A limited participation model of the monetary transmission mechanism in the United Kingdom

Shamik Dhar

and

Stephen P Millard

Bank of England, Threadneedle Street, London, EC2R 8AH.

The views expressed are those of the authors and do not necessarily reflect those of the Bank of England. We gratefully acknowledge comments on this paper received from Paul Tucker, Paul Mizen, Andrew Scott, Jens Larsen, Paul Fisher, Peter Andrews, two anonymous referees and seminar participants at the Bank of England.

Issued by the Bank of England, London, EC2R 8AH, to which requests for individual copies should be addressed; envelopes should be marked for the attention of Publications Group. (Telephone 020 7601 4030). Working Papers are also available from the Bank's Internet site at http://www.bankofengland.co.uk/wplist.htm

ISSN 1368-5562

Contents

Abstract

In this paper we develop a model of the UK economy in which monetary growth determines inflation, but in which multiple shocks obscure the relationship between money and inflation. It is a general equilibrium model, calibrated to match certain key features of the data, and it falls into the class of 'limited participation' models, popularised by Lucas (1990), Fuerst (1992) and Christiano and Eichenbaum (1992, 1995). Our version closely follows that of Christiano and Gust (1998), but with the addition of investment adjustment costs. We show that the model is able to capture important features of the monetary transmission mechanism in the United Kingdom, as embodied in the responses of variables to monetary policy shocks.

1. Introduction and overview

This paper arose out of answering the question of how to introduce money and monetary aggregates into a central bank's policy-making process in a rigorous fashion. In order to do this, we have attempted to construct a model of the UK monetary transmission mechanism that is structural in nature, theoretically rigorous, consistent with UK data and that also builds on previous research at the Bank of England (Thomas (1997a, b)).

All useful models are stylised representations, which inevitably abstract from one or other aspect of reality in order to focus on particular issues of interest. Our concern is with the information contained in monetary aggregates about future prices and activity: both corroborative and incremental. Therefore, the model we construct incorporates a relatively detailed treatment of money and of financial stocks and flows at the expense of, say, the labour market.

The model also falls into the category of so-called 'dynamic stochastic general equilibrium' (DSGE) models, and within that category it falls into another, known in the literature as 'limited participation' models (see, for example, Lucas (1990); Fuerst (1992); Christiano and Eichenbaum (1992, 1995)).[(1)] This begs two obvious questions: Why DSGE? And why limited participation?

The answer to the first question involves methodological issues, whereas the answer to the second is that, in the class of monetary DSGE models, they seem to fit important features of the data (in particular the liquidity effect) relatively well.

(1) For an overview of the DSGE literature see Cooley, (ed) (1995). Enthusiasts include Farmer (1993) and King and Rebelo (1998). For a more sceptical view, see, for example, Kirman (1992) and Hoover (1995).

1.1 Why DSGE?

In the last decade, DSGE models have become popular tools with which to address a whole range of policy issues, to the extent that they have now become the 'academic workhorse'. Early general equilibrium models involved well-informed agents trading in complete markets which cleared instantaneously at all times. Many second-generation models — including ours — relax some or all of these assumptions however while maintaining their general equilibrium characteristic. In particular, in all such models, rational agents solve well-defined dynamic optimisation problems subject to certain informational and technological constraints. In addition, known stochastic processes represent the evolution of uncertainty in the world.

These models arose out of Lucas's methodological critique of policy advice based on econometric models (Lucas (1976)). Although this critique put paid to the use of these econometric models for policy analysis in the academic community, they have remained popular in policy-making environments such as central banks and finance ministries.(2) Kydland and Prescott (1982) made the seminal contribution to applied DSGE modelling of the type described in this paper, and the field has burgeoned ever since.

Unlike macroeconometric models, a DSGE model tends to incorporate relatively few sources of uncertainty, and all its parameters can be characterised as 'deep' parameters as Lucas defined them. This means that they are 'structural' in the sense that all the model's predictions can be traced back to assumptions about the structure of the economy — the specification of agent's preferences or production technology, or the stochastic processes driving the shocks in the model. By contrast, the estimated parameters of macroeconometric models typically conflate a great deal of information about deep parameters and shock processes, making it more difficult to tell stories based on these 'primitives'.

