# Testing for convergence: evidence from nonparametric multimodality tests

Marco Bianchi

Bank of England, Threadneedle Street, London EC2R 8AH.

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### Abstract

The convergence hypothesis in growth theory implies that the frequency of the density distribution of GDP in a cross-section of countries tends to approach unimodality as we move forward in time. In this paper, we test the convergence theory in a crosssection of 119 countries by means of bootstrap multimodality tests and nonparametric density estimation techniques. By looking at the density distribution of GDP across countries in 1970, 1980 and 1989, we find increasing evidence for *bimodality*. The finding stands in contrast with the convergence prediction.

# 1 Introduction

There is a debate in growth economics as to whether less developed economies are catching up with richer economies — what is commonly known as the "convergence hypothesis". At the heart of the debate stands a fundamental controversy among researchers about the 'correct' answers to a number of relevant questions like (Quah, 1995b):

- Are poor countries becoming poorer and rich countries richer, or is there a tendency for the poor to catch up with the rich?
- Are countries converging towards each other only within groups or "clubs"?
- Is most of the world becoming middle class, or is it that the middle class is vanishing?

Much empirical work has been devoted in the literature to support or question different views about the convergence hypothesis. However, no widely accepted conclusion has been reached so far.

Conventional empirical analyses employing cross-section, timeseries or panel data techniques have found evidence to support the convergence hypothesis (see, among the others, Sala-i-Martin, 1994, 1995, and references therein). However, as recently pointed out by Quah (1995a,b), the limitation of conventional approaches is the modelling of the behaviour of an *average* or representative economy, rather than the *entire* cross-section of countries. Empirical evidence from cross section convergence regressions rely only on a few coefficients estimated from a linear model, without being able to clarify the dynamics of the entire income distribution.

In this paper, we follow the recommendation of Quah (1995b) to study "growth and convergence in terms of the dynamically evolving cross-country distribution of income".<sup>1</sup> Consistent with

<sup>&</sup>lt;sup>1</sup>Quah (1995), page 2.

previous studies (Quah, 1995a,b), we find substantial evidence against the convergence hypothesis between less and most developed economies, in favour of the convergence hypothesis within groups of economies or "clubs". We also find some evidence of a vanishing of the middle class (again, as suggested in Quah 1995a,b).<sup>2</sup>

The remainder of the paper is organised as follows. In Section 2 we briefly consider nonparametric techniques for the estimation of the density distribution of incomes across countries. We also discuss nonparametric multimodality tests. We report the empirical results in Section 3. Section 4 briefly summarises and concludes.

# 2 The Statistical Framework

Consider a random variable x with realizations  $x_i$ , i = 1, ..., n. In our application,  $x_i$  denotes the GDP per capita in US dollars in country i, in a cross-section of n countries. Also denote by f(x)the density of per capita incomes across countries.

With two groups of countries, say a group of "rich" and a group of "poor" countries, the convergence hypothesis predicts a catching-up of poor countries. In the presence of  $m^*$  groups of countries, the density of the frequency distribution has the form

$$f(x) = \sum_{i=0}^{m^*} p_i \cdot g_i(x; \mu_i, \sigma_i^2),$$
(1)

where  $p_i$ 's are mixture proportions with  $\sum_{i=0}^{m^*} p_i = 1$ , and  $g_i$  are unimodal densities with first and second moments  $\mu_i$  and  $\sigma_i^2$ . Assuming that the differences in the centrality parameters  $\mu_i$ 's (mean

 $<sup>^{2}</sup>$ It must be stressed nevertheless that our analysis, representing a purely statistical investigation of stylised facts, does not provide theoretical justifications or economic rationales as of why convergence may or may not occur. With this respect, interested readers are referred to the monograph by Barro and Sala-i-Martin (1994).

incomes in different groups) are "large" relative to the dispersion parameters  $\sigma_i^2$ 's (income variances in different groups), equation (1) implies that f(x) is multimodal with  $m^*$  modes (the modes of the density are said to be "well-separated" in this case).

According to the convergence hypothesis, if we start with a bimodal density in a given point in time, indicating the presence of two groups in a population of countries, there will be a tendency in the distribution to progressively move towards unimodality over time. Such a prediction indicates the way the convergence hypothesis can be tested empirically: we can estimate the density of the frequency distribution of incomes across countries at two (or more) distant points in time and evaluate at what points in time unimodality is most strongly rejected. We accomplish the former task by means of nonparametric kernel density estimators; the latter, by the bootstrap methodology.

