Shoe-leather costs reconsidered*

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Abstract

Lucas has recently suggested that the 'shoe-leather' costs of inflation may amount to as much as 1% of GNP in the United States when moving to the Friedman optimum. We assess his thesis using empirical evidence for the United Kingdom for the period 1870-1994. We find support for the Lucas proposition that interest rates should be specified in logs as a description of money demand dynamics, but not as a steady-state characterisation. Although Lucas's estimates can be corroborated, a semi-log interest rate specification implies smaller, though still tangible, welfare gain estimates: for example, 0.22% of GNP in perpetuity when moving from 6% to 2% nominal interest rates.

1 Introduction

Estimating the welfare costs of inflation has, on the whole, proved an elusive task for economists. But perhaps the area of least uncertainty surrounds the so-called 'shoe-leather' costs of inflation – the increased time and cost of making trips to the bank to replenish money balances whenever inflation increases. Since Bailey (1956), it has been customary to measure these 'shoe-leather' costs as the trapezoid of unsatisfied demand beneath a money demand schedule, which is foregone at any non-zero nominal interest rate. There is also a common perception that, measured in this way, shoe-leather costs are relatively trivial in macroeconomic terms.

Certainly, such a conclusion seems robust at the levels of inflation currently prevailing within developed economies. For example, Fischer (1981) and McCallum (1989) both estimate that the shoe-leather benefit from a 10 percentage point reduction in inflation is around 0.3% of GNP. But in today's low-inflation environment, a reduction in inflation of around 2 percentage points seems more apposite. Using the above calculus, such a reduction would deliver a welfare gain of only around 0.06% of GNP – a small number by most people's reckoning. Indeed, a recent study by Feldstein (1995) concludes that the shoe-leather savings from a move from 2% to zero inflation could well be *negative* once the indirect effect on seigniorage revenues is taken into account – a point first raised by Phelps (1973).[1]

That academic view chimes with anecdotal evidence. A recent survey by Shiller (1996) posed the question 'Why do people dislike inflation?' Among the general public, there was no mention of the extra time and cost expended by agents replenishing their money balances at high rates of inflation. There was, as you might expect, a greater recognition of these costs among economists. But that may well reflect reverse causality. Central banks, meanwhile, who are never shy to advertise the benefits of price stability, have rarely if ever ventured to suggest that the shoe-leather savings of low inflation amount to much in the aggregate and have concentrated on

(1) Feldstein (*op cit*) finds much larger distortions from inflation arising from the imperfect indexation of the US tax system – amounting perhaps to as much as 1% of GNP in perpetuity. That leads him to argue strongly for a zero inflation target.

the costs of unanticipated inflation.[2] Surely, then, the evidence – academic and anecdotal – is incontrovertible?

This received wisdom has recently been questioned by, among others, Lucas (1995) and Gillman (1995).[3] For example, Lucas estimates that the shoe-leather benefits of moving to a zero nominal rate of interest – of deflating at a rate equal to the real rate of interest, in line with Friedman's (1969) optimal rule – could amount to as much as 1% of US GNP in perpetuity. Gillman (1995) concludes that a 'conservative estimate range' of shoe-leather costs is 0.85%-3% of GNP. If we were to assume a discount rate of 5% and trend GNP growth of 2.5%, the net present value of a 1% GNP welfare gain would then amount to 40% of initial GNP. On 1995 US numbers, that is $2,900 billion. In Lucas's words, 'this is real money'.[4]

Lucas's argument has two strands. First, that mis-specification of existing simple money demand equations may lead to an underestimation of the welfare benefits which zero nominal interest rates confer. Second, that seigniorage losses are most unlikely to offset these benefits, so that second-best arguments cannot be used to justify positive nominal rates of interest.[5] This paper presents some empirical evidence for the United Kingdom on these issues. In particular, we seek to juxtapose the log and semi-logarithmic interest rate functional forms of a primitive money demand function. The former specification increases welfare costs markedly at low rates of inflation because each percentage point reduction in nominal interest rates has a proportionally greater impact upon real money holdings. In diagrammatic terms, the money demand function looks hyperbolic, so that the value of monetary services – the area under the demand function – increases as we move down the curve. By contrast, a semi-log money demand specification – the mainstay of much of the empirical literature since

(2) See, for example, the Governor of the Bank of England's first LSE lecture on 'The Case for Price Stability' (Leigh-Pemberton, 1992).
(3) For example, Dotsey and Ireland (1996), among others, offer general equilibrium analyses reaching a broadly similar conclusion.
(4) Such a calculation is necessarily illustrative, rather than definitive, because it is difficult to agree on whether and how to discount the utility of future generations.
(5) For example, optimal tax arguments would suggest a non-zero nominal interest rate, so as to equalise the marginal distortionary effects of various forms of taxation – inflation among them. But if money is not a final good but rather an intermediate good, then logic of the Diamond and Mirrlees (1971) type would suggest that a zero inflation tax could be optimal. We discuss tax effects in Section 5.

at least Cagan (1956) – implies the same response of money holdings to each percentage point reduction in nominal interest rates. This then delivers lower welfare cost estimates at low rates of inflation. The essence of Lucas's argument is that the log specification is to be favoured on theoretical and empirical grounds – hence larger welfare costs from not adhering to the Friedman rule.

To date, the extent of the empirical evidence on these specification issues is contained in some charts in Lucas (*op cit*). Here we consider that empirical evidence in a little more detail, and then use our empirical estimates to evaluate some welfare costs. By way of motivation, Charts 1 and 2 replicate those from Lucas (*op cit*) using UK data.(6) Both figures plot the ratio of the monetary base to money GNP (V, the inverse of velocity) against deposit interest rates (R) in the United Kingdom for the period 1870-1994. Chart 1 then plots the curve $V = \alpha R^{-\lambda}$ – a logarithmic specification for money demand, with the long-run income elasticity restricted to unity. As in Lucas, α is set such that the curves pass through the geometric mean of the data points, using three values of the long-run interest elasticity ($\lambda = 0.3$, 0.5 and 0.7).(7) Chart 2 does the same for the semi-logarithmic specification $V = \beta e^{-\gamma R}$, for $\gamma = 0.05$, 0.07 and 0.09. All of these elasticities are taken from Lucas.

(6) In fact, the charts are not a precise replication of those in Lucas in three important respects. First, we use a measure of the monetary base (M0), rather than M1. Second, we use here a longer sample – 1870-1994, rather than 1900-85 – which covers the whole of the Classical Gold Standard period. Including the data points at the beginning (1870-1900) and end (1985-94) of the sample considerably worsens the visual 'fit' of the data under either interest rate specification. And third, the interest rate we plot is a deposit rate, rather than a consol yield as in Lucas. We prefer the former because deposits are likely to be a closer substitute for cash than perpetual bonds. Section 3 discusses the data in greater detail.
(7) The geometric mean is sensible for very long-run data series as it minimises the effect of outliers on the mean.

