Uncertainty and Simple Monetary Policy Rules

An illustration for the United Kingdom

Simon Hall, Chris Salmon, Tony Yates and Nicoletta Batini

Bank of England, Threadneedle Street, London, EC2R 8AH.

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Abstract

We investigate the effects of additive and multiplicative uncertainty upon the stabilisation properties of a simple base money rule for monetary policy. Using a five-equation empirical model of the United Kingdom, we show that changes in the extent of additive uncertainty have no effect upon the ‘optimal’ degree of policy responsiveness to shocks to the economy. However, we find that policy-makers should respond by less to shocks in the face of multiplicative uncertainty. And as multiplicative uncertainty rises, so the optimal degree of policy reaction falls. This accords with Brainard’s (1967) theoretical analysis and could be interpreted as justifying a gradualist monetary policy.

Introduction

In 1990, Benjamin Friedman wrote that ‘the making of monetary policy... involves both skill and chance’. Few (and perhaps even fewer central bankers) would disagree with this. But if the world suddenly becomes more ‘chancy’, how should policy respond? We offer an answer to this question by supposing that policy-makers follow a simple policy rule (actually a variant of McCallum, 1988, 1990 a, 1990 b) and believe a simple empirical model of the United Kingdom. We look to see if the optimal degree of feedback varies according to how much uncertainty—proxied by the variances of model parameters—the policy-maker faces.

Renewed interest in parameter uncertainty is not unique to the Bank of England. Alan Blinder, in particular, has been arguing forcefully that more work is needed to tease out the policy implications of this form of uncertainty. He said in 1997: ‘[A]cademic economists could also be more helpful to policy-makers if they would...investigate the robustness of Brainard’s conservatism principle’. Recent papers by Estrella and Mishkin (1998), Sack (1998) and Wieland (1995, 1996, 1998) have explored its implications for US monetary policy. Sheutrim and Thompson (1998) consider its importance in Australia. Martin and Salmon (forthcoming) apply the approach of Sack (1998) to the United Kingdom.

The rest of this paper is organised as follows. Section 2 reviews the literature on parameter and model uncertainty. It illustrates the Brainard conservatism principle with a simple model of inflation forecasting due to Svensson (1997). Section 3 describes our simulation methods, the results of which are set out in Section 4. We conclude by drawing out some possible policy implications from our analysis.

2 Monetary policy and uncertainty(1)

To explain where our contribution fits in, it is useful briefly to review the literature. We begin by defining terms. We take additive uncertainty to be the component of a forecast error due to uncertainty about the outcome for an exogenous variable in the system—a residual in an econometric equation, for example. The second type of uncertainty discussed is multiplicative (or

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(1) Readers are referred to Batini, Martin and Salmon (1998) for a more complete discussion of the issues covered in this section.

parameter) uncertainty: this is uncertainty about the value of any or all of the observed parameters in the model of the economy.(2) Finally, we take model uncertainty to represent still more pervasive forms of uncertainty. This might take the form of a mis-specification of an equation, or set of equations in the system; an omitted variable; or a mistake in an assumption about the functional form of equation(s) in the model. Or there could be uncertainty about the model's core properties: for example, over whether money is non-neutral, neutral or super-neutral.(3)

Additive uncertainty was widely discussed in the early control literature, such as Phillips (1954) and Theil (1964). Theil (1964) derived the famous certainty-equivalence result that, in the presence of additive uncertainty, it was optimal for a policy-maker to act as if he were certain about the prospects for the economy.(4) At that time, there was a fair degree of confidence that econometricians could correctly identify the appropriate structural model, and thereby eliminate all but the additive uncertainty in the modelling process. Thus Phillips (1954) argued that 'it is quite likely...that a monetary policy based on the principles of automatic regulating systems would be adequate to deal with all but the most severe disturbances to the economic system' (page 315).

Multiplicative (or parameter*)* uncertainty was first analysed in 1962 by Holt, who showed that policy performance would deteriorate when model parameters are uncertain. But Brainard (1967) made the key breakthrough by working out the optimal response of policy-makers who face uncertainty of this sort (in a linear quadratic world with a known probability distribution of the uncertain parameters). He showed how it may be optimal for policy-

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(2) This could arise for a number of reasons. The underlying parameter might be stochastic, or alternatively any one of the following might be mis-measured: the underlying parameter, the magnitude it relates to (e.g. GDP) or, finally, some underlying equilibrium that determines the relationship between observable data (e.g. the NAIRU—see below).

(3) A further type of uncertainty that may have a bearing on the optimal form of policy arises when the public are uncertain about the incentives that policy-makers face. In particular, Kydland and Prescott (1977) and Barro and Gordon (1983) showed that otherwise optimal policies may not be time-consistent, such that it may not be in the interests of the policy-maker to stick to promises made to the public about monetary policy. These papers spawned a separate literature on the effects of uncertainty. For example, Canzoneri (1985) considered the effect of private sector uncertainty about the accuracy of central bank forecasts. Cukierman and Meltzer (1986), and later Lewis (1991) and Nolan and Schaling (1996), examined the importance of private sector uncertainty about central banker preferences. Although these forms of uncertainty are interesting and important, we leave these issues to one side.

(4) Certainty-equivalence also requires quadratic preferences, which may not be unrelated to the popularity of quadratics in monetary policy analysis.

makers to aim off in their policy-setting such that they do not expect to eliminate all of the gap between the target and actual value of their objective when they are uncertain about relationships within the economy. This has been interpreted as justifying a gradual monetary policy: with no further shocks, the policy-maker would close the gap period by period between the actual and target level of the objective, but in each period would not aim to close the gap completely.

Following Martin (1998), we adapt a simple model of inflation targeting due to Svensson (1997) to illustrate the original Brainard result and to contrast it with policy under certainty-equivalence. This illustration sets the scene in a model of the economy that approximates to a popular view of the transmission mechanism and, importantly, shows how the Brainard result carries through into a dynamic setting (Brainard's model was static). Assume that the policy-maker targets inflation only, and that the target is normalised to zero, as is the 'neutral' level of interest rates.(5) The policy tool is the nominal interest rate, i_t, and this affects inflation with a one-period lag (say one year). Inflation is persistent, so if inflation was high last period, it is likely to be high this period.(6)

Formally, the problem that the policy-maker faces is:

$$\min_{\{i_t\}_{t=0}^{\infty}} \; E_0 \sum_{t=0}^{\infty} b^t p_t^2 \qquad \textbf{(1)}$$

Where b is the discount rate, subject to:

$$p_{t+1} = ap_t - bi_t + e_{t+1} \qquad \textbf{(2)}$$

The 'economy' is collapsed into the reduced form **(2)**, with the parameter a capturing the persistence of inflation, and parameter b the policy multiplier. Expression **(3)** describes the nature of the uncertainty we consider in this model economy: if stochastic, a and b have mean values equal to their econometric estimates while the additive error e has mean zero, and there may be covariances between all three random variables.

