A Market for Intra-day Funds: Does it Have Implications for Monetary Policy?

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Abstract

The United Kingdom is due to move to a system of real-time gross settlement (RTGS) later this year. This in theory could lead to the creation of a market for intra-day funds and an extention of the yield curve back beyond its current shortest maturity of one day. This paper considers the form a market of this sort might take and provides an explicit derivation of the intra-day yield curve. The paper also considers the potential for spillover between the provision of intra-day liquidity to support the RTGS operation and a central bank's ability to implement monetary policy via its control over short-term interest rates.

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1 Introduction

On an average day, payment systems in the United Kingdom process 16 million transactions with a total value of over £160 billion. One system, the Clearing House Automated Payment System (CHAPS) regularly processes daily payments totalling more than £100 billion (Bank of England 1994). Put another way, it takes little over a week for transfers through CHAPS to exceed the entire value of the United Kingdom's annual GDP.

The scale of these flows and the size of the obligations they create between member banks make it essential that payments are based on sound settlement arrangements. To this end, the Association for Payment Clearing Services (APACS) decided in 1992 to adopt real-time gross settlement (RTGS) in the United Kingdom. This is due to be fully operational later this year.⁽¹⁾ This change will mean that transactions across settlement accounts at the Bank of England will be settled continuously during the business day - in "real" time. This is a fundamental change from the present settlement process, whereby interbank obligations are netted and settled at the end of the day.

The decision to change to real-time settlement was based on prudential concerns, in particular, about the size of intra-day credit exposures between settlement banks. But does the move to RTGS also have implications for monetary policy? Banks will require intra-day liquidity to cover the non-synchronous nature of payment flows in and out of their settlement accounts.⁽²⁾ This means that money is likely to have an explicit intra-day value (in the sense of there being a positive intra-day interest rate) for the first time in the United Kingdom. In these circumstances, it is possible that a private market for intra-day funds could develop, with the effect of extending the yield curve of interest rates back beyond its current shortest maturity of one day.

This paper considers the form a market for intra-day funds could take and its implications for the intra-day yield curve. It should be stressed, however,

(1) A detailed discussion of the development towards a RTGS system in the United Kingdom can be found in Bank of England (1994).

(2) Although the non-synchronous payment flows occur in the current net settlement system, the resulting intra-day credit exposures are not priced. The requirement to maintain positive intra-day balances on their settlement accounts at the Bank of England in an RTGS system means intra-day liquidity will need to be sought and provided explicitly.

that institutional arrangements in the United Kingdom make it unlikely that such a market will develop on any scale in the United Kingdom, at least in the short term. The theoretical form of the market is shown to depend critically upon the extent and timing of the Bank's provision of intra-day liquidity. The paper also considers the potential spillover between the Bank's provision of intra-day liquidity and its ability to implement monetary policy via its control over short-term interest rates, concluding that the arrangements for provision of intra-day liquidity do not pose a difficulty for the implementation of monetary policy. The rest of the paper is structured as follows: the next Section considers the general form an intra-day market could take, while Section 3 develops a simple model of such a market; Section 4 outlines the main findings and considers the possible implications for monetary policy; Section 5 concludes.

2 Intra-day market for funds

The movement to real-time settlement means banks will require explicit intra-day liquidity; they can no longer wait for payments to be netted off at the end of the day. But the scale and asynchronicity of payment flows means that it will not be feasible for banks to hold settlement balances sufficient to cover their largest net outflows. Banks are likely to want to finance at least part of their net outflows by borrowing intra-day funds. There are two possible sources for these funds: the central bank and the interbank market.

The Bank of England has agreed to meet this additional demand for liquidity by supplying intra-day liquidity to the CHAPS banks. A settlement bank will be able to obtain daylight funds by selling "eligible" assets to the Bank under same-day sale and repurchase agreements (repos).⁽³⁾ The Bank does not plan to charge for the provision of (fully-collateralised) intra-day funds.

