Coalition formation in international monetary policy games

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Abstract

It is well known from the analysis of monetary policy coordination of two countries that coordination often Pareto-dominates the outcome of the non-cooperative game. Hence both countries will have an incentive to form a union when it is certain that the other country will also join.

However, in an n-country model, free-riding incentives restrict the size of a stable coalition to less than n countries. Since the coalition members are bound by the union's discipline, an outsider can successfully export inflation without fearing that the insiders will try to do the same.

The formation of a large currency bloc is not sustainable since it would impose too much discipline on all participants. However, the co-existence of several smaller currency blocs may be a second-best solution to the free-riding problem of monetary policy coordination.

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1 Introduction

In the global monetary system we can observe the development of several currency blocs or, at least, areas where one currency plays a predominant role. The literature on optimal currency areas explains the existence of several (optimal) currency areas with asymmetries in trade structures, shocks or factor mobility. While historical evolution, geographical closeness and links in other policy areas undoubtedly play an important role in determining currency blocs, it is interesting to ask whether there are economic reasons other than asymmetries that could drive international currency arrangements.

A country which has to decide whether or not to join a union should consider the perceived gains and costs from joining. Some authors have linked the economic gains from joining a union to the elimination of inefficiencies arising from non-coordinated policies. This paper draws attention to some of the reasons why a country may want to stay out, despite the existence of inefficiencies.

In the model of international policy coordination used here, after a negative supply shock countries will try to export inflation via an appreciation of the exchange rate. In the non-cooperative Nash equilibrium, because all countries are doing so, none of them succeed but they all contract their money supply too much. This provides the 'classical' argument for benefits from coordination in Hamada's (1976) seminal article: all countries could do better by agreeing not to try to export inflation.

However, the situation changes when there are some countries that join a union while others remain outside. Since the union members are now bound by the coalition's discipline, an outsider can successfully export inflation without fearing that the insiders will try to do the same. These additional 'gains from staying out' are the reason why the largest stable coalition comprises some but not all countries, even in the case of symmetric shocks.

However, outsiders are willing to accept some discipline in a small union but not the larger discipline in a 'grand' coalition. As a result, the formation of several smaller blocs may be the outcome of individually optimal decisions.

This paper evaluates the stability of the union on the basis that a country cannot be forced to join. Hence participation must be incentive-compatible. A stability criterion proposed by D'Aspremont *et al* (1983) for cartels is used. It is found that over a large range of parameter values only three countries want to join a currency union. The next section reviews the literature on monetary policy coalitions in greater depth, concentrating on how the results in this paper relate to existing models of monetary unions. In Section 3 the basic shock stabilisation game is presented. Section 4 uses a two-stage game to analyse the stability of a coalition. Section 5 asks whether the outsiders would prefer to form a competing coalition. Section 6 concludes.

2 Monetary policy coalitions

The decision of a country to join a monetary union or to stay outside should be based on a 'balance' of *all* the costs and benefits involved. Academic research has used several approaches to the assessment of the costs and benefits of monetary union, though a complete survey is beyond the scope of this paper. Instead, this paper focuses on *one* type of cost only: the possibility to free-ride on the coalition's discipline when remaining outside. This complements existing research in a number of ways, and provides important insights into the feasibility of coalition formations and therefore the co-existence of currency zones such as the US dollar area and the euro area.

Gains from joining a coalition arise when participation in a currency union leads to credible commitment to a low-inflation policy ('buying reputation'). Alesina and Grilli (1993) develop a formal model in the setting of a 'multispeed' European Monetary Union (EMU) where countries differ in their degree of 'conservativeness', defined as the emphasis on the objective of price stability relative to that of full employment. However, reducing the EMU discussion to reputational considerations has one problem: if the 'toughest' country is to be a member of the coalition, an incentive other than reputation is needed. This paper entirely neglects reputational considerations but it must be stressed that, for a full assessment, reputational considerations are important, since lack of credibility can make monetary policy ineffective. In this sense the results in Alesina and Grilli complement the results presented here. In their model the constraint on the coalition size arises from restricted 'entry to the club'[1] whereas in the model here the restriction of coalition size arises from a lack of new members who want to join. In both cases, a free-riding incentive exists: in Alesina and Grilli, new members free-ride on the coalition's reputation for a low-inflation policy while in the model here the outsiders free-ride on the coalition's discipline.

The theory of optimum currency areas looks at the gains from joining a union that arise from the reduction of transaction costs. Mundell (1961) argued that fixing the exchange rate across regions was costly in the face of asymmetric disturbances and price rigidities. So studies – often empirical – have evaluated which countries ought to form a union based on 'economic similarities'.[2] This implies that *asymmetries* are the reason for an *asymmetric* outcome, where not all countries join. Consequently, policy-making in a planned union focuses mainly on the reduction of structural differences. An example of this in practice is the 'convergence criterium' for EMU, ie countries have to bring their economic performance into closer alignment before forming a coalition. However, the existence of an *asymmetric* equilibrium (where some countries want to form a coalition and others do not) in a *symmetric* model means that policy should not focus exclusively on the reduction of differences. That is the result shown in this paper. This is an important motivation for using a symmetric model – besides the apparent advantages of analytical tractability – when evaluating the issue of coalition formation.

The theory of optimum currency areas does not share the strategic aspects of the literature on international monetary policy coordination. At the core of the coordination literature lies the insight that shocks trigger responses from national policy-makers that may impose externalities on other countries. Coordination internalises these externalities and the joint elimination of the inefficiencies can benefit

(1) Other restrictions on 'entry to the club' may arise because of market power. In the context of regional trading agreements, Hughes Hallet and Braga (1994) show that insiders may gain more by exploiting market power than by accepting new members in the trading bloc.

(2) Asymmetries as a driving force for coalition formation are most evident in the empirical work on idiosyncratic shocks, for example in Bayoumi and Eichengreen (1992). However, this approach was also adopted in the coordination literature, for example Buiter *et al* (1995), Canzoneri (1982), Canzoneri and Henderson (1991) and Martin (1995). Their distinction between 'insiders' and 'outsiders' stems from asymmetries in the underlying economies.

all. Since benefits from coordination after a common shock could be demonstrated with only two countries, previous theory focused on two-country models, interpreting each country as a bloc. With two players only two outcomes are possible: full coordination or no coordination at all. However, when more than two players are involved game-theoretical structures like coalition formation can evolve.(3) Hence, a third possible outcome is the existence of a coalition implying partial coordination, where countries inside the coalition coordinate their policies and the outsiders play a non-cooperative Nash game. This is the possibility addressed in the current paper, and it does seem to match more closely with reality, where we observe bloc formation rather than the two extreme cases of full coordination or no coordination at all. The next section sets out the model used to derive the results.

3 The underlying economy

Since coalition formation is at the core of this paper a model is used with n countries. The issue of 'who wants to join' is tackled on the basis that no one can force a country to join a co-operative agreement. Hence, the decision (not) to participate in a policy union must be individually optimal for each member (outsider). In order to isolate the results based on this condition, a static framework is used, thereby neglecting issues arising from reputational considerations in infinitely repeated games. These are dealt with in depth elsewhere, for example in Canzoneri and Henderson (1991).

The model for an individual country's economy is based on that in Canzoneri and Henderson (1991). Their model is extended to the n-country case. All variables except for the interest rate represent deviations of actual values from zero-disturbance equilibrium values and are expressed in terms of logarithms. For simplicity, the deviation of the money supply (log) from its zero-disturbance value is referred to as 'money supply'; this convention is also applied to all other variables.

(3) Few authors use more than two countries. In the models by Canzoneri (1982) and (1991), and Martin (1995) a symmetric setup would induce all countries to join the union. Martin, however, sheds light on the main force driving the results here: free-riding incentives may restrict the coalition size; though his analysis differs in detail.

The domestic country's variables are indexed by i; $j = 1 \ldots n, j \neq i$ denote the foreign countries. The model is symmetric with respect to the economies' structure as well as to the type of exogenous shock, where the analysis is restricted to the case of a productivity shock, x, that affects all countries in the same way.

Monetary policy is effective because the monetary authorities have an information advantage arising from the timing of the game. Wage-setters fix nominal wages at the beginning of the period before they can observe the realisation of the shock. Monetary authorities set their policies after they know the shock.

Each country specialises in the production of one good. Output y_i increases in employment l_i (where $1-\alpha$ is the elasticity of output with respect to labour) and decreases with some (world) productivity disturbance x (independently distributed with mean 0):

$$y_i = (1-\alpha)l_i - x \qquad 0 < \alpha < 1 \tag{1}$$

Profit-maximizing firms hire labour up to the point where real wages $(w-p)$ are equal to the marginal product of labour. Labour demand is therefore determined by:

$$w_i - p_i = -\alpha l_i - x \tag{2}$$

Monetary policy is effective because of contractually fixed nominal wages. Home wage-setters set w at the beginning of the period so as to fix employment at the full-employment level ($l_i = 0$) if disturbances are zero and expectations are fulfilled. Wage-setters minimise the expected deviation of actual employment from full employment by setting the nominal wage w_i:

$$w_i = m_i^e \tag{3}$$

with m_i^e the expected money supply deviation and w_i the deviation from the full-employment wage level.(4) Actual labour demand might differ due to unexpected disturbances. It is assumed that the wage-setters guarantee that labour demanded is always supplied.

(4) Equations (**1**), (**2**) and (**4**) give $m = w + l$. Home wage-setters solve the optimisation problem $min_w E[l^2] = min_w E[(m-w)^2]$. This is obviously minimised by setting w equal to m^e. For the time being m_i^e will be set to zero, ie expected money supply takes the value determined when the disturbance is zero.

The market equilibrium for money is realized when the money supply m_i satisfies a simple Cambridge equation:

$$m_i = p_i + y_i \tag{4}$$

where p_i is the price of the home good.

The real exchange rate z defined as the relative price of the foreign good in terms of the domestic good is:

$$z_{ij} = (e_{ij} + p_j - p_i) \tag{5}$$

where e_{ij} is the nominal exchange rate, ie the price of the currency of country j in terms of the domestic currency.

The demand for the good produced in the home country is:

$$y_i = \delta \sum_{\substack{j=1 \\ j \neq i}}^{n} z_{ij} + (1-\beta)\epsilon y_i + \frac{1}{n-1} \sum_{\substack{j=1 \\ j \neq i}}^{n} \beta \epsilon y_j - (1-\beta)\nu r_i - \frac{1}{n-1} \sum_{\substack{j=1 \\ j \neq i}}^{n} \beta \nu r_j \tag{6}$$

Consumers spend a fraction ϵ of their income y_j on consumption. They spend a share β of their expenses on foreign goods and $(1-\beta)$ on the domestic good. Demand for the domestic good rises with y_j, $j = 1, \ldots, n$. A rise in the relative price of a foreign good shifts world demand from the foreign good to the home good by δ. The demand for all goods decreases with expected real interest rates, r_i. The residents in each country spend the amount ν less for each percentage point increase in the expected real interest rate.

The consumer price index q_i is an average of the home good's and the foreign goods' price levels weighted according to demand. Price increases abroad raise the domestic consumer price level through the share of imported goods.

$$q_i = (1-\beta)p_i + \beta \frac{1}{n-1} \sum_{\substack{j=1 \\ j \neq i}}^{n} (e_{ij} + p_j) \tag{7}$$

The expected real interest rate is:

$$r_i = i_i - q_i^e + q_i \tag{8}$$

where i_i is the nominal interest rate and q_i^e is the expected value of the consumer price index tomorrow based on the information available today.

International capital mobility and perfect substitutability of bonds give the condition of uncovered interest rate parity:

$$i_i = i_j + e_{ij}^e - e_{ij} \tag{9}$$

for all $i, j = 1, \ldots, n$. Only with this condition will private agents be indifferent between holding bonds of all countries.

Equation (**9**) is often based on a floating exchange rate regime, while governments use their money supply in order to optimise explicitly their individual loss function or the joint loss function in a coalition. However, this is compatible with the notion of a currency union that denotes a common currency or a fixed exchange rate regime. Exchange rate pegging can be viewed as a viable alternative to fully fledged coordination (see Giavazzi and Giovannini (1989));[(5)] then governments use the money supply to stabilize the exchange rate. The union has to stabilize $(n-1)$ exchange rates, but has n money supplies available as instruments. That leaves one money supply free for optimising the common loss function.

3.1 Policy-makers' objectives

The policy-maker in the home country has access to a single policy instrument, m_i, which is money growth. He evaluates the effects of monetary policy according to a loss function over the deviation of employment and inflation from the zero-disturbance equilibrium:

The objective function is:

$$L_i = \frac{1}{2}(\sigma l_i^2 + q_i^2) \tag{10}$$

The parameter σ denotes the relative weight of the full-employment objective. A low σ denotes a monetary authority for whom price stability is the ultimate goal. The policy-maker minimises the loss function subject to the restrictions arising from the economy.

(5) Of course, the possibility of speculative attacks has to be ruled out in a system of pegged exchange rates.

3.2 Reduced form of the economy's behaviour

Equations (**1**) to (**9**) can be reduced in the symmetric case to two equations for each country. They determine the constraints for the policy-maker's optimisation problem. The money supply m_i is free as an instrument for optimising the loss function.

The reduced forms for l_i, q_i are:[6]

$$l_i = m_i \tag{11}$$

$$q_i = \lambda m_i - \kappa \sum_{\substack{j=1 \\ j \neq i}} m_j + x \tag{12}$$

with:

$$\lambda = \alpha + \frac{\beta(1-\alpha)\left(1-\epsilon(1-\frac{n}{n-1}\beta)\right)}{\delta n + \nu(1-\frac{n}{n-1}\beta)^2} \qquad > 0$$

$$\kappa = \frac{\lambda - \alpha}{n-1} \qquad > 0$$

The explanation of the reduced form is as follows. Each country's employment l_i rises one-for-one with the domestic money supply (equation (**11**)).[7] Output rises with employment. Since real wages have to 'balance' the increase in employment (ie fall), the price of the domestic good rises. Hence, the price level rises with the money supply. The exchange rate depreciates. Consequently, the consumer price level, which is a weighted sum of the domestic goods price and the prices of the imported goods, rises. This is reflected in equation (**12**) as λ is positive.

A symmetric world productivity disturbance gives rise to a stabilisation game. Without policy intervention a negative disturbance ($x > 0$) would have no effect on employment because nominal output is unaffected. A negative productivity disturbance lowers real output and raises the output price by the same amount, since employment only remains constant if the real wage falls, ie the price of domestic goods rises (equations (**1**) and (**2**)). There is no change in the real exchange rate since real output falls in all countries by the same amount.

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(6) The reduced form is explicitly derived in Appendix A.

(7) Note that all variables are deviations from the long-run equilibrium. In the long-run equilbrium they are zero.

Consequently, the consumer price index rises. Real interest rates have to rise in order to equilibrate the goods markets. Since the real and the nominal exchange rate do not change, perfect substitutability on the international capital markets requires that the nominal interest rates in all countries change by the same amount.(8) In short, a negative productivity shock will leave employment unchanged and increase CPI inflation.

Each policy-maker – facing a loss function which increases in the square of employment and CPI deviations – now has an incentive to contract the money supply slightly to reduce inflation. He accepts the small loss from reducing employment below the full-employment level in favour of the significant gain from lowering inflation. Contractionary monetary policy in the home country improves the terms of trade, lowers the price of imports and thus lowers inflation. Abroad, the price of imports is increased, causing inflation. Thus, monetary policy creates an externality which is reflected in the negative sign ($\kappa > 0$) of foreign monetary policy in equation (**12**). If all policy-makers pursue anti-inflationary policies they enter into a competitive appreciation which leads to a contractionary bias in the losses. The exchange rate ultimately remains unchanged but all policy-makers have contracted too much with respect to their optimal money supply. This could be avoided if all countries coordinated on a less contractionary monetary policy.(9)

4 A non-cooperative game with coalition formation

The previous section outlined how policy-makers will react to a negative productivity shock if they do not co-operate at all. Since they impose negative externalities on each other there is scope for

(8) Whether nominal interest rates fall or rise depends on the size of the model parameters. When the real interest rate elasticity of goods demand is lower (higher) than the income elasticity of savings, nominal interest rates will rise (fall).

(9) Hamada (1976) pioneered the studies that uncoordinated policy making across countries may be inefficient. The result of shock stabilisation after a negative productivity shock was first formalized by Canzoneri and Gray (1985) and was then used as a work-horse by Canzoneri and Henderson (1988) and (1991), Persson and Tabellini (1995), among others.

improvement through co-operation. For this reason, the literature on international monetary policy coordination has argued that coordination is beneficial for all parties involved.

