Bank Capital and Risk-Taking

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Abstract

We study bank risk-taking and capitalisation in a continuous time model with closed-form solution, assuming uncertain cash flow, random regulatory audit and a constraint on equity issue. Capital reserves are built up towards a desired level as an insurance against the threat of liquidation. Risk-taking is a discontinuous function of the level of capital. We solve in steady-state for the liquidation rate and investigate the determinants of charter value. Minimum capital standards have little long-term impact on behaviour. Audit frequency is the principal tool for restraining moral hazard.

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1 Introduction

Capital regulation and supervision of the banking system is one of the principal public policy interventions in the workings of the economy. Yet the theoretical literature on banking provides little insight into many basic questions about the behavioural implications of our system of prudential regulation.[1] In what circumstances can banks be relied upon to behave prudently and choose, of their own accord, adequate levels of capitalisation? In what other circumstances is it necessary to monitor bank capital closely, to ensure that the probability of failure remains acceptably low? What is the relationship between the effort which regulators make in monitoring bank net worth and incentives to take risks or loot bank assets? What is the impact of regulatory capital requirements such as those of the 1988 Basle Accord or the 1991 FDICIA Act? Are there other policy interventions that might encourage prudent behaviour and restrain risk-taking?

Considerable analytical attention has been paid to the problem of moral hazard in banking, a cause of the excessive risk-taking which has exacerbated the scale of losses in both the savings and loans and other recent bank crises.[2] It is well known that the regulatory guarantee on the value of deposits removes incentives for depositors to monitor bank portfolio allocations or to seek a return which compensates for the risk of liquidation.[3] This regulatory guarantee also creates an incentive to transfer as much value as possible out of a failing institution into the hands of shareholders ('looting' or 'milking the property').[4] Yet there

(1) There are well known theoretical arguments for state-backed deposit insurance (Diamond and Dybvig (1983) argue that this is needed in order to offset the externality which can generate a bank run) or more generally for the provision of a bank 'safety net' in order to prevent systemic banking crises. In this paper we assume that a regulator guarantees the value of bank deposits, without discussing the merits of such an arrangement.

(2) We review some of this literature below. See Pyle (1995) for a survey and fuller references on the S&L crisis, and Krugman (1998) for an analysis of banking problems in South East Asia based on a model of moral hazard.

(3) The resulting moral hazard is familiar to practitioners. A former Deputy Governor of the Bank of England put it thus: "If the state guarantees the existence of individual banks, that can create incentives which encourage irresponsible behaviour. The prize for taking excess risk may - if things go well - be excess returns (and telephone number bonuses) while, if things turn out badly, the state steps in and picks up the tab." (Davies (1996)).

(4) Such transfers, both legal and fraudulent, have also contributed to the scale of recent banking crises (see Akerlof and Romer (1993) on the S&L crisis.)

are a number of aspects of bank behaviour which moral hazard alone cannot explain. Why do bank shareholders only rarely gamble with depositors' money? If shareholders have an incentive to loot, why do they not always extract maximum possible payouts from banks? Why is there a weak but significant *positive* association between bank capitalisation and the variability of asset returns?(5)

In this paper we address these many related questions by generalising the basic analysis of moral hazard to a dynamic setting in which there are constraints on the issue of equity capital and a random regulatory audit. In this setting bank capital is held as a form of self-insurance against poor asset returns, with the bank retaining earnings in order to build up capital reserves towards a desired level and so reduce the probability of losing ownership of the future profit stream.(6)

This self-insurance interpretation of bank capital has a number of implications, both for the relationship between capitalisation and risk-taking and for the design of regulatory policy. We show that the critically undercapitalised bank under immediate threat of closure, even if it is fundamentally profitable, is concerned only with survival, leading to the short-sighted risk-loving behaviour suggested by the basic model of 'moral hazard'. On the other hand a moderately undercapitalised bank is concerned with the future as well as the present and thus, in order to protect future profits (or 'charter value'), is risk-averse. According to our analysis bank regulation and supervision are fundamentally about identifying bad banks, closing those which are unprofitable, and closely monitoring those with low profits and high asset risk. Minimum capital standards, while reducing the exposure of the regulator, are relatively unimportant as determinants of bank behaviour.

In support of our argument we build what is virtually the first formal model of prudential bank capital in the literature.(7) This model allows

(5) See for example Demsetz, Saidenberg and Strahan (1996) table 5. As Berger, Herring and Szegoe (1995) point out this observation is a puzzle since, according to the conventional analysis, incentives for risk-taking decrease as the level of bank capital increases.

(6) We follow the conventions of the banking literature and use the term capital to refer to prudential capital, which provides protection against the risk of default, rather than to the total financing of an enterprise.

(7) Such models are conspicuous only by their absence from the reviews of the banking literature (Baltensperger (1980), Santomero (1984), Swank (1996) and Freixas and Rochet (1997)).

us also to endogenise the concept of charter value in a dynamic setting, analysing charter value as the present discounted value of economic rents earned by the bank, and showing how these rents are created by an interaction between costs of entry and deadweight costs of liquidation.

We proceed as follows. In Section 2 we discuss existing models of bank risk-taking. In Section 3 we present our model of capital holding and risk-taking for the case of a single bank. In Section 4 we consider industry equilibrium. In Section 5 we discuss implications for regulatory policy. In Section 6 we assess the robustness of our findings and indicate directions for future research. Section 7 concludes by restating the implications of our view of prudential capital as a form of of self-insurance. Our analysis is supported by three appendices: a digrammatic exposition of the literature on bank risk-taking in Appendix A; mathematical derivation of the solution of our model in Appendix B; and a detailed examination of the impact of parameter variation on capital holding and rates of liquidation in Appendix C.

2 Existing models of bank risk-taking

Moral hazard in banking has been formalized, using a simple static framework, in a number of different papers including Dothan and Williams (1980) and Furlong and Keeley (1989).(8) The principal prediction of these models is that, if the variance of returns on bank assets can be increased without reducing expected returns, shareholder value is maximised by increasing this variance to the maximum degree possible.

This result has a simple option interpretation. We can regard limited liability as an option held by shareholders to put losses onto the regulator whenever the bank is liquidated (Merton (1977)).(9) The value of this option is always increased by a widening of distribution of returns (the vega of the option is positive). It is also the case that, in

(8) The points made in this section are developed in greater detail in Appendix A.

(9) Option-pricing techniques are also used for valuing the exposure of bank deposit insurance schemes (eg Ronn and Verma (1986)). Crouhy and Galai (1991) show that if deposit insurance premia underprice risk then the bank will continue to maximise portfolio risk, as in the basic model of moral hazard.

this basic model, shareholder value is increased by making payment to shareholders out of bank capital since again this increases the value of the put option. Shareholders thus always have an incentive to loot the bank.

While an intuitively attractive explanation of excessive risk-taking by failing banks, this model is unsuccessful as an explanation of the relationship between capitalisation and risk-taking. It predicts that banks will always, regardless of the amount of capital they hold, seek to increase shareholder value by maximising portfolio risk and looting the banks assets.

A more satisfactory account of bank risk-taking emerges when allowance is made for bank 'charter value' ie a stream of future earnings with a positive present discounted value.(10) Provided charter value is sufficiently great then shareholders will have an incentive to avoid liquidation and the consequent loss of charter value, by maintaining adequate capital in the bank and reducing the riskiness of bank assets. Charter value thus restrains the moral hazard in banking (Marcus (1984)). New entry and other erosions of charter value have been suggested as an explanation of the increased level of bank failures in the United States during the 1980s (Keeley (1990)).(11) Our paper follows this line of analysis a step further, by endogenising both the decision to hold prudential capital and, in industry equilibrium, the level of charter value itself.

An alternative mechanism which may also restrain risk-taking by banks is divergence of interest between managers and shareholders. O'Hara (1983) formally models this divergence, showing that the costs to managers of losing their jobs can indeed induce risk-averse behaviour. Such divergence is a justification for assuming a risk-averse bank utility function as in Kahane (1977), Koehn and Santomero (1980), and Kim

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(10) Charter value is also sometimes referred to as 'franchise value', eg in Demsetz, Saidenberg and Strahan (1996). An alternative extension of the basic model of moral hazard is to introduce a role for warrants (Green (1984)). This is a less satisfactory explanation of bank behaviour since banks issue few warrants, and it does not explain why shareholders should refrain from decapitalising the bank and engaging in excessive risk-taking.

(11) Charter value also plays an essential role in the literature on the 'pre-committment' approach to the setting of regulatory capital standards (for a recent contribution see Kupiec and O'Brien (1997)). This is because banks must have something to lose if they are to have incentives to set adequate capital on their own account.

and Santomero (1988). Rochet (1992) analyses incentives for moral hazard in a static context when the bank maximises such a risk-averse utility function, showing that in this case risk-loving behaviour can still emerge when the probability of liquidation is sufficiently high.

3 Endogenous capital: one bank

Our principal assumption is that the bank is unable to issue new equity.[(12)] Some such assumption is clearly necessary in order to model prudential capital, since if equity can be raised without cost in all circumstances there would be no need to hold prudential capital reserves. Because of this constraint capital is not continuously controlled by shareholders and can be built up only out of retained earnings towards some target level. A further implication is that negative returns on assets can trigger temporary periods of financial distress during which time the bank has less capital than it desires. The bank may then respond by altering the riskiness of its assets, until such episodes are ended either by liquidation or by a rebuilding of prudential capital back up to desired levels.

In order to obtain an explicit analytical solution we assume that both the level and the expected return on bank assets are constant. The only portfolio decision made by our bank is about the uncertainty of cash flows. In Section 6 we consider the implications of relaxing this assumption.

The other feature of our model, which we introduce in order that our bank sometimes engages in extreme risk-taking, is that there is a random regulatory audit.[(13)] Whenever this audit reveals capital less than some minimum regulatory threshold the bank is liquidated, imposing deadweight costs because the assets of the bank are then

[(12)]In fact it is sufficient for our results to assume only that there is some fixed cost of making new capital issues. See section 4.2.
[(13)]We adopt the Poisson specification of Merton (1978). Section 3 of Marcus (1984) provides a brief discussion of bank risk-taking in a continuous time model with such a random audit. We are unable to make direct use of his analysis because it assumes that earnings are always retained within the bank.

resold for less than their full value.[14] Avoiding these deadweight costs provides the bank with an incentive to reduce the risk of liquidation by holding a margin of capital over and above the regulatory minimum.

We further assume that the cost of deposits (including any servicing costs and deposit insurance premia paid by the bank) is less than the shareholders discount rate so that, were it not for the need to reduce the risk of liquidation, deposit finance would always be preferred to shareholder capital for the financing of bank assets.[15] Bank runs are not possible in our model because returns to depositors are guaranteed by the regulator. As with other models of bank risk-taking we assume that managers maximise shareholder value, abstracting from any conflict of interest between shareholders and managers.