We find this approach attractive because the data demonstrate that the relationship between monetary aggregates and output and inflation has been highly unstable over the past three decades, during which time the

(2) These models are sometimes described as 'system of equations' models, where each behavioural equation is econometrically estimated and represents a specific economic relationship such as the *consumption function, the wage equation*, etc. Despite their susceptibility to the Lucas critique, these models remain the dominant forecasting and policy advice tool used in the United Kingdom. See, for example, Whitley (1994).

UK economy has been subjected to a variety of different shocks and successive governments have adopted a number of different monetary policy regimes. In order to be able to make sense of the monetary data, we need to aim to use models which are data-consistent and are capable of distinguishing the effects of different shocks, without falling foul of the Lucas critique.

1.2 Why limited participation?

Within the class of DSGE models, limited participation models have become a popular method of representing the monetary transmission mechanism. Their popularity originally sprang from the fact that they were able to replicate the 'liquidity effect' — the observation that short-term nominal interest rates tend to fall when the money supply expands. Other DSGE models could not reproduce this empirical observation because, in their stylised world of rational representative agents operating with full information in complete markets, higher monetary growth means high inflation in the future. Since agents are interested in real returns, they will demand a nominal return that will equal their desired real return plus the expected rate of inflation (the 'Fisher effect').

Limited participation models address this problem by introducing a degree of agent heterogeneity into the world, in particular restricting the information set of one group of agents relative to all others in one crucial dimension. In this case, the ability of some agents to adjust financial portfolios is limited; specifically, households are assumed to make all their financial decisions before they can observe current-period shocks (such as productivity or monetary policy shocks). Consequently, monetary injections lead to a temporary fall in interest rates because cash-rich banks have to convince other unconstrained agents (here firms) to borrow additional cash balances.

It is important to note that this story can also be told from the point of view of an interest rate setting central bank. Pegging the short-term nominal interest rate implies that the supply of reserves to the banking system is infinitely elastic at that rate. However, the liquidity effect now translates into the authorities having to cut interest rates if they want to expand the money supply (an action that will raise inflation expectations). While this might seem obvious to a central banker, it is not a feature of most DSGE models.

When the monetary authority cuts interest rates in the limited participation model, activity and employment rise in the short run, as do real wages and firms' demand for working capital loans. Banks supply these additional loans but, since households have already deposited their cash at the bank before the interest rate cut was observed, the banks have to come to the central bank for the extra reserves they need to back the loans. (We are assuming that the banks are always operating with a 'prudent' level of reserves and so need to increase their reserve holdings if they increase their loans.) The result is that broad and narrow measures of money expand following the interest rate cut, and inflation eventually rises.[(3)]

Whether money targeting or interest rate setting is assumed, the upshot is a model in which:

- there is a liquidity effect in the short run (interest rates and money move in opposite directions in response to nominal shocks), but nominal interest rates are determined by Fisherian fundamentals in the long run; and

- unanticipated monetary injections have real effects in the short run (because real interest rates and real wealth change temporarily), but monetary policy is neutral in the long run.[(4)]

However, replicating the liquidity effect is not the main reason we are attracted to the limited participation framework. Instead, the feature that appeals is that this is a model with an active (albeit rudimentary) banking sector which lends to agents who may be temporarily off their long-run demand for money curve. In other words, there is a rudimentary 'excess liquidity' story here, since the limited participation feature means that

(3) In fact, in many models interest rate setting leads to price level indeterminacy. There are multiple equilibria in which exogenous ('sunspot') shifts in price/inflation expectations can lead the price level almost anywhere. Christiano and Gust (1998) show that this can happen in the limited participation model for particular feedback coefficients in the policy rule.

(4) Neutrality implies that real variables in the model are invariant with respect to the price level in the long run. These models do not generally display superneutrality, however, since equilibrium consumption, labour supply and money balances depend on the steady-state inflation rate. The model also displays the Friedman rule property — the optimal long-run nominal interest rate is zero, so that the optimal inflation rate is the negative of the real interest rate – because the steady-state inflation tax is zero at that point. In what follows we assume that the central bank is following a rule which leads to a given steady-state inflation rate. This reflects the current situation in the United Kingdom where the Bank of England's goal (the inflation target) is mandated by the government.

some sectors take on more cash and hold onto it for longer than they would in a frictionless world. We explain this in more detail below, but it is broadly similar to the ‘buffer-stock’ story which motivated estimates of what previous research calls *M-M** (see Thomas (1997a, b)). The major difference is that it is couched within a fully-articulated general equilibrium model, and ‘excess money’ here has no ‘disequilibrium’ connotations — we think of ‘excess money’ as being the difference between short-run equilibrium money holdings and long-run equilibrium money holdings. Nevertheless, we can use the model to help us to ‘tell stories’ about econometric estimates of *M-M** developments in terms of the shocks which generate them, and therefore to assess any implications for inflation and output.

