WHICH INTER-DEALER MARKET PREVAILS?
AN ANALYSIS OF INTER-DEALER TRADING IN OPAQUE MARKETS

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Abstract

A common feature of dealeship markets is that dealers have a choice, when dealing with each other, between doing so directly and using an IDB. Using a three stage model we show that, for a dealer who has executed an undisclosed customer trade their choice depends on the number of firms who operate as dealers (market makers). Subject to a monotonicity constraint, a condition is derived determining which form of inter-dealer market will prevail. *Journal of Economic Literature* Classification Numbers G12, G13, D82.

1 INTRODUCTION

Most research in market microstructure presumes the existence of a single market mechanism.[1] A number of studies have compared the features of different market structures, with respect, in particular, to their liquidity characteristics and market viability.[2] Moreover, a few researchers have studied issues of market fragmentation and consolidation by considering competition between market structures.[3] The fact, however, that agents are observed to switch between market mechanisms and the considerations which determine such switching have not been investigated.

This paper focuses on the determinants of agents' choice of market mechanism, in the context of inter-dealer trading. In a number of dealership markets, inter-dealer trading accounts for a substantial percentage of total trading. In the foreign exchange market, over 80% of the trading volume in the spot market is between market makers (Lyons (1995)). In the U.K. government bond market, trading between the gilt-edged market makers amounts roughly to 47% of total turnover (Proudman (1995)), whereas in the London Stock Exchange equity market, inter-dealer trading accounts for 38% of total turnover (London Stock Exchange (1994)). Moreover, all of these markets offer dealers a choice between two trading mechanisms. Dealers can trade directly with each other on a bilateral basis, or can place an order though one of the inter-dealer brokers.

A number of authors have recognized the importance of inter-dealer trading in price formation. For example, Vogler (1995), abstracting from information asymmetries, compares a dealership market, endowed with an inter-dealer trading mechanism similar to an auction, to a one period standard auction market. In the context of the foreign exchange market, Lyons (1993) and Perraudin and Vitale (1994) consider the role of inter-dealer trading as a mechanism for the dissemination of information. The fact, however, that in various quote-driven financial markets, dealers are observed to conduct their inter-dealer trading through two different market mechanisms, the factors that determine their choice, at any point in time, and the implications for the public have not been analysed.[4]

[1] Examples of such work include Glosten and Milgrom (1985), Kyle (1985), Easley and O' Hara (1987, 1992) and Admati and Pfleiderer (1988). For an overview of the theoretical market microstructure literature see O' Hara (1995).

[2] See for example Glosten (1989), Pagano and Roell (1990, 1992) and Madhavan (1992).

[3] See for example Pagano (1989), Chowdhry and Nanda (1991) and Glosten (1994).

[4] An important exception is Garbade (1978) who considers explicitly the effect

We use a three-stage model to study dealers' choice of inter-dealer trading venue and we analyse the implications of such a choice for outside investors. At the first stage of the model, dealers decide whether to enter the market-making of the security. At the second stage of the model, one of the dealers trades with an informed outside customer. At the third stage, the dealer, who has executed the outside investor's order, decides whether she wishes to unwind her inventory trading directly with another dealer or whether to submit an order to the inter-dealer broker.

We model the direct inter-dealer market as a standard competitive quote-driven market. Price competition between dealers results in price schedules being set such that in equilibrium no dealer expects to make a positive surplus. The inter-dealer broking system, in contrast, is modelled as a typical order-driven trading mechanism. Dealers pay a fixed brokerage fee in order to submit their orders to the broker. The broker clears the market and determines the price. All dealers realize the impact their orders have on price and act strategically. In equilibrium, dealers expect to make a positive expected surplus.[5]

Trading motivation in the inter-dealer market is a result of the second stage dealer-customer trade. We assume that the customer possesses private information about the value of the asset and wishes to trade a fixed amount for liquidity reasons. Further, we suppose that the market is opaque, that is, the transaction details about the trade between the customer and the dealer remain undisclosed until the end of the inter-dealer trading session. Examples of opaque markets include the foreign exchange and U.K. government bond markets, where transactions made over the phone between agents remain undisclosed, and, to a lesser extent, the London Stock Exchange equity market, where trades above a

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of inter-dealer brokerage in quote-driven markets. By selecting out the best bids and offers in the market, brokerage services are shown to facilitate dealer search and to be thus more efficient in uncovering arbitrage opportunities than bilateral communication. Unlike our model, dealers in this work are not fully optimizing agents who learn from the trading process, but act according to pricing conventions and prespecified trading rules.

[5] The modeling of the brokered market is intended to capture, at its simplest, the basic order-driven structure of most inter-dealer broking systems. In general, inter-dealer brokers take the form of electronic order-driven trading systems, market makers have exclusive access to. Notably, in the foreign exchange market, a fraction of brokered inter-dealer trading is still conducted through traditional broking firms whose function is to maintain and make available the limit order book. However, the market share of electronic systems, such as Dealing 2000-2 and EBS, has grown dramatically. In a recent article in *The Economist* ('The foreign exchange market: illiquid lunch', 30.3.96) the market share of these two systems is estimated to be $40 - 45\%$ of all trades by value that go through London brokers.

certain size are subject to an hourly delay.[6]

Recent research in market microstructure has argued that in opaque markets, dealers have an incentive to offer customers better prices in return for valuable information. Naik et al. (1994) and Madhavan (1995), for example, provide models of dealers adjusting their quotes in order to attract more informative order flow and recoup in later trading rounds.[7] We follow their rationale and we show that the dealer, who trades with the customer, has an incentive to adjust her quotes in order to capture the customer's private information and use it later in the inter-dealer market. Conversely, we show that the customer has an incentive to precommit to reveal her information.

We demonstrate that the dealer's third stage choice of inter-dealer trading venue depends on the equilibrium number of dealers who enter the market-making industry. The reason for this lies in the in-built difference in the institutional structures of the two inter-dealer market mechanisms. In the direct market dealers compete in prices, whereas in the brokered market they compete in demand schedules. Moreover, as long as two or more dealers compete for the incoming inter-dealer order flow, the risk-sharing opportunities offered by the direct market are independent of the equilibrium number of dealers. In contrast, the risk-sharing opportunities offered by the brokered inter-dealer market are increasing in the number of dealers who enter the market-making industry. We show that, at any point in time, there exists a critical number of dealers such that if the equilibrium number of dealers exceeds this number the risk-sharing benefits of the brokered market outweigh the competitive benefits of the bilateral market. Then, the brokered market prevails. Otherwise, the direct market prevails.

The effects of transparency and information asymmetry on dealers' choice of inter-dealer trading venue are analysed. We find that, an increase in information asymmetry, or market transparency reduces the liquidity of the brokered order-driven market to a greater extent than

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[6] The LSE transparency regulations regarding block trades have changed frequently since 1987. The most recent change occured on January 1st 1996 and resulted in an increase in the size of trades which qualify for delayed publication and in a reduction of the publication delay from 90 minutes to an hour. See the two papers by the Securities and Investments Board (1994, 1996) for an overview of the transparency regulations and the debate surrounding it.

[7] Long-term competition between dealers is an important assumption in these models. Saporta (1995) has provided a model of a monopolist specialist who benefits from delayed publication without offering better quotes to the information-providing investor.

that of the direct quote-driven market. As a result, an increase in information asymmetry, or market transparency increases the critical number of market participants necessary for the order-driven system to prevail. These effects are of particular interest, in view of the recent London Stock Exchange (LSE) controversy over the introduction of an order-driven trading mechanism to which outside investors will have access, and the ongoing debate over LSE transparency regulations.[8]

Given that trading through the broker involves positive expected profits and a fixed transaction cost, we are able to derive the equilibrium number of dealers who enter the market-making of the security when inter-dealer trading is conducted through the broker. Subject to a monotonicity constraint on dealers' expected profitability, we obtain a condition which determines *ex ante* which inter-dealer market will prevail. In the absence of information asymmetry, we derive a simple characterization of the equilibrium number of dealers. This enables us to show that sufficient increases in asset volatility, in the customer's liquidity needs and in the aversion of dealers to risk can cause a shift of inter-dealer trading from the direct inter-dealer market to the brokered market and vice-versa. These potentially testable comparative static results, however, are not generalizable to the case with information asymmetry.

The price the investor is able to negotiate with the dealer, in the second stage of the model, depends on the type of inter-dealer market which prevails in the third stage of the model. We show that, in equilibrium, large (institutional) investors would either be better-off, or at least as well-off, when dealers have a choice between inter-dealer trading venues than when they do not. Moreover, as long as the brokerage fee is low relative to the size of the market order, small investors expect at least as low transaction costs, when dealers have a choice between inter-dealer trading venues than when they do not, but, for sufficiently small orders, at the cost of higher volatility.

The paper is organized as follows. In Section 2, we set out the three period model. In Section 3, we solve for the dealer's choice of inter-dealer trading venue. In Section 4, we determine the price negotiated between the dealer and the public and we show its dependance upon the dealer's choice of inter-dealer market. In Section 5, we consider the

[8] Controversy over the introduction of this trading system led to the removal of the chief executive of the London Stock Exchange, in January 1996 (see *The Economist*, 'Shaping up', 23.3.96). At the moment of writing, the LSE is planning to launch a public limit order book system for FTSE-100 stocks mid 1997.

factors that determine the number of dealers entering the market-making of the security, in equilibrium, and the implications of dealers having a choice between inter-dealer trading venues for the public. In Section 6, we summarize our findings and conclude. All proofs of results stated in the main text are in the Appendix.

2 THE MODEL

The model is constructed as a three-stage game.
Stage 1: At the first stage of the game $M+1 \geq 3$ out of an existing pool of identical dealers enter the market-making of a risky security. We denote the value of the risky security as X and we assume that X is normally distributed with zero mean and precision π_x. We also make the standard assumption that the return to the safe asset, or alternatively, the opportunity cost of money, is normalised to zero. In addition, each dealer has a negative exponential utility with coefficient of risk-aversion ρ and reservation utility normalised to -1.
Stage 2: At the second stage of the game, an outside investor wishing to trade W units of the security for liquidity reasons, approaches one of the $M+1$ dealers, whom we label dealer 1, and negotiates a price. If W is positive, the investor pays a price of $P_{,2}$ per unit and if W is negative she receives $P_{,2}$ per unit. The price negotiated depends on dealer 1's conjectures about future trading opportunities and on the relative bargaining power of the two counterparties.

We assume that the outside investor has private information about the value of the asset, in the form of a signal $S = X + \varepsilon$, where S and W are independently and normally distributed with zero means and precisions π_s and π_w respectively.[9] Moreover, the investor has a choice between precommitting to reveal or not to reveal her information to the dealer she decides to trade with. Benveniste et al. (1992) provide an example of a mechanism that dealers can employ to avoid manipulation of such precommitments. In environments where outside investors (or their brokers) trade repeatedly, manipulation can be avoided by employing the lack of anonymity which characterizes bilateral trading.[10]

[9] Although the assumption that the investor's order is independent of her information simplifies the analysis, it is not essential. Indeed our main conclusions would not change if we assumed that the investor's order is correlated to her private information in some prespecified manner. The essential assumption, here, is that the investor wishes to trade a *fixed* quantity and that she possesses private information.

[10] In a different context, Forster and George (1992) have shown that lack

We follow Naik et al. (1994) and resolve the issue of bargaining between the investor and the dealer by arguing that the bargaining power rests with the outside investor. As long as two or more dealers have entered the active market-making of the security, an outside investor who does not obtain a satisfactory deal from one dealer can approach another dealer and negotiate a better price. We imagine this process continuing, until the investor succeeds in obtaining a price that ensures that the dealer, who executes the trade, obtains the same expected utility, as the dealers who do not execute the trade.

Notice that it is a characteristic of a number of dealership markets, that dealers are prepared to honour trades of various sizes within their published quotes. In the London Stock Exchange, for example, approximately 60% of institutional trades and 35% of all trades occur within the narrowest bid-ask spread, the 'touch'(Wells (1993) and Stock Exchange Quarterly (1992)). This suggests that a substantial number of investors have the bargaining power to negotiate better prices than those actually publicly quoted. Similarly, despite the absence of any mandatory rules, foreign exchange dealers honour most orders either at or within their advertised quotes (Guillaume et al. (1994) and Flood (1991)).

Further, we assume that the market is opaque. The details of the executed transaction between dealer 1 and the outside investor are not published until the end of Stage 3 when all uncertainty is resolved.

Stage 3: At Stage 3 the M dealers who did not transact with the public at Stage 2 set quotes at which they wish to trade with dealer 1. Dealer 1 decides whether she wishes to adjust her position trading with one of the M dealers at these quotes, or whether to place an order with the inter-dealer broker at a fixed positive cost F.

We assume that the time interval between Stage 2 and Stage 3 is so short, that dealer 1 faces a choice between contacting a single dealer in the direct market, or placing a single order with the inter-dealer broker before all uncertainty is resolved. That is, dealer 1 cannot split her order between dealers or between the two inter-dealer markets (see Vogler (1995) for a similar assumption).

The modeling of the direct inter-dealer market is in the spirit of the competitive model of Glosten (1989). The dealers who did not trade with the public set price schedules at which they are prepared to honour trades of various sizes. Price competition and the assumption that the trading interval is short, jointly imply that, as long as two or more

of anonymity has important effects on price formation, efficiency and wealth distribution.

dealers compete for the entire incoming order flow ($M \geq 2$),[11] dealers set the lowest possible quotation schedules, such that their expected utility from trading equals their expected utility from not trading, that is, their reservation utility.[12]

Trading through the inter-dealer broker is organised as a continuous order-driven market. Both dealer 1 and each of the remaining M dealers pay fixed cost F and place orders simultaneously.[13] The inter-dealer broker clears the market and the transaction price is determined. The modeling of the order-driven mechanism at this stage is in the spirit of Kyle (1989). Dealers maximize their expected utility against their order schedule realizing the impact they have on price.

