Monetary Policy Uncertainty and Central Bank Accountability

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Abstract

Recently in many countries both political and monetary authorities have shown increasing interest in achieving and maintaining price stability. To that end, a number of institutional reforms have taken place in many developed and developing countries. In general, the reform process followed two distinct routes. The first is where the central bank is made independent by act of law and left largely to its own devices to achieve price stability. The second route is where the government introduces a target for a nominal magnitude, say an inflation or money target, and then makes the central bank accountable for achieving this target.

Both approaches have their proponents and critics. For instance, one well known concern with the creation of an independent central bank is that it might permit unaccountable and unelected officials to elevate their preferences (over the level of inflation and with respect to inflation versus output stabilisation) above society's. To put the point slightly differently, independence without accountability may permit the central banker to behave in an opportunistic manner which will not further society's objectives. This is the focus of our investigation.

The notion of accountability is somewhat difficult to pin down precisely. We approach this issue by noting that at one level accountability is simply a mechanism whereby agents' actions are made apparent to the principal. Within the context of our monetary policy 'game' this simply means that agents' (ie central bank officials') actions are more closely aligned with society's preferences. Or alternatively, penalty mechanisms can be more accurately calibrated in order to induce appropriate actions on the part of the agent. We show that if agents are unsure of how the central bank is going to act (ie there is uncertainty over the central bank's inflation versus output stabilisation preferences) their expectations of inflation are less accurate than they otherwise would be. And in general it is likely that inflation expectations are higher. From this, then, it follows that inflationary expectations may be reduced both by an increase in accountability, and/or an increase in the degree of central bank independence. Following on from this we also show that for a given target level of inflation the optimal degree of central bank accountability is higher, the lower is the degree of central bank independence.

We also show that accountability cannot get rid of all of the inflation bias on its own. This is not surprising since what creates the bias in the first place is the desire for a higher average output level. The key point is that a lack of accountability can react with this bias to create a worse problem than would exist had effective institutions been in place to ensure accountability.

Although it is difficult to test our theory directly, we show that it may well be consistent with real-world institutions. Central banks which have a higher degree of independence also appear to be less accountable.

1 Introduction

Recently, in many countries both political and monetary authorities have shown an increasing interest in the objective of monetary stability and the constitutional and legal position of the central bank. As pointed out by Persson and Tabellini (1993) recent policy reform, as well as historical experience, suggests two different routes to price stability.

The first way is the *legislative* approach; namely to create by law a very independent central bank (CB) with an unequivocal mandate to focus on price stability.(1) Interest in this approach is motivated by the success of the Deutsche Bundesbank in maintaining a low rate of inflation for several decades. Moreover, the statute of the European Central Bank is strongly influenced by the law governing the Bundesbank. And partly reflecting the Maastricht criteria, France and Spain have recently reformed their central bank laws making the Banque de France and the Banco de Espana more independent of government. Furthermore, countries in central and eastern Europe, such as the Czech Republic and Hungary, have increased the legal independence of their central banks. Finally, in Latin America there are also tendencies toward granting more independence to the central banks in countries like Argentina, Chile, Mexico and Venezuela. The academic case for such autonomy has been made by Rogoff (1985), Neumann (1991), Cukierman (1992), Lohmann (1992) and Eijffinger and Schaling (1995a, 1995b).

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(1) Note that, depending on whether the CB's goals are more precisely defined in terms of, say, a numerical target, it has - what Fischer (1994b, page 292) calls less goal independence. In what follows, we classify central banks as more independent if the CB law mentions price stability as the only or main *objective* of policy than central banks with a larger number of objectives in addition to price stability. This leaves the possibility of having differing degrees of goal independence. For instance, at one extreme the Deutsche Bundesbank has full goal independence, whereas the Reserve Bank of New Zealand has a very precisely specified inflation target, and hence no goal independence. Additional dimensions considered in the paper are procedures for appointing the board of the central bank, and the relationships between the CB and the government. Clearly, the latter dimension has major implications for its level of *instrument* independence.

The second way is the *targeting* or *contracting* approach; namely, let the political principal of the central bank impose an explicit, say, inflation target for monetary policy, and make the central bank explicitly accountable for the success in meeting this target. In varying degrees, Australia, Canada, Finland, Israel, New Zealand, Spain, Sweden and the United Kingdom have all recently made some progress along this route. Important theoretical work on this approach has been done initially by Walsh (1995) but also by Persson and Tabellini (1993). A discussion of the merits of this and other mechanisms for achieving price stability can be found in Canzoneri, Nolan and Yates (1996), Haldane (1995), Leiderman and Svensson (1995) and Schaling (1995).

As pointed out by Cukierman (1994, page 1,443), delegation of authority over instruments and/or goals of monetary policy to an independent central bank is sometimes criticised as creating a 'democratic deficit' on the grounds that it entrusts economic policy to technocrats who have not been elected by voters.(2) He states that a related objection to central bank independence is that independence without accountability may induce central bankers to behave in an opportunistic manner that will not lead to the achievement of society's policy objectives in various areas including, in particular, price stability. On this view the central bank is a bureaucratic agent with its own

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(2) According to Goodhart (1994, page 112) it is precisely this democratic deficit argument that has been used in the discussions of central bank independence in the United Kingdom. Moreover, since the credibility problem of monetary policy arises precisely because of the way politicians operate, this argument raises a dilemma. Central bank independence reduces the credibility problem at the cost of placing monetary policy in the hands of unelected officials. According to Cukierman this argument becomes more important in times of particularly large unexpected shocks. He suggests that one way of dealing with it while still reaping the benefits of delegation during tranquil times is to grant independence, but also to introduce escape clauses into the CB law. Within the *legislative approach* the situation in the Netherlands, where the Minister of Finance may give the Nederlandsche Bank a so-called directive in matters of monetary policy is - according to Lohmann (1992) - an example of such a construction. For the *contracting approach* examples are the 1989 Federal Reserve Bank of New Zealand Law, as well as the Roll *et al.* (1993) proposal for reforming the Bank of England charter.

private agenda which is not necessarily identical to that of society.[3] The latter view is succinctly illustrated by Fischer (1994b, page 293):

'An important reason to expose central bankers to elected officials is that, just as the latter may have an inflationary bias, the former may easily develop a deflationary bias. Shielded as they are from public opinion, cocooned within an anti-inflationary temple, central bankers can all too easily deny that cyclical unemployment can be reduced by easing monetary policy.'

However, unlike the well-developed literature on central bank independence very little theoretical work has been done on central bank *accountability*. The only contributions we are aware of are Havrilesky (1995), Briault, Haldane and King (1995) and Al-Nowaihi and Levine (1996).

This paper tries to fill that gap. Building on the Cukierman and Meltzer (1986) model (hereafter CM) we relate central bank accountability to uncertainty about inflation stabilisation preferences. Given the increasing interest in many countries in the objective of monetary stability we suggest that price stability may be reached by an appropriate mixture of accountability and conservativeness. Next, we show that - for a given target level of average inflation over the business cycle - optimal central bank accountability, is higher, the lower the degree of central bank conservativeness. This proposition is examined for 14 industrial countries (Australia, Belgium, Canada, France, Germany, Italy, Japan, The Netherlands, New Zealand, Spain, Sweden, Switzerland, United Kingdom and the United States). We employ the measures of central bank independence and accountability developed, respectively, by Eijffinger and Schaling (1993) and Briault, Haldane and King (1995). In accordance with our theory, we find that these aspects of the monetary regime tend to be negatively correlated.

