Should Uncertain Monetary Policy-Makers Do Less?

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Abstract

This paper examines the empirical importance of parameter uncertainty for monetary policy-making in the United Kingdom, following the method used by Brian Sack of the US Federal Reserve. Using a VAR model of the UK economy and an assumed quadratic loss function for the policy-maker, we calculate an optimal interest rate rule first ignoring parameter uncertainty, then assuming that the parameter uncertainty is given by the estimated standard errors on the VAR coefficients. We contrast these rules with the estimated interest rate equation from the VAR. The optimal rule accounting for parameter uncertainty results in a less aggressive path for official interest rates than when parameter uncertainty is ignored. However, the estimates of parameter uncertainty are not so large that the optimal rule matches all the characteristics of the actual path of official rates.

1 Introduction

Uncertainty is a key characteristic of the monetary policy environment. Were the lags in monetary policy long but 'certain', and were money velocity 'a known variable', the achievement of monetary targets could be reduced to a mechanical exercise. But most (monetary) economics has tended to treat uncertainty in a limited way by assuming that it can be adequately captured by the introduction of additive stochastic terms to otherwise deterministic and certain models of the economy. In combination with an assumption that policy-makers' preferences are quadratic, this has often led academics to model policy decisions as if they were made in a 'certainty equivalent' world.

Recently, there has been a resurgence of interest in the significance of broader forms of uncertainty for monetary policy. At the Bank this has been fuelled, in part, by the existence of an inflation target. Inflation targeting can be viewed as analogous to operating policy with an inflation forecast as an intermediate target (Svensson (1996, 1997), Haldane (1997)). According to this interpretation, policy should be adjusted in response to deviations of forecast inflation from target. But as everyone knows forecasting is an uncertain business.

One of the Bank's responses has been to publish a fan chart for the inflation forecast in each *Inflation Report* since February 1996, conditional on unchanged interest rates.[1] The fan chart shows the relative likelihood of possible outcomes for inflation, and is divided into 10% probability bands. The central band shows the range within which the actual outcome is most likely to fall, and successively wider bands indicate less likely outcomes. As Mervyn King (1997) commented, it is designed to 'summarise the information relevant to the MPC's (Monetary Policy Committee's) decision of whether or not to change interest rates'.

So does this mean that interest rates are changed automatically in response to changes in the central projection and associated probability distribution? Not necessarily; to quote from King again, one reason is that: 'as Bill Brainard showed 30 years ago, it may be sensible to move cautiously to the level of interest rates that would be necessary to equate expected

(1) The basis of the fan chart is described in Britton, Fisher and Whitley (1998).

inflation at the appropriate horizon with the target level, rather than move rates abruptly and so risk injecting volatility into the economy'.

Careful readers will have noted the '*may'* in this statement: as Mr King later commented in the same speech, we do not know how significant Brainard uncertainty is in the United Kingdom. This paper is part of the Bank's work to attempt to provide a quantitative answer to, and a framework for thinking about, this 'may'. Other papers that contribute to this analysis are Hall, Salmon, Yates and Batini (1999), and Martin (1999).

Renewed interest in parameter uncertainty is not unique to the Bank of England. Alan Blinder (1997), in particular, has been arguing forcibly that more work is needed to tease out the policy implications of this form of uncertainty, viz: '[A]cademic economists could also be more helpful to policy-makers if they would ... investigate the robustness of Brainard's conservatism principle'. Recent papers by Estrella and Mishkin (1998), Sack (1998a, 1998b) and Wieland (1995, 1996, 1998) have explored its implications for US monetary policy. Our analysis is based around Sack (1998a) and Goodhart's (1998) discussion of that paper.

The rest of the paper is organised as follows: Section 2 provides a stylised treatment of parameter uncertainty in an inflation targeting framework. It is based loosely on Svensson's (1996) inflation targeting model. It shows that in a dynamic setting multiplier uncertainty may imply an optimal monetary policy that involves smaller, more drawn out responses to shocks to the economy, compared with the case of no multiplier uncertainty. Section 3 then summarises Sack's approach and results. Section 4 explains how we adapt this approach for the United Kingdom. Section 5 sets out the results of this exercise for the United Kingdom. Finally, Section 6 offers some tentative conclusions.

2 Parameter uncertainty for inflation targeters: a simple theoretical model(2)

This section uses a stylised model based on Svensson (1996) to discuss the consequences of parameter uncertainty for optimal monetary policy when the authority sets nominal interest rates to meet an inflation target. It provides a simple framework within which to review the original Brainard

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(2) This section is based on Martin (1999).

(1967) result, to discuss the mechanisms driving the results, and to draw out possible implications for monetary policy.

2.1 Description of the model

The model is written in deviations from equilibrium values. Equilibrium output is the natural rate and is normalised to zero. The inflation target is also normalised to zero and the equilibrium real interest rate is defined as that rate consistent with output being at the natural rate and inflation being at target. This equilibrium real rate is also normalised to zero.

The path for inflation is assumed to follow the following simple process, which is essentially a reduced form of a backward-looking Phillips curve and an IS curve:

$$p_{t+1} = ap_t - br_t + e_{t+1} \qquad \textbf{(1)}$$

where e_t is a white noise disturbance. Following Svensson (1996), the policy-maker sets nominal interest rates with the aim of meeting the inflation target. Nominal interest rates, i_t , are related to real interest rates, r_t , and expectations of future inflation, p_{t+1} , (over the period for which the interest rate applies) by the Fisher equation.(3)

$$r_t = i_t - \mathrm{E}_t p_{t+1} \qquad \textbf{(2)}$$

In this model, the monetary authority’s objective is for real interest rates to deviate from their neutral level as a linear function of the deviation of inflation from target.

We shall assume that parameter uncertainty is characterised by:

$$\begin{pmatrix} a \\ b \end{pmatrix} \sim iid.\begin{pmatrix} \bar{a} \\ \bar{b} \end{pmatrix}, \begin{pmatrix} s_a^2 & r_{ab} \\ & s_b^2 \end{pmatrix} \qquad \textbf{(3)}$$

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(3) The policy-maker is assumed to be able to set the nominal interest rate at the end of period *t* that rules for one period.

with the additive error, e_t, independently distributed with mean zero and variance s_e^2.[(4)]

The policy-maker's objective is to set the domestic nominal interest rate to minimise the present discounted value of expected deviations of inflation from target, ie:

$$\mathrm{E}_0 \sum_{t=0}^{\infty} b^t p_t^2 \qquad \textbf{(4)}$$

subject to the reduced-form processes for inflation **(1)**.

Before solving for the optimal policy rules it is worth commenting on the form of this model. Compared with other simple models of monetary policy, eg Woodford (1999), one obvious difference is that inflation does not depend directly upon leads of inflation.[(5)] As Woodford *(op cit)* shows, such lead terms can create an incentive to smooth interest rates independent of parameter uncertainty. The analysis throughout this paper abstracts from issues raised by forward-looking expectations.

Another feature of our model is its simplicity; it includes just two independent variables—inflation and the real interest rate—and one policy objective—inflation. This is to simplify the exposition. The empirical analysis that follows makes use of a richer characterisation of the transmission mechanism that includes output and the exchange rate, and recognises that policy-makers also have output-smoothing objectives. But the policy rules we estimate empirically are straightforward generalisations of those derived in this section, and the intuition for the differences between an optimal policy that takes account of uncertainty about parameters, and one that does not, is identical in the theoretical and empirical sections of this paper.

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(4) Implicitly we have restricted the covariances between the multiplicative and additive processes to zero. This assumption is maintained in the empirical analysis. Readers are referred to Martin (1999) for a discussion of the theoretical implications of allowing these covariances to be non-zero.
(5) They enter indirectly, as the nominal interest rate is defined by the Fisher identity **(2)**. But under rational expectations and given the structure of the model, these expectations are predetermined at the time the policy-maker sets the nominal interest rate. Hence, there is no implicit behavioural link from expected future events to current inflation.

In more complex models of this kind, such as Sack (1998a) or the empirical part of this paper, the problem is solved using dynamic programming. But following Svensson (1996), it can be shown that in simple models such as this one, the multi-period problem reduces to a sequence of one-period problems where this period's interest rate is assigned to returning next period's inflation rate to target.[(6)] Therefore the policy-maker's objective function to be maximised each period can be taken to be:

$$\mathrm{E}_t p_{t+1}^2 = \mathrm{E}_t^2 p_{t+1} + \mathrm{var}_t\, p_{t+1} \qquad \textbf{(5)}$$

where we have made use of the fact that the expectation of the square of a random variable equals the square of its mean (the bias) plus its variance. This will be important in what follows: in many similar models with purely additive errors the variance of inflation will be given exogenously. Parameter uncertainty endogenises this variance.

Choosing the optimal level of the nominal interest rate requires the policy-maker to know two things: the optimal degree of monetary tightness, ie the level of the real interest rate that is required to meet the policy objective **(5)**, and the rational expectation of the next period's inflation given past shocks and policy settings. Once these are computed, the optimal nominal rate is the sum of these two quantities. We consider each of these three elements in turn.