In the presence of an alternative costless inter-dealer trading system, costly trading in the brokered market can only be viable, if the expected utility obtained by participating dealers is at least as large as their reservation utility level. One simple way to ensure that such a condition is satisfied, is to assume that the public order W is sufficiently large relative to F, so that $\frac{F}{W^2}$ approximates zero.

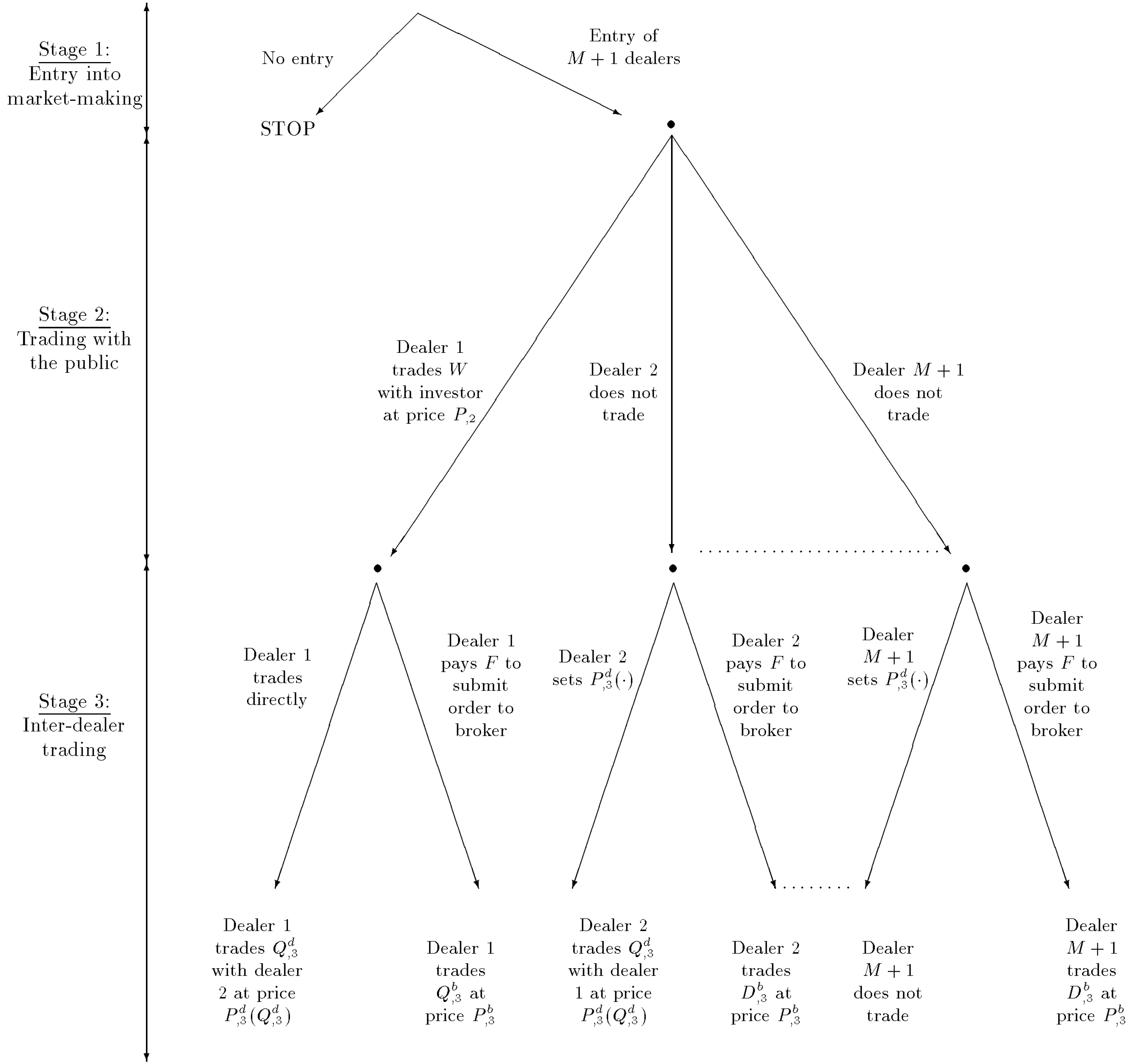
The game, the extensive form of which is depicted in Diagram 1, is solved by backward induction. For expositional simplicity, throughout our discussion of the second and third stage equilibria, we assume that dealer 1 sells W units at Stage 2 and buys $Q_{,3}$ units at Stage 3. Given the underlying symmetry of our problem, the discussion is analogous when the signs of the transactions are changed.

[11] As some form of competition is present in the inter-dealer trading of most liquid securities, we abstract from the special case where $M = 1$. The Stage 3 equilibrium outcome in such a case would be a version of the monopolistic equilibrium of Glosten (1989), the only difference being that the 'monopolist specialist' (the uninformed dealer) would be risk-averse.

[12] For justification of this equilibrium concept by Bertrand competition see for example the discussion in Subrahmanyam (1991) (footnote 10).

[13] We could also interpret the fixed cost F as the execution risk dealers face when placing orders in the order-driven market. By its very nature, trading in the quote-driven market is execution risk-free.

Diagram 1: The trading game



3 STAGE 3: INTER-DEALER TRADING

In this section we solve for dealer 1's choice of inter-dealer market. We suppose that the investor who traded with dealer 1 at Stage 2 had pre-committed to reveal her private information S. In Section 4, we show that this assumption is consistent with the investor's choice at Stage 2.

3.1 Direct inter-dealer market

Suppose that dealer 1 chooses to trade in the direct inter-dealer market. Price competition between dealers implies that all M dealers set the same price schedule $P^d_{,3}(\cdot)$ and that dealers 1 chooses to trade with one of them at random. In particular, dealer 1 chooses $Q^d_{,3}$ in order to maximize her expected utility of wealth, given by

$$-\mathrm{E}\left\{\exp\left[-\rho\left(P_{,2}W - P^d_{,3}Q^d_{,3} + X(Q^d_{,3} - W)\right)\right] \mid S\right\}. \qquad (\mathbf{1})$$

Assuming that $P^d_{,3}(\cdot)$ is twice differentiable, dealer 1's decision problem provides the following first order condition

$$P^{d'}_{,3}(Q^d_{,3})Q^d_{,3} + P^d_{,3}(Q^d_{,3}) + \rho\mathrm{Var}(X|S)Q^d_{,3} \; = \; \mathrm{E}(X|S) + \rho\mathrm{Var}(X|S)W. \quad (\mathbf{2})$$

Using standard Bayesian updating, we substitute $\mathrm{E}(X|S) = \dfrac{\pi_s}{\pi_x + \pi_s}S$ and $\mathrm{Var}(X|S) = \dfrac{1}{\pi_x + \pi_s}$ into equation (**2**) and we rewrite it as

$$\frac{\pi_x + \pi_s}{\pi_s}\left(P'(Q^d_{,3})Q^d_{,3} + P(Q^d_{,3})\right) + \frac{\rho}{\pi_s}Q^d_{,3} \; = \; Z, \qquad (\mathbf{3})$$

where $Z = S + \dfrac{\rho}{\pi_s}W$ is an unbiased estimator of S with precision $\pi_z = \dfrac{\pi_s^2\pi_w}{\pi_s\pi_w + \rho^2}$.

The dealers who did not trade at Stage 2 do not know W due to the lack of transparency in the market. However, they do know that they did not trade with the public at Stage 2. Thus, each of the M dealers, who did not trade with the public at Stage 2, infers that one of the other $M+1$ dealers must have traded and would be wishing to adjust her position through inter-dealer trading according to equation (**3**). It follows, that all the M dealers who did not trade with the public at Stage 2 set $P^d_{,3}(Q^d_{,3})$ such that their expected utility conditional on Z is equal to their reservation utility, that is, $P^d_{,3}$ satisfies

$$P^d_{,3}(Q^d_{,3}) = \mathrm{E}(X|Z) + 0.5\rho \mathrm{Var}(X|Z)Q^d_{,3}, \tag{4}$$

where $\mathrm{E}(X|Z) = \dfrac{\pi_z}{\pi_x + \pi_z}Z$ and $\mathrm{Var}(X|Z) = \dfrac{1}{\pi_x + \pi_z}$.

In the Appendix, we show that the direct inter-dealer market equilibrium quantity $Q^d_{,3}$ and price $P^d_{,3}$ are given by

$$Q^d_{,3} = \tau(W + \frac{\pi_s}{\rho}S), \tag{5}$$

$$P^d_{,3} = \frac{\rho}{2(\pi_x + \pi_s)}(1-\tau)(W + \frac{\pi_s}{\rho}S) \quad \text{where,} \tag{6}$$

$$\tau = \frac{\rho^2\pi_x - \pi_s\pi_w(\pi_x + \pi_s)}{2\pi_s\pi_w(\pi_x + \pi_s) + \rho^2(\pi_x + \pi_s) + \rho^2\pi_x}, \tag{7}$$

as long as

$$\begin{aligned} \tau &> 0 \qquad \text{or,} \\ \frac{\pi_s(\pi_x + \pi_s)}{\pi_x} &< \frac{\rho^2}{\pi_w}. \end{aligned} \tag{8}$$

Otherwise, the direct inter-dealer market does not open.

Noting that Z, the information inferred by the M dealers, is a simple linear transformation of $W + \dfrac{\pi_s}{\rho}S$, we observe that both the equilibrium order and price are linear in Z. The parameter τ lies between 0 and 0.5 and reflects the response of the M dealers to information asymmetry, market transparency and the coefficient of risk-aversion. In particular, we can show, that the greater the private information, π_s, or the lower the public information, π_x, the lower the τ and the lower the fraction of the customer order, dealer 1 lays off.[14] In the same way, we can show, that the higher the market transparency, π_w, or the lower the coefficient of risk aversion, ρ, the harder for dealer 1 to disguise her information signal, S, and the greater the fraction of the customer's order retained on her account.[15] Notice that we interpret the precision of the investor's

[14] These observation can be easily verified by showing that $\dfrac{\partial\tau}{\partial\pi_s} \leq 0$ and that $\dfrac{\partial\tau}{\partial\pi_x} \geq 0$.

[15] These observations can be easily verified by showing that $\dfrac{\partial\tau}{\pi_w} \leq 0$ and that $\dfrac{\partial\tau}{\partial\rho} \geq 0$.

order, π_w, as the degree of transparency in the market. This is due to the fact that the higher the precision of the investor's order (the higher the π_w), the better the idea the other dealers have about dealer 1's inventory and the more precise their inference about dealer 1's private information (the higher the π_z). As a result, dealers are more reluctant to trade in the direction of their more precise inference of dealer 1's information, Z, the higher the market transparency.

In the limit, as information asymmetry disappears ($\pi_s \rightarrow 0$), τ reaches its upper bound of 0.5 and dealer 1 places an order which is equal to half her inventory, W, at a price which is above the expected value of the asset. That is, even in the absence of adverse selection, the informed dealer, faced with an upward sloping supply curve, acts monopsonistically and retains a fraction of the customer's order on her account. Further, observe that the uninformed dealers are only prepared to make a market, as long as the information asymmetry between themselves and the informed dealer 1 is below a boundary level $\frac{\rho^2}{\pi_w}$.

3.2 Trading through the inter-dealer broker

Now suppose that dealer 1 decides to pay $F > 0$ in order to place an order through the inter-dealer broker. The structure of the indirect inter-dealer market is such that all market participants place their orders simultaneously. As a consequence, not only does dealer 1 maximize her expected utility against her demand, but so do the rest of the M dealers who maximize against their supply curves.

If each dealer conjectures that the order strategies of the other uninformed dealers are linear, in equilibrium, her conjectures turn out to be true. In the Appendix, we show that, in this linear rational expectations equilibrium, dealer 1's trade $Q^b_{,3}$ the trades of the other M dealers $D^b_{,3}$ and the market clearing price $P^b_{,3}$ are given by

$$Q^b_{,3} = (1 - 2\mathrm{T})(W + \frac{\pi_s}{\rho}S), \tag{9}$$

$$D^b_{,3} = -\frac{(1 - 2\mathrm{T})}{M}(W + \frac{\pi_s}{\rho}S), \tag{10}$$

$$P^b_{,3} = \frac{\rho}{\pi_x + \pi_s}\mathrm{T}(W + \frac{\pi_s}{\rho}S) \qquad \text{where,} \tag{11}$$

$$1-2\mathrm{T} = \frac{M(M-2)\rho^2\pi_x - M^2\pi_s\pi_w(\pi_x+\pi_s)}{(M^2-2)\pi_s\pi_w(\pi_x+\pi_s)+2(M-1)\rho^2(\pi_x+\pi_s)+M(M-2)\rho^2\pi_x}, \quad \mathbf{(12)}$$

as long as

$$\begin{aligned} (1-2\mathrm{T}) &> 0 \quad \text{or,} \\ \frac{\pi_s(\pi_x+\pi_s)}{\pi_x} &< \frac{M-2}{M}\left(\frac{\rho^2}{\pi_w}\right). \end{aligned} \quad \mathbf{(13)}$$

In the brokered equilibrium the orders of dealers and the clearing price are all linear in the informed dealer's inventory W and her signal S. Indeed, as it is shown in the Appendix, the uninformed dealers use the lack of Stage 2 trading and the conjectured linearity of order strategies to infer $Z = S + \frac{\rho}{\pi_s}W$ and in effect, form their expectations on the basis of the same information they would have inferred, under the direct inter-dealer system. Nevertheless, the resulting equilibria differ. This is due to the fact that in the brokered market dealers compete in demand schedules, whereas in the direct market they compete in prices.