#### 2.1 Nonparametric Density Estimation

The purpose of density estimation in statistics and data analysis is to evaluate where observations occur more frequently in a sample. In nonparametric density estimation, the "true" probability density function f(x) is estimated from a sample  $\{x_i\}_1^n$  of independent and identically distributed observations. The estimated density is constructed by centring around each observation  $x_i$  a kernel function K(u), with  $u = (x - x_i)/h$ , and averaging the values of this function at any given x. The estimator in its general form is defined as (see Silverman, 1986; Härdle, 1990)

$$\hat{f}_h(x) = (nh)^{-1} \sum_{i=1}^n K\left(\frac{x - x_i}{h}\right) = (nh)^{-1} \sum_{i=1}^n K(u), \quad (2)$$

where h > 0 is the bandwidth or window width and K(u) is the Gaussian kernel

$$K(u) = \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{1}{2}u^2\right).$$
(3)

Bandwidth h governs the degree of smoothness of the density estimate. With small values of h, wiggly estimates showing spurious structure in the data can often be obtained; with big values of h, on the contrary, important features of the underlying density can be smoothed away.<sup>3</sup> Figure 1 illustrates the construction of a kernel density estimator.



Figure 1: Construction of a nonparametric density estimate (solid line) by averaging the value of the kernel functions (Gaussian kernels represented by dotted lines), centred on the data points (x). Bandwidth: h = 0.5.

<sup>3</sup>Several data-driven bandwidth selectors have been proposed in the literature. As reviewed in Bianchi (1995), these aim to achieve an objective choice of the bandwidth by identifying a value of h which minimises some distance (or discrepancy) between the true and estimated density — such as for example the Mean Squared Error (MSE), its expected value, the Mean Integrated Squared Error (MISE), or the Taylor expansion of the MISE, the Asymptotic Mean Integrated Squared Error (AMISE). Other selectors, like those based on the bootstrap methodology, select a value of h consistent with the number of modes in the density, thus approaching the problem of bandwidth selection in the perspective of hypothesis testing.

### 2.2 Bootstrap Multimodality Tests

Well-known procedures for testing the number of modes in a density distribution include the parametric approach, based on maximum likelihood estimation, and the nonparametric approach, based on the bootstrap or resampling techniques (see Izenman and Sommer, 1988, for a comparison of the two approaches). We briefly consider in the following the nonparametric approach.

Bootstrap tests are built on the concept of *critical bandwidth* introduced by Silverman (1981, 1983, 1986). A critical bandwidth  $h_{crit}(m)$  is defined as the smallest possible h producing a density with at most m modes, what means that for all  $h < h_{crit}(m)$  the estimated density  $\hat{f}_h$  has at least m + 1 modes.

The idea of critical smoothing is naturally related with hypothesis testing. If the true underlying density has two modes, for example, then a large value of  $h_{crit}(1)$  is expected, because a considerable amount of smoothing is required to obtain a unimodal density estimate. This suggests that  $h_{crit}(m)$  can be used as a statistic to test

 $H_0$ : f(x) has m modes;  $H_1$ : f(x) has more than m modes. A 'large' value of  $h_{crit}(m)$  indicates more than m modes, thus rejecting the null.

How large is large in this context is assessed by the bootstrap, as discussed by the same Silverman in his works, and, among the others, by Efron and Tibshirani (1993), section 16.5. Given the vector of observations  $\mathbf{x} = (x_1, \ldots, x_n)'$ , a sample  $\mathbf{y}^* = (y_1^*, \ldots, y_n^*)'$  is obtained by resampling with replacement from  $\mathbf{x}$ . To ensure that the realizations obtained from the bootstrap have the same first and second moment properties of the observations  $\mathbf{x}$ , the following transformation is considered

$$x_i^* = \bar{y}^* + \left(1 + \frac{h_{crit}^2(m)}{s^2}\right)^{-1/2} (y_i^* - \bar{y}^* + h_{crit}(m)e_i), \quad (4)$$

where i = 1, ..., n,  $\bar{y}^* = \text{mean}(y^*)$ ,  $s^2$  is the sample variance of x, and  $e_i$  standard normal variables generated by the computer. A *p*-value for  $h_{crit}(m)$ , called the 'achieved significance level' (ASL) of the test, is obtained by generating a large number of samples from  $f_{crit}(m)$  and counting the proportion of samples for which  $h_{crit}^*(m) > h_{crit}(m)$ , where  $h_{crit}^*(m)$  is the smallest value of *h* producing a density estimate with *m* modes from the bootstrap data  $\mathbf{x}^*$ . We have formally

$$ASL_m = \operatorname{Prob.}\{h_{crit}^*(m) > h_{crit}(m)\}$$
(5)

where  $h_{crit}(m)$  is a fixed value obtained from the data x. Denoting by B the number of bootstrap replications, and defining the indicator variable<sup>4</sup>