3.1 The model

Time t is a continuous variable.[16] The bank holds a fixed amount of non-tradeable assets valued at an amount A.[17] These assets generate a perpetual but uncertain cash flow at a constant expected rate of RA per unit time and with a variance of cash flow per unit time of $\frac{1}{2}\sigma^2$. The prudential capital of the bank is its book equity or net worth, denoted by C, which depends upon the history of cash flows and of

(14)We do not explain this loss of value. It is reasonable to assume that, since outsiders are not fully informed, resale value will fall short of the fundamental value of the bank. Deadweight costs can also be due to the legal and other costs of re-organisation.

(15)Specifically, in order to obtain an analytical solution, we assume that debt carries a zero rate of interest. This is reasonable for the case where the 'endowment' effect on cash flow of unremunerated capital reserves is small relative to the total cash flow of the bank and where deposit insurance premia are invariant to bank net worth. In Section 6 we also consider the implications of relaxing this assumption.

(16)We are aware of two other papers which apply continuous-time diffusion techniques to the analysis of bank failure. In Talmor (1980) bank management face a gambler's ruin problem and determine capital at the level which reduces the ex-ante probability of bankruptcy to a desired level. Fries *et al* (1997) analyse optimal closure rules when the level of bank earnings evolve according to a diffusion process. In their model the bank holds no prudential reserves and can raise cash continuously from shareholders.

(17)The basis of this valuation is not important to our model. Were assets valued at a greater (or lesser) amount we can compensate simply by altering the expected return R, the recovery rate γ, and the liquidation threshold $\widetilde{C}$ without affecting our results.

dividend payments.[18] The bank raises the balance of finance for the holding of assets by issuing zero-cost short-term deposits of $D = A - C$. This infinitely elastic supply of deposits for the individual bank is not unreasonable given that deposits are fully insured by the regulator.

The bank pays dividends at a rate $\theta \mathrm{d}t$, subject to a constraint on raising new equity $\theta \geq 0$ and a condition that there be some finite upper bound on θ.[19] The bank also makes a choice over $\sigma \in [\sigma_1, \sigma_2]$. Cash flows impact on net worth C, and hence on deposits D, according to:

$$\mathrm{d}C = (RA - \theta)\mathrm{d}t + \sigma \mathrm{d}z = -\mathrm{d}D \qquad \mathbf{(1)}$$

Managers seek to maximise the value of the bank to its shareholders, measured by the expected discounted value of future dividends:

$$V(C) = \max_{\theta,\sigma} \mathbb{E}\left\{ \int_t^{\infty} \theta \exp(-\rho\tau)\mathrm{d}\tau \right\} \qquad \mathbf{(2)}$$

where $V(C)$ represents the market value of the bank's shares.

A random audit protects but does not eliminate the exposure of the regulator. If after audit the net worth of the bank is found to be below some required regulatory minimum $\widetilde{C}$, then shareholders lose control and the assets of the bank are sold for γA, where $0 \leq \gamma < R/\rho$.[20] After liquidation shareholders receive nothing, debt holders are repaid in full, and the regulators must pay the difference $D - \gamma A > 0$.[21] The audit is formally modelled as in Merton (1978) using a Poisson specification with parameter q, indicating that in each period δt there is a probability of $q\delta t$ of an audit being carried out. $1/q$ represents the 'audit half-life' ie the period after which half of a population of banks will have been audited.

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(18) We do not consider the possibility of other balance sheet liabilities, such as subordinated debt, playing a prudential role. Thus in our model capital, net worth and the book value of equity are all equivalent.

(19) This last assumption is required to rule out an unbounded and instantanous decapitalisation of the bank, which is worth more to shareholders than the loss of continuation value RA/ρ.

(20) The second inquality is required in order for liquidation to impose deadweight costs.

(21) In order that this inequality is always satisfied we further require that $\widetilde{C} < (1 - \gamma)A$.

3.2 The value of the bank

The value of the bank to shareholders (the value function) satisfies the ordinary differential equations:[22]

$$(\rho+q)V = \max_{\theta,\sigma\in[\sigma_1,\sigma_2]}[\theta+(RA-\theta)V_C+\frac{1}{2}\sigma^2 V_{CC}], \qquad C\le \widetilde{C} \quad (3)$$

$$\rho V = \max_{\theta,\sigma\in[\sigma_1,\sigma_2]}[\theta+(RA-\theta)V_C+\frac{1}{2}\sigma^2 V_{CC}], \qquad \widetilde{C}\le C \quad (4)$$

with smooth pasting at $\widetilde{C}$. Optimal policy is to pay dividends at as high a level as possible when C exceeds some desired level of capitalisation C^*, but otherwise to retain all earnings.

The optimal choice of σ is one of the extreme values σ_1, σ_2 depending on the sign of V_{CC}. We can show (see Appendix B): that $V_{CC}(C^*) = 0$; that when $\widetilde{C} < C < C^*$, $V_{CC} < 0$ so $\sigma = \sigma_1$; and that when $C < \widetilde{C}$, $V_{CC} > 0$ so $\sigma = \sigma_2$.

Substituting in the optimal choice of θ and σ we then obtain an explicit solution for the value function, which can be expressed in terms of undetermined constants M_1 and M_2 as:[23]

$$V = \begin{cases} (M_1+M_2)\exp(\bar{\mu}_1(C-\widetilde{C})), & C\le\widetilde{C} \\ M_1\exp(\mu_1(C-\widetilde{C}))+M_2\exp(\mu_2(C-\widetilde{C})), & \widetilde{C}\le C\le C^* \end{cases} \quad (5)$$

where $\bar{\mu}_1$ and $\bar{\mu}_2$ are respectively the positive and negative roots of the quadratic $(\rho+q) - RA\mu - \frac{1}{2}\sigma_2^2\mu^2 = 0$, while μ_1 and μ_2 are the positive and negative roots of the quadratic $\rho - RA\mu - \frac{1}{2}\sigma_1^2\mu^2 = 0$.

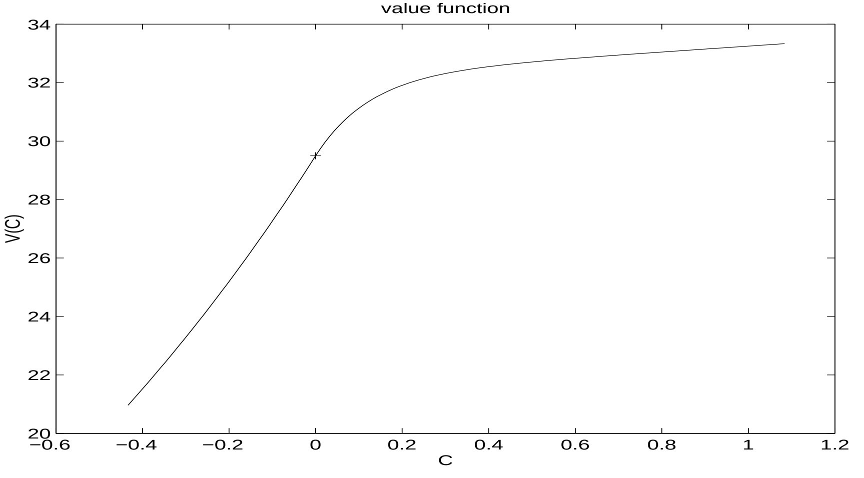
Chart 1 illustrates the value function $V(C)$ for $C \le C^*$.[24] Shareholders are indifferent at C^* as to whether an increment of cash

(22) All mathematical results are derived in Appendix B.

(23) In obtaining this result we make use of three boundary conditions: continuity and smoothness of V at $\widetilde{C}$ (ie smooth pasting) and the further condition, arising from the limited liability of shareholders, that as $C \downarrow -\infty$, $V(C) \downarrow 0$. Two further boundary conditions at C^*, $V_{CC}(C^*) = 0$ and $V_{CC}(C^*) = 1$, then determine M_1 and M_2.

(24) The parameter values of this chart imply that when minimising risk banks lose money about one year in twenty, approximately the observed performance of OECD banks, and when maximising risk lose money about one year in five.

Chart 1: Prudential capital reserves and the value of the bank to shareholders†



† Parameter values: we normalise by setting $RA = 1$, ie C and V are expressed as multiples of annual earnings, and $\widetilde{C} = 0$; other parameters are ρ=3% per annum, $\sigma_1 = 0.5$, and $\sigma_2 = 1$.

flow is paid out as a dividend or retained internally ($V_C(C^*) = 1$). $V(C^*) = RA/\rho$, indicating that the fully insured bank is worth the same to its shareholders as it would be in the absence of any deadweight costs of liquidation or constraints on capital issue.

As C falls below C^* the probability of liquidation, over any given time horizon, increases. Since internal funds serve to reduce the probability of liquidation and liquidation results in the loss of the shareholder's claim on the charter value of the bank, internal funds are valued at a premium over cash in the hands of shareholders ($V_C > 1$). This premium increases as C declines towards $\widetilde{C}$ ($V_{CC} < 0$) and then declines once C falls below $\widetilde{C}$ ($V_{CC} > 0$).

3.3 Capital

What determines the banks desired level of capital C^*?[25] C^* is given explicitly by:

$$C^* - \widetilde{C} = \frac{1}{\mu_1 - \mu_2} \ln[\frac{\mu_2^2}{\mu_1^2} \frac{\bar{\mu}_1 - \mu_1}{\bar{\mu}_1 - \mu_2}] \tag{6}$$

The bank therefore holds a margin of capital, over and above the required regulatory minimum $\widetilde{C}$, which is independent of $\widetilde{C}$. Following a change in $\widetilde{C}$ there is a corresponding increase in the desired level of capital C^* and a temporary shortage of capital, but once the level of capital has been built up towards the new target C^* there is no further impact on bank behaviour.[26]

To further understand the determinants of desired prudential capital holdings $C^* - \widetilde{C}$ we consider first the special case of continuous audit ($q = \infty$). In this case, which corresponds closely to the model of corporate behaviour analysed in Milne and Robertson (1996), $\bar{\mu}_1 = 0$ and the bank is liquidated at the instant when C falls to $\widetilde{C}$. Three factors now determine desired prudential capital. The first is the size of the bank. Multiplying RA and σ_1 by a common factor, ie a rescaling of the bank, results in a proportionate increase of $C^* - \widetilde{C}$. The second is the uncertainty of cash flows σ_1 which increases $(C^* - \widetilde{C})/RA$, because reducing the probability of liquidation to any given level requires more capital when cash flow is more uncertain. However, because capital is costly, desired prudential capital holdings do not increase linearly with σ_1.[27] Finally an increase in the discount rate, ρ, leads to a decline in desired prudential capital holdings. As shareholders become more impatient they prefer to receive cash in hand rather than pay for protection of future cash flows.