2.2 The path for desired real interest rates

The policy-maker sets the nominal interest rate but does so with the intention of influencing the real interest rate. The real interest rate defines the stance of monetary policy. If the authority can quantify with certainty the size of the multipliers a and b, and inflation is subject to random, uncorrelated disturbances, then the optimal, *additive-uncertainty*, rule for real rates in response to inflation deviations from target can be shown to be:

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(6) Because the real interest rate impacts on inflation with a one-period lag, r_t can influence inflation only in periods t+1, t+2 etc, and r_{t+1} influences inflation in t+2, t+3 etc, but not t+1. We can therefore assign the real interest rate set in period t to controlling inflation in period t+1. Martin (1999) provides a proof.

$$r_t = \frac{a}{b} p_t \tag{6}$$

Alternatively, suppose that the policy-maker knows the structure of the equations describing the economy, but does not know the size of the multipliers and has to estimate them. This will give point estimates of the multipliers (that we assume are equal to the parameter means) and the variances of the random variables in our model (as set out in **(3)**). In that case the optimal, *parameter-uncertainty,* rule for real interest rates is:

$$r_t = \left[\frac{\bar{a}\bar{b} + r_{ab}}{\bar{b}^2 + s_b^2} \right] p_t \tag{7}$$

In both cases deviations of real interest rates from neutral are a linear function of deviations of inflation from target.

2.3 The resulting path for inflation expectations

Under additive uncertainty, and following the rule **(6)**, the expected rate of inflation is given by:

$$\mathrm{E}_t p_{t+1} = \bar{a} p_t - \bar{b} \frac{\bar{a}}{\bar{b}} p_t + \mathrm{E}_t e_{t+1} = 0 \tag{8}$$

With parameter uncertainty, following the rule **(7)**, the expected rate of inflation is given by:

$$\mathrm{E}_t p_{t+1} = \left[\frac{\bar{a} s_b^2 - \bar{b} r_{ab}}{\bar{b}^2 + s_b^2} \right] p_t \tag{9}$$

Rational agents therefore expect inflation to be at target next period under additive uncertainty, whereas this is not necessarily the case under parameter uncertainty.

2.4 Nominal interest rates

For a given desired path of real rates, and given the current state of the economy, the policy-maker can infer the rational expectation of next period's inflation. This is sufficient information to choose a level of the nominal rate that delivers the desired real rate and validates the inflation expectation.

Under additive uncertainty the conditional expectation at time *t* of next period's inflation is zero given the desired path of real rates: agents expect that monetary policy will, on average, deliver the target level of inflation. To achieve the desired path for real rates, the nominal rate is set equal to the desired real rate plus expected inflation **(6)** plus **(8)**:

$$i_t = r_t + \mathrm{E}_t p_{t+1} = \frac{a}{b} p_t \qquad \textbf{(10)}$$

Rational agents expect inflation to be zero on average, so the nominal and real interest rates are equal.

Under parameter uncertainty the policy-maker sets the nominal interest rate as the sum of **(7)** and **(9)**:

$$i_t = \left[\frac{\bar{a}\left(\bar{b} + s_b^2\right) + \left(1 - \bar{b}\right) r_{ab}}{\bar{b}^2 + s_b^2} \right] p_t \qquad \textbf{(11)}$$

Again both rules are linear in deviations of inflation from target. Examination of the rules for real and nominal interest rates under parameter uncertainty shows that the response to inflation depends on the variance of the policy multiplier *b* and its covariance with the other model parameter *a*.

The authority cannot offset shocks until after they occur (since, given rational expectations, the best forecast is that the shock is zero). But once a disturbance has been observed, the optimal response under additive uncertainty is completely to offset it so that, in the absence of any new disturbance, inflation would be back at target (recall that there is a lag of one period from monetary policy to inflation). With this policy, inflation is only driven by the new shock each period, and nominal interest rates, i_t move solely in response to this period's shock, e_t, impacting on inflation next period. The policy-maker moves nominal interest rates aggressively to return the expectation of next period's inflation to target because in the case of additive uncertainty it is costless to move interest rates: in terms of the policy objective, interest rates can return the mean of the distribution of future inflation to target (reduce the bias), while the variance of the distribution is determined exogenously by the additive shocks.(7) Otherwise the policy-maker would be ignoring systematic deviations of inflation from target.

Under parameter uncertainty, the policy-maker's goal is again to minimise the expectation at time *t* of the squared deviation of inflation from target. In contrast to the case of additive uncertainty, both the bias and the variance of future inflation now depend on real rates because the more real interest rates deviate from neutral, the more uncertain the policy-maker is about the resulting effect on inflation. Hence both the bias and variance terms will depend on the authority's actions. But there is still only one instrument available to the policymaker. There must therefore be some trade-off between bias and variance, whereas with purely additive uncertainty the variance was given exogenously and therefore did not alter the policy-maker's incentives.

As we discuss next, the consequences for this trade-off of parameter uncertainty are threefold. It alters the deviation of real and nominal interest rates from neutral in response to deviations of inflation from target; it has consequences for the dynamic path of interest rates; and it has implications for the cumulative real and nominal interest rate responses.

2.5 Conservatism: a smaller response to inflation deviations from target

We define a conservative response to a shock as a deviation of the interest rate from the neutral level that is smaller in magnitude than the optimal response assuming only additive uncertainty. We can think of either real or nominal interest rate conservatism. Under the assumption that the covariance r_{ab} is zero, equation **(7)** shows that the authority desires smaller deviations of real rates from their 'neutral level' if there is parameter uncertainty than when there is only additive uncertainty. The result is a

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(7) Note that because expected inflation is back at target, the wedge between nominal and real rates and their neutral levels is the same (or more simply, $i_t = r_t$ since their respective neutral levels are normalised to zero). This will not in general be the case under parameter uncertainty (or if there was a preference for output smoothing).

path for real interest rates that does not offset inflationary shocks as soon as they are observed. This is because the welfare cost of the extra bias in inflation that arises by not completely offsetting the shocks is exactly matched by the welfare cost of the variance induced by moving real interest rates given the uncertain policy multiplier. This is the standard case of what Blinder (1997) has called 'Brainard conservatism'. However, as Brainard noted in his original 1967 article, large positive covariances can alter this result and lead to a more aggressive response to a given deviation of inflation from target if $r_{ab}>>0$. Part of the objective of this paper is to examine whether an optimising policy-maker who accounts for parameter uncertainty reacts more or less aggressively, ie whether covariances matter in the United Kingdom.

Equation **(11)** shows that the implications for nominal interest rates of real interest rate conservatism are ambiguous. Because the nominal rate is the sum of real rates and inflation expectations, there are two opposing effects on nominal rates. The first is from real rate conservatism. The second opposing effect comes from the fact that the public, when forming their expectations of inflation, know that the policy-maker is worried about Brainard uncertainty, and is operating a conservative policy. So their inflation expectations must rise after a positive shock to inflation. But this effect will only actually outweigh the real rate conservatism effect for exceptional parameter values, so nominal rate 'aggression' is unlikely to be a practical issue, more a theoretical possibility.(8)

2.6 Gradualism: a smoothed response of nominal and real interest rates to shocks

We define a gradualist response to a shock as one that is more phased in, or autocorrelated, than the response that would be optimal where the only uncertainty policy-makers face is additive.(9) From the discussion above, we noted that parameter uncertainty (at least under zero covariances) leads to a constant fraction of an inflationary shock being offset each period.

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(8) In particular, if the covariances are zero, nominal rate 'aggression' will occur only if the parameter b is greater than one. A crude calibration based on Rudebusch and Svensson (1998) suggests that a is fractionally less than one, and b is an order of magnitude smaller, suggesting that 'aggression' is unlikely.

(9)The optimal additive uncertainty response may itself be autocorrelated. This would arise in this model, for example, if policy-makers had a preference for output stabilisation as well as an inflation target.

Inflation is therefore autocorrelated, and since real interest rates are set in response to deviations of inflation from target, the real interest rate response to a one-off shock must also be autocorrelated. Since the nominal interest rate required to achieve the desired level of real rates is the sum of the real rate and the inflation expectation, nominal rates will also be autocorrelated, with the same autocorrelation coefficient as real rates.(10) In contrast, under purely additive uncertainty, shocks are offset straight away so inflation, real rates and hence nominal rates are not autocorrelated. Brainard uncertainty induces a more gradual response in real and nominal rates.

2.7 Caution

We define caution as a cumulative response to an inflationary shock that is less than the cumulative response under purely additive uncertainty. Again we can consider either real or nominal interest rate caution. If, in the absence of deviations of real interest rates from neutral, shocks to inflation would decay naturally with time, then operating a policy that only offsets part of a shock each period will allow this process of natural decay to help inflation return to target. Therefore by waiting to offset part of a shock, the cumulative real interest rate response is less. If no such process of natural decay occurs, part of a shock to inflation that is not immediately offset will persist, neither decaying nor growing, until it is offset by policy settings in subsequent periods. In that case the cumulative real interest rate response will be the same with or without parameter uncertainty.