Chart 1
M0/GNP 1870-1994, log specification

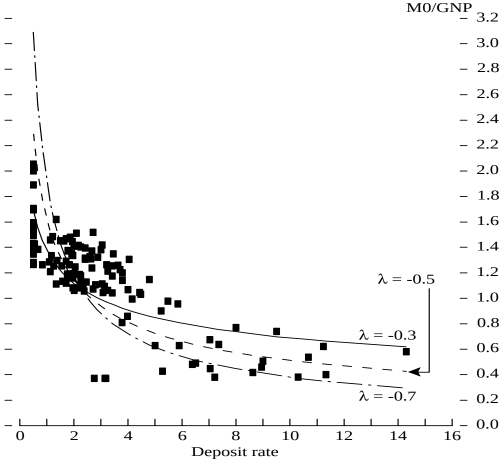
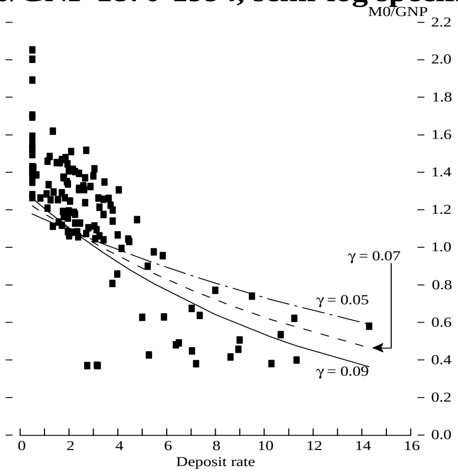


Chart 2
M0/GNP 1870-1994, semi-log specification



What is clear from Charts 1 and 2 is that the logarithmic formulation appears, on the face of it, to have much to recommend it empirically, at least by comparison with the semi-log function. This is in keeping with Lucas's findings for the United States. These stylised observations are enough to motivate us to re-consider the quantitative importance of the shoe-leather costs of inflation in the United Kingdom. Perhaps that holy grail for central bankers – a tangible welfare cost of inflation – has after all been hiding under their noses.

The paper is planned as follows. The next section outlines a textbook general equilibrium model of money demand, to shed some light on the theoretical case for and against the logarithmic money demand functional form. Section 3 discusses our methodology and data and Section 4 our results. Section 5 translates these estimates into some welfare costs of inflation; while Section 6 briefly concludes.

2 The McCallum-Goodfriend model

This section outlines a simple stylised general equilibrium model of money demand determination. The model is illustrative and is solved in order to examine plausible functional forms for interest rates in terms of satisfying consumers' utility-maximising demands for real money balances. Specifically, we examine whether a log or semi-log specification for interest rates seems plausible. The model is stylised in several respects – for example, it affords no role for credit. We use the model set out in McCallum and Goodfriend (1987). This introduces money into the utility function by assuming that money balances are leisure-enhancing because they save on shopping time. So we have a leisure function, l_t, of the form

$$l_t = \phi\ (C_t,\ M_t/P_t) \qquad \textbf{(1)}$$

where $\phi_1 < 0$ – higher consumption (C_t) implies more shopping time and hence less leisure time (*ceteris paribus*); and $\phi_2 > 0$ – higher real money balances (M_t/P_t) help to reduce shopping time and thereby free up more time for leisure. The function $\phi(.)$ defines transactions technology – saved shopping time – in the model. This notion of shopping-time technology is proving popular as a means of rationalising agents' money holdings among general equilibrium theorists, as an alternative to a cash-in-advance constraint or placing money directly in the utility function.

Otherwise the consumer's choice problem is entirely standard. Agents maximise an intertemporal utility function defined over leisure and consumption goods, which is separable across time:

$$\sum_{i=0}^{\infty} \beta^i U\left(C_{t+i}, l_{t+i}\right) \tag{2}$$

where U_1, $U_2 > 0$ and β is a discount rate satisfying $\beta \equiv 1/(1 + \rho)$ where ρ is the consumer's subjective rate of time preference. For simplicity, McCallum and Goodfriend (*ibid*) assume that labour is supplied inelastically so that agents allocate their time between only leisure and shopping each period. Agents receive real income, $\overline{y}_t$, each period, which they allocate in turn across consumption goods, nominal money balances (M_t) and one-period nominal bonds (B_t). So the household budget constraint is:

$$P_t \overline{y}_t + M_{t-1} + \left(1 + R_{t-1}\right)B_{t-1} = P_t C_t + M_t + B_t \tag{3}$$

where the LHS defines incomings and the RHS outgoings; P_t is the price of the consumption bundle and R_t defines the yield on the one-period bond.

The consumer's problem is then to maximise **(2)** subject to **(1)** and **(3)** by choice of M_t, C_t, and B_t.(8) The first-order conditions are:

$$0 = U_1\left[C_t, \phi\left(C_t, M_t / P_t\right)\right] + U_2\left[C_t, \phi\left(C_t, M_t / P_t\right)\right]\phi_1\left(C_t, M_t / P_t\right) - \lambda_t P_t \tag{4}$$

$$0 = U_2\left[C_t, \phi\left(C_t, M_t / P_t\right)\right] - \lambda_t + \beta\lambda_{t+1} \tag{5}$$

$$0 = \beta\lambda_{t+1}\left[1 + R_t\right] - \lambda_t \tag{6}$$

$$0 = \left(1 + R_{t-1}\right)B_{t-1} - P_t\left(C_t - \overline{y_t}\right) - \left(M_t - M_{t-1}\right) - B_t \tag{7}$$

(8) P_t, $\overline{y}_t$ and R_t are given exogenously.

where λ_t is the Lagrangian multiplier pertaining to **(3)**. Using **(4)** - **(7)** gives us the following Euler equation, as in McCallum (*ibid*).

$$U_2\left[C_t, \phi\left(C_t, M_t / P_t\right)\right]\phi_2\left(C_t, M_t / P_t\right) =$$

$$\left[U_1\left[C_t, \phi\left(C_t, M_t / P_t\right)\right] + U_2\left[C_t, \varphi\left(C_t, M_t / P_t\right)\right]\phi_1\left(C_t, M_t / P_t\right)\right] 1 - \left(1 + R_t\right)^{-1}$$

(8)