Introducing a random audit, ie a finite value q, leads to a reduction in desired prudential capital relative to the case of continuous audit, because there is now some possibility of escaping liquidation even when C falls below $\widetilde{C}$. The magnitude of this reduction of $C^* - \widetilde{C}$ depends

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(25) A more detailed analysis of the determinants of C^* is provided in Appendix C.

(26) The robustness of this result depends upon the magnitude of the shift in $\widetilde{C}$ (see Section 6)

(27) As σ_1 rises to extremely high levels, desired prudential capital will eventually begin to decline, but this does not occur for economically plausible parameter values.

upon the scope for increasing portfolio variance: the higher σ_2 the greater the decline of $C^* - \widetilde{C}$. As long as q is sufficiently high then desired prudential capital C^* is in fact very close to the level when there is continous audit. But the impact of q is non-linear. As q is reduced there is eventually a marked reduction of C^*. Ultimately, if supervision is sufficiently lax, the bank behaves in an imprudent manner, holding no prudential capital reserves and maximising portfolio risk.

This last point can be supported by an explicit analytical expression. Using (**6**) we obtain the further result that $C^* > \widetilde{C}$ if and only if:

$$q > (\frac{1}{2}\sigma_2^2/RA)(\rho/RA)\rho \qquad (\mathbf{7})$$

This inequality indicates that a bank will only ever behave prudently and hold capital if it has positive expected profits ($RA > 0$). It also indicates that the audit frequency needed to induce prudent behaviour is increasing in the coefficient of variation of bank earnings $\frac{1}{2}\sigma_2^2/RA$ and decreasing in the charter value RA/ρ. If (**7**) is not satisfied, ie if audit frequency is insufficiently high to promote prudent behaviour, then the bank will pay out capital at the maximum rate possible at all levels of C, both positive and negative.

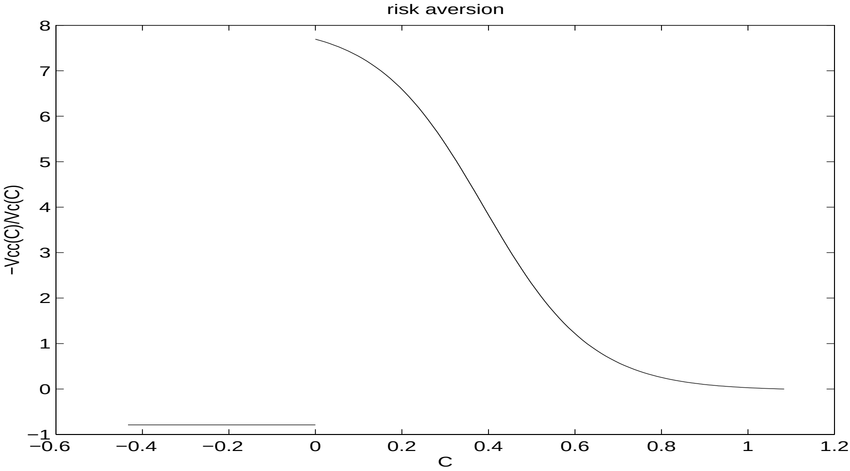
3.4 Risk-taking and looting

Our model explains both the observation that banks reduce portfolio risk as their capitalisation declines and the emergence of the 'gamble for resurrection' amongst critically undercapitalised institutions. This is illustrated in Chart 2 which displays a measure of risk-aversion, $-V_{CC}/V_C$, as a function of C.(28)

In order to understand the intuition underlying this chart consider again the case of continuous audit ($q = \infty$) when the bank is liquidated the instant C falls to $\widetilde{C}$. In this case there is no possibility of exploiting the regulatory guarantee and putting losses onto the regulator. There

(28)If the portfolio decision impacts both on expected return and on the variance of returns, then the mean-variance trade-off will be determined by $-V_{CC}/V_C$. Such a trade-off could be introduced into our model (as in Milne and Robertson (1996) or Green (1984)) but we would not then be able to obtain a closed-form analytical solution.

Chart 2: The level of prudential capital reserves and incentives for risk taking †



† Parameter values as in Chart 1.

is therefore no incentive for excessive risk-taking. Shareholders however still wish to protect their claim on the charter value of the bank. They therefore seek to self-insure against the possibility of liquidation triggered by a deterioration in cash flow through holding capital reserves.

For C close to C^*, the level of capital is sufficient to fully insure the bank against fluctuations of cash flow, and $V_{CC} \approx 0$. Thus the bank is effectively risk-neutral, behaving as if there were no threat of liquidation at all.

As C declines below C^* the capital of the bank becomes increasingly inadequate as self-insurance against the possiblity of a run of losses leading to default, which would trigger the loss to shareholders of the banks 'charter value' ie the entire stream of future expected earnings. As a result risk-aversion, measured by $-V_{CC}/V_C$, increases as capitalisation C declines; and increases sharply as C falls close to $\widetilde{C}$.

Consider now the case of random audit (finite q). It is only in this case that there is any possiblity of putting losses onto the regulator. When the bank capital exceeds the threshold $\widetilde{C}$ then there is no incentive to exploit this option as bank shareholders are unprotected from any short term losses. The behaviour of the bank is similar to the case of continuous audit. When $C < \widetilde{C}$, so the bank is 'critically undercapitalised', matters are different. Now, should an audit take place, the bank will be liquidated and shareholders are able to put losses onto the regulator. Banks are therefore subject to moral hazard and engage in a gamble for resurrection (in our model maximisation of σ) only when $C < \widetilde{C}$. Formally this risk loving behaviour appears as negative value for risk-aversion with $-V_{CC}/V_C < 0$.

Our model also predicts the looting of a bank with sufficiently negative net worth. As C declines the probability of liquidation before any payment of dividend becomes increasingly likely. Eventually, for net worth C below some lower bound $\widehat{C} \ll \widetilde{C}$, $V_C(C) < 1$, refecting the fact that the additional insurance against liquidation of retaining an increment of cash flow is now of less value to shareholders than an immediate distribution. Thus for a bank with extreme negative capitalisation value is increased by a transfer of assets out of the bank into the hands of shareholders. Dividends are paid at the maximum possible rate.

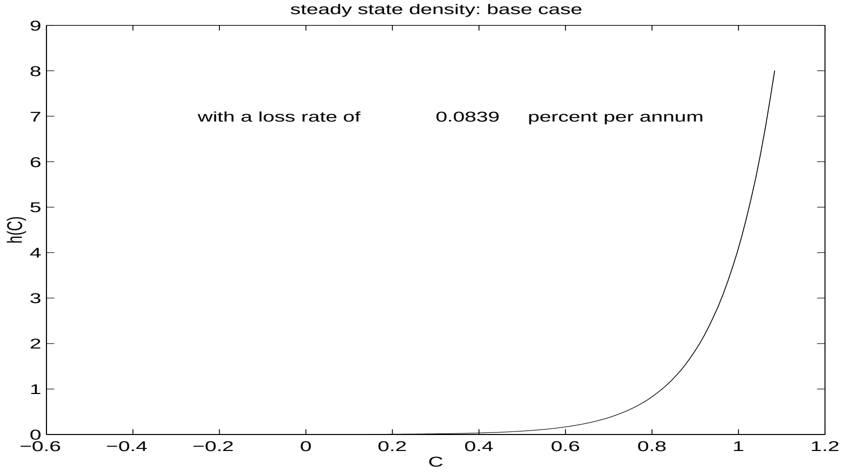
4 Endogenous capital: steady-state industry equilibrium

In this section we consider the steady-state equilibrium of a banking industry in which each individual bank behaves according to the model of the previous section. This allows us to analyse the aggregate rate of bank failure and, by assuming some fixed cost associated with the re-capitalisation of a bank and a negative relationship between the number of institutions and economic rents, to investigate the determination of bank charter value.

4.1 The aggregate rate of bank failure

We assume a continuum of identically sized small banks and that liquidated banks are immediately replaced by new banks of equivalent

Chart 3: The steady state distribution of banks, $-0.4 < C < C^*$, base case †



† Parameter values as in Chart 1.

size with capitalisation $C = C^*$. The total number of banks and total bank assets are therefore constant.

In the long run, regardless of initial conditions, the distribution of banks over the range $-\infty < C \leq C^*$ converges on the steady-state distribution $h(C)$:[29]

$$h/l = \begin{cases} q^{-1}\overline{\nu}_1 \exp(\overline{\nu}_1(C - \widetilde{C})), & C \leq \widetilde{C} \\ (q^{-1}\overline{\nu}_1 + \frac{1}{RA}) \exp(\nu(C - \widetilde{C})) - \frac{C-\widetilde{C}}{RA}, & \widetilde{C} \leq C \leq C^* \end{cases} \quad (8)$$

where $\nu = RA/\frac{1}{2}\sigma_1^2$ and $\overline{\nu}_1$ is the positive quadratic root $\left(RA + \sqrt{(RA)^2 + 2 * q * \sigma_2^2}\right)/\sigma_2^2$, .

Chart 3 shows this steady-state density $h(C)$ with parameter values as for charts 1 and 2. Although this is a steady-state the capitalisation of individual banks varies over time. Banks can expect to accumulate capital over time ie banks drift towards C^*. In addition the stochastic

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(29) We normalise the total number of banks assuming that $H = \int_{-\infty}^{C^*} h(C)\mathrm{d}C = 1$. Appendix B derives a formal expression for $h(C)$.

variation of cash flow leads to declines in capital C in some banks, ie banks diffuse towards $-\infty$. In steady-state the rate of upward drift and downward diffusion exactly offset each other so that the density of banks $h(C)$ at any level of capital C remains constant over time.