The parameter $1-2\mathrm{T}$ lies between 0 and 1 and reflects the sensitivity of dealers to information asymmetry, market transparency and the coefficient of risk aversion.[16] As information asymmetry becomes insignificant ($\pi_s \to 0$), dealer 1 buys back a fraction $\frac{M(M-2)}{2(M-1)+M(M-2)}$ of W at a price which is above the expected value of the asset. As $M \to \infty$, dealer 1 buys back a fraction of W at a clearing price which is equal to the other dealers' expected value of the asset.[17] That is, as long as information asymmetry is present, or M is finite, dealer 1 restricts her order monopsonistically and retains on her account a fraction of her Stage 2 order. In the absence of adverse selection and as $M \to \infty$, however, the brokered market equilibrium approximates the Walrasian equilibrium, which is characterized by perfect risk-sharing, at a price equal to the expected value of the asset.

[16] Indeed, by differentiating $(1-2\mathrm{T})$ with respect to the initial parameters π_s, π_x, π_w and ρ, we can readily verify that, in equilibium, the uninformed dealers' orders are increasing in information asymmetry and market transparency and are decreasing in risk-aversion.

[17] In order to demonstrate that as $M \to \infty$ the brokered market is 'semi-strong form' efficient, we first note that as $M \to \infty$, the parameter T goes to $\frac{\pi_s\pi_w(\pi_x+\pi_s)}{\pi_s\pi_w(\pi_x+\pi_s)+\rho^2\pi_x}$. We can then see that as $M \to \infty$, $P^d_{,3}$ goes to $\frac{\pi_s^2\pi_w}{\pi_s\pi_w(\pi_x+\pi_s)+\rho^2\pi_x}Z$, which is equal to $\mathrm{E}(X|P^b_{,3}) = \frac{\pi_z}{\pi_x+\pi_z}Z$.

Comparing inequality (**13**) with inequality (**8**), we observe that the indirect inter-dealer broker system is more sensitive to information asymmetry than the direct inter-dealer market.[18] The strategic behaviour displayed by the M dealers, in the brokered market, creates inefficiencies and makes the market less robust to problems of information asymmetry. Moreover, although the direct inter-dealer market operates with three or more dealers ($M \geq 2$), for the brokered market to open, four or more dealers must have entered the market-making of the security ($M \geq 3$).

3.3 Dealer 1's choice of inter-dealer market

In order to determine the dealer 1's choice of inter-dealer market we need to compare her terminal expected utility under the two inter-dealer market structures. This is equivalent to comparing the certainty equivalencies dealer 1 would obtain, when placing an order in the two inter-dealer markets.[19]

In the Appendix, we show that at the end of Stage 3, the certainty equivalence dealer 1 obtains can be written as

$$\mathrm{CE}_{1,3}(Z) = \mathrm{CE}_{1,3}(0) + \frac{\pi_s}{\pi_x + \pi_s} \int_0^Z \left[Q^r_{,3}(t) - W\right] dt \text{where,} \quad (\mathbf{14})$$

$$\mathrm{CE}_{1,3}(0) = -K + \frac{\rho}{2(\pi_x + \pi_s)} W^2 + P_{,2} W, \quad (\mathbf{15})$$

where K is the cost paid by dealer 1 to enter the inter-dealer market, that is, K is equal to zero, when dealer 1 trades bilaterally and is equal to F, when dealer 1 trades through the inter-dealer broker and where

$$Q^r_{,3}(Z) = \begin{cases} Q^d_{,3}(Z) & \text{when dealer 1 trades directly,} \\ Q^b_{,3}(Z; M) & \text{when dealer 1 trades indirectly.} \end{cases} \quad (\mathbf{16})$$

Notice that $Q^d_{,3}(Z) = \frac{\pi_s}{\rho} \tau Z$ and $Q^b_{,3}(Z; M) = \frac{\pi_s}{\rho}(1 - 2\mathrm{T})Z$ are simple rearrangements of the equilibrium orders given in (**5**) and (**9**), respectively. Notice also, that although the equilibrium order placed in the brokered market depends on the number of market makers, the equilibrium order placed in the direct market does not. Moreover, the constant

[18] See also Madhavan (1992) where the price dynamics of a quote driven system are compared to those of a continuous order-driven system.

[19] It is well known, that maximizing the expected value a negative exponential utility function, $\mathrm{U}(\mathcal{W}) = -\exp(-\rho\mathcal{W})$, conditional on some random variable I, is equivalent to the maximization of the quadratic function $\mathrm{E}(\mathcal{W}|I)\mathcal{W} - 0.5\rho\mathrm{Var}(\mathcal{W}|I)\mathcal{W}^2$, the maximized value of which we refer to as the 'certainty equivalence'.

term $\mathrm{CE}_{1,3}(0) + K$, is the certainty equivalence dealer 1 receives if there is no inter-dealer trading at Stage 3, that is, if the information inferred by the dealers who did not trade with the public at Stage 2 is equal to the unconditional mean of the asset.

For a fixed number of dealers, we can now determine dealer 1's choice of inter-dealer market, by subtracting the certainty equivalence she would attain if she chose to trade in the direct market, $\mathrm{CE}^d_{1,3}(Z)$, from the certainty equivalence she would attain if she chose to trade in the brokered market, $\mathrm{CE}^b_{,3}(Z)$. We thus obtain

$$\mathrm{CE}^b_{1,3}(Z) - \mathrm{CE}^d_{1,3}(Z) = -F + \frac{\pi_s}{\pi_x + \pi_s} \int_0^Z \left(Q^b_{,3}(t) - Q^d_{,3}(t)\right) dt. \quad \mathbf{(17)}$$

Substituting equations (**9**) and (**5**) into (**17**) and recalling that $\frac{F}{W^2}$ is close to zero, we can easily show that dealer 1 expects to achieve the higher expected surplus in the inter- dealer market where she can lay-off more of her original order.[20] The following proposition follows:

Proposition 1 *We let* $A = \frac{\pi_s(\pi_x + \pi_s)}{\pi_x}$ *and* $B = \frac{\rho^2}{\pi_w}$ *and we define the integer* M^* *to be*

$$M^* = 1 + \mathrm{Int}\left[\frac{C + \sqrt{C^2 - 8}}{2}\right] \quad \textit{where,} \quad \mathbf{(18)}$$

$$C = \frac{6AB + 2AB\pi_s{}^{-1}(2B - A)}{(B - A)(AB\pi_s^{-1} + A)}. \quad \mathbf{(19)}$$

Suppose that both market opening conditions (8) and (13) are satisfied. Then, the following statements hold:
(a) When the number of market makers who did not trade in the second stage M*, is less than* M^**, dealer 1 trades in the direct market. When* M *is greater or equal to* M^**, dealer 1 trades through the inter-dealer broker.*
(b) When $M = \frac{C + \sqrt{C^2 - 8}}{2}$*, dealer* 1 *is indifferent between inter-dealer markets.*
(c) When there is no information asymmetry in the market ($\pi_s = 0$)*, or when the market is completely transparent* ($\frac{1}{\pi_w} = 0$) M^* *reaches its lower bound, that is,* $M^* = 4$.

[20] See the proof of Proposition 1 for an explicit demonstration of this statement.

Proposition 1 is a direct consequence of the difference in the institutional make-up of the two inter-dealer markets. Trading in the direct market benefits dealer 1, to the extent that the positive effect of price competition between dealers outweighs the negative effect of the inefficient risk-sharing arising from bilateral trading. Conversely, trading through the inter-dealer broker benefits dealer 1, to the extent that the positive effect of risk-sharing with all the other M dealers outweighs the negative effect of imperfect competition. For every configuration of the initial parameters, there exists a critical number of market makers $M^* + 1$, such that if the number of dealers is equal or above $M^* + 1$ inter-dealer trading is mediated by the broker, whereas if the number of dealers is below $M^* + 1$ inter-dealer trading occurs directly.

In the unlikely circumstances where the positive effect of price competition between dealers exactly offsets the negative effect of inefficient risk sharing, dealer 1 is indifferent between markets. If the equilibrium concept used does not rule out mixed strategies, in such a case, the two inter-dealer markets coexist and dealer 1 chooses either of them with equal probabilities. However, this 'knife-edge' equilibrium is not robust to slight perturbations in the initial parameters of the model.[21]

Finally, Proposition 1 provides us with a lower bound in the minimum number of dealers required for the brokered market to prevail. This lower bound is reached when there is no private information or when market transparency is complete.[22] Notice that both $\pi_s = 0$ and $\frac{1}{\pi_w} = 0$ imply respectively, that dealer 1 does not have, or is not able to retain any private information about the value of the asset.

Turning to comparative statics, we obtain the following Corollary to Proposition 1:

Corollary 1 *Increases in the precision of dealer* 1*'s private information, π_s, and in the transparency of the market, π_w, increase the minimum number of dealers necessary for the brokered market to prevail, M^*. Increases in the precision of the public information, π_x, and the risk-aversion coefficient, ρ, decrease M^*.*

[21] See part (b) in the Proof of Proposition 2 in the Appendix, for a further discussion of this case. See also the paper by Pagano (1989) where the tendency of markets to consolidate or to fragment is examined in a model with no information asymmetries.

[22] Recall from the previous two subsections that when $\pi_s = 0$, $\tau = 0.5$ and $1 - 2\mathrm{T} = \frac{M(M-2)}{2(M-1)+M(M-2)}$. By subtracting the latter from the former we can check that in the absence of information asymmetry dealer 1 prefers the brokered market as long as $M > 4$.

Suppose that exactly M^*+1 dealers have entered the market-making of the security. Then, from Proposition 1, it follows that inter-dealer trading is mediated by the broker. Now suppose that an increase in the precision of private information, π_s, or an increase in market transparency, π_w, takes place. From our discussion of the two possible Stage 3 equilibria, it follows that either of these increases results in decreasing the depth of both inter-dealer markets, where the term 'market depth' has the standard Kyle (1985) definition of the sensitivity of prices to order flow. That is, *ceteris paribus*, dealers respond to increases in private information, or market transparency by posting higher price schedules in the direct market and submitting smaller market orders to the inter-dealer broker.[23] Each strategic dealer, however, is more sensitive to increases in information asymmetry, or market transparency than her price-taking counterpart. Consequently, the increase in information asymmetry, or transparency results in dealer 1 decreasing her Stage 3 optimal order to a greater extent when inter-dealer trading is conducted through the broker, than when inter-dealer trading is direct. As a result, the brokered market can only continue to prevail, if the relative loss in its liquidity is more than compensated by a sufficient increase in its absorbing capacity, corresponding to a sufficient increase in the number of dealers who enter the market-making of the security.

Corollary 1 also shows that as public information or the risk-aversion coefficient increase, M^* decreases. An increase in public information reduces information asymmetry having an effect in the same direction as a decrease in the precision of private information. Given that strategic dealers respond more aggressively to information asymmetry than price-taking ones, increases in public information reduce the minimum number of dealers required for the inter-dealer broker system to prevail. An increase in the risk-aversion coefficient has an effect in the same direction as a decrease in market transparency, as it implies that dealer 1 can disguise her private information better. Further, increases in the risk-aversion coefficient imply decreases in the risk-bearing capacity of each individual dealer. These in turn impose a greater toll on the absorbing capacity of the bilateral market than on that of the multilateral one.

[23] Note that the modeling of the inter-dealer broker system does not preclude the possibility of limit orders. In equilibrium, however, dealers know the price of the asset and will only submit market orders (see also discussion in Kyle (1989)).

4 STAGE 2: TRADING WITH THE PUBLIC

Having established the solution to the third stage problem, we turn to the dealers' price setting problem at Stage 2.

Long-run price competition between the $M+1$ dealers and the fact that bargaining power rests with the public imply that dealers set their price schedule, such that their expected gain from trading is equal to their expected gain from not trading. From our discussion of inter-dealer trading above, we know that dealers' payoffs depend on the number of dealers who enter the market. In particular, pure price competition and Proposition 1 jointly imply that dealer 1 sets a price such that

$$\mathrm{CE}_{1,3}(Z) = \begin{cases} 0 & \text{when } M+1 < M^* + 1, \\ \mathrm{CE}_{-1,3}(Z;M) & \text{when } M+1 \geq M^* + 1, \end{cases} \tag{20}$$

where $\mathrm{CE}_{1,3}(Z)$ is given by equation (**14**) and $\mathrm{CE}_{-1,3}(Z;M)$ denotes the per capita certainty equivalence obtained by the dealers who do not trade with the public at the second stage, when inter-dealer trading is conducted through the inter-dealer broker. In particular, we obtain the dealers' per capita certainty equivalence when $M \geq M^*$ by substituting the equilibrium trade and price, given in equations (**10**) and (**11**) respectively, into the expected utility function of each dealer who does not trade with the public at Stage 2. This yields

$$\mathrm{CE}_{-1,3}(Z;M) = -F + \frac{\pi_s}{\rho}\left(\frac{1-2\mathrm{T}}{M}\right) \times \left(\frac{\pi_s}{\pi_x+\pi_s}\mathrm{T} - \frac{\pi_z}{\pi_x+\pi_z} - \frac{\pi_s}{2(\pi_x+\pi_z)}\frac{1-2\mathrm{T}}{M}\right) Z^2, \tag{21}$$

where $(1-2\mathrm{T})$ is given by equation (**12**).

