Real Interest Rate Linkages: Testing for Common Trends and Cycles

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Abstract

This paper formed part of the Bank of England's contribution to a study by the G10 Deputies on saving, investment and real interest rates, see Jenkinson (1996). It investigates the existence of common trends and common cycles in the movements of industrial countries' real interest rates. Real interest rate movements are decomposed into a trend (random walk) element and a cyclical (stationary moving average) element using the Beveridge-Nelson decomposition. We then derive a common trends and cycles representation using the familiar theory of cointegration and the more recent theory of cofeatures developed by Vahid and Engle (1993). We consider linkages between European short-term real interest rates. Here there is evidence of German leadership/dominance - we cannot reject the hypothesis that the German real interest rate is the single common trend and that the two common cycles are represented by the spreads of French and UK rates over German rates. The single common trend remains when the United States (as representative of overseas rates) is added to the system , but German leadership is rejected in favour of US (overseas) leadership. We also find the existence of a single common trend in G3 rates after 1980.

Introduction

Real interest rates lie at the heart of the transmission mechanism of monetary policy. Increasingly attention has been paid to how different countries' real interest rates interact and how this interaction has developed through time. Economic theory would suggest that in a world where capital is perfectly mobile and real exchange rates converge to their equilibrium levels, *ex-ante* real interest rates (ie interest rates less the expected rate of inflation across the maturity of the asset) should move together in the long run.(1) The extent to which they move together in practice may therefore shed some light on either the degree of capital mobility or real exchange rate convergence, see Haldane and Pradhan (1992). For instance the increasing liberalisation of domestic capital markets during the 1980s would be expected to have strengthened the link among different countries' real interest rates in this period.

The aim of this paper is to investigate statistically the degree to which real interest rates have moved together both in the long run and over the cycle. Specifically we test for the existence of common 'trends' and 'cycles' in real interest rates for particular groups of countries, using familiar cointegration analysis and the more recent common feature techniques developed by Engle and Vahid (1993).

We first examine *short-term* real interest rates in the three major European economies (Germany, France and the United Kingdom), extending the analysis of previous studies (eg Katsimbris and Miller (1993)) that have examined linkages between short-term nominal interest rates. These studies have found evidence of German "dominance", with German rates Granger-causing movements in other European countries' rates. We investigate whether this holds in a real interest rate setting by examining whether German interest rates tend to drive common movements among other European rates, ie is the German rate the single common trend on which the other rates depend in the long run? Additionally, in common with other

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(1) The simplest theory of how real interest rates move together for two open economies is given by the real uncovered interest parity condition (UIP) which we can write as:

$$r_t = r^*_t - (E_t e_{t+1} - e_t) + \text{risk premium}$$

where r is the first country's real interest rate, r^* is the second country's real interest rate and e is the real exchange rate between the two countries. E_t is the expectations operator at time t. This condition equates the risk-adjusted real return on assets denominated in the currencies of both countries. Given perfect capital mobility, risk neutrality and real exchange rate convergence, the expected change in the real exchange rate and the risk premium will be zero in the long run, and real interest rates will be equalised across countries.

studies, we test how the addition of the United States to this European system affects the robustness of the results.

We then go on to consider a wider issue, namely whether the concept of a "world real interest rate" is sensible. This has been used as the dependent variable in a number of empirical studies, eg Barro and Sali-i-Martin (1990) and Driffill and Snell (1994) which have examined the structural determination of real interest rates. These studies have typically looked at long-term real interest rates and consequently we analyse linkages between *long*-term real interest rates of the major G3 economies (the United States, Germany and Japan). The existence of a single common trend among the three rates can be interpreted as a common world real interest rate.

The paper is organised as follows. In Section I we outline the techniques employed to test for the existence of common cycles and trends. In Sections II to IV we turn to our empirical analysis, outlining our use and choice of data along with our general method, before proceeding to analyse the European and G3 interest rate systems in turn. The final section draws some conclusions.

I Common trends and cycles - econometric theory and method

We begin by setting out exactly what we mean by a trend and a cycle. To do this we invoke the Beveridge-Nelson (1981) decomposition. This says that any time series can be decomposed into its *trend* element and its *cycle*. In a multivariate setting, this can be represented as:

$$y_t = C(1) \sum_{s=0}^{t} \varepsilon_s + C^*(L)\, \varepsilon_t + y_0 \qquad \textbf{(1)}$$

where y_t is the (n x 1) vector of variables under consideration (in this case the interest rates of the relevant country set) and ε_t is a white noise error term. The first term for each variable comprises a linear combination of random walks or stochastic *trends,* while the second term is a combination of stationary moving average processes which we define as *cycles*. By definition therefore, series that are stationary have no trend, and series which are pure random walks have no cyclical component.

In order to say more about *common* cycles and trends, we move to the dual representation of this system which is given by a finite VAR or vector autoregression. Inverting **(1)** yields :

$$A(L)\, y_t = \varepsilon_t$$

where $A(L) = I_n - A_1 L - A_2 L^2 - ....- A_p L^p$ and p is the lag length required to make the residuals white noise.

Any autoregressive time series of order p can be written in terms of its first difference, one lag level and p-1 lag differences. Rearranging **(1)** in this fashion gives

$$\Delta y_t = \Pi\, y_{t-1} + \sum_{i=1}^{p-1} \Gamma_i\, \Delta y_{t-i} + \varepsilon_t$$

or **(2)**

$$\Delta y_t = \Pi\, y_{t-1} + A^*(L)\, \Delta y_{t-1} + \varepsilon_t$$

where $\Pi = -I_n + \Sigma A_i = -A(1)$

$$\Gamma_i = \sum_{j=i+1}^{p} A_j = A^*_i$$

If the variables are integrated of order 1 but not cointegrated then $A(1)$ will be a zero matrix and we obtain a VAR model in differences. When the series are cointegrated, $A(1)$ will have rank r and can be decomposed into a product of two matrices of rank r : α and β. The α matrix is the (n x r) matrix of cointegrating vectors; β is the (n x r) factor loading matrix. Defining $z_{t-1} = \alpha' y_{t-1}$, (ie the vector of r cointegrating combinations), we can rewrite **(2)** as:

$$\Delta y_t = A^*(L)\Delta y_{t-1} - \beta z_{t-1} + \varepsilon_t \qquad \textbf{(3)}$$

Here z can be interpreted as describing the long run relationship(s) between the variables. Equation **(3)** is known as the Vector Error Correction Mechanism (VECM), and is familiar in cointegration analysis.

