

Incentive schemes for central bankers under uncertainty: inflation targets versus contracts

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Abstract

We look at the implications of uncertain monetary policy preferences for the targeting and contracting approach to monetary stability. It turns out that in the presence of uncertain preferences a linear incentive contract in the sense of Walsh (1995) performs better in terms of social welfare than an explicit inflation target as proposed by Svensson (1997). The reason is that, although both approaches can get rid of the inflationary bias, the impact of uncertain preferences on the variance of inflation will be considerably higher with an inflation target. Contrary to Beetsma and Jensen (1998) we find that the optimal inflation target depends on the degree of preference uncertainty. For the case of the Walsh contract, unlike Beetsma and Jensen, we find that it is optimal to offer a linear inflation contract to a central banker that does not depend on the degree of uncertainty about its preferences.

Keywords: nonlinearities, economic fluctuations, inflation targets, optimal contracts

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1 Introduction

In the literature on reform of monetary institutions, surveyed in Eijffinger and De Haan (1996), two ways have been advocated to reach low inflation. The first is the *legislative* approach, ie to establish by law an independent central bank with an exclusive mandate for price stability. The major academic contribution in this area is by Rogoff (1985). Rogoff develops a model where monetary policy is delegated to a central banker (the *agent*) who is more inflation-averse than society (the *principal*). The result is a lower inflation rate, but this is at the expense of a distorted response to output shocks. Thus the Rogoff model features the famous *credibility-flexibility* trade-off.

The second is the *targeting* or *contracting* approach to monetary policy. The idea is to let the political principal of the central bank impose an explicit inflation target for monetary policy and make the central bank governor publicly accountable for his success in meeting this target. Academic contributions in this area are Persson and Tabellini (1993), Walsh (1995) and Svensson (1997).

Walsh has shown that an optimal linear inflation contract can eliminate the inflationary bias without distorting stabilisation policy. Thus the credibility-flexibility trade-off is eliminated⁽²⁾ and makes inflation targeting superior to Rogoff's conservative central banker. Moreover, Svensson has shown that the linear contract can be mapped into an optimal *inflation target*. This means that these delegation arrangements are *equivalent*. They are both able to produce the pre-commitment outcome.

However, both Walsh (1995) and Svensson on the one hand and Rogoff (1985) on the other assume *symmetric information* in the principal-agent relationship. More specifically, they assume that both the political principal and the private sector know the central banker's preferences. This assumption was abandoned by Herrendorf and Lockwood (1997) and—most recently—by Beetsma and Jensen (1998) (hereafter BJ).

Herrendorf and Lockwood demonstrated that if a monopolistic trade union has more information about productivity shocks than the principal, maximum social welfare can be achieved by having a conservative central banker with perverse

(2) However, McCallum (1995) argues that the Walsh contract merely shifts the credibility problem from the policymaker to the principal.

preferences *on top of* the contract. Thus—notwithstanding an optimal contract—Rogoff’s conservative central banker is restored. BJ, building on earlier work on preference uncertainty by Briault, Haldane and King (1996) and Nolan and Schaling (1996), derive the important result that, if the private sector and the political principal have imperfect information about the central banker’s preferences, the equivalence between the linear Walsh contract and the Svensson inflation target breaks down. They show that the optimal linear inflation contract performs strictly better than the optimal inflation target when there is uncertainty about the central banker’s preferences.

However, they model the central banker’s preference uncertainty as ‘pure’ uncertainty. This implies that in their model, preference uncertainty has no effect on the private sector’s inflation expectations. Thus inflation expectations are invariant to this kind of monetary policy uncertainty. In this paper, building on Nolan and Schaling (1996), we investigate the effects of relaxing this restriction. Using a model that is otherwise similar to BJ, we find results that are very different—and sometimes exactly the opposite.

For instance, we find that it is optimal for the government to impose an inflation target on the central bank that depends on the degree of uncertainty about its preferences. This is in sharp contrast to BJ, who find that the optimal inflation target does *not* depend on the degree of preference uncertainty and is the same as in the Svensson model. In our model certainty equivalence does *not* hold, and the optimal inflation target with uncertain central bank preferences does *not* correspond to the optimal target derived by Svensson (1997).

Further, for the case of the Walsh contract we find that it is optimal to offer a linear inflation contract to a central banker that does *not* depend on the degree of uncertainty about its preferences. Again, this is in sharp contrast to BJ, who find that the optimal linear contract *does* depend on the degree of preference uncertainty. Here our result is the same as Walsh’s no preference uncertainty case. Hence, certainty equivalence *does* hold and the optimal linear inflation contract with uncertain central bank preferences is *identical* to the optimal Walsh contract.

Finally, we find that, in the case of uncertain central banker preferences, the optimal delegation arrangement is a combination of a linear inflation contract and a *quadratic contract*. Here the quadratic contract is equivalent to a Rogoff (1985)

conservative central banker. Again, this differs from BJ, who find that a combination of a linear inflation contract, an inflation target and a quadratic contract performs best. However, our result is in line with Herrendorf and Lockwood (1997), and hence suggests that their ‘restoration’ of Rogoff appears to be a robust result.

The paper is organised into six remaining sections followed by four appendices. In Section 2 we present the model. Section 3 looks at the optimal inflation target. In Section 4 we examine the linear inflation contract. Section 5 compares the inflation target with the linear contract. Section 6 considers quadratic contracts. Our conclusions are given in Section 7. The appendices provide the derivation of inflation expectations in the presence of non-linearities, the optimal inflation target, the optimal linear contract and the characterisation of quadratic contracts.