This section will analyse whether countries may prefer forming a coalition to full coordination. While other models, eg the model of Alesina and Grilli (1993), focus on the question of whether entry will be limited by the insiders, this paper focuses on the question of whether outsiders will refuse to enter. In the model here coalition members will always want other countries to join them. Hence an explicit assumption which ensures free entry into the coalitions is not needed.

The model is solved accounting for a possible coalition formation. Following Yi (1997), coalition formation can be formulated as a two-stage game:

- Stage 1: countries decide whether to join a coalition or not;
- Stage 2: countries engage in a shock stabilisation game (given the coalition).

The game is solved recursively. The second stage determines the equilibrium for a given insider-outsider structure. The stable coalition is analysed as the optimal outcome of the first stage.

A coalition is a subset of countries that optimise a common loss function. This common loss function is a weighted average of the individual countries' loss functions.

$$\mathcal{L} = \sum_{j=1}^{k} \alpha_j L_j$$

The relative weights are denoted α_j with $\sum_{j=1}^{k} \alpha_j = 1$ and are typically determined in a (co-operative) bargaining process. But as all members have the same economic structure and loss functions, there is no obvious reason why the result of such a bargaining process should be unequal weights. For the time being, we assume $\alpha_j = \frac{1}{k}$ for all $j = 1, \ldots, k$.(10)

(10) If weights are not equal, the results of the model probably change entirely. In the model here – if there was sequential entry – 'later' countries may receive larger bargaining weights since their incentive to free-ride by staying outside increases with coalition size. Thus the stable coalition size could be enlarged.

First, the reaction functions of the countries outside the coalition and of the coalition itself will be determined. The equilibrium is the intersection of the reaction functions. This yields the outcome of stage 2. It is dependent on n, the number of countries, and k, the number of coalition members. In Subsection 4.2 the equilibrium losses are analysed with respect to a change of n and k. This is extended to an analysis of the individually optimal choices in stage 1: the 'stability' of the coalition, using a concept drawn from the industrial organisation literature.

4.1 Stage 2: the optimal strategies and the equilibrium

It is assumed that k out of the n countries are members of the coalition C. Countries $1, \ldots, k$ are inside and countries $k+1, \ldots, n$ are outside the coalition.

The countries outside the coalition

In order to solve the policy-maker's optimisation problem when he is outside the coalition, the Nash strategy is calculated. l_i and q_i in the loss function are replaced by the reduced-form equations. This function is minimised with respect to m_i subject to given strategies of the other countries $m_j = \overline{m}_{j,nc}$ for all $j \neq i$ if j is an outsider, and $m_j = \overline{m}_{j,c}$ for all j if j is a coalition member. The symmetric set-up implies that all countries have the same degree of conservativeness σ. Since the structure is symmetric in every respect, it can be assumed that all countries outside the coalition have the same optimal money supply m^*_{nc}. Money supply of a non-member can be derived as a function of the coalition's money supply:(11)

$$m^*_{nc} = \frac{\lambda\kappa}{\sigma+\lambda^2-\lambda\kappa(n-k-1)} \sum_{j=1}^{k} \overline{m}_{j,c} - \frac{\theta}{\kappa} x = \theta \sum_{j=1}^{k} \overline{m}_{j,c} - \vartheta x \qquad \theta, \vartheta > 0 \tag{13}$$

Equation (**13**) needs some explanation. The optimal policy outside the coalition depends positively on the coalition policy, ie the money supplies of a non-member and a coalition member are strategic

(11) The results are derived in Appendix A.2.

complements.[12] This means that a less contractionary monetary policy of the coalition members triggers a less contractionary response from the non-members. The reason is that the coalition creates less competitive appreciation for the non-members because of the less contractionary policies adopted. Hence the countries outside the coalition also need to contract less, because they face less 'imported' inflation. This result will be referred to later.

The behaviour of the coalition

In order to solve the optimisation problem of the coalition members a further element of the structure of the game has to be clarified. The coalition can be involved in a Nash game or in a Stackelberg game with the non-members.

The Nash equilibrium assumes that both coalition members and outsiders have no information advantage, they 'move at the same time'. The coalition outcome represented by a Nash equilibrium cannot be achieved without a commitment technology: given the other coalition members' monetary policy each individual coalition member would like to export some inflation to the rest of the coalition.[13] But then the other coalition members would not enter into the agreement in the first place. So the coalition members are assumed to enter into a binding agreement that is known about by all players. But then it is not clear why the coalition would not use this commitment technology to behave as a Stackelberg leader. A less contractionary monetary policy of the coalition triggers a less contractionary response from an outsider. This in turn creates less externalities (imported inflation) for the coalition itself and lowers its losses. As Stackelberg followers, the outsiders know about the coalition's money supply before they set their own money supply. The coalition, which is the Stackelberg leader, can use this advantage by setting its money supply such that the combined effects of its own policy and the reaction of the outsiders create the smallest losses possible.

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(12) Strategic complements imply upward-sloping reaction functions, see Bulow *et al* (1985). The reaction function of a non-member is upward sloping since θ is positive. In a model where monetary policies act as strategic substitutes the stable coalition size will comprise all countries (see Yi (1997) for a game-theoretic proof). The reason is that the coalition discipline forces outsiders in this case into a reaction which leaves them worse off than the insiders.

(13) In game-theoretic terms, the countries which play co-operatively within a coalition are off their individual Nash reaction functions.

The Stackelberg concept, on the other hand, creates a conceptual problem when looking at the coalition *formation* process.[14] Assuming a Stackelberg leadership for the coalition implies a Stackelberg leadership for the single country at the 'early' stage of the coalition formation. However, a Stackelberg leadership of a single country has to be explained by a structural difference which is not captured by the symmetric model used here. Since it is not clear which of the two concepts should be chosen, both structures will be analysed.

It will be shown that the stable coalition size does not differ very much between the games. Though the results of the Stackelberg game are not robust to changes in the values of the model parameters, the basic argument for the existence of a stable coalition which does not include all countries remains valid in both games.

The Cooperation-Nash equilibrium

In the Cooperation-Nash[15] game the coalition solves its optimisation problem subject to a given money supply of the non-members. Exploiting the symmetry assumption $m^*_{j,c} = m^*_c$[16] for all $j = 1, \ldots, k$, this gives a coalition member's reaction function which depends on the non-members' money supply. By equating the reaction functions the equilibrium of the Nash game with a coalition is obtained:[17]

$$m^*_c = -\rho x \qquad \rho > 0 \qquad \mathbf{(14)}$$
$$m^*_{nc} = -\rho\kappa(n-k)k\theta + (n-k)\vartheta x = -\omega x \qquad \omega > 0 \qquad \mathbf{(15)}$$

The equilibrium policies in both games are linear functions of the shock x. If the shock is zero, the optimal policies are also zero since there is no need for stabilisation. If the shock is negative, ie $x > 0$, the optimal policy for all countries is a contractionary monetary policy since ρ and ω are both positive.

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(14) The Stackelberg concept gives in general a time-inconsistent result, ie the Stackelberg leader would, *ex post*, like to change his strategy and hence does not play an optimal response. A structural difference in the *timing* of the decision-making could explain such behaviour. One could argue that the coalition has to announce its policy at an early stage because all members have to coordinate on the optimal policy. It sets its money supply before the non-members react or it can credibly commit itself to its monetary policy.

(15) The notation of a *Cooperation-Nash equilibrium* and a *Cooperation-Stackelberg equilibrium* is an adaptation from Canzoneri and Henderson (1991), Chapter 3.

(16) Obviously, if weights are not symmetric, this assumption does not hold any more.

(17) ρ is quite a long expression which can be checked in Appendix A.2.

The coalition eliminates the negative externalities which the member countries impose on each other. The coalition members conduct a less contractionary, and thus less deflationary, policy. But if the coalition countries contract less, the inflation in the non-member countries is lower as well, since the currency of a coalition member appreciates less against *all* currencies. It has been shown already that the non-member country now has the possibility of contracting less. It increases employment without having inflation as high as it would be without the influence of the coalition policy. This clearly improves the loss function of the non-member. It means that the *coalition formation process produces positive spillovers* for non-members.

The Cooperation-Stackelberg equilibrium

In the Cooperation-Stackelberg game the coalition solves its optimisation problem under explicit consideration of the non-members' reaction functions. The optimisation problem yields the money supplies in the Stackelberg equilibrium:(18)

$$m_c^* = -\frac{(1+(n-k)\theta)(\lambda+\kappa-k\kappa(1+(n-k)\theta))}{\sigma+(\lambda+\kappa-k\kappa(1+(n-k)\theta))^2}x = -\varphi x \qquad \varphi > 0 \quad \textbf{(16)}$$

$$m_{nc}^* = -(\kappa k\varphi + 1)\vartheta = -\phi x \qquad \phi > 0 \quad \textbf{(17)}$$

As before, the equilibrium policies are linear functions of the shock x. If the shock is zero, the optimal policies are also zero as there is no need for a stabilisation game. If the shock is negative, ie $x > 0$, the optimal policy for all countries is a contractionary monetary policy since φ and ψ are both positive. The interpretation of the Nash game applies accordingly.

The analysis that follows considers whether the process of coalition formation might 'stop' at a certain point, since the spillovers from the coalition formation process might be high enough that a country prefers to stay outside.

4.2 Stage 1: the stability of the coalition in equilibrium

The number of countries, n, and the number of coalition members, k, are the parameters of interest with respect to coalition formation. The

(18) A detailed analysis is provided in Appendix B.2.

values of the optimal policies and the losses in equilibrium are dependent on n and k. A country's decision whether to join or leave the coalition may change when n and k change.

The factors ρ, ω, φ and ϕ in the optimal policy are nonlinear in n and k and it is difficult to analyse how the model parameters affect the outcome. One possible approach is to perform numerical simulations with specific values for the model parameters while varying n and k. The results reported are based on a simulation where n varies from 3 to 22 and k varies from 1 to 22.[19]

First, the loss functions for coalition members and non-members are analysed with respect to n and k. Then a stability condition adapted from the cartel formation literature in industrial organization[20] is used. The loss function of a non-member is denoted by $L_{nc}(n,k)$. If it joins the coalition (and no other country changes from one group to another), it will have the loss $L_c(n,k+1)$. If $L_{nc}(n,k)$ is smaller than $L_c(n,k+1)$, the country has no incentive to join the coalition – the coalition is called 'externally stable'. A similar condition holds for the coalition members. If $L_c(n,k)$ is smaller than $L_{nc}(n,k-1)$, the country has no incentive to leave the coalition. The coalition is called 'internally stable'. If both conditions are fulfilled, the coalition is stable, with size k.[21] If only external stability is fulfilled, the coalition is still stable in some sense, since it is possible that countries which join the coalition are committed to staying in. The commitment can arise from reputational considerations or from a formal international contract.

The stability conditions do not allow the coalition to block a further extension of the coalition. However, the coalition in this game would never want to limit entry since the coalition members' losses decrease

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(19) The parameter values were: $\alpha = 0.25$, $\beta = 0.5$, $\epsilon = 0.8$, $\nu = 0.05$, $\sigma = 1$ and $\delta = 0.3$. A robustness analysis was performed; the results did not change qualitatively in the Cooperation-Nash game. The results of the robustness analysis are discussed in detail in Appendix C.

(20) The stability condition used here is the one proposed by D'Aspremont *et al* (1983).

(21) This algorithm assumes that only the country under consideration takes a decision; all other countries remain in their 'group'. If the result is stability, there is no problem since no one actually will change. But if the result is instability, this algorithm might give an incorrect signal since all members of a group will take the decision to change and not only the country under consideration.

when new countries enter the coalition. Hence, a condition which ensures 'free entry' is not needed.[22]

4.2.1 The Cooperation-Nash game

The loss functions

The losses in equilibrium are determined through equilibrium policies:

$$
\begin{aligned}
L_c &= \sigma m_c^{*^2} + (\lambda m_c^* - \kappa(k-1)m_c^* - \kappa(n-k)m_{nc}^* + x)^2 \\
L_{nc} &= \sigma m_{nc}^{*^2} + (\lambda m_{nc}^* - \kappa k m_c^* - \kappa(n-k-1)m_{nc}^* + x)^2
\end{aligned}
$$

Losses decrease for all values of n with the number of coalition members: the more members a coalition has, the more externalities are internalised. As shown above, the coalition formation process has a positive externality for non-members: the less contractionary coalition policy evokes a less contractionary optimal policy on the part of the non-member. The country outside the union will be able to increase employment without increasing inflation. This will lower the losses for both parties.

The stability of the equilibrium

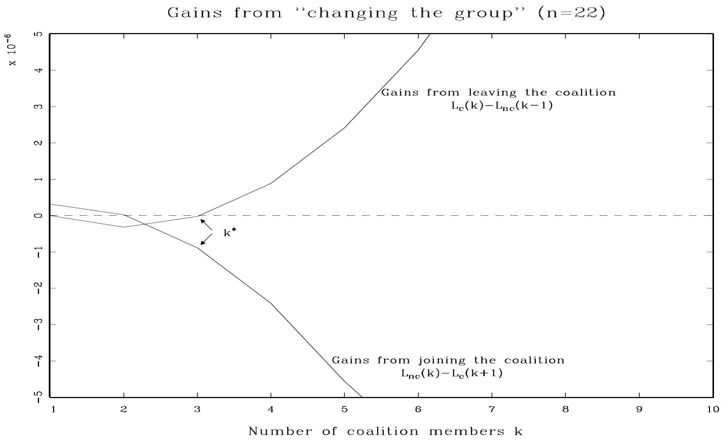
The stability analysis gives the following result (for a graphic illustration see Chart 1). The coalition is internally stable only for $k = 2$ and $k = 3$; this is true for all values of n. When the coalition size exceeds three, each coalition member individually could gain by leaving the coalition. The coalition is externally stable for all configurations where three or more countries are in the coalition. When the coalition size is one (in fact, then there is no coalition) or two, a country outside could reduce its losses by joining the coalition.

In other words, if *three* of the n countries are in the coalition, no country outside has an incentive to join. The coalition members do not want to leave the coalition in this situation either. Hence, there exists an equilibrium with a stable coalition which is not joined by all countries. This stable coalition size is three for all n.

---

(22) In contrast to the model here, coalition formation which is based on reputational considerations, as in Alesina and Grilli (1993), faces this problem. It is not in the interest of the coalition to admit a 'weaker' member which would worsen the 'stronger' members' positions.

Chart 1: *Nash game:* external and internal stability[(a)]



(a) Negative 'gains from changing the group' imply that changing does not pay and hence the group is stable. The convex graph shows the internal stability of the coalition where only coalition sizes of three or less are stable (negative gains from leaving). The concave graph shows the external stability where only coalition sizes of three or more are stable (negative gains from joining). Therefore, only a coalition of three countries fulfils both stability criteria.

A brief explanation why this result is possible even without asymmetries is as follows. When a country decides whether or not to join a coalition, two factors are involved. The country balances the gains from entering the coalition against the costs of giving up an optimal policy 'against' the coalition. The gains from entering arise from the elimination of competitive appreciations against the countries in the coalition. This is achieved through a less contractionary monetary policy. The gains from staying outside are given by the possibility of carrying out an optimal policy against the coalition. But the coalition – by contracting less – evokes a less contractionary policy from the non-members. As the size of the coalition is increased, the optimal amount of contraction declines and so the non-member countries also contract less. When the coalition has reached a certain size, the optimal response of the non-member is already less contractionary to a certain extent. There is not much to gain by joining the coalition. The countries then prefer to stay outside, as they are no longer engaged in an inflationary appreciation of any considerable extent.(23)

One may be surprised by the feature that the stable coalition size remains small even as the number of countries increases. However, this result has an equivalent in the cartel literature in a space of strategic complements, see D'Aspremont *et al* (1983). It pays for a country to join a coalition of up to three members because the gains from the additional discipline of the other members are higher than the costs from the discipline within the coalition imposed on the country which joins. With more than three members the costs imposed by the discipline within the coalition are too high. The reasoning is twofold. First the discipline costs within the coalition increase with the number of members because a coalition member has to adapt its monetary policy not only to two others but to three, four, ... etc. This means a country in the coalition is off its Nash reaction function, ie its optimal response, for all the other coalition members (and – having only one strategic variable – it drives the country off its individual reaction function towards the outsiders, as well). Secondly, the gains from joining the coalition decrease with the number of coalition members since the money supplies are strategic complements: with each 'step' towards more discipline within the coalition, losses outside are lower, too. Hence, there is less and less to gain by joining as the coalition size increases. In this way the result can be also explained that non-members are better off than coalition members (in the Nash and in the Stackelberg game).