The decision not to charge a positive interest rate on intra-day funds is based on the desire to supply the necessary intra-day liquidity at minimal additional expense and so minimise the cost to the economy of the move to real-time settlement (Bank of England 1992). But how does this decision relate to the

⁽³⁾ The Bank has indicated that it will be prepared to treat a fairly wide range of assets as "eligible" for this purpose, including Treasury bills, eligible bank and local authority bills, and gilts (see Bank of England 1994 for more details). As described in this paragraph, the transactions will take the form of sale and repurchase of assets rather than collateralised borrowing. For simplicity of exposition, however, in the rest of this paper the terminology used relates to borrowing/credit and interest rates/charges. While borrowing and repo differ significantly in legal substance, for the purpose of this analysis they are considered as identical in economic substance.

implementation of monetary policy? Monetary policy in the United Kingdom is implemented via the Bank's control over the interest rate at which it supplies short-term liquidity. The Bank wants to provide intra-day liquidity free of charge - and so minimise the burden of RTGS - but at the same time charge a positive interest rate on the liquidity it provides to the banks as part of its monetary policy.⁽⁴⁾ Can it do both?

Clearly, the Bank can only set the terms of intra-day liquidity independently of the current stance of monetary policy if the market for intra-day liquidity is completely separable from the market for one-day liquidity. In terms of central bank loans, this separability is ensured by the institutional restrictions the Bank plans to impose; the Bank will insist that all intra-day loans are repaid before the end of the business day. "Intra-day" loans are defined by the opening and closing of the business day, rather than by a maximum maturity. It is impossible for a bank to substitute a combination of intra-day loans for a one-day loan; if it wishes to borrow for a maturity which exceeds the business day (ie extends overnight), the bank has no option but to borrow for one day or longer at the (official) interest rate set by the Bank.

But some commentators have questioned whether this segmentation between intra-day and longer maturity loans will still hold if a parallel interbank market for intra-day loans emerges. The intuition underlying these doubts can be seen by thinking about two no-arbitrage conditions that can be expected to hold in the interbank market. The first condition states that there should be a fairly precise relationship between the interest rate at which the central bank is willing to supply loans of different maturities and the corresponding interest rate in the interbank market. These interest rates should only differ to the extent that the collateralisation requirements of the two types of loans differ; Bank of England loans of all maturities are fully-collateralised, whereas interbank loans are typically only partially collateralised, if at all. Thus, in equilibrium, the interbank interest rate of an additional £1 of borrowing should be equal to the corresponding central bank interest rate plus a mark-up reflecting the opportunity cost of the marginal bank holding an additional £1 of its assets in eligible assets.

The second condition is that the interbank yield curve should be arbitraged across its entire maturity spectrum, including intra-day loans. The institutional restrictions preventing arbitrage between intra-day and longer

⁽⁴⁾ For simplicity, the operations the Bank conducts for monetary policy purposes are assumed to be all loans of a maturity of 24 hours.

maturity loans by the central bank do not hold in the private interbank market. Thus, it should be possible, for example, to replicate (and consequently substitute) a 24-hour loan by two consecutive 12-hour loans and so on.

These two arbitrage conditions establish a link, via the interbank market, between the cost of intra-day funds supplied by the central bank and the interest rate on the longer maturity loans it supplies as part of its monetary policy operations. It is this link which has led some commentators to suggest the possibility of spill-over from intra-day liquidity into longer-maturity interest rates.

These concerns raise two basic questions: (i) will an intra-day funds market emerge; and (ii) if yes, what implications will it have for monetary policy? The answer to (i) depends on the cost to the banks of holding the collateral necessary to obtain intra-day funds from the central bank. If holding the necessary eligible assets does not impose a significant (opportunity) cost on the banks, there would be little incentive for an interbank market for daylight funds to emerge; banks will not borrow intra-day funds from other banks at a positive interest rate, if they can borrow funds from the central bank at zero cost. Research within the Bank of England and experience of flows within the existing CHAPS system point strongly to the conclusion that the banks typically already hold a sufficient quantity of eligible assets, suggesting, in practice, an intra-day market on any scale is not likely to emerge from the introduction of RTGS in the United Kingdom. In the United States, however, where institutional arrangements and payment systems developments are very different, many banks frequently incur daylight exposures in excess of their capital value (Humphrey 1989). The remainder of this paper examines the implications an intra-day market may have for monetary policy, if such a market were to emerge.

3 A simple framework

This section develops a simple framework in which it is possible to think about the form an intra-day market for funds could take and its possible implications for monetary policy.