The annual steady-state rate of bank liquidations, expressed as a proportion of the total population of banks, is then:[(30)]

$$l = \left[\begin{array}{r} q^{-1} + ((q^{-1}\overline{\nu}_1 + (RA)^{-1}))\{\exp(\nu(C^* - \widetilde{C})) - 1\}/\nu \\ -(RA)^{-1}(C^* - \widetilde{C}) \end{array} \right]^{-1} \qquad \mathbf{(9)}$$

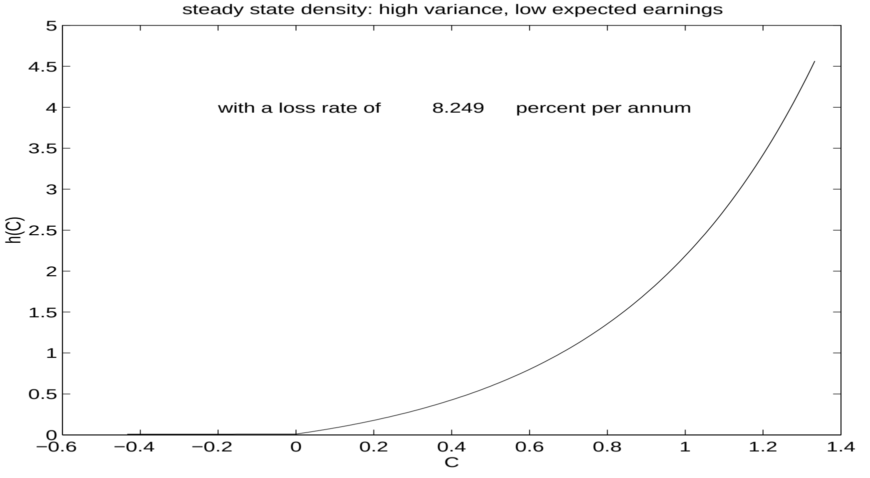
In Chart 3 this steady-state rate of liquidation l is less than 0.1% per annum ($l \approx 0.00084$).[(31)] Since $C^* - \widetilde{C} \simeq 1.1$ a bank located close to C^* would have to have cash flow over a one year period of less than $-RA$, compared to expected cash flow of $+RA$, in order for capital to fall below $\widetilde{C}$. With the parameter values used for Charts 1-3 this is an eventuality which would happen less than one year in four hundred. This is why l is small.

With alternative parameter values we can obtain a much higher rate of liquidation. Chart 4 shows the steady-state density function for the case where, relative to Chart 3, the standard errors of asset returns have increased fourfold and the rate of discount ρ has doubled reducing continuation value to half of what it was before. Relative to expected earnings the desired level of capitalisation is now higher ($C^*/(RA)$ increasing from around 1.2 to 1.4). Despite this increase in capitalisation the much higher rate of diffusion relative to expected earnings means that a much greater proportion of the steady-state density of banks is in the region $C < C^*/2$ and the rate of liquidation is approximately 100 times higher than before ($l \approx 0.083$). The combination of a low continuation value and a high rate of diffusion results in highly unstable banks.

[(30)] A further result is that the steady-state rate of payment out of deposit insurance or public funds is given by $\{(1-\gamma)A - \widetilde{C} + \overline{\nu}_1^{-1}\}l$. Here $\{(1-\gamma)A - \widetilde{C}\}l$ is the rate of payment that would be associated with a loss rate of l, were all liquidations to occur at $C = \widetilde{C}$. The overall rate of deposit insurance payout is however higher than this because the level of capital at which liquidation is triggered is distributed over the range $[-\infty, \widetilde{C}]$. The additional term $\overline{\nu}_1^{-1}$ corrects for this.

[(31)] Although this is not obvious from Chart 3, $h(C) > 0$ when $C \leq \widetilde{C}$. The rate of liquidation is q times the number of banks with capital less than $\widetilde{C}$.

Chart 4: The steady state distribution of banks, $-0.4 < C < C^*$, alternate case †



† Change in parameter values from Charts 1-3: ρ=6% per annum, $\sigma_1 = 2$, and $\sigma_2 = 4$.

4.2 The determination of charter value

We suppose that recapitalisation of an existing bank or the entry of a new and fully capitalised bank requires an entry cost of κ and that there is a negative relationship between current earnings RA and the total number of banks $H = \int_{-\infty}^{C^*} h(C)\mathrm{d}C$. With these assumptions the number of banks H adjusts until in steady-state industry equilibrium:[32]

$$V(C^*) = RA(H)/\rho = C^*(RA) + \kappa. \qquad \mathbf{(10)}$$

Equation (**10**) indicates that 'charter value', interpreted as the present value of current and future profits depends upon the number of banks, which in turn depends both upon entry costs κ (a lowering of entry costs leads to an increase in the number of banks and hence a decline in charter value) and also on any factors which increase C^*. Thus for

[32] We assume that the industry is viable ie that there is some $H > 0$ for which (**10**) is satisfied.

example an increase in the variance of bank earnings, or an increase in the frequency of audit would lead eventually to a decline in the population of banks and an increase in equilibrium charter value.

It is also apparent from (**10**) that we can relax our assumption about a contraint on new equity issue. It is sufficient for our analysis only that there be some fixed cost of making a new issue κ. Existing banks will not issue new equity because the cost to shareholders of doing this $(C^{*} - C) + \kappa$ exceeds the resulting benefit $V(C^{*}) - V(C)$.[(33)] Our only requirement, in order for the industry as a whole to be viable, is that κ is large enough so that (**7**) is satisfied in steady-state.

5 Implications for regulatory policy

In our model banks hold prudential capital to self-insure against the threat of falling below the required regulatory minimium level of capitalisation $\widetilde{C}$. In consequence an increase of $\widetilde{C}$ has only a temporary impact on bank behaviour. Over time earnings are retained in order to build the margin between actual capital and the regulatory minimum back towards desired levels. The bank is then once again fully insured against the threat of liquidation and the regulatory capital requirement no longer affects its decisions. We conclude that minimum capital requirements should be primarily interpreted not as tools to alter bank behaviour but, since an increase in $\widetilde{C}$ increases the average net worth of failing banks, as a method of reducing the expected payout on insured deposits.

During the adjustment period, when capital is in short supply following an increase in capital requirements, banks will typically become more risk-averse.[(34)] Assuming that increased risk aversion is associated with a widening of interest rate spreads and a reduction in the holding of risk assets, then this could result in a temporary reduction in the

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[(33)]This assumes that new entrants to the industry can obtain the same return on bank assets as incumbent firms. Existing banks, who already have charter value, are then crowded out of the market for new issues by new entrants, who obtain charter value through entry. If incumbent firms can obtain a better return on bank assets than new entrants, they may increase shareholder value by issuing new equity.
[(34)]Banks whose capital is below the new minimum will however become risk-loving.

aggregate supply of bank credit and a re-allocation of bank portfolios from risky to safe assets.[35]

The second policy instrument, the expected frequency of audit q, does have a major impact on risk-taking. Our analysis supports the following conclusions:

- An essential task of supervision is to distinguish profitable banks, with positive discounted expected earnings $RA/\rho > 0$, from unprofitable banks. Unprofitable banks, having nothing to lose, will always seek to exploit the regulatory guarantee and so need to be closed as soon as possible.
- For profitable banks the frequency of audit needed to induce positive holding of capital and prudent behaviour is inversely proportional to the expected discounted value of future earnings and directly proportional to the coefficient of variation for bank earnings. Thus, taking account of the direct and compliance costs of audit, efficient use of supervisory resources requires that they be concentrated on banks with relatively low earnings and relatively high variance of earnings. Banks with substantial and stable income streams can be lightly supervised and left largely to manage their own affairs.
- Closer supervision is also required for those banks which have greatest scope for risk-taking and looting when they are in difficulties (banks for which σ_2 is relatively high). This suggests, in particular, that supervisory effort should be focused on banks which have the least transparency in their affairs, especially perhaps banks which operate in a number of regulatory jurisdictions.

6 Robustness and further work

In order to obtain analytical results we assume that the level of bank assets is exogenous, rule out new equity issue by existing banks, impose

[35]Thakor (1996) and others have argued that the introduction of the Basle 1988 Accord was responsible for the the switch in US bank-portfolios from private sector loans to government securities in the early 1990s. If this hypothesis is correct then, according to our model, the switch will unwind as banks replenish their capital.

a zero cost of deposits, and adopt a rather stylised treatment of the process of regulatory audit and intervention. Weakening any of these assumptions, while making the model more realistic, would also introduce considerable solution difficulties requiring the use of numerical techniques even where a solution could be obtained at all. We must therefore consider how robust our findings would be to relaxation of our principal modelling assumptions.

Some of these assumptions probably matter less than might appear at first sight. For example we could have introduced a downward-sloping demand curve for bank loans, with R being a negative function of A. Banks would choose a level of assets A at which the benefits of increased marginal return to lending exactly offset the marginal costs of funding and the marginal costs of any increase in the uncertainty of returns. The qualitative features of such a model would be similar to those we report in this paper: prudential capital reserves would still be held as an insurance against liquidation, the bank would be increasingly risk-averse as net worth declined, and there would be gamble for resurrection when the bank was under immediate threat of liquidation.[36]

It would be more realistic to make regulatory audit depended upon the outcome of previous audits, with for example more frequent or even continuous audit being applied to a bank which was found at the most recent audit to have net worth below some threshold; and, in the event of capital C falling to the regulatory minimum C^*, to assume some intervention in the management of the bank rather than a complete loss of shareholder value. These specifications would be considerably more difficult to analyse than the one we have adopted, but it is clear that capital would still be used as a protection against default, suggesting that many of the qualitative features of our model would still apply.

Our assumption of a zero cost of deposits matters somewhat more to our results. In the case where deposits have a non-zero cost, then it will no longer be true that $C^* - \widetilde{C}$ will be independent of the level of $\widetilde{C}$. For example a large increase of $\widetilde{C}$, say for example to +50%, would

[36] See Milne and Robertson (1996) for further discussion of a related model. The outcome would however be qualitatively different were it possible to adjust A without reduction of expected return. Such an assumption, which would be appropriate for the case of an investment bank, would allow our bank to respond to declines in net worth by 'hunkering down' ie reducing the scale of operations in proportion to its capital. As a result it could always avoid liquidation.

force the bank to substantially reduce its gearing. If deposits involve a non-zero cost this would in turn lead to an increase in expected cash flow. We would then expect to see a modest decline in $C^* - \widetilde{C}$ (as discussed in Appendix C). Our assumption of a zero cost of deposits is reasonable provided that C is small enough, so that we can neglect these cash-flow effects.[(37)]

We impose a fixed constraint on equity issue as a simple way of modelling some of the consequences of financial distress for bank behaviour. This is not an artificial assumption: when a company or bank is in financial distress it is difficult to raise new equity because of fears that the underlying situation is worse than reported cash flows would indicate. Even when a company is not in financial distress problems of asymmetric information impose costs on new equity issue.[(38)] Nonetheless it would be desirable to investigate the extent to which our analysis generalises to models with a more fully articulated specification of information asymmetries.

Our results appear robust to other more obviously technical variations in model specification. We might have assumed jump stochastic declines in asset values as a more realistic treatment of bank credit risk, although again we would not then obtain a closed-form solution. Such a specification would introduce a smoothing of the discontinuity in attitude to risk shown in Chart 2, leading to the possibility of risk-loving behaviour when C narrowly exceeds $\widetilde{C}$. A similar smoothing would emerge in discrete time dynamic analysis (see Calem and Robb (1996) who report results, on the relationship between risk-taking and capitalisation in a discrete time model, which parallel our own findings).