Another attractive feature of the model we develop is that, by introducing inter-period as well as intra-period adjustment costs, we can generate nominal price stickiness endogenously, without having to impose it through a ‘menu cost’ assumption (though we could do this if we wished). This is important because sticky price models are probably the main competitor in this paradigm of monetary DSGE models. (See King and Watson (1996) and Christiano *et al* (1997) for opposing views on how well the models compare with each other.)

*1.3 Excess liquidity, money overhangs and M-M**

In two papers, Thomas (1997a, b) estimates models of the demand for money by persons, industrial and commercial companies (ICCs) and 'other financial institutions' (OFIs) using structural econometric modelling (SEM) techniques popularised by Johansen and Juselius (1994). The empirical results broadly confirm the findings outlined below — there is some evidence that sectoral money can help predict sectoral activity — but the economic interpretation is just as interesting. Thomas appeals to buffer-stock arguments (Laidler (1984), Millbourne (1988)), in which agents build up liquid balances in the face of monetary surprises.

Thomas' (1997a, b) description has a Wicksellian flavour (Wicksell (1935)): in the case of a positive 'monetary policy shock' the central bank temporarily lowers the short-term nominal (and real) interest rate below some equilibrium or 'natural' level (the level of real rates that would clear the market for savings and investment), and this causes agents in all sectors to spend more. This spending is financed by an expansion of bank lending, and so also broad money. Since planned investment exceeds planned saving at the lower rate of interest, agents will accumulate money balances in excess of those they would willingly hold. They are happy to do this in the short run because money acts as a temporary abode of purchasing power. Of course, other shocks that move the equilibrium real rate will also lead to changes in $M\text{-}M^*$. Thomas (*ibid*) presents his estimates of excess liquidity, $M\text{-}M^*$, in the personal and corporate sectors and shows that these can help to predict consumption and investment. He also outlines two potential channels through which this excess money might be dissipated, leading to positive real effects at first but to inflation later down the line.

The first is that excess money balances might simply be spent on goods and services which, given that portfolio rebalancing takes time, leads to a protracted effect on investment or consumption spending. This occurs with a mean lag of approximately six quarters. According to this channel, excess money balances in the personal or corporate sectors would be more informative than a build-up in the OFIs sector. However, the second channel of influence argues that, while an individual agent might be able to reduce surplus money holdings relatively easily by purchasing goods or assets, this may not be true in the aggregate. Agents may end up passing surplus money balances to one another, within and across sectors, until the transactions underlying this process raise the demand for money or reduce its supply (the latter through repayment of bank debt). This might take place through a rise in deposit rates relative to other yields, or it could take place via a rise in asset prices and wealth and hence in nominal demand.

Thomas does not specify the ultimate source of the portfolio frictions in a model, nor does he estimate how large they would have to be to generate the sort of persistence evident in the data. However, some key features of the Thomas story are maintained in the limited participation framework: principally costly portfolio adjustment which leads to persistent deviations of sectoral money holdings from long-run money demand (where the precise definitions of short-run and long-run money demand will be given later) as well as the notion of excess money circulating between unconstrained sectors, in this case firms and banks.

1.4 Structure of the paper

Our aim is to develop a model of the UK economy which we can use for conditional forecasting and policy analysis. The structure of the paper follows this aim. We first develop the model, concentrating on the maximisation problems featuring each of the groups of agents in the economy. We validate the model by analysing its ability to match the responses of endogenous variables in the data to a monetary policy shock; if the model can do this, then it can be used for policy analysis.