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(5) The 'neutral' rate is the level of interest rates that is consistent with the inflation target in the steady state.

(6) Specifically, it follows an AR(1) process.

$$\begin{pmatrix} a \\ b \\ e \end{pmatrix} \sim iid\left[\begin{pmatrix} \bar{a} \\ \bar{b} \\ 0 \end{pmatrix}, \begin{pmatrix} s_a^2 & r_{ab} & r_{ae} \\ & s_b^2 & r_{be} \\ & & s_e^2 \end{pmatrix}\right] \qquad \textbf{(3)}$$

Svensson (1997) shows how a problem of this form can be reduced to a one-period problem, because in each period the interest rate can be moved so as to meet the inflation target in the next period. But the optimal rule linking interest rates to inflation shocks is not invariant to the assumption made about uncertainty in the economy. We illustrate this next.

Additive uncertainty (non-stochastic parameters)

In this case the policy-maker knows the structure of the transmission mechanism between interest rates and inflation and can observe the constant multipliers *a* and *b*. Uncertainty is restricted to the additive errors, *e*. If this is the case, the optimal rule is:

$$i_t = \frac{a}{b} p_t \qquad \textbf{(4)}$$

Equation **(4)** is the optimal certainty-equivalent reaction function in this model. To understand its implications, it is instructive to solve for the dynamic path of interest rates as a function of additive shocks to the economy. This can be done by solving **(2)** and **(4)** simultaneously to give:

$$i_t = \frac{a}{b} e_t \qquad \textbf{(5)}$$

$$p_{t+1} = e_{t+1} \qquad \textbf{(6)}$$

From **(5)** and **(6),** it is clear that the policy-maker moves the policy tool exactly to negate the ‘second-round’ effects of a shock that would otherwise result, given the persistence of inflation. The policy instrument is moved in the same period as the shock arrives, to ensure that the next period’s inflation innovation is solely driven by the next period’s shock. Inflation is therefore identically and independently distributed each period. The policy response to a shock is instant and wholly contained within the initial period.

Multiplicative uncertainty (stochastic parameters)

Assume now that the policy multipliers are also stochastic, and that the policy-maker can observe—or estimate—the variance-covariance matrix **(3)**. Then the optimal feedback rule becomes:

$$i_t = \frac{1}{\bar{b}^2 + s_b^2}\left[\left(\bar{a}\bar{b} + r_{ab}\right)p_t + r_{be}\right] \tag{7}$$

This is equivalent to Brainard's original rule: the variance of the policy multiplier, s_b^2, reduces the response of the interest rate, i_t, to the deviation of inflation p_t from target (here normalised to zero). The covariance between the interest rate multiplier and the multiplier on inflation, r_{ab}, will increase or decrease the response to deviations of inflation from target depending on whether it is positive or negative. The covariance of additive shocks and the policy multiplier, r_{be}, affects the steady-state deviation of rates from their neutral level (the level that in the absence of shocks is consistent with the inflation target). The policy-maker will move less in response to a given deviation of inflation from target if the variance term is sufficiently large compared with the covariance of the multipliers on inflation and interest rates. Intuition has typically been that the variance effect will dominate any boost to policy reaction from covariance terms. In what follows, we abstract from covariance effects.

As Brainard shows, **(7)** can be re-written as follows,(7) where $V = s_b / \bar{b}$ is the coefficient of variation:

$$i_t = \frac{1}{1+V^2}\left(\frac{\bar{a}}{\bar{b}}\right)p_t \tag{8}$$

Equation **(8)** helps to interpret our results. It shows that the response to the expected gap between inflation and target—$(\bar{a} / \bar{b})p$—is (inversely) proportionate to the coefficient of variation of the policy parameter. As the coefficient of variation increases, the optimal policy response decreases.

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(7) To simplify, we have set the covariances to zero.

Although the theoretical result is well established,(8) there is less consensus over the empirical importance of parameter uncertainty.

Estrella and Mishkin (1998) estimate a simple Phillips curve for the United States using annual data from 1956-1996, which is a reduced-form variant of the Svensson model set out above. The equations are estimated by OLS, and the estimated standard errors are used as measures of Brainard uncertainty. They find that the optimal responsiveness of interest rates to inflation deviations from target is reduced by less than 5% over this period, once allowance is made for Brainard uncertainty.(9) This suggests small effects from parameter uncertainty.

By contrast, Sack (1998) concludes that parameter uncertainty is of material importance. He estimates a VAR model, and calculates optimal policy rules under the alternative assumptions of additive and multiplicative uncertainty. Again, the estimated standard errors are used as measures of Brainard uncertainty. He finds that the rule allowing for parameter uncertainty exhibits a considerably smaller initial reaction to shocks than the rule that allows only for additive uncertainty (this latter rule exhibits certainty equivalence). For example, the initial policy response to an inflation shock is approximately halved when allowance is made for parameter uncertainty. Martin and Salmon (1998) apply this methodology to the United Kingdom, with qualitatively similar results.

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(8) As Batini, Martin and Salmon (1998) report, the Brainard framework has been subject to two sets of theoretical criticism. First, Wieland (*op cit*) has argued that optimal Brainard rules do not take into account that current policy actions may influence future uncertainty (as measured, for example, by **(3)**), and that the true optimal rule should endogenise into decision-making the informational consequences of different courses of action. That is (Wieland argues), in choosing between different courses of action, policy-makers should consider what these actions will enable them to *learn* about the economy. Second, Sargent (1998) and Stock (1998) have raised a more fundamental objection, suggesting that it is unrealistic to assume that policy-makers can measure via a variance-covariance matrix how uncertain they are about the economy's characteristics. This amounts to saying that we need to consider model-uncertainty, as defined in Section 2, to which the Brainard approach is not applicable.