But it is possible that the short-run dynamic behaviour of the variables, embodied in the coefficients on the first differences given by the elements of the matrix polynomial $A^*(L)$, may also be related. This is what the common cycle analysis attempts to identify. In the same way as cointegration seeks to find a linear combination of the variables that is stationary (ie non-trended), we define a codependence/cofeature(2) vector as a linear combination of the variables that does not cycle (ie is not serially correlated).

A cycle is thus said to be common if a linear combination of the first differences can be found which is unforecastable. This motivates the search for linear combinations, $\tilde{\alpha}$, that remove all dependence on the past observations of the variables. Formally a cofeature vector $\tilde{\alpha}$ exists if:

$$E(\tilde{\alpha}'\Delta y_t \mid \Omega_t) = 0 \qquad \textbf{(5)}$$

where Ω_t = the information set containing all relevant information as of time t.

Premultiplying equation **(2)** by $\tilde{\alpha}'$, it can be shown that this requires

(2) Cofeature and codependence are used interchangeably here. The latter term is in fact older and was first introduced by Gourieroux and Paucelle (1989). But Engle and Vahid (1993) have recast the search for codependence in their general cofeature framework.

$$\tilde{\alpha}'\ \Pi = 0; \quad \tilde{\alpha}'\Gamma_i = 0 \qquad i = 1,\ldots,p-1 \tag{6}$$

ie not only must Π have reduced rank but so must all the Γs.

Exploiting the duality between the MA and VAR representations, it can be shown that the cointegrating vectors and codependence vectors must be linearly independent. A linear combination of a trend and a cycle can never be either solely a trend or cycle. Engle and Vahid (1993) show formally that, if y_t is a n-vector of $I(1)$ variables with r linearly independent cointegrating vectors ($r < n$), then if elements of y_t have common cycles, there can exist at most n-r linearly independent cofeature vectors that eliminate the common cycles.

The implication is that we may estimate the cofeatures that exist between variables by examining the cointegrating vectors, α, and the codependence vectors, $\tilde{\alpha}$, separately. Importantly though, should we find evidence of cointegrating vectors, then the cointegrating combinations, z_{t-s}, ($s = 1,\ldots,t-1$) should be included in the information set Ω_t, since details of how far variables are from some long-run equilibrium between the variables will be relevant in explaining the dynamic behaviour. It also follows that even in the absence of cointegration, a VAR with integrated variables can still be analysed for common features by looking for codependence vectors that eliminate common cycles.

Extracting Common Trends and Common Cycles

The existence of cointegrating and cofeature vectors allow us to place restrictions on the trend and cycles representation. This can be seen by inverting back to the vector moving average representation (ie $y_t = C(L)\varepsilon_t$). Importantly, the VAR model cannot be inverted directly if the variables are cointegrated since the coefficient matrix $A(1)$ of the VAR will be singular. But this singularity can be overcome by appropriate factorisation of the autoregressive polynomial $A(L)$ to isolate the unit roots in the system. Engle and Granger (1987) show that this yields:

$$y_t = C(1) \sum_{s=1}^{t} \varepsilon_s + C^*(L)\,\varepsilon_t + y_0$$

This is the multivariate Beveridge-Nelson decomposition of y_t we started with, but the matrices $C(1)$ and $C^*(L)$ are now of reduced rank. When all variables are $I(1)$ and there is no cointegration then the $C(1)$ matrix has full rank and the trend part of the decomposition is a linear combination of n

random walks, so that no linear combinations of y are stationary. If there are r cointegrating vectors then the rank of $C(1)$ is $k = n-r$ which can be decomposed into the product of two matrices of rank k. The trend part can then be reduced to linear combinations of $k\,(<n)$ random walks which are the *Common Stochastic Trends*. More formally, since $C(1)$ has rank k we can find a non-singular matrix G such that $C(1)\ G = [H\ 0_{nxr}]$ where H is an $n\ x\ k$ matrix of full column rank. Thus:

$$C(1)\ G\ G^{-1}\ \Sigma\varepsilon_s = H\ G^{-1}\ \Sigma\varepsilon_s = H\,\tau_t$$

where τ are characterised as random walks, and are the first k components of $G^{-1}\ \Sigma\varepsilon_s$

Similarly, if there are s codependence vectors, then there are only n-s independent stationary moving average processes so that the rank of $C^*(L)$ is $(n - s)$ - these are the *Common Stochastic Cycles*. We can write $C^*(L)$ as the product of two matrices with dimensions $n\ x\ (n\text{-}s)$ and $(n\text{-}s)\ x\ n$ with the left matrix having full column rank. That is $C^*_i = FC^{**}_i \quad i$. Hence we can write the cycle part as:

$$C^*(L)\ \varepsilon_t = F\ C^{**}(L)\,\varepsilon_t = Fc_t$$

Bringing the two components together implies the Common Trend - Cycle representation:

$$y_t = H\,\tau_t + F\ c_t \qquad \textbf{(7)}$$

where $\tau_t = \tau_{t-1} + \varepsilon_t = G^{-1} \sum_{s}^{t} \varepsilon_s$ are the common trends

and $\quad c_t = C^*(L)\ \varepsilon_t$ are the common cycles.

A Special Case

In the special case where the number of cointegrating vectors and the cofeature vectors sum to the number of variables, Vahid and Engle (1993) show that the common trend-cycle representation can be achieved directly without inverting the VECM model, using the cointegrating and cofeature vectors directly.

Define the ($n\,x\,n$) matrix $\quad A = \begin{bmatrix} \tilde{\alpha}' \\ \alpha' \end{bmatrix}$

where α' are the cointegrating vectors and $\tilde{\alpha}'$ are the cofeature vectors. A will have full rank and hence will have an inverse. By partitioning the columns of the inverse accordingly as $A^{-1} = [\tilde{\alpha}^{-} \mid \alpha^{-}]$ we can recover the common trend common cycle decomposition as:

$$y_t = A^{-1} A\, y_t = \tilde{\alpha}^{-}\tilde{\alpha}\; y_t + \alpha^{-}\alpha'\, y_t \qquad \textbf{(8)}$$
$$= \textit{trend} \quad + \quad \textit{cycle}$$

Thus the common cycle is given by the cointegrating combinations and the common trends by the codependence relationships; $\tilde{\alpha}^{-}$ and α^{-} are the matrices of loading vectors. This special case is useful as it will allow us to try and identify the common trends and cycles by placing restrictions directly on the cofeature and cointegrating vectors. When the special case does not hold and the VECM needs to be inverted directly, identifying the trends and cycles is more difficult, see Wickens (1996).