Again for reasons of tractability we have not taken into account conflicts of interest between managers and shareholders. Given that managers have an interest in the bank as a going concern, it is plausible that a similar divergence between risk-aversion for the undercapitalised bank and risk-loving behaviour by the critically undercapitalised bank would emerge in a more general model of this kind.

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(37) Our technical analysis would also have to be altered were $\widetilde{C}$ so large that the resale value of the assets of the bank γA are sufficent to fully repay deposits when $C = \widetilde{C}$. When depositors are fully repaid shareholders rather than regulators become the residual claimants on bank assets following liquidation. Our analysis does not allow for this possibility.

(38) Myers and Majluf (1984) is a well known analysis of this point.

In our model changes in minimum capital requirements have no long-run effect on bank behaviour, once the bank has adjusted by building up its own capital reserves. This suggests that the risk-weighting of regulatory minimum capital is also relatively unimportant and will not affect the asset allocation of the fully capitalised bank. This is a further topic for investigation within our framework.

Because it is dynamic, a model of capital holding such as ours can be used to make predictions about the behaviour of banks over the business cycle. The current state of the business cycle affects current and expected bank earnings and hence should influence bank decisions about capitalisation and bank attitudes towards risk-taking. Moreover, if an increase in risk-aversion leads to a desire to widen interest margins, reduce lending and contract the balance sheet, then reductions in bank capital will themselves contribute to the severity of cyclical downturns through a contraction of new lending. Such analysis should properly be pursued in a general equilibrium model. Our model does however make clear that the response to declines in bank capital will be non-linear, with large reductions in bank capital having a proportionately greater impact than small reductions.

7 Conclusion: capital reserves as self-insurance

We provide a complete closed-form solution of a model of bank capital and risk-taking. We conclude by restating the underlying economic intuition captured in this model: if there are constraints on raising external capital then prudential capital reserves are held as a form of self-insurance, despite the relatively high cost of capital relative to insured deposit finance, in order to reduce the probability of future liquidation. Shareholders wish to do this because liquidation incurs substantial deadweight costs, the loss of ownership of a stream of positive expected future profits (the ‘charter value’ of the bank.) Healthy banks with sufficient profits seek to hold a desired level of capital as a buffer against the risk of poor asset returns which might trigger liquidation. They retain earnings in order to increase capital reserves up to this desired level.

An implication of this analysis is that during episodes of capital shortage banks are under-insured and hence are increasingly risk-averse the more capital declines below desired levels. However once capital declines to below the regulatory minimum level of capital there is an immediate threat of loss of charter value through regulatory intervention. The bank then engages in a short-sighted 'gamble for resurrection' of the kind suggested by the simplest models of moral hazard. If capital declines further by a sufficient amount the shareholders then also have incentives to 'loot' the bank.

Analysing the steady-state properties of our model we are able to show that charter value is itself determined by costs of entry and the desired level of capital (see equation (**10**)). Imposing reasonable parameter values, we find that banks have sufficient charter value so that risk-loving behaviour occurs relatively infrequently. This helps to explain why moral hazard is an unusual and pathological symptom, not a chronic illness.

Our view of prudential capital as self-insurance underlies our finding that, once banks have fully responded by altering their actual levels of capital to new desired levels, changes in minimum capital regulations have little impact on bank behaviour. Our analysis also implies that supervisory resources should be concentrated on identifying and closing down bad banks, ie those with negative expected earnings, and monitoring those institutions with low positive expected earnings relative to the risks of their assets, since these are exactly the institutions which have the weakest incentives to properly self insure.

Appendix A
Diagrammatic exposition of existing literature

Existing models of the relationship between capitalisation and bank risk-taking share a common static structure in which bank managers maximise shareholder value and the level of capital is exogenously imposed.[39] In this appendix we exploit this common structure in a simple diagrammatic exposition of the literature.

A.1 The basic model of moral hazard in banking

In this appendix we assume that the assets of the bank are fixed at A. Capital C is exogenous. Debt finance of $D = A - C$ is raised at zero cost while returns to shareholders are undiscounted. Chart A1 represents the basic static model. In this chart, as in the subsequent Charts A2-A4, equity capitalisation C is represented on the horizontal axis.

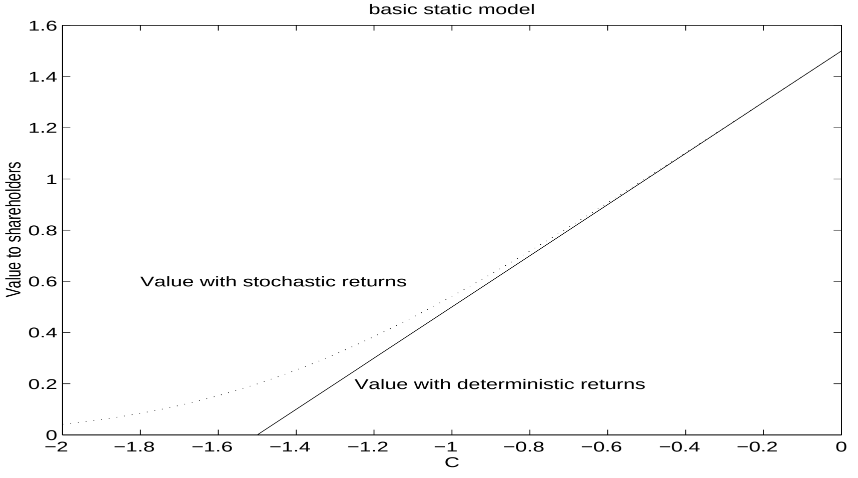
The value of the bank $V(C)$ is shown on the vertical axis for two special cases: the first where the returns on the banks assets are certain $R = R^e$; the second where there is uncertainty about returns $R = R^e + \varepsilon$. Since the expected return R^e in the two cases is identical, we are considering a mean-preserving spread in investment returns.

In this and subsequent charts the solid line represents the case where returns are certain while the dashed line represents the case where returns are uncertain. When returns are certain the value of the bank is given by $V^c = \max[0, C + AR^e]$. The maximisation operator reflects the limited liability of shareholders which prevents V^c falling below zero. V^c is greater than zero whenever $C > -AR^e$.

When returns are uncertain the value of the bank is given by $V^u = \int \max[0, C + A(R^e + \varepsilon)] df(\varepsilon)$. This integral has an option-value

[39] Other papers which we shall not review are O'Hara (1983), who presents a dynamic model of bank behaviour, allowing for conflicts of interest between shareholders and management, and Daripa and Varotto (1997) who provide an alternative (static) account of impact of such conflicts of interest on bank risk-taking. Neither analysis considers the relationship between capitalisation and risk-taking.

Chart A1: The value to shareholders of bank equity, as a function of equity capital, basic static model



interpretation since, as discussed by Merton (1977), whenever $\varepsilon < -(C/A + R^e)$ shareholders can put losses onto the regulators. If as assumed in Chart A1 ε is normally distributed this option value can be computed using the Black-Scholes formula.(40) More generally, when the distribution of ε is unbounded from below, $V^u > V^c$ for all finite C, and hence at all levels of capitalisation the value of the bank to its shareholders is increased by a mean-preserving increase in asset uncertainty (an increase in the variance of ε).

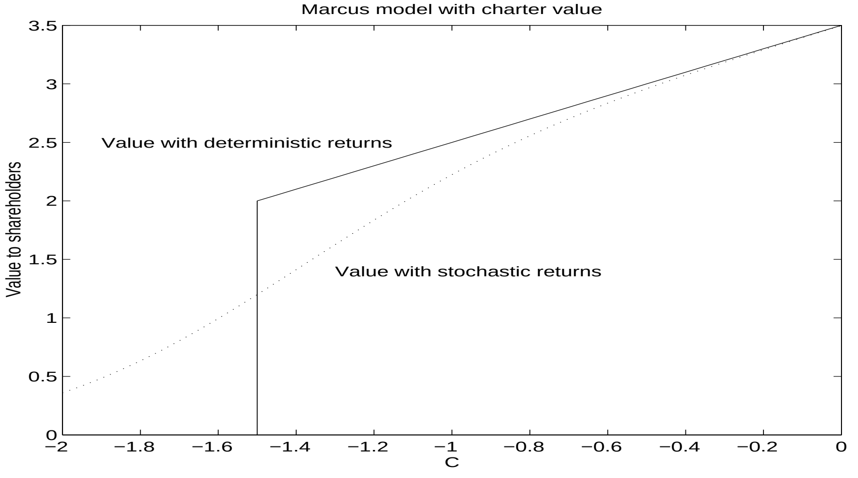
Finally note that, under these same assumptions, for all finite C , $\partial V^u/\partial C < 1$. This implies that at all levels of capitalisation total shareholder value is increased by extracting capital ΔC and paying it out to shareholders. Shareholders thus always have an incentive to loot the bank.

A.2 Generalisations

Chart A2 shows the effect of allowing for a continuation value, as in Marcus (1984) Section 2. In the case of certainty the value of the bank

(40) In the chart we assume normality, but in order to use the same computational technique as is applied in subsequent figures, we compute V^u using numerical integration.

Chart A2: The value to shareholders of bank equity, as a function of equity capital, static model with charter value



is a non-continuous function of capital. If capital falls below the level which triggers liquidation ($C < AR^e$) then the value of the bank is zero. In the case of uncertainty, when $C < -AR^e$ so that the option of putting losses onto the regulator is in the money, $V^c < V^u$ and the bank still has an incentive to maximise portfolio risk; but when $C > -AR^e$, ie when the put option is out of the money, $V^c > V^u$ and the bank has an incentive to minimise portfolio risk. Note that the continuation value must exceed a certain minimum bound, which is increasing in the maximum degree of asset risk, in order to induce risk-averse behaviour for $C > -AR^e$.

Chart A3 amends the basic model as in Green (1984) for the case where a proportion of the debt in the firm takes the form of warrants, which are convertible into equity whenever $C + A(R^e + \varepsilon) > W$ for some exogenously specified W. Again when C is low the bank continues to maximise portfolio risk ($V^c < V^u$). As C increases it becomes increasingly likely that warrants will be exercised, hence reducing the exposure of shareholders to upside risk. When C is sufficiently large shareholders prefer to minimise portfolio risk ($V^c > V^u$).