The paper is organised into four remaining sections followed by two appendices. In Section 2 we present the model. Section 3 contains the

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(3) King (1995, page 12) notes that in responding to an early draft of Fischer's (1990) paper on rules and discretion, Milton Friedman wrote about central bankers: 'From revealed preference, I suspect that by far and away the two most important variables in their loss function are avoiding *accountability* on the one hand and achieve public prestige on the other' [emphasis added].

derivation of the relationship between central bank conservativeness and central bank accountability. In Section 4 we confront the model with some cross-section evidence on inflation expectations, accountability and independence. Our conclusions are given in Section 5. The appendices provide the derivations of the private sector's forecasting rule and convexity and concavity of, what we term, the iso-inflation curve.

2 The Model

As pointed out by Briault *et al* (1995), the Oxford English Dictionary defines accountable as 'obliged to give a reckoning or explanation for one's actions; responsible'. In turn it defines responsible as 'legally or morally obliged to take care of something or to carry out a duty; liable to be blamed for loss or failure'. So the natural context in which to consider accountability is within a principal-agent relationship. And, in a monetary policy context, these roles are typically taken by the government (as principal) and the central bank (as agent).

In what follows, we model central bank accountability as a monetary policy game with uncertainty about the agent's inflation stabilisation preferences. We assume that there are two types of actors, *wage setters* and the *central bank.* Wage setters unilaterally choose the nominal wage every period, and the central bank controls monetary policy. The sequence of events is as follows. In the first stage wage setters sign one period nominal wage contracts [Gray (1976), Fischer (1977b)]. Wage setters know the domestic monetary regime *on average,* but there are random shocks to central bank preferences that cannot be observed at the time wage contracts are signed. However, they know the variance of the shock and take this information into account in forming their expectations. In the third stage stochastic shocks to productivity realise(4). Similarly these shocks cannot be observed at the time contracts are negotiated. As will be shown below the uncertainty associated with the second and third stages of the game is, respectively, of the *multiplicative* [Brainard (1967)] kind and the *additive* kind. In the fourth stage the central bank observes the value of the productivity shock and, given its own preferences, reacts to the productivity

(4) Strictly speaking we do not need the productivity shock to derive the key result in this paper. However, incorporating this supply-side disturbance makes it easier to map our paper into the existing literature on the credibility-flexibility trade-off.

shocks accordingly. In the fifth and final stage output is determined by competitive firms. This timing of events is summarised in Table A.

Table A: The timing of events

1	2	3	4	5
Nominal wage contracts signed	Shocks to CB preferences realise	Productivity shocks realise	CB sets monetary policy	Output determined

2.1 Accountability and uncertainty about inflation stabilisation preferences

Adopting the specification used in King (1995), output is described by a reduced-form Lucas surprise supply function:[(5)]

$$y = y^* + b(\pi - \pi^e) + \varepsilon \qquad \textbf{(2.1)}$$

where y is (the natural log of) output, y^* is the natural rate of output; inflation is denoted by π and nominal wage contracts signed at time t-1 are proxied by the expected inflation rate π^e.[(6)] ε is a white noise shock to productivity with zero mean and variance, σ_ε^2. The principal's (society's) loss function is given by

$$L = \alpha\, \pi^2 + (y - ky^*)^2 \qquad \textbf{(2.1.a)}$$

(5) In the CM analysis the slope coefficient b is normalised to unity for analytical tractability. We allow this parameter to take a wider set of values.
(6) This means that wage setters are forward looking and that there is no nominal rigidity in the model other than that of asymmetric information about productivity shocks.

where $0 < \alpha < \infty$ and α is society's relative weight of inflation stabilisation relative to output stabilisation. We assume that $k > 1$ so that the desired level of output is above the natural level.

The agent's (central bank's) loss function is similar:(7)

$$L = a_t \pi^2 + (y - ky^*)^2 \qquad \textbf{(2.2)}$$

Note that in both loss functions the target rate of inflation is normalised to 0.(8) Equation (**2.2.a**) specifies the stochastic behaviour of the parameter a_t

$$a_t = \alpha - x_t \qquad \text{where} \qquad x \sim N(0, \sigma_x^2).^{(9)} \qquad \textbf{(2.2.a)}$$

Hence, the CB shares society's preferences *on average*. However, at any particular point in time because of the shock x_t, the CB may be overly 'conservative' or advocate too loose a monetary stance (be too 'liberal'). We view σ_x^2 as reflecting the inverse of the level of central bank *accountability*. Clearly, the lower this variance the more *accurate* the *ex ante* specification of the penalty (for non-compliance) associated with a performance incentive contract that forces the agent to behave 'socially or responsibly'.(10) In the limiting case that $\sigma_x^2 \to 0$ (perfect accountability) and therefore $\alpha = a$, it is possible to write an optimal Walsh (1995) type contract between society (the principal) and the central bank (the agent) in which the latter's remuneration -

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(7) Unlike CM, we have output entering quadratically since in our set-up this is tractable.

(8) This means that the Svensson (1996) approach of designing a 'too low' inflation target is not considered here. He shows that the latter can mimic the incentive structure of the optimal [Walsh (1995)] contract. Moreover, under the alternative assumption that the monetary authority's private information concerns its target rate of inflation rather than its inflation stabilisation preferences, noisy announcements about the target might enhance the policy-maker's welfare. For an analysis along these lines see Garfinkel and Oh (1995).

(9) CM introduce persistence and dynamise their problem by letting $a_t = \alpha - e_t$ where $e_t = \rho e_{t-1} + x_t$ and $0 < \rho < 1$. For analytical tractability we set $\rho = 0$.

(10) We formalise this point in Nolan and Schaling (1996).

the central bank's transfer - declines in proportion to inflation.[11] An outcome then obtains with society experiencing zero inflation and optimal countercyclical stabilisation of supply shocks.

Importantly, as long as the mean value of a_t is sufficiently large, we can assume that a_t is positive.

For instance, as in CM, if $\bar{a}$ is equal to $2\sigma_x$, then a positive drawing of a_t will occur around 95% of the time. In each period, therefore, the CB's preferences are subject to a random shock distributed around $\bar{a}$. Finally, following Rogoff (1985) we view this mean value as the degree of central bank conservativeness.[12]

2.2 Time-consistent equilibrium under imperfect accountability

For simplicity, and with no loss of generality, we assume that the control variable of the central bank is inflation.[13] Substituting **(2.1)** into **(2.2)**, the first-order conditions for a minimum indicate

$$\pi = \frac{b^2}{a_t + b^2}\pi^e - \frac{b}{a_t + b^2}(\varepsilon - z) \qquad \textbf{(2.3)}$$

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(11) However, as pointed out by an anonymous referee this requires the summation of an ordinal an a cardinal utility. Therefore, the contract might be more appropriately expressed as a renuneration which declines in proportion to the sum of **(2.1.a)** and inflation.
(12) Note that at this stage whether the agent needs to be *more* conservative than the principal is an open question. We address this issue in Nolan and Schaling (1996).
(13) CM examine the case where the authorities can set the control variable only imperfectly. That is, $\pi_t = \pi^p + \Psi_t$, where $\Psi \sim N(0, \sigma_\Psi^2)$ and superscript p indicates a planned variable. We assume that there are no monetary control errors, that is $\sigma_\Psi^2 = 0$.