2 The model

We use a standard credibility model.⁽³⁾ In line with the credibility literature, output is described by a standard reduced-form Lucas supply function:

$$y = (\pi - E\pi) + \varepsilon \tag{1}$$

where y is (the natural log of) output; inflation is denoted by π and nominal wage contracts signed at time $t-1$ are proxied by the private sector’s expected inflation rate. ε is a supply shock with $E[\varepsilon] = 0$ and $E[\varepsilon^2] = \sigma_\varepsilon^2$. From these distributional characteristics it follows that the log of the natural rate of output is zero.

The welfare loss function of the government (and of society) is given as

$$L^G = \frac{1}{2} E^G \left[(y - y^*)^2 + (\pi - \pi^*)^2 \right] \quad y^* > 0 \tag{2}$$

where the government’s inflation target is π^* , and y^* denotes the government’s preferred value for output. Given this specification it follows that the

(3) See Walsh (1995), Svensson (1997).

government's relative weight of inflation stabilisation relative to output stabilisation—by convenient normalisation—is unity. This simplifies the algebra without affecting the generality of the results.

2.1 Delegation of monetary policy

For the remainder of the paper we assume that actual monetary policy is delegated to a central bank that has full *instrument* independence but uncertain preferences. More specifically, we assume that the central bank has the following loss function:

$$L^{CB} = \frac{1}{2} \left[(y - y^*)^2 + (1 - x)(\pi - \pi^*)^2 \right] \quad (3)$$

where x is a stochastic parameter unobservable by both the government and the private sector. Thus the central bank's weight on inflation stabilisation is continuously hit by the preference disturbance term x , which is a random variable with $E[x] = 0$ and $E[x]^2 = \sigma_x^2$. Moreover, it is assumed that x is independent of ε , hence $E[\varepsilon x] = 0$, and that $x < 1$ with probability one.

This specification follows Briault, Haldane and King (1996) and Nolan and Schaling (1996), and differs from other authors who have stressed the role of imperfect information about the central bank's preferences. For example, BJ constrain the sum of the weights on output and inflation to be the same as in society's loss function, but they assume that the relative weights on the output and inflation variances are random.⁽⁴⁾

We think that our specification is more plausible from an empirical point of view than the BJ approach. The reason is that our specification implies that inflation expectations (and the *level* of inflation) depend on the degree of uncertainty about monetary policy, σ_x^2 . More specifically, the higher this uncertainty the higher the (expected) inflation rate. This implication is in line with several empirical

(4) Beetsma and Jensen (1998) constrain the sum of the weights on $(y - y^*)^2$ and $(\pi - \pi^*)^2$ to be $(1 + x) + (1 - x) = 2$, as in society's loss function, but they assume that the relative weights on output and inflation variances are random. Using the notation of this paper, the BJ results are obtained by

$$L^{CB} = \frac{1}{2} \left[(1 + x)(y - y^*)^2 + (1 - x)(\pi - \pi^*)^2 \right].$$

studies. For instance, Davis and Kanago (1998), Ball and Cecchetti (1990), Brunner and Hess (1993) and Cukierman and Wachtel (1979) find a positive relationship between the *rate of inflation* and measures of *inflation uncertainty*.

Because BJ constrain the sum of the weights on output and inflation to be the same as in society's loss function, they have no link between inflation uncertainty and the *level* of inflation. Our set-up allows us to derive results that are very different—and sometimes the opposite—of their model, as will be shown below.

The timing of events is as follows. First monetary policy is delegated to the central banker. Then inflation expectations are formed. Next the supply shock, ε , hits the economy and, finally, monetary policy is selected by setting the inflation rate.

3 The optimal inflation target

We now consider the case of delegation with an inflation target. This means that the central bank no longer has goal independence; rather the government imposes its own inflation target for the central bank to adhere to. This is similar to the situation in the United Kingdom where the Bank of England sets short-term interest rates independently, but where the inflation target (for this period 2.5%) is set by the government. Under the above arrangements the central bank's loss function becomes

$$L^{CB} = \frac{1}{2} \left[(y - y^*)^2 + (1 - x)(\pi - \pi^b)^2 \right] \quad (4)$$

where π^b is the central bank's inflation target chosen by the government.

The central bank sets monetary policy, which involves the choice of the inflation rate. The central bank then sets inflation⁽⁵⁾ so as to minimise (4) subject to (1), having observed the supply shock and taking inflation expectations as given.

The first-order condition for a minimum implies the following reaction function

$$\pi = \pi^b + \frac{1}{2-x} (y^* - y^n) \quad (5)$$

The term y^n is the output level consistent with the inflation target. Upon substituting π^b for π in (1), it follows that $y^n = (\pi^b - \pi^e + \varepsilon)$. If y^* exceeds y^n , the central bank has an incentive to set inflation *above* the inflation target.

An important property of (5) is that the effects of preference shocks on inflation are *asymmetric*. To see this consider the following example.

Setting $y^* - y^n = 1$ and $\pi^b = 0$, if there is *no preference uncertainty*, implies that inflation is 0.50%. Now, if $x = +0.50$, inflation increases by 17 basis points to 0.67%. However a negative preference shock of the same size decreases inflation by 10 basis points to 0.40%. Hence, a positive preference shock has a more *inflationary* impact than the *disinflationary* impact of a negative preference shock.