Testing Procedure for Common Cycles

Having discussed the properties of common trends and cycles, it remains to describe how codependence and hence common cycles can be tested for. Vahid and Engle (1993) outline two methods; one based on canonical correlation analysis which is similar in spirit to the Johansen procedure for detecting cointegrating vectors, the other using an encompassing VAR approach. In this study we primarily choose the latter method which is described below. We however check the validity of the results obtained from this second method using the canonical correlation method.(3)

Reconsider the VECM model given by equation **(2)**:

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(3) See Engle and Vahid (1993) and Hamilton (1994) for details.

$$\Delta y_t = \Pi \, y_{t-1} + \sum_{i=1}^{p-1} \Gamma_i \, \Delta y_{t-i} + \varepsilon_t$$

Recall the existence of common cycles imposes the following restrictions on the unrestricted VECM:

$$\tilde{\alpha}' \Pi = 0, \quad \tilde{\alpha}' \Gamma_i = 0 \qquad i = 1, \ldots\ldots, p-1$$

If these restrictions are imposed and the resulting system encompasses the unrestricted VAR then the hypothesis that there are *s* cofeature vectors can be accepted. The codependence vectors themselves can also be estimated and, unlike the canonical correlation estimates, standard errors can be derived which facilitate hypothesis testing.

To make such a test operational the cofeature matrix $\tilde{\alpha}'$ is normalised, (this can be done since $\tilde{\alpha}'$ is only identified up to an invertible transformation so that any linear combination of its columns will be a cofeature vector), in the following way:

$$\tilde{\alpha} = \begin{bmatrix} I_s \\ \tilde{\alpha}^*_{(n-s)x\ s} \end{bmatrix}$$

Now $\tilde{\alpha}' \Delta y_t$ can be considered as pseudo-structural form equations for the first *s* elements of Δy_t.

If the system is completed by adding the unconstrained reduced-form equations for the remaining *n - s* elements of Δy_t the following system is obtained.

$$\begin{bmatrix} I_s & \tilde{\alpha}^{*\prime} \\ 0_{(n-s)x\ s} & I_{n-s} \end{bmatrix} \Delta y_t = \begin{bmatrix} 0_{sx\,(np+r)} \\ \Gamma_1^* \ldots . \Gamma_{p-1}^* \end{bmatrix} \begin{bmatrix} \Delta y_{t-1} \\ . \\ . \\ . \\ . \\ \Delta y_{t-p+1} \\ \alpha' y_{t-1} \end{bmatrix} + v_t \qquad \textbf{(9)}$$

where v_t is white noise, but its elements are possibly contemporaneously correlated. The test for the existence of at least *s* cofeature vectors is therefore a test of the above structural form encompassing the unrestricted reduced form **(2)**. The above system of equations can be estimated jointly using Full Information Maximum Likelihood (FIML). The estimates of the cofeature vectors can be obtained and an encompassing statistic derived (based on the ratio of the restricted and unrestricted likelihoods which has a χ^2 distribution), and the number of restrictions imposed on the parameters can be calculated. The unrestricted VECM has $n(np+r)$ parameters, whereas the pseudo-structural model has sn-s^2 parameters in the first s pseudo-structural equations and $(n - s)(np + r)$ parameters in the n-s equations which complete the system. The number of restrictions imposed by the assumption of s cofeature vectors is thus $s(np+r) - sn + s^2$.

An example of a trend-cycle decomposition

Consider the following simple VECM model:

$$\Delta y_{1t} = a_1 \Delta y_{1t-1} + a_2 \Delta y_{2t-1} - a_3 (y_{1t-1} - y_{2t-1}) + \varepsilon_{1t}$$
$$\Delta y_{2t} = b_1 \Delta y_{1t-1} + b_2 \Delta y_{2t-1} - b_3 (y_{1t-1} - y_{2t-1}) + \varepsilon_{2t}$$

where there is a homoegenous cointegrating relationship between y_1 and y_2 . Consider further that the following restrictions hold:

$2a_1 = -b_1$; $2a_2 = -b_2$; $2a_3 = -b_3$.

From **(6)** above these satisfy the conditions for a single common cycle. The pseudo-structural form is thus given by:

$$\Delta y_{1t} = -0.5\, \Delta y_{2t} + v_{1t}$$
$$\Delta y_{2t} = b_1 \Delta y_{1t-1} + b_2 \Delta y_{2t-1} - b_3 (y_{1t-1} - y_{2t-1}) + \varepsilon_{2t}$$

where $v_{1t} = \varepsilon_{1t} + 0.5\varepsilon_{2t}$

The cofeature vector implied by the restrictions is thus [1 0.5]. As there is one common trend and one common cycle between the two variables we can use the special case described above to form the A matrix and its inverse:

$$A = \begin{bmatrix} 1 & 0.5 \\ -1 & 1 \end{bmatrix} \qquad A^{-1} = \begin{bmatrix} 0.67 & -0.33 \\ 0.67 & 0.67 \end{bmatrix}$$

We can renormalise the cofeature vector (which is also the common trend) to be a weighted average of y_1 and y_2. As a result A and A^{-1} become:

$$A = \begin{bmatrix} 0.67 & 0.33 \\ -1 & 1 \end{bmatrix} \quad A^{-1} = \begin{bmatrix} 1 & -0.33 \\ 1 & 0.67 \end{bmatrix}$$

The two series can then be expressed in terms of the common trend and cycle as:

$$\begin{bmatrix} y_{1t} \\ y_{2t} \end{bmatrix} = \underbrace{\begin{bmatrix} 1 \\ 1 \end{bmatrix} \left[0.67 y_{1t} + 0.33 y_{2t} \right]}_{\text{Common Trend}} + \underbrace{\begin{bmatrix} -0.33 \\ 0.67 \end{bmatrix} \left[y_{2t} - y_{1t} \right]}_{\text{Common Cycle}}$$

II Empirical results

Measuring Real Interest Rates

For our measures of short-term European nominal interest rates we have used quarterly averages of three-month Euromarket rates from 1968 Q1 to 1994 Q3 except for France where a three-month interbank rate was used. The use of Euromarket rates is intended to avoid any problems associated with periods when exchange controls operate. In order to derive real interest rates we need some estimate of inflation expectations over the lifetime of the asset. More formally we can approximate *ex-ante* real interest rates by:

$$r_t^a = i_t^a - (E_t \Delta p_{t+1})^a$$

where r_t^a is the annualised *ex-ante* three-month real interest rate in time period (quarter) t, i_t^a is the three-month annualised nominal interest rate, and $(E_t \Delta p_{t+1})^a$ is the expected three-month (one quarter) change in the log of the consumer price level, annualised.