Much of this result is due to the strategic complementarity of money supplies in the model. Of course, one could imagine models where the policy variables are strategic substitutes. Yi (1997) shows in a game-theoretic context that in games with strategic substitutes a stable coalition often includes all countries, while in the case of strategic complementarity the 'grand' coalition can typically not be

(23) One may ask how results change if a second policy instrument is allowed for. If this policy is a strategic complement across countries it reinforces the 'small' coalition of the monetary policy game. If the policy is a strategic substitute across countries it allows for a larger stable coalition, as suggested by Yi (1997). Alternatively, the other policy instrument can be used as a 'threat' which enforces coordination in the monetary policy field. This last possibility is analysed in Kohler (1998).

sustained.[24] Consequently, if coordination on policies which are strategic substitutes was considered the stable coalition size would certainly be larger.[25]

Increasing costs of joining and decreasing gains from joining for an individual country explain why the stable coalition size is unique. Gains from joining the coalition decrease monotonically and costs of joining it increase monotonically with the coalition size (a graphical illustration is given below). Once the stable size is reached, ie the costs equal the gains, costs will always exceed the gains if the coalition size increases.

Coalition formation process

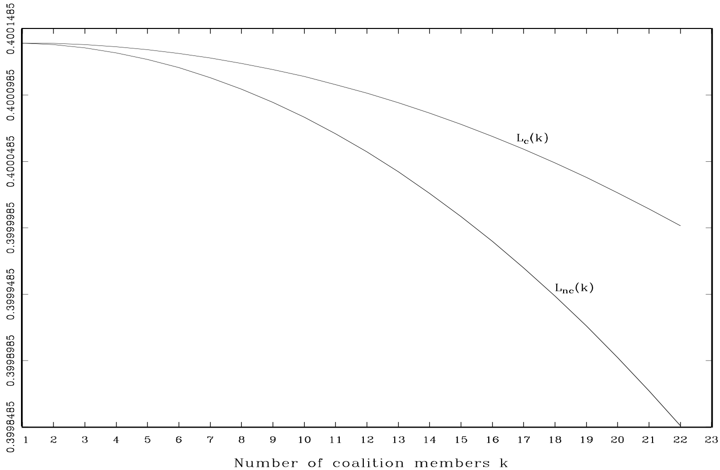
This section looks in more detail at the coalition formation process. For $k = 1$, ie the coalition consists of one country, which is the same as if there were no coalition, the coalition is externally unstable. Hence countries have an incentive to join the coalition. But there is a free-rider problem since the countries outside the coalition have smaller losses than the coalition members for any combination of n and k (see Chart 2 which shows the loss functions for $n = 22$ and k from 1 to 22). Every country would like the others to enter the coalition rather than joining itself. This may create an obstacle to coalition formation since every country would wait for the others to go ahead. But a similar problem is faced in the basic non-cooperative game: every country in the coalition could realise lower losses if it deviates from the coalition policy. On the other hand countries know that this is not an option – they have only the option to comply with coalition discipline or not to have any coalition. If it is assumed – as it is done throughout the paper – that there is some commitment mechanism which overcomes this dilemma, the same may be assumed with respect to enlarging the coalition to up to three members. For every country it would be better

(24) With strategic substitutes, reducing the coalition's losses will increase outsiders' profits. This will induce more players to join the coalition than in a game with strategic complements.

(25) An anonymous referee made the suggestion that transaction costs may play the role of strategic substitutes. In a model similar to the one used here, the removal of transaction costs would lower the relative price of goods involved in intra-union trade and consumers inside the union would typically substitute the more expensive outsider goods with those from within the union. While this set-up would increase gains from joining the union, there is no strategic complementarity in a game-theoretic sense. The reaction of the outsiders would be to join the union rather than to change a policy instrument, such as monetary policy.

to form a coalition of three countries than to form no coalition (or one with two members). This is true if the country is an insider or an outsider. This may be incentive enough to get coalition formation started.(26) The situation changes once the stable coalition size is reached: now the country is better off remaining outside and accepting a smaller coalition than joining the coalition. In the game here the outsider has then been somehow 'luckier' or 'smarter'.

Chart 2: *Nash game:* loss functions of insiders and outsiders



4.2.2 The Cooperation-Stackelberg game

The loss functions and the equilibrium behaviour in the model where the coalition takes a Stackelberg leader position are qualitatively and quantitatively almost identical to the results of the Nash game, so only the main results will be discussed. The results of the Stackelberg game are discussed in detail in Appendix C.

The Stackelberg game has *one* stable coalition size for each n. This stable coalition size is either two or three.

(26) Another solution to the dilemma that countries would rather be outsiders than coalition members would be to extend the model and assume that joining the union gives additional gains, such as transaction cost gains.

As in the Nash game, the non-members have lower losses than the union members. The reason is the same as before. Money supplies are strategic complements in the model here. The discipline within the coalition creates less imported inflation for the outsiders while at the same time they can export inflation to the union. The coalition cannot export inflation to the outsiders and restrain exported inflation to union members at the same time since each member has only one instrument available to direct at both objectives.

The losses in the Stackelberg equilibrium are, for all parties involved, always less than the losses in the Nash equilibrium.(27) The coalition would benefit from a less contractionary money supply of the outsiders which export inflation into the union. As Stackelberg leader the coalition can contract its money supply by less, thus triggering a less contractionary response from the outsiders who know which money supply is set by the union. As a result, both parties benefit because they have reduced inefficiencies arising from the competitive appreciation.

These results are in line with general results found by Dowrick (1986) where the Stackelberg equilibrium is Pareto superior to the Nash equilibrium in games with strategic complements. Though there are some quantitative differences between the Cooperation-Nash and the Cooperation-Stackelberg outcomes, they are very small. This may explain why the optimal coalition size only changes for extreme parameter values.

5 Bloc formation in a non-cooperative game

The main result of the previous section was that coalition formation will stop at three countries. The reason is that the coalition formation process itself causes positive spillovers for the outsiders: the increased discipline within the coalition reduces the negative externalities the

(27) The game-theoretic reasoning behind this is that the Stackelberg leader can never be worse off than in the Nash equilibrium. He could always pick his Nash strategy and hence realise the Nash losses. Therefore, if he deviates from the Nash equilibrium money supply he does so because he is able to lower his losses by choosing another money supply.

coalition countries create for *all* countries, independent of whether they are 'ins' or 'outs'. Countries will decide whether to join the union or not on the basis of whether it pays more to reduce imported inflation or to be able to export inflation.

But would countries which decided not to join the 'first' coalition prefer to form a second 'competing' coalition? The answer is provided in this section: the outsiders are willing to undergo some discipline in a small union but not the larger discipline in a big union. In order to deal with this question the rules for stage 1 of the game have to be modified: each country can join an existing coalition in stage 1 or form a new one. It will become evident that the hierarchical structure *between* the coalitions becomes important for the size of the stable coalitions. For analytical convenience the maximum number of coalitions will be restricted to two. However, the result could be extended to more coalitions.

5.1 The game structure

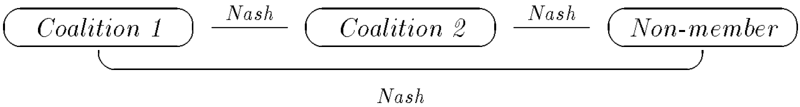
A second coalition is now permitted, which maximises the joint loss function of its members, like coalition 1. Coalition 1 consists of the countries $i = 1, \ldots k_1$ and optimises $\mathcal{L} = \sum_{i=1}^{k_1} \frac{1}{k_1} L_i$. Coalition 2 includes the countries $i = 1, \ldots, k_2$ and optimises $\mathcal{L} = \sum_{i=1}^{k_2} \frac{1}{k_2} L_i$. The remaining $n - k_1 - k_2$ countries play a non-cooperative Nash strategy against all other countries by minimising their individual loss functions.

As before, the coalitions can be involved in a Nash game or in a Stackelberg game with the non-members and with each other. Several game structures can be distinguished by combining the three groups – coalition 1, coalition 2 and the fringe – and the two behavioural assumptions – Nash or Stackelberg. Two model structures will be analysed:[28]

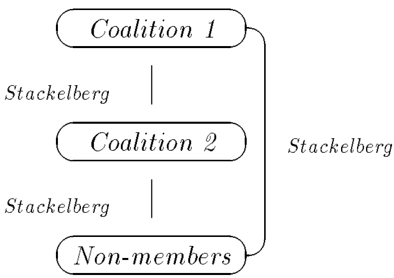
In the **Nash-Nash game** no country has an information advantage and both coalitions are formed simultaneously. This game describes,

[28] The structure where the two coalitions play a Nash game against each other, while they both behave as a Stackelberg leader against the outsiders, was also analysed. The results of the games with one coalition can be extended straightforwardly. All losses are somewhat lower than in the Nash-Nash game. But the stable coalition sizes are the same as in the game where the coalitions play Nash against the outsiders.

for example, a situation where the formation of (regional) miniblocs is envisaged. This may either be the result of the break-up of a larger, perhaps political bloc such as the former Soviet Union or Yugoslavia, or it may be the result of newly evolving regional structures.(29)



The **Stackelberg-Stackelberg game** takes account of the commitment structure within the coalition. This provides the opportunity for both coalitions to play a Stackelberg game against the outsiders. Yet the strategic behaviour between the two coalitions has to be clarified. Here the situation where the 'first' coalition plays Stackelberg against the 'second' will be considered. This requires that the money supply of the first coalition is known (or the first coalition can credibly commit to it) before the second coalition moves. The idea of a time structure for coalition formation may help to explain the different strategic positions of the two coalitions. This situation may arise when one coalition was established earlier than the other. This could allow stronger commitment or more timely movement of the first coalition. One example may be the creation of the European Union followed by the creation of EFTA by some of the outsiders (though these are examples of trade blocs rather than currency blocs).



(29) The hierarchical relation between groups are illustrated in the graphs. Two groups which lie on the same horizontal line do not have hierarchical differences and hence play a Nash game against each other. Groups which are connected by a vertical line play a Stackelberg game against each other where the top group is the Stackelberg leader while the lower group represents the Stackelberg follower.

5.2 Stage 2: equilibrium strategies and losses

The countries outside the coalition

As earlier, the outsider solves its optimisation problem subject to given strategies of the other countries $m_j = \overline{m}_{j,nc}$ for all $j \neq i$ if j is an outsider, $m_j = \overline{m}_{j,c_1}$ for all j if j is a member of coalition 1 and $m_j = \overline{m}_{j,c_2}$ for all j if j is a member of coalition 2. The money supply of a non-member can be derived as a function of the coalition's money supply:[(30)]

$$m^*_{nc} = \theta \sum_{j=1}^{k_1} \overline{m}_{j,c_1} + \theta \sum_{j=1}^{k_2} \overline{m}_{j,c_2} - \vartheta x \qquad \theta, \vartheta > 0 \tag{18}$$

As above, the coalition creates less competitive appreciation for the non-members by contracting less. Hence, the countries outside the coalition also need to contract less, because they face less 'imported' inflation. As a result, a less contractionary monetary policy of the coalition members triggers a less contractionary response from the non-members.

The equilibrium in the Nash-Nash game

The reaction functions of the coalitions are derived in Appendix A.3. They are upward-sloping with respect to the other countries' money supplies. This means that the money supplies are strategic complements for all players: a less contractionary money supply from coalition 2 or from the fringe triggers a less contractionary reaction from coalition 1.

Equating all three reaction functions gives the Nash equilibrium:

$$m_{c1} = -\tfrac{1}{\mu}(1 + \kappa k_2 \psi_2)(1 + \kappa(n - k_1 - k_2)\vartheta)\psi_1 x = -\rho_1 x \tag{19}$$

$$m_{c2} = -\tfrac{1}{\mu}(1 + \kappa k_1 \psi_1)(1 + \kappa(n - k_1 - k_2)\vartheta)\psi_2 x = -\rho_2 x \tag{20}$$

$$m_{nc} = -\tfrac{1}{\mu}(1 + \kappa k_1 \psi_1)(1 + \kappa k_2 \psi_2)\vartheta x = -\varrho x \qquad \rho_1, \rho_2, \varrho > 0 \tag{21}$$

As before, the losses in equilibrium decrease with the number of coalition members since more coalition members internalise more externalities. The union members contract their money supplies less because they reduce the competitive appreciation against the other

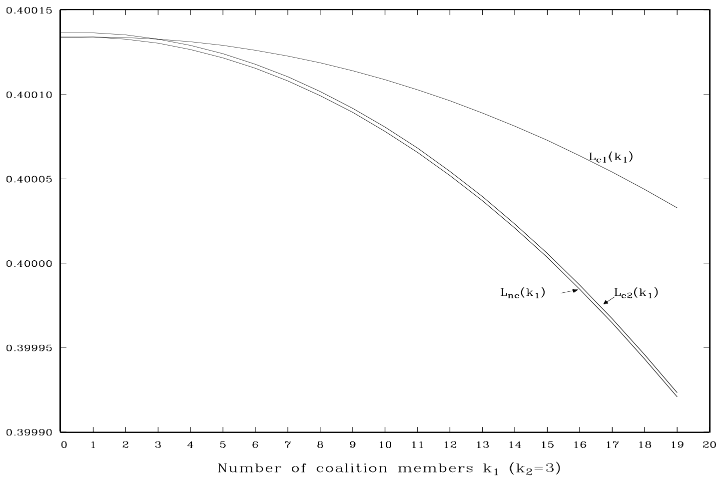
(30) The results are derived in Appendix A.3.

members. At the same time they reduce competitive appreciation against the outsiders since each country has only one money supply which it can set. So the process of coalition formation produces positive spillovers for outsiders which can contract less while they are still able to export inflation. This allows them to reduce inflation with lower output costs (compared with the situation without coalitions). This will lower the losses for both parties.

Members of a coalition benefit more from an increase in the size of the other coalition than from an expansion of their own coalition. The lowest losses are reached when all other countries are in the other coalition. The reason is clear: the more coalition members, the stronger is 'coalition discipline' from which all countries profit; the smaller the coalition size, the closer is the coalition policy to the individually optimal Nash response.(31)

Chart 3 shows the relative positions of the three groups with increasing coalition size k_1 for $n = 22$ and $k_2 = 3$. All groups benefit from the increasing coordination within coalition 1 which can be seen in the

Chart 3: Loss functions of insiders and outsiders



(31) This may create a similar problem to the one discussed in Section 4.2.2: everyone would rather someone else joined. But, as before, this may not be a feasible option: either the country in question joins or there is no coordination at all. Which country joins and which stays outside cannot be determined in this model.

downward-sloping functions. The smallest losses are always realized by the non-members. Again, this is a consequence of playing a game with strategic complements since non-members gain from the increased discipline within the coalition but are able to play an individually optimal response themselves. For the same reason coalition 2 benefits from the increasing size of coalition 1 by more than the members of coalition 1 themselves. This is reflected by a steeper decline of the loss function of coalition 2 than that of coalition 1.

The equilibrium in the Stackelberg-Stackelberg game

The optimisation problems and the reaction functions are derived in Appendix A.3. The reaction function of the first coalition (the Stackelberg leader) is independent of the other countries' reaction functions and hence is identical to the equilibrium policy. It determines successively the equilibrium policies of coalition 2 and of the fringe.

$$m_{c1} = -\frac{(1+f\theta)(1+\eta_2 k_2\kappa)(\lambda+\kappa-k_1\kappa(1+f\theta)(1+\eta_2 k_2\kappa))}{\sigma+(\lambda+\kappa-k_1\kappa(1+f\theta)(1+\eta_2 k_2\kappa))^2}x = -\omega_1 x \quad \textbf{(22)}$$

$$m_{c2} = -\eta_2(1+k_1\omega_1\kappa)x = -\omega_2 x \quad \textbf{(23)}$$

$$m_{nc} = -\theta(k_1\omega_1 + k_2\omega_2) + \vartheta)x = -\upsilon x \qquad \omega_1, \omega_2, \upsilon > 0 \quad \textbf{(24)}$$

Losses are reduced with increasing coalition size in the same way as described earlier. All three groups are now better off than in the previous case,(32) benefiting from the improvement of the Stackelberg leader upon his Nash strategies.

5.3 Stage 1: the stability of coalitions in equilibrium

The previous section has shown that the coalition formation produces positive spillovers for all countries and lowers their losses. These spillovers prevent countries from joining the coalition in stage 1. The stability conditions employed in the previous analysis will be extended to two coalitions and determine when a country would like to remain in the coalition, join another coalition or join the fringe.