The above example was constructed for a *given* level of expected inflation. To solve for private sector inflation expectations take the mathematical expectation (at time $t - 1$) of equation (5).⁽⁶⁾ This yields

$$\pi^e = \pi^b + \Phi y^* \quad (6)$$

where
$$\Phi = \frac{4 + \sigma_x^2}{4 - \sigma_x^2}$$

(5) Cukierman and Meltzer (1986) examine the case where the authorities can set the control variable only imperfectly. That is, $\pi_t = \pi^p + \psi_t$, where $\psi \sim N(0, \sigma_\psi^2)$ and p indicates a planned variable. We assume that there are no monetary control errors, that is $\sigma_\psi^2 = 0$.

(6) The technical issue here, as we show in Appendix A, is that this involves taking expectations in the presence of non-linearities.

It is useful to consider the limiting value of π^e for the case without preference uncertainty

$$\lim_{\sigma_x^2 \rightarrow 0} \pi^e = \pi^b + y^* \quad (7)$$

Equations (6) and (7) indicate that inflation expectations are proportional to the output bias y^* and the inflation target imposed by the government, a familiar conclusion in the literature. However (6) differs in a significant way from (7), as a result of uncertainty regarding the central bank's preferences. It can be seen that with uncertainty about inflation-stabilisation preferences, expected inflation is higher than in the standard case.

The reason is that *positive* shocks to x (more emphasis on output stabilisation) are *more inflationary* than negative shocks (more emphasis on inflation stabilisation) are *disinflationary*. With equal probabilities of positive and negative shocks to the central banker's preferences,⁽⁷⁾ expected inflation will be adjusted upwards and the more so the higher the variance of the shocks hitting the central bank's preferences.

This can be restated in a more technical way by noting that the expectation of inflation involves the expectation of a *convex* function of the preference shock, which will always be *higher* than the value of this function at the (zero) expected value of the preference shock. Hence the channel through which the uncertainty affects expected inflation is *Jensen's inequality effect*.⁽⁸⁾

Realised inflation under a given target follows upon inserting (6) back into (5)

$$\pi = \pi^e + \left(\frac{1 - \Phi(1-x)}{2-x} \right) y^* - \frac{1}{2-x} \varepsilon \quad (8)$$

(7) However, results do not depend on the *symmetry* of this distribution.

(8) Jensen's inequality states that $E\left(\frac{1}{2-x}\right)(\pi^e + y^*) + \pi^b E\left(\frac{1-x}{2-x}\right) > \pi^b + \frac{1}{2} E(y^* - (\pi^b - \pi^e + \varepsilon))$.

This effect is not present in the BJ model because they constrain the sum of the weights on output and inflation in the central banker's loss function to be the same as in society's loss function.

It is important to realise that now monetary policy will on average *not* coincide with policy in the absence of uncertainty about the central banker's preferences. If we set the preference shock equal to zero in (8), monetary policy will still react to the degree of preference uncertainty (through the variance term in Φ). In this respect certainty equivalence does not hold. The reason is that the central banker optimises its policy taking inflation expectations as given. Since private sector inflation expectations depend on the second moment of the distribution of x (see equation (6)) so does the central bank's monetary policy stance.

Combining (1) and (8) yields realised output as

$$y = \left(\frac{1 - \Phi(1 - x)}{2 - x} \right) y^* + \frac{1 - x}{2 - x} \varepsilon \quad (9)$$

From (8) and (9) it can be seen that inflation and output differ from their expected values when either x or ε differ from their (zero) expected values.

For the moment, assume that the supply shock is zero. Then the way the model works is extremely simple. Suppose there is a positive shock to the central banker's preferences ($\pi > \pi^e$). This implies that output will be above the natural rate. Conversely, if there is a negative realisation of x (more emphasis on inflation stabilisation) actual inflation will be lower than expected. Consequently, output will be below its natural level.

It is important to realise that the inflation surprise (be it positive or negative) is completely unaffected by the level of the imposed inflation target. Put differently, the delegation parameter π^b has no effect on how the uncertainty about the central banker's preferences is transmitted to inflation and output.⁽⁹⁾

To see this consider equation (6). A change in π^b changes inflation expectations one-for-one. Hence, the central banker's incentive to set inflation above the target is unaffected, because $(\pi^b - \pi^e + \varepsilon)$ in (5) is unaffected.

(9) This important result was derived earlier by BJ. Here we show that their result continues to hold when private sector inflation expectations reflect the uncertainty about the central banker's preferences (through Φ).

Moreover, the choice of inflation target has no implications for stabilisation policy. By inspecting (8) and (9), it can be seen that the term π^b does not affect the deviations of realised inflation from expectations ($\pi - \pi^e$) or output from its (zero) trend. Thus the variances of output and inflation are *independent* of the choice of inflation target.

For completeness these variances are

$$Var\pi = \left[\frac{(y^* + \Phi y^*)^2 + \sigma_\varepsilon^2}{4} \right] \frac{\sigma_x^2}{4} + \frac{\sigma_\varepsilon^2}{4} \left(1 + \frac{\sigma_x^2}{4} \right)^2 \quad (10)$$

$$Vary = Var\pi + \sigma_\varepsilon^2 - \left(1 + \frac{\sigma_x^2}{4} \right) \sigma_\varepsilon^2 \quad (11)$$

To find the optimal inflation target, the government chooses the value of π^b that minimises L^G subject to (6), (8) and (9).