The proposition below characterizes the outcome of the negotiation between dealer 1 and the investor:

Proposition 2 *Suppose that both market opening conditions (8) and (13) are satisfied. Then, at the second stage of the game the investor* *obtains a price $P_{,2}$ given by*

$$P_{,2} = \begin{cases} \dfrac{\pi_s}{\pi_x + \pi_s}(1-\tau)S + \dfrac{\rho}{2(\pi_x+\pi_s)}(1-\tau)W \\ \quad - 0.5\dfrac{\pi_s^2}{\rho(\pi_x+\pi_s)}\tau\dfrac{S^2}{W} & \text{when } M < M^*, \\ \dfrac{\pi_s}{\pi_x + \pi_s}2\mathrm{T}S + \dfrac{\rho}{2(\pi_x+\pi_s)}2\mathrm{T}W \\ \quad - 0.5\dfrac{\pi_s^2}{\rho(\pi_x+\pi_s)}(1-2\mathrm{T})\dfrac{S^2}{W} \\ \quad + \dfrac{1}{W}\left(\mathrm{CE}_{-1,3} + F\right) & \text{when } M \geq M^* \end{cases} \tag{22}$$

and where $\mathrm{CE}_{-1,3}$ *is given by (**21**).*

It is straightforward to demonstrate that the pricing equations given in (**22**) are concave in S (see Appendix). From Jensen's inequality it therefore follows, that an investor with a choice of a precommitment to reveal, or not to reveal S will reveal S.

Proposition 2 delineates the link between the choice of inter-dealer market and the prices obtained by the public. In the previous section we showed that, at Stage 3, dealer 1 acts monopsonistically and only buys back a fraction of her trade, W. It follows, that at the end of the trading game dealer 1 stays short of the asset by a fraction $(1-\tau)$ when $M < M^*$ and by a fraction 2T when $M \geq M^*$. The first term of the pricing equations is the compensation dealer 1 requires in order to stay short of the asset, when she knows it has been updated from 0 to $\dfrac{\pi_s}{\pi_x+\pi_s}S$. The second term of the pricing equations is the compensation she requires to bear inventory risk for the fraction of the asset she does not buy back.

The third term of the pricing equations is the 'payment' for early order flow. Owing to the lack of post-trade transparency, dealer 1 has an incentive to 'pay' the investor for the information she reveals and recoup later through inter-dealer trading. Indeed, given the normality assumption, in circumstances where the investor is sufficiently well-informed but has few liquidity needs (S is large relative to W), dealers would set negative prices.

The fourth term of the second pricing equation compensates dealer 1 for the opportunity cost of completing a trade with the public. Clearly, the opportunity cost of trading with the public is positive only when inter-dealer trading is conducted through the broker, that is, only when more than M^* dealers enter the market at Stage 1. One can verify that as long as inequality (**13**) is preserved, that is the brokered market is open, the term in brackets in equation (**21**) is non negative, that is, $\mathrm{CE}_{-1,3} + F \geq 0$.[24] Indeed, as we discuss in the next section, it is the size of this positive surplus which determines the existence and features of an equilibrium number of dealers exceeding $M^* + 1$.[25]

5 STAGE 1: ENTRY INTO MARKET-MAKING

Our analysis of Stage 3 trading suggests that, given a set of exogenous parameters, we can predict where inter-dealer trading will take place, as long as we know how many dealers have entered the active market-making of the security. Moreover, our analysis of Stage 2 trading suggests that the price obtained by the public, when market makers have a choice of inter-dealer trading venue, may differ from the price the public would have obtained, in the absence of such a choice.

In the first part of this section, we discuss the factors that determine the number of dealers who enter into active market-making. In the second part of the section, we focus on the implications for the public of market makers having a choice between inter-dealer trading markets.

5.1 The equilibrium number of dealers

In our discussion of Stage 2 trading, we argued that if the number of dealers in the market exceeds or is equal to $M^* + 1$, each dealer attains an expected utility equal to $-\exp(-\rho \mathrm{CE}_{-1,3})$, whereas if the number of dealers is between 3 and $M^* + 1$, each dealer attains her reservation utility of -1.

It follows that if an equilibrium number of active dealers greater or equal to $M^* + 1$ exists it is given by the greatest positive integer $M + 1 \geq M^* + 1$, such that the expected utility of those agents who

[24] See the proof of Proposition 3 in the Appendix, for an explicit demonstration of this result.

[25] Naik et al. (1994) obtain a pricing rule similar to (**22**) in their two period model of an opaque dealership market. In their model, inter-dealer trading is conducted multilaterally, costlessly and competitively (dealers receive their reservation utility). Unlike our paper, the focus of their analysis is a comparison of the prices obtained by an investor in a standard auction market and a dealership market with or without disclosure.

decide to become dealers is at least as large as the expected utility of those agents who do not enter the industry, that is

$$\mathrm{E}\left[-\exp(-\rho \mathrm{CE}_{-1,3})\right] \geq -1. \qquad \mathbf{(23)}$$

Substituting (**21**) into (**23**) and rearranging we obtain

$$\mathrm{E}\left[\exp(-\rho P Z^2)\right] \leq \exp(-\rho F) \qquad \text{where,} \qquad \mathbf{(24)}$$

$$P = \frac{\pi_s}{\rho}\left(\frac{1-2\mathrm{T}}{M}\right) \times \left(\frac{\pi_s}{\pi_x+\pi_s}\mathrm{T} - \frac{\pi_z}{\pi_x+\pi_z} - \frac{\pi_s}{2(\pi_x+\pi_z)}\frac{1-2\mathrm{T}}{M}\right). \qquad \mathbf{(25)}$$

Recalling that Z is normally distributed, we apply the standard formula for the moment generating function of a $\mathcal{X}^2$ variable to rewrite (**24**) as (Mood and Graybill (1963), page 226)

$$\left(1+2\rho P \sigma_z^2\right)^{-\frac{1}{2}} \leq \exp(-\rho F), \qquad \mathbf{(26)}$$

where $\sigma_z^2 = \mathrm{Var}(Z) = \dfrac{\pi_s \pi_w(\pi_x+\pi_s)+\rho^2\pi_x}{\pi_s^2\pi_w\pi_x}$. The expression is then rearranged as

$$1+2\rho P\sigma_z^2 \geq \exp(2\rho F). \qquad \mathbf{(27)}$$

Using Taylor's expansion on the right hand side of (**27**) and ignoring terms of order $O((\rho F)^2)$ the equilibrium condition is simplified to

$$\Pi = P\sigma_z^2 \geq F. \qquad \mathbf{(28)}$$

The approximation linearizes the preferences of the dealers over their expected surplus but does not remove the effect of risk aversion which is incorporated in the derivation of the per capita certainty equivalence $\mathrm{CE}_{-1,3}$. The approximation, however, is not valid for large coefficients of risk aversion or large F.

A necessary and sufficient condition for an equilibrium number of dealers exceeding M^*+1 to exist is described in the following proposition:

Proposition 3 *Suppose that both market opening conditions (8) and (13) are satisfied. Then, the brokered inter-dealer market prevails, that is, an equilibrium number of dealers $M+1 \geq M^*+1$ exists, if the per capita expected surplus, Π, evaluated at M^*+1, exceeds or is equal to the brokerage fee, F. Formally, the brokered market prevails if*

$$\Pi|_{M=M^*+1} \geq F. \qquad \mathbf{(29)}$$

*If the per capita expected surplus, evaluated at $M^* + 1$, is less than the brokerage fee (that is, (**29**)* is violated) and

$$\left.\frac{\partial \Pi}{\partial M}\right|_{M=M^*+1} \leq 0, \qquad (\mathbf{30})$$

then an equilibrium number of dealers $M + 1 \geq M^* + 1$ does not exist and the direct inter-dealer market prevails.

In the Appendix we show that dealers' per capita expected surplus, Π, given by the left hand side of (**28**), is either decreasing, or unimodal in the number of liquidity supplying speculators, depending on the choice of the other exogenous parameters. Adding extra speculators in the brokered market, not only cuts speculators' inventory costs by spreading inventory risks, but also lowers the price volatility of the asset[26] and decreases the market power of each individual speculator. As agents in general 'seek out' price risk the latter effect decreases dealers' expected surplus.[27] When the pool of speculators participating in the brokered market is small, under particular parameters specifications, the entry of an extra speculator causes the former effect to dominate. When the pool of speculators is sufficiently large, however, the latter effect always dominates.

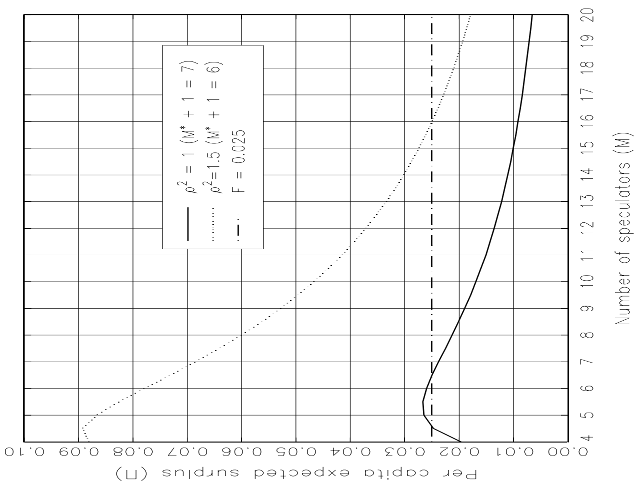
Figure 1 shows the variation of dealers' per capita expected surplus, Π, with the number of speculators M, for a choice of the exogenous parameters F, π_x, π_s, π_w and two choices of the risk-aversion coefficient, ρ. Recall that, by Proposition 1, the minimum number of speculators required for dealers to expect a positive per capita surplus, Π, is equal to 4. Note also that the parameters must satisfy the market opening condition (**13**) and the coefficients of risk-aversion and the brokerage fee chosen must validate approximation (**28**). Observe that the broken line curve is decreasing in the number of speculators M, whereas the solid curve is unimodal in M. Moreover, as predicted by Corollary 1, an increase in the risk-aversion coefficient, ρ, decreases the number of speculators M^* necessary for the brokered market to prevail.

It is clear from Figure 1, that as long as the surplus each dealer expects to extract, when exactly M^*+1 dealers enter the market-making industry, exceeds or is equal to the brokerage fee, F, the equilibrium

[26] This can be verified by calculating the variance of price given in (**11**) and taking its partial derivative with respect to M.

[27] Recall the standard result, that indirect utility functions are quasi-convex in price.

Figure 1: Per capita expected surplus vs number of speculators ($\pi_x = \frac{1}{3}$, $\pi_s = 1$, $\pi_w = 0.1$)



number of dealers will be at least as great as $M^* + 1$. This is not the case for the parameters specification yielding the solid curve, but is clearly the case for the parameter specification yielding the broken line curve. Also notice that since the monotonicity condition (**30**) is satisfied by both parameter specifications condition (**29**) is not only sufficient but also necessary. Thus, under the parameter specification yielding the solid curve, we can predict that inter-dealer trading will be conducted bilaterally.[28]

[28] Note that our modeling of the direct inter-dealer market does not allow us to determine the exact number of dealers who enter the market-making industry when (**30**) is satisfied and (**29**) is violated. Note also that there also exist circumstances when both inter-dealer markets coexist and trading may take place in either of them with equal probabilities. This of course occurs in the 'knife-edge' case, where the largest integer satisfying inequality (**28**) is equal to $M = 0.5\left[C + \sqrt{C^2 - 8}\right]$ (see

In the special case where there is no adverse selection, the equilibrium condition given in (**28**) reduces to

$$\Pi|_{\pi_s=0} = \frac{\rho M(M-2)}{2\pi_w \pi_x (M^2-2)^2} \geq F. \tag{31}$$

That is, the equilibrium number of dealers $M + 1 \geq 5$, is the greatest positive integer satisfying (**31**). We thus obtain the following corollary to Proposition 3.

Corollary 2 *In the absence of private information, there exists an equilibrium number of dealers $M^* + 1 \geq 5$ if and only if*

$$\frac{10}{529} \frac{\rho}{\pi_w \pi_x} \geq F. \tag{32}$$

The equilibrium number of dealers is increasing in the risk-aversion coefficient, ρ, the ex-ante volatility of the asset, $\frac{1}{\pi_x}$, the variance of the public's endowment, $\frac{1}{\pi_w}$, and is decreasing in the brokerage fee, F.

By Proposition 1 we know that in the absence of private information M^* attains its minimum value 4. This reduces inequality (**29**) to the simple inequality (**32**) of Corollary 2. Moreover, in the absence of adverse selection, inequality (**30**) is always satisfied. The entry of an extra speculator in the inter-dealer broker market always decreases dealers' per capita expected surplus.[29]

In the absence of information asymmetries, it is easy to conduct a comparative statics analysis on the equilibrium number of dealers. As long as the brokered market prevails, that is, (**32**) is satisfied, an increase in dealers' per capita expected surplus, given by the left hand side of (**31**), leads to an increase in the equilibrium number of dealers who enter the market-making industry. We can see that increases in the left hand side of (**31**) can be effected by increases in risk-aversion, the *ex ante* volatility of the asset and the liquidity requirements of the public. Similarly, as long as (**32**) is satisfied, decreases in the brokerage fee, F,

part (b) of Proposition 1). Given that the circumstances that will give rise to such a case are very unlikely we do not discuss this case further.

[29] Observe also, that in the absence of information asymmetry, the market is no longer opaque. Dealer 1, unable to extract any private information from trading with the public, is unable to conceal her Stage 2 trade W. The parameter π_w^{-1} while still capturing the degree of liquidity trading in the market can no longer be interpreted as the degree of market transparency.

increase the right hand side of (**31**) and decrease the equilibrium number of dealers entering the market-making industry. These results agree with those predicted by the models of Biais (1993) and Stoll (1978(a)) and have found empirical support in the work of Stoll (1978 (b)).