This has the general form:

$$g\left(\gamma, M_t, P_t, C_t, R_t\right) = 0 \qquad \textbf{(9)}$$

and, provided it is soluble, is hence interpretable as a structural money-demand function. For example, McCallum (*op cit*) shows that with utility taking a Cobb-Douglas form $U(C_t, l_t) = C_t^{\alpha}\, l_t^{(1-\alpha)}$ and transactions technology $\phi(C_t, M_t/P_t) = C_t^{-a}\, M_t/P_t^{a}$, where $0 < (a, \alpha) < 1$, we can then write:

$$\ln(M_t / P_t) = \ln\gamma + \ln C_t + \ln(1 + R_t^{-1}) \qquad \textbf{(10)}$$

where $\gamma \equiv a(1-\alpha)/(\alpha - a(1-\alpha))$. Note that the interest rate term in **(10)** can be rewritten as $\ln(1+R^{-1}) = -\log(R) + R$, using the approximation $\ln(1+R) = R$. That is, the interest rate term comprises both a log and a (wrongly signed) semi-log component. Note also that for small R – the cases we consider here – the second of these terms is small and can be ignored, giving a log-log money demand model. So in a system with sensible functional forms – those with standard diminishing marginal properties, U_{11}, $U_{22} < 0$, $\phi_{11} > 0$, $\phi_{22} < 0$ – a logarithmic money demand function obtains. A logarithmic form, similar to **(10)**, also obtains for a variety of other well-known utility and transaction-technology functions, such as the CES and translog functions, which also exhibit standard diminishing marginal properties.

We might ask, then, what primitive preference and technology functions would deliver a semi-logarithmic specification. Two functions able to deliver such a functional form are:

$$U\left(C_t, l_t\right) = \alpha(C_t \ln Ct - C_t) + (1-\alpha) l_t \quad \textbf{(11)}$$

$$\phi\left(C_t, M_t / P_t\right) = -aC_t + a[(M_t / P_t)\ln(M_t / P_t) - (M_t / P_t)] \quad \textbf{(12)}$$

which, using **(8)**, gives the money demand function:

$$\ln\left(M_t / P_t\right) = -1 + \gamma^* \ln C_t + \left[1 + Rt\right]^{-1} - \gamma^* \ln C_t \left[1 + Rt\right]^{-1} \quad \textbf{(13)}$$

where $\gamma^* = \alpha/(1\text{-}\alpha)a$. Ignoring the multiplicative last term gives an interest rate semi-elasticity:

$$\frac{-1}{\left[1 + R_t\right]^2} < 0 \quad \textbf{(14)}$$

which is negative as we would expect. The striking thing, however, is that neither the primitive preference ($U(.)$) nor technology ($\phi(.)$) functions, **(11)** and **(12)**, which deliver the semi-log reduced-form **(13)**, have the nice diminishing marginal properties that we would want: from **(11)** and **(12)**, U_{11}, $\phi_{22} > 0$ and U_{22} and ϕ_{11} are zero. This seems to suggest that any sensible preference and technology functions are likely to deliver a logarithmic money demand specification; and, conversely, that a semi-log functional form may involve placing restrictions on deep parameters which are at odds with standard marginal analysis.(9) That said, when the model is expressed in levels – the long-run equation, **(10)** – the differences between the log and semi-log specifications is unlikely to be that great. It is only when **(10)** is made dynamic that differences between the log and semi-log

(9) A log interest rate specification leads to non-satiable real balances as rates tend to zero, whereas the semi-log functional form does not lead to such indeterminacy. It is, of course, possible that some other functional form for interest rates can be found which satiates the demand for real money balances, but the behavioural implications of such functional forms may not be entirely clear (see Wolman, 1996).

specifications are likely to show up as acute under low rates of inflation. This is important when we come to account for our empirical results below.

3 Method and data

3.1 Method

Above we derived a money demand function, **(9)**, of the generic form:

$$g(\gamma,\ M_t,\ P_t,\ C_t,\ R_t) = 0 \qquad \textbf{(15)}$$

Let us assume that this function can then be straightforwardly rewritten as a log-linear representation across all variables with the exception of R_t and that it is soluble in M_t, thus:

$$m_t - p_t = \alpha_0 + \alpha_1 c_t + \alpha_2 h(R_t) \qquad \textbf{(16)}$$

where lower case letters now denote natural logs and $\alpha_0 \equiv \ln \gamma$. **(16)** is just an unrestricted version of **(15)**, where the consumption and interest rate elasticities are no longer equal to (1,-1) respectively. $h(.)$ is some general functional form which we need to specify to make **(16)** estimable. A particularly convenient functional form is the power transformation of Box and Cox (1964):[(10)]

$$\alpha_2\ h\ (R_t) = \alpha_2\ (R_t^{\theta} - 1) / \theta \qquad \textbf{(17)}$$

This particular transformation nests the log and semi-log specifications as special cases:

$$\begin{aligned} \alpha_2\ h\ (R_t) &= \alpha_2 R_t - \alpha_2 && \text{for } \theta = 1 \\ &= \alpha_2 \ln R_t && \text{for } \theta = 0 \end{aligned}$$

The primitive form of the money demand function is thus:

$$m_t - p_t = \alpha_0 + \alpha_1 c_t + \alpha_2 (R_t^{\theta} - 1) / \theta \qquad \textbf{(18)}$$

(10) For an application of this transformation in a similar context, see Mills and Wood (1977).

Equation **(18)** defines equilibrium real money balances, but to give it a convincing empirical form most researchers have posited some partial-adjustment mechanism for equilibrium real money balances:

$$\Delta\ (m_t\text{-}p_t)\ =\ -\ \beta\ (m_{t-1}\text{-}p_{t-1}\ -\ (m_t^{*}\text{-}p_t^{*})) \qquad \textbf{(19)}$$

where β is an adjustment, or error-correction, loading coefficient and * denotes an equilibrium value. Substituting **(18)** into **(19)** gives a conventional dynamic error-correction model:

$$\begin{aligned}\Delta\ (m_t\text{-}p_t)\ &= \gamma - \beta\ (m_{t-1}\text{-}p_{t-1}\ -\ \alpha_0\ -\ \alpha_1\ c_t\ -\ \alpha_2\ (R_t^{\theta}\text{-}1)/\theta) \\ &\quad + \boldsymbol{\delta(L)}\ \{\Delta(m\text{-}p)_t,\ \Delta c_t,\ \Delta(R_t^{\theta}\text{-}1)/\theta\}\ +\ \varepsilon_t \qquad \textbf{(20)}\end{aligned}$$

We have augmented **(20)** with some short-run dynamic terms in (m_t-p_t), c_t and R_t to pick up adjustment of money balances along the path to equilibrium. These short-run effects are captured here in **$\boldsymbol{\delta}$(*L*)** which is a vector polynomial in the lag operator (*L*).