The degree of persistence in inflation depends on various factors in the economy. For example, overlapping nominal contracts mean that shocks to inflation have an effect for some considerable time. In the limit, the shock to inflation might be permanent. Alternatively, other channels of monetary policy (eg the exchange rate) might play a role in offsetting inflationary shocks.(11)

The case of nominal interest rate caution is more complicated, because a conservative policy that only offsets a small part of an inflationary shock allows inflation expectations to rise and this must be accounted for by the policy-maker when setting nominal interest rates. If, over the horizon of the

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(10) Recall that, if inflation expectations are rational, expected inflation will be autocorrelated in the same way as actual inflation.
(11) See Martin (1999).

policy response, inflation expectations rose by a large enough amount, the cumulative nominal interest rate response might be larger under parameter uncertainty than under additive uncertainty even though the cumulative real interest response might be smaller.

Section 4 will examine the results for the UK economy in the context of conservatism, gradualism and caution. We cannot observe real interest rates, only nominal interest rates, and as such our discussion will focus on nominal interest rate conservatism, gradualism and caution.[(12)] Ideally we would be able to compute rational inflation expectations from the VAR but in practice the information set we have is not broad enough to do so convincingly.

3 Computation of the optimal rule with and without parameter uncertainty

The model set out in Section 2 is highly stylised. In particular it imposes a dynamic structure such that the control problem for the policy-maker can be reduced to a static one-period problem.

Sack (1998a) generalises the model to allow interest rates to have an impact on the economy over a number of periods. Rather than modelling the economy as a simple first-order difference equation in inflation and interest rates, he assumes that an n- vector y_t of relevant endogenous variables, not just inflation, follows a linear auto-regressive process with q lags. This approach captures uncertainty about a wider set of parameters than the simple model described above, and allows for a general covariance structure between them.

The 'relevant' data set comprises those variables targeted by monetary policy, the policy instrument and any other indicator variables that the authority takes into account when setting policy. In his study, Sack includes in this vector monthly data on industrial production growth (*ip*), unemployment (*u*), consumer price inflation (*inf*) and commodity price

(12) It is important to recognise that other forms of uncertainty, beyond just additive or parameter uncertainty, as characterised by Brainard, may have implications for policy-making. Batini, Martin and Salmon (1999), provide a summary of the wider literature on uncertainty and monetary policy.

inflation (*com*), and the federal funds rate (*i*) (in that order).[13] Estimation of the reduced form by OLS provides point estimates of the coefficients in the following equation:

$$y_t = c + \sum_{i=1}^{q} A_i y_{t-i} + e_t \tag{12}$$

where the elements of the vector of shocks e_t are likely to be correlated. Correct identification of the structural model (a task which gives rise to its own literature) gives *n* structural equations with the vector n_t of uncorrelated, structural disturbances to the system.

$$B_0 y_t = k + \sum_{i=1}^{q} B_i y_{t-i} + v_t \tag{13}$$

If the model **(13)** is identified correctly, then the first n-1 equations describe the structural form of the economy and the n-th equation is the estimated policy reaction function of the central bank.

3.1 The policy-maker's problem under additive uncertainty

Defining the state vector $x_t = (ip_t, u_t, inf_t, com_t, ip_{t-1}, u_{t-1}, inf_{t-1}, com_{t-1}, i_{t-1}, \ldots.. ip_{t-q}, u_{t-q}, inf_{t-q}, com_{t-q}, i_{t-q})$ allows the n-1 structural equations from **(13)** to be written as a first-order difference equation which forms the policy-maker's estimated constraint:

$$x_{t+1} = Fx_t + Hi_t + J + x_{t+1} \tag{14}$$

The policy-maker seeks to minimise the present discounted value of deviations of industrial production, unemployment and inflation from target, assuming quadratic loss:

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(13) Of these Sack assumes that industrial production, unemployment and price inflation are target variables, and commodity price inflation an indicator.

$$L = -\tfrac{1}{2}\mathrm{E}_t \sum_{s=1}^{\infty} b^s (x_{t+s} - x^*)' G (x_{t+s} - x^*) \qquad \textbf{(15)}$$

The vector x^* is the vector of targets and the matrix G contains zeros except for the first three diagonal elements which contain the preference weights (with the weight on inflation, $G(3,3)$ normalised to 1). Assuming a value function of the form $v(x) = x'\Lambda x + 2x'w + t$, it can be shown (see eg Sargent, 1987) that the optimal rule under additive uncertainty is a linear function of the state variables x_t:

$$i_t^a = -[H'\Lambda H]^{-1}[H'\Lambda F x_t + H'\Lambda J + H'w] \qquad \textbf{(16)}$$

where the constants in the value function are given by the expressions:

$$\Lambda = -G + bF'\Lambda F - bF'\Lambda H[H'\Lambda H]^{-1} H'\Lambda F \qquad \textbf{(17)}$$

$$w = \left[I - bF'\left(I - \Lambda H\left(H'\Lambda H\right)^{-1} H'\right)\right]^{-1}\left[Gx^* + bF'\Lambda\left(I - H\left(H'\Lambda H\right)^{-1} H'\Lambda\right)J\right] \qquad \textbf{(18)}$$

The matrix Λ is computed numerically due to the implicit form of **(17)**.

3.2 Introducing multiplicative uncertainty

With additive uncertainty and quadratic loss, uncertainty has no effect on the optimal rule because of certainty equivalence. But the problem is complicated when the multipliers are assumed to be random. To get round this problem, Sack replaces the state variable with its expected value in the previous period, $\hat{x}_t = \mathrm{E}_{t-1} x_t$. This imposes the informational restriction that the central bank cannot respond to contemporaneous shocks in the economy. The path of the new state variable is the same as that in the case of only additive uncertainty, given that both this path and the expectation operator are linear:

$$\hat{x}_{t+1} = F\hat{x}_t + Hi_t + J + x_{t+1} \qquad \textbf{(19)}$$

As in the simple one-target, one instrument case of section 2, expected loss now depends both on the squared deviations of expected variables from targets, and on the variance of the targeted variables. In terms of implementing this, the optimal rule is still a linear function of the (new) state variable, given by expression **(16)**, but the constants Λ and ω now depend both on estimates from the VAR and the variance-covariance matrices of the coefficient estimators from **(12)**:

$$\Lambda = -G - K + bF'\Lambda F - bF'\Lambda H\left[H'\Lambda H\right]^{-1} H'\Lambda F \qquad \textbf{(20)}$$

$$w = \left[I - bF'\left(I - \Lambda H\left(H'\Lambda H\right)^{-1} H'\right)\right]^{-1}\left[Gx^{*} - L + bF'\Lambda\left(I - H\left(H'\Lambda H\right)^{-1} H'\Lambda\right)J\right] \qquad \textbf{(21)}$$

where $K = G(1,1)\Sigma_{ip} + G(2,2)\Sigma_{u} + G(3,3)\Sigma_{inf}$, and for example, Σ_{inf} is the variance-covariance matrix of the coefficients on the state variables in the inflation equation. The matrix L is a similarly weighted combination of the vector of covariances of state vector variables with the estimated constant in each equation.(14)

Since the choice of weights and targets plays a crucial role in determining the coefficients in the optimal rule, Sack assumes that the past behaviour of the Fed was optimal and chooses the weights on *ip* and *u,* and the inflation target *inf** to minimise the mean square deviation between the optimal policy implied given historic data and the actual path of rates.

3.3 Sack's results for the United States

Sack's sample runs monthly from 1983:10 to 1996:12. The optimal interest rate in each period of the sample can be computed by substituting the vector of historical state variables into the derived rule. The generated paths are similar to the actual fed funds rate in that the broad thrust of policy according to the two rules is similar to actual policy.(15) Without parameter uncertainty, the generated path is more volatile, and wanders further from the actual historic path. Introducing parameter uncertainty reduces volatility and brings the two paths closer together.

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(14) As Sack notes, this solution technique ignores the fact that the variance of the disturbance x_t depends on the policy rule. An aggressive policy rule will increase this variance and as a result, the effect of parameter uncertainty will be underestimated.
(15) Sack acknowledges that this experiment violates the Lucas critique.

The tendency for the Fed to enact a gradualist response to shocks is evident from the impulse responses under the freely estimated reaction function from the VAR. Sack notes that, 'although the expected reaction speed varies across shocks, the observed policy maintains a restrained, deliberate speed of adjustment in the (federal) funds rate that is similar across all shocks, as if the Fed is simply reluctant to make aggressive funds rate changes' (page 14). The impulse responses of the optimal rules are somewhat different. In particular, according to the optimal, additive uncertainty only, rule responses to shocks should be more aggressive than has been the case.(16) The third set of impulse responses—showing the optimal response assuming both additive and parameter uncertainty—lies midway between the others: more gradual responses than the additive uncertainty only responses, but still quicker than in practice. From this Sack concludes that Brainard uncertainty offers a partial, but incomplete, explanation of interest rate smoothing in the United States.