(32) There are cases where the two respective games give the same outcome. In particular, when all countries are in the fringe or all countries are in one coalition the Nash-Nash and the Stackelberg-Stackelberg game are identical.

The coalitions are *internally stable* with size k_1^*, k_2^* if:

$$L_{c1}(k_1^*, k_2^*, n) < L_{nc}(k_1^* - 1, k_2^*, n) \text{ and } L_{c1}(k_1^*, k_2^*, n) < L_{c2}(k_1^* - 1, k_2^* + 1, n)$$
$$L_{c2}(k_1^*, k_2^*, n) < L_{nc}(k_1^*, k_2^* - 1, n) \text{ and } L_{c2}(k_1^*, k_2^*, n) < L_{c1}(k_1^* + 1, k_2^* - 1, n)$$

They are *externally stable* if:

$$L_{c2}(k_1^*, k_2^*, n) < L_{c1}(k_1^* + 1, k_2^* - 1, n) \text{ and } L_{nc}(k_1^*, k_2^*, n) < L_{c1}(k_1^* + 1, k_2^*, n)$$
$$L_{c1}(k_1^*, k_2^*, n) < L_{c2}(k_1^* - 1, k_2^* + 1, n) \text{ and } L_{nc}(k_1^*, k_2^*, n) < L_{c2}(k_1^*, k_2^* + 1, n)$$

They are *stable* when they are internally and externally stable. The results reported here are again based on a simulation where n varies from 3 to 22 and k_1 and k_2 vary from 1 to 22.(33)

5.3.1 The Nash-Nash game

The Nash-Nash game has a unique stable equilibrium where each coalition has three members if there are more than five countries in the world. In a Nash-Nash game with three countries, full coordination (ie all countries in one coalition) is the stable equilibrium. When there are four countries there will be two countries in each coalition in a stable equilibrium.(34) For $n = 5$ countries the 'last' country to enter is indifferent between the two coalitions, and so may switch between them in equilibrium. The stability conditions are only fulfilled with equality. However, the last country will clearly prefer joining one of the coalitions to remaining in the fringe. These results are independent of the total number of countries, in particular of the number of countries in the fringe, once the total number of countries exceeds five. The features of these results will be explained in detail later.

The stable coalition size remains small since the money supplies are strategic complements, as discussed earlier. By the same token, it can be explained why it is individually optimal to form two coalitions of three countries but not one coalition of six which may be Pareto-superior to the two 3-country blocs. It has to be borne in mind

(33) The parameter values for the simulation are: $\alpha = 0.25$, $\beta = 0.5$, $\epsilon = 0.8$, $\nu = 0.05$, $\sigma = 1$ and $\delta = 0.3$. A sensitivity analysis was performed; the results are given in Appendix B.

(34) For $n = 4$ countries some cases have a very high κ such that it pays off to extend the first coalition to the stable size of three rather than having two countries in each coalition.

that a coalition is only stable when it is *individually* optimal for each single country not to join the coalition (for a country in the fringe or in the other coalition) or not to leave the coalition (for a country in the coalition under consideration). Countries would undergo the increase in discipline from a 2 to a 3-country coalition, however, increasing coalition discipline and increasing free-riding possibilities at the same time prevent them from joining a coalition which has more than three countries. Therefore, the step from a 3 to a 6-country coalition is not incentive-compatible and countries would leave a coalition of 6 countries.

Charts 4, 5 and 6 illustrate the stability conditions (that is, the costs and gains from changing the group) as well as the dynamics of coalition formation.(35) The curves represent the different stability conditions for coalition 1. The two lower graphs represent the external stability conditions, ie:

$L_{c2}(k_1^*, k_2^*, n) - L_{c1}(k_1^* + 1, k_2^* - 1, n)$ *and*
$L_{nc}(k_1^*, k_2^*, n) - L_{c1}(k_1^* + 1, k_2^*, n)$.

The other two graphs describe internal stability, ie:

$L_{c1}(k_1^*, k_2^*, n) < L_{nc}(k_1^* - 1, k_2^*, n)$ *and*
$L_{c1}(k_1^*, k_2^*, n) < L_{c2}(k_1^* - 1, k_2^* + 1, n)$.

Coalition 1 is stable if all differences are negative. So for negative values of the graphs changes do not pay off and the corresponding equilibrium is stable. Positive values indicate that a country will gain from changing the group (coalition 1, coalition 2 or the fringe) and consequently the equilibrium is not stable.

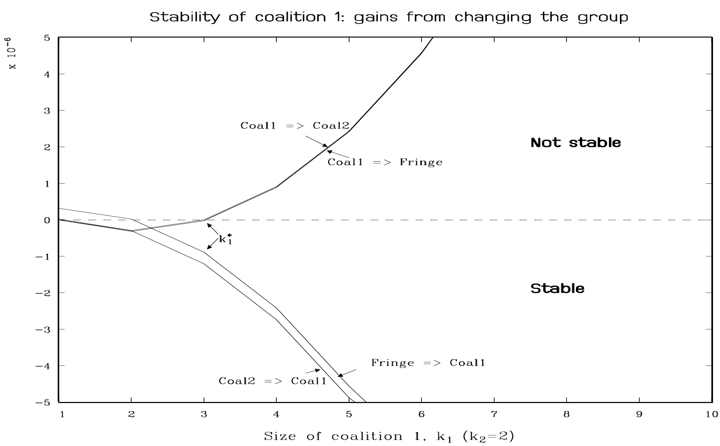
Chart 4 shows the 'gains from changing the group' for different sizes of coalition 1 when there are only two countries in coalition 2. When coalition 1 has more than two members there are no incentives to join coalition 1 any more. Members of coalition 2 prefer to remain in their own coalition which has only two members since they can benefit from the discipline in coalition 1 even if they are not members and they have to undergo less discipline themselves in their own smaller coalition. Members of the fringe would prefer to join the smaller coalition 2, if at

(35) All graphs are based on specific parameter values. Unless otherwise noted, the parameter values are $\alpha = 0.25$, $\beta = 0.5$, $\epsilon = 0.8$, $\nu = 0.05$, $\sigma = 1$, $\delta = 0.3$ and $n = 22$, $k_1 + k_2$ from 0 to 22.

all. The increasing decline of the graphs $Coal2 \Rightarrow Coal1$ and $Fringe \Rightarrow Coal1$ indicates that with increasing coalition size k_1 these disincentives to join coalition 1 become even larger, since the coalition-induced discipline which a joining member would have to undergo increases and outsiders benefit from the increasing coalition size. For three members in coalition 1 these members are indifferent between staying where they are or joining coalition 2 (and being its third member). For k_1^* more than three, coalition 1 members find it profitable to switch to the smaller coalition 2 or to join the fringe. Here, the internal stability condition $Coal1 \Rightarrow Coal2$ fails to hold with inequality for $k \geq 3$ and this is the reason why there is no stable coalition size for coalition 1 when there are only two members in coalition 2.

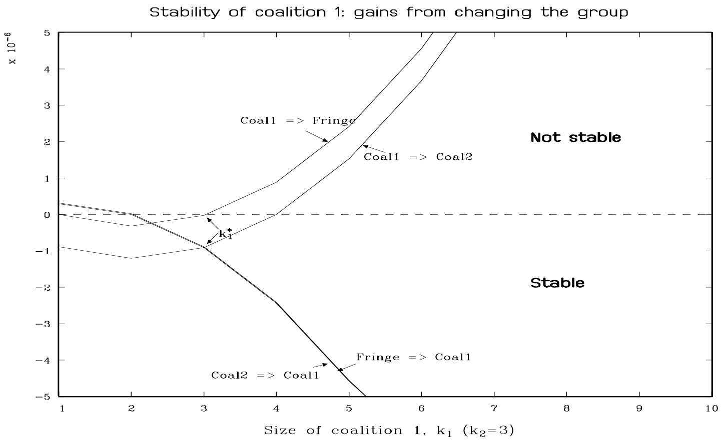
Chart 6 shows, on the other hand, the situation where there are $k_2 = 5$ countries in coalition 2, ie two more than the stable coalition size. Whereas the graphs of the switches between the fringe and coalition 1 are qualitatively the same as in chart 4, the graphs for the switches with coalition 2 have shifted. Members of coalition 2 benefit from

Chart 4: Stability of coalition 1 with varying k_1 (for $k_2 = 2$)[(a)]



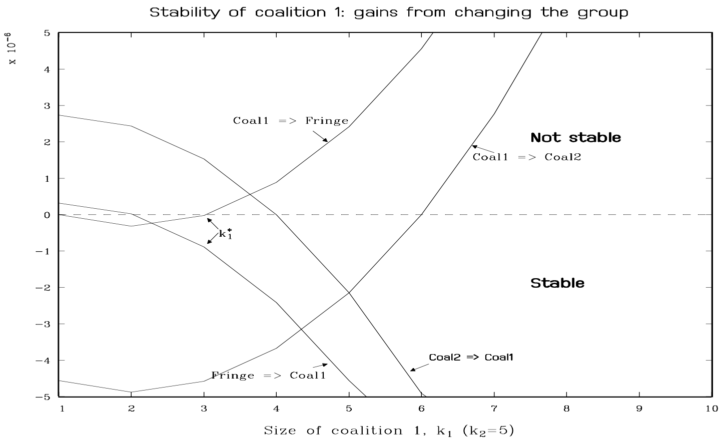
(a) Negative ‘gains from changing the group’ imply that the group is stable. The convex graphs show the internal stability conditions. The concave graphs show the external stability conditions. Here, no size of coalition 1 fulfils all stability criteria.

Chart 5: Stability of coalition 1 with varying k_1 (for $k_2 = 3$)[a]



(a) Only coalition size three fulfils all stability criteria.

Chart 6: Stability of coalition 1 with varying k_1 (for $k_2 = 5$)[a]



(a) No coalition size fulfils all stability criteria.

switching to coalition 1 until the latter has more than 4 members; this will prevent a stable coalition size below this number. On the other hand, coalition 1 members will prefer to switch to the fringe when there are more than three members in coalition 1. Again, there is no stable equilibrium.

Chart 5 finally shows a stable equilibrium for coalition 1. It is for exactly three countries in both coalitions and all other countries in the fringe; no country wishes to switch between the fringe and coalition 1, or between the two coalitions. The three charts only show the stability conditions for coalition 1. Since the model is symmetric the respective graphs for coalition 2 have exactly the same shape. As the stable equilibrium for coalition 1 is three, the stable equilibrium for coalition 2 is the same.

Comparison of all three charts shows that two graphs shift with increasing k_2 and two graphs do not change their values very much. The two graphs which remain unchanged are the potential gains from switching between the fringe and coalition 1. The reason is evident: both groups benefit in exactly the same way from an enlargement of coalition 2 against whom both groups play a Nash game. Hence, the relative positions which determine the gains from switches between the groups remain unchanged. The two graphs which determine the profitability of switching between coalition 1 and 2, however, shift with the size of coalition 2. The more countries there are in coalition 2, the less the incentive for a member of coalition 1 to join coalition 2. When coalition 2 has two members, a coalition 1 member for $k_1 = 3$ will be indifferent between joining either of the coalitions. Only when coalition 1 has more than three members will it be profitable to switch to coalition 2. A similar argument applies for the gains from switches between coalition 2 and coalition 1. Only when there are fewer members in coalition 1 than in coalition 2 will a member of coalition 2 not lose by switching to coalition 1. Hence, both graphs $Coal1 \Rightarrow Coal2$ and $Coal2 \Rightarrow Coal1$ shift to the right with increasing k_1. Whereas the first permits stable coalition sizes only for k_1 which is less than or equal to k_2, the latter shows stable coalition sizes which have at least k_2 members. In Chart 4 it is $Coal1 \Rightarrow Coal2$ which prohibits a stable coalition size of three which is the stable size implied by the conditions not to switch to the fringe. In Chart 6 it is $Coal2 \Rightarrow Coal1$ which is negative only above $k_1 = 4$ and therefore fails to give stability for a lower k_1.

The analysis above shows another feature: the symmetry of the game implies that the potential switches between the countries force a stable equilibrium to have the same coalition sizes for both coalitions (if stability has to be fulfilled with strict inequality). If coalition sizes are different, it would pay for a country to switch between the coalitions. Hence every combination which gives the same coalition sizes for both countries is stable against switches between the coalitions. It is the trade-off with the fringe which forces the system into a unique stable equilibrium of $k_1 = k_2 = 3$. This also explains why the stable coalition size is 3 in both cases, here and in the previous model where there is only one coalition.

Although the results have only been analysed for the case of two coalitions, one may speculate about what happens if there are three coalitions, four coalitions etc. The answer seems to be straightforward in the case where the coalitions play a Nash game against each other: the stable coalition size is determined by the trade-off between the fringe and co-operation within *one* of the coalitions. In the model here, the stable equilibrium of three countries in each coalition is 'dynamically' stable in the sense that it always pays for two countries to 'open' a new coalition and for a third one to join them. It pays as well for a fourth country which might have accidentally joined the coalition to leave it again. Hence including more coalitions should be straightforward, ie countries will prefer to split up in blocs of three countries to both options, staying in the fringe or forming a big coalition. In the former case, they gain from free-riding on the coalition discipline but they lose from incurring negative spillovers from non-cooperative policies with the other fringe countries. In the latter case, they gain from internalising the externalities with the other coalition members, however, they suffer too much coalition discipline. To join a smaller bloc seems to offer a 'balanced' solution to this cost-benefit analysis. Hence, there are mechanisms which exist in a symmetric world and are intrinsic to the process of coalition formation which can explain the existence of blocs that coordinate monetary policies. By contrast, the literature on optimum currency areas explains this phenomenon with asymmetries in the economic structures of the countries belonging to different blocs.[(36)]

[(36)] For a more detailed account on this literature, see eg Masson and Taylor (1992). A recent (empirical) study is Bayoumi and Eichengreen (1994).

5.3.2 The Stackelberg-Stackelberg game

In the Stackelberg-Stackelberg game the stable configuration is slightly different: coalition 1 consists of only two countries, coalition 2 of three countries in the stable equilibria. This is the result for more than four countries. For $n = 3$ countries the stable equilbrium has two countries in coalition 2 and none in coalition 1, for $n = 4$ countries both coalitions have a stable size of two countries.

The strategic position *between* the two coalitions is crucial for this result. In the two games where the coalitions play a Nash game against each other the stable coalition sizes are the same and determined by the trade-off with the fringe. This feature changes only when a difference in strategic positions of the coalitions is assumed.

The reason for the change in the stable coalition sizes is based on the argument outlined already above, ie that Stackelberg followers are better off than Stackelberg leaders in a game with strategic complements. Coalition 1, the 'double' Stackelberg leader, loses some of its strategic advantage against coalition 2 and the fringe which changes the relative positions sufficiently in order to result in a lower stable coalition size for the top coalition. The change in strategic 'equality' between the coalitions now allows an equilibrium which is not symmetric in the coalition sizes.

Extending this game to more than two coalitions is not as straightforward as in the Nash case. The top coalition is smaller since the Stackelberg follower's profit is higher in games with strategic complements. If another 'top' coalition is added, this effect certainly will be higher and the stable coalition size will be even smaller for this top coalition. Hence, a Stackelberg structure *amongst the coalitions* may set a limit to the maximum number of coalitions. However, a strict Stackelberg hierarchy of several blocs may be rather unrealistic and difficult to explain. An additional problem arises in justifying why one or two countries are a Stackelberg leader in a symmetric model. Therefore, an extension of this result to more than two coalitions may be questionable.

6 Conclusions

The analysis in this paper has shown that – in the framework of a standard international policy coordination model – the explicit possibility of coalition formation gives results different from the ones often assumed for coordination models with more than two countries. In the model here, countries will try to export inflation via an appreciation of the exchange rate as a response to a negative supply shock. Because all countries are doing so, none of them succeeds but all will have contracted their money supply too much. This provides the classic argument for benefits from international policy coordination: all countries could do better by agreeing not to try to export inflation. The situation changes when there is a union and outsiders. Since the union members are now bound by the coalition's discipline, an outsider can successfully export inflation without fearing that the insiders will try to do the same. Hence, there are additional 'gains from staying out' which arise even in the case of symmetric shocks.

The existence of a stable coalition size which does not include all countries has two important implications. First, it might be misleading to assume that only some countries join a coalition and to reduce the resulting two blocs to a two-country model. In particular, if the number of countries in the coalition bloc is higher than the stable coalition size, the resulting strategies are not optimal in the sense that some countries will prefer to leave the coalition. In fact, 'more' coordination is not always Pareto-superior to 'less' coordination if different coalition sizes are allowed for (and hence, different degrees of coordination). Second, in the discussion of asymmetric real world structures like 'hard-core EMU', it is not enough to focus only on asymmetric features, such as central bank preferences. Asymmetric results may also be produced by forces which evolve in a symmetric model, merely from the spillover effects of monetary policy.