A day is assumed to last for six periods (zero to five). The central banks' discount window is assumed to be open for two periods, from the beginning of period zero to the end of period one. During this time, the central bank is willing to supply two types of loan: intra-day loans and one-day loans. One-day loans are supplied as part of the central bank's monetary policy operations; the loans have a duration of exactly six periods and are charged

the official interest rate r.⁽⁵⁾ In contrast, the central bank is willing to supply intra-day loans at zero cost, for any maturity up to a maximum maturity given by the time remaining before the discount window is closed.

Commercial banks are assumed to have well-determined demand functions for both intra-day and one-day loans. The demand for one-day loans is a derived demand, stemming from the requirement that the clearing banks' *end-of-day* balances at the Bank of England must remain in credit. The banks' demand for intra-day funds is also a derived demand, stemming, this time, from the additional constraint imposed by the introduction of real-time gross settlement; banks' *intra-day* settlement balances must be sufficient to finance their net short position at any point during the day.

We abstract from the specific form these demand functions may take; all that is necessary is that the banking system demands both intra-day and one-day funds.⁽⁶⁾ In the analysis that follows, we assume that the banks' demand for both one-day and intra-day funds are perfectly interest inelastic; banks in each period have to borrow a certain quantity of funds irrespective of the interest rate prevailing at that time. This assumption allows us to abstract totally from bank behaviour and so develop an analysis of the intra-day funds market based solely on a set of restrictions necessary to ensure that no arbitrage opportunities exist in the market.⁽⁷⁾

The banks can borrow funds from either the central bank or from the interbank market. The only difference between the two markets is the degree of collateralisation. Central bank loans are fully collateralised. The opportunity cost to the banks of holding the eligible assets needed to borrow funds from the central bank is assumed to be strictly positive. In particular, the marginal cost to a representative bank of holding an additional £1 of its portfolio in eligible assets is assumed to be X%, where X > 0.⁽⁸⁾ In contrast,

(5) All interest rates and costs are quoted as annualised per period interest rates.

(6) See Poole (1968), Frost (1971), Baltsensperger (1980), Van Hoose (1991) for different formulations of banks' demand for overnight (or in this case, one-day) funds. Angelini (1994) generalises Baltsensperger (1980) to derive a demand function for intra-day loans.

(7) In practice, banks could delay or advance their intra-day needs through payment scheduling. The interest inelasticity assumption means this possibility can effectively be ignored by the model.

(8) This marginal cost is assumed to be constant. In addition, the marginal opportunity cost of holding the eligible assets needed to borrow central bank funds is assumed to be the same for both intra-day and one-day loans. This later assumption does not effect the analysis.

interbank loans are assumed to be less than fully collateralised. For simplicity, we assume here that interbank loans are totally unsecured.⁽⁹⁾

Figure 1 illustrates the main features of this system. At time t = 0, the central bank is willing to supply intra-day loans, at zero cost, for any maturity up to a maximum maturity of two periods - the time remaining before the discount window closes. It is also willing to supply one-day (six period) loans at the official interest rate r.⁽¹⁰⁾ A similar yield curve for official funds exists in the next period (t = 1), although now the central bank is only willing to supply intra-day funds for a maximum maturity of one period. At the instant in time at which the second period is just about to finish and the third period begin (denoted t = 2), the only option is for the bank to borrow one-day money. From this point on (t > 2) until the beginning of the next day, the central bank discount window is closed; the central bank is not willing to supply either intra-day or one-day loans.

What form does the corresponding interbank yield curve take? A simple no-arbitrage argument would indicate that, for those maturities and time periods in which the central bank is willing to supply funds, the interbank curve will lie above the official funds yield curve by an amount equal to the cost of the additional collateral needed to borrow central bank funds (X%). But what determines the interbank yield curve for those combinations of time periods and maturities for which there is not a corresponding central bank supply curve?

The "completeness" of the interbank yield curve depends critically on the times during which the payment system is open for settlement. One possibility is that the opening and closing of the settlement system is determined by the central bank's discount window; banks are only able to settle transactions in periods in which the discount window is open (periods 0 and 1).⁽¹¹⁾ In this case, the interbank yield curve illustrated in Figure 1 is fully specified. The institutional rules which ensure there is a segmentation

(9) An identical solution would be obtained if interbank loans were only partially collateralised or collateralised using assets "freely" held by the banks. The important thing is that banks cannot switch costlessly between the two markets; there is an additional (marginal) cost associated with accessing central bank funds equal to X%.