where $z = (k-1)y^*$. Taking expectations through yields[14]

$$\pi^e = F(.)\frac{b}{\bar{a}}z \tag{2.4}$$

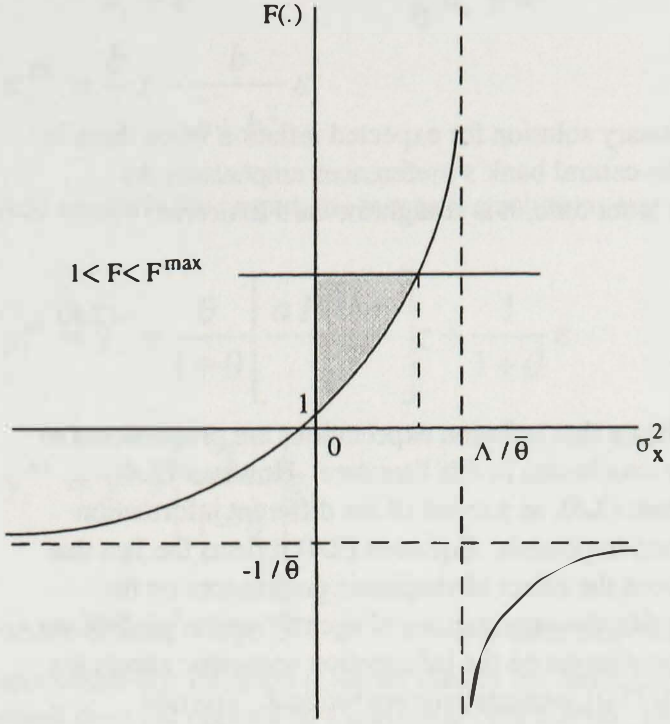
where

$$F(.) = \frac{\bar{a}[(\bar{a}+b^2)^2 + \sigma_x^2]}{\bar{a}(\bar{a}+b^2)^2 - b^2\sigma_x^2} \tag{2.5}$$

This function is depicted in Figure 2.2.

(14) See Appendix A for details on the derivations in this Section.

Figure 2.2 The F(.) function



In order to ensure that $1 < F(.) < F(.)^{\max}$, ie that values of F are bounded between unity and an upper limit $F(.)^{\max}$, where $F^{\max} < \infty$,[(15)] we assume that $0 < \sigma_x^2 < \frac{1}{4}\bar{a}^2 < \frac{\Lambda}{\bar{\theta}}$ where $\bar{\theta} \equiv b^2 / \bar{a}$ and $\Lambda \equiv (\bar{a} + b^2)^2$. Note that the first upper limit on the variance of x_t follows from our requirement of a_t being positive in about 95% of drawings ($\sigma_x < \frac{\bar{a}}{2}$), and that the second

(15) We are grateful to an anonymous referee for drawing our attention to this point.

(higher) limit is the vertical asymptot at $\sigma_x^2 = \frac{\Lambda}{\bar{\theta}}$. Here the first limit is binding.

Solving for the same discretionary solution for expected inflation when there is no uncertainty surrounding the central bank's preferences emphasises the importance of **(2.4)**. For this latter case, it is straightforward to derive

$$\pi^e = \frac{b}{\bar{a}} z \qquad \textbf{(2.6)}$$

Equations **(2.4)** and **(2.6)** indicate that inflation expectations are proportional to the output bias, z, a familiar conclusion in this literature. However **(2.4)** differs in a significant way from **(2.6)**, as a result of the different information sets that the agents are assumed to possess. Equation **(2.4)** reflects the fact that agents have had to 'guess' about the effect of stochastic preferences on the inflation rate. And assuming that the expectations of agents 'are the predictions implied by the model itself, contingent on the information economic agents are assumed to have' [Fischer (1977a)], expectations are rational. The real problem here, as we show in Appendix A, is that this involves taking expectations in the presence of nonlinearities.

From **(2.5)**, it is clear that, if the variance is not too large, $F(.) > 1$. Moreover, in the limiting case that $\sigma_x^2 \to 0$ and therefore $\alpha = a$, then $F(.) = 1$, and **(2.4)** effectively collapses to the discretionary case given by **(2.6)**.

Using the previous results, it is straightforward to write the final solutions for output and inflation, under the case of uncertainty about inflation stabilisation preferences, ie *imperfect accountability* (denoted by superscript 'IA' and compare these to the (discretionary) case of *perfect accountability* ('PA'). Respectively,

$$\pi_t^{IA} = \frac{b}{a_t + b^2}[1 + \bar{\theta} F(.)]z - \frac{b}{a_t + b^2}\varepsilon$$

$$\pi_t^{PA} = \frac{b}{\bar{a}} z - \frac{b}{\bar{a} + b^2}\varepsilon \tag{2.7}$$

And similarly for output, the respective solutions are

$$y_t^{IA} = y^* + \frac{\theta}{1+\theta}\left[\frac{\bar{a} F(.) a_t}{\bar{a}}\right] z + \frac{1}{1+\theta}\varepsilon$$

$$y_t^{PA} = y^* + \left[\frac{1}{1+\bar{\theta}}\right]\varepsilon \tag{2.8}$$

where $\theta \equiv b^2 / a_t$, $\bar{\theta} \equiv b^2 / \bar{a}$. It is clear that with imperfect accountability, inflation is higher than in the discretionary case. Note that this result does not depend on a particular distribution. Hence our result is general and follows from Jensen's inequality.(16)

An important point arising from **(2.7)** is that although preference uncertainty exacerbates the existing inflation bias problem, it cannot generate an inflation bias on its own. This fairly intuitive point is clearly a function of the multiplicative nature of the problem. Since $\bar{a}$ captures societal preferences,

(16) As we show in Appendix A, the result depends on the concavity of $\frac{1}{(a_t + b^2)}$, which implies $E_{t-1}[1/(a_t + b^2)] \geq 1/(\bar{a} + b^2)$ and hence $\pi^e \geq (b / \bar{a})z$. Hence, as pointed out by an anonymous referee the story can also be told with a log-normal or uniform distribution. We leave this for further research.

the CB's actions executed according to a_t will no longer permit optimal stabilisation.(17)

2.3 Accountability and the inflation bias

In the above analysis, we have essentially treated the variance of a_t to be a measure of (the inverse of) accountability. That is, we have assumed that the institutions of accountability will lead the central bank to act more as society would wish, with less 'discretionary' changes to $\bar{a}$ than would otherwise be the case. We also demonstrated that this bias interacts with the familiar output bias to push inflation further from its optimal value. Straightforward comparative statics demonstrate that rising uncertainty, or falling accountability, further hinder the pursuit of price stability. That is:

$$\frac{\partial F(.)}{\partial \sigma_x^2} = \frac{\bar{a}(\bar{a}+b^2)^3}{[\bar{a}(\bar{a}+b^2)^2 - b^2\sigma_x^2]^2} > 0 \qquad \textbf{(2.9)}$$

The interpretation of this expression, which is unambiguously positive, is straightforward and follows on from our remarks above. To the extent that increasing uncertainty threatens risk-averse agents' real wages, they will build in an inflation rate hedge to their nominal contracts. Similarly,

$$\frac{\partial^2 F(.)}{(\partial \sigma_x^2)^2} = \frac{2b^2\bar{a}(\bar{a}+b^2)^3}{[\bar{a}(\bar{a}+b^2)^2 - b^2\sigma_x^2]^3} > 0 \qquad \textbf{(2.10)}$$

as long as $\bar{a}(\bar{a}+b^2)^2 - b^2\sigma_x^2 > 0$, which is exactly the condition we imposed on the variance of the shock to preferences in the context of equation (2.5). Thus, the inflation penalty becomes steeper the further accountability is eroded.

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(17) For an elaboration on this point see Nolan and Schaling (1996).

Using equations (2.9) and (2.10) we obtain Figure 2.2.

From this figure we derive proposition 2.1.

PROPOSITION 2.1: The greater monetary policy uncertainty (the higher σ_x^2), the higher expected inflation.

If we interpret accountability in the narrow sense of providing incentives for the policy-maker to reduce the degree to which they arbitrarily shift their priorities between inflation and output, then the above comparative statics suggest a potentially powerful case for increased accountability.