When inequality (**32**) is violated, Corollary 2 implies that fewer than 5 dealers enter the market-making industry. That is, in the absence of information asymmetry, a sufficient increase in the brokerage fee, or sufficient decreases in dealers' aversion to risk, in the volatility of the asset, or in the public's liquidity needs can lead to a shift of inter-dealer trading from the brokered market to the direct inter-dealer market and vice versa.

It would be interesting to conduct a similar equilibrium analysis in the presence of information asymmetry. Although, it is straightforward to see that, as long as (**29**) is satisfied, a decrease in F increases the equilibrium number of dealers, the comparative statics effects of the other, more interesting exogenous parameters are difficult to derive.

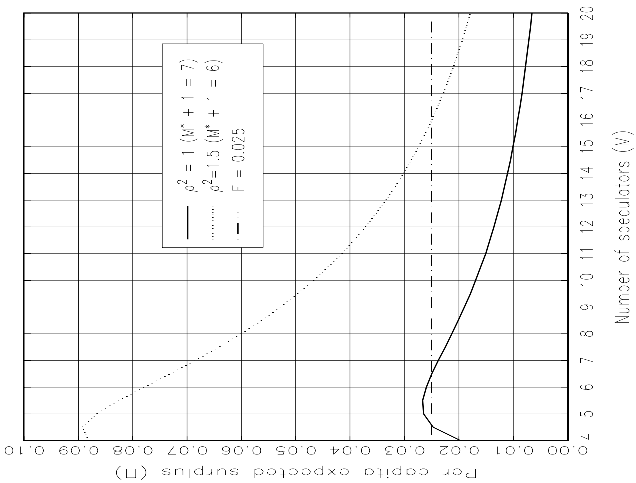
It is possible, however, to show that the results of Corollary 2 are not generalizable. For example, Figure 2 shows the variation of the per capita expected surplus, Π, with the precision of public information, π_x, for a choice of the initial parameters π_s and ρ and two choices of M and π_w. Notice that the parameters chosen must satisfy the market opening condition (**13**). In addition, since dealers can only expect a positive surplus if the brokered inter-dealer market prevails, our choice of M must be greater than M^* across the range of parameters chosen. From the Figure it is clear that in the presence of information asymmetry, there exist non empty parameter spaces such that the per capita expected surplus is no longer monotonically increasing in the unconditional volatility of the asset, $\dfrac{1}{\pi_x}$.

Similarly, when information asymmetry is present, the circumstances under which condition (**29**) ceases to hold are more difficult to establish. Obviously, for a sufficiently large brokerage fee, F, inequality (**29**) is violated. Derivation of the other comparative statics effects, however, involves the calculation of the following 4×1 vector

$$\frac{\mathrm{d}\,\Pi|_{M=M^*+1}}{\mathrm{d}\mathbf{p}} = \left.\frac{\partial \Pi}{\partial M}\right|_{M=M^*+1} \frac{\mathrm{d}M^*}{\mathrm{d}\mathbf{p}} + \left.\frac{\partial \Pi}{\partial \mathbf{p}}\right|_{M=M^*+1}, \qquad (\mathbf{33})$$

where $\mathbf{p} = (\pi_s, \pi_w, \rho, \pi_x)$ is the vector of the initial parameters. Once more, we see that the results of Corollary 1 are not generalizable. For

Figure 2: Per capita expected surplus vs precision of public information ($\pi_s = 0.2,\ \rho = 1$)



example, monotonicity constraint (**30**) and Corollary 1, imply that

$$\left.\frac{\partial \Pi}{\partial M}\right|_{M=M^*+1} \frac{\partial M^*}{\partial \pi_x} \geq 0. \tag{\textbf{34}}$$

Our discussion of Figure 2 implies that there exist non empty parameter spaces such that a decrease in the unconditional volatility of the asset leads to an increase in dealers' per capita expected surplus, that is, $\frac{\partial \Pi}{\partial \pi_x} > 0$. From (**33**) it thus follows, that there exist non empty parameter spaces such that a decrease in the unconditional volatility of the asset may lead to an increase in the left hand side of (**29**) and thus to a shift of inter-dealer trading to the brokered market.

5.2 Expected transaction costs and price volatility when dealers have a choice between inter-dealer markets

We now turn our attention to the implications of a choice in inter-dealer trading venues for the public. By definition, dealership markets involve bilateral trading amongst dealers and between dealers and the public. Is the public, however, better-off when dealers can also trade with each other through a broker, having, thus, access to an order-driven trading mechanism?

Suppose that the inter-dealer brokered market did not exist. That is, all trading was conducted bilaterally. Suppose also that $M^e \geq M^* + 1$ dealers have entered the market-making of the security. It follows from Proposition 2 that *ex ante* to receiving any private information an outside investor wishing to trade W units of the security expects to be able to negotiate the following unit price

$$\mathrm{E}(P_{,2}) = \frac{\rho}{2(\pi_x + \pi_s)}(1-\tau)W - \frac{\pi_s}{2\rho(\pi_x + \pi_s)}\frac{\tau}{W}, \qquad \textbf{(35)}$$

with volatility

$$\mathrm{Var}(P_{,2}) = \frac{\pi_s}{(\pi_x + \pi_s)^2}(1-\tau)^2 + \frac{\pi_s^2}{2\rho^2(\pi_x + \pi_s)^2}\frac{\tau^2}{W^2}. \qquad \textbf{(36)}$$

Now suppose that an inter-dealer brokered system is introduced. Given that $M^e \geq M^* + 1$, Proposition 1 predicts that the brokered system will prevail over the direct inter-dealer system. From Proposition 2 and equilibrium condition (**28**), it then follows that the investor expects to negotiate a price given by

$$\begin{aligned} \mathrm{E}(P_{,2}) &= \frac{\rho}{2(\pi_x + \pi_s)}2\mathrm{T}W \\ &\quad - \frac{\pi_s}{2\rho(\pi_x + \pi_s)}\frac{(1-2\mathrm{T})}{W} + \frac{F^+}{W} \quad \text{where,} \qquad \textbf{(37)} \\ F^+ &= F - \Pi|_{M=M^e}, \qquad \textbf{(38)} \end{aligned}$$

with volatility

$$\mathrm{Var}(P_{,2}) = \frac{\pi_s}{(\pi_x + \pi_s)^2}(2\mathrm{T})^2 + \frac{\pi_s^2}{2\rho^2(\pi_x + \pi_s)^2}\frac{(1-2\mathrm{T})^2}{W^2} + \frac{2(P\sigma_z^2)^2}{W^2}. \qquad \textbf{(39)}$$

Recall from our discussion of dealer 1's choice of inter-dealer market that dealer 1 chooses to trade in the inter-dealer market where she can execute the order of the greater magnitude. It follows from equations (**5**) and (**9**) that dealer 1 chooses to trade in the inter-dealer brokered market as long as $\tau < (1 - 2\mathrm{T})$. By inspection of equations (**35**) and (**37**), (**36**) and (**39**), we can see that, as long as the liquidity needs of the investor are large ($\mid W \mid \gg 0$), the introduction of the brokered system decreases both the mean and the variance of price. Note also, that when the equilibrium number of dealers M^e is less than $M^* + 1$, investors expect to gain the same utility regardless of whether dealers have access to the order-driven mechanism or not.

We thus obtain the following proposition:

Proposition 4 *Suppose investors have mean-variance utility functions. Then ex ante to receiving private information, investors who wish to execute large orders for liquidity reasons (that is, investors with large endowments* $\mid W \mid$*) are either better-off or no worse-off when dealers can choose between inter-dealer trading venues.*

Proposition 4 follows from the fact that the bargaining power between the dealer and the customer is assumed to rest with the customer. This implies that, as long as the customer's order is large enough so that the per unit fixed costs are insignificant, the greater benefits dealers can derive from an expanded choice set are passed on to the customers, in the form of lower expected prices. Moreover, the customer expects that an expansion of the dealers' choice set will lead to a reduction in price volatility, as long as the components of price volatility which relate to the discount she obtains in return for her private information (second component of equations (**36**) and (**39**)) and to the volatility in dealers' per capita expected surplus (third component of (**39**)) are insignificant relative to the size of her order. This reduction occurs via the component of price volatility which relates to the volatility of the fraction of the order retained by the dealer after the completion of the inter-dealer trading session (first component of equations (**36**) and (**39**)).

What about investors wishing to execute small orders? As long as the ratio $\dfrac{F^+}{\mid W \mid}$ is sufficiently small, it is easy to check that the introduction of the brokered system lowers expected transaction costs, but –for sufficiently small $\mid W \mid$– at the cost of increased price volatility.

6 CONCLUSION

An interesting feature of opaque dealership markets is that an investor negotiating a price with a dealer has an incentive to precommit to reveal her information. The dealer, on the other hand, has an incentive to compensate the investor for the information she reveals, as she can recoup later through inter-dealer trading. Moreover, the dealer has a choice between inter-dealer trading venues.

In this paper, we focus on a dealer, who having completed an undisclosed order with an informed investor, chooses where to unwind her inventory. We suppose that the dealer can choose between trading in a typical quote-driven inter-dealer market and a continuous order-driven market, run by a broker. This choice reflects the existence of inter-dealer brokers in a number of opaque quote-driven markets, such as the foreign exchange market, the UK government bond market and the London Stock Exchange. Although the information released through the dealer's order flow is identical, the difference in the market structure of the two inter-dealer markets results in different equilibria. We show that the dealer chooses to unwind her inventory in the market where she can execute the largest order. Given a set of exogenous parameters, we predict that the dealer trades through the broker if the number of dealers who have entered the market-making of the security exceeds a critical number M^*. This is shown to be increasing in information asymmetry and transparency and decreasing in risk-aversion. Otherwise, the dealer trades directly at the quotes of another dealer.

In equilibrium, the price a dealer negotiates with the public is such that her expected utility from trading is equal to her expected utility from not trading. The choice of inter-dealer market, therefore, determines dealers' payoffs. When inter-dealer trading is conducted bilaterally, dealers expect to receive their reservation utility. Otherwise, they receive a positive expected gain. Since trading through the broker is costly, the equilibrium number of dealers who enter the market-making of the security, when inter-dealer trading is conducted through the broker, is, thus, endogenously derived.

In the absence of information asymmetry we show that sufficient increases in the risk-aversion coefficient, the volatility of the asset and liquidity trading shift inter-dealer trading from the direct market to the broker. This is an empirically testable implication of the model, applicable to markets where trading on private information is not significant, such as the U.K. government bond market (see Proudman (1995) for empirical evidence that the trading process in gilts markets reveals little information). These results are shown not to be generalizable, however, when information asymmetry is present.

When dealers have a choice of inter-dealer trading venue, large investors expect at least as low transaction costs and price volatility as when they do not. This may explain the wide-spread use of these two similar inter-dealer mechanisms in a variety of opaque quote-driven markets catering for large (institutional) investors. In general, however, the model predicts that, as long as the brokerage fee is low relative to the size of the order, all investors can expect at least as low transaction costs when dealers can choose where to conduct their inter-dealer trading as when they do not. For sufficiently small orders, however, price volatility may increase.

Further research could test empirically the connection between the choice of inter-dealer trading venue, the number of dealers actively making a market in the security, the degree of transparency of the market and the underlying information structure. In view of the recent emergence of order-driven electronic systems running in parallel with the traditional quote-driven ones (for example *Tradepoint* running in parallel with the London Stock Exchange), theoretical research could analyse the relationship between informed investors' conjectures about the number of agents submitting orders to the order-driven systems, their liquidity needs and their choice of trading venue.

7 APPENDIX

In all the proofs that follow, we assume that dealer 1 buys a a quantity $Q^d_{,3} \geq 0$ at Stage 3 and sells a quantity $W \geq 0$ at Stage 1. Due to symmetry, the proofs are analogous when the signs of the transactions are changed.

Section 3.1

Proof of direct inter-dealer market equilibrium: Each of the M dealers who did not trade with the public at Stage 2 sets a price schedule given by equation (4). Substituting into (4) the expressions for the conditional expectation and conditional variance, we obtain

$$P^d_{,3} = \frac{\pi_z}{\pi_x + \pi_z} Z + \frac{\rho}{2(\pi_x + \pi_z)} Q^d_{,3}. \tag{40}$$

Substituting into equation (**40**) the expression for Z given in equation (**3**), we obtain a first order differential equation in $P^d_{,3}$, which we can write as

$$P^{d'}_{,3}(Q^d_{,3}) + \frac{P^d_{,3}(Q^d_{,3})}{Q^d_{,3}} \left(1 - \frac{\pi_s(\pi_x + \pi_z)}{\pi_z(\pi_x + \pi_s)}\right) = -\frac{\rho(\pi_s + 2\pi_z)}{2\pi_z(\pi_x + \pi_s)}. \tag{41}$$

Multiplying both sides of the equation by the integrating factor and taking integrals yields

$$P^d_{,3}(Q^d_{,3}) = \frac{\rho(\pi_s + 2\pi_z)}{2\pi_s(\pi_x + \pi_z) - 4\pi_z(\pi_x + \pi_s)} Q^d_{,3} + D(Q^d_{,3})^{-1 + \frac{\pi_s(\pi_x + \pi_z)}{\pi_z(\pi_x + \pi_s)}}, \tag{42}$$

where D is an arbitrary constant.

