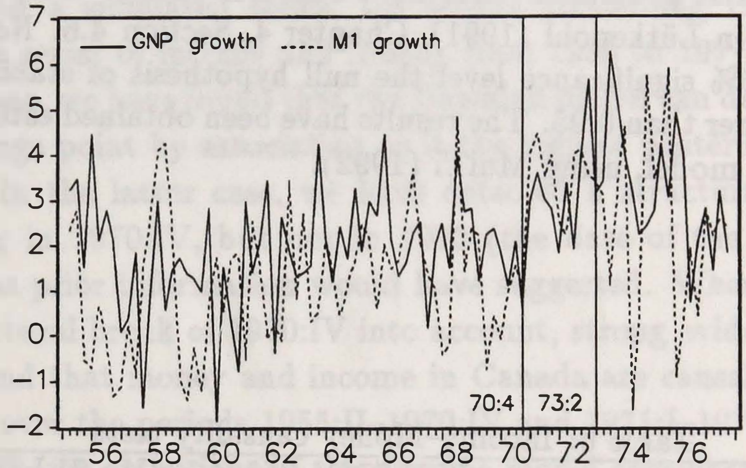


Figure 10: Canadian GNP and M1 series with two candidate dates for a structural change: 1973:2 (the date of the oil shock) or 1970:4 (the date detected as a change point by piecewise scatterplot smoothing).

Table 5: Prediction tests for structural change

1973:II			1970:IV		
Date	One period	Several periods	Date	One period	Several periods
1974:III	0.67	0.66	1971:I	0.94	0.93
1974:IV	0.98	0.91	1971:II	1.00	0.98
1975:I	0.98	0.98	1971:III	0.99	0.98
1975:II	0.82	0.97	1971:IV	0.99	0.99
1975:III	0.35	0.95	1972:I	0.88	0.98
1975:IV	0.21	0.90	1972:II	0.85	0.97
1976:I	0.09	0.85	1972:III	0.96	0.97
1976:II	0.24	0.79	1972:IV	1.00	0.99
1976:III	0.36	0.73	1973:I	1.00	1.00
1976:IV	0.31	0.64	1973:II	1.00	1.00

p-values of F prediction tests for structural change computed as described in Lütkepohl (1991), Chapter 4, Section 4.6. Note: we reject at 5% significance level the null hypothesis of stability for values bigger than 0.95. The results have been obtained estimating a VAR(1) model, using MulTi (1992).

Table 6: Income–Money causality tests

H_0	55:II-77:IV	55:II-70:IV	71:I-77:IV
ΔGNP_t NGC by $\Delta M1_t$	0.01	0.47	0.15
$\Delta M1_t$ NGC by ΔGNP_t	0.07	0.39	0.96

p-values of Granger causality tests obtained by estimating VAR(1) models in different subsamples.

5 Conclusions and Extensions

We have investigated in this paper the impact of structural breaks on Granger causality tests. As in Lütkepohl (1989), we have found that the impact can be remarkable. The pitfalls of Granger causality tests can be avoided, however, by detecting the number and the dating of the structural changes.

The device that we have considered for the detection of the structural breaks is the Bayesian approach developed by Kashiwagi (1991). The Bayesian model prescribes to combine likelihoods obtained by fitting a large number of switching regressions with flat prior probabilities, in order to derive posterior probabilities of different dates for being structural changes in the relationship between a *pair* of time series. This information is taken into account by testing for Granger causality in the appropriate subsamples, where no structural instabilities can be found.

We have presented two applications of the procedure, the first concerning a simulated series, the second concerning Canadian quarterly series of income and money from 1955 to 1977. In the former case, we have found that the Bayesian model can detect the true change point by associating to it the highest posterior probability. In the latter case, we have detected a structural break occurring in 1970:IV, but not in 1973 (the date of the first oil shock), as prior information would have suggested. When taking the structural break of 1970:IV into account, strong evidence has been found that money and income in Canada are causally independent over the periods 1955:II–1970:IV and 1971:I–1977:IV.

Two major extensions of the baseline model considered in the paper would require: (i) to relax the assumption of independent and identically distributed disturbances in the switching regression model, allowing for the presence of autocorrelation, and (ii) to tackle the problem of causality in the presence of structural instabilities for systems of more than two equations. The case of breaks in slope parameters rather than abrupt shifts in the mean of the series, on the other hand, can be treated within the proposed switching regression model.

Appendix

Data and programs. The Canadian GNP and M1 series are derived from Hsiao (1979). The following programs, written in the GAUSS language, were used to derive the results in the paper. The programs are available upon request from the author.

File	Output
`luetk.prg`:	Results reported in Table 1.
`var.prg`:	Results in Table 4.
`var1.prg`:	Figure 3 and 7.
`ols-rec.prg`:	Figure 5 and Table 3.
`ssbayes5.prg`:	Figure 5 and 9.
`ols-fit.prg`:	Figure 2 and 8.