(9) In their model, the optimal rule relates interest rates to the deviation in unemployment from the NAIRU and inflation from target. The certainty-equivalent coefficients relating interest rates to these terms are (1.70,-2.56); they change to (1.59, -2.49) when allowance is made for parameter uncertainty.

3 Method

We proceed by analysing how sensitive the stabilisation properties of McCallum's simple policy rule for monetary growth are to parameter uncertainty. Our rationale is that Haldane, McCallum and Salmon (1996) (HMS) show that the McCallum rule exhibits good policy stabilisation properties across of range alternative models of the UK economy. That study suggests that the rule is robust to model-uncertainty. But HMS do not test the rule's robustness to parameter uncertainty within any of the particular models that they consider, and this is what we do here. Our contribution in relation to the Sack (1998) and Martin and Salmon (1998) work is that they focus on optimal rules, rather than the 'simple' rule considered in this paper. We examine:

- whether the 'optimal' degree of feedback—or rather, the feedback in the simple rule that delivers the best stabilisation properties—falls when additive uncertainty is introduced into our model;
- whether this 'optimal' degree of feedback falls when parameter uncertainty is introduced into our model; and
- how uncertainty about particular parameters in the model influences the 'optimal' degree of feedback in the policy rule. The issue is whether uncertainty about some relationships in the economy is more important for the operation of monetary policy than uncertainty about others (in terms of the optimal degree of reaction to news).

According to optimal control theory, the answers should be 'no', 'yes' and 'yes' respectively. But as the McCallum rule is a simple rule, results from optimal control theory are not guaranteed to carry through.(10) Next, we set out the loss function, the policy rule, the model used in the analysis and describe in more detail the simulation exercises carried out.

3.1 Loss function

We assume a standard quadratic loss function, with the simplifying assumption that the discount factor is unity, such that current and all future deviations carry equal weight with the monetary authorities. The loss function is:

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(10) For example, Currie and Levine (1985) demonstrate formally that certainty-equivalence is not guaranteed for simple rules of the type that we are analysing.

$$L = \sum_{t=0}^{\infty} (x_t - x_t^{*})^2 \tag{9}$$

While choice of a quadratic loss is in some ways arbitrary, Brainard's result relies upon it.(11)

3.2 Policy rule

We assume that the policy-maker follows a McCallum-type rule for monetary policy. The instrument (or the intermediate target) of policy is base money, *b*, and this is set to follow:

$$\begin{aligned} \Delta b_t &= 0.01 - (1/16)[x_{t-1} - b_{t-1} - x_{t-17} + b_{t-17}] \\ &+ l[x_t^{*} - E_{t-1}(x_t)] \end{aligned} \tag{10}$$

where x is nominal GDP, and $E_{t-1}(x_t)$ is the authorities' forecast of nominal GDP based on information available at time t-1. All variables are in logs. The constant term is set to imply a fixed component of growth for base money equivalent to 1% a quarter, or 4% per year, which would be appropriate for a regime targeting 2% inflation, 2% real growth and with a broadly stable velocity of money. The second term in the rule corrects the fixed path for slow-moving changes in the velocity of base money (to capture secular, as opposed to cyclical, changes in velocity).

The rule is designed to stabilise nominal GDP. This is achieved by the last term, which allows policy to respond to some proportion, l, of the news in the economy. This final term marks the rule out from Friedman's (1959) k% rule: with $l=0$, the rule collapses back to something close to a Friedman rule. News is defined by the difference between actual and target nominal GDP. Target nominal GDP grows linearly, by 1% per quarter, the same rate as baseline money growth. Once again, the rationale is consistency with about 2% growth in potential output and a 2% inflation target. The target can be thought of as a levels target, since the path for target GDP is pre-determined at

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(11) Quadratic loss contains two key simplifications: first, it implies that undershoots and overshoots of a target are treated symmetrically; second, it implies that policy-makers exhibit a constant (unit) absolute coefficient of risk-aversion. Chadha and Schellekens (1999) suggest that the first of these simplifications is not important for the Brainard result, but that the assumption of symmetric preferences is.

the beginning of time, and is not adjusted in the light of actual behaviour. Thus, the target value is not adjusted up (down) following a period of above (below)-target growth.(12)

This rule differs from McCallum's original specification in one important respect: policy-makers feed back on deviations of *expected* nominal GDP from target, rather than on deviations of *actual* nominal GDP from target. This forward-looking aspect of our rule is consistent with current monetary policy practice in the United Kingdom. As the minutes of the Monetary Policy Committee of the Bank of England show, decisions on interest rates place much weight on the inflation rate expected in 18 months' to 2 years' time. But an important methodological reason for modifying McCallum's original rule, is that by embedding model-based forecasts of nominal GDP in the policy rule, we make policy dependent on assumptions about the parameters governing the process that generates nominal GDP.(13)

3.3 The model

We combine the policy rule in **(10)** above with a small empirical model of the economy, which closely resembles model B in HMS. The model is given by equations **(11)-(14)** below (t-ratios in parentheses); variable definitions can be found in the Data Annex. The sample period is 1959 Q2 to 1993 Q4.(14)

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(12) We have no particular preference for a levels target over growth rate of mixed levels/growth targets. We choose to focus on the levels target simply for expositional convenience.

(13) McCallum himself (1995) concedes that a more general rule of this form would be operational for policy. His main concern is that posited policy rules should assume that policy-makers know only the past values of real variables when forming their expectation of the target variable and do not have access to current period information. Our modification in consistent with McCallum's stricture.

(14) Although this means our model does not take account of the most recent developments in the economy, our interest is in comparing the in-sample stabilisation performance of alternative feedback rules, which should not be sensitive to small changes in the sample period.