 $I_{m,b} = \begin{cases} 1 & \text{if } f_{h_{crit}(m)}(x^*) \text{ has more than } m \text{ modes} \\ 0 & \text{otherwise,} \end{cases}$ 

an estimate for the p-value or achieved significance level of the test is given by

$$\widehat{ASL}_{m} = B^{-1} \sum_{b=1}^{B} I_{m,b}.$$
 (6)

We fail to reject the null hypothesis of m modes in the density whenever the p-value is larger than standard levels of significance.

<sup>&</sup>lt;sup>4</sup>It has been proven by Silverman that the event  $h_{crit}^*(m) > h_{crit}(m)$  is equivalent to the event that  $\hat{f}_{h_{crit}}(m)(x^*)$  has more than m modes. This results implies that it is not necessary to compute  $h_{crit}^*(m)$  for each bootstrap sample; one needs only to check the proportion of cases when  $f_{h_{crit}}(m)(x^*)$ has more than m modes.

## **3** Empirical Results

In this section, we consider per capita GDP at constant US dollars for n = 119 countries, measured in 1970, 1980 and 1989. Prior to testing for multimodality using kernel density estimation and the bootstrap methodology, we standardise the data by dividing income differentials from the mean value by the sample standard deviation. For the standardised data, the critical bandwidths are shown in Table 1 and Figure 2.

The results of nonparametric multimodality tests are shown in Table 1. It appears that the most likely hypothesis is that the underlying density has  $m^* = 2$  modes in 1980 and 1989, whereas the unimodality hypothesis is not rejected in 1970 at a 5% level of significance. This provides evidence against the convergence hypothesis.

In the three years, by selecting a value of the bandwidth consistent with bimodality, we obtain the density estimates reported in Figure 3. The densities are all skewed to the right, indicating the presence of a large mass of "poor" countries, with a small proportion of "rich" countries. In 1980 and in 1989, a pronounced mode appears in the long right-end tail of the distribution suggesting the formation of "clubs" or clusters in the data, in support of the so called "polarisation hypothesis".

Also, in the decade from 1970 to 1980, we notice the mode centred about -0.5 shifts to the left and the mode centred about 1.5-1.6 shifts to the right; this indicates a widening gap between less and more developed countries. In the decade from 1980 to 1989, there are no shifts in the modes, but a larger proportion of poor countries (larger than in 1980), together with a smaller proportion of middle-income countries, is now observed; this result appears consistent with the hypothesis of vanishing of the middle class.

|        | $h_{crit}(m)$ |       |       | ÂSL, | a; B = | 10000 |       |
|--------|---------------|-------|-------|------|--------|-------|-------|
| GDP in | m = 1         | m = 2 | m = 3 |      | m = 1  | m = 2 | m = 3 |
| 1970   | 0.495         | 0.290 | 0.265 |      | 0.07   | 0.33  | 0.05  |
| 1980   | 0.565         | 0.220 | 0.150 |      | 0.00   | 0.36  | 0.63  |
| 1989   | 0.570         | 0.190 | 0.160 |      | 0.00   | 0.68  | 0.51  |

Table 1: Bootstrap tests for multimodality.



Figure 2: Critical bandwidths obtained by kernel density estimation in 1970 (top panel), 1980 (middle panel) and 1989 (bottom panel) are the values of h where jumps in the step function occur. Note that the number of modes in the estimated densities are a decreasing function of the window width.



Figure 3: Density estimates of standardised income distributions in 1970, 1980 and 1989. Selected bandwidths are the critical bandwidths consistent with bimodality: 0.29, 0.22 and 0.19 respectively.

## 4 Conclusions

Recent theoretical work in growth theory, such as for example Baumol (1986), Esteban and Ray (1994) and Quah (1994, 1995a,b), has rationalised phenomena like the formation of convergence clubs, polarisation and poverty traps. According to these models, convergence clubs endogenously form and the distribution of income across countries has a tendency to polarise towards a bimodal distribution.