Chart A4 amends the basic model as in Rochet (1992) to allow for limited liability in a mean-variance optimisation model of bank

Chart A3: The value to shareholders of bank equity, as a function of equity capital, static model with equity warrants

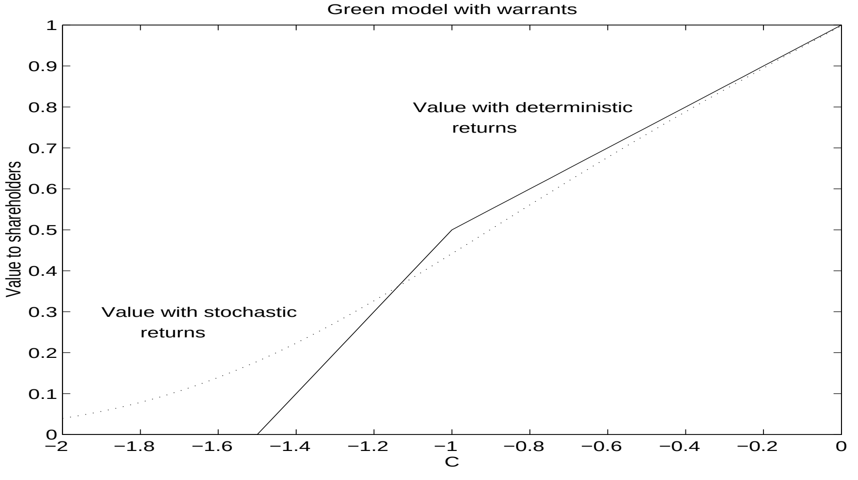
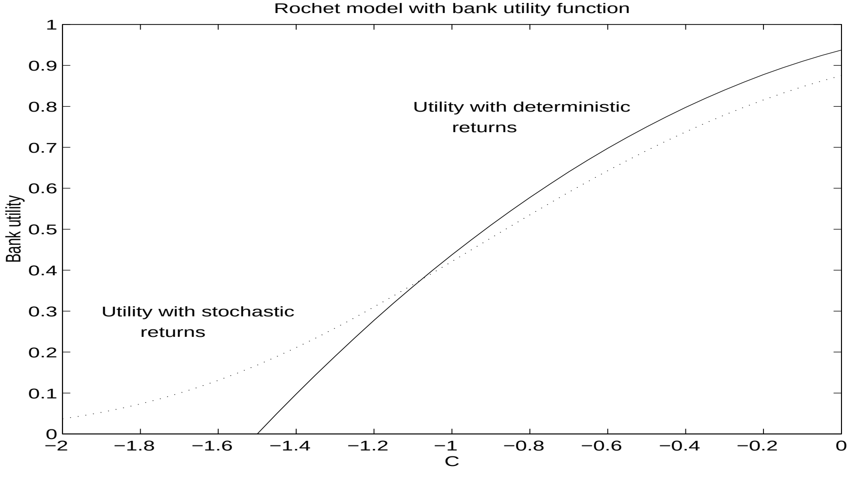


Chart A4: The value to shareholders of bank equity, as a function of equity capital, with risk-averse shareholders



behaviour.[41] Here we have assumed that shareholder value is a quadratic function $u(C + A(R^e + \varepsilon))$ of the return to shareholders. For low values of C the effect of limited liability dominates and shareholders prefer to maximise portfolio risk ($V^c < V^u$). Again, when C is sufficiently large, shareholders prefer to minimise portfolio risk ($V^c > V^u$).

In all three generalisations of the basic model of moral hazard banks only engage in extreme risk-taking behaviour when capital is below some critical level. The Green model however fails to explain why banks should hold capital in the first place, since as in the basic model, shareholders will always benefit from a transfer of capital out of the bank.[42] The Rochet model combining limited liability and a non-linear shareholder utility function is most appropriate in the case where shares are closely held and represent a large proportion of shareholder wealth. It is more difficult to justify such a utility function in the usual case when shares are widely traded and shareholders can diversify away the risk of individual bank earnings, although as noted in the main text this could be thought of as a way of capturing conflicts of interest between bank managers and shareholders.

Appendix B
Mathematical derivations

In this appendix we provide derivations of all the mathematical results reported in the main text.

(41) The mean-variance model with unlimited liability was developed by Kahane (1977). Other mean-variance models of bank behaviour are Koehn and Santomero (1980) and Kim and Santomero (1988). A feature of these models, not captured in our diagram, is that an exogenous increase in capitalisation can lead to an increase in portfolio risk, as banks respond by reallocating their portfolio towards riskier assets.

(42) Green (1984) considers the behaviour of a non-bank corporate. It is also unsatisfactory as a model of bank behaviour since in practice banks issue little convertible debt or warrants.

B.1 Optimal dividend policy and value function curvature

Proposition 1 of Milne and Robertson (1996) applies directly to this model. Hence optimal policy is to retain all earnings provided capital C is less than some threshold $C^* > \widetilde{C}$. Proposition 2 of Milne and Robertson (1996) also applies when $\widetilde{C} < C < C^*$ implying in turn that in this region $V_{CC} < 0$.

We now establish that V_{CC} is both continuous and positively signed for $C < \widetilde{C}$. Assume instead that there is a discontinuity for some $C < \widetilde{C}$. The infinite variation of the diffusion process requires continuity of both V and of V_C. Hence any discontinuity in V_{CC} must involve a change of σ and a multiplication (or division) of V_{CC} by (σ_1/σ_2). But since $\sigma_1/\sigma_2 > 0$, V_{CC} cannot have changed sign and hence there cannot in fact have been a change in σ. This contradiction implies that V_{CC} is continuous for $C < \widetilde{C}$. This in turn implies that V_{CC} is single signed for $C < \widetilde{C}$ since a switch in σ itself must generate a discontinuity in V_{CC}

The function $V(C)$ therefore takes the simple exponential form for $C < \widetilde{C}$. But, in order that $V(C) \downarrow 0$ as $C \downarrow -\infty$, it is then necessary that $V_{CC} > 0$ when $C < \widetilde{C}$.

B.2 The value function

Assuming $C < C^*$ so all earnings are retained, the value function satisfies:

$$(\rho + q)V = \max_{\sigma}[RAV_C + \frac{1}{2}\sigma^2 V_{CC}], \qquad C \leq \widetilde{C} \qquad \textbf{(B1)}$$

$$\rho V = \max_{\sigma}[RAV_C + \frac{1}{2}\sigma^2 V_{CC}], \qquad \widetilde{C} \leq C \leq C^* \qquad \textbf{(B2)}$$

with solution (where $\bar{\mu}_1$,$\bar{\mu}_2$,μ_1, and μ_2 are quadratic roots):

$$V = \bar{M}_1 \exp(\bar{\mu}_1(C - \widetilde{C})) + \bar{M}_2 \exp(\bar{\mu}_2(C - \widetilde{C})) \qquad C \leq \widetilde{C} \qquad \textbf{(B3)}$$

$$V = M_1 \exp(\mu_1(C - \widetilde{C})) + M_2 \exp(\mu_2(C - \widetilde{C})) \qquad \widetilde{C} \leq C \leq C^* \quad \textbf{(B4)}$$

The unknowns M_1,M_2,$\overline{M}_1$,$\overline{M}_2$, and $C^*$are derived from the five boundary conditions:

$$\lim_{C\downarrow-\infty}[\bar{M}_1\exp(\bar{\mu}_1(C-\widetilde{C}))+\bar{M}_2\exp(\bar{\mu}_2(C-\widetilde{C}))]=0 \quad \textbf{(B5)}$$

$$\bar{M}_1+\bar{M}_2=M_1+M_2 \quad \textbf{(B6)}$$

$$\bar{\mu}_1\bar{M}_1+\bar{\mu}_2\bar{M}_2=\mu_1 M_1+\mu_2 M_2 \quad \textbf{(B7)}$$

$$V_C(C^*)=\mu_1 M_1\exp(\mu_1(C^*-\widetilde{C}))+\mu_2 M_2\exp(\mu_2(C^*-\widetilde{C}))=1 \quad \textbf{(B8)}$$

$$V_{CC}(C^*)=\mu_1^2 M_1\exp(\mu_1(C^*-\widetilde{C}))+\mu_2^2 M_2\exp(\mu_2(C^*-\widetilde{C}))=0 \quad \textbf{(B9)}$$

(**B5**) is a limiting condition which ensures that the extremely undercapitalised bank is worth nothing to its shareholders. The smooth pasting conditions at $\widetilde{C}$ ((**B6**) and (**B7**)), and smoothness at C^* ((**B8**)) all follow from the infinite local variance of the diffusion process. (**B9**) is a further optimality condition on the choice of C^*

(**B5**) implies that $\bar{M}_2=0$. (**B9**) implies that:

$$\exp[(\mu_1-\mu_2)(C^*-\widetilde{C})]=-\frac{\mu_2^2 M_2}{\mu_1^2 M_1} \quad \textbf{(B10)}$$

Substituting $\bar{M}_2=0$ into (**B6**) and (**B7**) yields:

$$-\frac{M_2}{M_1}=\frac{\bar{\mu}_1-\mu_1}{\bar{\mu}_1-\mu_2} \quad \textbf{(B11)}$$

and substitution of (**B11**) into (**B10**) then yields:

$$C^*-\widetilde{C}=\frac{1}{\mu_1-\mu_2}\ln[\frac{\mu_2^2}{\mu_1^2}\frac{\bar{\mu}_1-\mu_1}{\bar{\mu}_1-\mu_2}] \quad \textbf{(B12)}$$

The roots of the quadratics are:

$$\bar{\mu}_1,\bar{\mu}_2=\frac{-RA\pm\sqrt{(RA)^2+2(\rho+q)\sigma_2^2}}{\sigma_2^2} \quad \textbf{(B13)}$$

$$\mu_1,\mu_2=\frac{-RA\pm\sqrt{(RA)^2+2\rho\sigma_1^2}}{\sigma_1^2} \quad \textbf{(B14)}$$

$\bar{\mu}_1 > \mu_1 > 0 > \mu_2 > \bar{\mu}_2$ so $C^* - \widetilde{C}$ is real. $C^* - \widetilde{C}$ is positive if and only if:

$$\frac{\mu_2^2}{\mu_1^2}\frac{\bar{\mu}_1 - \mu_1}{\bar{\mu}_1 - \mu_2} > 1 \qquad \textbf{(B15)}$$

which yields after re-arrangement and substitution of (**B14**):

$$\bar{\mu}_1 > \frac{\mu_1\mu_2}{\mu_2 + \mu_1} = \frac{\rho}{RA} \qquad \textbf{(B16)}$$

or after further substitution of (**B13**):

$$\frac{q}{\rho} > \frac{\frac{1}{2}\sigma_2^2}{RA}\frac{\rho}{RA} \qquad \textbf{(B17)}$$

It is not possible to have a finite $C^* \leq \widetilde{C}$, since for $-\infty < C \leq \widetilde{C}$, $V_{CC} > 0$. Thus if (**B17**) is not satisfied, $C^* = -\infty$, and shareholders remove capital from the bank as rapidly as possible at all levels of C.