Equation **(20)** is clearly non-linear. We approach its estimation in two – complementary – ways. The first is to impose values of θ on the model and then to conduct a grid search across the θs. Doing this linearises **(20)** and thus allows conventional linear estimation and inference to be conducted. The mechanics of this linear estimation procedure are as follows. First, the stationarity of the variables is assessed using standard unit-root tests. Then cointegration among the variables is assessed using Johansen's (1988) maximum likelihood technique; this gives estimates of the α's, as well as allowing us to test $\beta = 0$ (or, equivalently, whether or not cointegration exists). Finally, the dynamics, **$\boldsymbol{\delta}$(*L*)**, constant and error-correction loading coefficient are all estimated having substituted the Johansen-estimated error-correction mechanism into **(20)**. All of this procedure is then repeated by searching over a grid of θs between -1 and +1. A grid-search comparison then allows us to compare the different functional representations – and in particular to test the restrictions $\theta = 1$ and $\theta = 0$ implied by the semi-log and logarithmic special cases.

Our second approach is to estimate **(20)** in one step using non-linear least squares, while allowing for cointegration between real money balances, real GNP and interest rates (specified as a general power transformation). That amounts, in practice, to minimising ($\boldsymbol{\varepsilon'\varepsilon}$), where ε is the vector of residuals from **(20)**, by choice of the parameter vector **(α, β, θ, δ)**. This is achieved using Gauss-Newton iterative methods.[11] Reassuringly, both methods yield similar results which are reported below and allow us to conclude that the non-linear estimates are not biased by incorrect choice of starting value, specifically that the non-linear results correspond to the maximised log-likelihood of the linear method.

As well as corroboration, the (linear) grid-search method provides some information that cannot be derived from the non-linear estimation results. In particular, because the non-linear method does not distinguish between the short and long-run intercepts of the money demand function we would not be able to estimate the steady-state welfare costs of inflation if we focused exclusively on the non-linear estimates.

3.2 Data

We use annual average data for the United Kingdom for the period 1870-1994. Monetary and interest rate data are from Capie and Webber (1985). For money we use the monetary base, the demand for which is where we could expect the welfare costs of inflation to be largest at non-zero nominal rates of interest.[12] The monetary base series consists of cash in the hands of the public, banks' till money and bankers' balances at the Bank of England. In contrast with the series reported by the Bank of England since March 1981, this series includes as part of till money reserves of notes and coin held in the Banking Department. The Capie and Webber series only runs to 1982, so growth rates of M0 (as published by the Bank) are used post-1982 to break-adjust the level of the monetary base.

As the best proxy for the opportunity cost of holding non-interest bearing money balances, we use the rate of interest paid on bank deposit accounts. From 1870 to 1944, the data were obtained from *The Economist* and from 1945 from the Bank of England's *Statistical Abstract*. From a theoretical perspective, we would like to have used consumption as a scale variable but

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(11) For this method to give sensible answers we require that $Cov(\varepsilon_t, \varepsilon_{t-i}) \approx 0 \ \forall\, i$, which further justifies the inclusion of the dynamic terms in **(15)** to mop up any serial correlation.
(12) Authors have sometimes used M1, but this is not available as a time series over our full sample. Moreover, an increasingly large proportion of M1 is these days interest-bearing. An earlier vintage of this paper also presented estimates using an M3 measure of money and consol yields as interest rates. But these changes do not alter markedly the analysis.

we take GNP[13] because of its likely robustness for the whole sample. This should not greatly affect the estimates. Table A presents some summary statistics on these data.

Table A
Summary statistics, 1870-1994

	M0 (%)	Deposit rate (%)	Δ Deposit rate	Deflator (%)	Real GNP (%)
μ	4.28	3.31	0.01	3.56	1.96
σ	5.61	36.96	1.32	6.44	3.33

The levels of these series were found, using ADF tests with two lags, to be I(1).[14] We therefore present the summary statistics in Table A as log-differences, except interest rates which are also shown in levels and in simple first-differences. As we would expect, the time-series behaviour of money and prices is very similar. That explains why we focus on real money balances in the remainder of this paper. The average growth of real GNP is just below 2%. The average change in the deposit rate (in first differences) is very small. A few large changes in the deposit rate explain its high standard deviation.

4 Results

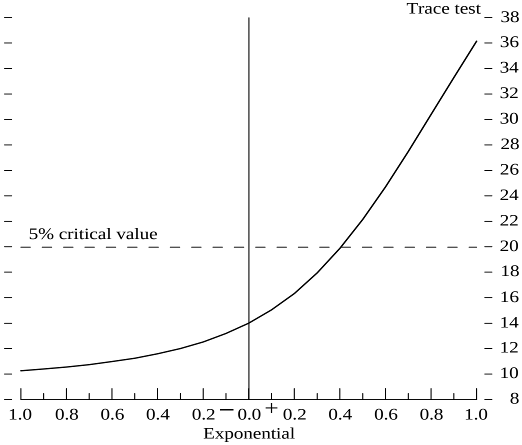
Following the theory outlined in Section 2, we begin with some base money demand estimates of equation **(20)**.[15] As the variables are I(1), cointegration among the variables in the equilibrium money demand equation, **(18)**, was first assessed using Johansen's (1988) methodology. The implied income elasticity could be restricted to unity ($\alpha_1 = 1$) in each case, as in Charts 1 and 2. Cointegration was then tested separately for a range of θs lying in the grid

(13) Since we use aggregate money data consumption would perhaps not be the best indicator of domestic transactions; GDP would be the 'ideal' scale variable, but reliable data are not available over the full sample period.
(14) Unit root test statistics are available from the authors on request. An ERM impulse dummy variable was also found to be necessary for 1992, without which the estimated interest elasticity was distorted.
(15) It is not the purpose of this paper to estimate a forecasting equation for narrow money demand, rather to examine the costs of inflation implied by money demand estimates. See Janssen (1998) for an overview of narrow money demand studies in the United Kingdom.

(-1,1), searched in intervals of 0.05. This grid encompasses the semi-log ($\theta = 1$) and logarithmic ($\theta = 0$) special cases.