3 Expected inflation and the accountability-conservativeness trade-off

An obvious question flows from the above: to what degree can central bank accountability substitute for central bank conservativeness in the pursuit of price stability? The above has demonstrated the essential complementarity of the output and accountability biases. This would seem to suggest that to some extent the pursuit of price stability can be conducted independently by increasing accountability, regardless of the mean value of inflation aversion. And this is essentially what the foregoing has indicated. By the same token, the above also indicates that the inflation bias cannot be eradicated by accountability alone. Indeed, even appointing a conservative central banker, with a very high mean value for a_t will not get rid of this bias completely.

Still, the question of over which interval, and to what degree, one can be substituted for the other is of clear policy relevance. An indication of the answer may be derived by drawing what we call the *iso-inflation expectations curve*. This relationship is closely related to the idea of an isoquant (or iso-utility curve) which is found in microeconomic theory. Whereas an isoquant shows the different proportions of inputs that can be combined to keep output fixed, an iso-inflation expectations curve shows the combinations of the degree of conservativeness, and the degree of accountability that delivers a certain constant level of expected inflation.

We formalise this idea by computing the total differential of the reaction function of wage setters. This reaction function, given by equation (2.4), is of the form $\pi^e = \pi^e(\bar{a}, \sigma_x^2)$, hence setting the total differential of expected inflation equal to zero yields

$$\frac{\partial \pi^e}{\partial \bar{a}} d\bar{a} + \frac{\partial \pi^e}{\partial \sigma_x^2} d\sigma_x^2 = 0 \qquad \textbf{(3.1)}$$

which can be rewritten as

$$\frac{d\bar{a}}{d\sigma_x^2} = \frac{-\partial \pi^e / \partial \sigma_x^2}{\partial \pi^e / \partial \bar{a}} \qquad \textbf{(3.2)}$$

Of course, $d\bar{a} / d\sigma_x^2$ is the marginal rate of substitution of σ_x^2 for $\bar{a}$; it is the inverse of their relative marginal effects on inflation expectations.

Taking account of (2.4) it can be shown that

$$\frac{\partial \pi^e}{\partial \bar{a}} = \pi^{eNU} \frac{\partial F(.)}{\partial \bar{a}} + F(.) \frac{\partial \pi^{eNU}}{\partial \bar{a}} < 0 \qquad \textbf{(3.3)}$$

where superscript 'NU' denotes no uncertainty

and

$$\frac{\partial \pi^e}{\partial \sigma_x^2} = \pi^{eNU} \frac{\partial F(.)}{\partial \sigma_x^2} > 0 \qquad \textbf{(3.4)}$$

where these expressions reflect the partial effects of conservativeness and accountability on expected inflation. It follows straightforwardly that

$$\frac{d\bar{a}}{d\sigma_x^2} = \frac{-\pi^{eNU}(\partial F / \partial \sigma_x^2)}{\pi^{eNU}(\partial F / \partial \bar{a}) + F(.)(\partial \pi^{eNU} / \partial \bar{a})} > 0 \qquad (3.5)$$

This means that the MRS is positive, and that the iso (expected) inflation curve will be upward sloping. Therefore, conservativeness and *accountability* (being the inverse of σ_x^2) are inversely related. The above discussion can be summarised in

PROPOSITION 3.1: For a given level of expected inflation, the higher central bank accountability (the lower σ_x^2), the lower the degree of central bank conservativeness.

The curvature of the iso-inflation curve depends on the first derivative of the MRS, ie how the amount of uncertainty needed to replace a unit quantity of $\bar{a}$, changes as σ_x^2 increases, π^e being constant. It turns out that the relevant derivative is [18]

$$\frac{d^2\bar{a}}{d(\sigma_x^2)^2} = \frac{1}{(\partial\pi^e/\partial\bar{a})^3}\left[\frac{\partial^2\pi^e}{\partial(\sigma_x^2)^2}\left(\frac{\partial\pi^e}{\partial\bar{a}}\right)^2 - 2\frac{\partial^2\pi^e}{\partial\bar{a}\partial\sigma_x^2}\frac{\partial\pi^e}{\partial\bar{a}}\frac{\partial\pi^e}{\partial\sigma_x^2} + \frac{\partial^2\pi^e}{\partial\bar{a}^2}\left(\frac{\partial\pi^e}{\partial\sigma_x^2}\right)^2\right] < 0 \qquad (3.6)$$

if

$$\frac{\partial^2\pi^e}{\partial\bar{a}\,\partial\sigma_x^2} < K \qquad (3.7)$$

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(18) This is derived in Appendix B.1.

where

$$K = -\frac{1}{2}\left[\frac{\partial^2 \pi^e}{\partial \bar{a}^2}\frac{d\bar{a}}{d\sigma_x^2} + \frac{\partial^2 \pi^e}{\partial (\sigma_x^2)^2}\frac{d\sigma_x^2}{d\bar{a}}\right]$$

From equations **(2.9)** and **(3.4)** it follows that

$$\frac{\partial^2 \pi^e}{\partial (\sigma_x^2)^2} = \pi^{eNU}\frac{\partial^2 F(.)}{\partial (\sigma_x^2)^2} > 0 \qquad \textbf{(3.8)}$$

Moreover, in Appendix B.2.1 we show that $\frac{\partial^2 \pi^e}{\partial \bar{a}^2} > 0$ and $\frac{\partial^2 \pi^e}{\partial \bar{a}\,\partial \sigma_x^2} < 0$.

This implies that $K < 0$, and for the iso-inflation curve to be concave in the $(\bar{a}, \sigma_x^2)$ space **(3.7)** boils down to the requirement that the cross partial be sufficiently negative.

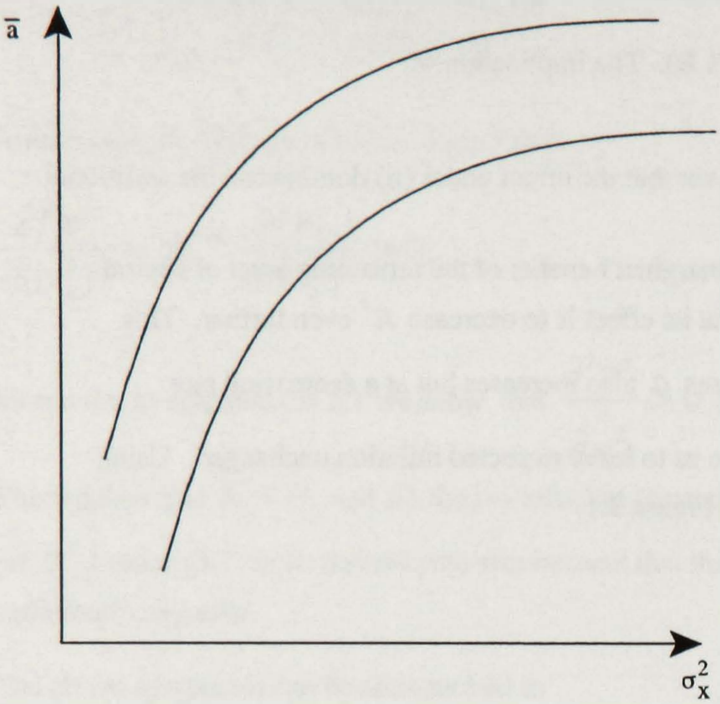
The above discussion can be summarised in

PROPOSITION 3.2: If increased uncertainty sufficiently increases the benefits of central bank conservativenes ($\frac{\partial^2 \pi^e}{\partial \bar{a}\,\partial \sigma_x^2} < K$) the iso-inflation curves are concave to the origin.

