Caution and gradualism in monetary policy under uncertainty

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The views expressed are those of the author, not necessarily those of the Bank of England. I would like to thank Nicoletta Batini, William Brainard, Willem Buiter, Jagjit Chadha, Alec Chrystal, Andrew Haldane, Steve Millard, Simon Price, Chris Salmon, Philip Schellekens and Paul Tucker for their helpful comments. I would also like to thank Jayne Willis for research assistance.

Issued by the Bank of England, London, EC2R 8AH, to which requests for individual copies should be addressed: envelopes should be marked for the attention of Publications Group. (Telephone 020-7601 4030). Working papers are also available from the Bank's Internet site at http://www.bankofengland.co.uk/wplist.htm

ISSN 1368-5562

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Abstract

This paper explores the theoretical implications of parameter uncertainty for the optimal monetary policy reaction function. The policy-maker sets the nominal interest rate to meet an inflation target in a simple dynamic model of the economy. The paper looks at how parameter uncertainty in the transmission mechanism affects the optimal nominal and real interest rate relative to the case when the parameters are known. Its chief contribution is to show that three consequences are identified: *conservatism* (smaller deviations of real and nominal interest rates from some neutral level in response to inflationary shocks), *gradualism* (increased autocorrelation in real and nominal interest rates) and *caution* (a smaller cumulative policy response). The paper examines the sensitivity of these effects to different specifications of the transmission mechanism; in particular the introduction of an exchange rate channel. The paper also considers situations in which a more aggressive response may be called for.

1. Introduction

The certainty equivalence principle states that an optimising policy-maker can ignore uncertainties about disturbances to the economy when setting policy and proceed as if in a certain world. This principle has played an important role in policy discussions since its introduction by Simon (1956) and Theil (1958). But Brainard (1967) showed that when optimising policy-makers are uncertain, not only about disturbances but also about various elasticities in the transmission mechanism, there were circumstances under which they should decide what should be done according to certainty equivalence, and then do less. This is what Blinder (1997) calls the Brainard conservatism principle, and it has been of considerable interest to policy-makers recently (see for example the minutes of the Bank of England's MPC meetings (1998), Goodhart (1998), Vickers (1998), and Cecchetti (1998)).

This paper reviews Brainard's contribution in the context of a simple open-economy dynamic model, which encompasses the closed-economy model used by Svensson (1997a) to discuss inflation targeting. The model is a linear IS-LM economy (both open and closed variants are considered) with a quadratic objective function for the policy-maker.[1] The framework is simple but nevertheless useful for discussing several policy-relevant issues.

In a certainty equivalent model with only additive uncertainty and no parameter uncertainty, the optimising policy-maker responds fully to offset shocks immediately after they have been observed. The additive disturbances result in a deadweight loss to the policy-maker, but do not alter the authority's incentives from those faced in a model without uncertainty.

Introducing Brainard uncertainty (ie uncertainty about the parameters in the transmission mechanism) can result in inflation deviating from target for longer than in the certainty equivalent model. This consequence is optimal: aggressively setting interest rates to achieve expected inflation equal to target can increase the likelihood of missing the target by a large amount when there are uncertain elasticities. This is because the variance of future inflation outturns is positively related to the aggressiveness of policy. In

(1) Chow (1977) and Svensson (1997b) discuss optimal policy rules for dynamic models with random coefficients. Svensson (1998) considers inflation targeting in an open-economy framework.

this way, parameter uncertainty, unlike additive uncertainty, changes the incentives facing the policy-maker, and so policy is necessarily set differently from a world of purely additive shocks.

The extension of the basic Tinbergen (1952) and Theil (1958) approach to one which accounts for estimated second moments as well as first moments could be argued to be a narrow (if important) interpretation of the uncertainty facing economic policy-makers. The econometric model's structure is taken as given and for a stable regime, a large data set and efficient estimation techniques, the uncertainty about parameters can be reduced to some level. According to Knight (1937), this is a model of 'risks' that can be assessed probabilistically, and Brainard's analysis provides optimal policy in the face of such risks. It could be argued that there is much uncertainty about the form of the true model, eg relevant variables and correct lag specifications as well as unknown coefficients. This has been labelled 'Knightian uncertainty', or model uncertainty. Techniques to deal with model uncertainty were developed in other disciplines but have only recently been applied to problems of monetary policy, by Onatski and Stock (1999), and Sargent (1999). The tendency for approaches based on model uncertainty to advocate an activist solution is discussed in Batini, Martin and Salmon (1999).

In the model presented below, there are three separate effects of parameter uncertainty on policy. The first effect is *conservatism* which, following Blinder, we define as a smaller policy response to a given deviation of inflation from target at a point in time than under additive uncertainty. The second effect is *gradualism*. Here we adopt Goodhart's (1996) definition: gradualism is the smoothing of the response of interest rates to an inflationary disturbance such that 'instead of adjusting interest rates by a large enough jump, whenever inflation begins to deviate from its desired path, the authorities prefer to make relatively small changes' (page 1). A similar definition is given by Sack (1998) who writes that 'gradual monetary policy would smooth the response of the funds rate to a change in the state of the economy, resulting in higher serial correlation of funds rate changes than expected from the dynamic behaviour of the economy' (page 4). Given a disturbance to inflation, gradualism (based around these definitions) is interpreted as smoothing the policy response required to return inflation to target over a longer horizon than in a certainty equivalent world. Finally we define a third effect: *caution*. A policy-maker is said to be cautious if the cumulative policy response to a disturbance to inflation is less than under purely additive uncertainty. The three concepts are obviously related, as is made clear below.

In this simple model the nominal interest rate is the policy-maker's instrument. However output and inflation are influenced by the real interest rate, linked to the nominal interest rate by the Fisher equation. The policy-maker needs to decide the desired stance of monetary policy (ie the level of the short-maturity real interest rate relative to some 'equilibrium' level consistent with the inflation target), which in conjunction with observable expectations of future inflation, enables the policy-maker to decide the optimal level of the nominal interest rate. The aim of this paper is to characterise the implications of parameter uncertainty on conservatism, gradualism and caution for both the nominal interest rate (the instrument) and the real interest rate (the monetary stance), for both the closed and open variants of the model described below.

The main findings are:

- For a wide range of parameters in our simple model, Brainard uncertainty results in conservatism, gradualism and caution in desired real interest rate and in the nominal interest rate instrument.

- The conservatism results are almost identical to those in Brainard's static model. However a more conservative real interest rate response will lead to a larger deviation of inflation expectations from target. This will be reflected in the nominal instrument, but for a wide range of plausible parameter values this effect will not outweigh real interest rate conservatism, resulting in conservatism in the nominal interest rate.

- Gradualism in the desired monetary stance (ie the real interest rate) is accompanied by gradualism in the instrument. This is because as long as the authority wishes to keep monetary conditions tight (or loose), it must use the nominal interest rate to do so.

- If shocks to inflation have a permanent effect on the level of inflation, the desired real interest rate in the closed-economy variant may not display caution. Rather, the cumulative response can be the same as in the benchmark additive-uncertainty model. But even when inflationary shocks have a permanent effect in the open-economy model, the real interest rate again displays caution. This is because the open-economy variant's transmission mechanism operates through both the domestic real interest rate and the real exchange rate, and the optimal monetary policy leads to the real exchange rate having a stabilising effect on inflation.

- But it is possible that, under parameter uncertainty, caution in the desired real interest rate can be accompanied by a larger total nominal interest rate response than under purely additive uncertainty. This is because inflation expectations rise in response to an inflationary disturbance, and could drive a sufficiently large wedge between the cumulative real interest rate response and the cumulative nominal interest rate response.

The rest of the paper is organised as follows. Section 2 describes the open-economy model and how it nests the closed-economy variant, as well as the policy-maker's preferences. Section 3 considers the closed-economy variant under additive uncertainty, then introduces parameter uncertainty and examines the implications for conservatism, caution and gradualism. Section 4 reverts to a particular case of the open-economy model (for which an analytical solution is obtainable), to provide a comparison of the effects of parameter uncertainty when there is an exchange rate channel of monetary policy, as well as the direct real interest rate channel. Section 5 discusses 'optimal' policy prescriptions where an aggressive response is called for. Section 6 concludes.