Goodhart (1998) explores further the implications of Sack's results. On the assumption that the Fed can only move interest rates in increments of 25 basis points, he calculates time series of changes in the fed funds rate implied by the two optimal rules. He then compares the average size and pattern of rate changes with the actual changes enacted by the Fed. The Fed's behaviour suggests that it has a preference for moving rates by small increments (ie, it would prefer two 25 basis points changes to one 50 basis points change), and a dislike for reversing policy (ie, increasing rates for the first time following a series of cuts). Goodhart finds that the optimal rule allowing for additive uncertainty implies rate changes should have been larger on average than was the case, and that policy reversals should have been much more common. The optimal rule allowing for parameter uncertainty also suggests that policy reversals should be frequent, but implies smaller-sized rate changes, which match history, would have been optimal. Goodhart concludes that parameter uncertainty can explain the desire by the Fed for small rate changes, but not its dislike for policy reversals.

(16) Although as Sack notes even these impulse responses show some gradualism. This is optimal given the lag structure of the economy and the Fed's dual inflation and output objectives.

4 Analysing parameter uncertainty in the United Kingdom

In contrast to the United States, there have been many changes in the framework of UK monetary policy over the past two decades which complicate applied analysis. Modellers have the unenviable trade-off of either estimating a model over a long sample period, but over different policy regimes, or estimating a model over one regime only, in which case the sample period will be short. We chose to estimate a VAR over several regimes.(17)

4.1 UK policy regimes: implications for our analysis

Our sample period is 1980 Q2 to 1997 Q2. This spans the period from the introduction of the Medium Term Financial Strategy (MTFS) to the granting of operational independence to the Bank of England. Over the sample period at least five regimes for monetary policy can be identified:
(i) monetary targeting, which started in the late 1970s and ran in various forms to 1986, (ii) informal exchange rate targeting (shadowing the Deutsche Mark) for around a year from mid-1986, (iii) 'eclecticism' from then until 1990, during which nominal GDP and narrow money (M0) growth were prominent indicators, (iv) official exchange rate targeting (ERM) from 1990-92, and finally (v) direct inflation targeting.(18) Associated with these regimes is a long list of intermediate targets for policy: broad money growth (£M3 then M4); the DM/£ exchange rate; M0 and nominal GDP growth; sterling/ECU rate, and 'forecast inflation'.

This creates a problem in determining what variables to include in the VAR. We chose to include just the DM/£ exchange rate from the list of intermediate targets, along with output, RPIX inflation and base interest rates. The logic is that, following the introduction of the MTFS, all the other intermediate targets were designed to achieve a domestic inflation objective and so, in principle, their impact on policy should be captured by including the final inflation objective alone.(19) But the periods of exchange rate

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(17) We also attempted to estimate a VAR on post 1992 data, (which can be regarded as a single regime) to provide a cross-check on our results. But we did not obtain usable results from that exercise.
(18) We therefore interpret our results as being some kind of average over regimes.
(19) This assumes that intermediate targets were set consistently with the final objective of monetary policy, whereas *ex post* evidence clearly suggests this has not always been the case. The most obvious example of this is the unexpected shift in broad money velocity during the early and mid 1980s which unexpectedly changed the money-inflation link.

targeting are separable in the sense that the objective of policy can be characterised as the convergence of domestic inflation upon a foreign nominal anchor, effectively the German inflation rate.[(20),(21)] Output is included on the grounds that excessive output volatility is undesirable: as King (1998) notes, 'shocks of various kinds will mean that inflation will often deviate from the target and the MPC is required to take action to bring inflation back to the target. But it will do so gradually, if to do otherwise would have damaging consequences for employment or output'.

To identify the optimal policy rules we have to define the objectives of policy. We assume that the policy-makers want to minimise squared deviations of output from trend and inflation from target. We measure the output gap by a Hodrick-Prescott filter. The inflation target is set to 2½% from 1992 Q4. Prior to this the inflation target was not explicit, but we assume that it fell during the 'monetarist experiment' and during ERM membership, and remained broadly constant between 1986 and 1990 at around 3½%. The Data Annex provides more detail on both of these targets. Each of the targets for policy is defined such that the deviations from target, which enter the VAR, are stationary. For the exchange rate we assume the objective is to keep the bilateral exchange rate constant at the current rate. This is a weak way of specifying a objective for the prevailing rate, that takes account that the DM central rate implicit during ERM membership (2.95), differed from the level informally targeted in 1986/87 (3.00). An alternative would have been to assume a fixed target for the exchange rate, but in that case the policy rules would have continually been seeking to return the exchange rate to, say, 3.00, even in periods when policy clearly was attempting no such thing.

The remaining issue is how to enter the policy instrument, the nominal base rate, into the VAR. Over the sample period base rates are non-stationary, and it would be statistically invalid to enter them in the VAR in levels. To render interest rates stationary we could have taken first differences or

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(20) Use of the DM/£ rate can be viewed as an approximation to an actual ECU target during ERM entry, but the two exchange rates were highly correlated; the bilateral rate is a good proxy.
(21) An alternative modelling strategy would have been to include all intermediate targets for policy since 1981, a list that would include: broad money (£M3 then M4); the DM/£ exchange rate; M0 and nominal GDP; sterling/ECU rate, and 'forecast inflation'. But this would have greatly increased the dimension of the VAR and could have led to a degrees of freedom problem. Furthermore it is doubtful whether such a large-dimension VAR would be statistically robust.

de-trended the data. The latter option is preferable. Had we included the first difference of nominal interest rates in the VAR then we would have left undetermined the 'neutral' *level* of nominal interest rates. Suppose each of the objectives of policy were at target, then with a first-difference specification this would imply that the desired change in nominal rates was zero, so that whatever level of interest rates prevailed that would have been the appropriate 'neutral' level. By contrast, by entering interest rates as deviations from trend we define the neutral rate as the trend in nominal rates, so when objectives equal target values nominal rates should be set to their neutral level. We de-trended base rates using the Hodrick-Prescott filter (again, see the Data Annex for details).(22)

Charts 4.1 to 4.4 plot the data that enter the VAR. According to our de-trending procedure monetary policy was tight at the start of the sample period, was then loosened substantially during the rest of the first half of 1980s, averaging close to a neutral rate. Policy became very loose after 1987, and then was substantially tightened at the end of the decade, remaining so during membership of the ERM. Policy was made looser, and became absolutely 'loose' thereafter, and has been close to, if slightly above, neutral since early 1995 (see Chart 4.1). These general patterns accord well with perceptions: tight monetary policy contributed to the disinflations and recessions of the early 1980s and 1990s, while loose policy helped to fuel the late 1980s boom.

(22) An alternative would have been to set the trend equal to target inflation. This approach would have had two drawbacks: first it would not have allowed for any lack of credibility on the part of monetary authorities (that would result in a difference between the actual and perceived inflation target). Second, it would not have allowed for any trends in the real interest rate. There is considerable evidence that real rates do contain trends and, in particular, that trend real rates rose during the 1980s (see the G10 deputies report, 'Savings, Investment and Real Interest Rates', for a discussion).

Chart 4.1
Base rates: deviation from trend

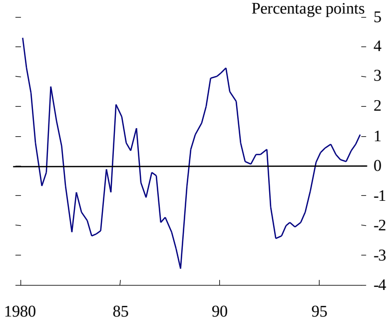
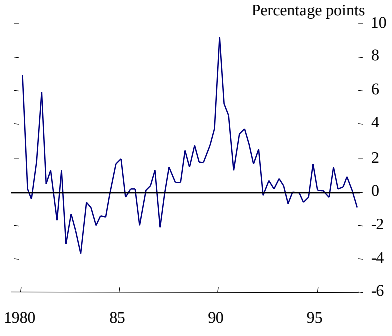


Chart 4.2 shows inflation relative to target. Two observations are worth making. First, inflation has averaged 0.8 percentage points above a changing target over the sample; this is consistent with a period of trend disinflation. Second, the significant error in policy over the sample occurred in the late 1980s when inflation over-shot target by as much as 9 percentage points in 1990 Q2. This coincided with a local peak in quarterly RPIX inflation of 12.5%.

Chart 4.2
Inflation: deviation from target



Charts 4.3 and 4.4 show the output gap and change in the exchange rate. Chart 4.3 suggests that the most significant deviations in output from trend occurred at the very beginning of the sample period and then during the late 1980s/early 1990s boom bust. This accords with other estimates of the output gap, see eg Thomas, Dhar, and Pain (1998).

Chart 4.3
Output: deviation from trend

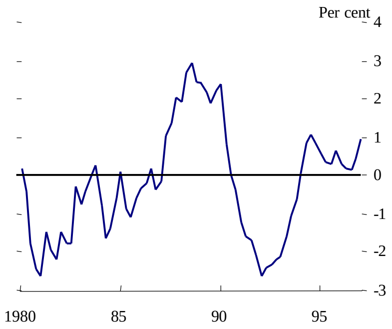
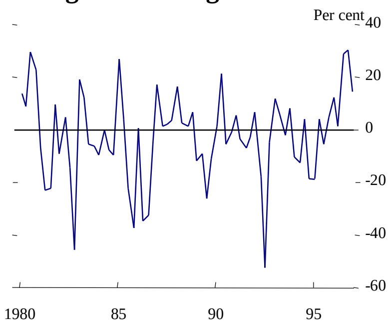


Chart 4.4
Change in exchange rate



4.2 VAR diagnostics

In estimating the VAR we had to determine lag length and the appropriate identification technique. Given that the study uses OLS standard errors as a proxy for parameter uncertainty, we did not want to inflate the standard errors by over-parameterising the model. AIC and SBC information criteria both suggested a lag length of 1. As 1 lag was sufficient to eliminate vector autocorrelation, both in the individual equation residuals and in the vector residuals, we adopted that length (see Table 4.1 below).