(10) The effective marginal cost of official one-day funds is thus equal to r + X%. This effective cost is denoted R%.

(11) This would presumably also broadly coincide with the opening and closing of domestic financial markets.

Figure 1: Official funds and interbank yield curves



Official yield curve

•• Effective cost (= interbank yield curve) R = r + X

100

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in the official funds yield curve between intra-day and one-day loans also ensure that the interbank yield curve is segmented. In practice this will be the position in the United Kingdom following the introduction of RTGS.

But it is interesting to consider what would happy if the operating hours of the RTGS system are not restricted to those of the discount window. Developments directed towards addressing Herstatt risk, associated with the settlement of foreign exchange transactions mean that there is increasingly an incentive to extend the operating hours of RTGS systems beyond those of the respective country's domestic financial markets. For example, the Federal Reserve Board recently took action to extend the operating hours of the Fedwire funds transfer to 18 hours per day. Summers (1994) suggests that the principal motive for this extension was to improve the management of Herstatt risk. A similar extension in operating hours has already occurred in Japan (Nambara 1994), while the Working Group on EC Payments Systems expressed a similar intention in their report to the Committee of Governors of the EU Central Banks (1993).

The possible outcome if the operating hours of the RTGS systems are extended beyond those of the central banks' discount windows can most easily be considered by taking the limiting case in which the RTGS system is open for the entire 24-hour period. Although the central bank's discount window is only open for two periods, the banks are free to trade (and settle) with each other in all six periods. In this limiting case, there is nothing to stop the banks arbitraging along the entire length of the intra-day yield curve. A "complete" interbank intra-day yield curve should exist at all times during the day. The question is, what determines the position and slope of the yield curve when there is no corresponding central bank supply?

It is possible to determine an explicit expression for each of the different interbank interest rates by using the methodology of replicating contracts. The cashflows corresponding to each interbank borrowing contract can be replicated via a combination of *borrowing* from the central bank (when the discount window is open) and *on-lending* these funds in the interbank market during the periods in which the funds are not required. The price of these replicating contracts can then be used to derive the implied (no-arbitrage) interbank interest rate.

The form these replicating contracts needs to take is relatively straightforward. Suppose, for example, a bank wishes to borrow funds at t = 2 for three periods. Since the discount window is just about to close, the bank's only option is to borrow central bank funds for six periods. In order to replicate an interbank loan for three periods taken out at t = 2, the bank

can "hold" these funds to finance its borrowing requirements in the first three periods, and then on-lend the (then unwanted) funds in the interbank market at the prevailing interest rate for the remaining three periods before the one-day loan needs to be repaid. A standard no-arbitrage argument implies that the corresponding interbank interest rate should reflect the net cost of these two transactions.

The replicating contracts are encompassed by a generating equation of the form:⁽¹²⁾

$$i_{wt} = 1 / w [\alpha X + 6(r + X) - \beta i \beta_s - \gamma i \gamma_\lambda + \delta i \delta_s]$$
(1)

where $i_{wt} = is$ the "no arbitrage"

s = the period t at which the discount window closes (s = 2) w = [1,6] t = [0,5] X is the marginal cost of collateral R (= r + X) is the effective cost of official one-day loans

(<i>s</i> - <i>t</i>	if s - t > 0			
$\alpha = (0)$	if $s - t \le 0$			
(1	if $w = 1$ and $t = 0$			
(<i>t</i> - <i>s</i>	if $w > \alpha$ and $t - s > 0$			
$\beta = ($				
(0	$\text{if } w \le \alpha \text{ or } t - s \le 0$			
(t + w	if $t + w \le 5$			
$\gamma = (t + w - 6)$	if $t + w > 5$			
(2	if $w = 1$ and $t = 0$			
(t+w)	if $t + w \le 5$			
$\lambda = (t + w - 6)$	if t + w > 5			
(2	if $w = 1$ and $t = 0$			
(w + (t-s) - 6) $\delta = ($	if $w + (t - s) - 6 > 0$			
(0	$\text{if } w + (t - s) - 6 \le 0$			

(12) The "no-arbitrage" interest rates generated by equation (1) are calculated as an arithmetic mean, and so exclude the possibility of compounding. An explicit treatment of compounding would not affect the qualitative analysis.