The optimal inflation target, denoted by π^{b*} , is:⁽¹⁰⁾

$$\pi^{b*} = \pi^* - \Phi y^* \quad (12)$$

As before it is useful to consider the limiting value of π^{b*} for the case without preference uncertainty

$$\lim_{\sigma_x^2 \rightarrow 0} \pi^{b*} = \pi^* - y^* \quad (13)^{(11)}$$

If $\sigma_x^2 > 0$, equation (12) reveals that it is optimal for the government to impose an inflation target on the central bank which depends on the degree of uncertainty about its preferences. This is in sharp contrast to BJ, who find that the optimal inflation target does *not* depend on the degree of preference uncertainty and is the same as in the Svensson model. So here—contrary to BJ—certainty equivalence

(10) See Appendix B for the derivation.

(11) This is the Svensson (1997) result on the ‘inflation-target conservative’ central bank where the inflation target equals the socially best inflation rate less the inflation bias.

does *not* hold, and π^{b*} does not correspond to the optimal target derived by Svensson in the absence of uncertainty about central bank preferences.

The intuition behind our result is simple. First, from (10) and (11) we know that the choice of inflation target has no effect on the stochastic components of the government's loss function. So the inflation target is chosen to equate *expected inflation* with the socially desirable inflation rate. Since expected inflation depends on σ_x^2 (through Φ), the optimal inflation target also depends on σ_x^2 . Finally, comparing (12) with (13), it can be seen that with preference uncertainty the optimal inflation target is 'stricter' (lower) than the Svensson target. The reason is as follows. As is clear from (6), central banker uncertainty implies higher inflation expectations. To offset this additional inflation bias the degree of 'target conservativeness' has to increase as well.

4 The optimal linear inflation contract

Having characterised the optimal inflation target, we now turn to the case where delegation takes the form of a simple linear inflation contract.⁽¹²⁾ The contract adds a linear cost to the central bank's loss function. Let the added linear cost to inflation be $f_1(\pi - \pi^*)$, where f_1 is a constant.

Then the central bank is assigned the loss function

$$L^{CB} = \frac{1}{2} \left[(y - y^*)^2 + (1 - x)(\pi - \pi^*)^2 \right] + f_1(\pi - \pi^*) \quad (14)$$

The central bank chooses inflation so as to minimise (14) subject to (1) having observed the supply shock and taking inflation expectations as given.

From the first-order condition we obtain the following reaction function:

$$\pi = \pi^* + \frac{1}{2 - x} \left(y^* - (\pi^* - \pi^e + \varepsilon) \right) - \frac{f_1}{2 - x} \quad (15)$$

(12) Walsh (1995) has shown that, in the absence of uncertainty about central bank preferences, such a contract can remove the inflation bias under discretion while still allowing the central bank to react optimally to supply shocks. Thus the trade-off between credibility and flexibility as emphasised by Rogoff (1985) and others is eliminated, and the pre-commitment outcome can be attained.

As before, the term $(\pi^* - \pi^e + \varepsilon)$ is the output level consistent with the socially optimal inflation rate π^* , ie the output level that follows upon substituting π^* for π in (1). If y^* exceeds this level, then the central bank has an incentive to set inflation above the social optimum.

Comparing (15) and (5) it can be seen that the central bank's reaction function under a linear inflation contract looks very similar to the reaction function under the optimal inflation target. In both cases the effects of preference shocks on inflation are *asymmetric*, with positive shocks increasing inflation by more than negative shocks decrease it. There are two differences. The first is that in (15) the government-imposed inflation target π^b is replaced by the socially optimal inflation rate. Second, under a linear contract inflation decreases with f_1 . The reason is that an increase in f_1 raises the central banker's marginal loss from higher inflation.

Taking expectations (at time $t - 1$) of the central bank's reaction function under a linear contract gives inflation expectations as

$$\pi^e = \pi^* + \Phi(y^* - f_1) \quad (16)$$

where the term Φ has the same meaning as in the private sector's forecasting rule under an optimal inflation target.⁽¹³⁾

From expression (16) it is clear that inflation is expected to exceed its social optimum whenever $y^* > f_1$, in which case the marginal benefits of inflation are greater than the marginal costs, ie the incentive to inflate dominates the disciplining effect of the contract.

Inserting (16) back into (15) actual inflation is

$$\pi = \pi^e + \left(\frac{1 - \Phi(1 - x)}{2 - x} \right) (y^* - f_1) - \frac{\varepsilon}{2 - x} \quad (17)$$

(13) The derivation is similar to the case of an optimal inflation target; for more details see Appendix A. Note that, unlike BJ, uncertainty about the central banker's preferences increases inflation expectations. The intuition is similar to the case of the optimal inflation target. Positive preference shocks are more inflationary than negative shocks are disinflationary. Hence, with equal probabilities of positive and negative shocks, expected inflation will be adjusted upwards, and the more so the higher the variance term in Φ .

As in the case of the optimal inflation target, it is important to realise that—unlike BJ—monetary policy does not coincide with policy in the absence of uncertainty about the central banker’s preferences. If we set the preference shock equal to zero in (17) monetary policy will still react to the variance term in Φ . So in this respect certainty equivalence does *not* hold. The reason is that the central banker optimises its policy by taking inflation expectations as given. Since inflation expectations depend on the variance of the preference shocks (through Jensen’s inequality; see expression (16)) so does the central bank’s reaction function.

Combining (17) and (1) yields realised output:

$$y = \left(\frac{1 - \Phi(1 - x)}{2 - x} \right) (y^* - f_1) + \frac{1 - x}{2 - x} \varepsilon \quad (18)$$

As in the case of the optimal inflation target, inflation and output deviate from their expected values when either x or ε differ from their (zero) expected values. But compared with (8) and (9), the solutions for inflation and output differ in an important aspect: the delegation parameter f_1 now affects how uncertainty about the central banker’s preferences feed into inflation and output. Here we show that their result still holds if private sector inflation expectations depend on the uncertainty about preferences.