If outsiders can join a competing bloc, several coalitions may coexist as an outcome of individually optimal decisions. Countries are always willing to undergo some amount of coalition discipline in exchange for the reduction of externalities arising from competitive appreciation. However, since the amount of discipline imposed on a single member increases with coalition size too large a coalition is not attractive for an individual country. There are two results. First, countries will find it profitable to form several coalitions of a smaller size rather than forming one big coalition or not forming a coalition at all. If the coalitions are in the same strategic position the coalition sizes in a stable equilibrium are the same. The specific stable coalition size is then determined by the trade-offs between entering a coalition or staying in the fringe. An extension to more than two coalitions seems to be straightforward, and will lead ultimately to the result that all countries will form a coalition with some other countries. These coalitions, however, will be of a rather small size (in the model here, they will have three members) when there are many countries.

Second, if there are differences in the strategic position of the coalitions the result changes. Due to the strategic complementarity of international monetary policy the 'leading' coalition turns out to be the group with the largest losses. Consequently, fewer countries will want to join it in a stable equilibrium than the other coalition. In this case, an extension to more than two countries is not straightforward.

Appendix A
Deriving the equilibrium strategies

A.1 Deriving the reduced form

The reduced form of the economy is calculated in two steps. First, the reduced form for employment is derived, then the reduced form for the CPI.

Reduced form for l_i
Substituting equation (**3**) into (**2**) gives:

$$p_i = m_i^e + \alpha l_i + x \tag{25}$$

In order to simplify the analysis it is assumed that the expected money supply (more precisely, its deviation) for wage-setters is $m_i^e = 0$. Substituting (**1**) and (**25**) into (**4**) yields:

$$l_i = m_i - m_i^e = m_i \tag{26}$$

Thus, employment changes one-for-one with the domestic money supply and is not affected by monetary policy abroad.

Reduced form of q_i
Deriving the reduced form for the CPI takes a bit longer. Substituting equation (**5**) into (**7**) gives:

$$q_i = p_i + \frac{1}{n-1}\beta \sum_{\substack{j=1 \\ j \neq i}}^{n} z_{ij} \tag{27}$$

Now the right-hand side of this equation will be expressed in terms of the money supplies. First, p_i is expressed in terms of m_i by substituting (**26**) into (**25**):

$$p_i = \alpha m_i + x \tag{28}$$

Next, $\frac{1}{n-1} \sum_{\substack{j=1 \\ j \neq i}}^{n} z_{ij}$ is expressed in terms of m_i. Summing (**6**) from $j = 1, j \neq i$ to n, which means that there are double sums on the right-hand side; dividing it by $(n-1)$ and subtracting this equation from itself (ie (**6**) for country i); and collecting terms yields:

$$\begin{aligned}(y_i \;-\; \tfrac{1}{n-1}\textstyle\sum_{\substack{j=1\\j\neq i}}^n y_j)\left(1-(1-\beta)\epsilon+\tfrac{1}{n-1}\beta\epsilon\right)\\ = \;\delta\tfrac{n}{n-1}\textstyle\sum_{\substack{j=1\\j\neq i}}^n z_{ij}-(r_i-\tfrac{1}{n-1}\textstyle\sum_{\substack{j=1\\j\neq i}}^n r_j)\left((1-\beta)\nu-\tfrac{1}{n-1}\beta\nu\right) \quad \textbf{(29)}\end{aligned}$$

Equation (**8**) is summed for all countries $j, j \neq i$, divided by $(n-1)$ and subtracted from equation (**8**) for country i. Note that $z^e_{ij} = 0$ which excludes speculative bubbles (for an explanation, see Canzoneri and Henderson (1991)).[37] Together with equations (**9**) and (**5**) this yields:

$$r_i - \tfrac{1}{n-1}\textstyle\sum_{\substack{j=1\\j\neq i}}^n r_j = \left(\tfrac{\beta n}{n-1}-1\right)\tfrac{1}{n-1}\textstyle\sum_{\substack{j=1\\j\neq i}}^n z_{ij} \qquad \textbf{(30)}$$

Using equations (**1**) and (**26**) (for all countries $j = 1, \ldots, n$) for the left-hand side of equation (**29**) and equation (**30**) for the right-hand side, then solving for the z_{ij}'s gives:

$$\frac{1}{n-1}\sum_{\substack{j=1\\j\neq i}}^n z_{ij} = \frac{(1-\alpha)\left(1-(1-\beta)\epsilon+\frac{1}{n-1}\beta\epsilon\right)}{\delta n+\left(1-\frac{\beta n}{n-1}\right)\left((1-\beta)\nu-\frac{1}{n-1}\beta\nu\right)}\left(m_i-\frac{1}{n-1}\sum_{\substack{j=1\\j\neq i}}^n m_j\right) \quad \textbf{(31)}$$

Substituting equations (**31**) and (**28**) into (**27**) the reduced form for q_i is obtained:

$$\begin{aligned} q_i \;&=\; \underbrace{\left(\alpha+\frac{\beta(1-\alpha)\left(1-\epsilon(1-\frac{n}{n-1}\beta)\right)}{\delta n+\nu(1-\frac{n}{n-1}\beta)^2})\right)}_{\lambda}m_i-\underbrace{\frac{\beta(1-\alpha)\left(1-\epsilon(1-\frac{n}{n-1}\beta)\right)}{\delta n+\nu(1-\frac{n}{n-1}\beta)^2}\frac{1}{n-1}}_{\kappa}\sum_{\substack{j=1\\j\neq i}}^n m_j+x \\ &=\; \boldsymbol{\lambda \mathrm{m_i}-\kappa\sum_{\substack{j=1\\j\neq i}}^{n}\mathrm{m_j}+\mathrm{x}} \qquad \textbf{(32)}\end{aligned}$$

[37] Furthermore, note that $z_{ij} = -z_{ji}$. This gives:

$$\sum_{\substack{j=1\\j\neq i}}^n z_{ij} - \frac{1}{n-1}\sum_{\substack{j=1\\j\neq i}}^n\sum_{\substack{l=1\\l\neq j}}^n z_{jl} = \frac{n}{n-1}\sum_{\substack{j=1\\j\neq i}}^n z_{ij}$$

The sign of the coefficients

It will be shown below that the signs of the coefficients λ and κ in equation (**32**) are positive.

λ is positive since α is positive and it can easily be shown that the second term is positive (α, ϵ and β are positive and smaller than one).

κ can be rewritten as $\kappa = \frac{\lambda - \alpha}{n-1}$. This fraction is positive because λ is α plus a positive term and n is larger than 2.

A.2 Solving the equilibrium with one coalition

The countries $j = 1, \ldots, k$ are members of the coalition C, the countries $i = k+1, \ldots, n$ are not in the coalition. The equilibrium is derived in three steps:

- the reaction function of a country outside the coalition;
- the reaction function of a coalition member; and
- the equilibrium through equating the reaction functions.

Two types of behaviour of the coalition against the non-members can be distinguished. The cooperation-Nash equilibrium implies that the coalition members co-operate (ie minimise a joint loss function) amongst the members and then play a Nash game against the non-members. The cooperation-Stackelberg equilibrium implies Stackelberg behaviour of the coalition against the non-members.

A.2.1 The cooperation-Nash equilibrium

The optimisation problem which has to be solved by the monetary authority of a country can be summarised as follows. Outside the coalition L_i is minimised with respect to the country's own money supply; in the coalition L_i is minimised with respect to the money supplies of all coalition members.

$$min_{m_i,m_j} L_i = \frac{1}{2}\Big(\sigma m_i^2 + (\lambda m_i - \kappa \sum_{\substack{j=1 \\ j \neq i}}^{n} m_j + x)^2\Big)$$

The reaction function of a country outside the coalition

A country which is not in the coalition sets its own money supply so as to minimise its losses. It takes the other money supplies as given (Nash conjectures).

$$min_{m_i}\ L_i \qquad \text{s.t.}\ m_j = \overline{m_j} \quad \forall\, j \neq i$$

The first-order condition $\frac{\partial L_i}{\partial m_i} = 0$ gives $m^*_{i,nc}$ as a function of all other money supplies.

The symmetric set-up implies that all countries have the same degree of conservativeness σ. Hence, it can be assumed that all countries outside the coalition have the same optimal money supply m^*_{nc}. We can write the money supply of a non-member as a function of the coalition's money supply:

$$m^*_{nc} = \underbrace{\frac{\lambda\kappa}{\sigma + \lambda^2 - \lambda\kappa(n-k-1)}}_{\theta} \sum_{j=1}^{k} \overline{m}_{j,c} - \underbrace{\frac{\lambda}{\sigma + \lambda^2 - \lambda\kappa(n-k-1)}}_{\vartheta} x \qquad \textbf{(33)}$$

The reaction function of a coalition member

The coalition solves its optimisation problem subject to a given money supply of the non-members:

$$min_{m_j \in C}\ \mathcal{L} = \sum_{j=1}^{k} \frac{1}{k} L_j \qquad \text{s.t.}\ m_i = \overline{m}_{i,nc} \quad \forall\, i = k+1, \ldots, n$$

The first-order condition gives $\frac{\partial \mathcal{L}}{\partial m_j} = \frac{1}{k}\frac{\partial L_j}{\partial m_j} + \sum_{\substack{h=1 \\ h \neq j}}^{k} \frac{1}{k}\frac{\partial L_h}{\partial m_j} = 0$. Together with the symmetry assumption for the coalition money supplies $m^*_{j,c} = m^*_c$ for all $j = 1, \ldots, k$ the coalition member's reaction function dependent on the non-members' money supplies is obtained:

$$m^*_c = \frac{\lambda - \kappa(k-1)}{\sigma + \lambda^2 + \kappa^2(k-1)^2 - 2\kappa\lambda(k-1)} \left(\kappa \sum_{i=k+1}^{n} \overline{m}_{i,nc} - x\right) \qquad \textbf{(34)}$$

The equilibrium

Replacing the non-members' money supply in equation (**34**) with equation (**33**) gives the equilibrium money supply of a coalition member:

$$m_c^* = -\underbrace{\frac{(\lambda-\kappa(k-1))\big(\sigma+\lambda^2+\kappa\lambda\big)}{(\sigma+\lambda^2)\eta+\kappa^3\lambda(k-1)(n-1)+\kappa^2\lambda^2(k(n-k)-2(n-1))}}_{\rho}\,x$$

$$\text{with:}\quad \eta = \sigma + \lambda^2 + \kappa^2(k-1)^2 - \kappa\lambda(k+n-3)$$

and the equilibrium money supply of a non-member:

$$m_{nc}^* = -\underbrace{\frac{\lambda(\kappa k\rho + 1)}{\sigma + \lambda^2 - \kappa\lambda(n-k-1)}}_{\omega}\,x$$

The sign of the coefficients

It will be shown that the signs of the coefficients θ, ϑ, ρ and ω are positive.

The nominator of θ is positive because λ and κ are positive. The denominator is positive as κ can be rewritten as $\frac{\lambda-\alpha}{n-1}$.

ϑ can be written as $\frac{\theta}{\kappa}$ and is positive as θ and κ are positive.

The nominator of ρ is positive because λ and κ are positive and κ can be rewritten as $\frac{\lambda-\alpha}{n-1}$. The denominator of ρ is positive for all feasible values of k that is, k between 1 and n. For the proof two cases will be distinguished: the first case where $\lambda \geq 2\alpha$ and the second case where $\lambda < 2\alpha$.

- **First case:** $\lambda \geq 2\alpha$.

  The denominator of ρ can be rewritten as:

$$\underbrace{\sigma(\eta+\lambda^2)}_{\tau_1} + \underbrace{\kappa^3\lambda(k-1)(n-1) + \kappa^2\lambda^2(k(n-k)-2(n-1))}_{\tau_2}$$

  τ_1 can be written as:

$$\tau_1 = \sigma\Bigg(\sigma + \frac{1}{(n-1)^2}\Big(\underbrace{\lambda(\lambda-2\alpha)}_{\geq 0 \text{ for } \lambda\geq 2\alpha}\underbrace{\big(n(n-k)+k(k-1)\big)}_{>0 \text{ since } 1\leq k\leq n}$$
$$+\lambda\alpha\big(n\underbrace{(3n-k-4)}_{>0 \text{ for } n>2}+k+1\big)+\alpha^2(k-1)^2\Big)\Bigg)$$

τ_1 is positive for all feasible values of k and n when $\lambda \geq 2\alpha$.

τ_2 can be written as:

$$\frac{\lambda}{(n-1)^2}\Big(\underbrace{\lambda^2\alpha n(n-k)}_{\geq 0 \text{ since } k\leq n}+\underbrace{\lambda\alpha^2 n(k-2)}_{\geq 0 \text{ for } k\geq 2}+\underbrace{\alpha^2 k(\lambda-\alpha)}_{>0 \text{ since } \lambda>\alpha}+\alpha^3\Big)$$

τ_2 is positive for all $k \geq 2$ as well as for $k = 1$ which yields $\frac{\lambda}{(n-1)^2}\lambda\alpha(n-1)(\lambda n-\alpha)$, which is positive as $\lambda > \alpha$ and $n \geq 3$.

Hence, the denominator is positive for all k between 1 and n when $\lambda \geq 2\alpha$.

- **Second case:** $\lambda < 2\alpha$.

  The denominator of ρ can be interpreted as a convex[38] parabola in k. If the parabola is downward sloping in $k = n$ all feasible values of k are left of the minimum, on the monotonically decreasing part of the parabola. Hence, the necessary and sufficient condition that the denominator of ρ is positive is that it takes a positive value in the lowest feasible point, $k = n$.

  The derivative of the parabola in $k = n$ can be written as:

$$\frac{\partial \text{ denominator}}{\partial k}\bigg|_{k=n} = \underbrace{-\frac{\lambda\alpha}{n-1}(n\lambda+\alpha)}_{\text{intercept } <0}+\underbrace{(\lambda-2\alpha)}_{<0 \text{ for } \lambda<2\alpha}\sigma$$

  This expression can be interpreted as a linear function in σ with a negative intercept. Since σ can take any non-negative value a sufficient condition for a negative derivative is that $\lambda < 2\alpha$.
  The value of the denominator in $k = n$ is:

$$\frac{1}{n-1}\underbrace{(\sigma^2(n-1)+\sigma\lambda(\lambda-\alpha)+\sigma\lambda^2(n-1))}_{>0 \text{ since } \lambda>\alpha}+\frac{1}{n-1}\underbrace{\lambda\alpha^2(\lambda-\alpha)}_{>0} > 0$$

  Hence, for $\lambda < 2\alpha$, the denominator of ρ is always positive. q.e.d.

ω is positive since λ, κ and ρ are positive.

(38) The coefficient of k^2 is $\sigma\kappa^2 \geq 0$.

A.2.2 The cooperation-Stackelberg equilibrium

In this game the coalition takes explicit account of the reaction of the non-members.

The reaction function of a country outside the coalition

A country outside the coalition has to solve the same problem as above. Hence, the reaction function of a non-member is as in equation (**33**).

The reaction function of a coalition member

The coalition solves its optimisation problem subject to the reaction functions of the non-members which are dependent on the coalition's money supply:

$$min_{m_j \in C}\ \mathcal{L} = \sum_{j=1}^{k} L_j \qquad \text{s.t. } m_i = \theta \sum_{j=1}^{k} m_{j,c} - \vartheta x \quad \forall\, i = k+1, \ldots, n$$

The first-order condition together with the symmetry assumption for the coalition money supplies $m_{j,c}^* = m_c^*$ for all $j = 1, \ldots, k$ gives the coalition member's money supply. Since this is already independent of the non-members' money supplies it is the equilibrium money supply of a coalition member.

$$m_c^* = -\underbrace{\frac{(1+(n-k)\theta)(\lambda+\kappa-k\kappa(1+(n-k)\theta))}{\sigma+(\lambda+\kappa-k\kappa(1+(n-k)\theta))^2}}_{\varphi}\, x \tag{35}$$

The equilibrium

Replacing the coalition members' money supply in equation (**33**) with equation (**35**) gives the equilibrium money supply of a non-coalition member:

$$m_{nc}^* = -\underbrace{(\kappa k \varphi + 1)\vartheta}_{\phi}\, x \tag{36}$$

The sign of the coefficients

It will be shown that the signs of the coefficients φ and ϕ are positive.