The economic intuition behind this proposition is broadly as follows. We know that $\frac{\partial \pi^e}{\partial \bar{a}} < 0$, so that the effect of decreasing accountability (increasing σ_x^2) at the price of being more conservative is two-fold: (i) expected inflation rises in response to the additional uncertainty - ie $\frac{\partial \pi^e}{\partial \sigma_x^2} > 0$ [see equation **(3.4)**];

but (ii) expected inflation falls because the additional σ_x^2 is associated with an increase in conservativeness which in itself reduces inflation expectations ($\frac{\partial \pi^e}{\partial \bar{a}} < 0$, see appendix B). The implication of $\frac{\partial \pi^e}{\partial \bar{a} \partial \sigma_x^2} < K$ is, however that the effect under (ii) dominates: the additional uncertainty so raises the marginal benefits of the remaining level of central bank conservativeness that its effect is to decrease π^e even further. This means that as σ_x^2 increases $\bar{a}$ also increases but at a decreasing rate. ($d^2 \bar{a} / d(\sigma_x^2)^2 < 0$) so as to leave expected inflation unchanged. Using (3.5) and (3.6) we obtain Figure 3.1.

Figure 3.1 Iso-inflation curves



The above figure shows iso-inflation expectations curves which are concave to the origin. The level of $\bar{a}$ is on the vertical axis, and σ_x^2 is on the vertical axis. Clearly, as uncertainty about inflation stabilisation preferences rises, there is required an offsetting increase in the degree of conservativeness. However, the extent to which further increases in conservativeness are required to hold inflation expectations constant, falls as accountability declines. This has important implications which we have alluded to above. Increasing the accountability of monetary policy can only achieve a certain amount *vis a vis* price stability. We interpret the iso-inflation curves to indicate that both types of institutional reform (emphasising that the CB focuses increasingly on the objective of price stability, and increasing the degree of accountability of the central bank) are important. We now move on to confront the model with some cross-country evidence.

4 Some cross-section evidence on inflation expectations, accountability and independence

Finally we turn to some empirical evidence on the relationship between central bank independence and accountability. To do this we use an index of accountability for 14 developed countries developed by Briault *et al* (1995). The remainder of this section draws heavily on their work.

This index is based on four criteria: (a) whether the central bank is subject to external monitoring by parliament (as for example in France, the United States and the United Kingdom); (b) whether the minutes of meetings to decide the setting of monetary policy are published (as in the United Sates and the United Kingdom); (c) whether the central bank publishes an inflation or monetary policy report of some kind, in addition to standard central bank bulletins; and (d) whether there is a clause that allows the central bank to be overriden in the event of certain shocks. If the central bank law mentions an explicit escape clause - for example New Zealand - a country receives a numerical value of 2.0. If overriding the central bank is not *a priori* excluded they assigned a value of 1.0. Finally, if no provision exists they assigned 0. The other characteristics are simply given zero/unity values, and added to a base level of one. These are obviously simple proxies. But they cover most of the main features of accountability, as defined earlier.

Following well-known cross-sectional work on central bank independence [Alesina and Summers (1993)] in Chart 1 we plot central bank independence against the accountability index. We use Eijffinger and Schaling's (ES) (1993, 1995c) measures of central bank independence.(19) The ES index determines the degree of policy independence using three criteria:

(a) Is the bank the sole final policy authority (2.0), is this authority not entrusted to the CB alone (1.0), or is it entrusted completely to the government (0)?

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(19) The independence measures for New Zealand and Spain are from Eijffinger and Van Keulen (1994), who extend Eijffinger and Schaling's (1993) twelve country sample with another eleven countries.

(b) Is there no government official (with or without voting power) on the bank board (1.0)?

(c) Are more than half of the board appointments made independent of the government (1.0)?

In quantifying central bank independence, feature (a), the final authority in policy making, is assessed in conjuction with the policy goals of monetary policy. That is, the extent to which a central bank is regarded to be the sole policy authority, also depends on the presence of statutory requirements concerning monetary stability.[20] Hence, the ES index of policy independence picks up the strength of the 'conservative bias' of the monetary regime as embodied in the CB law.[21]

Chart 1
Central bank independence and accountability

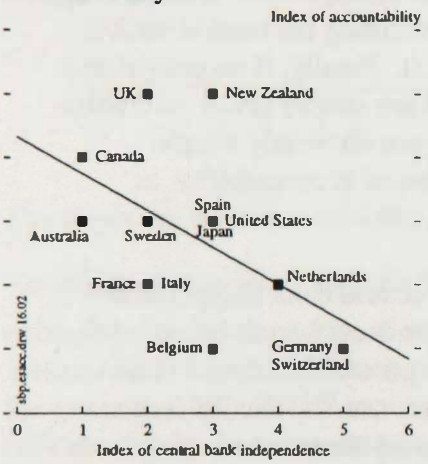
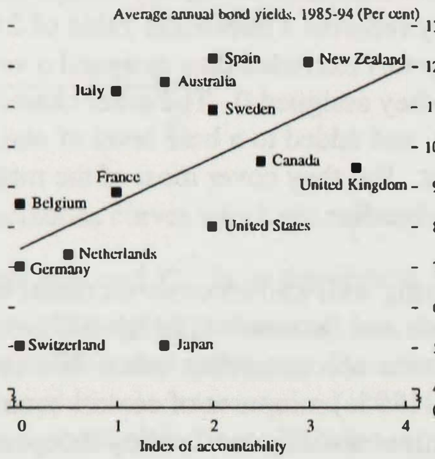


Chart 2
Accountability and average bond yields



The correlation is clearly negative. As pointed out by Briault *et al* (1995), Chart 1 is inconsistent with a purely democratic or political explanation of

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(20) For more details see Eijffinger and Schaling (1993).
(21) For a detailed analysis on the distinction between central bank *independence* and *conservativeness*, see Eijffinger and Hoeberichts (1996).

accountability, which would assert that independence and accountability should run in paralell.[22] Instead, in accordance with proposition 3.1, it suggests that accountability and transparency may have served as partial substitutes for independence in some of these countries rather than as complements.[23]

Chart 2 plots the accountability index against the average level of bond yields over the past decade - a crude proxy for inflation expectations[24] - for our 14 countries. The correlation is clearly positive. This suggests that several countries - being constrained on the central bank independence dimension - have responded by trying to to reduce inflation expectations by increasing the accountability and transparency of monetary policy. Note that this is exactly what proposition 2.1 suggests. Indeed, Chart 2 could perhaps be characterised as two main clumps: low inflation expectations/low accountability in the bottom left-hand corner; and high inflation expectations/high accountability in the top right. It is particularly striking to note how many inflation target countries lie in the second of these.

5 Conclusions

William Brainard (1967) has indicated that multiplicative, or multiplier uncertainty has important implications for both the *conduct* of decision making (by both principal and agent), and the *efficiency* of policy. And our analysis concurs with this conclusion. We deduced such multiplicative uncertainty from the degree to which the central bank shifts priorities between output and

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(22) For instance, Eijfinger and de Haan (1996, page 57) do not believe in a long run trade-off between independence and accountability. Defining accountability according to the Lohmann (1992) model, their argument is that a central bank that is continuously conducting a policy which lacks broad political support, will sooner or later be overridden.

(23) Care has to be taken in interpreting Chart 3. Strictly speaking, our approach refers to optimal institutional design (conditioned on a given expected inflation rate). The plot on the other hand, shows the relationship between actual independence and accountability. Of course, in reality optimal and actual regimes may differ. Assessing the optimality of real-world institutions is complex and requires sophisticated econometric techniques see, for example, Eijffinger and Schaling (1995a, b) who employ a latent variables method (LISREL) to conduct such an analysis.

(24) For a more sophisticated presentation of the Bank's method for deriving inflation expectations from bond prices see Breedon (1995).

inflation goals. Eradicating such uncertainty may provide a powerful rationale for procedures requiring the central bank to be accountable for inflation outturns. Our analysis allowed us to come to two key conclusions.