For the moment, suppose that the supply shock is zero. From (15) it can be seen that the contract affects the inflation rate through two channels.⁽¹⁴⁾ The first channel operates *directly* by increasing the marginal loss of inflation. Consider a marginal increase in f_1 starting from $f_1 = 0$. The direct effect is then given by the term $-f_1 / 2 - x$. So this effect is *uncertain* and ultimately depends on the realisation of x . The second, *indirect* channel operates through inflation expectations. Here it is equal to $-\Phi f_1$ (see equation (16)).⁽¹⁵⁾ The combined direct and indirect effects of the contract imply that any deviation of x from its (zero) expected value will have *smaller* effects on actual inflation and output. Thus, with a contract in place, ie $f_1 > 0$,—in contrast to the case of an optimal

(14) This decomposition follows BJ.

(15) In the BJ model the direct effect is *deterministic* and—using the notation of this paper—equal to $-f_1 / 2$. BJ have no effect from preference uncertainty on expected inflation so $\Phi \rightarrow 1$ and the indirect effect in their model is $-f_1$.

inflation target—the transmission of preference shocks to inflation and output is dampened.

Now consider the effects of a supply shock. The transmission of this shock to inflation and output is affected by the preference shock. Consider a positive realisation of x . This means that there is more emphasis on output stabilisation, which increases the willingness of the central bank to accommodate supply shocks. Hence the higher is x , the more such shocks are dampened before they pass on to output. The flipside of the coin is that inflation volatility is less well cared for. Of course, if x is negative the reverse is true. Then the central bank dampens the impact effect of the supply shock on inflation, and output is allowed to fluctuate more.

Under a linear contract, ie if $f_1 > 0$, it is clear that the role of fluctuating preferences in the transmission of supply shocks to inflation and output is significantly reduced. Thus there will be less scope for the central bank to impose its ‘own’ preferences on society and distort stabilisation policy by ‘overstabilising’ either inflation or output.

This can also be seen from the expressions for the variances of inflation and output

$$Var\pi = \left[\frac{(y^* - \pi^* + \pi^e - f_1)^2 + \sigma_\varepsilon^2}{4} \right] \frac{\sigma_x^2}{4} + \frac{\sigma_\varepsilon^2}{4} \left(1 + \frac{\sigma_x^2}{4} \right)^2 \quad (19)$$

$$Vary = Var\pi + \sigma_\varepsilon^2 - \left(1 + \frac{\sigma_x^2}{4} \right) \sigma_\varepsilon^2 \quad (20)$$

Through the first term on the right-hand side of (19) the contract affects the variability of inflation and, through (20), of output.

To find the *optimal* linear inflation contract, the government chooses f_1 so as to minimise L^G subject to (16), (17), and (18). Using the first-order condition, the optimal value of f_1 , denoted by f_1^* is given by:⁽¹⁶⁾

(16) See Appendix C for the derivation.

$$f_1^* = y^* \quad (21)$$

This expression reveals that it is optimal to offer a linear inflation contract to a central banker that does *not* depend on the degree of uncertainty about its preferences. This is in sharp contrast to BJ, who find that the optimal linear contract *does* depend on the degree of preference uncertainty. Our result is the same as in Walsh's no preference uncertainty case. Hence, certainty equivalence *does* hold. The value f_1^* corresponds to the optimal Walsh contract.

5 The linear Walsh contract versus the Svensson inflation target

In this section we compare the outcomes for inflation and output under a Walsh contract with the outcomes under a Svensson inflation target. In the absence of uncertainty about the central banker's preferences, Svensson has shown that the linear Walsh contract is equivalent to the Svensson inflation target. Both achieve optimal stabilisation without an inflationary bias. We will show that this equivalence no longer holds when disturbances to the central banker's preferences are introduced.

Optimal linear inflation contract

Under the optimal linear inflation contract the inflationary bias is eliminated so that expected inflation is equal to the inflation target (this can be seen from (16)). Thus the credibility problem is solved. To see how the linear contract performs with respect to *stabilisation policy* we need to look at the variances of inflation and output. By substituting $f_1 = y^*$ into (19) and (20) we get

$$\begin{aligned} \text{Var}\pi &= \frac{\sigma_\varepsilon^2}{4} \left(1 + \frac{3\sigma_x^2}{4} + \frac{\sigma_x^4}{16} \right) \\ \text{Var}y &= \frac{\sigma_\varepsilon^2}{4} \left(1 - \frac{\sigma_x^2}{4} + \frac{\sigma_x^4}{16} \right) \end{aligned} \quad (22)$$

Optimal inflation target

The optimal Svensson inflation target also solves the credibility problem. So in this respect it is identical to the linear inflation contract. However, its stabilisation properties are different from the linear inflation contract.