Substituting into (**B8**), using (**B11**), yields:

$$M_1 = \exp(-\mu_1(C^* - \widetilde{C}))\frac{\mu_2}{\mu_1}(\mu_2 - \mu_1) \qquad \textbf{(B18)}$$

and:

$$M_2 = -\exp(-\mu_2(C^* - \widetilde{C}))\frac{\mu_1}{\mu_2}(\mu_2 - \mu_1) \qquad \textbf{(B19)}$$

Substituting into $V(C^*) = M_1 \exp(\mu_1(C^* - \widetilde{C})) + M_2 \exp(\mu_2(C^* - \widetilde{C}))$ then yields:

$$V(C^*) = \frac{\mu_2 + \mu_1}{\mu_2} = \frac{RA}{\rho} \qquad \textbf{(B20)}$$

Finally substituting for $\overline{M}_2$, M_1, and M_2 in (**B6**) yields $\bar{M}_1$.

B.3 The steady-state density

Under the assumption that all liquidations are matched by the creation of a new banks at C^* the total population of banks remains a constant (H). The time differential for the density of banks $\mathrm{d}h(C)/\mathrm{d}t$ satisfies:

$$\frac{\mathrm{d}h}{\mathrm{d}t} = -qh - RAh_C + \frac{1}{2}\sigma_2^2 h_{CC}, \qquad C \leq \widetilde{C} \qquad \textbf{(B21)}$$

$$\frac{\mathrm{d}h}{\mathrm{d}t} = -RAh_C + \frac{1}{2}\sigma_1^2 h_{CC}, \qquad \widetilde{C} \le C \le C^* \qquad \textbf{(B22)}$$

which in steady-state can be equated to zero ($\mathrm{d}h/\mathrm{d}t = 0$) with steady-state solution:

$$h = \overline{N}_1 \exp(\overline{\nu}_1(C - \widetilde{C})) + \overline{N}_2 \exp(\overline{\nu}_2(C - \widetilde{C})), \qquad C \le \widetilde{C} \qquad \textbf{(B23)}$$

$$h = N_1 \exp(\nu(C - \widetilde{C})) + N_2, \qquad \widetilde{C} \le C \le C^* \qquad \textbf{(B24)}$$

where $\nu = RA/(\frac{1}{2}\sigma_1^2)$, and $\overline{\nu}_1, \overline{\nu}_2$are the roots
$\left(RA \pm \sqrt{(RA)^2 + 2 * q * \sigma_2^2}\right)/\sigma_2^2$

The four unknowns $\overline{N}_1$, $\overline{N}_2$, N_1, and N_2 can be solved using the normalisation:

$$H = \int_{-\infty}^{C^*} h(C)\mathrm{d}C = 1 \qquad \textbf{(B25)}$$

and the three boundary conditions:

$$\lim_{C \downarrow -\infty} [\overline{N}_1 \exp(\overline{\nu}_1(C - \widetilde{C})) + \overline{N}_2 \exp(\overline{\nu}_2(C - \widetilde{C}))] = 0 \qquad \textbf{(B26)}$$

$$\overline{N}_1 + \overline{N}_2 = N_1 + N_2 \qquad \textbf{(B27)}$$

$$l \equiv q \int_{-\infty}^{\widetilde{C}} h\mathrm{d}C = F(\widetilde{C}) \equiv -RAh(\widetilde{C}) + \frac{1}{2}\sigma_1^2 h_c(\widetilde{C}) \qquad \textbf{(B28)}$$

These boundary conditions can be interpreted as the requirement that the density asymptotes to zero as $C \downarrow -\infty$; continuity of the density function at $\widetilde{C}$; and the requirement that the steady-state rate of liquidations l equals the steady-state rate of flow of banks F from the region where $C > \widetilde{C}$ to the region where $C < \widetilde{C}$.

It is convenient to express all other magnitudes in terms of l. (**B26**) implies that $\overline{N}_2 = 0$. Carrying out the integration in (**B28**) establishes that $\overline{N}_1 = q^{-1}\overline{\nu}_1 l$. Substitution of (**B24**) into (**B28**) yields $F(\widetilde{C}) = N_2 RA$ and hence establishes that $N_2 = -l/RA$. Finally substitution into (**B27**) establishes that $N_1 = (q^{-1}\overline{\nu}_1 + (RA)^{-1})l$.

Integration of (**B23**) and (**B24**) shows that the total population of banks is given by:

$$\int_{-\infty}^{C^*} h\mathrm{d}C = \overline{N}_1/\overline{\nu}_1 + (N_1/\nu)\{\exp(\nu(C^* - \widetilde{C})) - 1)\} + N_2(C^* - \widetilde{C}) = 1 \qquad (\mathbf{B29})$$

yielding in turn, after substitution for $\overline{N}_1$, N_1, and N_2, an expression for the steady-state rate of liquidation:

$$l = \left[\begin{array}{r} q^{-1} + ((q^{-1}\overline{\nu}_1 + (RA)^{-1}))\{\exp(\nu(C^* - \widetilde{C})) - 1\}/\nu \\ -(RA)^{-1}(C^* - \widetilde{C}) \end{array} \right]^{-1} \qquad (\mathbf{B30})$$

We can also compute the steady-state rate of call upon deposit insurance. In the event of liquidation occurring at C, the holders of debt are paid $A - C$ while the deposit insurance or other public funds must pay $A - \gamma A + C$, which we have assumed is greater than 0.

The steady-state rate of safety net payments is then given by:

$$\begin{aligned} S &= +ql\int_{-\infty}^{\widetilde{C}} [(1-\gamma)A - \rho C]q^{-1}\overline{\nu}_1 \exp(\overline{\nu}_1(C^* - \widetilde{C}))\mathrm{d}C & (\mathbf{B31}) \\ &= \{(1-\gamma)A - C + \overline{\nu}_1^{-1}) \exp(\overline{\nu}_1(C^* - \widetilde{C}))]_{-\infty}^{\widetilde{C}}\}l & (\mathbf{B32}) \\ &= \{(1-\gamma)A - \widetilde{C} + \overline{\nu}_1^{-1}\}l & (\mathbf{B33}) \end{aligned}$$

Appendix C
Variation of parameter values

In this appendix we examine in more detail the effect of altering parameter values on the desired level of capitalisation C^* and the steady-state liquidation rate l. There are seven exogenous parameters in our model: the expected rate of return on bank assets RA; the lower and upper variances σ_1^2 and σ_2^2; the discount rate ρ; the audit frequency q; the minimum net worth $\widetilde{C}$; and the proportion of asset value recovered in liquidation γ. The main text has discussed the impact of variation of $\widetilde{C}$ (in the long run this simply results in a corresponding increase in C^*) and of a change in γ (this alters the exposure of the regulator but does not affect bank behaviour provided shareholders lose everthing in liquidation).

This leaves five parameters. In fact the the dimensionality of the model can be reduced further. Note first that the solution does not depend upon the scale of the bank (a proportionate increase in RA, in σ_1, and in σ_2 leads to a corresponding increase in $C^* - \widetilde{C}$ but has no impact on incentives for risk-taking or the steady-state rate of liquidation). For this reason, in the charts of both the main text and this appendix, we normalise by measuring capital as a multiple of expected earnings RA.[43]

Chart C1 and C2 show the impact of varying the Poisson frequency of audit q on desired capitalisation C^* and on the steady-state rate of loss of bank assets l. All other parameters are as in Charts 1-3 of the main text. This base case itself is marked with a '$*$' in all of Charts C1-C8.

The impact of altering q is non-linear. Reducing q from 2 to 1 has a relatively small impact on either C^* or l. But reductions below 0.5 have a substantial effect, leading to a sharp fall in C^* and corresponding rise of l. Note also that C^* falls to zero and l asymptotes to $+\infty$ as q declines to the minimum level given by equation (**7**) of the main text.

In the two Charts C3 and C4 we show the impact of a change in the discount rate ρ. Increased shareholder impatience, as is to be expected, leads to a reduction in C^* and a rise in steady-state liquidation l. Note that liquidation rates are very sensitive to the discount rate ρ, in part a reflection of the fact that ρ appears twice in (**7**).

We also show, in Charts C5-C6, the effect of altering the expected rate of return RA. The solid line indicates that C^*/RA falls quite sharply as RA rises, mainly due to the increase of RA. C^* itself however actually rises slightly. This can be seen from a comparison of the two lines in Chart C6. The dashed line is drawn to show C^*/RA in the case where C^* remains unchanged at its base value. The gap between

[43] A further reduction in dimensionality emerges because the model solution is independent of the unit of time. In this appendix we ignore this further reduction of dimensionality, but the reader should be aware that the four sets of parameter variations reported here do not have independent effects on capitalisation and loss rates.