First, accountability makes economic sense. That is, a less accountable policy-maker may well impose his or her preferences above those of society at any particular point in time. As we showed above, when this occurs an additional upward inflation bias is imparted into the system. Second, although accountability is a valuable, and perhaps key, institutional requirement in the pursuit of stable prices, it cannot, by itself, eradicate the inflation bias in the standard discretionary outcome. In short, if there exists an output bias, it may be a necessary but not sufficient condition for price stability.

Appendix A: the derivation of equation (2.4)

From the main text, substituting (2.1) into (2.2) and taking the first order conditions for a minimum, we get

$$\pi = \frac{b}{a_t + b^2}(b\pi^e + z - \varepsilon) \tag{2.3}$$

Taking expectations across this expression:

$$E(\pi) = (bz + b^2\pi^e)E\left(\frac{1}{a_t + b^2}\right) - bE\left(\frac{\varepsilon}{a_t + b^2}\right) \tag{A.1}$$

This expression requires us to take the expected value of ratios of random variables in one case, and as a special case, when the numerator is one. This can be achieved through a Taylor-series expansion. We have to make some assumptions about the rational function of which the most important are:

(i) around the point of expansion, the function is differentiable to the required order, and that

(ii) the remainder converges to zero as the order of expansion increases, that is $R_n(x) \to 0$ as $n \to \infty$

(ii) is especially crucial since $R_n(x)$ may tend to zero for some values of x and not for others. In general then, the expansion is of the form

$$\phi(x) = \frac{\phi(x_0)}{0!} + \frac{\phi'(x_0)}{1!}(x - x_0) + \frac{\phi''(x_0)}{2!}(x - x_0)^2 + \ldots + R_n$$

where x_0 is the point of expansion. For the above problem the expansion takes place around the respective means of the two random variables

(μ_x, μ_y). Remembering to take account of the cross partial effects, we can re-write the last expression as

$$\phi(x-\mu_x, y-\mu_y) = \frac{\phi(\mu_x,\mu_y)}{0!} + \frac{1}{1!}(x\frac{\partial\phi}{\partial x} + y\frac{\partial\phi}{\partial y}) + \frac{1}{2!}(x^2\frac{\partial^2\phi}{\partial x^2} + 2xy\frac{\partial^2\phi}{\partial x\partial y} + y^2\frac{\partial^2\phi}{\partial y^2}) + \ldots + R_n$$

provided $R_n(x) \to 0$. And provided *x* and *y* are sufficiently small, we can take the left-hand side to be approximated by a quadratic (or cubic) expression.

Our problem is to expand $\phi(z) = x / y$ about the respective means. Assuming conditions (i) and (ii) hold and assuming that the first two moments of *E(x/y)* exist then we can write down an expression for such an expected value. Calculating the relevant partial and cross partial derivatives, entering them into the above expression and assuming the following orthogonality condition holds

$$\mu_y x = \mu_x y = 0$$

then we can, taking expectations through, write

$$E\left(\frac{X}{Y}\right) \approx \frac{\mu_X}{\mu_Y} - \frac{1}{\mu_Y^{\,2}}\mathrm{cov}[X,Y] + \frac{\mu_X}{\mu_Y^{\,3}}\mathrm{var}[Y] \qquad \textbf{(A.2)}$$

This is the (second-order) approximation used in the paper, therefore we can write

$$E\left(\frac{1}{a_t + b^2}\right) \approx \frac{(\bar{a} + b^2)^2 + \sigma_x^2}{(\bar{a} + b^2)^3}. \qquad \textbf{(A.3)}$$

Using this same procedure we now find

$$bE\left(\frac{\varepsilon}{a_t + b^2}\right) \approx 0 \tag{A.4}$$

So substituting **(A.3)** and **(A.4)** in **(A.1)** gives

$$\pi^e = (bz + b^2\pi^e)\frac{[(\bar{a}+b^2)^2 + \sigma_x^2]}{(\bar{a}+b^2)^3} \tag{A.5}$$

And rearranging gives

$$\pi^e = zb\left(\frac{(\bar{a}+b^2)^2 + \sigma_x^2}{(\bar{a}+b^2)^3 - b^2[(\bar{a}+b^2)^2 + \sigma_x^2]}\right) \tag{A.6}$$

As $\sigma_x^2 \rightarrow 0$, this expression reduces to

$$\pi^e = z\frac{b}{\bar{a}} \tag{2.6}$$

For the case of the Walsh (1995) model. This becomes clearer if we re-write **(A6)**, by simplifying the denominator, as:

$$\pi^e = \frac{\bar{a}[(\bar{a}+b^2)^2 + \sigma_x^2]}{\bar{a}(\bar{a}+b^2)^2 - b^2\sigma_x^2}.\frac{b}{\bar{a}}z \tag{A.7}$$

which is expression **(2.4)** in the text. Again, it is clear that as $\sigma_x^2 \rightarrow 0$ the first part of the expression on the right hand side collapses to unity, and **(A7)** is equivalent to **(2.6)**.

Appendix B: Convexity and concavity in the iso inflation curve

B.1 The evolution of the marginal rate of substitution

The shape of the iso inflation curve (ie how the amount of accountability needed to replace a unit quantity of conservativeness changes as the former changes, expected inflation being constant) depends on the value of the first derivative of the MRS, ie on

$$\frac{d^2 \bar{a}}{d(\sigma_x^2)^2} = \frac{d}{d\sigma_x^2}\left(-\frac{\partial\pi^e / \partial\sigma_x^2}{\partial\pi^e / \partial\bar{a}}\right) \tag{B.1}$$

Bearing in mind that, from (3.5) $\frac{\partial\pi^e}{\partial\bar{a}}$ and $\frac{\partial\pi^e}{\partial\sigma_x^2}$ are functionally related to each other (a fact which we will use later on) the differential at **(B.1)** may be written

$$\frac{d^2 \bar{a}}{d(\sigma_x^2)^2} = -\frac{1}{[\partial\pi^e / \partial\bar{a})^2}\left[\frac{\partial\pi^e}{\partial\bar{a}}\frac{d}{d\sigma_x^2}\left(\frac{\partial\pi^e}{\partial\sigma_x^2}\right) - \frac{\partial\pi^e}{\partial\sigma_x^2}\frac{d}{d\sigma_x^2}\left(\frac{\partial\pi^e}{\partial\bar{a}}\right)\right] \tag{B.2}$$

But

$$\frac{d}{d\sigma_x^2}\left(\frac{\partial\pi^e}{\partial\sigma_x^2}\right) = \frac{\partial^2\pi^e}{\partial(\sigma_x^2)^2} + \frac{\partial^2\pi^e}{\partial\bar{a}\partial\sigma_x^2}\frac{d\bar{a}}{d\sigma_x^2} \tag{B.3}$$

and

$$\frac{d}{d\sigma_x^2}\left(\frac{\partial\pi^e}{\partial\bar{a}}\right) = \frac{\partial^2\pi^e}{\partial\bar{a}\partial\sigma_x^2} + \frac{\partial^2\pi^e}{\partial\bar{a}^2}\frac{d\bar{a}}{d\sigma_x^2} \tag{B.4}$$

Substituting **(3.5)** into **(B.3)** and **(B.4)** gives

$$\frac{d}{d\sigma_x^2}\left(\frac{\partial \pi^e}{\partial \sigma_x^2}\right) = \frac{\partial^2 \pi^e}{\partial (\sigma_x^2)^2} - \frac{\partial^2 \pi^e}{\partial \bar{a}\, \partial \sigma_x^2} \frac{\partial \pi^e / \partial \sigma_x^2}{\partial \pi^e / \partial \bar{a}} \tag{B.5}$$

and

$$\frac{d}{d\sigma_x^2}\left(\frac{\partial \pi^e}{\partial \bar{a}}\right) = \frac{\partial^2 \pi^e}{\partial \bar{a}\, \partial \sigma_x^2} - \frac{\partial^2 \pi^e}{\partial \bar{a}^2} \frac{\partial \pi^e / \partial \sigma_x^2}{\partial \pi^e / \partial \bar{a}} \tag{B.6}$$