The variances of inflation and output under an optimal inflation target can be found by substituting $\pi^b = \pi^* - \Phi y^*$ into (10) and (11). This yields

$$\begin{aligned} \text{Var}\pi &= \frac{\sigma_\varepsilon^2}{4} \left(1 + \frac{3\sigma_x^2}{4} + \frac{\sigma_x^4}{16} \right) + \frac{4\sigma_x^2 y^{*2}}{(4 - \sigma_x^2)^2} \\ \text{Var}y &= \frac{\sigma_\varepsilon^2}{4} \left(1 - \frac{\sigma_x^2}{4} + \frac{\sigma_x^4}{16} \right) + \frac{4\sigma_x^2 y^{*2}}{(4 - \sigma_x^2)^2} \end{aligned} \quad (23)$$

Comparing (22) and (23), it is clear that in the presence of uncertainty about the central banker's preferences the Walsh contract unambiguously yields a lower variance of inflation and a lower variance of output than the Svensson inflation target. This means that the optimal linear inflation contract yields a lower expected loss to the government and, therefore, is *strictly superior* to the optimal inflation target.⁽¹⁷⁾

The intuition behind this result is simple. In Section 3 we showed that with an inflation target, the delegation parameter π^b has no effect on how the uncertainty about the central banker's preferences is transmitted to inflation and output. Put differently, the choice of the inflation target has no implications for stabilisation policy. Thus, the variances of output and inflation are *independent* of the choice of inflation target.

From Section 4 we know that this is no longer true under a linear contract. With the optimal contract in place, ie $f_1^* = y^*$ —contrary to the case of an optimal inflation target—the transmission of preference shocks to inflation and output is dampened. Because the role of fluctuating preferences in the transmission of supply shocks to inflation and output is significantly reduced, the variances of inflation and output will be lower than in the case of an optimal inflation target.

6 Quadratic contracts

Even though the optimal linear contract is superior to the optimal inflation target, it does not achieve the pre-commitment solution. The reason is that the interaction between the supply shocks and preference uncertainty generates excess

(17) This result is similar to BJ. However, in their model stabilisation policy under a linear inflation contract is at the cost of expected inflation being *below* the socially optimal inflation rate. This trade-off is not present in this paper.

macroeconomic variability. We address this problem by expanding the set of admissible contracts to include quadratic contracts. A quadratic contract penalises quadratic (squared) deviations of inflation from some target.

Combination of a linear and a quadratic contract

More specifically, to moderate the excess macroeconomic variability under the linear inflation contract we examine the combination of a linear and a quadratic contract. Let the added quadratic cost to inflation be $f_2(\pi - \pi^*)^2$, where f_2 is a constant. Then the central bank is assigned the loss function

$$L^{CB} = \frac{1-x}{2}(\pi - \pi^*)^2 + \frac{1}{2}(y - y^*)^2 + f_1(\pi - \pi^*) + \frac{f_2}{2}(\pi - \pi^*)^2 \quad (24)$$

From (24) it can be seen that, if $f_2 > 0$, the quadratic contract is mathematically equivalent to Rogoff's conservative central banker. Of course, if this parameter is negative, we have a *liberal* central banker.

The optimal delegation arrangement is found in two steps. The first step is to solve for the optimal linear contract, taking f_2 as given. The second step is to solve for the optimal value of f_2 . As was shown in the previous section, the variances of inflation and output do not depend on the parameters of the Walsh contract when the parameter f_1 is optimally chosen. Adding a quadratic penalty on inflation does not change the optimal Walsh contract. Therefore in this section we have two independent instruments for two problems. We use the linear penalty term to eliminate the inflationary bias and we use the quadratic term to minimise the welfare loss to society caused by the variance of inflation and output. The quadratic term then brings a trade-off between the variance of inflation and the variance of output, as is well-known from the Rogoff model.

From Appendix D it follows that the optimal combination of a linear and a quadratic contract is

$$f_1^{**} = y^*, \quad 0 < f_2^* < \frac{9\sigma_x^2}{4 + 3\sigma_x^2} \quad (25)$$

So to mitigate excess macroeconomic variability due to the interaction between supply shocks and preference shocks, the linear Walsh contract needs to be augmented with a *conservative* central banker. This result was first found by

Herrendorf and Lockwood. Here we show that their result—which follows from information asymmetries between a monopolistic trade union and the political principal with respect to productivity shocks—continues to hold in our setting.⁽¹⁸⁾ The Walsh contract *cum* conservative central banker result is also found by BJ. Thus the Herrendorf-Lockwood result appears to be robust.

Combination of an inflation target and a quadratic contract

As was shown in the previous section, the optimal inflation target performs worse—in terms of social welfare—than the optimal linear contract. More specifically, because the inflation target has no effect on the transmission of uncertainty about the central banker’s preferences to inflation and output, macroeconomic variability is higher than with an optimal linear contract. Thus social welfare with an optimal inflation target will be even further away from the first-best pre-commitment outcome than in the case of a linear contract.

To mitigate this sub-optimal volatility, we now consider a combination of an optimal inflation target and a quadratic contract. Under this delegation scheme the central banker minimises

$$L^{CB} = \frac{1-x}{2}(\pi - \pi^b)^2 + \frac{1}{2}(y - y^*)^2 + \frac{f_2}{2}(\pi - \pi^*)^2 \quad (26)$$

As before, the optimal delegation arrangement is found in two steps. The first step is to solve for the optimal inflation target, taking f_2 as given. The second step is to solve for the optimal value of f_2 . As was shown in the previous section, the variances of inflation and output do not depend on the inflation target π^b . So adding a quadratic penalty on inflation does not change the optimal inflation target. This means that, as before, we now have two independent instruments for two problems. We use the inflation target to eliminate the inflationary bias, and we use the quadratic contract to minimise the welfare loss to society caused by the excess macroeconomic variability.