In order to restrict the pricing rules that obtain in equilibrium, we employ the second order condition of dealer 1's maximization problem. In particular, dealer 1's second order condition is given by

$$P^{d''}_{,3}(Q^d_{,3}) + 2P^{d'}_{,3}(Q^d_{,3}) + \frac{\rho}{\pi_x + \pi_s} > 0. \tag{43}$$

Substituting (**42**) into (**43**) and rearranging we obtain

$$\alpha(\alpha - 1)D(Q^d_{,3})^{\alpha - 2} + 2\lambda + \frac{\rho}{\pi_x + \pi_s} > 0, \tag{44}$$

where we have let $\alpha = \frac{\pi_s(\pi_x + \pi_z)}{\pi_z(\pi_x + \pi_s)}$ and $\lambda = \frac{\rho(\pi_s + 2\pi_z)}{2\pi_s(\pi_x + \pi_z) - 4\pi_z(\pi_x + \pi_s)}$. It is clear that $\alpha > 2$ if and only if $\lambda > 0$. We distinguish between the following cases:

Case 1(a): Assume $\alpha > 2$ (which implies $\lambda > 0$) and $D > 0$. Clearly, the second order condition (**44**) is satisfied for all $Q^d_{,3} > 0$.
Case 1(b): Assume $\alpha > 2$ and $D < 0$. Then the first term of (**44**) is negative, whereas the other two terms are positive. We can see that there exists an order of size $Q^* > 0$ such that for all $Q^d_{,3} \geq Q^*$ the second order condition is violated.
Case 2(a): Assume $\alpha < 2$ (which implies $\lambda < 0$) and $D > 0$. The same argument as in Case 2(a) applies.
Case 2(b): Assume $\alpha < 2$ and $D < 0$. This implies that $P^d_{,3}(-Q^d_{,3}) > P^d_{,3}(Q^d_{,3})$, that is that dealers set negative bid-ask spreads. By standard arbitrage such negative spreads are going to be eliminated.

It follows, that only if Case 1(a) obtains, the second order condition is always satisfied, that is we require that $\alpha > 2$ ($\lambda > 0$) and $D > 0$. Finally, we observe that the pricing rules defined by equation (**42**) are bounded below by the linear pricing rule. In equilibrium, Bertrand competition ensures, that dealers set the lowest possible price schedule such that they can expect to obtain their reservation utility. As a result, the price set by the M dealers is given by

$$P^d_{,3}(Q^d_{,3}) = \lambda Q^d_{,3} \quad \text{where,} \tag{45}$$

$$\lambda = \frac{\rho\left(\rho^2 + 3\pi_s\pi_w\right)}{2\pi_x\rho^2 - 2\pi_s\pi_w(\pi_x + \pi_s)} \quad \text{as long as,} \tag{46}$$

$$\lambda > 0. \tag{47}$$

We can see that condition (**47**) reduces to condition (**8**) in the main text.

In order to obtain the equilibrium order $Q^d_{,3}$, we notice that dealer 1's first order condition given in equation (**3**) can be rearranged to yield

$$Q^d_{,3} = \frac{Z - \frac{\pi_x+\pi_s}{\pi_s}P^d_{,3}}{\frac{\rho}{\pi_s} + \frac{\pi_x+\pi_s}{\pi_s}P^{d\prime}_{,3}}. \tag{48}$$

Substituting in the equilibrium pricing rule we obtain

$$Q^d_{,3} = \frac{\rho}{\pi_s}\tau Z \quad \text{where} \tag{49}$$

$$\tau = \frac{1}{2\lambda + \frac{\rho}{\pi_x+\pi_s}}. \tag{50}$$

Substituting into τ the expression for λ in (**46**) yields expression (**7**) in the text. Substituting the definition of Z in (**49**), we obtain equation

(**5**) in the main text. Finally, from equation (**50**), we note that $\lambda = \frac{\rho}{2(\pi_x + \pi_s)} \frac{1-\tau}{\tau}$; substituting this and (**49**) into (**45**) yields equation (**6**) in the main text. □

<u>**Section 3.2**</u>

Proof of indirect inter-dealer market equilibrium: We assume that the informed dealer 1 conjectures that the other M dealers submit linear orders and we show that in equilibrium her conjecture is true. Dealer 1 conjectures that each of the M dealers submits order $D^b_{,3}$ given by

$$D^b_{,3} = -\mu P^b_{,3} \tag{51}$$

Market clearing implies that

$$-\mu M P^b_{,3} + Q^b_{,3} = 0 \qquad \text{or,} \tag{52}$$

$$P^b_{,3} = \frac{Q^b_{,3}}{\mu M}, \tag{53}$$

where $Q^d_{,3}$ is dealer 1's order. Negative exponential utility implies that dealer 1 chooses $Q^d_{,3}$ to maximize a quadratic function given by

$$-F + (Q^b_{,3} - W)\mathrm{E}(X|S) - \frac{(Q^b_{,3})^2}{\mu M} - \frac{\rho}{2}\mathrm{Var}(X|S)(Q^b_{,3} - W)^2 + P_{,2}W. \tag{54}$$

Rearranging dealer 1's first order condition yields

$$\frac{\pi_x + \pi_s}{\pi_s} Q^b_{,3} \left(\frac{2}{\mu M} + \frac{\rho}{\pi_x + \pi_s} \right) = Z, \tag{55}$$

where $Z = S + \frac{\rho}{\pi_s} W$ is the, by now, familiar unbiased estimator of S with precision $\pi_z = \frac{\pi_s^2 \pi_w}{\pi_s \pi_w + \rho^2}$.

Given that each of the other M dealers is identical, we focus on the decision problem of the representative uninformed dealer. We assume that each uninformed dealer conjectures that the other $M-1$ uninformed dealers submit linear orders and show that in equilibrium their conjecture is true. Market clearing implies that

$$-(M-1)\mu P^b_{,3} + D^b_{,3} + Q^b_{,3} = 0 \qquad \text{or,}$$

$$P^b_{,3} = \frac{D^b_{,3} + Q^b_{,3}}{\mu(M-1)}. \tag{56}$$

Each uninformed dealer takes into consideration how her order $D^b_{,3}$ affects the equilibrium market price and learns from it. Moreover, each uninformed dealer knows that all the other uninformed dealers are identical to herself and that in equilibrium, the informed dealer's order will be absorbed equally by all the uninformed dealers. Knowledge of their own order $D^b_{,3}$ implies that the uninformed know dealer 1's order $Q^b_{,3}$, since in equilibrium $Q^b_{,3} = -MD^b_{,3}$. From equation (**55**), observing $Q^b_{,3}$ is equivalent to observing Z. It follows that each uninformed dealer maximizes a quadratic function given by

$$-F + D^b_{,3}\mathrm{E}(X|P^b_{,3}) - \frac{D^b_{,3}Q^b_{,3}}{\mu(M-1)} - \frac{(D^b_{,3})^2}{\mu(M-1)} - \frac{\rho}{2}\mathrm{Var}(X|P^b_{,3})(D^b_{,3})^2, \quad \mathbf{(57)}$$

where $\mathrm{E}(X|P^b_{,3}) = \dfrac{\pi_z}{\pi_x + \pi_z}$ and $\mathrm{Var}(X|P^b_{,3}) = \dfrac{1}{\pi_x + \pi_z}$. Recalling that from market clearing, $Q^b_{,3} = (M-1)\mu P^b_{,3} - D^b_{,3}$, we can rearrange the first order condition of the representative uninformed dealer to yield

$$D^b_{,3}\left(\frac{1}{\mu(M-1)} + \frac{\rho}{\pi_x + \pi_z}\right) = \frac{\pi_z}{\pi_x + \pi_z}Z - P^b_{,3}. \quad \mathbf{(58)}$$

Substituting (**55**) into the above equation and noting that $Q^b_{,3} = -MD^b_3$, we obtain

$$D^b_{,3} = -\mu P^b_3 \qquad \text{where,} \quad \mathbf{(59)}$$

$$\mu = \frac{(M-2)\pi_x\rho^2 - M\pi_s\pi_w(\pi_x + \pi_s)}{\rho(M-1)\left(\rho^2 + (M+1)\pi_s\pi_w\right)}. \quad \mathbf{(60)}$$

For these order strategies to be well defined, they must satisfy the second order conditions of the agents' maximization problems. It is easy to see that these in turn require that $\mu > 0$. That is for the conjectured linear order strategy to be an equilibrium order strategy condition (**13**) in the main text must be satisfied.

In order to obtain the equilibrium pricing rule we note that from equation (**55**) dealer 1's equilibrium order satisfies

$$Q^b_{,3} = \frac{Z}{\frac{2}{\mu M}\frac{\pi_x + \pi_s}{\pi_s} + \frac{\rho}{\pi_s}} \quad \mathbf{(61)}$$

Substituting this expression in the market clearing condition (**52**) we obtain

$$P^b_{,3} = \frac{\pi_s}{\pi_x + \pi_s}\mathrm{T}Z \qquad \text{where,} \quad \mathbf{(62)}$$

$$T = \frac{1}{2 + \frac{\rho}{\pi_x+\pi_s}\mu M}. \quad (63)$$

Substituting the expression for Z into (**62**), we obtain the equilibrium price (**11**) in the main text. Substituting (**60**) into (**63**) and rearranging we obtain the expression for $1-2\mathrm{T}$ given in (**12**). From equation (**63**) it is clear, that $\mu = \frac{\pi_x+\pi_s}{\rho}\frac{1-2\mathrm{T}}{\mathrm{T}}$. Substituting this and (**62**) into (**59**) we obtain equation (**10**) in the main text. Finally, we obtain equation (**9**) by simply noting that $Q^b_{,3} = -MD^b_{,3}$.□

<u>**Section 3.3**</u>

Proof of expression 14 of dealer 1's certainty equivalence: In what follows $P(Q)$ denotes the price that dealer 1 has to pay to obtain her optimal order Q at Stage 3.

Regardless of which market dealer 1 chooses, her certainty equivalence $\mathrm{CE}_{1,3}$ is given by

$$\begin{aligned}\mathrm{CE}_{1,3} &= -K + \mathrm{E}(X|S)(Q(Z)-W) + P_{,2}W - \\ &\quad P(Q(Z))Q(Z) - \frac{\rho}{2}\mathrm{Var}(X|S)(Q(Z)-W)^2, \quad (64)\end{aligned}$$

where K is defined in the main text. Since $Z = S + \frac{\rho}{\pi_s}W$, we can write $\mathrm{E}(X|S)$ as $\frac{\pi_s}{\pi_x+\pi_s}(Z - \frac{\rho}{\pi_s}W)$. Substituting this into (**64**) we obtain

$$\begin{aligned}\mathrm{CE}_{1,3}(Z) &= -K + \frac{\pi_s}{\pi_x+\pi_s}(Z - \frac{\rho}{\pi_s}W)(Q(Z)-W) + P_{,2}W \\ &- P(Q(Z))Q(Z) - \frac{\rho}{2}\mathrm{Var}(X|S)(Q(Z)-W)^2. \quad (65)\end{aligned}$$

Differentiating (**65**) with respect to Z yields

$$\mathrm{CE}'_{1,3}(Z) = \frac{\pi_s}{\pi_x+\pi_s}(Q(Z)-W) + Q'(Z)\times \quad (66)$$

$$[\mathrm{E}(X|S) - P(Q(Z)) - P'(Q(Z))Q(Z) - \rho\mathrm{Var}(X|S)(Q(Z)-W)] \quad (67)$$

Notice that the term in brackets in equation (**67**) is nonetheless but the first order condition of dealer 1's maximization problem. Given that dealer 1's optimum order Q must satisfy this first order condition, regardless of which inter-dealer market is chosen, equation (**67**) becomes

$$\mathrm{CE}'_{1,3}(Z) = \frac{\pi_s}{\pi_x+\pi_s}(Q(Z)-W). \quad (68)$$

Integrating (**68**) with respect to Z yields expression (**14**) in the main text. Evaluating (**65**) at $Z = 0$ yields expression (**15**) in the main text.□

Proof of Proposition 1: Substituting equations (**5**) and (**9**) in equation (**17**) yields an expression for the difference in expected utility dealer 1 obtains when choosing the brokered inter-dealer market over the direct one. Specifically, we obtain

$$\mathrm{CE}_{1,3}^{b} - \mathrm{CE}_{1,3}^{d} = -F + \frac{\pi_s}{\rho} \int_0^Z \left[(1-2\mathrm{T}) - \tau\right] t dt. \qquad (\mathbf{69})$$

After some rearranging we can show that expression (**69**) is positive if and only if

$$-\frac{F}{W^2} + \frac{1}{2W^2}\frac{\pi_s}{\rho}(S + \frac{\rho}{\pi_s}W)^2 \left[(1-2\mathrm{T}) - \tau\right] \qquad (\mathbf{70})$$

is positive. Recalling that we have assumed that $\frac{F}{W^2}$ is close to zero it follows, that dealer 1 chooses to deal in the inter-dealer market where she can place the larger order. That is, the brokered market prevails if $(1-2\mathrm{T}) - \tau > 0$, the inter-dealer market prevails if $(1-2\mathrm{T}) - \tau < 0$ and dealer 1 is indifferent between markets if $(1-2\mathrm{T}) - \tau = 0$. Defining $A = \dfrac{\pi_s(\pi_x + \pi_s)}{\pi_x}$ and $B = \dfrac{\rho^2}{\pi_w}$, we rewrite $(1-2\mathrm{T})$ and τ given in equations (**12**) and (**7**) respectively as

$$1 - 2\mathrm{T} = \frac{B - \frac{M}{M-2}}{\frac{M^2-2}{M(M-2)}A + \frac{2(M-1)}{M(M-2)}\frac{AB}{\pi_s} + B}, \qquad (\mathbf{71})$$

$$\tau = \frac{B - A}{2A + \frac{AB}{\pi_s} + B}. \qquad (\mathbf{72})$$

Subtracting (**72**) from (**71**) we obtain

$$(1-2\mathrm{T}) - \tau = \frac{\mathrm{Num}}{\mathrm{Den}}, \qquad \text{where,}$$

$$\begin{aligned}
\mathrm{Num} &= \left[(B-A)(AB\pi_s^{-1} + A)\right] M^2 \\
&\quad - \left[6AB + 2AB\pi_s^{-1}(2B - A)\right] M + 2(B-A)(AB\pi_s^{-1} + A), \\
\mathrm{Den} &= \left[(M^2-2)A + 2(M-1)AB\pi_s^{-1} + M(M-2)B\right] \times \\
&\quad \left[2A + AB\pi_s^{-1} + B\right]. \qquad (\mathbf{73})
\end{aligned}$$