In order to proxy inflation expectations over the next three months, we take a simple four-quarter moving average of quarterly inflation:

$$(E_t \Delta p_{t+1}) = \left[\frac{1}{4} \sum_{i=0}^{3} \Delta p_{t-i} \right]$$

For long-term nominal interest rates in the G3 countries we used ten-year government bond yields. To proxy inflation expectations over the lifetime of the bond, it seemed appropriate to employ a more forward looking method. We therefore took a two-year centred moving average of CPI inflation. Our measures of short and long-term real interest rates are shown in Charts 2.1 and 2.2.

Clearly more elaborate methods of modelling inflation expectations can be employed. More general ARIMA processes are an obvious alternative,see Driffill and Snell (1994) for example. Another possibility is the use of survey data which has been used for example by Haldane and Pradhan (1992). We leave testing the sensitivity of our results to changes in the measure of inflation expectations for future work.

Chart 2.1: European short-term real interest rates

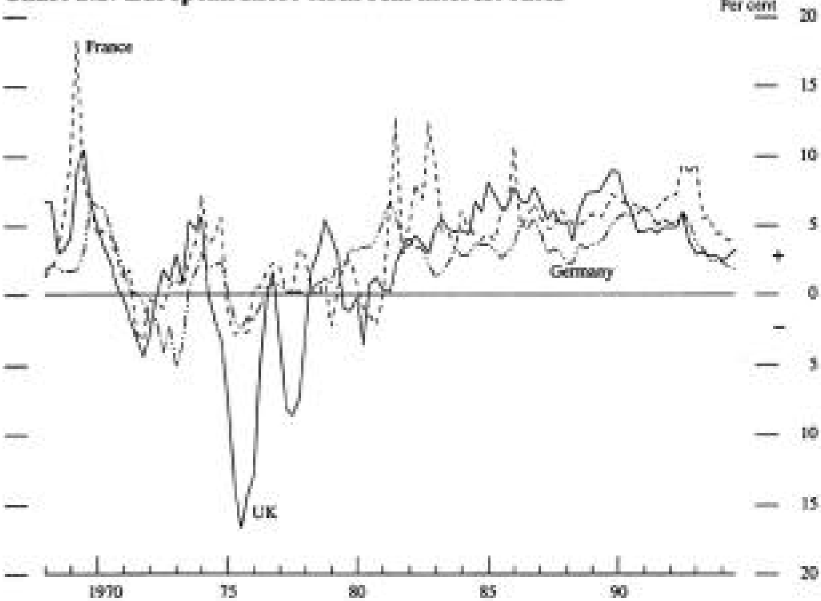
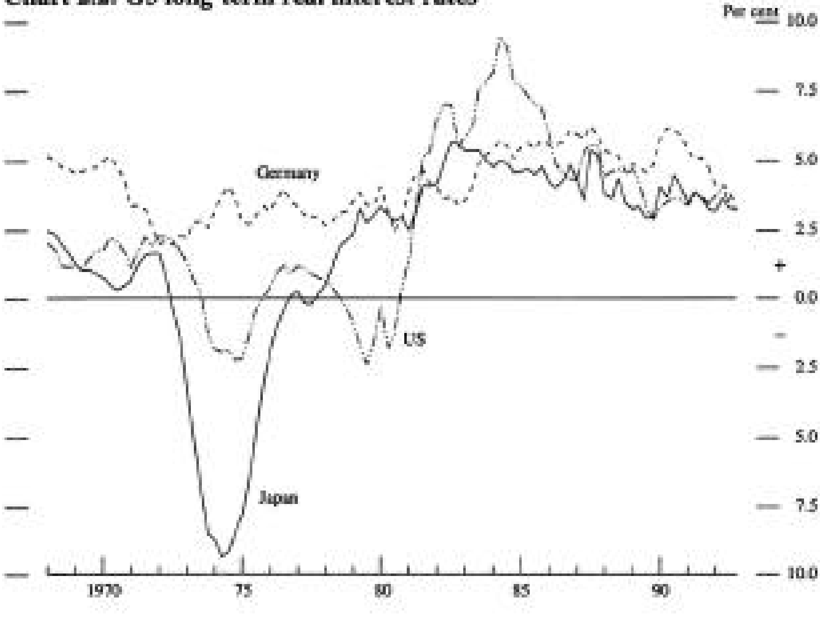


Chart 2.2: G3 long-term real interest rates



Time Series Properties of the Data

(i) Unit root tests - are real interest rates stationary or non-stationary?

As a starting point we examine the univariate time series properties of the data. The results of Augmented Dickey-Fuller (unit root) tests, shown in Table 2.A below, indicate that the interest rate data are borderline stationary/non-stationary.(4) However given that the power of ADF tests are notoriously low when the root is close to unity and given that the work on "near-integrated" processes of Phillips (1987) suggests borderline stationary-non stationary variables should be treated as non-stationary, we treat real interest rates as $I(1)$ variables in this study.(5)

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(4) The standard ADF tests were run both with and without a constant. But these do not necessarily relate to sensible alternative hypotheses. The former attempts to distinguish between a random walk with no drift and a series which is stationary around a zero mean, while the latter attempts to distinguish between a random walk with drift and a stationary series around a non-zero mean. However, one might wish to test the hypothesis that real interest rates were random walks with no drifts against the alternative that they are stationary around a constant mean, see Bhargava (1986). This requires setting the ADF statistics from the regressions with a constant against a different set of critical values as shown in the table.
(5) The fact that real interest rates may be non-stationary raises some theoretical problems as discussed in Rose (1988).