2. The model

The issue of parameter uncertainty is considered in an amalgam of the Svensson (1997a,b) model of inflation targeting in a closed economy and the Dornbusch (1976) model of a small open economy.

All variables (except interest rates) are in logs, so inflation is the difference between the current and previous period's price level.[2] Inflation p_t is described by a backwards-looking Phillips curve:

$$p_{t+1} = ap_t + dy_t + e_{t+1} \qquad \textbf{(1)}$$

Output, y_t, is decreasing in the real interest rate, r_t, and increasing in the real exchange rate, q_t, as in a simple IS specification:[3]

$$y_t = -qr_t + gq_t \qquad \textbf{(2)}$$

Eliminating output gives:

$$p_{t+1} = ap_t - br_t + cq_t + e_{t+1} \qquad \textbf{(3)}$$

where $b = dq$ and $c = dg$. This gives the basic reduced-form process for inflation that we shall use.

To rule out arbitrage opportunities, the exchange rate is driven by UIP. We focus on the real exchange rate, which is driven by real interest differentials:[4][5]

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(2) The price level, real output etc are to be taken as end-period values, and inflation rates and interest rates are to be taken as ruling over a period. So, for example, p_{t-1} is the price level at the end of period *t*-1, and p_t the price level at the end of period *t*. The inflation rate p_t rules over period *t*. The nominal interest rate i_t is set at the end of period *t* to rule over the next period.

(3) Note that equations **(2)** and **(4)** do not include dynamics or an error term. This would considerably complicate the model presented here at the expense of clarity. Martin and Salmon (1999) present an empirical model which places less restrictions on the dynamics and number of shocks.

(4) $E_t x_{t+1}$ refers to the expectation of variable *x* at *t*+1 conditional on time *t* information.

(5) It is assumed that real UIP is an arbitrage condition, and therefore **(4)** is an identity with no uncertainty about the coefficients in this equation.

$$\mathrm{E}_t q_{t+1} - q_t = r_t - r_t^{\,*} \qquad \textbf{(4)}$$

The real interest rate set at the end of period t to rule over $t+1$ is equal to the nominal interest rate in the same period minus expected inflation over period $t+1$, where expectations are formed at the end of period t.

$$r_t = i_t - \mathrm{E}_t p_{\,t+1} \qquad \textbf{(5)}$$

To tie down the exchange rate, the foreign real interest rate is normalised to zero.[6]

This model is very simple. One limitation is that it does not account for direct exchange rate effects on the price level (ie the nominal exchange rate does not enter the domestic price index directly) so, for example, there is relatively little import penetration. The results on parameter uncertainty are unlikely to be seriously affected by this. Another limitation is that the model does not have a forward-looking component to the Phillips curve. To extend the model to incorporate this would seriously complicate the analysis and so we do not consider it here.

We shall assume that additive and parameter uncertainty is characterised by:

$$\begin{pmatrix} a \\ b \\ c \\ e \end{pmatrix} \sim iid.\begin{pmatrix} \bar{a} \\ \bar{b} \\ \bar{c} \\ 0 \end{pmatrix},\begin{pmatrix} s_a^2 & r_{ab} & r_{ac} & r_{ae} \\ & s_b^2 & r_{bc} & r_{be} \\ & & s_c^2 & r_{ce} \\ & & & s_e^2 \end{pmatrix} \qquad \textbf{(6)}$$

The inflation target, $\bar{p}_t$, is normalised to zero. The policy-maker's objective is to set the domestic nominal interest rate to minimise the present discounted value of expected deviations of inflation from target subject to the reduced-form processes for inflation and the real exchange rate, **(3)** and **(4)**. The problem can be solved using dynamic programming, but Svensson (1997a,b) notes that the solution often coincides with solving a sequence of

[6] A more satisfactory model would allow for foreign shocks (which might be correlated with domestic shocks), by letting the foreign real interest rate follow a random process. This is not pursued here because it complicates the analysis.

one-period problems where the policy-maker's objective function to be minimised each period is :[7]

$$E_t p_{t+1}^2 \quad \textbf{(7)}$$

Choosing the optimal level of the nominal interest rate requires the policy-maker to know two things: the optimal degree of monetary tightness ie the level of the real interest rate that is required to meet the policy objective **(7)**, and the rational expectation of the next period's inflation as a result of past shocks and policy settings. Once these are computed, the optimal nominal rate is the sum of these two quantities.

This paper exploits the simple structure of this variant of the Svensson model to derive some intuitive conclusions about the effect of parameter uncertainty on optimal policy. The closed-economy variant is an example of the general linear-quadratic model (so called because the objective function is quadratic and the constraint is linear). Chow (1977) generalised Brainard's analysis to this general linear-quadratic framework and showed that the presence of conservatism in the optimal rule would depend on a range of model parameters. Aggressively moving the instrument still injects variance into future inflation, but there is the possibility of offsetting factors, and so one avenue is to test empirically whether this effect dominates in more general models. This approach was adopted by Sack (1998), Shuetrim and Thompson (1998) and Martin and Salmon (1999). It is less clear how parameter uncertainty would affect a purely forward-looking model like Woodford (1999).

3. The closed economy

3.1 The benchmark: additive uncertainty

This section analyses the baseline closed economy (based on Svensson (1997a)) with additive uncertainty, with which we can contrast the effects of parameter uncertainty. The policy-maker knows the structure of the transmission mechanism between real interest rates and inflation, embodied

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[7] See Appendix 1 for details.

in **(3)**. But since the economy is closed, the coefficient on the real exchange rate c equals zero. It is also assumed that the authority can quantify with certainty the size of the multipliers a and b. But inflation is subject to random, serially uncorrelated disturbances:

$$p_{t+1} = ap_t - br_t + e_{t+1} \qquad \textbf{(8)}$$

Recall that, in order to set the optimal nominal interest rate, the authority needs to compute the desired monetary stance (deviation of the real interest rate from equilibrium) and the associated inflation expectation. The desired path for the real interest rate minimises expected squared deviations of inflation from target:

$$\mathrm{E}_t p_{t+1}^2 = \mathrm{E}_t \left(ap_t - br_t + e_{t+1}\right)^2 \qquad \textbf{(9)}$$

Differentiating with respect to r_t and setting the result equal to zero gives:

$$r_t = \frac{a}{b} p_t \qquad \textbf{(10)}$$

To see the dynamic path of real interest rates in this model as a function of additive shocks to the economy, we need to find the equilibrium inflation process, ie the process accounting for the authority's desired path for the real interest rate. This is obtained by substituting **(10)** in the process for inflation **(8)**:

$$p_{t+1} = e_{t+1} \qquad \textbf{(11)}$$

Given this equilibrium process for inflation, the desired path for the real interest rate in terms of the additive shocks is:

$$r_t = \frac{a}{b} e_t \qquad \textbf{(12)}$$

To achieve the desired path for the real interest rate given that inflation expectations are zero, the nominal interest rate is set equal to the desired real rate plus expected inflation:

$$i_t = r_t + \mathrm{E}_t p_{t+1} = \frac{a}{b} e_t \qquad \textbf{(13)}$$

Once a disturbance has been observed, the optimal response is completely to offset it so that, in the absence of any new disturbance, inflation would be back at target. With this policy, inflation is driven only by the new shock each period, and nominal interest rates i_t move solely in response to this period's shock e_t, impacting on inflation next period. The policy-maker moves the nominal interest rate aggressively to return the expectation of next period's inflation to target; otherwise the policy-maker would be ignoring systematic deviations of inflation from target.

3.2 Parameter uncertainty

One possible situation is that the policy-maker knows the structure of the equations describing the economy, but does not know the size of the multipliers and has to estimate them. This will give point estimates and variances of the multipliers a and b, and covariances between random variables in our example (as set out in **(6)**). First assume that the covariances are zero (the case where they are not is discussed later in Section 5).