Equation (11): Aggregate demand curve

$$Dy_t \quad = \quad 0.0042 - 0.042\,Dy_{t-1} + 0.085\,Dy_{t-2}$$

(0.22) (-0.47) (0.94)

$$+ 0.018\,(Db\text{-}Dp)_{t-1} + 0.080\,(Db\text{-}Dp)_{t-2} + 0.072\,Dg_t$$

(0.36) (1.68) (1.04)

$$+ 0.084\,\Delta g_{t-1} - 0.037\,\Delta g_{t-2} - 0.155\,\Delta g_{t-3}$$

(1.16) (-0.51) (-2.25)

$$+ 0.028\,\Delta q_{t-1} + 0.025\,Dq_{t-2} + 0.762\,Dy^{*}_{t}$$

(1.51) (1.34) (4.64)

$$- 0.013\,Dy^{*}_{t-1} - 0.043\,Dy^{*}_{t-2}$$

(-0.69) (-0.25)

$\overline{R}^2 = 0.21$; SEE = 0.0097; Durbin’s h = -0.158

Equation (12): Cost mark-up pricing

$$\Delta p_t = -0.004 - 0.111\,Dp_{t-1} + 0.176\,Dp_{t-2}$$

(-2.57) (-1.35) (2.36)

$$+ 0.284\,\Delta w_t + 0.309\,\Delta w_{t-1} + 0.213\,\Delta w_{t-2}$$

(4.30) (4.47) (2.80)

$$+ 0.046\,Dpim_{t-1} + 0.048\,\Delta pim_{t-2} + 0.057\,\Delta pim_{t-3}$$

(1.62) (1.59) (1.92)

$\overline{R}^2 = 0.68$; SEE = 0.00851; Durbin’s h = -1.80

Equation (13): Wage-bargaining

$$[(\Delta w_t - E_{t-1}(\Delta p_t)] = 0.0073 + 0.191\,(y - \bar{y})_t - 0.127\,(y - y)_{t-1}$$

(3.33) (2.09) (-1.35)

$$+ 0.015\,\Delta w_{t-1} + 0.220\,\Delta w_{t-2} - 0.093\,\Delta w_{t-3} - 0.247\,\Delta w_{t-4}$$

(0.18) (2.63) (-1.12) (-3.10)

$\bar{R}^2 = 0.12$; SEE = 0.0111; DW = 1.93

Equation (14): Price expectations

$$E_t\,(\Delta p_{t+1}) = 0.0027 + 0.317\,\Delta p_t + 0.391\,\Delta p_{t-1} + 0.164\,\Delta b_t$$

(1.76) (4.32) (5.04) (2.87)

$\bar{R}^2 = 0.51$; SEE = 0.0105; Durbin’s h = -2.18

Equation **(11)** is our aggregate demand curve.(15) It links real output to real money balances (*via* the effect of real interest rates upon investment and consumption), the real exchange rate and overseas output (*via* net exports) and some measure of government spending (*via* fiscal policy). Since this system does not directly model the LM curve, real money balances are substituted for the real interest rate. As in HMS, movements in the real exchange rate are treated as exogenous.(16)

Equations **(12)**, **(13)** and **(14)** form the wage-price system that defines the supply side of the model. Agents enter into wage bargains at the beginning of each period. They bargain over expected real wages (equation **(13)**). Agents’ bargaining power is greater—and thus their real wage is higher—the smaller is the pool of unemployed workers; or, as in this model, the closer output is to

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(15) Equation **(11)** differs slightly from HMS, including only lagged terms in real money balances. This ensures a recursive structure in the model, and allows us to solve the model under different assumptions about the uncertainty facing policy-makers.

(16) A drawback of treating the exchange rate as exogenous is that we fail to take account of an important transmission channel of monetary policy. But from a modelling perspective, it yields a benefit, as we can partition out of the model the various (relative) supply and demand shocks to the real exchange that Clarida and Gali (1994) and Astley and Garratt (1998) suggest are the predominant source of real exchange rate movements in the short as well as long run.

trend (this effect is captured by the term in ($y - \bar{y}$).(17) Trend productivity is proxied by a time trend (or the constant in this regression). There is also nominal wage inertia, which is typical of overlapping-contracts models of the wage-setting process proposed by, for example, Taylor (1980).

Agents need to form expectations about inflation. We assume a simple, adaptive expectations-formation process, **(14)**: expectations are based on past outcomes for inflation and past outcomes for base money (the policy outcome). So although expectations are backward-looking, they are based on some observation of monetary policy. This mechanism has some characteristics of New-Classical monetary transmission—a link from money to prices that short-circuits output and works directly through the expectations-formation process. There are, of course, Keynesian aspects to the transmission of policy in this model: monetary policy affects prices indirectly via real money balances, output and therefore prices.

Finally, prices are determined as a cost mark-up over nominal wages and import prices, **(12),** which are assumed to be exogenous.

This model is not the only model of the United Kingdom available, is almost certainly not the best and may not even be a particularly good representation. But it is convenient. First, it retains a certain theoretical structure, which helps us when we come to use the model to talk about uncertainty about particular parts of the economy, as we can identify a 'policy multiplier' and an 'output gap'. A purely empirical VAR would not enable us to do this. Second, it is reasonably small, which is a virtue given the nature of the simulations we perform later in the paper. Conversely, readers might observe that the fit on some of the equations is not good. But this is not a problem for us. In our simulations, we shall shock the central estimates of the parameters to simulate policy-makers making 'mistaken' estimates of these parameters. We begin with a model that has quite large standard errors, which provides us with a benchmark that corresponds to a 'plausible' amount of uncertainty faced in the real world—'plausible' in the sense that this is the amount of uncertainty pervading when we estimate a model of a particular, conventional, structural form. But we repeat these experiments with both smaller and larger standard errors, and examine the sensitivity of our results to the degree of uncertainty introduced.

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(17) A result like this could be rationalised within a Lindbeck and Snower (1986) insider-outsider model of the wage-bargaining process.

3.4 Simulation analysis

The method used to simulate the effects of uncertainty is as follows. Note that the model could be written in general terms (and in vector form) as:

$$x_{t+1} = b(L)M_t + g(L)Y_t + d(L)Z_t + e_{t+1} \qquad \textbf{(15)}$$

where *x*, as before, is nominal GDP (our target variable), *L* is the lag operator, *M* is a vector of terms in base money (our instrument), b is a vector of coefficients defining the link between the instrument and the target (in our case the elasticity of nominal GDP with respect to base money and its lags); *Y* is a vector of other endogenous variables determined within the system; *Z* is a vector of exogenous variables that affect nominal GDP; and e is a vector of additive stochastic disturbances.