Chart C1: Audit frequency q and desired capital C^*

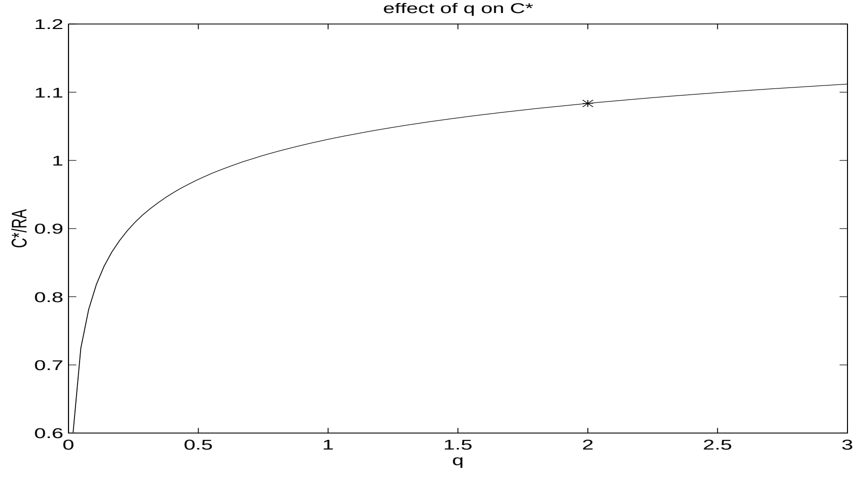


Chart C2: Audit frequency q and liquidation rate l

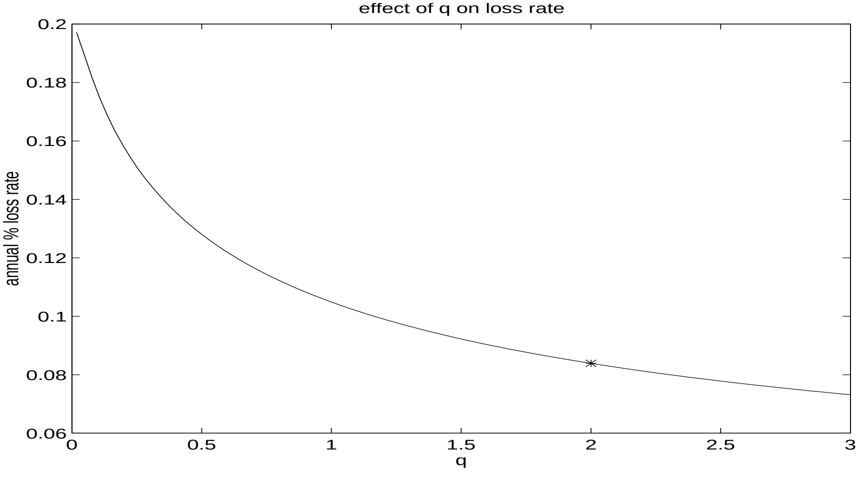


Chart C3: Discount rate ρ and desired capital C^*

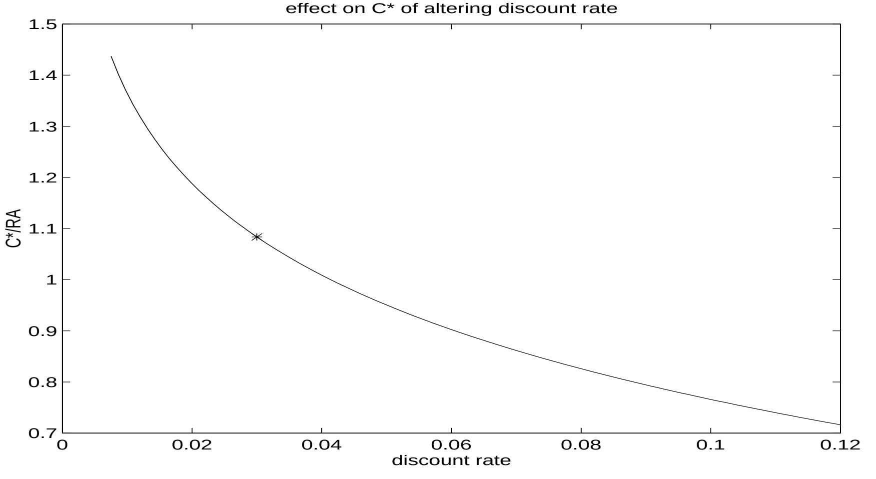
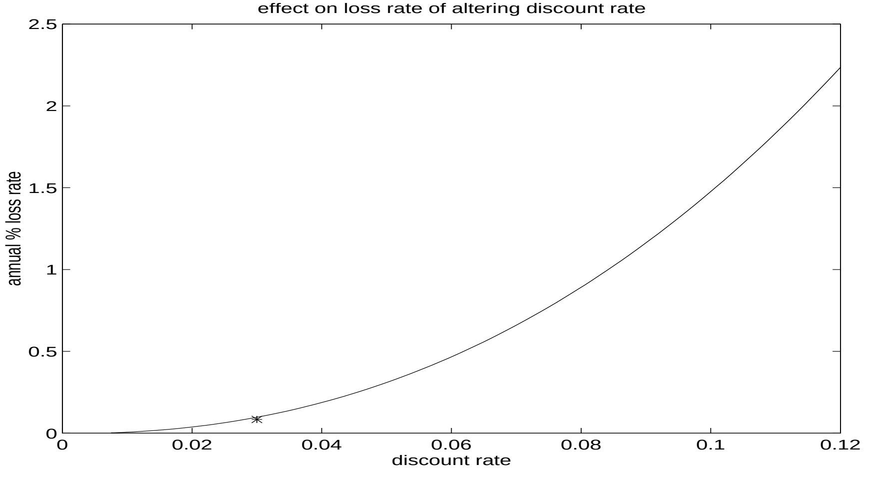


Chart C4: Discount rate ρ and liquidation rate l



the solid and dotted lines indicates that an increase of RA leads to a small increase in C^*.

Since we normalise capital by RA the increase in RA is equivalent to a reduction in both the standard errors $\sigma_1$and σ_2. We therefore also see from C5 that a reduction of variances leads to a reduction in the desired level of capital C^* relative to expected cash flow RA.

We note from Chart C6 that, because of the higher expected rate of return on assets relative to the uncertainty of cash flow, the steady-state loss rate l declines sharply as RA increases or as the standard errors $\sigma_1$and σ_2 decline.

The final two charts, C7 and C8, show the impact of increasing the higher variance σ_2. The impact is approximately linear, with a modest reduction in C^*/RA, and a more marked impact on the loss rate l. As the variance rises fourfold relative to the base case, the loss rate approximately triples.

As a final remark we point out that, since the model is non-linear, the combined effect of altering two parameters can be very different from the sum of the effects of altering two parameters seperately. In the alternative to the base case reported in section 4 of the main paper, a halving of RA and a multiplication of σ_2 by four leads to a one-hundred fold increase in the steady-state loss rate l. This is considerably greater than the sum of the individual effects illustrated in Charts C6 and C8. This large multiplicative impact arises because the increase in σ_2 effectively pushes RA into the rapidly rising region on the extreme left-hand side of Chart C6.

Chart C5: Expected cash flow RA and desired capital C^*

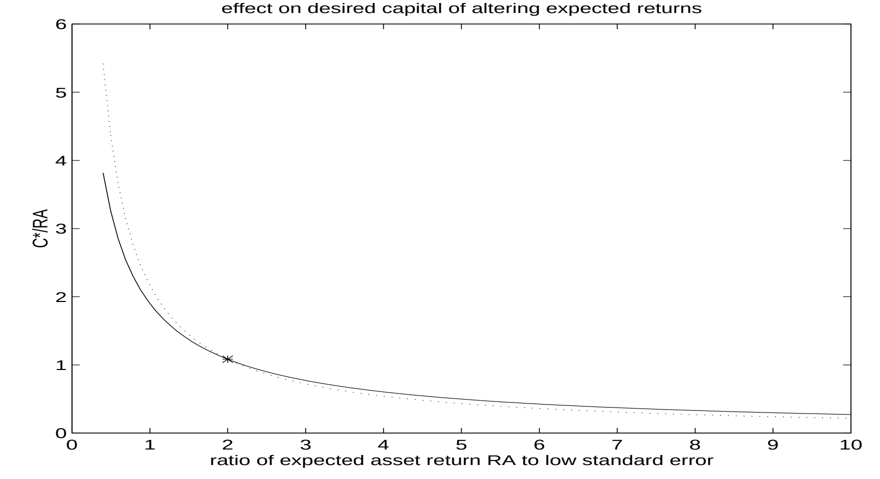


Chart C6: Expected cash flow RA and liquidation rate l

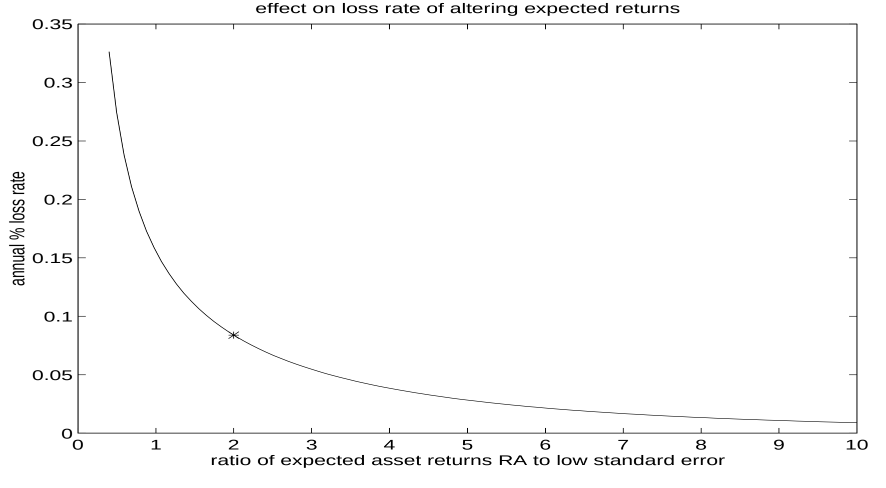


Chart C7: Maximum variance σ_2 and desired capital C^*

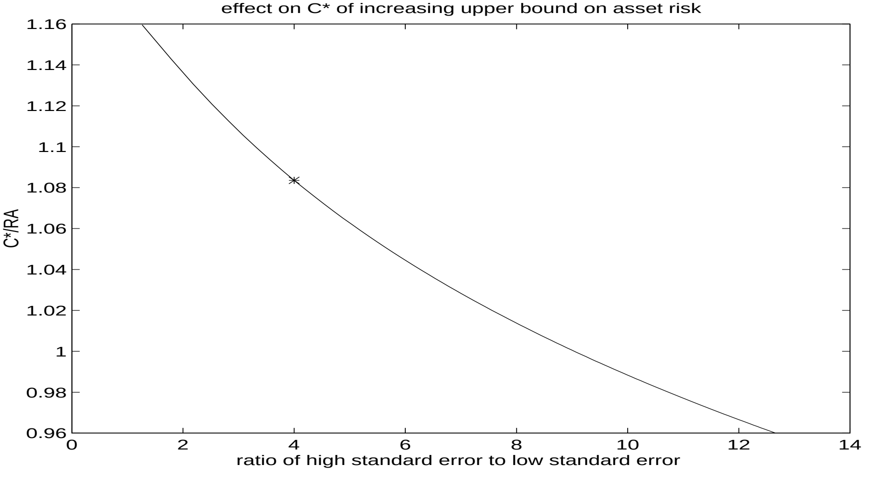
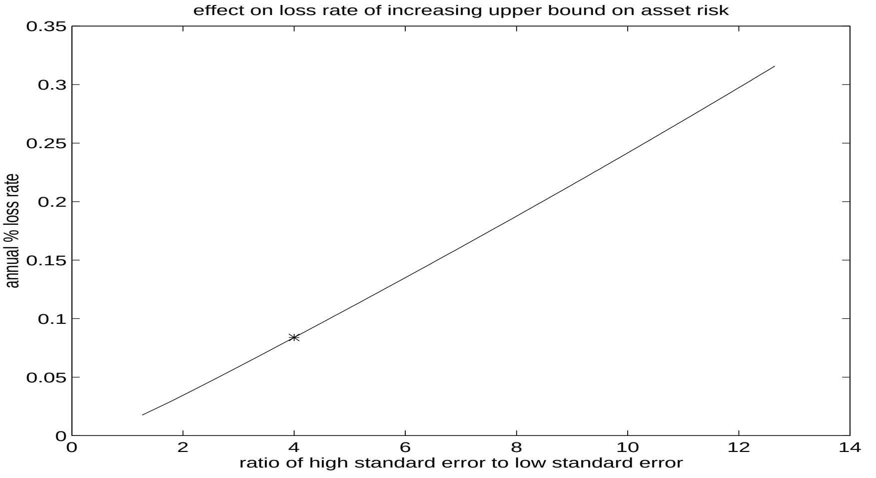


Chart C8: Maximum variance σ_2 and liquidation rate l



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