Table 2.A: Unit root tests:

Short rates 1969 Q3 to 1994 Q3

Country	ADF t-statistic no constant	ADF t-statistic with constant
United Kingdom	-2.1021	-2.2143
France	-2.1827	-2.93
Germany	-1.6774	-2.699

Long rates 1968 Q3 to 1992 Q4

Country	ADF t-statistic no constant	ADF t-statistic with constant
United States	-1.08	-1.76
Japan	-1.5	-1.73
Germany	-0.77	-1.71

Critical values (no constant, H_o random walk with no drift): 5%=-1.943,1%=-2.586
Critical values (H_0 random walk with no drift) : 5%= -1.943, 1%= -2.586
Critical values (H_0 random walk with drift): 5%= -2.89, 1%= -3.496

A possibility is that the non-stationarity over the sample period may be the result of a deterministic regime shift, for example in response to the oil price shocks during the 1970's. A rise in the real price of oil may have led to a one-off shift in the marginal product of capital in oil-importing countries. This obviously has implications for the cointegration analysis we employ below.(6)

(ii) Lag length

In any VAR framework the chosen lag length can have important implications for the results. This is particularly so for the common trend/common cycle analysis, since all inferences in both the cointegration and common cycle stages are conditional on the number of lags specified. There are no definitive procedures for choosing the lag length; the Akaike Information Criteria is one method that is frequently employed. But using this method

(6) Cointegration between variables whose non-stationarity is primarily due to deterministic regime shifts may be an example of the recently developed concept of "co-breaking", see Hendry (1996).

sometimes leaves serially correlated residuals. Here we choose lag length on the basis of both the Akaike Information Criteria and evidence of white noise errors.

(iii) Constants in the VAR

A further problem is whether to include a constant term in the VAR and whether, if one is included, to restrict it to the long run solution or cointegrating vector. Given the non-monotonic (or lack of drift) path of real interest rates it seems unlikely that, if not $I(1)$, they would be stationary about a deterministic trend (ie it does not seem sensible to test whether real interest rates are difference stationary processes as opposed to trend stationary processes). Thus a constant, if included in the VAR, should probably be restricted to the long run. Here they may have the natural interpretation of time invariant risk premia. In our work we include a restricted constant in the VAR.

III European short rates

A number of recent studies have examined the links between European interest rates. In particular, several papers such as De Grauwe (1989) and Karfakis and Moschos (1990) have investigated the possibility of asymmetric links between European nominal interest rates and whether German rates tend to lead other European rates. In our analysis we start off with a general unrestricted representation of a European real interest rate system from which we then progressively test down to see if the German dominance hypothesis is congruent with the data. We begin by testing for the number of cointegrating relationships using the Johansen procedure, the results of which are shown in Table 3.A. In what follows a "*" and "**" denote rejection of the null hypothesis at the 5% and 1% levels respectively.

Table 3.A: Cointegration results

H_0 : rank = p	Eigenvalue test	Critical value	Trace test	Critical value
$p=0$	20.05**	17.9	38.69**	24.3
$p\leq 1$	15.78**	11.4	18.64**	12.5
$p\leq 2$	2.861	3.8	2.861	3.8

Notes: (a) Constant restricted to the long-run
(b) 3 lags in the VAR

Both the eigenvalue and trace test support the existence of two cointegrating vectors, which suggest the existence of a single common trend. The estimates of the unrestricted cointegrating vectors derived via the Johansen procedure were given by:

$$\alpha' = \begin{bmatrix} 1 & -0.15 & -0.46 \\ -0.39 & 1 & -0.44 \end{bmatrix}$$

where the variables are ordered $[Rs_g, Rs_f, Rs_{uk}]$.

Testing for common cycles using the canonical correlation method yielded the result shown in Table 3.B:

Table 3.B: Common feature results

Null hypothesis	Test statistic	Critical value
s > 0	4.6*	12.59
s > 1	39.0	23.68
s > 2	80.67	36.42

The data support the existence of one cofeature vector. This was confirmed using the encompassing VAR method. From the earlier discussion the existence of a single cofeature vector imposes $(np+r) -n + 1$ restrictions on the VAR which given n=3 and p=2 implies 6 restrictions in total. Table 3.C shows these overidentifying restrictions could not be rejected at conventional significance levels. The cofeature vector was given by:

$$\tilde{\alpha}' = [1 \quad 0.37 \quad 0.44]$$

Given that the number of cofeature vectors and cointegrating vectors add up to the number of the variables we are able to employ the special case outlined earlier to derive the common trends and cycles. The single common trend is given by the cofeature vector. If we make the normalisation that the sum of the $\tilde{\alpha}_i$'s equals unity we can thus express the common trend or real interest rate as:

$$R_{common} = 0.55\, Rs_g + 0.20\, Rs_f + 0.24\, Rs_{uk}$$

Thus Germany has the dominant "share" of the common trend. In general the weights resemble absolute GDP shares which would help us interpret the common trend as some sort of "European real interest rate". We therefore test for the restrictions that the weights equal average GDP shares for the three countries across the sample period[7] which were 0.24, 0.34 and 0.42 for the United Kingdom, France and Germany respectively. This implies two further overidentifying restrictions which were acceptable at the 5% level (the encompassing test statistic was given by $\chi^2(8) = 12.6245$ with an associated p-value of 0.1257). Thus our common trend or common "European real interest rate" is given by:

$$Reur = 0.42\, Rs_g + 0.34\, Rs_f + 0.24\, Rs_{uk}$$

(7) We took simple averages of GDP commonly denominated in dollars over the period 1970-1991 (prior to German unification).

Chart 3.1 shows the common trend relative to the three countries' real interest rates.