Table 4.1
Testing the quarterly VAR for autocorrelation with 1 lag

Variable			p-value
exchange rate	:AR 1- 5 F(5, 59) =	1.2301	[0.3065]
inflation	:AR 1- 5 F(5, 59) =	0.96393	[0.4473]
output	:AR 1- 5 F(5, 59) =	0.35987	[0.8738]
official rates	:AR 1- 5 F(5, 59) =	0.38529	[0.8569]
Vector	:AR 1- 5 F(80,164) =	1.2337	[0.1311]

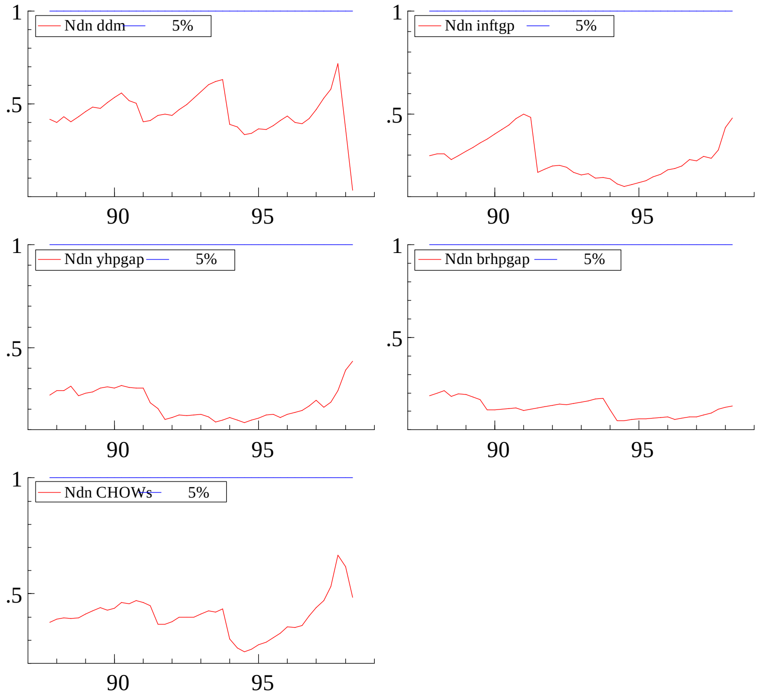
The normality assumption (Table 4.2 below) was violated in the inflation and official rate equations. The latter problem is not surprising given the discrete nature of interest rate changes. We acknowledge this but do not offer a solution (dummy variables would considerably complicate the calculation of optimal rules and, as Hamilton (1994) notes on page 298, the OLS estimators of population parameters are consistent estimators even if the innovations are non-normal).

Table 4.2
Testing for normality

Variable			p-value	
exchange rate	:Normality Chi^2(2)=	1.3173	[0.5175]	
inflation	:Normality Chi^2(2)=	11.961	[0.0025]	**
output	:Normality Chi^2(2)=	2.3208	[0.3134]	
official rates	:Normality Chi^2(2)=	13.35	[0.0013]	**
Vector normality	Chi^2(8)=	31.214	[0.0001]	**

Chart 4.5 shows that the model does not fail the break-point F tests at 5%, either by equation or as a system.

Chart 4.5: Testing for stability



4.3 Identification

The simplest method of identification, which Sack uses, is to decompose the variance-covariance matrix of residuals into its lower triangular square root or Choleski factor, following Sims (1980). This imposes a recursive response to disturbances. But this would have the unsatisfactory implication for our model that either base rates do not respond to exchange rate innovations in a quarter or *vice versa*.

Hence we adopted an alternative approach, introduced by Sims (1986), discussed in Leeper, Sims and Zha (1996) and applied to a small open economy (Canada) in Cushman and Zha (1997). This method applies non-recursive restrictions to the contemporaneous relationships between the variables. Leeper, Sims and Zha distinguish between three types of data: information variables; private sector variables and policy instruments.

Information variables are typically asset prices which are set in auction markets and are assumed to respond to all other variables. The exchange rate is an example of an information variable. Private sector variables are determined in goods markets, and respond only sluggishly to information elsewhere in the economy. Output and inflation are examples of private sector variables. Finally, instruments are set by policy-makers and respond to whatever information policy-makers have access to.

According to this schema the exchange rate should respond to shocks in all other markets. Output and inflation should move more slowly: we assume output responds to all other shocks with a lag; we allow inflation also to respond to within-period output shocks. We assume interest rates respond to exchange rate and inflation shocks within period, but to output shocks with a lag (see Table 4.3). This ensures that the interest rate and exchange rate both respond within period to shocks in other markets, which we believe is a desirable property. Not allowing policy to respond to within-period output shocks reflects the delay in published statistics.(23) The assumption that inflation does not respond to the exchange rate within the quarter is somewhat arbitrary but provides the final restriction to separately identify the four shocks.

Table 4.3
Quarterly VAR identifying restrictions

	Shock			
Variable	Output	Inflation	Exchange rate	Base rates
Output	**x**	0	0	0
Inflation	**x**	**x**	0	0
Exchange Rate	**x**	**x**	**x**	**x**
Base Rates	0	**x**	**x**	**x**

There is however a cost to this identification scheme. As discussed in Section 3.1, in order to identify the optimal rules we minimise the loss function **(17)**, taking the reduced-form (non-policy block) of the model as a given constraint. But because we allow the exchange rate to respond within

(23) One can make the argument that the monetary authorities have the same information set as the foreign exchange markets, so should respond as quickly to output shocks. But some restriction is necessary to distinguish two shocks. Swapping the identifying assumption around, and allowing policy-makers to respond more quickly to shocks than the foreign exchange markets leaves the results qualitatively unchanged, and has minimal quantitative impact.

the period to policy, the reduced-form should not be invariant to changes in the policy rule. Another way of thinking about this is that, if the reduced-forms for each of the VARs we calculate are the same, yet the structural equation for interest rates—the interest rate reaction functions are different in each system—then, implicitly, the *structural forms* for the non-policy variables must be changing in each system. Given the identification system we use, these changes will be wholly contained in the exchange rate equations. The functional form for these equations will be identical in each model, but some parameter values will differ. Hence, in considering the properties of the optimal rules, we need to recognise that some of the differences in the optimal rules may be accounted for by differences in the central parameter estimates in each exchange rate equation, rather than whether or not parameter uncertainty is accounted for.

An alternative approach would have been to 'switch off' the contemporaneous response of the exchange rate to policy. But this would only have been a superficial gain—we know that the exchange rate does respond within the quarter—which would have resulted in each model being misspecified. Moreover, as we discuss in Section 5.1 below, the distortion introduced by keeping the reduced-form fixed appears limited.

4.4 Choice of policy-maker's preference weights

The optimal rule is derived from the objective function and the constraints, and as such depends on the relative weights placed by the policy-maker on the terms in the objective function. Sack proposes that these variables are data-determined by the following procedure. For any choice of relative weights, an associated optimal rule can be calculated. The optimal interest rate in each period is then computed by substituting the vector of state variables into the derived rule. By grid-searching over the parameters, we can find the relative weights that minimise the sum of squared deviations between the optimal policy and the actual official rate.

For our UK model, it transpires that the objective function over which we are grid-searching is quite flat, so the relative weights do not make that much difference to optimal policy paths. For the results quoted here, the weight on inflation deviations is normalised to 1, that on output deviations equals 0.5 and the weight on exchange rate deviations equals 0.1. We include the exchange rate in the objective function because we are essentially capturing an average of regimes and must account for the ERM and shadowing the Deutsche Mark.

5 Results

We begin this section by looking at the coefficients on the optimal rules, and the historical paths for official interest rates that these rules would have implied. We then look at the evidence for conservatism, gradualism and caution in nominal interest rates. Finally, following Goodhart (1998), we bracket interest rate changes into discrete steps of 25 basis points and look at the distribution of the changes.

5.1 Predicted optimal rules

Having selected the variables to include in the objective function and the VAR describing the economy, we can calculate the optimal rule under additive uncertainty and under parameter uncertainty. The table below shows the coefficients in the optimal rules.