Substituting **(B.5)** and **(B.6)** into **(B.2)** yields

$$\frac{d^2 \bar{a}}{d(\sigma_x^2)^2} = -\frac{1}{(\partial \pi^e / \partial \bar{a})^2}\left[\frac{\partial \pi^e}{\partial \bar{a}}\left(\frac{\partial^2 \pi^e}{\partial (\sigma_x^2)^2} - \frac{\partial^2 \pi^e}{\partial \bar{a}\, \partial \sigma_x^2} \frac{\partial \pi^e / \partial \sigma_x^2}{\partial \pi^e / \partial \bar{a}}\right) - \frac{\partial \pi^e}{\partial \sigma_x^2}\left(\frac{\partial^2 \pi^e}{\partial \bar{a}\, \partial \sigma_x^2} - \frac{\partial^2 \pi^e}{\partial \bar{a}^2} \frac{\partial \pi^e / \partial \sigma_x^2}{\partial \pi^e / \partial \bar{a}}\right)\right] \tag{B.7}$$

$$= \frac{-1}{(\partial \pi^e / \partial \bar{a})^2}\left[\frac{\partial \pi^e}{\partial \bar{a}} \frac{\partial^2 \pi^e}{\partial (\sigma_x^2)^2} - \frac{\partial^2 \pi^e}{\partial \bar{a}\, \partial \sigma_x^2} \frac{\partial \pi^e}{\partial \sigma_x^2} - \frac{\partial \pi^e}{\partial \sigma_x^2} \frac{\partial^2 \pi^e}{\partial \bar{a}\, \partial \sigma_x^2} + \frac{\partial^2 \pi^e}{\partial \bar{a}^2}\left[\frac{\partial \pi^e}{\partial \sigma_x^2}\right]^2 \frac{1}{\partial \pi^e / \partial \bar{a}}\right]$$

$$= -\frac{1}{(\partial \pi^e / \partial \bar{a})^3}\left[\frac{\partial^2 \pi^e}{\partial (\sigma_x^2)^2}\left(\frac{\partial \pi^e}{\partial \bar{a}}\right)^2 - 2\frac{\partial^2 \pi^e}{\partial \bar{a}\, \partial \sigma_x^2} \frac{\partial \pi^e}{\partial \bar{a}} \frac{\partial \pi^e}{\partial \sigma_x^2} + \frac{\partial^2 \pi^e}{\partial \bar{a}^2}\left(\frac{\partial \pi^e}{\partial \sigma_x^2}\right)^2\right]$$

which is equation **(3.6)** in the text.

B.2.1 Cross and second partial derivatives of expected inflation

In this part of the Appendix we prove that $\dfrac{\partial^2 \pi^e}{\partial \bar{a} \partial \sigma_x^2} < 0$ and $\dfrac{\partial^2 \pi^e}{\partial \bar{a}^2} > 0$.

As regards the cross partial, by taking the first derivative of (3.3) with respect to σ_x^2 we get

$$\frac{\partial^2 \pi^e}{\partial \bar{a} \partial \sigma_x^2} = \pi^{eNU} \frac{\partial^2 F(.)}{\partial \bar{a} \partial \sigma_x^2} + \frac{\partial \pi^{eNU}}{\partial \bar{a}} \frac{\partial F(.)}{\partial \sigma_x^2} \tag{B.8}$$

All signs in this equation are known, apart from the second term on the right-hand side. Hence, the sign pattern is (+)*(?) + (-)*(+). Therefore, pinning down the sign of the cross partial involves an examination of

$$\frac{\partial^2 F(.)}{\partial \bar{a} \partial \sigma_x^2} = \frac{\partial [\partial F(.) / \partial \bar{a}]}{\partial \sigma_x^2} \tag{B.9}$$

Next, with respect to our second task (which was to prove that $\dfrac{\partial^2 \pi^e}{\partial \bar{a}^2} > 0$) again we focus on (3.3). Differentiating the 'marginal benefits' of central bank conservativeness with respect to $\bar{a}$ we get

$$\frac{\partial^2 \pi^e}{\partial \bar{a}^2} = \pi^{eNU} \frac{\partial^2 F(.)}{\partial \bar{a}^2} + 2 \frac{\partial F(.)}{\partial \bar{a}} \frac{\partial \pi^{eNU}}{\partial \bar{a}} + F(.) \frac{\partial^2 \pi^{eNU}}{\partial \bar{a}^2} \tag{B.10}$$

Here the sign pattern is (+)*(?) + 2*(?)*(-) + (+)*(?). Hence, we need to know the signs of the first and second derivatives of *F*(.) with respect to $\bar{a}$.

B.2.2 First and second derivatives of $F(.)$

In summing up, so far it can be clearly seen that we need to take a closer look at (i) $\partial F(.) / \partial \bar{a}$ and (ii) the derivatives of (i) with respect to σ_x^2 and $\bar{a}$. The latter will be derived using implicit differentiation of equation (2.5).

First we write (2.5) in the equivalent form

$$F - \frac{\bar{a}[(\bar{a}+b^2)+\sigma_x^2]}{\bar{a}(\bar{a}+b^2)^2 - b^2\sigma_x^2} = 0 \qquad \textbf{(B.11)}$$

Now we no longer have an explicit function F of σ_x^2. Rather, the function (2.5) is only implicitly defined by equation (**B.11**). Using the parameter abbreviations $\Lambda \equiv (\bar{a}+b^2)^2$ and $\bar{\theta} \equiv \frac{b^2}{\bar{a}}$, (**B.11**) collapses to

$$F - [\frac{\Lambda + \sigma_x^2}{\Lambda - \bar{\theta}\sigma_x^2}] = 0 \qquad \textbf{(B.12)}$$

In general (**B12**) can be denoted by

$$G(F, \Lambda, \sigma_x^2, \bar{\theta}) = 0 \qquad \textbf{(B.13)}$$

Totally differentiating (**B.13**) yields

$$G_f dF + G_\Lambda d\Lambda + G_{\sigma_x^2} d\sigma_x^2 + G_{\bar{\theta}} d\bar{\theta} = 0 \qquad \textbf{(B.14)}$$

where G_i denotes the partial derivative of the function G with respect to argument i. **(B.14)** is an important equation which will be used to determine the sign of $\partial F(.) / \partial \bar{a}$.

Upon dividing through by $d\bar{a}$, noting that $G_f = 1$ and solving for $\frac{dF}{d\bar{a}} = \frac{\partial F}{\partial \bar{a}}$. we get

$$\frac{\partial F}{\partial \bar{a}} = -\frac{d\Lambda}{d\bar{a}} G_\Lambda - \frac{d\sigma_x^2}{d\bar{a}} G_{\sigma_x^2} - \frac{d\bar{\theta}}{d\bar{a}} G_{\bar{\theta}} \tag{B.15}$$

From the definitions of Λ and $\bar{\theta}$ it follows that $\frac{d\Lambda}{d\bar{a}} = 2(\bar{a} + b^2) = 2\sqrt{\Lambda} > 0$ and $\frac{d\bar{\theta}}{d\bar{a}} - = \frac{-\bar{\theta}}{\bar{a}} = -g'(\bar{\theta}) < 0$ whereas of course $\frac{d\sigma_x^2}{d\bar{a}} = 0$. Substituting the latter results in **(B.15)** yields

$$\frac{\partial F}{\partial \bar{a}} = -2\sqrt{\Lambda} G_\Lambda + g'(\theta) G_{\bar{\theta}} \tag{B.16}$$

Now, pinning down the sign of the first derivative of F with respect to $\bar{a}$ boils down to determining the signs of the partial derivatives of G with respect to the arguments Λ and $\bar{\theta}$.