We assume that the authorities estimate **(15)** on the basis of observed period *t* outcomes and their estimates of the multipliers in the economy, and form $E_t(x_{t+1})$, their forecast of nominal GDP, such that:

$$E_t(x_{t+1}) = \hat{b}(L)M_t + \hat{g}(L)Y_t + \hat{d}(L)Z_t + e_{t+1} \qquad \textbf{(16)}$$

where $\hat{b}$ is the OLS estimate of b (etc.). Policy-makers feed this estimate of nominal GDP into the policy rule **(10)** and, together with the feedback coefficient, this determines the value of the policy variable (base money). We simulate the effect of policy-makers' uncertainty about the economy by assuming that the true bs, gs and ds are generated initially by a random draw from a distribution with a mean equal to $\hat{b}$, $\hat{g}$and $\hat{d}$ respectively and a variance proportional to the OLS estimated variances. So in our analysis, uncertainty about parameters (the difference between the true and estimated model parameters) is scaled by the econometric uncertainty in the estimated model equations.

We examine the impact of uncertainty as follows. First, we make an assumption about the nature of the uncertainty facing policy-makers: is uncertainty additive or multiplicative (i.e. are the shocks to the es or the bs, gs and ds)? Second, we draw 100 randomly generated sets of values for the 'true' parameters of the economy and simulate these economies under the assumption that policy-makers figure out policy from the central estimates of these parameters. Third, we repeat our simulations of these randomly generated economies with different values for the feedback term, l, searching for the 'optimal' value, (i.e. the one that minimises the loss function **(9)**, on average, across the 100 simulations). Fourth, we repeat with different sets of 'true' parameters drawn from distributions with ever-increasing variances. Of interest is whether the 'optimal' feedback value varies when we change the *nature* of uncertainty (additive, multiplicative etc.) and when we change the *amount* of uncertainty, as represented by the variance of the distribution from which the 'true' parameters are drawn.

We make four different sets of assumptions about the nature of the uncertainty facing policy-makers.

Our benchmark experiment is a deterministic policy simulation. This assumes that there is no parameter or additive uncertainty at all. In this case, the policy-makers' model of the economy and the true economy are one and the same, and there are never any expectational errors. We assume that there is a 10% step increase in the policy-makers' target for nominal GDP in 1959 Q1 (which persists, so all future target values of nominal GDP increase by 10%) to identify the optimal policy activism that would exist in the absence of uncertainty. All subsequent experiments retain this shock.

Our second set of experiments introduces additive uncertainty alone to the benchmark experiment. We shock equations **(11)** to **(14)** period by period by stochastically generated additive shocks, with variances that are proportional to the historic OLS residuals. The aim is to establish whether the rule exhibits certainty-equivalence when the economy is subjected to additive shocks. This will be the case if the optimal l is invariant to their introduction.

Our third set of experiments assumes that there is generalised parameter uncertainty but no additive uncertainty. Specifically, we assume that there is uncertainty about all parameters in the model, and that the variance of the true parameters is proportional to the estimated OLS variances. Thus, for example, we identify optimal ls on the assumption that the parameter variances equal the OLS estimates and, separately, that they are twice as large as the OLS estimates.

The fourth set of experiments assumes that there is specific, (rather than generalised), parameter uncertainty. Thus, for example, in one experiment we assume that there is uncertainty about the policy multipliers (the coefficients on base money in **(11)**), but that all other parameters in the economy are

known with certainty. Again, we assume that there are no additive stochastic shocks.
There are two further points to be made about these simulations. First, we assume that policy-makers think that the world is best represented by a model with fixed parameters, when the true model parameters are stochastic. An alternative would have been to assume that both the true model and the policy-makers' representation of it are fixed, but that policy-makers make an initial error in estimating model parameters. We chose not to do this, as this would have necessitated making some assumption about how policy-makers learn. It is probably fair to say that although there are many examples of learning rules in the literature, there is no generally accepted model: any investigation of the interaction of uncertainty and learning would need to check for robustness. We leave this task for future research, and instead isolate the effect of uncertainty when learning would be impossible since the parameters are stochastic.

Second, note that for simplicity, we have chosen to abstract from the issue of covariances between the stochastic processes in the model. But if the covariances implicit in our model are large, then our results could be biased.

Finally, we should note that we are of course vulnerable to the Lucas critique, since we do not explicitly model the possible endogeneity of the *bs*, *gs* and *ds* with respect to the policy rule. However, this does not reduce the validity of the comparisons we wish to make, as there is no reason why it should affect one form of uncertainty more than others.

4 Results

Table A reports the feedback parameters that minimise the policy-maker's loss function for varying degrees of additive uncertainty. The uncertainty coefficient (x) measures the extent to which the economy is subjected to additive shocks. The coefficient scales the variance of the shocks in relation to the OLS-estimated variances of historic residuals identified by estimation of the model, hence a coefficient of x implies that a residual series is randomly drawn from the distribution $N(0, x\ s_u^2)$. For instance, a coefficient of zero implies that there are no additive shocks: in this case the actual economy **(15)** and the monetary authority's estimate **(16)** are one and the same. And a coefficient of unity implies that the variances of the additive shocks are equal to the OLS estimates.

Table A

Policy activism and additive uncertainty

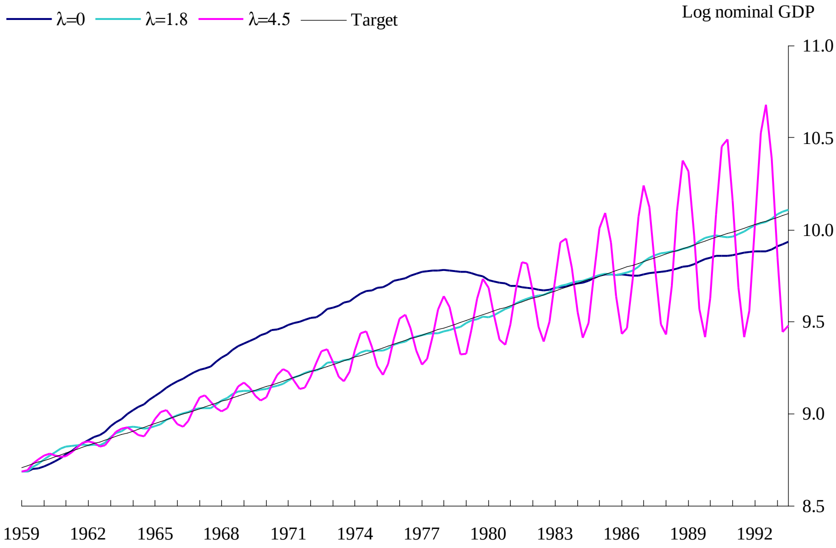
Uncertainty coefficient x [(a)]	Optimal *l*	Minimum loss	*With l= 1.8:* Loss	Extra loss
0	1.8	0.028	0.028	0.00%
1	1.7	0.356	0.359	0.83%
2	1.7	0.706	0.713	0.95%
3	1.7	1.049	1.059	0.98%
4	1.7	1.376	1.390	1.00%
10	1.6	3.424	3.461	1.08%
100	1.6	33.078	33.447	1.11%

(a) x is the scaling factor applied to estimated equation residuals. A value of zero implies no additive uncertainty. A value of one implies additive shocks with variances equal to their OLS estimates, etc.