The policy-maker's goal is again to minimise the squared deviation of inflation from the target expected at time *t*. At this point it is useful to note that the expectation of a random variable equals its squared bias plus the variance:[8]

$$\mathrm{E}_t p_{t+1}^2 = \left\{\mathrm{bias}_t^2 p_{t+1}\right\} + \left[\mathrm{var}_t\, p_{t+1}\right] \qquad \textbf{(14)}$$

Substituting in **(8)**, this expression can be expanded to:

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[8] In this case, the bias of a random variable p_t is defined as $\mathrm{E}_t\left(\mathrm{E}_t(p_{t+1}) - p^*\right)^2$ and measures how far expected inflation is from target. Equation **(14)** follows from the fact that:

$$\mathrm{E}_t\left(p_{t+1} - p^*\right)^2 = \mathrm{E}_t\left(\mathrm{E}_t(p_{t+1}) - p^*\right)^2 + \mathrm{E}_t\left(p_{t+1} - \mathrm{E}_t(p_{t+1})\right)^2$$

where the second term is the variance.

$$\left\{\bar{a}^2 p_t^2 + \bar{b}^2 r_t^2 - 2\bar{a}\bar{b} p_t r_t\right\} + \left[s_a^2 p_t^2 + s_b^2 r_t^2 + s_e^2\right] \quad \textbf{(15)}$$

Note that both the bias and the variance terms depend on the real interest rate, hence both terms will depend on the authority's actions. But there is only one instrument available so there must be some trade-off between bias and variance. This is in contrast to the additive uncertainty case, where the variance term depends only on the exogenous variance of the additive error and so is independent of the policy-maker's actions. In this case there is no trade-off and the policy-maker can eliminate the bias in inflation.

3.3 Conservatism: scaling down the certainty equivalent response

To compute the desired process for real interest rates, the loss function **(15)** is minimised with respect to the real rate. This gives:

$$r_t = \frac{\bar{a}\bar{b}}{\bar{b}^2 + s_b^2} p_t \quad \textbf{(16)}$$

To facilitate comparison with the certainty equivalent case, let *v* denote the coefficient of variation, $\frac{s_b}{\bar{b}}$, and define the parameter, *g*, such that $g = \frac{1}{1+v^2}$. Then **(16)** can be re-written so that:

$$r_t = g\frac{\bar{a}}{\bar{b}} p_t \quad \textbf{(17)}$$

The coefficient *g* indicates the 'gap' identified by Brainard (1967, page 415), and allows the response under parameter uncertainty to be written as a fraction of the certainty equivalent response (since *g* has to lie between zero and one). This fraction is determined purely by the coefficient of variation, *v*, ie the relative size of the uncertainty (measured by the standard deviation) and mean of the policy multiplier. When uncertainty is large relative to the mean then *g* will be small. As uncertainty decreases relative to the mean of the policy multiplier, *g* tends to one and the optimal response approaches that under certainty equivalence. Equation **(17)** shows that the authority desires smaller deviations of the real interest rate from the 'neutral level' than when there is only additive uncertainty. Unlike the additive uncertainty case, where it is costless to move the real interest rate, any deviation of the real interest rate from neutral injects variance into future inflation. So in the absence of covariances, it is never optimal to completely offset a shock in any period. The result is a path for the real interest rate that does not completely offset inflationary shocks as soon as they are observed. The proportion of the shock that is offset each period is determined by the real interest rate that equates the marginal benefit of a further reduction in the bias with the marginal cost of the variance induced in future inflation.[9] This is the standard case of what Blinder (1997) has called 'Brainard conservatism'.

Substituting the interest rate rule back into the equation for inflation and taking expectations gives:

$$\mathrm{E}_t p_{t+1} = \bar{a}(1-g)p_t \qquad \textbf{(18)}$$

The optimal nominal rate is the sum of the real rate **(17)** and expected inflation **(18)**:

$$i_t = \frac{g\bar{a}}{\bar{b}} p_t + \bar{a}(1-g)p_t = \frac{g\bar{a}}{\bar{b}}\left(1+\bar{b}v^2\right)p_t \qquad \textbf{(19)}$$

Equation **(19)** shows that the implications for the nominal interest rate of real interest rate conservatism could be ambiguous. Because the nominal interest rate is the sum of the real interest rate and inflation expectations, there are two opposing effects on the nominal rate. The first is from real rate conservatism. The second, opposing effect comes from the fact that rational inflation expectations rise when the policy-maker follows a conservative real rate policy. The net effect will still be nominal interest rate conservatism unless the elasticity *b* is sufficiently large, and a plausible parameter estimate based on Rudebusch and Svensson (1999) suggests that that this is unlikely to be the case.

[9] The relative cost and benefit of moving the real interest rate is captured by the coefficient of variation *n*.

3.4 Gradualism: smoothing the dynamic response to shocks

Continuing to assume that the covariances are zero, we shall compare the autocorrelation of nominal and real interest rates, and inflation, for the rules derived under parameter uncertainty and the benchmark model. To do this, we shall assume that the elasticities in the economy take on their mean values.[10] Then under rule **(17)** inflation follows a first-order autoregressive process (as can be seen from **(18)**), and both nominal and real interest rates are proportional to inflation. Clearly then nominal and real rates follow an AR(1) process, and in fact these processes can be shown to be:

$$r_{t+1} = \bar{a}(1-g)r_t + \frac{g\bar{a}}{\bar{b}}e_{t+1} \qquad \textbf{(20)}$$

$$i_{t+1} = \bar{a}(1-g)i_t + \frac{g\bar{a}}{\bar{b}}\left(1 + bv^2\right)e_{t+1} \qquad \textbf{(21)}$$

Parameter uncertainty (with zero covariances) leads to gradualism in nominal and real interest rates because only a constant fraction of the shock is offset each period. This result depends on: i) the presence of autocorrelation in either the economy or the shock process; ii) the fact that the cost of moving the real interest rate is strictly positive because of the uncertainty injected into future inflation, so that in any period it is too costly completely to offset the remaining bias in inflation; and iii) the fact that the loss function is convex so penalises for example a 2 percentage point deviation of the real rate from neutral more than twice as heavily as a 1 percentage point deviation. If there is no autocorrelation, the remaining part of the shock has no impact on inflation next period even if it is too costly completely to offset a shock in any given period. If it is costless to move the interest rate then there is no reason not completely to offset the shock straightaway— as in the case of no parameter uncertainty presented in Section 3.1. But costs can arise for reasons other than parameter uncertainty, for example if the quadratic loss function contained an output-smoothing objective or imposed that deviations of interest rates from neutral entered the loss

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[10] This is a reasonable assumption, first because we want to compare the effect of rules derived under additive uncertainty and under parameter uncertainty, and second because we do not want to have to keep track of previous outturns for coefficients.

function directly. Finally the quadratic loss function penalises extreme deviations making small movements more desirable (and because of persistence, it becomes a sequence of small movements). This is not such a restrictive assumption, as taking a second-order Taylor expansion to many loss functions will lead to this property.

As the variance of the policy multiplier goes to zero, the degree of autocorrelation goes to zero and gradualism disappears. Recall that it is possible for the Brainard conservatism principle to hold for the real interest rate but, in extreme cases, not for the nominal rate. But for either nominal or real interest rates, the degree of gradualism (ie the autocorrelation coefficients in **(20)** and **(21)**) are equal. This is because as long as some component of a disturbance remains to be offset, both the nominal and real interest rate need to be away from their neutral level.