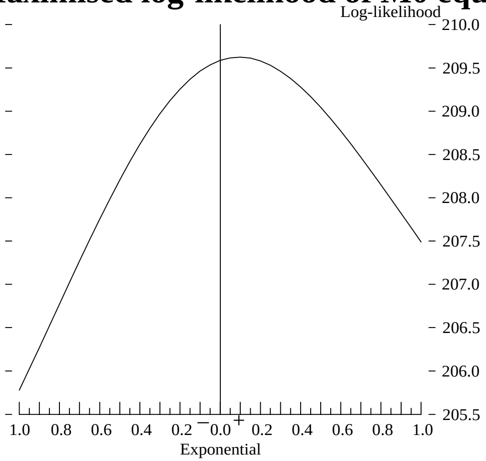
Chart 3 plots the trace test statistic for cointegration against values of θ, together with the 5% critical value.[16] Two points emerge. First, in neither case is the evidence of cointegration overwhelming. Second, as we can reject cointegration of the log-linear representation at 5% and not the semi-log form, there is stronger evidence of a well-defined long-run relation using the semi-log form ($\theta = 1$) than using the log representation ($\theta = 0$). Although we should not take this result too literally for, from Ermini and Granger (1993), we know that cointegration in levels must imply cointegration in logs, this empirical evidence, albeit partial, lends some weight to the conventional semi-logarithmic long-run money demand specification of **(18)**.

Chart 3
Johansen cointegration test based on trace statistic



(16) The eigenvalue cointegration test statistic gives a very similar picture.

Chart 4
Maximised log-likelihood of M0 equation



With the long-run equilibrium of the model estimated using Johansen for various θ, the short-run dynamics (**δ(.)**) were then estimated with the respective θ now imposed upon the dynamics in **(20)**.[17] The maximised log-likelihoods of **(20)** which resulted from this grid search are plotted in Chart 4. They peak at $\theta \approx 0.15$. Because this is much closer to 0 than to 1, the figure is clearly supportive of the log over the semi-log specification. We can test this formally by conducting likelihood ratio tests of the restrictions $\theta = 0$ and $\theta = 1$, against an alternative of $\theta = 0.15$.[18] These test statistics are distributed as a $\chi^2(1)$:

$H_0 : \theta = 0$ $\qquad \chi^2(1) = 0.08$
$H_0 : \theta = 1$ $\qquad \chi^2(1) = 4.26^*$

where * denotes rejection of the null at 5%. There is therefore strong evidence from the full dynamic model *against* the semi-logarithmic

(17) In each case we included the estimated Johansen long-run estimates even where evidence of cointegration was borderline. The significance of the ECM terms in the full dynamic model is evidence that this approach is justified and that cointegration does indeed exist for each θ.

(18) If we use the freely-estimated maximised log-likelihood – the 'true' maximised log-likelihood – from the non-linear model for these tests, it gives almost identical results.

specification – which can be rejected at 5% – and in favour of the logarithmic form.

This means that the peak maximum likelihood estimate of θ is not significantly different from the level of θ implied by the logarithmic specification, but is significantly different from that implied by the semi-logarithmic specification. The same conclusion obtains if we free up θ and conduct non-linear estimation of **(20)** in one step.

Table B

Base money demand 1872-1994 (t-ratios – where available – in parentheses):

	Restricted linear estimates		Unrestricted non-linear estimates
	$\theta = 0$	$\theta = 1$	
γ	-0.10 (2.16)*	-0.08 (3.22)*	-0.12 (1.21)
β	-0.019 (2.12)*	-0.048 (3.19)*	-0.014 (-0.80)
α_0	-5.48	-1.66	
α_1	1	1	1
α_2	-0.82	-18.58	-2.28
δ_0 ($\Delta(m-p)_{t-1}$)	0.19 (2.31)*	0.20 (2.38)*	0.18 (2.05)*
δ_1 (Δy_t)	0.38 (2.91)*	0.34 (2.53)*	0.38 (2.88)*
δ_2 ($\Delta[R_t^{\theta}-1]/\theta$)	-0.05 (4.43)*	-1.05 (3.17)*	-0.09 (0.82)
θ	0	1	0.16 (0.48)
Log-likelihood	209.59	207.49	210.02
Autocorrelation $\chi^2(1)$	0.94	2.76	1.23

* denotes significance at the 5% level

The non-linear least squares freely-estimated results are reported in Table B, alongside restricted linearised estimates for θ = 0 and 1. The non-linear estimates clearly have dynamic coefficients which are closer to the log specification. We do not, however, present detailed estimates of the welfare gains of reducing interest rates when $\theta = 0.16$, because it is unclear how to interpret such a specification of the interest rate term in a money demand equation economically.

What these results suggest is a clear distinction between the short and long-run functional form. In the long run, money demand seems to be better captured by a semi-logarithmic specification – though we might question both long-run specifications as a full description of equilibrium money holdings. This is evident from Chart 3 and from the (implausibly) low loading coefficient under the log specification. At first blush, such a result appears inconsistent with Charts 1 and 2, but if we trace out the locus $V = \beta e^{-\gamma R}$ using our freely-estimated value for γ ($\gamma = 0.019$),(19) then the visual fit appears much better (see Chart 5). So part of the reason for the apparently poor visual fit in Chart 2 was the imposition of semi-elasticities which were too low. Turning to the short-run behaviour of money demand, the reverse appears to be true: the dynamic effects of interest rates are better captured by a logarithmic formulation – as is evident from Table B and Chart 4.

Taking these results together, we arrive at a model of money holdings which is not intuitively unreasonable. In equilibrium, the allocation of wealth between monetary and non-monetary assets depends linearly upon the level of the interest rate, since this measures the absolute interest income foregone on money holdings. We suggest though that the speed with which money balances are adjusted following a change in interest rates depends on where interest rates are starting from. At low rates of interest, the dynamic effect of a percentage point interest rate change on an individual's income flow is that much greater; the proportional income loss from an interest rate fall is that much larger. The incentive to switch between monetary and non-monetary assets – for a given percentage point interest rate change – is therefore also greater, with the opposite being true if interest rates are initially high.

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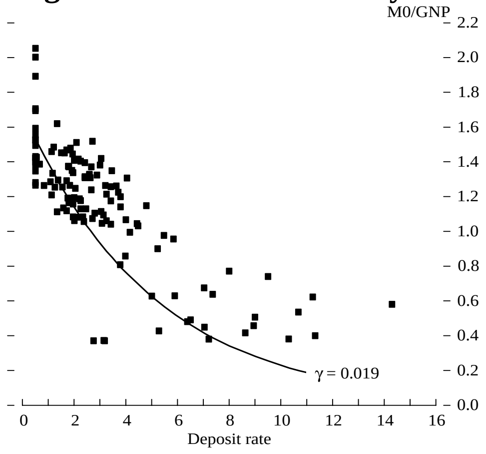
(19) This value of γ corresponds to the value for $-\alpha_2$ obtained with the semi-log specification (see Table B). α_2 has been rescaled to accommodate the different scale of the M0/GNP ratio in Chart 5.