(i) No uncertainty

In the case of our benchmark scenario of no additive shocks (x=0), the optimal feedback coefficient is 1.8. In practical terms, this means that given a 1% projected excess of the level of nominal GDP above target, the optimal response would be to decrease base money growth by 1.8%. Chart 1 illustrates that if the feedback parameter is less than 1.8 (say l=0, which corresponds to policy following a modified constant money-growth rule), then the policy-maker could induce a quicker return to target by reacting more aggressively to the deviation in nominal GDP from target. By contrast, a more aggressive reaction (say l=4.5), combined with autocorrelated effects of policy on the economy, starts to induce cycles and possibly instabilities in nominal GDP.

Chart 1
Impact of feedback variations on nominal GDP



(ii) Additive uncertainty

When we augment the benchmark model with additive stochastic shocks, simulated nominal GDP deviates from policy-makers’ forecasts. According to the principle of certainty-equivalence, such shocks should not have any impact upon the optimal degree of policy feedback. While the McCallum-style rule we use is not an optimal rule, we still might expect certainty-equivalence to prevail on average, as the shocks to the economy average out to zero across the simulations. Table A shows this is indeed broadly the case. When we introduce shocks with variances equal to the OLS estimates (the second shaded row of Table A), the optimal degree of feedback does fall marginally to 1.7. But as the final column of the table suggests, the reduction in loss from decreasing feedback from 1.8 to 1.7 is small—less than 1%. Increasing the variance of the additive shocks does not alter this broad certainty-equivalence result: for instance, with shocks with variances equal to 100 times the OLS estimates, the optimal *l* is 1.6, and the policy-makers’ loss would increase by only 1.11% if they set policy using the certainty-equivalent feedback l of 1.8.(18)

The table also shows that, as we would have expected, policy-makers' loss increases as the economy is subject to more variable shocks. The optimal policy with no shocks results in loss of just 0.028%; this increases slightly more than ten-fold to 0.356% when shocks (with historically calibrated variances) are introduced to the model. Thereafter, losses broadly increase in line with the extent of uncertainty (so, for example, the minimum loss achievable doubles as the variance of the residuals doubles).

(iii) Generalised parameter uncertainty

Table B reports results for experiments in which we introduce varying degrees of parameter uncertainty ($x > 0$).(19) The optimal degree of feedback and the associated total welfare loss vary with the extent of uncertainty. A key result is that the optimal degree of feedback falls as the extent of parameter uncertainty rises. And not surprisingly, as the world becomes more uncertain (x increases), the minimum loss that the policy-maker can achieve increases.

Table B
Policy activism and multiplicative uncertainty

Uncertainty	**Optimal l**	**Minimum**	***With l= 1.8:***	
coefficient x(a)		**loss**	**Loss**	**Extra loss**
0	1.8	0.028	0.028	0.00%
1/3	1.8	0.062	0.062	0.00%
1/2	1.7	0.083	0.083	0.48%
1	1.6	0.159	0.166	4.24%
2	1.2	0.664	0.964	45.18%
3	1.0	1.079	3.960	267.04%
4	0.8	2.553	52.913	1972.45%

(a) $x$is the scaling factor applied to estimated parameter variances. A value of zero implies no parameter uncertainty, a value of one implies parameter variances equal to their estimated value, etc.

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(18) The departure from complete certainty-equivalence is probably an artefact of our relatively small number of simulations (100).

(19) The uncertainty coefficient x now measures the extent to which the parameters describing the evolution of the actual economy vary from those used by the policy-maker in deriving his forecast of the target variable (nominal GDP). The coefficient scales the OLS-estimated variances of the parameter in the policy-maker's model of the economy. A coefficient of x implies that each actual parameter in the economy $U=\{b,\ g\ s\}$ is randomly drawn from the distribution $N(\hat{U}, x s_e^2)$.

Consider the two shaded rows of Table B. These contrast results for simulations in which there is no parameter uncertainty with the case in which we assume that the OLS-estimated parameter variances proxy the uncertainty faced by the policy-maker. Optimal policy is less activist in this world: a policy-maker would reduce money growth by 1.6% for every expected 1% deviation in the level of nominal GDP above target, compared with 1.8% in the certain world. Furthermore, the costs of keeping l at 1.8 are now much greater: with this amount of uncertainty, the policy-maker's loss would be more than 4% higher than it need be.

As the actual policy instrument in the United Kingdom is the short-term interest rate, it is instructive to translate this effect into interest rate space. For this, we require estimates of the (semi-) elasticity of M0 with respect to rates. Janssen (1998) suggests that a 5 percentage point rise in short-term interest rates is required to reduce real money growth by 1% (assuming 5% average nominal interest rates). This implies that a 0.2% decrease in quarterly M0 growth would require the policy-maker to be approximately 100 basis points *less* aggressive in raising interest rates in response to a shock that resulted in a 1% expected deviation in nominal GDP above target. But these results need to be treated cautiously. In particular, this calculation only takes into account the *direct* impact of interest rates upon base money growth and holds output constant. There will be an important *indirect* effect via output, as increases in interest rates will tend to reduce output growth.

The remaining rows in Table B show how the optimal feedback varies for other assumed variances of the economy parameters. If we assume that the OLS estimates *overestimate* the uncertainty facing policy-makers, then we see the difference from the base-case certain world become smaller. With a variance of $^1/_3$ of the OLS estimates, the optimal policy reaction is identical (to one decimal place) to the base-case, and the extra loss induced by estimation error is small. Conversely, if we concluded that the OLS estimates *underestimated* the degree of uncertainty, then the effect on optimal feedback becomes more marked. In our most extreme case, where the variance of the actual parameters was four times greater than the OLS estimates, a policy-maker's optimal response to an expected deviation in nominal income from target would be under half that of the base-case.