The general form of the generating equation is fairly intuitive. Each of the five terms in the parenthesis refers to one of the five types of transactions which are needed to replicate the cashflows for the thirty-six different interbank contracts possible in any given day.⁽¹³⁾ The first two terms of (1) denote the "effective" cost of borrowing funds from the central bank; X% per period for intra-day loans and r + X% per period for one-day loans. The parameter α denotes the maximum maturity for which the bank is able to borrow intra-day funds, ie the time remaining before the discount window closes. If the bank wishes to borrow money beyond the end of period one, it must borrow one-day (six period) funds at an effective cost of r + X% per period.

The next two terms capture the extent to which the cost of this one-day loan may be offset by the bank on-lending the funds during the periods in which they are not required; either in the initial β period(s) after borrowing the funds until they are required and/or for the γ period(s) after the funds cease to be required until they need to be repaid. The banks are assumed to be price-takers in the interbank market; they on-lend the funds at the interest rate prevailing in the interbank market at that time.

The final term in the equation denotes the additional cost faced by a bank if, in order to replicate the cashflow from an interbank lending contract, it would also be forced to borrow from the central bank the following day. Suppose, for example, a bank wished to borrow funds for five periods at the beginning of period five. The bank would have to borrow one-day funds from the central bank at the beginning of period two and on-lend these funds on the interbank market until they are required, at the beginning of period five. However, these funds would have to be repaid at the beginning of period two the following day, at which point the bank would have to borrow additional funds for a further two periods. In terms of (1), these five transactions are replicated by the parameter values: $\alpha = 0$, $\beta = 3$, $\gamma = 0$, $\delta = 2$.⁽¹⁴⁾

(13) A bank can borrow for one of six different maturities at six different points in time during the day.

(14) Equation (1) is constructed such that the bank borrows these additional ("next day") funds from the interbank market, rather than from the central bank. This assumption is made purely to simplify the form of equation (1). Given the perfect certainty assumption used to close the model (see below), it is identical to the bank borrowing one-day money from the central bank and on-lending these funds once they are no longer required.

The equation can be used to derive a system of simultaneous equations, which can be solved for the 36 (no-arbitrage) interbank rates possible in any day. These interest rates satisfy two types of no-arbitrage condition. First, by construction, banks will be indifferent between borrowing funds from the interbank market and borrowing funds from the central bank. Second, the interest rates satisfy a pure (zero risk premium) version of the expectations theory of the term structure.⁽¹⁵⁾

However, two further problems need to be addressed before the system can be solved for the implied interbank interest rates. First, the forward-looking nature of the replicating contracts means that an additional condition is required to close the system. The simplest condition is to assume an environment of no change, such that the model parameters are held constant and interbank interest rates follow an identical daily cycle. This implies that interbank interest rates in day s + 1 are equal to the corresponding (defined in terms of both maturity w and period t) interest rate in day s.⁽¹⁶⁾

The second problem is that the system, in its current form, is under-identified. This under-identification arises because the central bank is only willing to lend funds during the period in which the discount window is closed (the "night") for a fixed maturity which spans the entire night. The no-arbitrage conditions ensure that the weighted average of interbank interest rates are such that a bank would be indifferent between borrowing funds from the interbank market and the central bank for the entire night. But there is nothing to tie down the relationship between the interbank interest rates during the night.

This problem could be overcome by introducing a greater degree of economic behaviour into the model. In particular, by explicitly modelling the process determining the banks' demand for intra-day funds. But this would complicate the model considerably. As an alternative solution, we exploited the simplistic nature of the model to derive a further set of conditions which can be used to impose additional restrictions on the system. These symmetry conditions highlight groups of interbank interest rates which are identical in

(15) This can be seen by re-writing the expressions generated by equation (1) to derive the corresponding expectations theory conditions (see the Appendix).