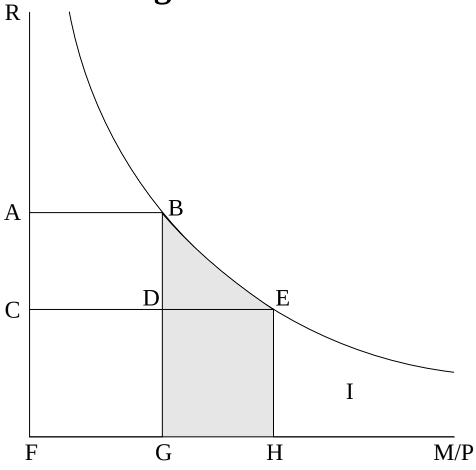
Chart 5
M0/GNP 1870-1994, semi-log specification using estimated semi-elasticity



5 The welfare costs of inflation

Finally, we can attempt to translate these empirical money demand estimates into some welfare costs of inflation, as Chart 6 illustrates. It plots real money demand (M/P) as a function of nominal interest rates (R). So a fall in nominal interest rates, A-C, reduces unsatisfied demand by the trapezoid BGHE. When moving to the Friedman optimum of zero interest rates (F), we need to add in the extra welfare triangle, I. As both sets of empirical results in Section 4 led to us prefer either the logarithmic (in the dynamic) specification or semi-logarithmic (in the long-run) specification, rather than one of the intermediate cases, we calibrate the welfare costs for each of these cases. For a logarithmic money demand function, this welfare triangle is likely to be significant as money demand asymptotes on the horizontal axis: that is the essence of Lucas's argument. So we begin by looking at the welfare estimates implied by a logarithmic long-run money demand function – despite the fact that we have found some empirical evidence to reject it – before turning to consider a semi-log money demand specification.

Chart 6
Measuring welfare costs



We take as a base case a nominal rate of interest of 10%, corresponding to, say, a 3% real rate and 7% annual inflation – numbers which are close to their averages in the United Kingdom since the 1970s.[20] We then compare this base case with: (a) nominal interest rates of 5% – corresponding to an inflation objective of around 2%, which is similar to that currently prevailing in many industrialised countries; and (b) nominal interest rates of zero, corresponding to Friedman's (1969) rule of deflating at a rate equal to the real rate of interest. Lucas's estimates imply that the costs of inflation really show up when moving from (a) to (b), but even more when moving from zero inflation to (b).

(20) We assume that real rates have been 3% over the full sample because nominal deposit rates and inflation are found to be cointegrated with a unit coefficient and an intercept not significantly different from 0.03. But this is not critical in any way to our results, and different real rate assumptions could be plugged in to generate different inflation-reduction experiments.

The long-run solution for the demand for narrow money implied by our *logarithmic* estimates is:(21)

$$M_t \,/\, P_t = e^{-5.48}\; Y_t\; R_t^{\;-0.82} \qquad \textbf{(20)}$$

The gain in consumer surplus which results from moving from nominal interest rate R_1 to R_2 is:

$$w(R_1, R_2) = \int_{M/P(R_1)}^{M/P(R_2)} \psi(M/P)\, d(M/P) \qquad \textbf{(22)}$$

where ψ (*M/P*) is the inverse of the money demand function **(21)**. **(22)** just measures the area under this inverse demand curve between R_1 and R_2.(22)
$\psi(.)$ is from **(21)**:

$$R_t \;=\; (e^{-5.48}\; P_t\; Y_t / M_t)^{1.22} \qquad \textbf{(23)}$$

By normalising $P_t Y_t = 1$, we can calibrate money holdings and welfare costs as a percentage of nominal GNP using **(22)** and **(23)**.(23) Using **(23)**, we can then map out the relationship between R and M/P. This is shown in the first two columns of Table C. At $R = 10\%$ base money holdings are around 3% of GNP and at $R = 5\%$ they are 5% of GNP. Over the period since 1970, average interest rates have typically been in the 5%-10% range, while average base money holdings have been around 5%-6% of GNP, suggesting that our calibrations are about right.

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(21) For our calibration of the welfare costs of inflation, we focus on the *long-run* narrow money demand estimates, because the welfare implications of changes in inflation ultimately occur when inflation reaches a new steady state. It should be kept in mind, however, that adjustment to a new steady state is likely to be lengthy, as the low loading coefficients on the ECM terms (β) in Table B show.

(22) It thus comprises the box of unsatisfied demand under the money demand function stressed by Tower (1971) as well as the conventional Harberger triangle of Bailey (1956) – hence the trapezoid BEHG.

(23) Of course, we are implicitly assuming that all M0 balances are held for transaction purposes and it is possible that any welfare gains may then be overstated if this is not the case. But in the long run we would expect cash to be held for transaction purposes and our finding of a unit coefficient on GNP in the long-run money demand equation supports this.

Table C
Welfare gains – base money, log specification

Nominal interest rate (%)	Base money/GNP	Interest rate range (R_1 - R_2) (%)	Gross welfare gains w(R_1, R_2) (% of GNP)	Seigniorage losses s(R_1, R_2) (% of GNP)	Net welfare gains w(.)-ξs(.) (% of GNP)
10	0.03	10-5	0.15	0.032	0.138
5	0.05	5-4	0.04	0.009	0.041
4	0.06	4-3	0.055	0.012	0.051
3	0.07	3-2	0.07	0.015	0.067
2	0.10	2-1	0.11	0.024	0.10
1	0.18	1-0	0.87	0.185	0.80
0.01	8.22	5-0	1.15	0.245	1.06
0	∞	10-0	1.3	0.278	1.20

Table C assesses the series of gross and net welfare gains arising from a reduction in steady-state nominal interest rates from 10% to 0%.(24) Column 4 shows the gross gains of making the reduction in nominal interest rates indicated in column 3, while columns 5 and 6, respectively, show the associated seigniorage losses and *net* welfare gains. These gains are calculated using the long-run estimates (α_0, α_1 and α_2) in Table B and then evaluating the definite integral **(22)** using **(21)**. Looking first at a shift from 10% to 5% nominal interest rates, this leads to a gross welfare gain of 0.15% of GNP. This number is very much in line with estimates from the United States (McCallum (*op cit*), Fischer (*op cit*)). But as Lucas' paper intimates, and as we would expect from a log specification, the welfare gains of moving from 5% to 0% nominal interest rates are an order of magnitude larger at 1.15% of GNP. This is in line with Lucas's (1995) welfare cost estimate of 1% of GNP in the US and Gillman's conservative range of 0.85%-3% of GNP. On the basis of this result alone, Lucas's thesis seems to have some empirical support.