Chart 2

Relationship between feedback and multiplicative uncertainty

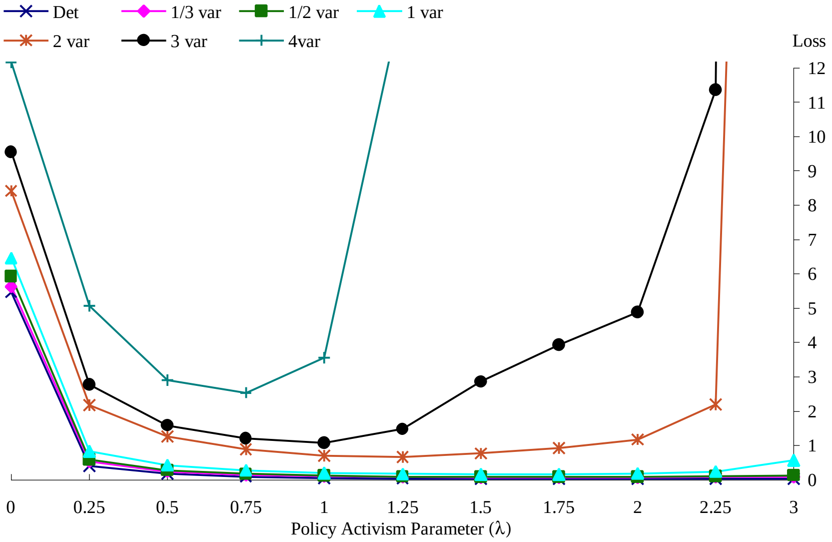

Chart 2 presents the data in a different way. Each line plots the loss for varying levels of policy activism (*l*), given differing assumed amounts of parameter variance. The chart clearly illustrates the key finding that as the variance increases, the optimal degree of feedback falls. The chart also shows that the costs of being overly aggressive increase more quickly as the variance of the parameters increases. For example, if the variance of the actual parameters were four times greater than the OLS estimates, the cost to a policy-maker of retaining a feedback parameter at 1.8 would be a near 2000 per cent increase in loss relative to the minimum achievable.

Table C contrasts the standard deviation of quarterly real output growth and (quarterly) inflation across experiments. It provides an alternative measure of the costs to the policy-maker of mistakenly retaining the baseline feedback of 1.8. The first three columns of results compare standard deviations when the variance factor, *x*, is set equal to unity. The contrast between the additive and multiplicative experiments is clear: when there is additive uncertainty, the cost of retaining the deterministic feedback parameter is negligible (a $2^1/_2$% or so increase in the standard deviations). These costs approximately triple when there is multiplicative uncertainty. The three right hand columns report similar results when the variance factor, *x*, is set equal to four. The point is not that we necessarily believe that the world is so uncertain; rather, these three columns illustrate how sensitive estimates of the costs of ignoring parameter uncertainty are to the maintained assumption about the extent of that uncertainty.

Table C

Costs of not modifying policy to take account of uncertainty

	Unit shock variances (*x*=1)			**Four-fold shock variances (*x*=4)**		
	St. deviation (%)		**Cost**	**St. deviation (%)**		**Cost**
	Baseline $\boldsymbol{l}^{(a)}$	**Optimal** $\boldsymbol{l}$	**(% change)**	**Baseline** $\boldsymbol{l}^{(a)}$	**Optimal** $\boldsymbol{l}$	**(% change)**
Inflation:						
Det.	0.510	0.510	n/a	0.510	0.510	n/a
Add.	1.896	1.849	2.52%	3.737	3.646	2.49%
Mult.	1.331	1.242	7.24%	17.797	4.047	339.73%
Output:						
Det.	0.569	0.569	n/a	0.569	0.569	n/a
Add.	1.390	1.360	2.22%	2.633	2.573	2.36%
Mult.	1.040	0.973	6.92%	14.290	2.363	504.87%

(a) Baseline $\boldsymbol{l}=1.8$.

(iv) Specific parameter uncertainty simulations

It is also instructive to consider the policy implications that arise if we partial out uncertainty only about specific parameters. We focus on two groups of parameters:

(a) policy multipliers; and
(b) output gap multipliers.

Discussions of the merits of a gradualist monetary policy have often been couched with reference to uncertainties about particular aspects of the economy. For instance, one of the arguments that Friedman put forward in favour of a k% money growth rate rule was that the direct policy multipliers are ‘long’ and ‘variable’ which we could interpret as ‘long’ and ‘uncertain’. More recently, there has been much debate in the United States and United Kingdom about both the level of the NAIRU and the output gap and their relation to inflation. Wieland (1998) has examined the implications for the optimal monetary policy rule once allowance is made for uncertainty about the NAIRU-inflation relationship. Our Phillips curve, which gives a relation between the output gap and inflation, is summarised by the wage and price expressions **(12)-(14),** and it is this output gap that we treat as uncertain in the experiments below. There is clearly a mapping to some implicit NAIRU, and so this experiment can be thought of as analogous to one examining NAIRU uncertainty, though the two concepts are not identical.(20)

Table D reports the parameters that are varied in each experiment (together with their mean and variance estimates).

Table D

Uncertainty about specific parameters

Parameter	Experiment	Equation No.	Est. Mean	Est. Variance	Coefficient of variation
Base money	(a)	11, 1st lag	0.0182	0.0024	2.692
Base money	(a)	11, 2nd lag	0.0797	0.0023	0.602
Output gap	(b)	13, no lag	0.1912	0.0084	0.479
Output gap	(b)	13, 1st lag	-0.1271	0.0088	-0.738

Note: Experiments partial out uncertainty about (a) policy multipliers and (b) the output gap.

Table E gives the results. It shows that uncertainty about the policy multipliers (base money) has a more significant impact upon policy-makers' optimal degree of reactiveness than uncertainty about the output gap. This result accords with the theoretical Brainard/Svensson result for fully optimal rules **(8)**, because the coefficients of variation are greater for the base money parameters. Remember, the basic insight of the Brainard result is that changing the instrument to try to bring the objective of policy back to target also entails a cost, because this will increase the variance of the target. A higher coefficient of variation, other things being equal, implies that the variance costs of policy reaction will increase relative to gains from attempted

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(20) Given the openness of the United Kingdom and the importance of the exchange rate in the transmission mechanism, an obvious additional experiment would have been to vary the exchange rate parameters only. We chose not to do this, because we treat the exchange rate as an exogenous variable, and felt that any results we generated would be misleading: such an experiment would capture uncertainty about the response of output to the real exchange rate, but not about the response of the exchange rate to domestic developments.

stabilisation, such that it will become optimal to respond less actively to policy shocks.