The denominator of φ is positive since it is the sum of the positive σ and a squared expression. The nominator of φ is positive if:

$$\lambda+\kappa-k\kappa(1+(n-k)\theta) = \frac{\sigma(\lambda\frac{n-k}{n-1}+\alpha\frac{k-1}{n-1})+\frac{1}{n-1}(\lambda\alpha(\lambda n-\alpha))}{\sigma+\lambda^2-\lambda\kappa(n-k-1)} > 0$$

The denominator of the second expression is positive as proved above for θ, the nominator is positive since $\lambda > \alpha$ and $n > 3$.

ϕ is positive since it is the sum of positive expressions.

A.3 Solving the equilibrium with two coalitions

The number of countries in coalition 1 (C_1) is k_1, the number of countries in coalition 2 (C_2) is k_2 and the number of countries in the fringe (NC) is f; and $n = f + k_1 + k_2$ where n is the total number of countries.

A.3.1 The Nash-Nash game

The reaction function of a country outside the coalition

As before, a country which is not in a coalition acts according to its Nash reaction function. It sets its own money supply so as to minimise its losses while taking all other money supplies as given.

$$\begin{aligned} m^*_{nc} &= \underbrace{\tfrac{\lambda\kappa}{\sigma+\lambda^2-\lambda\kappa(f-1)}}_{\theta}\Big(\sum_{j=1}^{k_1}\overline{m}_{j,c_1} + \sum_{j=1}^{k_2}\overline{m}_{j,c_2}\Big) - \underbrace{\tfrac{\lambda}{\sigma+\lambda^2-\lambda\kappa(f-1)}}_{\vartheta} x \\ &= \theta\sum_{j=1}^{k_1}\overline{m}_{j,c_1} + \theta\sum_{j=1}^{k_2}\overline{m}_{j,c_2} - \vartheta x \end{aligned} \tag{37}$$

The reaction function of a member of coalition 1

The coalition solves its optimisation problem subject to a given money supply of the non-members and of coalition 2:

$$min_{m_i \in C_1}\, \mathcal{L} = \sum_{i=1}^{k_1}\frac{1}{k_1}L_i \qquad \text{s.t. } m_j = \overline{m}_j \quad \forall\, j \notin C_1$$

The first-order condition gives $\frac{\partial \mathcal{L}}{\partial m_{i,c_1}} = \frac{1}{k_1}\frac{\partial L_i}{\partial m_{i,c_1}} + \sum_{\substack{h=1\\h\neq i}}^{k_1}\frac{1}{k_1}\frac{\partial L_h}{\partial m_{i,c_1}} = 0$. Together with the symmetry assumption for the money supplies $m^*_{i,c_1} = m^*_{c_1}$ for all $i = 1,\ldots,k_1$, the coalition 1 member's reaction function is obtained as a function of the non-members' and the coalition 2 money supplies:

$$\begin{aligned} m^*_{c_1} &= \underbrace{\tfrac{\lambda-\kappa(k_1-1)}{\sigma+(\lambda-\kappa(k_1-1))^2}}_{\psi_1}\kappa(\sum_{j=1}^{k_2}\overline{m}_{j,c_2}+\sum_{j=1}^{f}\overline{m}_{j,nc})-\psi_1 x \\ &= \psi_1\kappa\sum_{j=1}^{k_2}\overline{m}_{j,c_2}+\psi_1\kappa\sum_{j=1}^{f}\overline{m}_{j,nc}-\psi_1 x \end{aligned}$$

The reaction function of a member of coalition 2

Coalition 2 solves its optimisation problem subject to a given money supply of the non-members and of coalition 1:

$$min_{m_i\in C_2}\,\mathcal{L}=\sum_{i=1}^{k_2}\frac{1}{k_2}L_i \qquad \text{s.t. } m_j=\overline{m}_j \quad \forall\, j\notin C_2$$

The optimisation problems of the two coalitions are mirrors. Consequently, the same is true for the resulting reaction functions.

$$\begin{aligned} m^*_{c_2} &= \underbrace{\tfrac{\lambda-\kappa(k_2-1)}{\sigma+(\lambda-\kappa(k_2-1))^2}}_{\psi_2}\kappa\,(\sum_{j=1}^{k_1}\overline{m}_{j,c_1}+\sum_{j=1}^{f}\overline{m}_{j,nc})-\psi_2 x \\ &= \psi_2\kappa\sum_{j=1}^{k_1}\overline{m}_{j,c_1}+\psi_2\kappa\sum_{j=1}^{f}\overline{m}_{j,nc}-\psi_2 x \end{aligned}$$

The equilibrium

Equating all three reaction functions yields the Nash equilibrium:

$$\begin{aligned} m_{c_1} &= \underbrace{-\frac{1}{\mu}(1+\kappa k_2\psi_2)(1+\kappa f\vartheta)\psi_1\,x}_{\rho_1} \\ m_{c_2} &= \underbrace{-\frac{1}{\mu}(1+\kappa k_1\psi_1)(1+\kappa f\vartheta)\psi_2\,x}_{\rho_2} \\ m_{nc} &= \underbrace{-\frac{1}{\mu}(1+\kappa k_1\psi_1)(1+\kappa k_2\psi_2)\vartheta\,x}_{\varrho} \end{aligned}$$

with: $\mu = (1+\kappa k_1\psi_1)(1+\kappa k_2\psi_2) -$
$(1+\kappa f\vartheta)(\kappa k_1\psi_1(1+\kappa k_2\psi_2)+\kappa k_2\psi_2(1+\kappa k_1\psi_1))$

The sign of the coefficients

It will be shown that the signs of the coefficients θ, ϑ, ψ_1, ψ_2, ρ_1, ρ_2 and ϱ are positive.

θ and ϑ are identical to the coefficients in the game with one coalition if k_1+k_2 is replaced by k.

The denominator of ψ_1 is positive since it is the sum of two positive expressions. The nominator is positive since $\kappa = \frac{\lambda-\alpha}{n-1}$. The proof that ψ_2 is positive corresponds to the proof for ψ_1.

ρ_1, ρ_2 and ϱ are positive if μ is positive since the nominators are positive. μ can be split up in its nominator and its denominator. The following variables will be used: $den_{\psi_1} = \sigma + nom^2_{\psi_1}$, $den_{\psi_2} = \sigma + nom^2_{\psi_2}$, and $f = n - k_1 - k_2$. Here and subsequently, den_{var} stands for 'denominator of variable *var*', nom_{var} stands for 'nominator of variable *var*'.

$$\begin{aligned}
\mu &= \frac{1}{den_{\psi_1}\, den_{\psi_2}\, den_\vartheta}\Big[\left(\sigma+nom^2_{\psi_1}\right)\left(\sigma+nom^2_{\psi_2}\right)\Big(\sigma+\lambda\big(\lambda-\kappa(f-1)\big)\Big)\\
&\quad -\kappa^2 k_1 k_2 nom_{\psi_1} nom_{\psi_2}\Big(\sigma+\lambda\big(\lambda-\kappa(f-1)\big)\Big) - \kappa^2\lambda f\,(k_1 nom_{\psi_1}\\
&\quad (\sigma+nom^2_{\psi_2}) + 2k_1k_2\kappa nom_{\psi_1} nom_{\psi_2} + k_2 nom_{\psi_2}(\sigma+nom^2_{\psi_1}))\Big]\\
&= \frac{1}{den_{\psi_1}\, den_{\psi_2}\, den_\vartheta}\ \left[\sigma^3+\sigma^2\left(nom^2_{\psi_1}+nom^2_{\psi_2}+\lambda(\lambda-\kappa(f-1))\right)+\right.\\
&\quad \sigma\ \Big(nom^2_{\psi_1} nom^2_{\psi_2} + \lambda(\lambda-\kappa(f-1))(nom^2_{\psi_1}+nom^2_{\psi_2})\\
&\qquad -\kappa^2 k_1 k_2 nom_{\psi_1} nom_{\psi_2} - \kappa^2\lambda f(k_1 nom_{\psi_1} + k_2 nom_{\psi_2})\Big)\\
&\quad +\lambda(\lambda-\kappa(f-1))\left(nom^2_{\psi_1} nom^2_{\psi_2} - \kappa^2 k_1 k_2 nom_{\psi_1} nom_{\psi_2}\right) - \kappa^2\lambda f\\
&\quad \left.\Big(k_1 nom_{\psi_1} nom^2_{\psi_2} + 2k_1k_2\kappa nom_{\psi_1} nom_{\psi_2} + k_2 nom^2_{\psi_1} nom_{\psi_2}\Big)\right]\\
&= \underbrace{\frac{1}{den_{\psi_1}\, den_{\psi_2}\, den_\vartheta}}_{\geq 0,\ see\ \psi_1,\psi_2,\vartheta}\Big[\underbrace{\sigma^3}_{\geq 0} + \sigma^2\underbrace{\left(nom^2_{\psi_1}+nom^2_{\psi_2}+\lambda(\lambda-\kappa(f-1))\right)}_{\geq 0,\ see\ \psi_1,\psi_2,\vartheta}\\
&\qquad +\sigma T_a + T_b\Big]
\end{aligned}$$

Since the denominators of ψ_1, ψ_2 and ϑ are positive, μ is positive if T_a and T_b are positive. T_a can be rewritten as:

$$\begin{aligned} T_a &= \lambda(\lambda+\kappa)\Big(nom_{\psi_1}(\underbrace{\lambda-\kappa(n-k_2-1)}_{\geq 0}) + nom_{\psi_2}(\underbrace{\lambda-\kappa(n-k_1-1)}_{\geq 0}) \\ &\quad + nom_{\psi_1} nom_{\psi_2}(\lambda+\kappa)(\underbrace{\lambda-\kappa(k_1+k_2-1)}_{\geq 0})\Big) \geq 0 \end{aligned}$$

T_a is positive because the nominators of ψ_1 and ψ_2 are positive and because $\kappa = \frac{\lambda-\alpha}{n-1}$ and $k_1 + k_2 \leq n$ with $0 \leq k_1, k_2 \leq n$.

T_b can be rewritten as:

$$T_b = nom_{\psi_1} nom_{\psi_2}(\lambda+\kappa)^2 \lambda(\underbrace{\lambda-\kappa(n-1)}_{\geq 0}) \geq 0$$

T_b is positive because the nominators of ψ_1 and ψ_2 are positive (see ψ_1, ψ_2) and because $\kappa = \frac{\lambda-\alpha}{n-1}$.

A.3.2 The Stackelberg-Stackelberg game

The reaction function of a country outside the coalitions

A country outside the coalition minimises its own losses given the money supplies of all other countries. Hence, the reaction function is the same as in equation (**37**).

The reaction function of a country in coalition 2

Coalition 2 acts as a Stackelberg leader towards the fringe, ie it takes the non-members' reaction functions into account when minimising the coalition's losses. However, it takes the other coalition's money supplies as given.

$$\begin{aligned} min_{m_i \in C_2} \quad & \mathcal{L} = \sum_{i=1}^{k_2} \frac{1}{k_2} L_i & \\ \text{s.t.} \quad & m_{j,c_1} = \overline{m}_{j,c_1} & \forall j \in C_1 \\ & m_{j,nc} = \theta \sum_{j=1}^{k_2} \overline{m}_{j,c_2} + \theta \sum_{i=1}^{k_1} m_{i,c_1} - \vartheta x & \forall j \in NC \end{aligned}$$

The first-order condition gives: $\frac{\partial \mathcal{L}}{\partial m_{i,c_2}} = \frac{1}{k_2}\frac{\partial L_i}{\partial m_{i,c_2}} + \sum_{\substack{h=1\\h\neq i}}^{k_2} \frac{1}{k_2}\frac{\partial L_h}{\partial m_{i,c_2}} = 0$. Together with the symmetry assumption for the money supplies $m^*_{i,c_2} = m^*_{c_2}$ for all $i = 1, \ldots, k_2$ the coalition 2 member's reaction function is obtained dependent on the money supplies of coalition 1:

$$m^*_{c_2} = \underbrace{\frac{(\lambda+\kappa-k_2\kappa(1+f\theta))(1+\theta f)}{\sigma+(\lambda+\kappa-k_2\kappa(1+f\theta))^2}}_{\eta_2} \kappa \sum_{j=1}^{k_1} \overline{m}_{j,c_1} - \eta_2 x = \eta_2 \kappa \sum_{j=1}^{k_1} \overline{m}_{j,c_1} - \eta_2 x \tag{38}$$

The reaction function of a country in coalition 1

Coalition 1 acts as Stackelberg leader against coalition 2 and the fringe ie it minimises its loss function taking the reactions of all other groups into consideration.

$$min_{m_i \in C_1} \mathcal{L} = \sum_{i=1}^{k_1} \frac{1}{k_1} L_{i,c_1}$$

$$\text{s.t. } m_{j,c_2} = \eta_2 \kappa \sum_{i=1}^{k_1} m_{i,c_1} - \eta_2 x \qquad \forall j \in C_2$$

$$m_{j,nc} = \theta \sum_{i=1}^{k_1} m_{i,c_1} + \theta \sum_{j=1}^{k_2} \left(\eta_2 \kappa \sum_{i=1}^{k_1} m_{i,c_1} - \eta_2 x \right) - \vartheta x \ \ \forall j \in NC$$

The first-order condition gives: $\frac{\partial \mathcal{L}}{\partial m_i} = \frac{1}{k_1}\frac{\partial L_i}{\partial m_i} + \sum_{\substack{h=1\\h\neq i}}^{k_1} \frac{1}{k_1}\frac{\partial L_h}{\partial m_i} = 0$. Together with the symmetry assumption for the money supplies $m^*_{i,c_1} = m^*_{c_1}$ for all $i = 1, \ldots, k_1$ the coalition 1 member's reaction function is obtained:

$$m^*_{c1} = - \underbrace{\frac{(1+f\theta)(1+\eta_2 k_2 \kappa)(\lambda+\kappa-k_1\kappa(1+f\theta)(1+\eta_2 k_2\kappa))}{\sigma+(\lambda+\kappa-k_1\kappa(1+f\theta)(1+\eta_2 k_2 \kappa))^2}}_{\omega_1} x = \omega_1 x \tag{39}$$

The equilibrium

Since equation (**39**) is not dependent on other countries' money supplies it is already the equilibrium money supply of a country in coalition 1. Replacing $m_{j,c1}$ in equation (**38**) with equation (**39**) gives coalition 2's equilibrium money supply. Correspondingly, m^*_{nc} is calculated by replacing the coalitions' money supplies in equation (**37**).

$$m^*_{c_1} \ = \ -\omega_1 x$$

$$
\begin{aligned}
m^*_{c_2} &= -\underbrace{\eta_2(1+k_1\omega_1\kappa)}_{\omega_2}x \\
m^*_{nc} &= -\underbrace{(\theta(k_1\omega_1+k_2\omega_2)+\vartheta)}_{\upsilon}x
\end{aligned}
$$

The sign of the coefficients

It will be shown that the coefficients of the reaction functions, θ, η_2 and ω_1, and the coefficients of the equilibrium policies, ω_1, ω_2 and υ, are positive.

It has been shown above that θ and ϑ are positive.

The denominator of η_2 is positive since it is the sum of two positive terms. The nominator is positive if:

$$
\begin{aligned}
\lambda+\kappa-k_2\kappa(1+\theta f) &> 0 \\
\frac{\sigma(\lambda+\kappa(1-k_2))+\lambda(\kappa+\lambda)(\kappa(1-f-k_2)+\lambda)}{\underbrace{\sigma+\lambda^2-\lambda\kappa(f-1)}_{>0 \text{ see } \theta}} &> 0 \\
\sigma\Big(\lambda-(\lambda-\alpha)\underbrace{\tfrac{k_2-1}{n-1}}_{\leq 1}\Big)+\lambda(\kappa+\lambda)(\lambda-\underbrace{\kappa(n-k_1-1))}_{\leq\lambda \text{ since } \kappa=\frac{\lambda-\alpha}{n-1}} &> 0
\end{aligned}
$$

The denominator of ω_1 is positive as it is the sum of two positive terms. The nominator of ω_1 is positive if:

$$
\begin{aligned}
\lambda+\kappa-k_1\kappa\Big(1+f\theta\Big)\Big(1+\eta_2k_2\kappa\Big) &\geq 0 \\
\frac{1}{den_{\eta_2}}\Big[\Big(\lambda+\kappa-k_1\kappa(1+f\theta)\Big)den_{\eta_2}-k_1\kappa(1+f\theta)^2k_2\kappa\Big(\lambda+\kappa-k_2\kappa(1+f\theta)\Big)\Big] &\geq 0
\end{aligned}
$$

Since the denominator of η_2 is positive (for proof see η_2) this term is positive if the nominator is positive. The nominator can be rewritten as:

$$
\sigma\Big(\underbrace{\lambda+\kappa-k_1\kappa(1+f\theta)}_{=T_1\geq 0,\text{ see } \eta_1}\Big)+\Big(\underbrace{\lambda+\kappa-k_2\kappa(1+f\theta)}_{=T_2\geq 0,\text{ see } \eta_2}\Big)\Big(\underbrace{T_1*T_2-k_1k_2\kappa^2(1+f\theta)^2}_{:=T_{III}}\Big)
$$

This expression is positive if T_{III} is positive.

$$
T_{III} = \underbrace{\frac{1}{den_\theta}}_{\geq 0}\Big[\sigma\Big(\underbrace{\lambda-\kappa(k_1+k_2-1)}_{\geq 0}\Big)+(\lambda+\kappa)\lambda\alpha\Big] \geq 0
$$

T_{III} is positive because the denominator of θ is positive and because $\kappa = \frac{\lambda - \alpha}{n-1}$ and $0 \leq k_1, k_2 \leq n$ with $k_1 + k_2 \leq 0$.