From **(B.12)** it follows that

$$G_{\Lambda} = \frac{(1+\bar{\theta})\sigma_x^2}{[\Lambda - \bar{\theta}\sigma_x^2]^2} > 0 \tag{B.17}$$

and

$$G_{\bar{\theta}} = \frac{-\sigma_x^2[\Lambda + \sigma_x^2]}{[\Lambda - \bar{\theta}\sigma_x^2]^2} < 0 \tag{B.18}$$

Substituting (**B.17**) and (**B.18**) into (**B.16**) and rearanging results in

$$\frac{\partial F}{\partial \bar{a}} = \frac{-\sigma_x^2\{2\sqrt{\Lambda}(1+\bar{\theta}) + g'[\Lambda + \sigma_x^2]\}}{[\Lambda - \bar{\theta}\sigma_x^2]^2} < 0 \tag{B.19}$$

Hence, the first part of our investigation has been succesful; we know that the first derivative of F with respect to $\bar{a}$ is negative.

In order to derive the first derivative of (**B.19**) with respect to σ_x^2 and $\bar{a}$ - which gives us the needed cross and second partials - we apply the implicit function theorem again. We write (**B.19**) in the equivalent form

$$\frac{\partial F}{\partial \bar{a}} + \frac{\sigma_x^2\{2\sqrt{\Lambda}(1+\bar{\theta}) + g'[\Lambda + \sigma_x^2]\}}{[\Lambda - \bar{\theta}\sigma_x^2]^2} = 0 \tag{B.20}$$

which defines the first derivative of F with respect to $\bar{a}$ implicitly. As with (**B.12**), in general (**B.20**) can be denoted by

$$\mathrm{H}(\mathrm{f}', \sigma_x^2, \Lambda, \theta, \mathrm{g}') = 0 \tag{B.21}$$

Totally differentiating (**B.21**) yields

$$H_{f'}\,df' + H_{\Lambda}\,d\Lambda + H_{\sigma_x^2}\,d\sigma_x^2 + H_{\bar{\theta}}\,d\bar{\theta} + H_{g'}\,dg' = 0 \qquad \textbf{(B.22)}$$

(B.22) is another important equation. It will be used to determine the signs of $\partial^2 F / \partial \bar{a} \partial \sigma_x^2$ and $\partial^2 F / \partial \bar{a}^2$ which are needed to sign **(B.8)** and **(B.9)**, which in turn have implications for the convexity/concavity of the iso-inflation curve.

First, we turn to the more simple case of the cross partial. Dividing through by $d\sigma_x^2$, noting that $H_{f'} = 1$ and solving for $\dfrac{df'}{d\sigma_x^2} = \dfrac{\partial^2 F}{\partial \bar{a} \partial \sigma_x^2}$ we get

$$\frac{\partial^2 F}{\partial \bar{a} \partial \sigma_x^2} = -H_{\Lambda}\frac{d\Lambda}{d\sigma_x^2} - H_{\sigma_x^2} - H_{\bar{\theta}}\frac{d\bar{\theta}}{d\sigma_x^2} - H_{g'}\frac{dg'}{d\sigma_x^2} \qquad \textbf{(B.23)}$$

where $\dfrac{d\Lambda}{d\sigma_x^2} = \dfrac{d\bar{\theta}}{d\sigma_x^2} = \dfrac{dg'}{d\sigma_x^2} = 0$,

therefore

$$\frac{\partial^2 F}{\partial \bar{a} \partial \sigma_x^2} = -H_{\sigma_x^2} \qquad \textbf{(B.24)}$$

From **(B.20)** it follows that

$$H_{\sigma_x^2} = \frac{\{2\sqrt{\Lambda}(1+\bar{\theta}) + g'[\Lambda + 2\sigma_x^2]\}[\Lambda - \bar{\theta}\sigma_x^2] + 2\bar{\theta}\sigma_x^2\{2\sqrt{\Lambda}(1+\bar{\theta}) + g'[\Lambda + \sigma_x^2]\}}{[\Lambda - \bar{\theta}\sigma_x^2]^3} > 0 \qquad \textbf{(B.25)}$$

Recall, that all we needed to pin down the sign of $\partial^2 \pi^e / \partial \bar{a} \partial \sigma_x^2$ in equation (**B.8**) was the sign of (**B.9**). However, by combining (**B.24**) and (**B.25**) we know that $\partial^2 F / \partial \bar{a} \partial \sigma_x^2 < 0$. Thus, the sign pattern of the RHS of (**B.8**) becomes (+)*(-) + (-)*(+). This implies that $\dfrac{\partial^2 \pi^e}{\partial \bar{a} \partial \sigma_x^2} < 0$

QED.

Now, we move on to determine the sign of $\dfrac{\partial^2 \pi^e}{\partial \bar{a}^2}$. We already know that $\dfrac{\partial F(.)}{\partial \bar{a}} < 0$, so all that remains to be done is to find the second derivative of F with respect to $\bar{a}$.

Turning again to (**B.22**), upon dividing through by $d\bar{a}$ and solving for $\dfrac{df'}{d\bar{a}} = \dfrac{\partial^2 F}{\partial \bar{a}^2}$ we get

$$\frac{\partial^2 F}{\partial \bar{a}^2} = -2\sqrt{\Lambda} H_\Lambda + g' H_{\bar{\theta}} - \frac{dg'}{d\bar{a}} H_{g'} \qquad \textbf{(B.26)}$$

From (**B.20**) it follows that

$$H_{\Lambda} = \frac{\Lambda^{-1/2}(1-4\Lambda)(1+\bar{\theta}) - g^{'}[\Lambda + (2+\bar{\theta})\sigma_x^2]}{[\Lambda - \bar{\theta}\sigma_x^2]^3} < 0^{(25)} \qquad \textbf{(B.27)}$$

$$H_{\bar{\theta}} = \frac{\sigma_x^2[\{2\sqrt{\Lambda} + [\Lambda + \sigma_x^2]\frac{dg^{'}}{d\bar{\theta}}\}[\Lambda - \bar{\theta}\sigma_x^2] + 2\sigma_x^2\{2\sqrt{\Lambda}(1+\bar{\theta}) + g^{'}[\Lambda + \sigma_x^2]\}]}{[\Lambda - \bar{\theta}\sigma_x^2]^3} > 0^{(26)} \qquad \textbf{(B.28)}$$

and

$$H_{g^{'}} = \frac{\sigma_x^2[\Lambda + \sigma_x^2]}{[\Lambda - \bar{\theta}\sigma_x^2]^2} > 0 \qquad \textbf{(B.29)}$$

whereas

$$\frac{dg^{'}}{d\bar{a}} = \frac{dg^{'}}{d\bar{\theta}}\frac{d\bar{\theta}}{d\bar{a}} = \frac{-\bar{\theta}}{\bar{a}^2} < 0 \qquad \textbf{(B.30)}$$

Taking account of **(B.27)** - **(B.30)** it can be easily verified that the right-hand side of **(B.26)** is greater than zero, hence $\frac{\partial^2 F}{\partial \bar{a}^2} > 0$.

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(25) With $\bar{a} > 0, b > 1$ (b is proxied by $\frac{\lambda}{1-\lambda}$, where $0 < \lambda < 1$ and λ is the production elasticity of labour [see Schaling (1995) Chapter 4]) $\Lambda > .25$.

(26) $\frac{dg^{'}}{d\bar{\theta}} = \frac{1}{\bar{a}} > 0$.

Now, we are ready to establish the sign pattern of **(B.10)**. It is (+)*(+) + 2* (-)*(-) + (+)*(+). This implies that $\dfrac{\partial^2 \pi^e}{\partial \bar{a}^2} > 0$ QED.

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