Given that a necessary condition for the inter-dealer broker system to open is that $M > 2$, it must be that, when conditions (**8**) and (**13**) are both satisfied, the denominator of (**73**) is positive. Hence, in order to determine the sign of (**73**) we need only consider the sign of its numerator which further implies that we need only consider the sign of the following quadratic polynomial in M

$$\begin{aligned} G(M) &= M^2 - CM + 2 \qquad \text{where,} \\ C &= \frac{6AB + 2AB\pi_s^{-1}(2B - A)}{(B - A)(AB\pi_s^{-1} + A)}. \end{aligned} \tag{74}$$

It follows, that if $G(M)$ is positive, (**70**) is positive and inter-dealer trading occurs through the broker, whereas if $G(M) < 0$, (**70**) is negative and inter-dealer trading occurs directly. If $G(M) = 0$ dealer 1 is indifferent between inter-dealer markets. We now consider in turn statements (a), (b) and (c) of the proposition:

(a) A necessary condition for either of the two inter-dealer markets to open is that $A < B$. We thus observe that the denominator of C is positive. Secondly, we can show that $C^2 > 8$, which in turn implies that $G(M)$ has two real positive roots given by $\frac{C \pm \sqrt{C^2 - 8}}{2}$. Thirdly, observing that $G(0) > 0$ and verifying that $G(1)$, $G(2)$ and $G(3)$ are negative, it follows that the smallest root of $G(M)$ given by $m_1 = \frac{C - \sqrt{C^2 - 8}}{2}$ is less than 1. Given that a necessary condition for the direct market to open is that $M \geq 2$, we can then see that $G(M) \geq 0$ if and only if M exceeds the largest root of $G(M)$ given by $m_2 = \frac{C + \sqrt{C^2 - 8}}{2}$. Hence, if M is greater or equal to the integer given by $M^* = 1 + [m_2]$, $G(M)$ is positive and dealer 1 chooses to trade through the inter-dealer broker. If M, however, is less than $M^* = 1 + [m_2]$, then $G(M)$ is negative and dealer 1 trades in the direct market.

(b) From our discussion so far, it follows, that if the configuration of the initial parameters is such, that when $M = m_2$, dealer 1 is indifferent between the two inter-dealer markets. In fact, if mixed strategies are allowed and $M = m_2$, dealer 1 chooses between inter-dealer market with equal probabilities. In such a case both inter-dealer markets coexist. However, this equilibrium is not only unlikely but is also not robust to slight perturbations in the parameters of the game. For example, if an extra dealer exits the market-making of the security, dealer 1 chooses to trade in the direct inter-dealer market with probability one. Similarly, if an extra dealer enters the market-making industry, dealer 1 chooses to trade through the broker with probability one. More significantly, our

findings so far, are based on the assumption that the fraction $\frac{F}{W^2}$ is too close to zero to be taken into account. For sufficiently small increases in $\frac{F}{W^2}$ however, one can see from (**70**) that even when $M = m_2$ (which is equivalent to $1 - 2\mathrm{T} = \tau$) dealer 1 chooses to trade in the direct market.
(c) Using the definitions of A and B we rewrite (**74**) as

$$C = \frac{6\pi_s + 4B - 2\frac{\pi_s(\pi_x+\pi_s)}{\pi_x}}{B - \frac{\pi_s^2}{\pi_x} - \frac{\pi_s^2(\pi_x+\pi_s)}{B\pi_x}}. \tag{75}$$

Clearly, evaluating (**75**) at $\pi_s = 0$ yields $C = 4$ which in turn yields a value of M^* given by $1+[3.41] = 4$. In the proof of Corollary 1 below, we show that M^* is increasing in π_s. By a continuity argument, it therefore follows, that for all $\pi_s > 0$, $M^* > 4$. Moreover, we can clearly see from (**75**), that as $B \to \infty$, $C \to 4$ and $M^* \to 4$. In the proof of Corollary 1 below we show that M^* is decreasing in π_w. By a continuity argument, it then follows that for all $\frac{1}{\pi_w} > 0$, $M^* > 4$.□

Proof of Corollary 1: In the proof of Proposition 1 we have shown that $M^* = 1 + 0.5\left[C + \sqrt{C^2 - 8}\right]$. For the purpose of this corollary we assume that M^* is continuous and differentiable. A simple application of the chain rule reveals that

$$\frac{\partial M^*}{\partial \pi_s} = \frac{\partial M^*}{\partial C}\frac{\partial C}{\partial \pi_s}, \tag{76}$$

$$\frac{\partial M^*}{\partial \pi_x} = \frac{\partial M^*}{\partial C}\frac{\partial C}{\partial \pi_x}, \tag{77}$$

$$\frac{\partial M^*}{\partial \frac{\rho^2}{\pi_w}} = \frac{\partial M^*}{\partial C}\frac{\partial C}{\partial \frac{\rho^2}{\pi_w}}. \tag{78}$$

Since M^* is increasing in C, it follows from above equations that we need only consider how C varies with the initial parameters of the model.
<u>Variation of C with π_s:</u> We rewrite expression (**73**) as

$$C = \frac{4\pi_s + 4B - 2\frac{\pi_s^2}{\pi_x}}{B - \frac{\pi_s^2}{\pi_x} - \frac{\pi_s^2\pi_w}{\rho^2} - \frac{\pi_s^3\pi_w}{\rho^2\pi_x}}. \tag{79}$$

The sign of $\frac{\partial C}{\partial \pi_s}$ can be found by applying the standard formula for calculating derivatives of fractions and concentrating on the sign of the numerator of the result (the denominator being the square of the denominator of C and thus being always positive). Thus:

$$\mathrm{sgn}[\frac{\partial C}{\partial \pi_s}] = \mathrm{sgn}[4(-\beta)\gamma + 4(\gamma+\epsilon)\delta] \quad \text{or,} \tag{80}$$

$$\mathrm{sgn}[\frac{\partial C}{\partial \pi_s}] = \mathrm{sgn}[4\gamma(\delta-\beta) + 4\delta\epsilon] \quad \text{where,} \tag{81}$$

$$\beta = \frac{\pi_s}{\pi_x} - 1 \tag{82}$$

$$\gamma = B - \frac{\pi_s^2}{\pi_x} - \frac{\pi_s^2 \pi_w}{\rho^2} - \frac{\pi_s^3 \pi_w}{\rho^2 \pi_x} > 0, \tag{83}$$

$$\gamma + \epsilon = B + \pi_s - \frac{\pi_s^2}{2\pi_x} > \gamma \text{ and} \tag{84}$$

$$\delta = \frac{2\pi_s}{\pi_x} + \frac{2\pi_s \pi_w}{\rho^2} + \frac{3\pi_s^2 \pi_w}{\rho^2 \pi_x} > \beta. \tag{85}$$

It therefore follows that $\frac{\partial C}{\partial \pi_s}$ is positive, which in turn implies that $\frac{\partial M^*}{\partial \pi_s}$ is positive.

Variation of C with π_x: We rewrite (**79**) as

$$C = \frac{L - N\pi_x^{-1}}{H - J\pi_x^{-1}} \qquad \text{where,} \tag{86}$$

$$L = 4\pi_s + 4B, \tag{87}$$

$$N = 2\pi_s^2, \tag{88}$$

$$H = B - \frac{\pi_s^2 \pi_w}{\rho^2} \text{ and,} \tag{89}$$

$$J = \pi_s^2 + \frac{\pi_s^3 \pi_w}{\rho^2}. \tag{90}$$

It follows that

$$\frac{\partial C}{\partial \pi_x^{-1}} = \frac{LJ - NH}{(H - J\pi_x^{-1})^2} > 0, \tag{91}$$

since $LJ - NH > 0$. Therefore, $\frac{\partial C}{\partial \pi_x} < 0$, which in turn implies that $\frac{\partial M^*}{\partial \pi_x} < 0$.

Variation of C with $\frac{\rho^2}{\pi_w}$: We rewrite (**73**) as

$$C = \frac{6\pi_s + 4B - 2A}{B - \frac{\pi_s^2}{\pi_x} - \frac{1}{B}\frac{\pi_s^2(\pi_x+\pi_s)}{\pi_x}} \tag{92}$$

and we differentiate with respect to B, applying the standard formula for calculating derivatives of fractions. Once more, we concentrate on the sign of the numerator of the derivative and we use the necessary condition $A < B$ to show that $\frac{\partial C}{\partial B} < 0$. This in turn implies that $\frac{\partial M^*}{\partial \frac{\rho^2}{\pi_w}} < 0$.□

Section 4

Proof of Proposition 2: We distinguish between two cases.
Case 1: $M < M^*$ From equation (**20**) we know that when $M < M^*$ dealers 1 offers a Stage 2 price such that $\text{CE}_{1,3} = 0$. Using equation (**14**) it then follows that the price the investor obtains satisfies

$$\frac{\pi_s}{\pi_x + \pi_s}\int_0^Z \left[Q^d_{,3}(t) - W\right]dt + \frac{\rho}{2(\pi_x+\pi_s)}W^2 + P_{,2}W = 0, \tag{93}$$

Substituting the optimal Stage 3 order $Q^d_{,3} = \frac{\pi_s}{\rho}\tau Z$ (see equation (**5**)) into (**93**), evaluating the integral and rearranging we obtain

$$P_{,2}W = -\frac{\pi_s^2}{2\rho(\pi_x+\pi_s)}\tau Z^2 + \frac{\pi_s}{\pi_x+\pi_s}WZ - \frac{\rho}{2(\pi_x+\pi_s)}W^2. \tag{94}$$

Substituting the expression for Z into (**94**) we obtain

$$\begin{aligned} P_{,2}W &= -\frac{\pi_s^2}{2\rho(\pi_x+\pi_s)}\tau\left(S^2 + \frac{\rho^2}{\pi_s^2}W^2 + \frac{2\rho}{\pi_s}SW\right) \\ &\quad + \frac{\pi_s}{\pi_x+\pi_s}W\left(S + \frac{\rho}{\pi_s}W\right) - \frac{\rho}{2(\pi_x+\pi_s)}W^2. \end{aligned} \tag{95}$$

Further rearranging of (**95**) yields

$$P_{,2}W = \frac{\pi_s}{\pi_x+\pi_s}(1-\tau)SW + \frac{\rho}{2(\pi_x+\pi_s)}(1-\tau)W^2 - \frac{\pi_s^2}{2\rho(\pi_x+\pi_s)}\tau S^2. \tag{96}$$

Dividing both sides of (**96**) with W yields the first pricing equation in (**22**).

Case 2: $M \geq M^*$ From equation (**20**) we know that when $M \geq M^*$ dealer 1 offers a Stage 2 price such that her expected utility from trading with the public is equal to her expected utility from not trading, that is $\text{CE}_{1,3} = \text{CE}_{-1,3}$. Using the expression for $\text{CE}_{1,3}$ given in (**14**) it then follows that dealers set their Stage 2 quotes such that

$$-F + \frac{\pi_s}{\pi_x + \pi_s} \int_0^Z \left[Q^b_{,3}(t) - W \right] dt + \frac{\rho}{2(\pi_x + \pi_s)} W^2 + P_{,2} W = \text{CE}_{-1,3}, \tag{97}$$

is satisfied. Substituting $Q^b_{,3} = \frac{\pi_s}{\rho}(1 - 2\text{T})Z$ (see equation (**9**)) into (**97**), evaluating the integral and rearranging we obtain

$$\begin{aligned} P_{,2} W &= \frac{\pi_s^2}{2\rho(\pi_x + \pi_s)}(1 - 2\text{T})Z^2 + \frac{\pi_s}{\pi_x + \pi_s} WZ - \frac{\rho}{2(\pi_x + \pi_s)} W^2 + \\ &\quad (\text{CE}_{-1,3} + F) \,. \end{aligned} \tag{98}$$

Substituting the expression for Z into (**98**) and rearranging yields

$$\begin{aligned} P_{,2} W &= \frac{\pi_s}{\pi_x + \pi_s} 2\text{T}SW + \frac{\rho}{2(\pi_x + \pi_s)} 2\text{T}W^2 - \frac{\pi_s^2}{2\rho(\pi_x + \pi_s)}(1 - 2\text{T})S^2 \\ &+ (\text{CE}_{-1,3} + F) \,. \end{aligned} \tag{99}$$

Dividing both sides of (**99**) with W yields the second equation in (**22**).□

Proof of concavity of pricing equations w.r.t. S**:** We distinguish between two cases:

Case 1: $M < M^*$ Differentiating twice the first pricing equation in (**22**) w.r.t. S yields

$$\frac{\partial^2 P_{,2}}{\partial S^2} = -\frac{1}{W} \frac{\pi_s^2}{2\rho(\pi_x + \pi_s)} \tau. \tag{100}$$

Given that when $M < M^*$, the inter-dealer market opens as long as $\tau > 0$ (see condition (**8**) in the main text), it is obvious that (**100**) is negative.