Chart 1: Closed economy, additive uncertainty

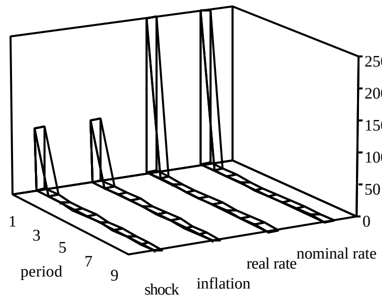
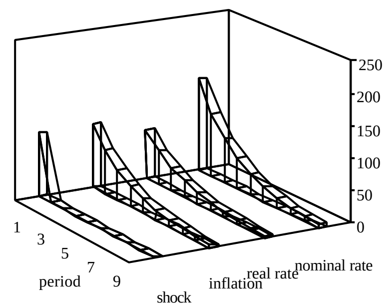


Chart 2: Closed economy, parameter uncerainty



Notes: *Charts 1 and 2 show the impulse responses of the key variables in the model to a one-period positive inflation shock (of magnitude +100 units, measured on the vertical axis). Time is measured from back to front of the charts. From the left, the first profile is the shock. The next profile shows the response of inflation, which returns to target more quickly under additive uncertainty (Chart 1) than under parameter uncertainty (Chart 2). The next profile shows the real interest rate, with a sharper, more aggressive response under additive uncertainty. The final profile is the nominal interest rate which again shows a smoother response.*

Expressions **(20)** and **(21)** allow us to compare the impulse response of policy to a unit additive shock ($e_0 = 1$ and $e_t = 0$ for t>0). We shall consider the case where the interest rate multiplier b is random, and all covariances are zero.[(11)] A graphical example is shown above to give a flavour of the results (for a 100 arbitrary unit shock).

3.5 Caution: the magnitude of the total response

The benchmark cumulative (real and nominal) interest rate response is simple to calculate, as the response lasts for only one period and equals $\frac{a}{b}$. The cumulative real interest rate response to a unit shock under parameter uncertainty can also be calculated because, since real rates follow an AR(1) process, the impulse response follows a geometric progression with parameter $\bar{a}(1-g)$. The cumulative real interest rate response (ie the sum of this geometric progression) equals $\frac{\bar{a}}{b}\frac{1}{1+(1-\bar{a})v^2}$, so that the cumulative real response is lower with parameter uncertainty than under additive uncertainty if $\bar{a}$ <1. If the coefficient on lagged inflation in the Phillips curve ($\bar{a}$) is less than 1 (in expectation if there is model uncertainty), then the part of a shock to inflation that is not offset straightaway decays naturally with time. So by waiting to offset part of a shock, the cumulative response is reduced. If $\bar{a} = 1$, then any part of a shock to inflation that is not immediately offset will persist, neither decaying nor growing, until it is offset by policy settings in subsequent periods. So the cumulative response will be the same with or without parameter uncertainty. Finally, if $\bar{a} > 1$ (but not so large that the system cannot be made stable by the use of policy), then the part of a shock that is not offset immediately will be magnified and therefore the cumulative response will have to be greater.

[(11)] This may seem at odds with the statement that b is held at its mean value, so it is important to spell out exactly what is happening here. The experiment is to consider the effect of a unit additive shock to the economy, holding the coefficients a and b at their expected values, but under a policy rule designed assuming that the coefficients are random. Furthermore, given that the covariances are zero, the policy rule is unaffected by the variance of the coefficient a since this variance is just deadweight loss to the policy-maker.

The degree of persistence in inflation depends on various factors in the economy. For example, overlapping nominal contracts mean that shocks to inflation have an effect for some considerable time. In the limit, shocks to inflation might be permanent. If the process for inflation means that additive disturbances affect the level of inflation permanently, then the cumulative real interest rate response to a single shock will be the same whether it is done in one period (as it would in the benchmark model) or over many periods (as with parameter uncertainty). Once a shock is in the system it has a permanent effect on the level of inflation, and the only way to get it out of the system is by using monetary policy. Carrying out the tightening over many periods when $\bar{a} = 1$ is a case of gradualism *without* caution. But if shocks to inflation die out gradually over time then, by smoothing the desired path for the real interest rate in the face of parameter uncertainty, the cumulative real rate response will be less than in the benchmark model where the shock is offset before it begins to die out. In this case, parameter uncertainty results in caution as well as gradualism in the real interest rate. In this model, whether or not caution is optimal depends on the persistence of the underlying process for inflation.[12]

The cumulative nominal rate response equals:

$$\frac{\bar{a}}{\bar{b}}\frac{1+\bar{b}v^2}{1+(1-\bar{a})v^2} \qquad \textbf{(22)}$$

This will be smaller than the cumulative nominal interest rate response under purely additive uncertainty if:

$$\bar{a} < 1 - \bar{b} \qquad \textbf{(23)}$$

This is a tighter condition than that required for real interest rate caution, which only requires $\bar{a} < 1$. Because not all of the shock is offset in the first period, inflation will deviate from target in a systematic manner, hence expected inflation will rise in the second period and thereafter. The

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[12] It probably also relies on the fact that there is only one channel of monetary policy in the closed economy model (via the real interest rate). The introduction of other, indirect channels (eg an exchange rate channel) could affect this result, as will be seen in the open-economy model of Section 4.

requirement for caution in the nominal interest rate is that this cumulative, systematic, deviation of inflation from target is not so large that it offsets the caution effect in real rates. The systematic deviation of inflation from target will be smaller, the smaller the coefficient ($\bar{a}$) on lagged inflation.

It is difficult to form a view about the relative sizes of $\bar{a}$ and $\bar{b}$, because from the data we observe inflation conditional on actual monetary policy. It may well be that the model is too stylised to attempt to parameterise the monetary transmission mechanism in terms of two parameters. The empirical study by Martin and Salmon (1999) found no evidence for nominal interest rate caution in the United Kingdom, but this does not rule out the possibility of real interest rate caution.(13)

4. Open-economy model

4.1 Introducing another channel to the transmission mechanism

Opening the economy up to trade and capital flows with a much larger foreign economy means that there are now two routes for the transmission mechanism of monetary policy: directly through the domestic real interest rate effect on output, and via the real exchange rate effect on net trade hence output. When real activity depends on the real exchange rate as well as the real interest rate, the policy-maker moves the real interest rate less in an open economy than in a closed economy when faced with any given shock to the domestic economy. This is because a disturbance which leads to an increase in the domestic real interest rate leads to an expected depreciation of the real exchange rate following the uncovered interest parity condition (UIP). This induces an instant jump appreciation which reduces the external demand for domestic goods. The real appreciation is not a substitute for moving the real interest rate from neutral, as after all it is the real interest differential that drives the real exchange rate movement. But because a non-neutral real interest rate affects output and hence inflation both directly and

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(13) The Martin and Salmon (1999) study was unable to recover *ex ante* real interest rates to examine real interest rate caution.

indirectly through the real exchange rate, the deviation of the real interest rate from neutral can be smaller.

In the open economy, holding other things equal, when disturbances to inflation have permanent effects ($\bar{a} = 1$), parameter uncertainty will lead to cautious policy whereas caution in a closed economy arose only with lower inflation persistence. Recall that in the closed-economy case, if shocks to inflation did not die away naturally then even if the response to shocks was smoothed, the same cumulative response would be required. With an exchange rate channel and an inflation target, the exchange rate acts to help stabilise inflation as agents anticipate monetary policy. Any component of a shock that is not offset immediately by the interest rate response, even if this component does not decay naturally, will be offset by a stronger real exchange rate effect on real output and hence inflation. Therefore, under parameter uncertainty, the policy-maker's decision to smooth the interest rate response over several periods allows him to take advantage of the real exchange rate effect and so rely less on the direct effect of real interest rates. An alternative way to look at it is that rather than assuming inflation dies away naturally over time, the exchange rate provides a plausible story as to why this might be the case.