In this paper, we have empirically examined the convergence hypothesis from the perspective of income distributions in a crosssection of countries. By means of purely statistical techniques such as nonparametric density estimation and bootstrap multimodality tests, we have tested for the number of modes and estimated, consistently with the detected number of modes, the income distribution of a cross-section of 119 countries in 1970, 1980 and 1989. We have found strong evidence for bimodality (ie polarisation and clubs formation) occurring in the seventies, associated with a process of vanishing of the middle class in the eighties.<sup>5</sup>

Overall, the empirical evidence suggested in our study supports the view of clustering and stratification of growth patterns over time, in contrast with the convergence hypothesis.

<sup>5</sup>It is worthwhile noticing here that our method, although pertinent to testing for convergence, does not provide detailed information about intradistribution dynamics. Quah (1995a,b), however, has already provided results for the latter.

# Appendix

## Income Data

The data set is taken from the database of the University of Pennsylvania, called Penn World Table (PWT), June 1993. The following countries were included in the cross section.

| Country        | Per cap | Per capita GDP |      |  |
|----------------|---------|----------------|------|--|
|                | 1970    | 1980           | 1989 |  |
|                |         |                |      |  |
| ALGERIA        | 1837    | 2778           | 2778 |  |
| ANGOLA         | 1100    | 627            | 657  |  |
| BENIN          | 1144    | 1111           | 953  |  |
| BOTSWANA       | 863     | 1871           | 3218 |  |
| BURKINA FASO   | 399     | 473            | 541  |  |
| BURUNDI        | 324     | 463            | 518  |  |
| CAMEROON       | 867     | 1275           | 1293 |  |
| CAPE VERDE IS. | 686     | 988            | 1269 |  |
| CENTRAL AFR.R. | 699     | 663            | 559  |  |
| CHAD           | 549     | 425            | 380  |  |
| CONGO          | 1579    | 1829           | 2216 |  |
| EGYPT          | 1105    | 1572           | 1829 |  |
| GABON          | 3692    | 4789           | 3618 |  |
| GAMBIA         | 599     | 878            | 645  |  |
| GHANA          | 1012    | 921            | 815  |  |
| GUINEA         | 351     | 424            | 360  |  |
| GUINEA-BISS    | 653     | 440            | 659  |  |
| IVORY COAST    | 1320    | 1563           | 1282 |  |
| KENYA          | 577     | 889            | 887  |  |
| LESOTHO        | 386     | 917            | 958  |  |
| MADAGASCAR     | 1123    | 959            | 672  |  |
| MALAWI         | 429     | 541            | 504  |  |
| MALI           | 389     | 498            | 544  |  |
| MAURITANIA     | 985     | 958            | 860  |  |
| MAURITIUS      | 2348    | 3892           | 5363 |  |
| MOROCCO        | 1296    | 1866           | 2043 |  |
| MOZAMBIQUE     | 1458    | 896            | 755  |  |
| NAMIBIA        | 2602    | 2417           | 2054 |  |
| NIGER          | 752     | 694            | 467  |  |
| NIGERIA        | 769     | 1196           | 742  |  |

| RWANDA         | 626   | 733   | 659   |
|----------------|-------|-------|-------|
| SENEGAL        | 1104  | 1087  | 1081  |
| SEYCHELLES     | 1666  | 2825  | 3426  |
| SIERRA LEONE   | 1050  | 1001  | 908   |
| SOMALIA        | 845   | 836   | 865   |
| SOUTH AFRICA   | 3146  | 3512  | 3316  |
| SWAZILAND      | 2415  | 3015  | 2182  |
| TOGO           | 626   | 726   | 628   |
| TUNISIA        | 1398  | 2473  | 2743  |
| UGANDA         | 764   | 513   | 901   |
| ZAIRE          | 644   | 450   | 403   |
| ZAMBIA         | 1091  | 930   | 722   |
| ZIMBABWE       | 1063  | 1176  | 1292  |
| BARBADOS       | 4758  | 6534  | 7727  |
| CANADA         | 10175 | 14231 | 17690 |
| COSTA RICA     | 2796  | 3694  | 3572  |
| DOMINICAN REP. | 1496  | 2305  | 2293  |
| EL SALVADOR    | 1737  | 1923  | 1738  |
| GUATEMALA      | 2003  | 2540  | 2099  |
| HAITI          | 788   | 980   | 793   |
| HONDURAS       | 1207  | 1491  | 1351  |
| JAMAICA        | 2670  | 2274  | 2413  |
| MEXICO         | 3950  | 5707  | 5165  |
| PANAMA         | 2497  | 3291  | 2650  |
| PUERTO RICO    | 5784  | 6768  | 9051  |
| TRINIDAD&TOBAG | 6725  | 11242 | 8355  |
| U.S.A.         | 12725 | 15097 | 18354 |
| ARGENTINA      | 4165  | 4745  | 3615  |
| BOLIVIA        | 1614  | 1908  | 1597  |
| BRAZIL         | 2401  | 4254  | 4133  |
| CHILE          | 3687  | 3900  | 4024  |
| COLOMBIA       | 2097  | 2892  | 3150  |
| ECUADOR        | 1762  | 3181  | 2805  |
| GUYANA         | 1706  | 1965  | 1184  |
| PARAGUAY       | 1439  | 2516  | 2235  |
| PERU           | 2648  | 2889  | 2177  |
| SURINAME       | 3048  | 3969  | 2367  |
| URUGUAY        | 3870  | 4955  | 4320  |
| VENEZUELA      | 7624  | 7233  | 5692  |
| BANGLADESH     | 919   | 1098  | 1254  |