In Section 2, we outline the details of the limited participation model: we set up the maximisation problems, derive first-order conditions, calibrate the model so that it reproduces historical values of observables in steady state, and log-linearise the model around this steady state. Given this representation of the model, we derive log-linear decision rules specifying the dynamic paths of all the endogenous variables in terms of the endogenous state variables and exogenous variables (or ‘shocks’). These decision rules are then used to simulate the model. The technicalities are spelled out in some detail, but at all points we try to highlight the intuition underlying them.

In Section 3, we confront the model with UK data by comparing impulse responses from the analytical model with those generated by a structural vector autoregressive (VAR) model of the monetary transmission mechanism (Dhar, Pain and Thomas (1998)). We find that the analytical model predictions are broadly consistent with those of our structural VAR. In Section 4, we demonstrate how the model relates to the earlier work of Thomas (1997b) carried out at the Bank. We do this by analysing the responses of investment and ICCs' M-M^* to a monetary policy shock, matching these responses in the model with the responses estimated by Thomas (1997b). We conclude with some observations on future directions that this work could take.

2. The model

2.1 The structure of the model

The model we use is similar to that of King and Watson (1996) and Christiano and Gust (1998). There are four types of agent in the model: households, firms, banks and a monetary authority.

The timing and direction of flows of funds in the limited participation set-up are extremely important. At the beginning of the period, the households have all the money, M_{t-1}, and decide how much spending money, S, to hold in order to buy consumption goods, C, in the period and how much to deposit with the banks. The banks then lend money out to the firms as working capital, out of which the firms pay wages. The money lent out will consist of the deposits of the households plus new reserves from the monetary authority, X, which are injected into the economy via open market operations (OMOs). We assume that OMOs are carried out in the repo market. The banks hand over X worth of (their own privately-issued one-period) bonds[(5)] in return for the new reserves; at the end of the period they pay back $(1+R)X$ to the central bank in return for the bonds (where R is the nominal repo rate). This is then passed on to the consumers as a lump-sum transfer.

In an interest rate setting environment, the demand for new reserves will be determined by the demand for loans such that the banks maintain their desired reserve ratio. As our main focus is not on the credit channel of

(5) The fact that these repo transactions are in bank bills rather than gilts is not important for our analysis. In order to focus on monetary rather than fiscal policy issues, we have chosen not to model the government explicitly (ie there is no government spending or taxation). However, it would be relatively straightforward to introduce one while preserving Ricardian equivalence and Modigliani-Miller neutrality with respect to financial structure. In that situation, gilt repo transactions could take place and the implications for real and nominal variables would be exactly the same as here.

monetary transmission we are not concerned with what determines this ratio. Hence, we assume that this is constant and fixed at 100%.(6) Given this assumption, we can note that X adds not only to reserves but also to bank money. During the period, the households purchase goods with their retained money, S, and any wages received, Wh. W is the nominal wage and h represents total hours worked. At the end of the period, the firms pay interest to the banks, $(1+R)(M - S)$ and dividends, D, to the households (which are assumed to own the firms). Note, we have assumed that competition in the financial sector drives the loan rate down to the repo rate, ie, the banks make zero profits in equilibrium. The banks pay interest on the households' deposits, $(1+R)(M_{t-1} - S)$, and $(1+R)X$ to the central bank in return for the earlier posted bonds. This amount is then redistributed lump sum to the households. (We think of this as the government redistributing seignorage revenue.) Hence, the households end up holding all the money at the end of the period. These 'flows of funds' are illustrated in Charts 1 and 2.

(6) Which examines the effects of shocks to reserve requirements. See Cooley, Nam (1998).