Table 3.C: Pseudo-structural form

Equation 1 for ΔRs_{ft}

Variable	Coefficient	Standard error	*t*-value
ΔRs_{ft-1}	0.0682	0.096	0.710
ΔRs_{ft-2}	0.051	0.077	0.659
ΔRs_{ukt-1}	-0.234	0.103	-2.279
ΔRs_{ukt-2}	-0.067	0.096	-0.698
ΔRs_{gt-1}	0.043	0.182	0.234
ΔRs_{gt-2}	-0.077	0.206	-0.375
$ECM1_{t-1}$	-0.358	0.101	-3.557
$ECM2_{t-1}$	0.137	0.112	1.213

Equation 2 for ΔRs_{ukt}

Variable	Coefficient	Standard error	*t*-value
ΔRs_{ft-1}	0.153	0.073	2.091
ΔRs_{ft-2}	-0.011	0.057	-0.201
ΔRs_{ukt-1}	0.213	0.078	2.738
ΔRs_{ukt-2}	0.127	0.071	1.797
ΔRs_{gt-1}	-0.161	0.134	-1.204
ΔRs_{gt-2}	-0.393	0.158	-2.492
$ECM1_{t-1}$	0.185	0.078	2.363
$ECM2_{t-2}$	0.216	0.088	2.448

Equation 3 for ΔRs_{gt}

Variable	Coefficient	Standard error	*t*-value
ΔRs_{ft}	-0.369	0.235	-1.574
ΔRs_{ukt}	-0.437	0.266	-1.646

LR test of over identifying restrictions: $\chi^2(6) = 11.3104$ [0.0792]

Just as the cofeature vector yields the common trend, the two common cycles are similarly given by the two cointegrating vectors. For now we keep these as unrestricted and therefore not identified in any structural sense. The two vectors are simply normalised with respect to the German and French rates, but equally could be scaled up or down by any factor which would simply alter the loading coefficient of each cycle in each country's real interest rate. Chart 3.1 also shows the two common cycles using this particular normalisation.

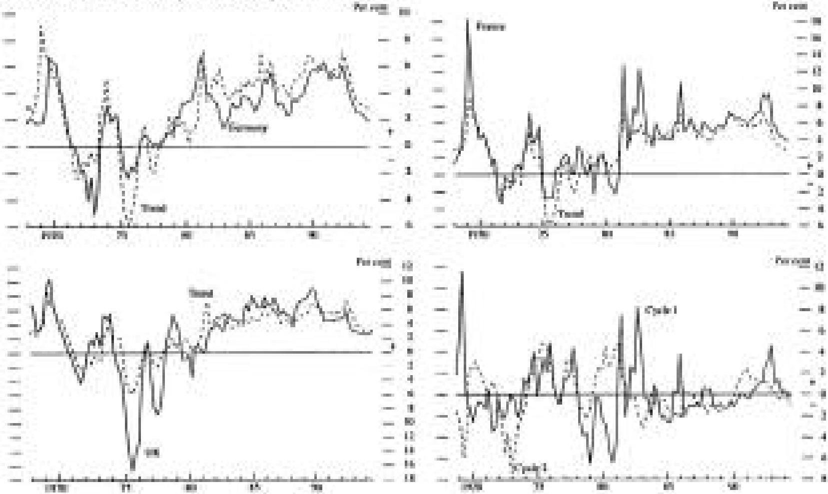
To see the importance of the trend and cycles for each real interest rate we write down the common trend-cycle representation as:

$$\begin{bmatrix} Rs_g \\ Rs_f \\ Rs_{uk} \end{bmatrix} = \begin{bmatrix} 0.8 \\ 0.94 \\ 1.43 \end{bmatrix} \left[0.42\, Rs_g + 0.34\, Rs_f + 0.24\, Rs_{uk} \right] \qquad \textit{CommonTrend}$$

$$+ \begin{bmatrix} -0.59 & -0.18 \\ -0.14 & 0.65 \\ -0.84 & -0.61 \end{bmatrix} \begin{bmatrix} Rs_g - 0.15\, Rs_f - 0.46\, Rs_{uk} \\ -0.39\, Rs_g + Rs_f - 0.44\, Rs_{uk} \end{bmatrix} \qquad \textit{Common Cycles}$$

The loading vectors for the trend show that in equilibrium the French real interest rate grows roughly in line with the common trend while the United Kingdom and Germany are significantly above and below in steady state. The loading vectors for the cycle imply that only the first cycle is important for the German real interest rate and only the second cycle is important for the French rate. Both cycles seem to be important to the UK rate, but in both cases the United Kingdom rate tends to move in the opposite direction to its European partners.

Chart 3.1: Trends and cycles in European short rates



Testing for German dominance

Without further identifying restrictions on the cointegrating and cofeature relationships, we can say little more about the nature of common cycles and trends in European real interest rates. We therefore seek to impose some additional restrictions on the cofeature and cointegrating vectors, which enable us to *identify* the comovements. For example, we might wish to investigate single country dominance. An obvious hypothesis to test is that of German leadership such that German rates tend to drive movements in other European rates which has been the focus of previous studies. In our framework this would entail testing whether the German real interest is the common trend among the three interest rates. This will imply certain identifying restrictions on both the cointegrating and cofeature vectors.

Given that there are two cointegrating vectors then at least two restrictions are required on each long run relationships for exact identification (by definition these are untestable). Real UIP, as noted earlier, would suggest that real interest rates should be equalised in the long run, after accounting for risk premia. We thus excluded the French rate from one of the cointegrating vectors and the UK rate from the other. Additionally we imposed equality between the two remaining rates in each relationship. This implies two overidentifying restrictions which are testable using the Johansen and Juselius (1994) switching algorithm. Table 3.D (a) shows these restrictions are acceptable at the 5% level. Looking at the constants we can see a positive risk premium for French rates over German rates, and a small risk premium for German rate above UK rates. Since the latter was unlikely we tested for a third overidentifying restriction testing whether this premium was zero. Table 3.D (b) shows this restriction was easily accepted.

Thus we cannot reject the hypothesis that *ex-ante* real interest rates will move one for one across countries in the long run, with short-term dynamics driven by the UK and French interest rate differentials with respect to Germany. The loading vectors (not shown) implied that the UK-German differential only entered the UK real interest rate equation, while the French-German differential only entered the French equation. This was confirmed when the VECM was estimated with the identified cointegrating vectors.