ω_2 is positive as η_2 and ω_1 are positive.

υ is positive as it is the sum of positive expressions.

Appendix B
Simulation analysis

B.1 Simulation analysis with one coalition

B.1.1 The cooperation-Nash game

The results of the simulation analysis are presented in the following. There is no *a priori* reason for a specific parameter value, hence values in the middle of the plausible parameter ranges were chosen. The elasticity of the demand with respect to the interest rate was set relatively low ($\nu = 0.05$), while the relative importance of the two policy objectives, σ, was set to 1. Since the parameter values are chosen arbitrarily, a sensitivity analysis was performed. The 'standard values', that is the values if not noted otherwise, are:

$$\alpha = \beta = \tau = 0.5, \quad \nu = 0.05, \; \delta = 0.3, \quad \sigma = 1,$$

For the cooperation-Nash game, the results of the simulation analysis were already discussed in detail in Section 4. Therefore, attention will be restricted to the results of the robustness analysis.

All parameter values except for σ may vary between 0 and 1. The results have been tested by increasing the parameter values in steps of 0.1 from 0.1 to 0.9. σ, the relative weight of the employment target, may be any positive number. Consequently, the results have been tested for $\sigma = 0.1$ which attaches ten times as much importance to the inflation target as to the employment target. σ was increased in steps of 0.5 up to a level where results no longer changed. The analysis was stopped at $\sigma = 10$ in the univariate analysis and at $\sigma = 3.1$ in the multivariate analysis. The robustness analysis was performed as a univariate and a multivariate analysis.

Univariate analysis

The univariate analysis varies one parameter at a time while setting the other parameters to their standard values. Table A shows the results of the univariate analysis where the number of countries n was varied from 3 to 20. The numbers in the tables represent the stable coalition size. In the univariate analysis the stable coalition size was always unique, at three.

Table A: *Nash game:* univariate sensitivity analysis

Parameter		*Number of countries n =*							
		3	4	5	6	7	8	9	10-20
α	0.1-0.9	3	3	3	3	3	3	3	3
β	0.1-0.9	3	3	3	3	3	3	3	3
δ	0.1-0.9	3	3	3	3	3	3	3	3
ϵ	0.1-0.9	3	3	3	3	3	3	3	3
ν	0.1-0.9	3	3	3	3	3	3	3	3
σ	0.1-10	3	3	3	3	3	3	3	3

Table B: *Nash game:* multivariate sensitivity analysis (σ, α, β)

Parameter			*Number of countries n =*									
σ	α	β	3	4	5	6-7	8	9-10	11	12-13	14	15
1.1-3.1	0.1-0.9	0.1-0.9	3	3	3	3	3	3	3	3	3	3
< 1.0	all except	all except	3	3	3	3	3	3	3	3	3	3
0.6	0.1	0.9	3	3	4	3	3	3	3	3	3	3
0.1	0.1	0.6	3	4	4	4	4	3	3	3	3	3
		0.7	3	4	4	4	4	4	4	3	3	3
		0.8	3	4	4	5	4	4	4	4	4	3
		0.9	3	4	4	5	5	5	4	4	4	4
0.1	0.2	0.6	3	4	4	4	3	3	3	3	3	3
		0.7	3	4	4	4	4	4	3	3	3	3
		0.8	3	4	4	4	4	4	4	4	3	3
		0.9	3	4	4	5	5	4	4	4	4	4
0.1	0.3	0.6	3	3	4	3	3	3	3	3	3	3
		0.7	3	4	4	4	4	3	3	3	3	3
		0.8	3	4	4	4	4	4	4	3	3	3
		0.9	3	4	4	4	4	4	4	4	3	3
	0.4	0.7	3	3	4	3	3	3	3	3	3	3
		0.8	3	4	4	4	4	3	3	3	3	3
		0.9	3	4	4	4	4	4	3	3	3	3
	0.5	0.8	3	3	4	3	3	3	3	3	3	3
		0.9	3	4	4	4	3	3	3	3	3	3

Multivariate analysis

The multivariate analysis was performed in two sets. While Table B shows the results of a simultaneous variation of σ, α and β, Table C presents the results of the variation of δ, ϵ and ν. The number of countries n ranges from 3 to 20; the results from 3 to 15 are reported here.

Table C: ***Nash game:*** **multivariate sensitivity analysis** (ϵ, ν, δ)

Parameter			*Number of countries n =*							
ϵ	ν	δ	3	4	5	6	7	8	9	10-20
0.1-0.5	0.1-0.9	0.1-0.9	3	3	3	3	3	3	3	3
0.6-0.9	0.1-0.9	0.1-0.9	3	3	3	3	3	3	3	3

While the stable coalition size is three for all values of $\alpha, \beta, \delta, \epsilon$ and ν, there is a higher stable coalition size when σ is extremely low ($\sigma = 0.1$) and β is relatively high at the same time. When σ is very low, the objective of the inflation target is relatively much more important than the employment objective. This implies that the optimal policy is more contractionary with a lower σ in both cases, the non-coordinated and the coordinated policies. However, σ affects the coordinated policies a bit more than the non-coordinated policies which can be verified when comparing the two coefficients ρ and ω. When σ is very low this means that the difference between non-coordinated and coordinated policies becomes smaller and, hence the gains from free-riding are smaller. This creates an incentive for one more country to enter the coalition in these cases.

B.1.2 The cooperation-Stackelberg game

The results of the simulation[39] of the cooperation-Stackelberg game – qualitatively very similar to the Nash case – are summarised in the following.

[39] The numerical simulations were performed for the same 'default' parameter values as in the cooperation-Nash game: $\alpha = 0.5$,$\beta = 0.5$,$\epsilon = 0.8$,$\nu = 0.05$,$\sigma = 1$ and $\delta = 0.3$.

The loss functions

The losses in equilibrium are determined through equilibrium policies:

$$
\begin{aligned}
L_c &= \sigma {m_c^*}^2 + (\lambda m_c^* - \kappa(k-1)m_c^* - \kappa(n-k)m_{nc}^* + x)^2 \\
L_{nc} &= \sigma {m_{nc}^*}^2 + (\lambda m_{nc}^* - \kappa k m_c^* - \kappa(n-k-1)m_{nc}^* + x)^2
\end{aligned}
$$

Losses decrease for all values of n with the number of coalition members. The explanation is as in the Nash game: the more members a coalition has, the more externalities are internalised. This lowers losses for the countries inside and outside the coalition.

The stability of the coalition

Each n has one stable coalition size, as in the cooperation-Nash game. The coalition is internally stable for all k smaller or equal to the stable coalition size; it is externally stable for all $k \geq k^*$. Hence, the graphic illustration of the 'gains from changing the group' is the same as in Chart 1.

The stable coalition size is always either two or three. The stable coalition size is two for all n smaller than six. For $n \geq 7$ the stable coalition size switches to three. These results are not perfectly robust to changes of the parameter values, ie the 'point' where the stable coalition size switches from two to three may vary with the parameter values. The results of the sensitivity analysis are summarised below.

Univariate sensitivity analysis

The results of the univariate sensitivity analysis are described in Table D. The analysis was performed for the number of countries n from 3 to 20. The numbers in the tables indicate the stable coalition size k^* which is unique for every parameter set. α and σ have the highest impact on whether the stable coalition size is two or three. For $\alpha = 0.05$, the stable coalition size is always two; for $\alpha = 0.95$, the stable coalition size is three for all $n \geq 4$. That is, for low α the stable coalition tends to comprise only two members, even for a high number of countries n.

If the weight of the unemployment target in the loss function σ is high the stable coalition is more likely to comprise only two members. For $\sigma = 0.1$, the stable coalition size is always three while for a high σ the stable coalition size is only three for a high number of countries n. A

Table D: *Stackelberg:* univariate sensitivity analysis

Parameter		*Number of countries n =*												
		3-4	5	6	7	8	9	10-11	12	13	14-15	16	17	18-20
α	0.1	2	2	2	2	2	2	2	2	2	2	2	2	2
	0.2	2	2	2	2	2	2	2	3	3	3	3	3	3
	0.3	2	2	2	2	2	3	3	3	3	3	3	3	3
	0.4	2	2	2	2	3	3	3	3	3	3	3	3	3
	0.5	2	2	2	3	3	3	3	3	3	3	3	3	3
	0.6	2	2	3	3	3	3	3	3	3	3	3	3	3
	0.7-0.9	2	3	3	3	3	3	3	3	3	3	3	3	3
β	0.1-0.5	2	2	2	3	3	3	3	3	3	3	3	3	3
	0.6-0.9	2	2	3	3	3	3	3	3	3	3	3	3	3
δ	0.1-0.2	2	2	3	3	3	3	3	3	3	3	3	3	3
	0.3-0.9	2	2	2	3	3	3	3	3	3	3	3	3	3
ϵ	0.1-0.4	2	2	3	3	3	3	3	3	3	3	3	3	3
	0.5-0.9	2	2	2	3	3	3	3	3	3	3	3	3	3
ν	0.1-0.9	2	2	2	3	3	3	3	3	3	3	3	3	3
σ	1[40]	2	2	2	3	3	3	3	3	3	3	3	3	3
	2	2	2	2	2	2	3	3	3	3	3	3	3	3
	3	2	2	2	2	2	2	3	3	3	3	3	3	3
	4	2	2	2	2	2	2	2	3	3	3	3	3	3
	5	2	2	2	2	2	2	2	2	3	3	3	3	3
	6	2	2	2	2	2	2	2	2	2	3	3	3	3
	7	2	2	2	2	2	2	2	2	2	2	3	3	3
	8	2	2	2	2	2	2	2	2	2	2	2	3	3
	9–10	2	2	2	2	2	2	2	2	2	2	2	2	3

high σ stresses the importance of the employment target and hence the optimal monetary policy will be less contractionary. Consequently, the externalities of uncoordinated monetary policy are lower, which decreases the gains from coordination. Hence, incentives to join the coalition are lower. This is particularly true for the Stackelberg case where the outsiders profit more from the coalition formation than in the Nash case.

(40) The results for σ smaller than one are contained in the multivariate sensitivity analysis.

Multivariate sensitivity analysis

The analysis has been performed in two sets: simultaneous variation of σ, α and β and simultaneous variation of δ, ϵ and ν. All parameter combinations yield a stable coalition size of either two or three. Details are available on request from the author. The results of the multivariate analysis reconfirm the results of the univariate analysis summarised in table D. The explanations given above also apply here.

The coalition formation process

The coalition formation process is equivalent to the one in the cooperation-Nash game. For $k = 1$ the non-cooperative Nash equilibrium is obtained. It is externally unstable and, hence, additional countries will join the coalition until the stable coalition size is reached. The non-members have lower losses than the Stackelberg leader, ie the coalition. This result is – for models of duopolies in industrial organization – shown to be generally valid in Stackelberg games with strategic complements by Dowrick (1986). Hence, every country would like the others to go ahead with the coalition formation. This again might be an obstacle to getting the coalition formation started at all.

Comparison of Stackelberg and Nash outcome

The Stackelberg leader, ie the coalition, has losses at least as low as in the Nash game since he can always realize the cooperation-Nash losses. The simulation results indeed show that the Stackelberg leader money supplies are always higher (less contractionary) than the Nash money supplies for the same n and k. However, the money supplies are only slightly higher than in the Nash game. In particular, for very low and very high k the results of the Nash and of the Stackelberg game are very close to each other.

Dowrick (1986) shows that in a general framework of strategic complements, the Stackelberg outcome is Pareto superior to the Nash outcome, which means that not only the Stackelberg leader but also the Stackelberg follower improves upon its Nash outcome. In the model here the Stackelberg money supplies of the non-members are less contractionary than in the cooperation-Nash game since the non-members react with a higher money supply to the higher money supply of the coalition. Additionally, the losses of the non-members improve more upon the cooperation-Nash equilibrium than the losses of the coalition.

B.2 Simulation analysis with two coalitions

B.2.1 The Nash-Nash game

As in the simulations with one coalition the 'standard values', ie the values if not noted otherwise, are:

$$\alpha = \beta = \tau = 0.5, \quad \nu = 0.05, \; \delta = 0.3, \quad \sigma = 1,$$

The sensitivity analysis was performed by varying all parameters between 0.1 and 0.9, increasing in steps of 0.1. σ, the relative weight of the employment target, starts with 0.1 and increases in steps of 0.5 to 10 (univariate) and from 1 to 10 in steps of 1 (multivariate).

For the Nash-Nash game, the results of the simulation analysis were already discussed in detail in Section 5.2. Therefore, attention will be restricted to the results of the robustness analysis.

Table E: *Nash-Nash:* univariate sensitivity analysis

Parameter		*Number of countries n =*						
		4	5	6	7	8	9	10
α	0.1-0.9	(2,2)	(2,3),(3,2)	(3,3)	(3,3)	(3,3)	(3,3)	(3,3)
β	0.1-0.9	(2,2)	(2,3),(3,2)	(3,3)	(3,3)	(3,3)	(3,3)	(3,3)
δ	0.1-0.9	(2,2)	(2,3),(3,2)	(3,3)	(3,3)	(3,3)	(3,3)	(3,3)
ϵ	0.1-0.9	(2,2)	(2,3),(3,2)	(3,3)	(3,3)	(3,3)	(3,3)	(3,3)
ν	0.1-0.9	(2,2)	(2,3),(3,2)	(3,3)	(3,3)	(3,3)	(3,3)	(3,3)
σ	0.1-10	(2,2)	(2,3),(3,2)	(3,3)	(3,3)	(3,3)	(3,3)	(3,3)

Univariate analysis

Table E shows the results of the univariate analysis where the number of countries n was varied from 3 to 20; the results from 3 to 10 are reported here. The pairs of numbers in the tables represent the stable coalition sizes. While the first number indicates the stable size of coalition 1, the second number represents the stable size of coalition 2. In the univariate analysis the stable coalition size is always unique, at three, when the number of countries exceeds five. For $n = 4$ countries the stable coalition sizes are two for each coalition, and for $n = 5$

Table F: *Nash-Nash:* multivariate sensitivity analysis (σ, α, β)

Parameter			*Number of countries $n =$*					
σ	α	β	4	5	6	7	8	9
1	0.1-0.2	0.1-0.7	(2,2)	(2,3),(3,2)	(3,3)	(3,3)	(3,3)	(3,3)
		0.8-0.9	(3,1),(1,3)	(2,3),(3,2)	(3,3)	(3,3)	(3,3)	(3,3)
	0.3	0.1-0.8	(2,2)	(2,3),(3,2)	(3,3)	(3,3)	(3,3)	(3,3)
		0.9	(3,1),(1,3)	(2,3),(3,2)	(3,3)	(3,3)	(3,3)	(3,3)
	0.4-0.9	0.1-0.9	(2,2)	(2,3),(3,2)	(3,3)	(3,3)	(3,3)	(3,3)
2-10	0.1-0.9	0.1-0.9	(2,2)	(2,3),(3,2)	(3,3)	(3,3)	(3,3)	(3,3)

countries the 'fifth' country is indifferent between coalition 1 or 2 when both already have two members.

Multivariate analysis

Table F shows the results of a simultaneous variation of σ, α and β; Table G presents the results of the variation of δ, ϵ and ν. The number of countries n ranges from 3 to 20; the results from 3 to 9 are included. The pairs of numbers in the tables represent the stable coalition sizes.