(16) Mathematically, this assumption allows interest rates which are dependent on interest rates prevailing in day s + 1 to be solved by referring back to the corresponding interest rate in day s.

respect of every cost characteristic and so can be restricted to take the same value.⁽¹⁷⁾

4 Results

The model can now be solved to compute the set of 36 "no-arbitrage" interest rates possible in any 24-hour period. Assuming that X = 0.5% and R = 7%, these "no-arbitrage" interest rates take the values:



(17) See the Appendix for details. The inclusion of the symmetry conditions meant that the indifference conditions no longer encompassed the restrictions relating to the expectations theory of the term structure and so needed to be explicitly imposed when solving for the implied interbank interest rates - see the Appendix for details.

Figure 2: Intra-day term structures



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The implied term structures are illustrated in Figure 2.⁽¹⁸⁾ The most striking feature of the term structures is the way in which they continuously move through the course of the 24 hours; both the shape and position of the yield curves adjust continuously. These interest rate dynamics occur despite the complete absence of shocks or uncertainty.

The intuition underlying these dynamics can be illustrated by considering why the interest rate on a loan of a given maturity varies through the course of the day. Consider, for example, the difference between the interest rate on a three-period interbank loan initiated in period t = 2 ($i_{32} = 10.3\%$) to that initiated in period t = 5 ($i_{35} = 3.8\%$).

In order to replicate the cashflows from either of the interbank loans, a bank would need to borrow one-day money from the central bank just before the discount window closes at t = 2. In the case of the loan initiated in period t = 2, these funds are held for the first three periods, before being on-lent in the interbank market. The reverse is true for the interbank loan taken out in period t = 5; the funds from the one-day loan are initially on-lent to the interbank market and are only held by the bank in the final three periods before the loan is repaid. These holding and borrowing patterns are summarised in Figure 3.

Figure 3



If we define "day" and "night" respectively as the periods when the discount window is open and closed, it can be seen that for the loan taken out in period t = 2, the funds are held by the bank for three periods during the night, and lent out for one period during the night and two periods during the day. In contrast, for the loan beginning in period t = 5, the bank holds the funds

⁽¹⁸⁾ Figure 2 also illustrates the effect of reducing the "effective" cost of official one-day funds from 7% to 5%. The non-linearity in the resulting shifts in the yield curves occur because the significance of the reduction depends on the day/night composition of the loan (see below).

for one period during the night and two periods during the day, and lends them out for three periods during the night.

The importance of the day/night composition of a loan stems from the fact that it is cheaper to hold funds during the day than it is during the night. During the day, the shadow cost of borrowing from the interbank market is given by the effective cost of borrowing intra-day funds from the central bank, ie X%. But the cost of borrowing funds during the night is always strictly greater than X%; the central bank charges a positive interest rate on funds borrowed for periods which include the night. The interest rate on the two loans differ due to the differences in their day/night composition. The greater the proportion of a loan used during the night, the more expensive is the loan. Hence $i_{32} > i_{35}$.

This basic intuition can be used to explain the general dynamics observed in the yield curves. The yield curves move continuously through the course of the day and night because one of the key state variables in the system, the residual time before the day finishes (the discount window closes) or starts (the discount window re-opens), is continuously moving. This has the effect of continuously changing the day/night composition of any given loan.

A number of other conclusions can be drawn from the derived term structures. First, the six-period (24-hour) interbank interest rate is always equal to the official interest rate (net of collateral costs). The provision of intra-day liquidity free of charge does not effect the central banks' ability to control the 24-hour rate; there are no "spillover" effects.

This no "spillover" result rests on two features of the model. First, the central bank imposes the institutional restriction that intra-day loans must be repaid before the discount window closes.⁽¹⁹⁾ This restriction ensures that it is impossible to substitute a combination of intra-day loans from the central bank for a one-day loan from the central bank. An important point to note in this context, is that the necessary condition is that the discount window closes (intra-day loans have to be repaid) at some point during the day; the *length* of time for which the discount window is closed is irrelevant.

(19) This condition of repayment before the discount window closes is not as strict as it might appear. In practice, if a bank did not have sufficient funds to repay the loan at this time, the central bank could choose to advance it a 24-hour loan (at the official interest rate) from which the intra-day loan could be repaid.