Table 3.D: Test of overidentifying restrictions on the cointegrating vectors

(a)

Rs_f	Rs_{uk}	Rs_g	*Constant*
1.00	0.00	-1.00	-1.53
0.00	1.00	-1.00	0.13

LR-test: $\chi^2(2) = 2.866$ [0.2386]

(b)

Rs_f	Rs_{uk}	Rs_g	*Constant*
1.00	0.00	-1.00	-1.57
0.00	1.00	-1.00	0.00

LR-test: $\chi^2(3) = 2.8874$ [0.4093]

Our definition of German dominance, would additionally imply that only the German rate entered the cofeature vector ie that it is the single common trend. It would require that neither lags of the first differences of each real interest rate nor either of the cointegrating vectors affect the German real interest rate in the unrestricted VAR. Thus German rates will follow a martingale and will tend to lead other rates but not vice versa.(8) Together this implies some eight overidentifying restrictions. As is shown in Table 3.E the German dominance restrictions cannot be rejected at the 5% level. We then make similar restrictions to test whether French and UK dominance are also acceptable; the table shows that these are rejected as would be expected from the significance of the cointegrating vectors in the French and UK equations.

Table 3.E: Testing for single country dominance

(i) German dominance

LR test of over identifying restrictions: $\chi^2(8) = 14.755$ [0.0641]

(ii) UK dominance

LR test of over identifying restrictions: $\chi^2(8) = 34.7327$ [0.0000] **

(iii) French dominance

LR test of over identifying restrictions: $\chi^2(8) = 25.1838$ [0.0014] **

(8) This is similar to testing for Granger-causality. But in VECMs there are certain subtleties in the exact conditions for Granger-causality to hold, so our dominance tests are not entirely equivalent.

Together, the total restrictions suggest the following forms for the matrices A and A^{-1} which determine the common trend-cycle representation (see Section I):

$$A = \begin{bmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix} \qquad A^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$$

This implies the following trend-cycle decomposition for each real interest rate:

$$Rs_{uk} = \underset{\text{trend}}{Rs_g} + \underset{\text{cycle}}{(Rs_{uk} - Rs_g)}$$

$$Rs_f = \underset{\text{trend}}{Rs_g} + \underset{\text{cycle}}{(Rs_f - Rs_g)}$$

$$Rs_g = \underset{\text{trend}}{Rs_g}$$

The German real interest rate is thus purely a stochastic trend which is common across the country set. The two common cycles are simply the interest differentials.

Adding the US rate to the European short rate system

As a test of robustness we follow previous research and test for the effect of the addition of the real US short rate as representative of overseas rates to the system. The choice of lag length was more problematic in this case. The AIC criterion and autocorrelation tests suggested either a lag length of two or three. Furthermore the cointegration tests were highly sensitive to the inclusion of a constant in the VAR. Table 3.F shows the results of the Johansen test with two lags in the VAR and the constant restricted to the long-run:

Table 3.F: Cointegration test

H_0 : rank = p	Eigenvalue test	Critical value	Trace test	Critical value
$p = 0$	24.78*	23.8	60.79**	39.9
$p \leq 1$	21.37*	17.9	33.27**	24.3
$p \leq 2$	12.46*	11.4	14.64*	12.5
$p \leq 3$	2.183	3.8	2.183	3.8

The cointegration tests support the existence of a single common trend among the four interest rates. The canonical correlation test in Table 3.G for common cycles shows that there is one cofeature vector, so that once again we are able to use the special case common trend-cycle decomposition.

Table 3.G: Common feature results

Null hypothesis	Test statistic	Critical value
$s > 0$	5.96*	9.49
$s > 1$	31.56	18.31
$s > 2$	58.26	28.87
$s > 3$	104.86	41.34

We test for the hypothesis of German leadership against that of US leadership. To facilitate this we first make identifying restrictions on our cointegrating vectors. These took the form of spreads above US rates. The fact that the cointegrating vectors have been defined as spreads over US rates does not conflict with the notion of German dominance since it could be that in the VECM French and UK rates feed off their spread above the US rate which in turn feeds off its spread over German rates and not vice versa (the US-German spread enters the US equation in the VECM significantly but not the German equation). This ensures that the test for German versus US (overseas) leadership are nested within the same VECM. The encompassing implications are described by the matrices:

$$A = \begin{bmatrix} 1 & 0 & 0 & 0 \\ -1 & 1 & 0 & 0 \\ -1 & 0 & 1 & 0 \\ -1 & 0 & 0 & 1 \end{bmatrix} \quad A^{-1} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 \end{bmatrix}$$

for US dominance where the order of variables is $[Rs_{ik} Rs_g \; Rs_f \; Rs_{uk}]$ versus those for German dominance where the first row of A changes to [0 1 0 0]. This yielded the encompassing test statistics for the resulting restricted pseudo-structural form shown in Table 3.H:

Table 3.H: Encompassing statistics

US Dominance
LR test of over-identifying restrictions: $\chi^2(7) = 8.13273$ [0.3210]

German dominance
LR test of over-identifying restrictions: $\chi^2(7) = 22.4032$ [0.0022] **

As can be seen the hypothesis of US leadership is not rejected at the 5% level whereas German leadership is decisively rejected. Again this can be interpreted as a stronger form of a Granger causality test where US rates Granger cause German rates but not *vice versa*.

Thus it appears that the German leadership hypothesis is not robust to the inclusion of an overseas interest rate, indeed its leadership is supplanted by foreign leadership. This is line with the results of Katsimbris and Miller (1993) who examined nominal interest rate linkages.

IV Long-term real interest rates in the G3

Several recent studies have looked at the determinants of real interest rates. For this researchers have typically used a "world" real interest rate as the dependent variable, consisting of a weighted average of different countries' real interest rates. Driffill and Snell (1994) have considered whether the concept of a world real interest rate is sensible using principal components techniques. We investigate this issue by testing for the existence of a single common trend among long-term real interest rates in the G3 countries.

Analogous to the short-rate system we apply the Johansen procedure for testing the number of cointegrating vectors. The results are shown in Table 4.A below for the whole sample period from 1968 Q3 to 1992 Q4. They suggest the existence of only one cointegrating vector and hence two common trends in the data. Additionally we tested for the number of cofeature vectors using the encompassing VAR test and found that one was present (ie two common cycles). Thus over the whole sample period there appears to be little evidence of much co-movement between G3 real interest rates both in the short and long run.