Table 5.1
The optimal rules

	inf(t)	***e(t)***	***y(t-1)***	***inf(t-1)***	***e(t-1)***	***br(t-1)***	***const***
Additive uncertainty	0.109	0.099	83.852	-0.132	-0.069	-0.561	0.023
Parameter uncertainty	0.109	0.099	47.545	0.046	-0.031	-0.227	0.117
memo:							
Estimated (actual)	0.110	0.099	-2.222	0.051	-0.034	0.572	-0.008

Recall that in our identification scheme we have restricted the contemporaneous response of official rates to output to be zero. Because of scaling we cannot compare the coefficient eg on inflation to that on output. But it is possible to compare the relative magnitude of the coefficients on one variable across rules. Interestingly, the contemporaneous response coefficients on inflation and the exchange rate are the same in both rules. But three out of four coefficients on the lagged state variables have the same sign and are smaller in magnitude for the rule under parameter uncertainty, in line with the predictions of the simple Brainard model. The exception is the coefficient on lagged inflation, which is perversely signed in the additive uncertainty rule.

The coefficients on the parameter-uncertainty rule are generally closer to those in the actual estimated reaction function. The two main differences are that the actual rule implies a much smaller, and initially perverse, reaction to output changes, and that the actual rule for interest rates puts a large weight on the value of lagged interest rates. Other things being equal, this will impart a degree of smoothness to the series for actual interest rates.

In Section 4.3 we noted that differences in the three rules could arise simply because of differences in the structural form for the exchange rate equation in each model. Looking at these coefficients it is apparent that the only significant difference across models is in the exchange rate response to inflation. In particular, mirroring the additive uncertainty interest rate response, the contemporaneous exchange rate response to inflation in the additive uncertainty model is differently signed (negative) to the responses in the other two models. The other important difference is that the cumulative exchange rate response to inflation is greater in both additive and parameter-uncertainty models than in the actual estimated model. These differences suggest that we should be cautious in interpreting different interest rate responses to inflation shocks across our models as reflecting the effects of parameter uncertainty.

Charts 5.1 and 5.2 below show the interest rates implied by the optimal rule (under either purely additive uncertainty or accounting for multiplier uncertainty) given the actual state of the economy each quarter. Chart 5.3 shows the optimal interest rate from the two optimal rules, allowing the state of the economy to respond endogenously to the chosen rule in the face of the observed non-policy shocks; ie it shows dynamic forecasts.

Note that the optimal rule accounting for parameter uncertainty is smoother than the rule under only additive uncertainty. In both cases, the optimal rules do not match actual rates in the early part of the sample, which may be because we fail to represent the actual reaction function well in the early period. The rule also predicts that official rates should have risen earlier towards the end of the 1980s (the so-called ‘policy mistake’) and, according to the one step ahead forecasts, should have either come down sooner (additive-uncertainty rule) or peaked lower (parameter-uncertainty rule) during the early 1990s. This may be related to the fact that we cannot vary the weights in the objective function, and therefore do not place a high enough weight on the exchange rate during the ERM period.[24]

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(24) By examining the historical behaviour of the non-policy variables in response to alternative rules using an estimated reduced-form model we are clearly subject to the Lucas critique. We acknowledge this but offer no solution.

Chart 5.1
Actual versus optimal interest Rates (one step ahead of forecasts)

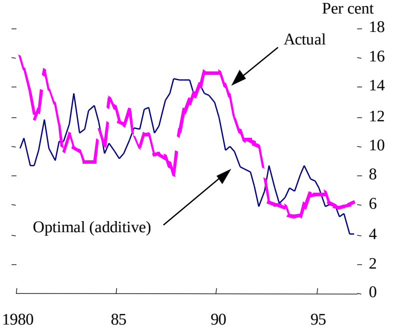


Chart 5.2
Actual versus optimal interest Rates (one step ahead of forecasts

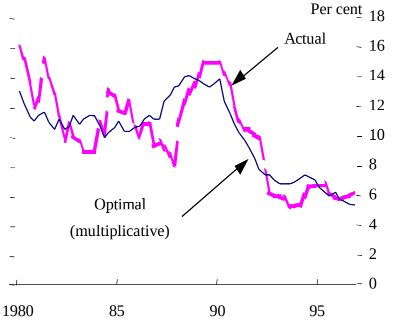
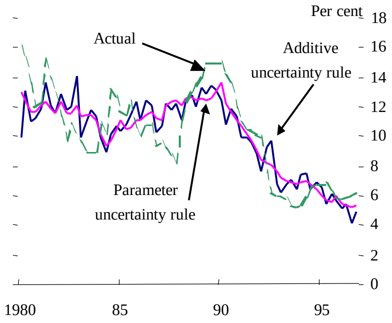


Chart 5.3
Optimal interest rates: dynamic forecasts

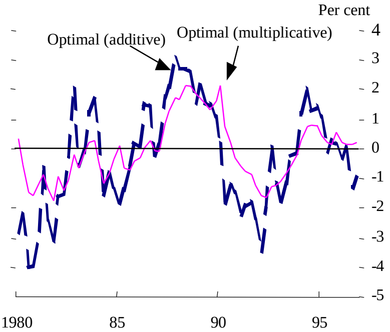


5.2 Nominal interest rate conservatism

In Section 2.5 we showed that parameter uncertainty could lead to conservatism. Chart 5.4 shows the deviations of optimal (additive and parameter-uncertainty) interest rates from this neutral level.(25) This chart suggests that the nominal conservatism principle applies to the United Kingdom. Deviations of nominal interest rates from neutral are smaller under parameter uncertainty than when only additive uncertainty is considered.

(25) This chart is of one step ahead forecasts; a comparable chart based on dynamic forecasts tells a similar story.

Chart 5.4
Deviations of optimal rates from neutral level



Furthermore, the differences between the two series are quite marked, which implies that conservatism might be a material concern. For example, following a positive shock to inflation in 1988 Q2 the optimal additive rule points to an immediate increase in base rates of 100 basis points (to 3.1 percentage points above neutral). According to the parameter-uncertainty rule, rates should have been increased by 40 basis points (to 1.7 percentage points above neutral). And in 1992 Q3, following contractionary shocks, the additive rule points to a 110 basis points cut in rates (to 3.5 percentage points below neutral); the parameter-uncertainty rule points to a mere 30 basis points cut (to 1.6 percentage points below neutral). Table 5.2 presents some summary statistics for the three rules: note that the maximum and minimum deviations from neutral, and standard error for the additive rule, are all greater than for the parameter-uncertainty rule.

Table 5.2:
Deviations of interest rates from trend—some summary statistics

	Estimated (actual)	Additive uncertainty	Parameter uncertainty
Mean	0.00	-0.30	-0.04
Standard error	1.69	1.74	1.04
Minimum	3.29	3.06	2.17
Maximum	-3.46	-4.02	-1.73

Note: The first row reports the average gap between trend interest rates and the level implied by each rule; the second row reports the standard deviation of these 'gaps'. The third and fourth rows report the largest negative and positive deviations that occur over our sample period

As we discussed in Section 2.2, conservatism arises in the presence of parameter uncertainty because large movements in interest rates away from neutral, or trend, will increase the variance of the targets of monetary policy (in our model primarily the stabilisation of inflation and output around their target levels). A higher variance increases the probability of missing these targets by a significant amount, and policy-makers therefore will choose to move interest rates less in response to a shock. Our finding that the standard error and maximum deviations in the path for parameter-uncertainty interest rates from trend are smaller than for the additive-uncertainty rule is consistent with the idea that policy-makers will choose to move rates less in response to a shock.

5.3 Gradualism

Section 2.6 shows that parameter uncertainty can lead to gradualist, phased-in, policy responses. Charts 5.5 to 5.8 show the impulse responses of base rates under the estimated and optimal rules to the four shocks identified in our model.[26] In response to output, the estimated response

(26) Forecast error variance decompositions suggest that interest rate and exchange rate shocks are the most important drivers of the variance of interest rates in the estimated model. This may be because the identification scheme wrongly attributes some interest rate shocks as exchange rate shocks (both shocks impact immediately upon both interest rates and exchange rates). Output shocks are more important than inflation shocks. Of the influences on the variance of output the most important are output shocks, followed by interest rate shocks. Inflation variance is influenced significantly by all four shocks. The exchange rate variance is almost wholly accounted for by exchange rate and interest rate shocks.

appears more drawn out than the rule accounting for parameter uncertainty, and the response under the parameter-uncertainty rule is more drawn out than the response under additive uncertainty.(27) This is consistent with the notion of gradualism. Table 5.3 shows how the total magnitude of the nominal interest rate response is spread out over time for the three rules and for various shocks.(28) The first three columns of the table suggest that gradualism is apparent in the optimal parameter-uncertainty response to the output shock, but is less evident after one year.(29)

Chart 5.6 shows the responses to an inflation shock. Gradualism in the actual response is again apparent; although around half of the total response occurs in the first year (see column 6 of table 5.3). The parameter-uncertainty response shows a similar pattern— still gradualist but less so than in response to output (compare columns 2 and 5). The difficulty is in comparing these two responses with the optimal additive response. It is positive in the first period but then goes negative. As we discussed earlier, comparison with the additive-uncertainty rule may be complicated by the different exchange rate reaction implicit in the additive uncertainty model. Nevertheless, in Table 5.3 we cumulate the magnitude of responses (ie add together the initial positive and subsequent negative responses), and by this measure the additive response is the quickest of the three.

---

(27) Our identification scheme means that policy-makers cannot respond immediately to an output shock; hence the initial responses are all zero. As policy-makers can immediately respond to other shocks, the initial responses in Charts 5.6 to 5.8 are non zero.