This section derives the optimal rule for a policy-maker who seeks to minimise the present discounted value of inflation, using the nominal interest rate, in a variant of the small, open economy model due to Dornbusch (1976).(14)

We saw earlier that the open economy can be described in reduced form by:

$$p_{t+1} = ap_t - br_t + cq_t + e_{t+1} \quad \textbf{(24)}$$

$$\mathrm{E}_t q_{t+1} - q_t = r_t - r^* \quad \textbf{(25)}$$

where the foreign interest rate is normalised to zero. We again wish to calculate the path for the real interest rate, r_t , that minimises the expected squared deviation of inflation from target, and we also want to derive the associated real exchange rate and inflation expectation and hence the

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(14) Svensson (1998) considers an open-economy inflation-targeting model.

optimal level of the nominal interest rate. Even for this simple model, the closed-form solutions for the real interest rate and real exchange rate are unobtainable. However, an approximation to the solution can be obtained for the case $\bar{a} = 1$, and that is a case of particular interest when discussing caution in open and closed economies. In both the certainty equivalent case and under parameter uncertainty (with zero covariances), the real exchange rate follows the process **(26)**:[15]

$$q_t = -\bar{b}^{-1} p_t \qquad \textbf{(26)}$$

4.2 Optimal policy under additive uncertainty

In this case, the process for the real interest rate can be shown to be:

$$r_t = \frac{1}{\bar{b}}\left(1 - \frac{\bar{c}}{\bar{b}}\right) p_t \qquad \textbf{(27)}$$

Even though the response to a unit shock to domestic inflation is smaller than in the closed economy when $\bar{a} = 1$, this path for the real interest rate is sufficient to result in inflation expectations being at target:

$$\mathrm{E}_t p_{t+1} = 0 \qquad \textbf{(28)}$$

Again, with a pure inflation target and no parameter uncertainty, it is optimal to offset any disturbances completely. This can be done with a smaller deviation of the real interest rate from the neutral level because of the tightening effect via the real appreciation. Since inflation expectations are at target, the real and nominal interest rates again coincide so the optimal deviation of the nominal interest rate from neutral equals the desired deviation of the real interest rate from neutral.

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(15) Appendix 2 solves the open-economy model under parameter uncertainty for the case when $\bar{a} = 1$. To recover the certainty equivalent case, set any variance terms to zero.

Because domestic shocks are offset completely as soon as they are observed, real and nominal interest rates and inflation are not autocorrelated in the face of domestic shocks as in the closed economy.

4.3 Open economy with parameter uncertainty

In this case, it can be shown that the desired path for the real interest rate is:

$$r_t = \frac{g}{\bar{b}}\left(1-\frac{\bar{c}}{\bar{b}}\right)p_t \qquad \textbf{(29)}$$

This expression **(29)** disentangles several components affecting desired real rates: the coefficient g was defined earlier, and approaches zero as uncertainty about b increases. This scales down the real interest rate response. The term in brackets premultiplying domestic inflation is another scaling effect due to the real exchange rate channel $\left(1-\frac{\bar{c}}{\bar{b}}\right)$.

Substituting **(29)** into the inflation process gives:

$$\mathrm{E}_t p_{t+1} = (1-g)\left(1-\frac{\bar{c}}{\bar{b}}\right)p_t \qquad \textbf{(30)}$$

Comparing this expression with **(18)** for the closed economy again shows that parameter uncertainty scales up expected future inflation. But the real exchange rate channel reduces expected inflation. From the Fisher equation, the optimal nominal interest rate is:

$$i_t = \frac{g}{\bar{b}}\left(1+\bar{b}v^2\right)\left(1-\frac{\bar{c}}{\bar{b}}\right)p_t \qquad \textbf{(31)}$$

This is again very similar to the closed-economy case, with the exchange rate channel scaling down the response.

4.4 Conservatism

In response to a domestic shock, again we get Brainard conservatism in the real interest rate, and in the nominal interest rate for sensible parameter values. For the real interest rate, the scale of the reduction is again measured by the coefficient g, and for the nominal interest rate it is again $g(1+\bar{b}v^2)$. But at the same time the real exchange rate effect $\left(1-\frac{\bar{c}}{\bar{b}}\right)$ also scales down the response to shocks relative to the closed-economy case.

4.5 Gradualism

Again inflation follows an AR(1) process and nominal and real interest rates are proportional to inflation, so they themselves follow an AR(1) process.[16] The expressions for real and nominal rates expressed as autoregressive processes of domestic shocks are therefore:

$$r_{t+1} = (1-g)\left(1-\frac{\bar{c}}{\bar{b}}\right)r_t + \frac{g}{\bar{b}}\left(1-\frac{\bar{c}}{\bar{b}}\right)e_{t+1} \quad \textbf{(32)}$$

$$i_{t+1} = (1-g)\left(1-\frac{\bar{c}}{\bar{b}}\right)i_t + \frac{g}{\bar{b}}\left(1+bv^2\right)\left(1-\frac{\bar{c}}{\bar{b}}\right)e_{t+1} \quad \textbf{(33)}$$

Clearly the real interest rate and nominal interest rate autocorrelations are equal in the open-economy case, but smaller than their counterparts in the closed-economy model - by a factor of $\left(1-\frac{\bar{c}}{\bar{b}}\right)$ for $\bar{a}=1$ (equations **(20)** and **(21)**). The policy-maker is less gradualist relative to the closed-economy case because of the continual stabilising effect of the real exchange rate while domestic policy is responding to inflationary disturbances, which allows policy to return to neutral sooner than in the absence of an exchange rate channel.

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(16) Note we are carrying out the experiment assuming that the model parameters take on their mean values, as described in footnote 11.

Chart 3: Open economy, additive uncertainty

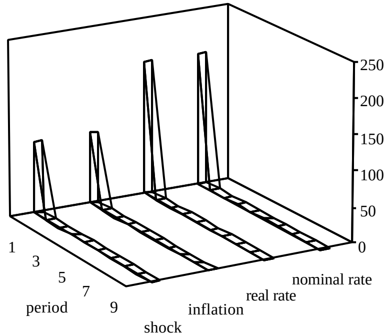
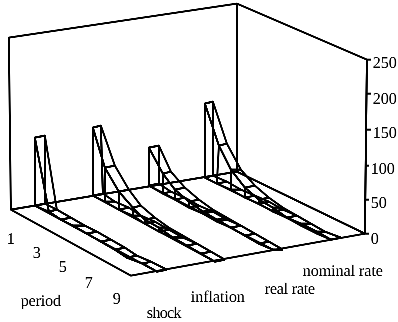


Chart 4: Open economy, parameter uncertainty



Notes: *Charts 3 and 4 contrast additive and parameter uncertainty in the open-economy model. The effect of parameter uncertainty is again to smooth the response of the real interest rate to inflation shocks, and since this allows inflation expectations to deviate from target for a while, a wedge is driven between the real and nominal interest rate response. The dampening effect of the real exchange rate on the desired real interest rate scales down the optimal response of the real and nominal interest rate relative to the closed-economy case (Charts 1 and 2).*

Charts 3 and 4 above show the open-economy responses of inflation, and real and nominal interest rates, under additive uncertainty and parameter uncertainty.

4.5 Caution

From expressions **(32)** and **(33)**, we saw that the autocorrelation in the real and nominal interest rate is reduced in the open economy. This is because any inflationary disturbance not offset directly by the action of the real interest rate is eroded by the induced movements in the real exchange rate (this is seen in the scaling factor $\left(1-\frac{\bar{c}}{\bar{b}}\right)$). Clearly the effect of this is to reduce the necessary cumulative response for both nominal and real rates. In the closed economy and when $\bar{a}=1$ (ie when inflationary disturbances have permanent effects on inflation), we found that the cumulative real response was the same under parameter uncertainty and additive uncertainty, and caution was only evident for $\bar{a} < 1$. But because of the dampening effect of the real exchange rate each period in the open economy there will be real rate caution even when $\bar{a} = 1$, because over time the real exchange rate will have helped to offset the shock. To see this, note that in response to a unit inflation shock, the path for the real interest rate is a geometric progression in $(1-g)\left(1-\frac{\bar{c}}{\bar{b}}\right)$. The cumulative real interest rate response under parameter uncertainty is:

$$\frac{1}{\bar{b}}\left(1-\frac{\bar{c}}{\bar{b}}\right)\left(\frac{1}{1+\frac{v^2\bar{c}}{\bar{b}}}\right) \tag{34}$$

compared with the following under purely additive uncertainty:

$$\frac{1}{\bar{b}}\left(1-\frac{\bar{c}}{\bar{b}}\right) \tag{35}$$

So in the open economy with $\bar{a} = 1$, parameter uncertainty induces caution since **(34)** is always strictly less than **(35)**.