Table 4.A: Cointegration results 1968 Q1-1992 Q4

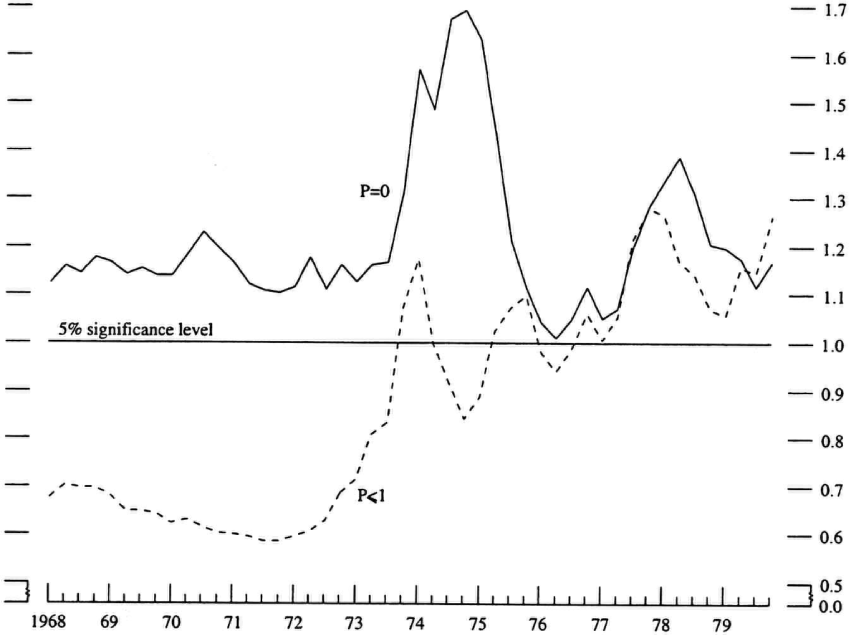
H_0 : rank = p	Eigenvalue test	Critical value	Trace test	Critical value
$p=0$	25.7*	22.0	39.24*	34.9
$p \leq 1$	10.75	15.7	13.54	20.0
$p \leq 2$	2.787	9.2	2.787	9.2

Notes: (a) Constant restricted to long-run
(b) 4 lags in VAR

However inspection of the time-series of the real interest rates (see Chart 2.1) suggests that stronger linkages may exist after 1980. This may reflect the different responses of the authorities to the oil price shocks of the 1970s. And the financial liberalisation of capital markets is likely to have led to an increase in international capital mobility. Other researchers have found stronger linkages between the United States and other countries' nominal interest rates after 1980, see Madjtahedi (1987) for example.

To investigate whether the degree of linkage between real interest rates has changed across time, we applied the Johansen procedure over successively shorter sample periods, beginning at 1968 Q3 and moving the start period forward until 1980 Q1 (the latest start period that would give us enough observations to get a sensible estimate of the number of common trends). Chart 4.1 shows the recursively computed trace test statistics for the rank p of the long-run matrix (ie the number of cointegrating vectors). The statistics for $p = 0$ and $p \leq 1$ are plotted relative to their 5% significance levels. As can be seen the existence of a single common trend (two cointegrating vectors) appears more likely from the late 1970s' onwards (ie we are able to reject the hypothesis that the rank of the long run matrix is less than or equal to one from the late 70's onwards).

Chart 4.1: Recursive cointegration tests



Post-1980 results

Given these results we investigated more closely the sub-sample 1980 Q1 to 1992 Q4. As implied by chart 4.1 the Johansen test indicated two cointegrating vectors. The two resulting unrestricted vectors were entered into a VECM to test for the number of common cycles. The encompassing VAR test statistic implied the existence of one cofeature vector and hence two common cycles, as shown in Table 4.B. Thus, given that the number of cointegrating and cofeature vectors span the dimension of the system we were able to obtain the trend-cycle decomposition using just the estimates of these vectors. These define the *A* matrix of Section I.

Table 4.B: Common feature results

Test for a single cofeature vector (two common cycles)
LR test of overidentifying restrictions: $\chi^2(9) = 13.713$ [0.1329]

As before the estimated cofeature vector gives us the linear combinations of variables that make up the common trend. And the estimated cointegrating vectors give us the linear combinations of variables that make up the common cycles. Again we proceeded to place identifying restrictions on these vectors to test various hypotheses. Unlike the European short-rate system the common trend did not appear to define a weighted average of the three real interest rates. We tested to see if the coefficients corresponded to GDP shares but this failed the encompassing VAR test as is shown in Table 4.C. We also tested the restrictions on both cointegrating and cofeature vectors implied by single country dominance as we did for the short-rate system but these also failed the encompassing test.(9)

Table 4.C: Hypothesis tests on the common trends and cycles

(i) Is the common trend a (GDP share) weighted average of G3 rates?

LR test of over identifying restrictions: $\chi^2(11) = 31.4336$ [0.0009] **

(ii) Is the Japanese rate the common trend ?

LR test of over identifying restrictions: $\chi^2(11) = 23.5331$ [0.0149] *

(iii) Is the US rate the common trend ?

LR test of over identifying restrictions: $\chi^2(11) = 28.1552$ [0.0031] **

(iv) Is the German rate the common trend ?

LR test of over identifying restrictions: $\chi^2(11) = 24.4001$ [0.0111] *

Thus although there is some evidence that the degree of short and long-run co-movement between the three real interest rates increased in the latter half of the sample period we are unable to say much about the nature of the common trends and cycles. These results therefore provide little support for a world long-term real interest rate that is some weighted average of individual countries' real interest rates.

(9) The only acceptable restrictions on the cofeature vector were those that defined a common trend in the spread of US rates over Japanese rates.

V Conclusion

There appear to be significant cross-country linkages between real interest rates both cyclically and in the long run. Employing cointegration and cofeature analysis allowed common cycles and common trends to be identified. There is also evidence of a single “European” short term real interest rate (represented by the single common trend in the short rate system), with Germany the dominant player. Indeed the hypothesis that Germany is the common trend driving European real interest rates cannot be rejected. But in common with other studies this result does not seem robust to the inclusion of US (overseas) rates, and the hypothesis that US rates determine the trend in European rates could not be rejected. Linkages between long term rates among the G3 appear stronger in the post-1980 period, where the results supported the notion of a single common trend. This would be consistent with the effect of financial liberalisation in increasing capital market integration. But there is little evidence that this common trend is some weighted average of individual countries’ real interest rates.

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