From Appendix D it follows that the optimal combination of an inflation target and a quadratic contract is

(18) As can be seen in Appendix D, the combination of an optimal linear Walsh contract and a quadratic contract gives the Rogoff result that the variance of inflation decreases with f_2 whereas the variance of output increases with f_2 (for σ_x^2 sensibly small).

$$\pi^{b**} = \pi^* - \Phi y^*, \quad f_2^{**} > f_2^* \quad (27)$$

From (27) it follows that the Svensson optimal inflation target (which already depends on the degree of preference uncertainty) also has to be augmented with a conservative central banker. Moreover, because the additional variance that the inflation target generates can unambiguously be reduced by making the central banker more conservative, the central banker has to be even more conservative under a Svensson target than in the case of a linear Walsh contract. This is proved in Appendix D.

7 Concluding remarks

In this paper we have investigated the effects of uncertain central bank preferences on the optimal institutional design of monetary policy. We have studied this in the context of a model that goes beyond ‘pure’ uncertainty. This means that we have relaxed the Beetsma and Jensen restriction that the central banker’s preference uncertainty has no effect on private sector inflation expectations.

We show that monetary policy with uncertainty will on average *not* coincide with policy in the absence of uncertainty. In this respect certainty equivalence does not hold. The reason is that the central bank optimises its policy, taking inflation expectations as given. Since private sector inflation expectations now depend on the degree of preference uncertainty (being the variance of the preference shock) so does monetary policy. Hence, through the expectations channel, preference uncertainty now has *systematic* (as opposed to transitory) effects on policy. The implications of allowing the uncertainty to play a bigger role are substantial.

First, the optimal inflation target that has to be imposed on the central banker now depends on the degree of preference uncertainty, and has to be *stricter* (lower) the higher the uncertainty. This is in sharp contrast to BJ, who find that the optimal inflation target does *not* depend on the degree of preference uncertainty and is the same as in the Svensson model.

Further, for the case of the Walsh contract we find that it is optimal to offer a linear inflation contract to a central banker that does *not* depend on the degree of uncertainty about its preferences. Again, this is in sharp contrast to BJ, who find that the optimal linear contract *does* depend on the degree of preference

uncertainty. Here our result is the same as in Walsh for the case without preference uncertainty.⁽¹⁹⁾

The policy implications of our analysis are clear. It suggests that—in the presence of uncertainty about the central banker’s preferences—the best institutional arrangement is to select a central banker who is more inflation-averse than society, grant the central bank *instrument independence* and make it *accountable* for its monetary policy performance by imposing a linear inflation contract.

Moreover, this delegation arrangement has the appealing property that the optimal linear inflation contract is *independent* of the degree of preference uncertainty. There is no need for the government (the principal) to make delegation arrangements conditional on this uncertainty. In other words the legislator needs *less* information than in the case of the optimal inflation target (where the target *does* depend on σ_x^2) and still reaps the rewards of better stabilisation policy than in the case where policy is set by a central bank with an optimal inflation target.

(19) Moreover, it is interesting to realise that the results in this paper can also be seen as a kind of sensitivity analysis of stochastic credibility models. We have shown that the Herrendorf-Lockwood result is very robust. This holds both in a context where a monopolistic trade union has more information about the economy than the political principal, and in both the present model and the BJ approach. However, as was also shown in the paper, the BJ results are not robust. A more plausible modelling of preference uncertainty between both the political principal and the private sector on the one hand, and the central bank on the other, enabled us to reverse most of their results. Of course, this is by itself telling us something about the fragility of results when one moves away from the core Barro-Gordon model.

Appendix A

Private sector inflation expectations

Optimal inflation target

Taking expectations across equation (5):

$$E(\pi) = E\left(\frac{1}{2-x}\right)(\pi^e + y^*) + \pi^b E\left(\frac{1-x}{2-x}\right) \quad (\text{A.1})$$

This expression requires us to take the expected value of ratios of random variables. This can be achieved through a Taylor series expansion. Our problem is to expand $\phi(z) = x/y$ about the respective means. Assuming that the first two moments of $E(x/y)$ exist we can write down the expression

$$E\left(\frac{X}{Y}\right) \approx \frac{\mu_x}{\mu_y} - \frac{1}{\mu_y^2} \text{cov}[X, Y] + \frac{\mu_x}{\mu_y^3} \text{var}[Y] \quad (\text{A.2})$$

This is the (second-order) approximation used in the paper, therefore we can write

$$E\left(\frac{1}{2-x}\right) \approx \frac{4 + \sigma_x^2}{8} \text{ and } E\left(\frac{1-x}{2-x}\right) \approx \frac{4 - \sigma_x^2}{8} \quad (\text{A.3})$$

So substituting (A.3) in (A.1) and rearranging gives

$$\pi^e = \frac{4 + \sigma_x^2}{4 - \sigma_x^2} y^* + \pi^b \quad (\text{A.4})$$

which is equation (6) in the text. Again, it is clear that as $\sigma_x^2 \rightarrow 0$ the first part of the expression on the right-hand side collapses to unity, and (A.4) is equivalent to (7).

Linear inflation contract

The derivation of private sector expectations for the case of a linear inflation contract is similar to the case of an optimal inflation target (see above).

Appendix B

The optimal inflation target

Since the choice of inflation target does not affect the stochastic components of the government's expected loss, its problem is

$$\text{Min}_{\pi^b} E^G \left[\frac{1}{2} (\pi^e - \pi^*)^2 \right] \quad (\text{B.1})$$

subject to

$$\pi^e = \pi^b + \Phi y^* \quad (6)$$

The first-order condition (FOC) for minimising (B.1) with respect to π^b is

$$(\pi^e - \pi^*) \frac{\partial \pi^e}{\partial \pi^b} = (\pi^e - \pi^*) = 0 \quad (\text{B.2})$$

It follows that the FOC can be written as

$$\pi^e = \pi^* \quad (\text{B.3})$$

Substituting (6) for π^e yields

$$\pi^{b*} = \pi^* - \Phi y^* \quad (\text{B.4})$$

which is equation (12) in the main text.