Table E

Uncertainty about specific parameters

Uncertainty coefficient *x* [(a)]	**Base Money**		**Output gap**		***Memo: Baseline***	
	Optimal *l*	**Loss**	**Optimal *l***	**Loss**	**Optimal *l***	**Loss**
0	1.8	0.028	1.8	0.028	1.8	0.028
1/3	1.8	0.030	1.8	0.047	1.8	0.062
1/2	1.8	0.031	1.8	0.057	1.7	0.083
1	1.7	0.034	1.8	0.089	1.6	0.159
2	1.7	0.041	1.8	0.156	1.2	0.664
3	1.6	0.048	1.8	0.231	1.0	1.079
4	1.5	0.057	1.8	0.313	0.8	2.553

(a) *x* is the scaling factor applied to estimated parameter variances. A value of zero implies no parameter uncertainty. A value of one implies parameter variances equal to their estimated value, etc.

We can also compare the absolute welfare losses that arise in the presence of different types of uncertainty. For each level of feedback, uncertainty about the output gap parameters leads to greater loss than uncertainty about the policy multipliers. This arises because the variance of the output gap parameters (Table D) is greater. But because the central estimates of the output gap parameters are relatively large (so that the coefficients of variations are small), the benefits from active policy are greater when there is uncertainty about the output gap. As such, the optimal feedback parameter is higher when there is output gap uncertainty.

5 Conclusions

We set out to test the effect of uncertainty on the optimal monetary policy response when policy is constrained to follow a simple rule. To do this, we took a simple empirical model of the economy and assumed that policy followed a McCallum-style rule, where policy-makers set base money as a function of the deviation of expected nominal GDP from target. We built in a form of uncertainty by assuming that policy-makers estimate their model of the economy assuming that model parameters are fixed, and simulating in situations when model parameters are in fact stochastic.

Our first set of results—which shows how optimal policy is affected by the presence of purely additive uncertainty (where there are simply shocks to equation residuals)—confirms that the McCallum rule, when applied to our model, approximates certainty-equivalence: the optimal degree of feedback to expected deviations in nominal GDP from target is barely affected by the extent of additive uncertainty. We do observe some deviation from certainty-equivalence, but we believe that this is because of the relatively small number of simulations that we performed. We would have been surprised if certainty-equivalence had not prevailed.

Our second, and chief, set of results compares the policy reactions when there is no uncertainty with the case when there is generalised parameter uncertainty. Assuming that OLS variances provide our best metric of parameter uncertainty, our results suggest that policy-makers should view parameter uncertainty as a material issue. The optimal l, taking account of parameter uncertainty, is just over 10% less than the optimal l derived on the assumption that there is no parameter uncertainty (l=1.6 compared with 1.8). In order to assess the implications of this for nominal interest rates, we would need to specify a complete model for the economy that included these rates. That task is beyond the scope of the paper. Janssen's (1998) single-equation analysis of the demand for M0 may offer some clues however; it suggests that the adjustment in the interest rate necessary to change quarterly M0 growth by 1.6 percentage point could be materially less than the adjustment necessary to reduce money growth by 1.8 percentage point. But it is important to remember that these calculations only account for the *direct* effects of interest rates upon M0 growth. An additional issue is that, as Tables B and C show, the effect of parameter uncertainty does depend crucially upon the assumed extent of uncertainty: if our OLS estimates overstate (understate) genuine parameter uncertainty, then the effects upon policy will be commensurately smaller (greater) than the OLS estimates imply. We interpret Table B as providing a *prima facie* case that parameter uncertainty may be of material importance in the United Kingdom. In this sense, our results are closer in spirit to Sack (1998) and Martin and Salmon (1998) than to Estrella and Mishkin (1998).

The final set of results considers the effects of uncertainty about some specific parameters. These results reaffirm that when we think about uncertainty, we need to scale any variance measure we have by the central parameter estimate. In our model, uncertainty (as measured by the coefficient of variation) is greatest for the direct policy lags, and uncertainty about these parameters has more impact upon the optimal feedback parameter than uncertainty about the output gap.

We think these results make a useful point about measurement error. Wieland (1998) shows that measurement errors about the NAIRU—and, by extension of his argument, the output gap—can be thought of as inducing uncertainty about the parameters relating observable unemployment (output) data and inflation. NAIRU (output gap) uncertainty has generated much debate in policy-making circles about how much notice monetary policy-makers should take of observable labour market (output) data (see, for example, the minutes from the March 1998 meeting of the Bank of England's Monetary Policy Committee). Our results indicate that policy-makers should pause before concluding that such data contain no usable information. As long as unemployment (output) is an important determinant of inflation in the short run, such that the mean coefficient on employment (or output) in a reduced-form equation for inflation is 'large', then policy-makers should place some weight on the data, even if the variance of the parameter estimate—uncertainty about the underlying equilibria—is significant.

Data Appendix

The data cover 1955 Q1 to 1993 Q4. Unless otherwise stated, they are seasonally adjusted and in logs.

Base money, b_t:	Official base money data are only available from 1963 Q3. To create a longer time-series, we spliced Capie and Weber's (1984) estimates of M0 for the period before 1963 Q3 onto (non seasonally adjusted) official M0 data. This long run of M0 data was then seasonally adjusted using the Bank's own seasonal adjustment programme for monetary aggregates (GLAS). Further details and a copy of this series is available from the authors upon request.
Interest rates, i_t:	Quarterly averages of end-month short-term interest rates: 1955-72, the Bank rate; 1973-81, the MLR; and 1982 onwards, clearing banks' base rate (not in logs).
Real output, y_t:	ONS data—GDP at market prices.
Price Level, p_t:	ONS data—GDP deflator.
Nominal GDP, x_t:	$y + p$.
Real government expenditure, g_t:	ONS data—General Government Final Expenditure
Import prices, pim_{t}:	ONS data—Import price deflator.
Foreign output, y^*_t:	OECD data—Total OECD output.
Real exchange rate, q_t:	Nominal US/UK exchange rate adjusted by the ratio of foreign to domestic prices.

Wages, w_t:	ONS data—ratio of wages and salaries of the personal sector to the workforce in employment.
Trend output, $\bar{y}_t$:	Fitted values from a linear trend through y.

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