Because of this real interest rate caution and the dampening effect of the real exchange rate on inflation expectations, the model will exhibit nominal interest rate caution for a wider range of parameter values than in a closed economy. (This can be seen by summing the cumulative nominal response.)

4.6 Over and undershooting

In the Dornbusch (1976) model, the sluggish adjustment of prices following a change in the money stock led the nominal exchange to overshoot its new long-run equilibrium. The model presented here is similar but with an interest rate instrument and inflation target. In this model the degree of overshooting depends on parameter uncertainty.[17]

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[17] To simplify things in this section, assume that the equilibrium domestic and foreign price levels and the equilibrium nominal exchange rate are constants, and the domestic and foreign inflation targets equal zero.

To understand the nominal exchange rate we need to understand the behaviour of the price level and of the real exchange rate. The path for the price level is determined by the size of an inflationary shock and the rate at which it is offset (ie the policy rule). Under additive uncertainty, a unit shock raises inflation from zero to one in the first period, and inflation is returned to zero in the second period. So the price level adjusts from a starting level of zero before the first period to one at the end of the first period. But under parameter uncertainty, the inflationary shock is offset gradually, so the price level rises by one in the first period, then by slightly less than one in the second period, and so on. The adjustment of the price level is therefore more drawn out under parameter uncertainty.

Real interest rate differentials drive the real exchange rate. In the certainty equivalent case, given a one-off disturbance to inflation, the policy-maker influences real rates for one period only (tightening policy) before returning policy to neutral. Because this leads to an expected depreciation (via real UIP), the real exchange rate jump-appreciates immediately, then depreciates to equilibrium as the real interest rate is returned to neutral. Under parameter uncertainty, the real interest differential persists for some time as the shock is offset gradually so the real exchange rate jump-appreciates on the news and depreciates back to its equilibrium rate over several periods.

For a constant foreign price level, the nominal exchange rate is the sum of the domestic price level and the real exchange rate (all in logs). In the certainty equivalent case, for a one-off disturbance to inflation the price level adjusts upwards immediately, and the real exchange rate immediately falls (appreciates). So the nominal exchange rate might jump-appreciate or depreciate depending on which effect is the larger. But in the long run the nominal exchange rate must depreciate to its new long-run level because of nominal UIP. The possibility arises that the nominal rate might jump in the 'wrong' direction (see Charts 5 and 6 below). Parameter uncertainty will certainly smooth movements in the exchange rate, because of gradualism in the real interest rate (avoiding something akin to what Goodfriend (1991) has described as 'whipsawing the financial markets'), but a similar 'wrong-way' jump could occur. In all cases, the exchange rate does not jump straight to its new equilibrium level, which is in the spirit of Dornbusch's model.

Chart 5: Open economy, additive uncertainty

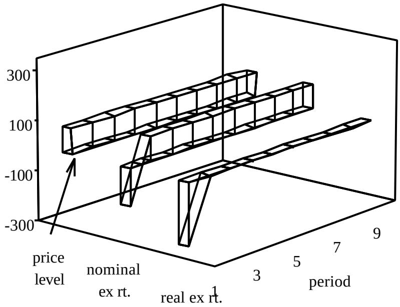
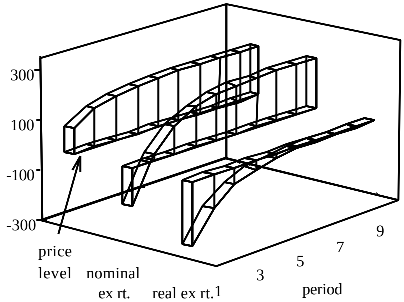


Chart 6. Open economy, parameter uncertainty



Notes: *In Charts 5 and 6, we again measure the magnitude of the response on the vertical axis (again the response is to a 100 unit shock), but to focus on the detail of the initial exchange rate response, time is now measured from left to right on the horizontal axis. In both charts, the profile on the left is the price level, which adjusts immediately to its new level under additive uncertainty. The middle profile on both charts is the nominal exchange rate and the rightmost profile is the real exchange rate.*

5. When a more aggressive policy might be optimal

This section focuses on the question: when does theory suggest that the policy-maker should follow an aggressive interest rate policy?

5.1 Covariances: when Brainard conservatism breaks down

So far the covariances between parameters and the additive error have been assumed to be zero. But as Brainard noted, the size and sign of covariances has implications for optimal policy. This section asks if we can draw any conclusions about such covariances, back in the closed-economy variant of the model.

Following Chow (1977), the optimal policy when covariances are non-zero can be shown to be of the form (see Appendix 1):

$$r_t = \frac{\bar{a}\bar{b} + r_{ab}}{\bar{b}^2 + s_b^2} p_t + m_t \qquad \textbf{(36)}$$

The term m_t is a function of, among other things, the covariance between b and the additive error. A non-zero covariance between the disturbance and the policy multiplier (r_{be}) could lead to real rates being kept away from neutral in the steady state via the term m_t , although as is explained in Appendix 1, the sign of this coefficient will be difficult to determine. A large positive covariance between the policy multiplier and the coefficient on lagged inflation (r_{ab}) could overturn the conservatism result leading to a larger response to a given deviation of inflation from target than in the certainty equivalent case.[18]

Such covariances will also influence the behaviour of expected inflation, as can be seen by substituting the policy rule **(36)** in the process for expected inflation:

$$\mathrm{E}_t p_{t+1} = \frac{\bar{a} s_b^2 - \bar{b} r_{ab}}{\bar{b}^2 + s_b^2} p_t - b m_t \qquad \textbf{(37)}$$

We have shown that the effects of pure variance terms can have opposing effects on the optimal nominal interest rate, on the one hand because of Brainard conservatism in the real interest rate, and on the other because of a systematic increase in inflation expectations. With both variance and covariance effects operating on the desired real interest rate and corresponding inflation expectations, the sum effect on the optimal nominal rate is complicated, and may lead to a more aggressive nominal interest rate response. It is therefore important to understand when non-zero covariances may arise, and what economic interpretation the resulting optimal policies can be given.

Consider the interpretation offered earlier that elasticities are random because they are econometric estimators, with associated distributions. In

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[18] Brainard noted this point in his original 1967 article.

the simplest case of ordinary least squares (OLS) regression, the model can be written as:

$$Y = Xb + e \quad \textbf{(38)}$$

The standard OLS assumptions are that the mean of the residual vector is zero (so $E(e) = 0$), and the residuals are uncorrelated with constant variance, hence the variance-covariance matrix of disturbances is diagonal (so $E(ee') = Is^2$). The matrix of regressors X is also assumed to be of full rank. Then the vector of OLS parameter estimates $\hat{b}$ has variance-covariance matrix $(X'X)^{-1}s^2$. This has the variance of parameter estimates on the diagonal and covariances of parameter estimates in the off-diagonal entries. There is nothing in the standard regression assumptions mentioned above to guarantee that this covariance matrix has zero off-diagonal entries: this would require unrealistic restrictions on the matrix of regressors X. Furthermore, Turnovsky (1977, page 342) shows that even if such an equation, estimated in levels, did have a diagonal variance-covariance matrix this need not be the case for the same equation rewritten in deviations from equilibrium.

So econometric interpretations do not rule out the possibility of non-zero covariances. And the algebra of the previous section recommends that the policy-maker exploits such covariances. The reason for this exploitation is examined below for the case when only $r_{ab} \neq 0$, as the intuition is far simpler than the case of $m_t \neq 0$.