Appendix C

The optimal linear contract

The government's problem is

$$\text{Min}_{f_1} E^G \left[\frac{1}{2} (\pi^e - \pi^*)^2 + \frac{1}{2} (\text{Var}\pi + \text{Vary}) \right] \quad (\text{C.1})$$

subject to (16), (20) and (21)

The first-order condition (FOC) for minimising (C.1) with respect to f_1 is

$$E^G (\pi^e - \pi^*) \frac{\partial \pi^e}{\partial f_1} + \frac{1}{2} \left[\frac{\partial \text{Var}\pi}{\partial f_1} + \frac{\partial \text{Vary}}{\partial f_1} \right] = 0 \quad (\text{C.2})$$

Substituting for π^e from (16) and using that $\partial \pi^e / \partial f_1 = -\Phi$ we get

$$E^G \left(-\Phi^2 (y^* - f_1) \right) + \frac{1}{2} \left[\frac{\partial \text{Var}\pi}{\partial f_1} + \frac{\partial \text{Vary}}{\partial f_1} \right] = 0 \quad (\text{C.3})$$

Then expanding $\frac{\partial \text{Vary}}{\partial f_1}$ as $\frac{\partial \text{Vary}}{\partial \text{Var}\pi} \frac{\partial \text{Var}\pi}{\partial f_1} = \frac{\partial \text{Vary}}{\partial f_1}$ and rearranging we get

$$-\left(\Phi^2 (y^* - f_1) \right) + \left[\frac{\partial \text{Var}\pi}{\partial f_1} \right] = 0 \quad (\text{C.4})$$

Finally, using that, from (20), $\frac{\partial \text{Var}\pi}{\partial f_1} = \frac{-\sigma_x^2 (\Phi + 1) (y^* - f_1)}{4}$

the government's FOC can be written as

$$-\left[\frac{8\Phi^2 + 2(1 + \Phi)\sigma_x^2}{8} \right] (y^* - f_1) = 0 \quad (\text{C.5})$$

It can be easily seen that the optimal linear contract then is

$$f_1^* = y^* \tag{C.6}$$

which is equation **(21)** in the main text.

Appendix D

Quadratic contracts

Optimal quadratic contract with optimal Walsh contract

$$\text{Var}\pi = \frac{\sigma_\varepsilon^2}{(f_2 + 2)^2} \left(1 + \frac{3\sigma_x^2}{(f_2 + 2)^2} + \frac{\sigma_x^4}{(f_2 + 2)^4} \right) + \frac{y^{*2}\sigma_x^2(f_2 + 2)^2}{((f_2 + 1)(f_2 + 2)^2 - \sigma_x^2)^2} \quad (\text{D.1})$$

$$\text{Vary} = \frac{\sigma_\varepsilon^2}{(f_2 + 2)^2} \left((f_2 + 1)^2 - \frac{(2f_2 + 1)\sigma_x^2}{(f_2 + 2)^2} + \frac{\sigma_x^4}{(f_2 + 2)^4} \right) \quad (\text{D.2})$$

The optimal value for f_2 is found at

$$f_2 = \frac{6\sigma_x^2}{(f_2 + 2)^2 + 3\sigma_x^2} + \frac{6\sigma_x^2}{(f_2 + 2)((f_2 + 2)^2 + 3\sigma_x^2)} \equiv G(f_2) \quad (\text{D.3})$$

Since $G(0) = \frac{9\sigma_x^2}{4 + 3\sigma_x^2}$, $G(\infty) = 0$, and G is monotonically decreasing in f_2 , there is a unique solution to (D.3).

Optimal quadratic contract with optimal Svensson target

$$\text{Var}\pi = \frac{\sigma_\varepsilon^2}{(f_2 + 2)^2} \left(1 + \frac{3\sigma_x^2}{(f_2 + 2)^2} + \frac{\sigma_x^4}{(f_2 + 2)^4} \right) + \frac{y^{*2}\sigma_x^2(f_2 + 2)^2}{((f_2 + 1)(f_2 + 2)^2 - \sigma_x^2)^2} \quad (\text{D.4})$$

$$\text{Vary} = \frac{\sigma_\varepsilon^2}{(f_2 + 2)^2} \left((f_2 + 1)^2 - \frac{(2f_2 + 1)\sigma_x^2}{(f_2 + 2)^2} + \frac{\sigma_x^4}{(f_2 + 2)^4} \right) + \frac{y^{*2}\sigma_x^2(f_2 + 2)^2}{((f_2 + 1)(f_2 + 2)^2 - \sigma_x^2)^2} \quad (\text{D.5})$$

The optimal value for f_2 is found at

$$f_2 = G(f_2) + \frac{\sigma_x^2 y^{*2} (f_2 + 2)^6 \left((2f_2 + 3)(f_2 + 2)^2 + \sigma_x^2 \right)}{\sigma_\varepsilon^2 \left((f_2 + 1)(f_2 + 2)^2 - \sigma_x^2 \right)^3 \left((f_2 + 2)^2 + 3\sigma_x^2 \right)} \quad (\mathbf{D.6})$$

Comparing **(D.6)** with **(D.3)**, it is clear that the solution to **(D.6)** is found at a higher value for f_2 . So we need a more conservative central bank under a Svensson inflation target than with a linear Walsh contract.

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