There may, however, be grounds for scepticism. One reason why we may distrust these results is that they ignore countervailing costs – costs which could prove prohibitive if the economy were to undergo a steady-state deflation. Some such costs are real – for example, arising out of Phillips

(24) We present the analysis in terms of interest rate reductions rather than inflation rate reductions, but the two clearly have a one-for-one mapping having made some assumption about the steady-state level of real interest rates.

curve convexities – but are difficult to quantify. Others – of a monetary nature – *are* quantifiable. One such cost is the loss of seigniorage revenue resulting from lower nominal interest rates.(25) These revenue losses mean that distortionary taxes have to be raised elsewhere, with attendant welfare costs (Phelps (1973)). In his recent work, Feldstein (1995) finds that (the welfare effects of) these seigniorage losses may more than offset direct welfare gains at low rates of inflation. Note that because seigniorage income (S_t) is equal to $r_t M_t$ per period, the welfare loss from reduced seigniorage as we change nominal interest rates is:

$$dS_t / dr_t = \xi \{ dr_t M_t + dM_t r_t \} \qquad \mathbf{(24)}$$

where the first term in brackets on the RHS of **(24)** captures the tax rate effects of a change in interest rates, and the second term the offsetting effects of a change in the tax base. The term ξ scales seigniorage losses, reflecting the fact that they need to be made good by raising distortionary taxes elsewhere in the economy; lump-sum taxes are not a practical option. These distortionary taxes in turn impose a deadweight welfare burden of ξ per pound. In order to calibrate ξ we ideally need to simulate the welfare effects of an across-the-board tax increase in a fully general equilibrium model – as in, for example, Ballard, Shoven and Whalley (1985) for the United States. To simplify matters, we know that if the tax system were to comprise a single linear tax levied at a rate τ, then $\xi = 1/(1-\tau)-1$. Setting $\tau = 0.25$ gives $\xi = 0.33$ which is used here as our estimate.(26)

Making these assumptions, and using **(21)**, we can then calculate the marginal seigniorage loss from disinflating. Setting this marginal cost against the gross marginal welfare gains outlined earlier allows us to estimate the 'optimal' nominal rate of interest in this partial equilibrium setting. Seigniorage losses, and the resulting net benefits from disinflating, are given in the final two columns of Table C. From these, it is clear that the optimal level of nominal interest rates is zero – the Friedman optimum. Nowhere are marginal seigniorage losses sufficient to counterbalance marginal shoe-leather gains, as nominal interest rates are reduced. Indeed, net welfare benefits rise as nominal interest rates fall and these net benefits still amount to a sizable

(25) In Chart 6, for example, this is the area ABCD-DEGH when moving interest rates from A to C.
(26) This is in line with the mid-point of Ballard, Shoven and Whalley's (*op cit*) calibrated estimates for the United States.

1% of GNP when moving from 5% to 0% nominal interest rates – still extremely large by comparison with previous estimates.(27)

These estimates still leave some questions unanswered. For example, around three-quarters of the welfare gain comes when moving from 1% to 0% nominal interest rates. Technically, of course, this is not that surprising since this is, in effect, a 100% proportional change under the log specification. Intuitively, though, it is difficult to understand how such a minor numerical shift could have such far-reaching welfare implications, amounting to 0.8% of GNP, even after seigniorage losses are subtracted.(28) The welfare gains from a near-logarithmic specification of $\theta = 0.16$ would, for instance, only lead to a gain of 0.35% of GNP.

The money/GNP ratios in Table C add to this uneasiness at rates of interest below 1%. Even though implied base money holdings are high as interest rates approach 1%, they are not entirely implausible (though, in truth, they still seem unlikely). The average deposit rate between 1870-1913 was around 2% and average holdings of base money were around 12%-13% of GNP at that time – as in Table C. At 1% nominal interest rates base money holdings would need to rise to 18% of GNP to deliver the implied welfare benefits. This means that average money balances in the United Kingdom would have to grow at least four-fold from their current level. Or, put differently, currency holdings per head would have to rise from around £460 currently to around £1,830! Against a backdrop of rapid ongoing innovation in transactions technology, and given the inconvenience and risk involved, such a rise in cash holdings sounds highly implausible in our view. Moreover, at rates of interest below 1% the money/GNP ratios become less plausible still; they are required to head-off to infinity.

Part of the problem may simply be a Lucas critique: we have little or no evidence on money-interest relationships at near-zero interest rates. Mulligan and Sala-i-Martin (1996) question Lucas' estimates on just these grounds. They then draw on micro-level evidence on individuals' asset holdings, which in principle ought to have a direct mapping with interest rate

(27) Wolman (*op cit*) obtains similar welfare benefits for the United States (between 0.47% and 0.88% of output) when reducing inflation from 5% to 0%.
(28) We analyse the robustness of our welfare gain estimates in the Appendix, using a different sample for our base money demand equation, and a broad money (M3) demand equation estimated over the full sample period.

elasticities. From this evidence, they conclude that money holdings are actually likely to be rather interest-insensitive at low rates of interest. This follows from the reduced incentive to substitute into interest-bearing financial assets when aggregate interest rates are low.

A further consideration is that maintaining average nominal interest rates of zero would require interest rates to be as often negative as positive. But nominal interest rates are truncated at zero.(29) So the appropriate comparative static comparison may not be between positive and zero average interest rates, since the latter outcome is unattainable. If this is so, then a good chunk of Lucas' welfare benefits are lost. All of these factors – taken together with the absence of a well-defined long-run log-linear money demand model – cast doubt on the robustness of the logarithmic shoe-leather estimates.

Table D
Welfare gains – base money, semi-log specification

Nominal interest rate (%)	Base money/GNP	Interest rate range (R_1 - R_2) (%)	Gross welfare gains $w(R_1, R_2)$ (% of GNP)	Seigniorage losses $s(R_1, R_2)$ (% of GNP)	Net welfare gains $w(.)-\xi s(.)$ (% of GNP)
10	0.03	10-5	0.32	-0.079	0.35
5	0.08	5-4	0.07	0.014	0.064
4	0.09	4-3	0.06	0.035	0.053
3	0.11	3-2	0.05	0.065	0.003
2	0.13	2-1	0.04	0.105	0.0
1	0.16	1-0	0.02	0.159	-0.037
0.01	0.19	5-0	0.25	0.378	0.083
0	0.19	10-0	0.57	0.299	0.433

Table D presents the same welfare cost calculus using the semi-logarithmic money demand model. As we would expect, it suggests lower gross welfare benefits when moving from 5% to 0% nominal interest rates. These gains are now around 0.25% of GNP and this comes without a corresponding take-off in money holdings as a percentage of GNP, or in gross welfare benefits, as we approach zero nominal interest rates. In fact, the welfare gains