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| CHINA          | 825   | 1241  | 2290  |
|----------------|-------|-------|-------|
| HONG KONG      | 4456  | 8801  | 14035 |
| INDIA          | 704   | 763   | 1042  |
| INDONESIA      | 700   | 1252  | 1798  |
| IRAN           | 4212  | 3148  | 3046  |
| ISRAEL         | 5718  | 7494  | 8431  |
| JAPAN          | 7500  | 10292 | 14045 |
| JORDAN         | 1412  | 2600  | 2280  |
| KOREA, REP.    | 1688  | 3123  | 6209  |
| MALAYSIA       | 2117  | 3772  | 4470  |
| MYANMAR        | 392   | 475   | 576   |
| PAKISTAN       | 997   | 1076  | 1340  |
| PHILIPPINES    | 1368  | 1869  | 1727  |
| SINGAPORE      | 3155  | 6958  | 10240 |
| SRI LANKA      | 1315  | 1851  | 2218  |
| SYRIA          | 2201  | 4286  | 3705  |
| TAIWAN         | 2387  | 4827  | 8209  |
| THAILAND       | 1508  | 2146  | 3231  |
| YEMEN          | 586   | 1031  | 1615  |
| AUSTRIA        | 7565  | 10586 | 12378 |
| BELGIUM        | 8453  | 11354 | 13097 |
| CYPRUS         | 3757  | 5294  | 7827  |
| CZECHOSLOVAKIA | 3825  | 5583  | 6171  |
| DENMARK        | 9675  | 11234 | 13579 |
| FINLAND        | 8247  | 10985 | 14371 |
| FRANCE         | 9621  | 11798 | 13664 |
| GERMANY, WEST  | 9557  | 12013 | 13937 |
| GREECE         | 4234  | 5895  | 6622  |
| HUNGARY        | 3382  | 5051  | 5623  |
| ICELAND        | 7086  | 11909 | 13092 |
| IRELAND        | 4884  | 6785  | 8406  |
| ITALY          | 7669  | 10445 | 12367 |
| LUXEMBOURG     | 10000 | 12029 | 16079 |
| MALTA          | 2367  | 4387  | 6482  |
| NETHERLANDS    | 9228  | 11323 | 12434 |
| NORWAY         | 8129  | 12249 | 14647 |
| POLAND         | 2999  | 4465  | 4583  |
| PORTUGAL       | 3323  | 5048  | 6281  |
| SPAIN          | 6017  | 7495  | 9305  |
| SWEDEN         | 10643 | 12290 | 14534 |

| SWITZERLAND    | 13274 | 14653 | 16799 |
|----------------|-------|-------|-------|
| TURKEY         | 2179  | 2853  | 3370  |
| U.K.           | 7695  | 10028 | 13050 |
| U.S.S.R.       | 2873  | 4270  | 5457  |
| YUGOSLAVIA     | 3337  | 5641  | 5090  |
| AUSTRALIA      | 10917 | 12622 | 14904 |
| FIJI           | 2501  | 3557  | 3541  |
| NEW ZEALAND    | 9352  | 10260 | 11811 |
| PAPUA N.GUINEA | 1740  | 1658  | 1445  |

### Programs

The following GAUSS programs were used to derive the results. The programs, which can be used to replicate the results in the paper, are available upon request from the author (but free of charge only to academic institutions and/or non-profit organisations).

| File         | Output of the program   |
|--------------|---|
| hcrit1.prg:  | Figure 2, and $h_{crit}(m)$ , $m = 1, 2, 3$<br>reported in Table 1. |
| boot-h1.prg: | p-values in Table 1.  |
| denest.prg:  | densities plotted in Figure 3.                                      |

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