Chart 1: Beginning-of-period flows of funds

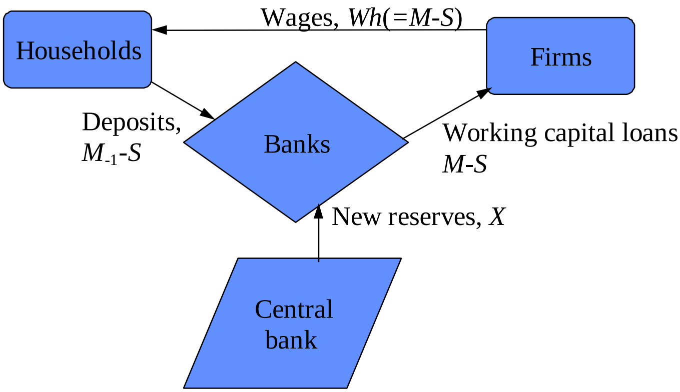
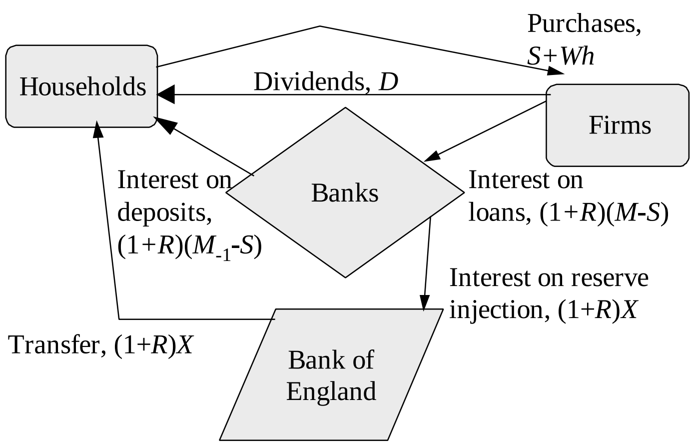


Chart 2: End-of-period flows of funds



2.2 Households

2.2.i The formal problem

As all the households are identical we consider the problem of a representative consumer. He begins the period with the stock of money M_{t-1} and chooses consumption, hours of work, cash holdings and deposits in order to maximise the discounted value of his present and future expected streams of utility, subject to a cash-in-advance constraint and asset accumulation constraints.

Formally:

$$\text{Maximise}\, E_0 \sum_{t=0}^{\infty} b^t U(C_t, l_t)$$

$$\text{Subject to}\, P_t C_t = S_t + W_t h_t \qquad \textbf{(1)}$$

$$M_t + V_t x_{t+1} = (1 + R_t)(M_{t-1} - S_t + X_t) + V_t x_t + D_t x_t$$

$$1 = l_t + H_t + h_t$$

where b is his discount factor, U is a twice differentiable utility function, C is real consumption, l is leisure, h is total hours worked, P is the aggregate price level, M is end-of-period money holdings, x is the proportion of the representative firm owned by the consumer (equal to unity in equilibrium), V is the value of a unit share in the representative firm (equal to the stock market value of the whole firm), S is ‘spending money’ (decided upon before any shock is realised), W is the nominal wage, R is the nominal interest rate, X is this period’s injection of reserves, and D is dividends received from the firm. H is the cost to each household associated with altering the cash held over for purchasing goods (equivalently bank deposits) *between* periods, measured in time units of labour (ie minutes per quarter); this inter-period adjustment cost enables the model to generate persistence in real and nominal variables independently of the persistence in the shock processes.

The first constraint is a cash-in-advance constraint: all purchases of consumption or investment goods are made with cash held over, S, or are paid for out of current-period wages. The second constraint describes the evolution of the household’s financial assets: end-of-period money consists of deposits plus interest earned, the transfer from the central bank (equal to new reserves plus interest earned) and dividend payments made by firms. The final constraint says that the consumer has to divide his time between leisure, paid employment and adjusting his financial portfolio.

2.2.ii Functional forms

Following Christiano and Gust (1998), we adopt the following functional forms for utility and adjustment costs:

$$U_t = \ln\left(C_t - q_0 e^{gt} \frac{(1-l_t)^{1+q}}{1+q} \right)$$

$$H_t = d\left(e^{c\left(\frac{S_t}{S_{t-1}}-1-m\right)} + e^{-c\left(\frac{S_t}{S_{t-1}}-1-m\right)} - 2 \right) \qquad \textbf{(2)}$$

where g is the (exogenous) rate of labour-augmenting technical progress, and m is the (exogenous) trend rate of nominal expansion. This will equal the steady-state inflation rate plus the steady-state real growth rate, g, so that real money balances grow at the same rate as output.

These functions have several important features. In particular, log utility and separability of consumption and labour jointly mean that the marginal propensity to consume out of wealth is independent of the real interest rate and equal to $(1-b)/b$. Moreover, this functional form means that the wealth elasticity of labour supply is zero: an assumption that makes understanding the labour market response to shocks simpler (as we do not have to worry about movements in the labour supply curve) and ensures a large response of output and employment to shocks (again, since the labour supply curve does not shift). The functional form for inter-period nominal adjustment costs ensures that H and $\partial H/\partial S$ are both zero when evaluated at the steady state. In other words, in equilibrium both the cost of changing nominal deposits and the cost of changing them ‘quickly’ are zero.