Consider again the one-period closed-economy problem that we have been discussing so far, where the policy-maker seeks to minimise:

$$\mathrm{E}_t p_{t+1}^2 \quad \textbf{(39)}$$

subject to:

$$p_{t+1} = ap_t - br_t + e_{t+1} \quad \textbf{(40)}$$

Recall the formula relating expectation to variance, $E_t\pi_{t+1}^2 = E_t^2\pi_{t+1} + \text{var}_t\,\pi_{t+1}$, so the per-period loss equals the sum of the square of the bias plus the variance of inflation. With parameter uncertainty, the optimal rule will usually reduce bias only at the expense of increased variance.

The per period loss can be expressed as the sum of **(41)** and **(42)**:

$$\text{bias}_t^2 = \bar{\alpha}^2\pi_t^2 + \bar{\beta}^2 r_t^2 - 2\bar{\alpha}\bar{\beta}\pi_t r_t \tag{41}$$

$$\text{var}_t\,\pi_{t+1}^2 = \sigma_\alpha^2\pi_t^2 + \sigma_\beta^2 r_t^2 + \sigma_\varepsilon^2 - 2\rho_{\alpha\beta}\pi_t r_t \tag{42}$$

When variances and covariances are zero, **(42)** does not depend on the real interest rate so does not alter the policy-maker's incentives and we are back in a certainty equivalent world. The variation in the bias term as the real rate changes is:

$$\frac{\partial \text{bias}_t^2}{\partial r_t} = -2\bar{\beta}\left(\bar{\alpha}\pi_t - \bar{\beta}r_t\right) \tag{43}$$

Therefore if inflation is above target by some amount, there is a marginal benefit to increasing interest rates away from their neutral level (so square-bias is reduced). In a model where the coefficients α and β are non-random, this marginal benefit should be driven to zero and we end up with the path for real interest rates that was derived earlier $r_t = \frac{\bar{\alpha}}{\bar{\beta}}\pi_t$.

The marginal change in variance with respect to the real interest rate is:

$$\frac{\partial\, \text{var}_t\,\pi_{t+1}}{\partial r_t} = 2\left(\sigma_\beta^2 r_t - \rho_{\alpha\beta}\pi_t\right) \tag{44}$$

If the covariances are zero but the variances are not, then any deviation of the real interest rate from neutral increases the variance of future inflation. And the further the real interest rate is from neutral, the greater is the impact of a marginal increase in the deviation of the real interest rate from neutral.

Because the coefficient on the real interest rate is random, the further the real rate is from neutral, the higher the variance of future inflation.

But when the covariance r_{ab} is sufficiently large and positive, Brainard conservatism breaks down and the interest rate response to inflation deviations is larger than without parameter uncertainty. This occurs because the covariances between coefficients mean that deviations of the real interest rate from neutral result in a reduced variance of future inflation as well as reducing the bias in future inflation.

One possible interpretation of this case (noted by Brainard (1967), page 419) is that the parameter uncertainty is costly to the policy-maker, but if the coefficients are correlated then various uncertainties can be played off against each other to reduce this cost. This is analogous to the role played by variances and covariances when managing risk ie hedging. For example, suppose that the policy multiplier and the inflation persistence parameter estimates have been found to be positively correlated with a sufficiently large covariance for it to be optimal for the real interest rate to be more, rather than less, responsive to shocks. Because of the positive correlation between the multipliers, then in the outcome where inflation persists in the system (ie the true *a* turns out to lie above the point estimate) that impact of the real interest rate on inflation will be strongest (true *b* also lies above the point estimate), ie when it is needed most. Conversely, this covariance means that in the outcome where inflation does not persist, a given level of the real interest rate will have less effect on inflation. It is as if the policy-maker is insured by running an aggressive policy, because the correlation of the random coefficients effectively ensures inflation is hit hard precisely when it matters most.

Whether this is likely to occur is an empirical issue (although it would seem to be an optimistic way of controlling the economy). Applications that calculate optimal rules assuming parameter uncertainty can be measured by OLS standard errors can provide further clues: the study by Sack (1998) for the United States found no evidence of a more activist optimal rule under parameter uncertainty. A similar study at the Bank (Martin and Salmon, 1999) also found little evidence that, when parameter uncertainty was accounted for, an activist policy was optimal. In contrast, Shuetrim and Thomson's 1998 study of the Australian economy found a more activist rule when parameter uncertainty was accounted for.

6. Conclusion

The consequences of parameter uncertainty are often expressed in terms of reduced (or increased) responses of the policy variable to deviations of the target variable from equilibrium, embodied in Blinder's Brainard conservatism principle. Central banks also need to consider the dynamic sequence of the level of interest rates. In the model presented here, parameter uncertainty can result in both caution (less cumulative real and nominal interest rate response) and gradualism (autocorrelation in real and nominal interest rates) for a range of parameter values. Conditions where the Brainard conservatism principle may break down were noted by Brainard himself, although the empirical evidence seems to be against these conditions prevailing.

Gradualism arises because it is no longer costless to influence the real interest rate with the nominal rate. Parameter uncertainty increases the variance of the distribution of future inflation when monetary policy is used to return expected inflation to target. The optimal response each period is to equate the marginal cost of the extra variance injected by policy with the marginal cost of only partially returning inflation to target that period. As a consequence a new shock is not completely offset in the first period after it is observed and, because of persistence, inflation will still be away from target next period (even in the absence of other shocks). So the optimal response to shocks is smoothed over more periods than in the model under purely additive uncertainty, leading to the gradualism result in both real and nominal interest rates.

In the closed economy, caution arises if the effect of a shock dies away naturally with time, so the gradualism required in the face of parameter uncertainty leads to a smaller cumulative real interest rate response. The cumulative real interest rate response will be the same with or without multiplier uncertainty if shocks have a permanent effect on the level of inflation rather than dying away naturally. However, because the real rate remains closer to neutral in the face of shocks, inflation expectations must rise. A sufficiently large increase in inflation expectations could mean that the cumulative nominal interest rate response required to achieve the desired cumulative real rate response is larger than that in the same model with just additive uncertainty.

Finally, in the open-economy variant of the model considered here, the real exchange rate plays a part in the monetary transmission mechanism, and can appreciate when policy is tight and depreciate when policy is loose. The real interest rate needs to deviate less from neutral, and may return to neutral more quickly, but the Brainard conservatism principle is not affected. Because of the real exchange rate channel, caution is likely to arise for a larger range of parameter values than in the closed economy. The dampening effect of the real exchange rate channel allows the real interest rate to return to neutral more quickly and so leads to a less gradualist response than in the closed economy.

Appendix 1: Comparison of the dynamic programming approach to the one-period solution

1. Chow's general solution of the linear-quadratic problem

This section follows chapter 10 of Chow's 'Analysis and control of dynamic systems' (1977), giving a brief summary of some of the results presented there and their applicability to the problem set out in this paper.

Let y_t denote the vector of target variables (inflation, output etc) and x_t the vector of control variables. A_t and C_t are the random matrices of elasticities linking these variables and the vector b_t is a vector of additive errors. The equation describing the economy is:

$$y_t = A_t y_{t-1} + C_t x_t + b_t \qquad \textbf{(A1.1)}$$

The loss function is:[19]

$$W = \mathrm{E}_0 \sum_{t=0}^{T} b^t (y_t - a_t)' K_t (y_t - a_t) \qquad \textbf{(A1.2)}$$

where E_0 is the conditional expectations operator given information at time zero, a_t is a vector of targets, K_t is a diagonal matrix of preference weights and b is a discount factor less than one and greater than zero.