(29) It is theoretically possible that under deflation agents would be prepared to accept *negative* interest rates in return for the security provided by bank accounts. But we ignore that possibility here.

of reducing nominal interest rates to 3% are actually higher than was found for the logarithmic case. It is only at rates of interest below this that gross welfare gains fall, as the interest elasticity of money demand heads toward zero. As a result, the optimal level of nominal interest rates is no longer zero; it is around 2%, equating marginal seigniorage costs and marginal shoe-leather benefits. Below 2% nominal interest rates, seigniorage losses more than counterbalance shoe-leather gains. That in turn equates to an optimal rate of inflation of close to zero, given our earlier assumption about real rates.(30)

All in all, the semi-log formulation appears to yield much the more plausible welfare cost estimates. Shoe-leather gains are more evenly spread; they are not clumped around zero nominal interest rates. Money/GNP ratios are not required to head-off to infinity. Cash balances per head seem more reasonable. And the implied optimal rate of inflation is close to zero. These comparative static conclusions from the welfare analysis corroborate those drawn from the earlier econometric analysis.

6 Concluding remarks

Theoretical evidence seems to favour a logarithmic primitive money demand function. Using this implied functional form, we arrive at welfare estimates in line with those found by Lucas − around 1% of initial GNP. But these estimates are questionable, at least at the limits of the functional form. Around three-quarters of the welfare gain comes when moving from 1% to 0% nominal interest rates; and at the same time the money-GNP ratio is required to head-off to infinity. Taken alongside the weaker econometric evidence on the existence of a long-run log-linear money demand relationship, this would lead us to be cautious in reading Lucas' welfare results too literally.

However, we have found greater empirical support for a log-linear description of money demand dynamics following an interest rate change. The short-run adjustment in money balances following an interest rate change does seem to depend crucially on the level from which interest rates are starting out. This

(30) We assessed the sensitivity of these estimates by looking at some sub-samples estimates, in particular looking at the period from 1970 onwards. This slightly lowered the implied welfare benefits.

sounds like an intuitively plausible description of agents' portfolio allocation decisions. In the long run, it is the absolute amount of interest income foregone which dictates agents' money holdings. But in the shorter run, proportional changes in interest income are important in determining the speed with which equilibrium money balances are restored. This short-run/long-run distinction is important when understanding the dynamic response we would expect from narrow money in response to an interest rate change – currently a key issue as the rate of inflation has fallen among developed economies.

As for welfare costs, the present value of the welfare gains implied by even our semi-log specification are still non-trivial. Nominal interest rates in the United Kingdom are currently around 6%. Our partial equilibrium analysis suggests an optimal nominal interest rate of around 2%. The net welfare gain when moving from 6% to 2% nominal interest rates is around 0.22% of GNP.(31) With a discount rate of 5% and real growth of 2.5%, this gives a net welfare gain equal to around 9% of initial GNP. In 1995, that would have amounted to around £60 billion. And that is more than twice the current size of the Bank of England's balance sheet.

(31) This result is derived from the integral of the net welfare gain from 6% to 2%. To the results in Table D the reader should add 0.07%, as the net gain of moving from 6% to 5% interest rates.

Appendix

To test the sensitivity of the welfare gain estimates obtained from reducing nominal interest rates in the steady state we present alternative estimates based on a base money demand equation over the period 1970-94.(32)

Overall, Table E shows that with the log and semi-log interest rate specification for the base money demand function the estimated welfare gains of reducing nominal interest rates are well below those obtained over the full sample. And the welfare gains remain larger with the log than with the semi-log specification. The optimal interest rate appears to be 3% for the semi-log formulation and 0% for the log model. Lowering nominal rates from 5% to 0% in the semi-log model now leads to welfare losses due to the decline in seigniorage benefits.

Table E
Welfare gains – base money, log (semi-log) specification (1970-1994)

Nominal interest rate (%)	Base money/GNP	Interest rate range (R_1 - R_2) (%)	Gross welfare gains $w(R_1, R_2)$ (% of GNP)	Seigniorage losses $s(R_1, R_2)$ (% of GNP)	Net welfare gains $w(.)-\xi s(.)$ (% of GNP)
10	0.02 (0.02)	10-5	0.08 (0.07)	0.05 (0.06)	0.06 (0.05)
5	0.03 (0.03)	5-4	0.02 (0.01)	0.013 (0.02)	0.017 (0.004)
4	0.04 (0.03)	4-3	0.025 (0.009)	0.015 (0.03)	0.02 (0.001)
3	0.04 (0.04)	3-2	0.03 (0.007)	0.019 (0.03)	0.024 (-0.003)
2	0.06 (0.04)	2-1	0.04 (0.005)	0.026 (0.04)	0.034 (-0.007)
1	0.09 (0.04)	1-0	0.14 (0.002)	0.086 (0.04)	0.11 (-0.01)
0.01	1.52 (0.05)	5-0	0.26 (0.03)	0.16 (0.16)	0.21 (-0.02)
0	∞ (0.05)	10-0	0.34 (0.10)	0.21 (0.22)	0.27 (0.03)

Welfare gain estimates derived from an M3 demand function (over the full sample) are given in Table F.

(32) Welfare gains can also be derived from an M3 demand equation, but since part of M3 is interest-bearing we consider these estimates to be less reliable.

Table F
Welfare gains – broad money M3, log (semi-log) specification (1872-1994)

Nominal interest rate (%)	Base money/GNP	Interest rate range (R_1 - R_2) (%)	Gross welfare gains $w(R_1, R_2)$ (% of GNP)	Seigniorage losses $s(R_1, R_2)$ (% of GNP)	Net welfare gains $w(.)-\xi s(.)$ (% of GNP)
10	0.42 (0.40)	10-5	0.60 (0.90)	1.69 (-0.079)	0.05 (0.92)
5	0.51 (0.52)	5-4	0.14 (0.13)	0.39 (0.014)	0.011 (0.12)
4	0.54 (0.55)	4-3	0.15 (0.10)	0.41 (0.035)	0.011 (0.09)
3	0.58 (0.58)	3-2	0.16 (0.08)	0.45 (0.065)	0.013 (0.06)
2	0.65 (0.61)	2-1	0.18 (0.05)	0.52 (0.11)	0.014 (0.015)
1	0.78 (0.64)	1-0	0.28 (0.02)	0.78 (0.16)	0.022 (-0.035)
0.01	2.61 (0.68)	5-0	0.91 (0.38)	2.54 (0.38)	0.07 (0.25)
0	∞ (0.68)	10-0	1.51 (1.28)	4.23 (0.30)	0.12 (1.17)

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