(28) Table 5.3 shows the proportion of the cumulative response to a shock after five years that should arise *n* quarters after the shock. As some of the responses lie above and below the zero axis at various points in time, we consider the proportion of the total *magnitude* of the response that has been completed.

(29) Note also, in response to an output shock, the initial response of the additive uncertainty rule is around 1.5 times larger than the initial parameter uncertainty response which is consistent with conservatism affecting the latter.

Chart 5.5
Base rate response to output

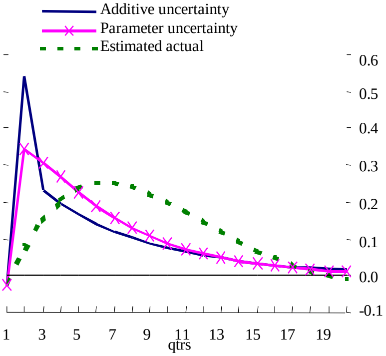


Chart 5.6
Base rate response to inflation

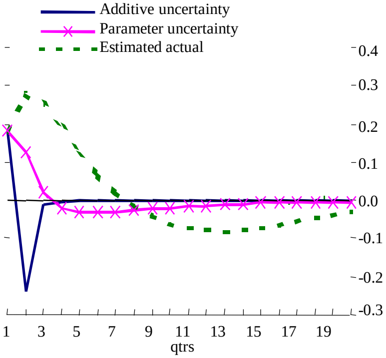


Chart 5.7
Base rate response to exchange rate

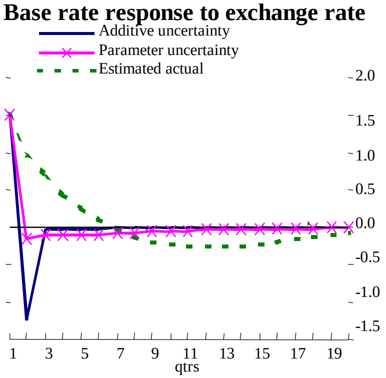
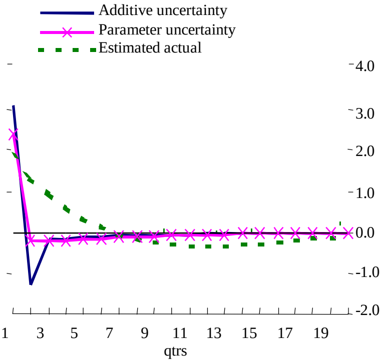


Chart 5.8
Base rate response to base rate



The response of interest rates to policy shocks shows that the optimal rules do not account for the degree of autocorrelation in official rates. Sack observed a similar pattern for his US results and interpreted them as suggesting that policy-makers have additional motives for smoothing interest rates that are not captured in the specified objective function.

It is difficult to place an interpretation on the exchange rate response since we are not modelling the foreign economy. But one tentative explanation is as follows. An important determinant of the exchange rate is the interest rate differential compared with overseas. Uncovered interest rate parity suggests that the spot exchange rate will appreciate in a jump in anticipation of a previously unexpected increase in interest rates. Our model is backward looking and such moves in exchange rates will be identified as exchange rate shocks, when in fact they are endogenous responses to expected future interest rate changes. Assuming this type of shock is the most frequent 'exchange rate shock', the impulse-response function for interest rates will then mimic the response to genuine interest rate shocks, as identified in Chart 5.8. Hence the patterns in Chart 5.7 replicate those observed in 5.8: a gradual actual response to exchange rate shocks, compared with an immediate non-persistent response under each of the optimal rules.

Table 5.3
Proportion of total response occurring in quarters after the shock

	Output shock			**Inflation shock**			**Base rate shock**		
Horizon (qtrs)	**Add rule**	**Mult Rule**	**Est rule**	**Add rule**	**Mult Rule**	**Est Rule**	**Add rule**	**Mult rule**	**Est rule**
1	0.01	0.01	0.01	0.42	0.32	0.10	0.57	0.59	0.22
2	0.28	0.17	0.04	0.97	0.53	0.25	0.81	0.64	0.38
3	0.39	0.31	0.10	0.99	0.57	0.38	0.84	0.69	0.49
4	**0.49**	**0.43**	**0.18**	**0.99**	**0.60**	**0.48**	**0.87**	**0.74**	**0.56**
5	0.57	0.54	0.27	1.00	0.65	0.55	0.89	0.79	0.59
6	0.64	0.63	0.37	1.00	0.71	0.59	0.90	0.83	0.61
7	0.70	0.70	0.47	1.00	0.76	0.60	0.92	0.86	0.61
8	**0.75**	**0.76**	**0.56**	**1.00**	**0.80**	**0.61**	**0.93**	**0.89**	**0.63**
9	0.79	0.81	0.65	1.00	0.84	0.63	0.95	0.91	0.67
10	0.83	0.85	0.73	1.00	0.87	0.66	0.95	0.93	0.70
11	0.86	0.88	0.80	1.00	0.90	0.70	0.96	0.94	0.74
12	**0.89**	**0.90**	**0.85**	**1.00**	**0.92**	**0.75**	**0.97**	**0.96**	**0.78**
13	0.91	0.93	0.90	1.00	0.94	0.79	0.98	0.97	0.82
14	0.93	0.94	0.93	1.00	0.95	0.83	0.98	0.97	0.86
15	0.95	0.96	0.96	1.00	0.97	0.87	0.99	0.98	0.90
16	**0.96**	**0.97**	**0.98**	**1.00**	**0.98**	**0.91**	**0.99**	**0.99**	**0.93**
17	0.97	0.98	0.99	1.00	0.98	0.94	0.99	0.99	0.95
18	0.98	0.99	1.00	1.00	0.99	0.96	1.00	0.99	0.97
19	0.99	0.99	1.00	1.00	1.00	0.98	1.00	1.00	0.99
20	**1.00**	**1.00**	**1.00**	**1.00**	**1.00**	**1.00**	**1.00**	**1.00**	**1.00**

5.4 Caution

As discussed in Section 2.7, depending on the persistence of the underlying process in the economy, gradualism might imply caution as well. Table 5.4 shows the cumulative magnitude of the response for each of the two optimal rules as a proportion of the total estimated response:

Table 5.4
Relative cumulative responses

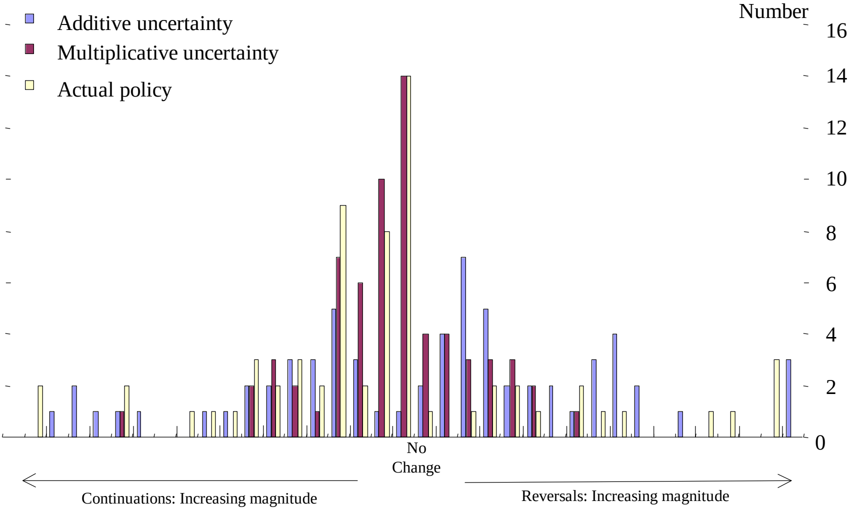
Shock	Additive Uncertainty Rule	Parameter Uncertainty Rule	Estimated Uncertainty Rule
Output	0.80	0.85	1.00
Inflation	0.23	0.31	1.00
Base rate	0.65	0.48	1.00

For output and inflation, the additive rule implies a smaller total response than the rule under parameter uncertainty. The table provides no evidence to suggest parameter uncertainty should impart caution.

5.5 Size and direction of interest rate changes

Following Goodhart (1998) we have constructed time series for interest rates on the assumption that interest rates can only be changed in increments of 25 basis points. This allows us to analyse the average size and direction of interest rate changes implied by rules. Chart 5.9 compares the histograms of the size of rate changes implied by the rules, distinguishing between changes that would have represented a continuation in policy (ie a rise following previous rises, or a cut following previous cuts) and changes that would have been a policy reversal (ie the first rise (cut) after a period of cuts (rises)). ‘No change’ lies in the centre, and the magnitude of continuations (both increases and decreases) increases to the left. The magnitude of reversals (increases and decreases) increases to the right.

Chart 5.9
Distribution of continuations and reversals



This pictures contrasts a little with that for the United States. Actual rate changes are dispersed over the whole distribution, and there were as many large rate changes in practice as either of the two rules would have implied. Nevertheless there is a bunching of actual rate changes towards the centre of the histogram: small rate changes are still the norm. It is also evident that the bulk of the mass of actual changes lies in the left-hand side of the histogram—continuations have been more common than reversals. Turning to the two optimal rules it is clear that the parameter-uncertainty distribution is most bunched around the centre—pointing to predominance of small changes—and that both distributions appear to be broadly symmetrically distributed around zero.