2.2.iii First-order conditions

Solving the household's maximisation problem leads to the following conditions, which must hold at all times:

$$q_0 \exp(gt)(h_t + H_t)^q = \frac{W_t}{P_t}$$
$$E_{t-1}\Lambda_t = bE_{t-1}\{(1+R_t)\Lambda_{t+1}\}$$
$$E_{t-1}\Lambda_t V_{t-1} x_t = bE_{t-1}\{(V_t + D_t)\Lambda_{t+1} x_t\} \qquad \textbf{(3)}$$

where

$$\Lambda_t = \frac{U_{C,t}}{P_t} + U_{H,t} H'\left(\frac{S_t}{S_{t-1}}\right)\frac{1}{S_{t-1}} - bU_{H,t+1} H'\left(\frac{S_{t+1}}{S_t}\right)\frac{S_{t+1}}{S_t^{\,2}}$$

The first of these conditions is a labour supply curve and relates the marginal disutility of working an extra hour to the marginal benefit of doing so. The second condition is the dynamic supply of savings curve; Λ is the marginal value (shadow price) of savings/wealth, a complex expression involving leads and lags of holdings of 'spending money'. This condition states that the utility loss involved in forgoing £1 today must at all times be equal to the discounted utility of that £1 tomorrow, where these valuations take into account the cost of adjusting deposits as well as the utility of consumption. The twist with this condition is that it is evaluated *ex ante*, ie on the basis of information available in period t-1; this is the limited participation feature of the model as it means that households are unable to participate in the financial market after the shocks have happened, whereas firms and banks will be able to. The final condition is an arbitrage condition on share-holding. It states that the marginal utility cost of purchasing x shares in a representative firm at the beginning of the period (at a cost of V_{t-1} per share) will equal the benefit derived from holding the shares over the period; this will equal the dividends paid out plus the proceeds from selling the shares, weighted by the marginal value of a dollar in the next period and discounted back to today by b (since the proceeds can only be spent in the next period). Again the expectation is taken with respect to time t-1 information. We can use this equation to calculate the value of a share in the representative firm (the stock market value of the economy).

2.3 Firms

2.3.i The formal problem

We assume that the households own the firms. Again all firms are identical and the representative firm's problem is to maximise the *utility-weighted* present discounted value of current and future dividend streams that it sends to the households. We assume that there are 'costs of adjustment' involved when investments are made; this enables us to model equity prices in a non-trivial way, since if there were no adjustment costs the stock market value of the economy would always equal (the nominal value of) the capital stock. The firm's costs are wages, for which a fraction, J, of the money has to be borrowed from the banks, and investment, which is financed by retained earnings. Following Christiano and Gust (1998), we introduce J in order to represent shocks to the demand for credit which, in this model, are the same as money demand shocks; a shock to J will alter the amount of money required to finance a particular level of real activity, the velocity of money.[7] If we were concerned with modelling the determinants of velocity, J would be endogenous, but as we are not we assume it is exogenous following the process outlined below.

(7) There is no obvious interpretation of J; it is introduced solely to represent shocks to the demand for money. We could set its steady-state value in order to match average velocity but, since velocity is trending in UK data, we simply normalise it to unity. Of course, this means that the firm can be required to borrow more than 100% of its working capital requirement — a clearly unrealistic feature of the model — but such a situation could be interpreted as one in which the firm was asked to pay a risk premium for its borrowing.

Formally, the representative firm's problem can be written as:

$$\text{Maximise } E_0 \sum_{t=0}^{\infty} b^t \Lambda_{t+1}\left(P_t\left(Y_t - I_t\right) - W_t h_t\left(1 + R_t J_t\right)\right)$$

$$\text{Subject to } Y_t = f\left(K_{t-1}, h_t, A_t\right)$$

$$K_t = (1-d)K_{t-1} + f\left(\frac{I_t}{K_{t-1}}\right)K_{t-1} \quad \textbf{(4)}$$

$$\ln(A_t) = r_A \ln(A_{t-1}) + e_{A,t}$$

$$\ln(J_t) = \ln(J_{t-1}) + e_{J,t}$$

Notice that because dividends are sent to households at the end of the period and consumption is subject to a cash-in-advance constraint, they will be spent one period after the profit is made. That is why profits are multiplied by L_{t+1} and not L_t. If there were no cash-in-advance constraint or costs of adjusting capital, the firms' problem would reduce to maximising profits each period.