References

[1] **Bianchi, M** 1995, *Time Series Modelling in the Presence of Structural Change.* PhD thesis, Université Catholique de Louvain, Louvain–la–Neuve, Belgium.

[2] **Haase, K and Lütkepohl, H and Claessen, H and Moryson, M and Schneider W** 1992, *MulTi: A menu-driven Gauss program for Multiple Time series analysis.* (Available from Prof. Lütkepohl, Institut für Statistik und Ökonometrie, Humboldt Universität zu Berlin, Germany), Berlin.

[3] **Hsiao, C** 1979, 'Autoregressive modeling of canadian money and income data'. *Journal of the American Statistical Association*, (74):553–560.

[4] **Kashiwagi, N** 1991, 'Bayesian detection of structural changes'. *Annals of the Institute of Statistical Mathematics*, 43(1):77–93.

[5] **Lütkepohl, H** 1989, 'The stability assumption in tests of causality between money and income'. *in Walter Krämer editor*, pages 75–86.

[6] **Lütkepohl, H** 1991, *Introduction to Multiple Time Series Analysis.* Springer-Verlag, Berlin.

Bank of England Working Paper Series

Publication date in italics

1	Real interest parity, dynamic convergence and the European Monetary System *(June 1992)*	Andrew G Haldane Mahmood Pradhan
2	Testing real interest parity in the European Monetary System *(July 1992)*	Andrew G Haldane Mahmood Pradhan
3	Output, productivity and externalities—the case of banking *(August 1992)*	R J Colwell E P Davis
4	Testing for short-termism in the UK stock market *(October 1992)*	David Miles
5	Financial deregulation and household saving *(October 1992)*	Tamim Bayoumi
6	An investigation of the effect of funding on the slope of the yield curve *(January 1993)*	D M Egginton S G Hall
7	A simple model of money, credit and aggregate demand *(April 1993)*	Spencer Dale Andrew G Haldane
8	Bank credit risk *(April 1993)*	E P Davis
9	Divisia indices for money: an appraisal of theory and practice *(April 1993)*	Paul Fisher Suzanne Hudson Mahmood Pradhan
10	The effect of official interest rate changes on market rates since 1987 *(April 1993)*	Spencer Dale
11	Tax specific term structures of interest rates in the UK government bond market *(April 1993)*	Andrew J Derry Mahmood Pradhan
12	Regional trading blocs, mobile capital and exchange rate co-ordination *(April 1993)*	Tamim Bayoumi Gabriel Sterne
13	Temporary cycles or volatile trends? Economic fluctuations in 21 OECD countries *(May 1993)*	Gabriel Sterne Tamim Bayoumi
14	House prices, arrears and possessions: A three equation model for the UK *(June 1993)*	F J Breedon M A S Joyce

15	Tradable and non-tradable prices in the UK and EC: measurement and explanation *(June 1993)*	C L Melliss
16	The statistical distribution of short-term libor rates under two monetary regimes *(September 1993)*	Bahram Pesaran Gary Robinson
17	Interest rate control in a model of monetary policy *(September 1993)*	Spencer Dale Andrew G Haldane
18	Interest rates and the channels of monetary transmission: some sectoral estimates *(September 1993)*	Spencer Dale Andrew G Haldane
19	The effect of futures trading on cash market volatility: evidence from the London stock exchange *(December 1993)*	Gary Robinson
20	M0: causes and consequences *(December 1993)*	F J Breedon P G Fisher
21	An empirical analysis of M4 in the United Kingdom *(December 1993)*	P G Fisher J L Vega
22	A model of building society interest rate setting *(June 1994)*	Joanna Paisley
23	Deriving estimates of inflation expectations from the prices of UK government bonds *(July 1994)*	Mark Deacon Andrew Derry
24	Estimating the term structure of interest rates *(July 1994)*	Mark Deacon Andrew Derry
25	Potential credit exposure on interest rate swaps *(August 1994)*	Ian Bond Gareth Murphy Gary Robinson
26	New currencies in the Former Soviet Union: a recipe for hyperinflation or the path to price stability? *(September 1994)*	C L Melliss M Cornelius
27	Inflation, inflation risks and asset returns *(November 1994)*	Jo Corkish David Miles

28	The construction of RPIY *(February 1995)*	R Beaton P G Fisher
29	Pricing deposit insurance in the United Kingdom *(March 1995)*	David Maude William Perraudin
30	Modelling UK inflation uncertainty: the impact of news and the relationship with inflation *(April 1995)*	M A S Joyce
31	Measuring core inflation *(April 1995)*	Danny T Quah Shaun P Vahey
32	An assessment of the relative importance of real interest rates, inflation and term premia in determining the prices of real and nominal UK bonds *(April 1995)*	David G Barr Bahram Pesaran
33	Granger causality in the presence of structural changes *(May 1995)*	Marco Bianchi