Case 2: $M \geq M^*$ Differentiating twice the second pricing equation in (**22**) w.r.t. S yields

$$\frac{\partial^2 P_{,2}}{\partial S^2} = -\frac{1}{W} \frac{\pi_s^2}{2\rho(\pi_x + \pi_s)}(1 - 2\text{T}) + \frac{1}{W} \frac{\partial^2 \text{CE}_{-1,3}}{\partial S^2}, \tag{101}$$

where $\text{CE}_{-1,3}$ is given by equation (**21**) in the main text. It follows that

$$\frac{\partial^2 P_{,2}}{\partial S^2} = -\frac{1}{W}\frac{\pi_s^2}{2\rho(\pi_x+\pi_s)}(1-2\text{T}) + \frac{1}{W}\frac{\pi_s}{\rho}\left(\frac{1-2\text{T}}{M}\right)\left(\frac{\pi_s}{\pi_x+\pi_s}\text{T} - \frac{\pi_z}{\pi_x+\pi_z} - \frac{\pi_s}{2(\pi_x+\pi_z)}\frac{1-2\text{T}}{M}\right). \quad \textbf{(102)}$$

Recalling that a necessary and sufficient condition for the brokered market to be chosen is that $\text{T} < \frac{1}{2}$ and that $M \geq M^* \geq 4$ (see condition (**13**) and Proposition 1, respectively) we notice that

$$\frac{\pi_s^2(1-2\text{T})}{2\rho(\pi_x+\pi_s)} > \frac{\pi_s^2(1-2\text{T})\text{T}}{M\rho(\pi_x+\pi_s)} \quad \text{since,} \quad \textbf{(103)}$$

$$\frac{\text{T}}{M} < \frac{1}{2}. \quad \textbf{(104)}$$

From (**103**) and (**103**) it then follows that when $M \geq M^*$, $\frac{\partial^2 P_{,2}}{\partial S^2} < 0$.□

<u>**Section 5**</u>

Proof that Π is decreasing, or umimodal in M for $M \geq 4$: In what follows we make the assumption that M varies continuously.

Recalling that dealers will trade in the inter-dealer market when $M \geq M^* \geq 4$, we need only consider the variation of Π for $M \geq 4$. We rewrite Π, defined in the l.h.s. of (**28**), in terms of the initial parameters of the model. This yields

$$\frac{N1}{\zeta D1^2} \geq F \quad \text{where,} \quad \textbf{(105)}$$

$$\zeta = 2\rho\pi_w\pi_x, \quad \textbf{(106)}$$

$$N1 = acM^2 + (ad+bc)M + bd, \quad \text{and} \quad \textbf{(107)}$$

$$D1 = eM^2 + fM + g, \quad \text{where,} \quad \textbf{(108)}$$

$$a = \rho^2\pi_x - \pi_s\pi_w(\pi_x+\pi_s) > 0, \quad \textbf{(109)}$$

$$b = -2\rho^2\pi_x < 0, \quad \textbf{(110)}$$

$$c = \pi_s^2\pi_w^2(\pi_x+\pi_s) + 2\rho^2\pi_s\pi_w\pi_x + \rho^4\pi_x - \rho^2\pi_s^2\pi_w > 0, \quad \textbf{(111)}$$

$$d = 2\pi_s\pi_w(\pi_x+\pi_s)(\pi_s\pi_w+\rho^2) > 0, \quad \textbf{(112)}$$

$$e = \rho^2\pi_x + \pi_s\pi_w(\pi_x+\pi_s) > 0, \quad \textbf{(113)}$$

$$f = 2\rho^2\pi_s > 0. \quad \textbf{(114)}$$

It is clear from the equations above that the numerator of Π is a quadratic continuous polynomial in M whereas its denominator is a quartic continuous polynomial in M. It thus clearly follows that as $M \to \infty$, $\Pi \to 0$.

In order to demonstrate that Π is either unimodal, or decreasing in M, we need to show that for $M \geq 4$, Π has a maximum of one turning point.

Note that in the Proof of Proposition 1 below we demonstrate, by use of the market opening condition (**13**), that $\Pi \geq 0$ for all $M > 2$. This implies that N_1 is positive for $M > 2$. After some algebra it is also possible to show that $(ad + bc) < 0$. Hence the quadratic polynomial N_1 has a negative real root and a positive real root and a negative minimum value at $M = -\dfrac{ad+bc}{2ace} > 0$. Given that $N1$ is positive for all $M > 2$, it must be that $M = -\dfrac{ad+bc}{2ace} < 2$. This observation is used later on in the proof (see Case 2(b)).

Taking the partial derivative of (**105**) with respect to M yields

$$\frac{\partial \Pi}{\partial M} = \frac{1}{\zeta D1^3}\left(-2N1\frac{\partial D1}{\partial M} + D1\frac{\partial N1}{\partial M}\right). \qquad (\mathbf{115})$$

In the proof of Proposition 3 below, $D1$ is rewritten is such a way that it is obvious that for $M > 2$ it is positive (see equation (**127**)). Therefore, $\dfrac{1}{\zeta D1^3}$ is positive. It hence follows that Π is unimodal, or decreasing in M for $M \geq 4$ if and only if the term in brackets in (**115**) has a maximum of one real root greater or equal to 4.

It turns out that the term in brackets in (**115**) is a cubic polynomial in M equal to

$$\phi(M) = C_1M^3 + C_2M^2 + C_3M + C4, \qquad \text{where,} \qquad (\mathbf{116})$$

$$C_1 = -2ace < 0, \qquad (\mathbf{117})$$

$$C_2 = -3(ad + bc)e > 0, \qquad (\mathbf{118})$$

$$C_3 = (ad + bc)f - 4ebd + 2acg, \qquad (\mathbf{119})$$

$$C_4 = -2fbd + (ad + bc)g > 0. \qquad (\mathbf{120})$$

We distinguish between the following cases:

Case 1: $C_3 \geq 0$ From Descartes' rule of signs it follows that $\phi(M)$ has a unique positive real root. If this real root is less or equal to 4, Π is decreasing in $M \geq 4$, whereas if it is greater to 4, Π is unimodal in M.

Case 2(a): $C_3 < 0$ and $C_2^2 - 3C_1C_3 < 0$ In this case, $\phi'(M)$ has a negative discriminant and thus no real roots. Since $C_1 < 0$ it follows that $\phi'(M) < 0$, for all M. Since, $\phi(0) = C_4 > 0$ and $\phi'(M) < 0$ it follows that ϕ has a unique positive real root. If this root is less or equal to 4, Π is decreasing for all $M \geq 4$, whereas if it is greater than 4, Π is unimodal for $M \geq 4$.

Case 2(b): $C_3 < 0$ and $C_2^2 - 3C_1C_3 \geq 0$ In this case, $\phi'(M)$ has two real positive roots given by:

$$r_1 = \frac{-C_2 + (C_2^2 - 3C_1C_3)^{0.5}}{3C_1} \quad \text{and} \tag{121}$$

$$r_2 = \frac{-C_2 - (C_2^2 - 3C_1C_3)^{0.5}}{3C_1} \tag{122}$$

with $r_2 \geq r_1$. This implies that ϕ has three positive real roots. Demonstrating that $r_2 < 4$, is equivalent to showing that the two smaller positive real roots are less than 4, in which case there would be a maximum of one positive real root greater or equal to 4. This in turn would imply that Π is either unimodal, or decreasing for $M \geq 4$. Indeed, from (**117**) and (**118**) it follows that $-\frac{C_2}{3C_1} = -\frac{(ad+bc)}{2ace}$, which we know is less than 2 (see observation in the beginning of proof). It thus follows that $r_2 \leq -2\frac{ad+bc}{2ace} < 4.\square$

Proof of Proposition 3: Note that dealers will enter the market as long as the surplus they expect to extract by entering the industry remains non negative, that is, as long as $P\sigma_z^2 \geq F$ is not violated (see inequality (**28**) in the main text). Writing (**28**) in terms of the initial parameters of the model, we rewrite the condition for entry in the market-making industry, when $M \geq M^*$, as

$$\frac{(Q1 + Q2) \times Q3}{D1} \geq F \qquad \text{where,} \tag{123}$$

$$Q1 = (M+2)\pi_s^2\pi_w^2(\pi_x + \pi_s) + 3M\rho^2\pi_s\pi_w(\pi_x + \pi_s), \tag{124}$$

$$Q2 = M\rho^4\pi_x - (M-2)\rho^2\pi_s\pi_w(\pi_x + \pi_s), \tag{125}$$

$$Q3 = \frac{[(M-2)\rho^2\pi_x - M\pi_s\pi_w(\pi_x + \pi_s)]}{2\rho\pi_w\pi_x} \quad \text{and} \tag{126}$$

$$D1 = \left[(M^2-2)\pi_s\pi_w(\pi_x+\pi_s) + 2(M-1)\rho^2(\pi_x+\pi_s) + M(M-2)\rho^2\pi_x\right]^2. \tag{127}$$

Let the l.h.s. of (**123**) be equal to $\Phi(M)$. First note that as long as condition (**13**) is satisfied the expected surplus $\Phi(M)$ is positive. Secondly, observe that the numerator of $\Phi(M)$ is a quadratic equation in M, whereas its denominator is a quartic equation in M. Thirdly, note that $\Phi(M)$ is continuous in M. Clearly, it follows, that as $M \to \infty$, the expected surplus $\Phi(M)$ goes to zero. Hence, as long as $\Phi(M^*+1) \geq F$, there must exist at least one positive value $M^e+1 \geq [M^e]+1 \geq M^*+1$ which satisfies the equality $\Phi(M^e+1) = F$. Since the equilibrium number of dealers is the integer part of the largest root of the equation $\Phi(M) = F$, it further follows, that as long as $\Phi(M^*+1) \leq F$, there exists a unique integer $M^{*e}+1 \geq M^*+1$, such that for all $M+1 > M^{*e}+1$, $\Phi(M+1) < F$.

So far we have shown that $\Phi(M^*+1) \geq F$ is a sufficient condition for an equilibrium number of dealers greater or equal to M^*+1 to exist. To prove the proposition we need to show that if (**30**) is true, $\Phi(M^*+1) \geq F$ is also a necessary condition for an equilibrium number of dealers greater or equal to M^*+1 to exist. That is, if $\Phi(M^*+1) < F$, then an equilibrium number of dealers greater than M^*+1 does not exist. Suppose that M^*+1 dealers have entered the market. Suppose also that (**29**) is violated. Given that condition (**30**) is true and Π is either unimodal, or decreasing in $M \geq 4$, the entry of an additional dealer decreases Π, that is $\Pi(M^*+2) < \Pi(M^*+1) < F$. Using the same argument, we can see that $\Pi(M^*+n) < \Pi(M^*+2) < \Pi(M^*+1) < F$ for all $n = 3, 4, ...$ It thus follows that if (**30**) is true and (**29**) is violated, an equilibrium number of dealers greater or equal to M^*+1 does not exist.□

Proof of Corollary 2: From part (c) of Proposition 1, we know that in the absence of information asymmetry $M^*+1 = 5$. Further, recall that in the absence of private information the equilibrium condition is given by (**31**). Substituting $M = 5$ into (**31**) and rearranging yields (**32**). From Proposition 1, it therefore follows that (**32**) is a sufficient condition for an equilibrium number of dealers greater or equal to 5 to exist. In order to show that (**32**) is also necessary we need to show that

$$\left.\frac{\partial \Pi|_{\pi_s=0}}{\partial M}\right|_{M=5} \leq 0. \tag{128}$$

Observe that

$$\frac{\partial \Pi|_{\pi_s=0}}{\partial M} = \frac{\rho\left(-M^3+3M^2-2M+2\right)}{\pi_w \pi_x (M^2-2)^3}. \tag{129}$$

Evaluating (**129**) at $M = 5$ we can see that (**128**) is true. That is, in the absence of information asymmetry, (**30**) is always true. The comparative statics results follow directly by inspection of (**31**).□

Proof of Proposition 4: Suppose the investor has a coefficient of risk-aversion r. Then her utility function *ex ante* to the signal is given by

$$W\mathrm{E}(P_{,2}) - r\mathrm{Var}(P_{,2})W^2 \tag{130}$$

Clearly from our discussion in section 5.2 it follows that, as long as the equilibrium number of dealers who have entered the market is greater or equal to M^*+1 and $\mid W \mid\gg 0$, the introduction of the inter-dealer broker system lowers the mean and variance of price. Hence (**130**) increases and the investor is better-off. If however less than M^*+1 dealers have entered the market the investor's expected utility remains unchanged.

Now consider the case where the equilibrium number of dealers is $M^e < M^* + 1$. Suppose that the only way dealers can trade with each other is through the inter-dealer broker. Then, the per unit price an outside investor, wishing to trade W units of the security, expects to obtain is given by (**37**), whereas the volatility of price is given by (**39**). Suppose direct bilateral trading is now introduced. Given that $M^e < M^* + 1$, by Proposition 2 it follows, that the expected price is now given by (**35**), whereas the price volatility is given by (**36**). Since when $M^e < M^* + 1$, $\tau > 1 - 2\mathrm{T}$, it follows that the investor expects lower transaction costs. As long as she is wishing to trade a large quantity ($W \gg 0$), the volatility of price is also lower. Hence, the introduction of direct trading increases (**130**) and makes the investor better-off. If, however, more than M^* ($M^e \geq M^* + 1$) dealers had entered the market the investor's expected utility would remain unchanged.

Finally, suppose $M^e = 0.5\left[C + \sqrt{C^2 - 8}\right]$ (see Proposition 1, part (b)), then $\tau = 1 - 2\mathrm{T}$ and the utility an investor, wishing to trade $\mid W \mid\gg 0$, expects to obtain, when she does not have a choice of inter-dealer trading system, is the same as the utility she expects to obtain, when she has a choice between systems.□

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