The second feature ensuring there are no "spillover" effects is that the central bank loans outstanding during the night have a minimum interest charge equal to the interest on a one-day loan. This means banks do not have the ability to borrow overnight funds (at r% per period) for shorter maturities for less than the cost of a one-day loan.⁽²⁰⁾ If this were possible, a bank would never choose to borrow for six periods; it would always prefer to borrow overnight funds for the exact length of its borrowing requirements or at most until the discount window re-opens (a maximum of four periods), after which it would borrow intra-day funds free of charge. Allowing banks to substitute a proportion of the one-day loan with intra-day loans in this way would cause the one-day rate to fall below the official interest rate.

The second feature worth noting is that, due to the day/night composition of the loans, interbank interest rates during the night (ie during the period when the discount window is closed and intra-day loans are not available) tend to be considerably higher than those during the day. This may detract from the effectiveness of the current world trend to extend the opening hours of settlement systems in reducing Herstatt risk. The increased cost of settling during the night means that banks will have an incentive to settle during their domestic banking hours, even if this entails greater risk.

This observation has implications for the optimal length for which the discount window is kept open (and intra-day funds are available free of charge). A central bank can extend the period for which cheap intra-day funds are available, and so reduce the cost to banks of settling outside domestic banking hours, by simply extending the period for which the discount window is kept open. The important point to note in this context is that making intra-day funds available free of charge for twenty-three hours a day (rather than say eight hours a day), does not have any implications for the ability of the central bank to control one-day (24 hour) interest rates. As long as the discount window closes (intra-day loans have to be re-paid) at some point during the day, the central bank will retain perfect control over the one-day interest rate.

(20) This is ensured in the model by the assumption that the central bank is only willing to supply two types of loan: intra-day loans and one-day loans. Obviously, an identical result would occur if the central bank made overnight loans for a maturity less than one day, but the total cost of these loans was equal to that of a one-day loan.

5 Conclusions

Real-time gross settlement is due to be fully operational in the United Kingdom later this year. This will mean that money for the first time in the United Kingdom is likely to have an explicit intra-day value. This in turn raises the possibility that an interbank market for intra-day funds could emerge. This paper considers the form such a market could take and provides an explicit derivation of the intra-day yield curve from a simple theoretical model. The paper also considers the potential for spillover between the provision of intra-day liquidity and a central banks' ability to implement monetary policy via its control over short-term interest rates. Given the particular design features of RTGS in the United Kingdom, an intra-day market is not likely to emerge on any scale in the United Kingdom at least in the short term.

In the context of monetary policy, the paper concludes that, as long as official intra-day loans have to be repaid at some point during the day, the terms on which intra-day liquidity is provided do not affect a central bank's ability to control short-term interest rates. In particular, the planned provision of intra-day liquidity free of charge and in unlimited quantity by the Bank of England in the United Kingdom RTGS system will not impair the Bank's control over short-term interest rates.

The paper also shows that the term structure of intra-day interbank interest rates is highly sensitive to the relationship between the length of time for which the discount window is open (and hence intra-day funds are available) and the opening hours of the RTGS system. This suggests that the length of time for which intra-day funds are available may be important in minimising the cost of eliminating Herstatt risk in an environment in which settlement systems have extended opening hours.

Appendix

Equation (1) generates a series of 36 simultaneous "no-arbitrage" conditions. The complete system of 36 equations is listed below:

(A1) $i_{10} = X$ $(A19) 4 i_{40} = 2 X + 6 R - 4 i_{44}$ (A2) $i_{11} = X$ $(A20) 4 i_{41} = X + 6 R - 3 i_{35}$ (A3) $i_{12} = 6 R - 5 i_{53}$ $(A21) 4 i_{42} = 6 R - 2 i_{20}$ (A4) $i_{13} = 6 R - i_{12} - 4 i_{44}$ $(A22) 4 i_{43} = 6 R - i_{12} - i_{11}$ (A5) $i_{14} = 6 R - 2 i_{22} - 3 i_{35}$ $(A23) 4 i_{44} = 6 R - 2 i_{22}$ $(A24) 4 i_{45} = 6 R - 3 i_{32} + i_{12}$ (A6) $i_{15} = 6 R - 3 i_{32} - 2 i_{20}$ (A7) $i_{20} = X$ $(A25) 5 i_{50} = 2 X + 6 R - 3 i_{35}$ (A8) $2i_{21} = X + 6R - 5i_{53}$ $(A26) 5 i_{51} = X + 6 R - 2 i_{20}$ (A9) $2 i_{22} = 6 R - 4 i_{AA}$ $(A27) 5 i_{52} = 6 R - i_{11}$ $(A10) 2 i_{23} = 6 R - i_{12} - 3 i_{35}$ $(A28) 5 i_{53} = 6 R - i_{12}$ (A11) $2 i_{24} = 6 R - 2 i_{22} - 2 i_{20}$ $(A29) 5 i_{54} = 6 R - 2 i_{22} + i_{12}$ $(A30) 5 i_{55} = 6 R - 3 i_{32} + 2 i_{22}$ (A12) $2i_{25} = 6R - 3i_{32} - i_{11}$ (A13) $3 i_{30} = 2 X + 6 R - 5 i_{53}$ $(A31) 6 i_{60} = 2X + 6R - 2 i_{20}$ (A32) $6 i_{61} = X + 6 R - i_{11}$ $(A14) 3 i_{31} = X + 6 R - 4 i_{44}$ $(A33) 6 i_{62} = 6 R$ (A15) $3 i_{32} = 6 R - 3 i_{35}$ $(A34) 6 i_{63} = 6 R$ (A16) $3 i_{33} = 6 R - i_{12} - 2 i_{20}$ $(A35) 6 i_{64} = 6 R$ (A17) $3 i_{34} = 6 R - 2 i_{22} - i_{11}$ $(A36) 6 i_{65} = 6 R$ $(A18) 3 i_{35} = 6 R - 3 i_{32}$

These "no arbitrage" conditions are consistent with a pure (zero risk premium) version of the expectation theory of the term structure. That is, equations (A1)-(A36) can be re-written to define a set of "term structure" conditions of the form:

(A37)
$$n_{i_{n,m}} = \sum_{\substack{j=0 \\ j=0}}^{(n-1)} i_{1,m+j}$$
 when $m + j \le 5$
$$= \sum_{\substack{j=0 \\ j=0}}^{(n-1)} i_{1,(m+j-6)}$$
 when $m + j > 5$

for n: 16 m: 05

As suggested in Section 4, in order to identify the system an additional set of restrictions is required. The "symmetry" conditions were obtained by grouping sets of interbank loans of equal maturity, which had identical day/night composition. Following the discussion in Section 4, these interest rates can be restricted to equal one another. The full set of symmetry conditions are:

(A38) $i_{10} = i_{11}$ (A39) $i_{12} = i_{13} = i_{14} = i_{15}$ (A40) $i_{22} = i_{23} = i_{24}$ (A41) $i_{32} = i_{33}$ (A42) $i_{54} = i_{55}$ (A43) $i_{63} = i_{64} = i_{65}$

The inclusion of the symmetry conditions means that the term-structure conditions are no longer encompassed by the no-arbitrage conditions, and so have to be explicitly imposed when solving the model.

Substituting the "symmetry conditions" into the "term structure" conditions, and then substituting these into the "no-arbitrage" conditions, we can rewrite and summarise our system of "no-arbitrage" conditions as follows:

(A44) $i_{10} = i_{11} = i_{20} = X$ $i_{12} = i_{13} = i_{14} = i_{15} = i_{22} = i_{23} = i_{24} = i_{32} = i_{33} = i_{42} = 1/4 (6R - 2X)$ (A45) $i_{21} = i_{25} = i_{40} = i_{44} = i_{45} = 1/4 X + 3/4 R$ (A46) $i_{30} = i_{35} = 1/2 (R + X)$ (A47) $i_{31} = i_{34} = i_{60} = i_{61} = i_{62} = i_{63} = i_{64} = i_{65} = R$ (A48) (A49) $i_{41} = i_{43} = 9/8 R - 1/8 X$ (A50) $i_{50} = i_{53} = i_{54} = i_{55} = 1/10 X + 9/10 R$ (A51) $i_{51} = i_{52} = 6/5 R - 1/5 X$

These equations can be solved for values of X and R to derive the 36 "no arbitrage" interbank interest rates possible in any given day.

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