A major difficulty in interpreting these results is that the data period is quarterly. Given interest rate decisions are made more frequently (currently once a month in the United Kingdom and every six weeks in the United States) this creates a time aggregation problem. Hence, as an aid to interpretation we have aggregated Goodhart's results for the United States to the quarterly frequency. Table 5.5 compares the sizes of rates changes in the United Kingdom and United States. Focus first on actual policy. It is apparent that actual changes in the United States have, on average, been smaller than in the United Kingdom. Differences in the sample period might contribute to this, but our supposition is that policy regime changes (eg exiting the ERM) and policy errors (eg acting too late to contain inflation during 1988/89) explain the greater incidence of large changes in the United Kingdom. This interpretation is consistent with the finding that the parameter-uncertainty rule suggests that a greater proportion of changes in the United Kingdom should have been 25 basis points or less: the optimal rules do not allow for mistakes or regime changes. The additive rule suggests that most changes in the United Kingdom should have been 75 basis points or more.

Table 5.5
Proportion of interest rate decisions that are:

	No Change, or 25 basis points		50 basis points		75 basis points or more	
	US	**UK**	US	**UK**	US	**UK**
Actual (estimated)	55	**38**	18	**21**	27	**41**
Parameter uncertainty	45	**57**	29	**21**	27	**21**
Additive uncertainty	39	**17**	31	**30**	31	**53**

Table 5.6 shows the relative frequency of policy reversals. Again they are more common in the United Kingdom than the United States, and the same explanations are probably relevant. If so, this implies that correlation between the United Kingdom actual and parameter uncertainty lines in the table is artificially high, because some actual reversals will have been the result of factors extraneous to the calculation of the optimal rule (ie regime changes, policy errors).

Table 5.6
Proportion of interest rate decisions that are:

Per cent	Continuations		No change		Reversals	
	US	**UK**	US	**UK**	US	**UK**
Actual (estimated)	55	**54**	31	**21**	14	**24**
Parameter uncertainty	55	**48**	12	**21**	33	**30**
Additive uncertainty	55	**41**	8	**0**	37	**58**

Interpretation of our results is clearly harder than of Sack's results for the United States, but we conclude that the UK results are consistent with the notion that Brainard uncertainty can contribute to explaining the preference of policy-makers to enact small changes, but not their dislike of policy reversals.

6 Conclusions

The task that we set ourselves was to examine whether Brainard uncertainty matters empirically for the United Kingdom. In answering this question we have found it useful to distinguish three concepts: conservatism, gradualism and caution.

We have found some evidence of nominal interest rate *conservatism*: Chart 5.4, which plots deviations of the optimal interest rate from neutral under additive and parameter uncertainty over the sample, shows that the deviations were smaller when parameter uncertainty was considered. And there is also evidence of nominal interest rate *gradualism* from the impulse response to output shocks in particular, but also to inflation shocks (Charts 5.5 and 5.6, as well as Table 5.3). In response both to output and inflation shocks, the optimal response for the parameter-uncertainty rule is more gradual in the <u>first year</u> than the optimal response for the additive rule. This carries over when we transform the series to ones where the policy-maker can move only in integer multiples of 25 basis points.

There is no evidence of nominal rate caution. Hence Brainard uncertainty seems to lead optimally to initially smaller and more drawn-out responses by policy-makers, but ultimately to just as large a total change in nominal interest rates.

The difficulty in interpretation is that there is no simple statistical hypothesis test to decide whether these effects are significant. But we are reassured by the range of evidence presented in Section 5 above, and importantly that the results accord with (our) intuition. We would have been surprised, for example, if the impulse responses had implied gradualism running into two to three years.

Finally, along with Goodhart (1998), we find that Brainard uncertainty may provide an explanation for policy-makers to prefer a series of small changes to large changes, but it does not provide an explanation of policy-makers' apparent dislike of policy reversals.

Data Annex

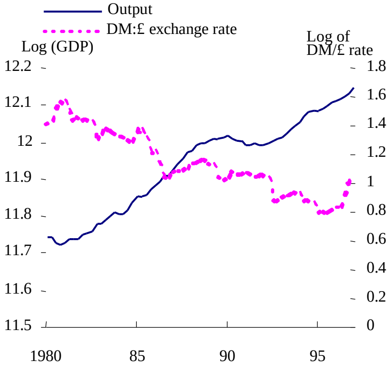
The raw data underlying the quarterly VAR are as follows.

Official base rates	Quarter average
RPIX inflation	Annualised quarterly change in log of seasonally adjusted RPIX price index
Real GDP	Measured at 1995 market prices
Exchange rate	Log of quarter average DM/£ exchange rate

They are plotted in charts A1 and A2 over the sample period 1980:Q2 to 1997:Q2. All the data appear non-stationary.

Chart A1
Base rates and inflation

Chart A2
Output and the Exchange rate (logs)



The data do not cointegrate; so to estimate the VAR we need to transform these data into stationary time series. The choice of whether to difference or de-trend data is always controversial and perhaps especially so when the objective, as in this paper, is to estimate policy reaction functions. We adopt the following hybrid approach.

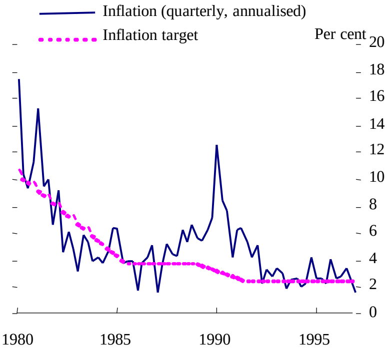
The underlying target inflation rate can be interpreted as a policy choice. Since 1992 this has been made explicit, and had been specified as 2½% annual inflation.[(30)] Before 1992 there was no explicit inflation target. Prior to this, as we have discussed, there existed several policy regimes. Broadly speaking, two disinflationary regimes can be identified: the 'monetarist experiment' of the early 1980s and ERM membership that could be characterised as an attempt to 'import' German inflation. Thus we have assumed that the inflation target fell between 1976 and 1987, remained constant to 1990 and then fell again during ERM membership to 1992, reaching the explicit target after exit from the ERM.

To quantify this characterisation of (the first implicit and then explicit) target, inflation was regressed on a time trend between 1976 and 1987, the

(30) The precise definition of the target has changed periodically since its introduction in October 1992, but specification has always included 2½% annual inflation as either a medium-term objective or reference level. To characterise the target as 2½% annual inflation since 1992 is a reasonable approximation.

fitted value being the assumed target. The level of the target at the end of money targeting (3½%) is assumed constant until ERM entry, after which it declines linearly to 2.5%, which we take to be the explicit target post ERM membership. Chart A3 compares this target with actual inflation outcomes over the VAR sample period.

Chart A3
Inflation target and outcomes



Although the trend in base rates and inflation are clearly linked by the Fisher identity they are not necessary identical. First, the underlying real rate may have trends. There is evidence in the United Kingdom that the long-run equilibrium rate may have increased and then fallen again during the 1980s/90s on account of fiscal developments (G10 Group of Deputies, 1995). Second, the inflation expectations and the inflation target may differ; not only for cyclical reasons but because the credibility of the policy-makers may have been less than complete. For this reason we choose to identify the trend in base rates separately to the inflation target using the Hodrick-Prescott filter.

We de-trended output using the same filter and the resultant underlying and trend series are shown in Charts A4 and A5. We discuss the interpretation of the interest rate and output 'gaps' in Section 4 of the main text.

Chart A4
Base rates and trend

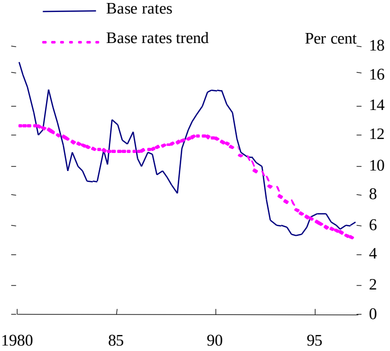
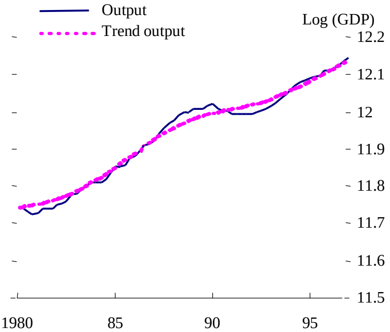


Chart A5
Output and trend



Finally, we differenced the DM/£ exchange rate (see Chart 4.4 in the main text). Our main motivation for including this rate was that for sub-samples (mid-1986 to mid-1987 and 1990 Q4-1992 Q3) the intermediate objective of monetary policy was, in effect, to keep this exchange rate constant. The underlying trend in the rate is likely to reflect differences in underlying UK and German inflation, as well as persistent real exchange rate shocks. Most of these influences are beyond the scope of this VAR, so there would have been little point in including the deviation in the exchange rate from some estimated trend.

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