Chow solves the finite-period problem by backwards induction starting at period *T*. This generates the optimal feedback control equation:

$$\hat{x}_t = M_t y_{t-1} + m_t \qquad \textbf{(A1.3)}$$

[19] Chow's loss function is undiscounted, but here we allow for discounting as we want to extend the analysis from a finite horizon model to an infinite horizon model.

where the parameters M_t and m_t are defined as:

$$M_t = -\left(\mathrm{E}_{t-1}C_t'H_tC_t\right)^{-1}\mathrm{E}_{t-1}C_t'H_tA_t \qquad \textbf{(A1.4)}$$

$$m_t = -\left(\mathrm{E}_{t-1}C_t'H_tC_t\right)^{-1}\left[\mathrm{E}_{t-1}\left(C_t{}'H_tb_t\right) - \mathrm{E}_{t-1}\left(C_t{}'h_t\right)\right] \qquad \textbf{(A1.5)}$$

and the parameters H_t and h_t are defined recursively as:

$$H_{t-1} = K_{t-1} + b\mathrm{E}_{t-1}A_t'H_tA_t - b\mathrm{E}_t\left(A_t'H_tC_t\right)\left(\mathrm{E}_{t-1}C_t'H_tC_t\right)\mathrm{E}_{t-1}\left(C_t'H_tA_t\right)$$
(A1.6)

$$h_{t-1} = K_{t-1}a_{t-1} + b\mathrm{E}_{t-1}\left(A_t + C_tM_t\right)'h_t - b\mathrm{E}_{t-1}\left(A_t'H_tb_t\right) - bM_t'\left(\mathrm{E}_{t-1}C_t'H_tb_t\right)$$
(A1.7)

With starting values $H_T = K_T$ and $h_T = K_Ta_T$.

The ‘solution’ in **(A1.3) - (A1.7)** is difficult to interpret. As Chow notes, the optimal feedback rule **(A1.3)** is a function of conditional expectations and as such depends on all observations of variables up to that point. The conditional expectations enter into the recursive equations **(A1.6)** and **(A1.7)**, equations that in general will have no analytical solution. The control rule is also not time-invariant.

2. Mapping the simple ‘Brainard-Svensson’ model into Chow’s general solution

For the simple model set out in the main text, Chow’s solution can be greatly simplified. The vector, y_t, simplifies to the scalar, p_t, the vector, x_t, simplifies to the scalar, r_{t-1}, and the vector b_t to the error term, e_t. The random matrices A_t and C_t correspond with the random scalars a and b, and these are assumed to be independently and identically distributed with means $\bar{a}$ and $\bar{b}$ and variances s_a^2 and s_b^2 respectively, and covariances r_{ab}. A consequence of this is that the conditional expectation at time t-1 is the same as the unconditional expectation (the means noted above), providing that population variances are known (as assumed in the text). The matrix of preference weights, K_t, becomes the scalar 1, and the vector of targets, a_t, becomes the scalar zero.

2.1 The case where the additive error does not co-vary with a or b

Given the above assumptions, $h_T = 0$ and therefore from the recursive relation **(A1.7)** $h_t = 0$ for all *t*. Therefore $m_t = 0$ for all *t*. This dispenses with the time-varying shifter in the optimal rule **(A1.3)**. It remains to simplify the coefficient M_t.

Note that the matrix H_T simplifies to a scalar in the model presented in the main text, since it is equal to the preference matrix K_t which we noted equals the scalar 1. Hence H_t is a scalar for all *t*. In fact all the matrices in the definition of M_t are scalars. Since, from **(A1.6)**, the H_t are non-random scalars, they cancel out of the expression for M_t: so basically for the model set out in the main text the optimal rule is independent of H_t. Evaluating the conditional expectations in **(A1.4)** leads us to the time-invariant optimal rule:

$$r_t = \frac{\bar{a}\bar{b} + r_{ab}}{\bar{b}^2 + s_b^2} p_t \qquad \textbf{(A1.8)}$$

This rule is independent of the time horizon *T*, and so the infinite horizon model, where *T* tends to infinity, has the same optimal rule, which coincides with the one derived by the simple Svensson quick fix in the paper.

2.2 The case where the covariance of the additive error and the parameters a *and* b *is non-zero.*

The coefficient *M* is unchanged. But now h_{T-1} does not equal zero and in turn depends on the coefficient H_T. As *T* becomes large the fixed points of **(A1.6)** and **(A1.7)** will in general be non-zero and depend on the variance and covariance of *a* and *b*, as well as the covariance of the additive error and

b. This does not affect the coefficient M_t, but does affect the additive shifter in the optimal rule m_t which will in general be non-zero. An analytical solution to **(A1.7)** is possible, and hence an expression for g_t can be obtained.

For the open economy case where $\overline{a} = 1$ and the world real interest rate is held constant at zero, the dynamic programming solution can again be shown to coincide with the quick-fix solution. This is essentially because the only state variable is inflation so the above arguments relating to the closed economy case still go through.

Finally, when allowing the horizon to go to infinity we need to check that the objective function still converges:

$$\lim_{T\to\infty} E_0 \sum_{t=0}^{T} b^t p_t^2 < +\infty \qquad \textbf{(A1.9)}$$

It can be shown that, for a range of rules including those considered in this paper, and for uncertainty about coefficients that is white noise and serially uncorrelated such that the resulting process for inflation is 'not too unstable', this objective function converges to a finite value. Since the optimal rule is one of these rules, the requirement **(A1.9)** is satisfied.

Appendix 2: Solving the open-economy model for a = 1

The model in reduced form is:

$$p_{t+1} = p_t - br_t + cq_t + e_{t+1} \qquad \textbf{(A2.1)}$$

$$\mathrm{E}_t q_{t+1} - q_t = r_t - r^*_t \qquad \textbf{(A2.2)}$$

with the foreign real interest rate held constant at zero. The optimal paths for the real exchange rate and the real interest rate are assumed to be linear processes for q_t and r_t of the form:[20]

$$r_t = f\, p_t \qquad \textbf{(A2.3)}$$

$$q_t = k\, p_t \qquad \textbf{(A2.4)}$$

Substituting these expressions into **(A2.1)** and **(A2.2)** and taking expectations gives:

$$\mathrm{E}_t p_{t+1} = \left(1 - \bar{b}f + \bar{c}k\right) p_t \qquad \textbf{(A2.5)}$$

$$\mathrm{E}_t p_{t+1} = \frac{(f+k)}{k} p_t \qquad \textbf{(A2.6)}$$

Equating coefficients on inflation gives:

$$\bar{c}k^2 - \bar{b}fk - f = 0 \qquad \textbf{(A2.7)}$$

Using the quadratic formula we obtain:

$$k = \frac{1}{2\bar{c}}\left\{\bar{b}f \pm \sqrt{\bar{b}^2 f^2 + 4\bar{c}f}\right\} = \frac{\bar{b}f}{2\bar{c}}\left\{1 \pm \sqrt{1 + \frac{4\bar{c}}{\bar{b}^2 f}}\right\} \qquad \textbf{(A2.8)}$$

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[20] This assumption is validated later on.

Recalling that for small x, the binomial expansion of $\sqrt{1+x} \cong 1+\frac{1}{2}x$, (assuming that $4\bar{c} << \bar{b}^2 f$)we obtain that:[21]

$$k \cong \bar{c}^{-1}bf + \bar{b}^{-1} \quad \text{or} \quad k \cong -\bar{b}^{-1} \qquad \textbf{(A2.9)}$$

The second root corresponds to the minimum of the solution, so the process for the real exchange rate is:

$$q_t = -\bar{b}^{-1}p_t \qquad \textbf{(A2.10)}$$

To find the optimal real interest rate policy rule, substitute **(A2.10)** into **(A2.5)** and minimise the expected squared deviation of inflation from target with respect to f to obtain:

$$f = \frac{\bar{b} - \bar{c}}{\bar{b}^2 + s_b^2} \qquad \textbf{(A2.11)}$$

Therefore the optimal path of the real interest rate under parameter uncertainty, with $\bar{a} = 1$ and in the open economy is:

$$r_t = \frac{\bar{b} - \bar{c}}{\bar{b}^2 + s_b^2} p_t \qquad \textbf{(A2.12)}$$

These processes for the real exchange rate and the real interest rate enable the policy-maker to determine inflation expectations and hence to set the optimal nominal interest rate (see main text).

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[21] Anticipating the result for f, this assumption will be valid providing that the effect of interest rates on inflation is sufficiently large relative to the effect of the real exchange rate on inflation, and uncertainty about the policy multiplier is not too large.

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