Table G: *Nash-Nash:* multivariate sensitivity analysis (ϵ, ν, δ)

Parameter			*Number of countries $n =$*					
ϵ	δ	ν	4	5	6	7	8	9
0.1	0.1	0.1-0.5	(3,1),(1,3)	(2,3),(3,2)	(3,3)	(3,3)	(3,3)	(3,3)
		0.6-0.9	(2,2)	(2,3),(3,2)	(3,3)	(3,3)	(3,3)	(3,3)
	0.2-0.9	0.1-0.9	(2,2)	(2,3),(3,2)	(3,3)	(3,3)	(3,3)	(3,3)
0.2-0.3	0.1	0.1-0.3	(3,1),(1,3)	(2,3),(3,2)	(3,3)	(3,3)	(3,3)	(3,3)
		0.4-0.9	(2,2)	(2,3),(3,2)	(3,3)	(3,3)	(3,3)	(3,3)
	0.2-0.9	0.1-0.9	(2,2)	(2,3),(3,2)	(3,3)	(3,3)	(3,3)	(3,3)
0.4	0.1	0.1	(3,1),(1,3)	(2,3),(3,2)	(3,3)	(3,3)	(3,3)	(3,3)
	0.1-0.9	0.1-0.9	(2,2)	(2,3),(3,2)	(3,3)	(3,3)	(3,3)	(3,3)
0.5-0.9	0.1-0.9	0.1-0.9	(2,2)	(2,3),(3,2)	(3,3)	(3,3)	(3,3)	(3,3)

If n exceeds four countries the results of the 'standard' equilibrium which are discussed in Section 5.2 do not vary with the parameter values. Only for $n = 4$ countries are there some cases where the stable coalition sizes are three and one, rather than two and two. This situation occurs when α is very low and β is very high or when δ is very low. Both situations imply that κ, which indicates the impact of negative spillovers from non-coordinated policies, is high. When negative externalities imposed by non-coordinated policies are large it pays to extend the first coalition to the stable size of three rather than having two countries in each coalition.

B.2.2 The Stackelberg-Stackelberg game

The results of the simulation of the Stackelberg-Stackelberg game are summarised in the following.(41) Though the results differ quantitatively only little from the Nash-Nash case, they are qualitatively different.

The losses in equilibrium

Both coalitions profit more from an increase in the other coalition than from an extension of their own coalition. Because of the asymmetry of the strategic position of the coalitions, the countries in the fringe profit slightly more from an extension of coalition 1 than of coalition 2. However, the lowest losses are realised where almost all countries concentrate in only one coalition. All losses decrease with the extension of the coalitions and, consequently, increasing coordination.

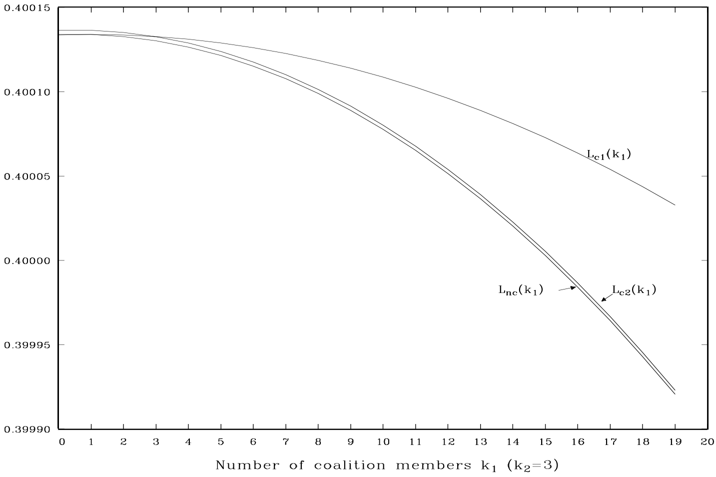
The relative positions of the three groups with increasing coalition size k_1 are shown in Chart 7. Again, functions are downward-sloping, ie all groups profit from the increasing coordination within coalition 1. The lowest losses are always realised by the non-members which gain from the increased 'discipline' within the coalition but are able to play an individually optimal response themselves. For the same reason coalition 2 benefits from the increasing size of coalition 1 more than the members of coalition 1 themselves.

A comparison with the Nash-Nash game shows that all three loss functions are slightly lower in the Stackelberg-Stackelberg than in the Nash-Nash game.(42) Hence there is a clear ranking of the two games

(41) Again, the results reported are based on a simulation of the model with the parameter values: $\alpha = 0.5, \beta = 0.5, \epsilon = 0.8, \nu = 0.05, \sigma = 1$ and $\delta = 0.3$. All charts refer to the case of $n = 22$ countries.

(42) There are combinations of coalition sizes where the strategic differences do not matter. When all countries are concentrated in one of the groups the two games represent exactly the same situation.

Chart 7: Loss functions of insiders and outsiders

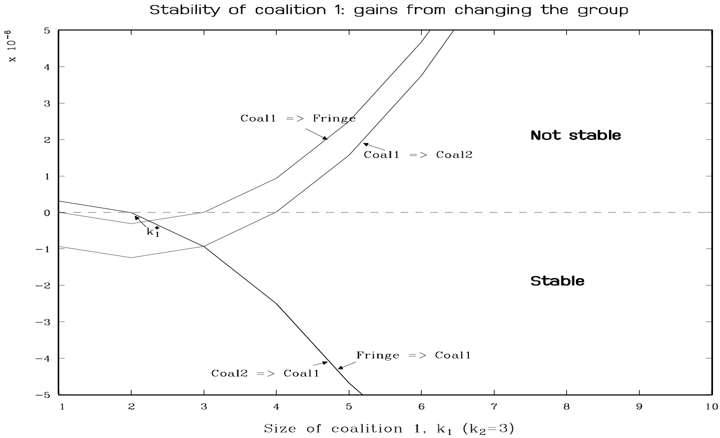


when comparing situations with the same coalition sizes: the more 'hierarchical' the structure is, the better it is for all groups. However, with a more hierarchical structure, the differences between the three groups become larger. The countries outside the coalitions profit most from the differences in strategic positions and this affects the stability of the equilibrium.

The stability of the equilibrium

As in the Nash-Nash game the stability conditions between the coalition and the fringe remain almost unchanged with increasing coalition size of coalition 2. They imply a stable coalition size around three where it does not pay to join or leave the coalition. The gains from switching between the two coalitions are dependent on the coalition sizes. Essentially, it will usually be preferable to be in the coalition with fewer members. As above, for a small size of coalition 2 only a few countries will be in coalition 1, so its internal stability will be a problem. When coalition 2 is larger, countries will prefer to switch to the smaller coalition 1. Here, an external stability condition fails to hold for smaller k_1. However, one has to take the different strategic position of the two coalitions into account, which makes it preferable to be in coalition 2 (the Stackelberg follower) rather than coalition 1. This changes the stable equilibrium to a situation where there are only

Chart 8: Stability of coalition 1 with varying k_1 (for $k_2^* = 3$)[a]



[a] Negative 'gains from changing the group' imply that changing does not pay and, hence, the group is stable. The convex graphs show the internal stability conditions. The concave graphs show the external stability conditions. A coalition size of two for coalition one fulfils all stability criteria.

two countries in the 'leading' coalition 1 and three countries in coalition 2. Chart 8 illustrates the stability conditions for the stable case where there are three countries in coalition 2.

It has been shown above that for some coalition sizes all three groups are best off in the Stackelberg-Stackelberg game. However, when the *stable* equilibrium outcomes are compared, ie $k_1^* = k_2^* = 3$ in the Nash case and $k_1^* = 2, k_2^* = 3$ in the Stackelberg case, the lowest losses for all three groups are realized in the stable equilibrium of the Nash-Nash game. The Stackelberg-Stackelberg structure is the least preferable structure when taking the stability of coalitions into account, since fewer countries will coordinate their policies within coalitions. This reduction of coordination outweighs the reduction of losses through the Stackelberg structure.

Sensitivity analysis

As in the other games, a univariate and a multivariate sensitivity analysis were performed.

Table H: *Stackelberg-Stackelberg:* univariate sensitivity analysis

Parameter		*Number of countries n =*						
		4	5	6	7	8	9	10
α	0.1-0.9	(2,2)	(2,3)	(2,3)	(2,3)	(2,3)	(2,3)	(2,3)
β	0.1-0.8	(2,2)	(2,3)	(2,3)	(2,3)	(2,3)	(2,3)	(2,3)
	0.9	(1,3)	(2,3)	(2,3)	(2,3)	(2,3)	(2,3)	(2,3)
δ	0.1	(1,3)	(2,3)	(2,3)	(2,3)	(2,3)	(2,3)	(2,3)
	0.2-0.9	(2,2)	(2,3)	(2,3)	(2,3)	(2,3)	(2,3)	(2,3)
ϵ	0.1-0.9	(2,2)	(2,3)	(2,3)	(2,3)	(2,3)	(2,3)	(2,3)
ν	0.1-0.9	(2,2)	(2,3)	(2,3)	(2,3)	(2,3)	(2,3)	(2,3)
σ	0.1	(1,3)	(2,3)	(2,3)	(2,3)	(2,3)	(2,3)	(2,3)
	0.6-10	(2,2)	(2,3)	(2,3)	(2,3)	(2,3)	(2,3)	(2,3)

Univariate analysis

Table H shows the results of the univariate analysis where the number of countries n was varied from 3 to 20; the results from 3 to 10 are included. The pairs of numbers in the tables represent the stable coalition sizes. The first number indicates the stable size of coalition 1, and the second number represents the stable size of coalition 2. In the univariate analysis the stable coalition size is always unique, with two members in the 'leader' coalition and three members in the 'follower' coalition when the number of countries exceeds four. For $n = 4$ countries the stable coalition sizes are either two for each coalition or one member in the 'leader' coalition and three members in the 'follower' coalition. The latter case occurs when the negative externalities, κ, are very high (δ very low or β very high) or when σ is very low, ie the employment target has a low priority which stresses the importance of avoiding imported inflation by joining the coalition.

Multivariate analysis

As above, the multivariate analysis was performed in two sets. Table I shows the results of a simultaneous variation of σ, α and β, and Table J presents the results of the variation of δ, ϵ and ν. The number of countries n ranges from 3 to 20; the results from 3 to 10 are included. The pairs of numbers in the tables represent the stable coalition sizes.

Table I:
***Stackelberg-Stackelberg:* multivariate sensitivity analysis (σ,α,β)**

Parameter			*Number of countries n =*						
σ	α	β	4	5	6	7	8	9	10
1	0.1-0.3	0.1-0.7	(2,2)	(2,3)	(2,3)	(2,3)	(2,3)	(2,3)	(2,3)
		0.8-0.9	(1,3)	(2,3)	(2,3)	(2,3)	(2,3)	(2,3)	(2,3)
	0.4-0.5	0.1-0.8	(2,2)	(2,3)	(2,3)	(2,3)	(2,3)	(2,3)	(2,3)
		0.9	(1,3)	(2,3)	(2,3)	(2,3)	(2,3)	(2,3)	(2,3)
	0.6-0.9	0.1-0.9	(2,2)	(2,3)	(2,3)	(2,3)	(2,3)	(2,3)	(2,3)
2	0.1	0.1-0.8	(2,2)	(2,3)	(2,3)	(2,3)	(2,3)	(2,3)	(2,3)
		0.9	(1,3)	(2,3)	(2,3)	(2,3)	(2,3)	(2,3)	(2,3)
	0.2-0.9	0.1-0.9	(2,2)	(2,3)	(2,3)	(2,3)	(2,3)	(2,3)	(2,3)
3-10	0.1-0.9	0.1-0.9	(2,2)	(2,3)	(2,3)	(2,3)	(2,3)	(2,3)	(2,3)

Table J:
***Stackelberg-Stackelberg:* multivariate sensitivity analysis (ϵ, ν, δ)**

Parameter			*Number of countries n =*						
δ	ν	ϵ	4	5	6	7	8	9	10
0.1	0.1-0.2	0.1-0.9	(1,3)	(2,3)	(2,3)	(2,3)	(2,3)	(2,3)	(2,3)
	0.3	0.1-0.8	(1,3)	(2,3)	(2,3)	(2,3)	(2,3)	(2,3)	(2,3)
		0.9	(2,2)	(2,3)	(2,3)	(2,3)	(2,3)	(2,3)	(2,3)
	0.4-0.5	0.1-0.7	(1,3)	(2,3)	(2,3)	(2,3)	(2,3)	(2,3)	(2,3)
		0.8-0.9	(2,2)	(2,3)	(2,3)	(2,3)	(2,3)	(2,3)	(2,3)
	0.6-0.7	0.1-0.6	(1,3)	(2,3)	(2,3)	(2,3)	(2,3)	(2,3)	(2,3)
		0.7-0.9	(2,2)	(2,3)	(2,3)	(2,3)	(2,3)	(2,3)	(2,3)
	0.8-0.9	0.1-0.5	(1,3)	(2,3)	(2,3)	(2,3)	(2,3)	(2,3)	(2,3)
		0.6-0.9	(2,2)	(2,3)	(2,3)	(2,3)	(2,3)	(2,3)	(2,3)
0.2-0.9	0.1-0.9	0.1-0.9	(2,2)	(2,3)	(2,3)	(2,3)	(2,3)	(2,3)	(2,3)

If n exceeds four countries the results of the 'standard' equilibrium, discussed above, do not vary with the parameter values. Again, for $n = 4$ countries, there are some cases where the stable coalition sizes are three and one rather than two and two. In short, these cases occur in the same situations as discussed in the univariate analysis, for the same reasons.

References

Alesina, A and Grilli, V (1993), 'On the feasibility of a one- or multi-speed European Monetary Union', *Discussion Paper Series 792*, CEPR 1993.

Bayoumi, T and Eichengreen, B (1992), 'Shocking aspects of European Monetary Union', *Working Paper 3949*, NBER 1992.

Bayoumi, T and Eichengreen, B (1994), 'One money or many? Analyzing the prospects for monetary unification in various parts of the world', *Princeton Studies in International Finance 76*, Princeton University.

Buiter, W H, Corsetti, G, and Pesenti, P A (1995), 'A centre-periphery model of monetary coordination and exchange rate crises', *Discussion Paper Series 1201*, CEPR.

Bulow, J I, Geanakoplos, J D, and Klemperer, P D (1985), 'Multimarket Oligopoly: Strategic substitutes and complements', *Journal of Political Economy*, 93, pages 488–511.

Canzoneri, M B (1982), 'Exchange intervention policy in a multiple country world', *Journal of International Economics*, 13, pages 267–89.

Canzoneri, M B and Henderson, D W (1988), 'Is sovereign policymaking bad?', *Carnegie-Rochester Conference Series on Public Policy*, 28, pages 93–140.

Canzoneri, M B and Henderson, D W (1991), *Monetary policy in interdependent economies: A Game-theoretic approach*, MIT Press.

Canzoneri, M B and Gray, J A (1985), 'Monetary policy games and the consequences of non-cooperative behavior', *International Economic Review*, 26 No. 3, pages 547–64.

D'Aspremont, C, Jacquemin, A, Gabszewicz, J J, and Weymark, J A (1983), 'On the stability of collusive price leadership', *Canadian Journal of Economics*, 16, pages 17–25.

Dowrick, S (1986), 'von Stackelberg and Cournot duopoly: choosing roles', *Rand Journal of Economics*, 17, pages 251–60.

Giavazzi, F and Giovannini, A (1989), *Limiting exchange rate flexibility*, MIT Press.

Hamada, K (1976), 'A strategic analysis of monetary independence', *Journal of Political Economy*, 84, pages 677–700.

Hughes Hallett, A and Braga, P (1994), 'The new regionalism and the threat of protectionism', *Journal of the Japanese and International Economies*, 8 (4), pages 388–421.

Kohler, M (1998), 'Optimal currency areas and customs unions: are they connected?', *Working paper 89*, Bank of England.

Martin, P (1995), 'Free-riding, convergence and two-speed monetary unification in Europe', *European Economic Review*, 39, pages 1345–64.

Masson, P and Taylor, M (1992), 'Common currency areas and currency unions: An analysis of the issues', *Discussion Paper 617*, CEPR.

Mundell, R A (1961), 'A Theory of Optimum Currency Areas', *American Economic Review*, 51, pages 657–65.

Persson, T and Tabellini, G (1995), 'Double-edged incentives: Institutions and policy coordination', in Grossman and Rogoff, eds., *Handbook of International Economics*, Vol. III, Elsevier, chapter 38, pages 1973–2030.

Yi, S-S (1997), 'Stable coalition Structures with Externalities', *Games and Economic Behaviour*, 20, pages 201–237.