Production is a concave and twice differentiable function of capital and labour. We assume that f is a strictly convex adjustment cost function. The production frontier is shifted around by A, which is modelled as an AR(1) process. The money demand shock, J, is modelled as a random walk.

2.3.ii Functional forms

We assume a constant-returns-to-scale Cobb-Douglas form for the production function. This ensures that profits are zero in equilibrium.

$$Y = A_t K_{t-1}^{a}(e^{gt} h_t)^{1-a} \quad \textbf{(5)}$$

In terms of the investment adjustment cost function we assume only the following values for the function and its first two derivatives along a balanced growth path:

$$f\left(\frac{Ie^{g}}{K}\right)=\frac{Ie^{g}}{K},\; f'\left(\frac{Ie^{g}}{K}\right)=1 \text{ and } \frac{Ie^{g}}{K}\,\frac{f''\left(\frac{Ie^{g}}{K}\right)}{f'\left(\frac{Ie^{g}}{K}\right)}=-1 \qquad \textbf{(6)}$$

The first two conditions amount to assuming that there are no average or marginal costs of adjustment close to the balanced growth path (similar to the assumption we make about portfolio adjustment costs). The final condition sets the elasticity of the investment/capital ratio with respect to Tobin's *q* equal to unity. This value was taken from King and Watson (1996) and is in line with Chirinko's (1993) overview of empirical investment functions.

2.3.iii First-order conditions

Solving the representative firm's problem leads to the following set of first-order conditions:

$$\frac{W_t}{P_t}(1+R_t J_t)=(1-a)e^{(1-a)gt}A_t\left(\frac{K_{t-1}}{h_t}\right)^{a}$$

$$E_t\,\frac{P_t\Lambda_{t+1}}{f'\left(\frac{I_t}{K_{t-1}}\right)}=bE_t P_{t+1}\Lambda_{t+2}\left(\left(\frac{1-d+f\left(\frac{I_{t+1}}{K_t}\right)}{f'\left(\frac{I_{t+1}}{K_t}\right)}-\frac{I_{t+1}}{K_t}\right)+ae^{(1-a)g(t+1)}A_{t+1}\left(\frac{K_t}{h_{t+1}}\right)^{a-1}\right) \qquad \textbf{(7)}$$

The first equation describes the representative firm's demand for labour; the marginal cost of labour, including loan costs (*RJ*), is set equal to the marginal product of labour. The second condition is the dynamic investment demand curve. It says that firms balance the cost of installing one extra unit of capital against the marginal benefit. The cost today is $\frac{P_t}{f'\left(\frac{I_t}{K_{t-1}}\right)}$, which needs to be weighted by L_{t+1}, as it would have been paid out as dividends to the consumer who would have been able to carry it into the next period as cash. Hence, they would be worth $\dfrac{P_t \Lambda_{t+1}}{f'\left(\dfrac{I_t}{K_{t-1}}\right)}$. The benefit consists of two parts. Next period the investment good will produce more output and so generate higher dividends $P_{t+1} MPK_{t+1}$ (where *MPK* is the marginal product of capital). These dividends mean extra cash in hand at the beginning of period $t+2$ and they will be worth $\Lambda_{t+2} P_{t+1} MPK_{t+1}$. In addition, the undepreciated part of the installed capital will still be around next period. This is valuable to the firms as it eases the burden of capital adjustment costs. The gain from this will be

$$\left(\frac{1 - d + f\left(\dfrac{I_{t+1}}{K_t}\right)}{f'\left(\dfrac{I_{t+1}}{K_t}\right)} - \frac{I_{t+1}}{K_t} \right) P_{t+1} \Lambda_{t+2} .$$

Note that if we multiply this second equation through by K_t we obtain:

$$E_t \frac{P_t K_t \Lambda_{t+1}}{f'\left(\dfrac{I_t}{K_{t-1}}\right)} = b E_t \Lambda_{t+2} \left(\frac{P_{t+1} K_{t+1}}{f'\left(\dfrac{I_{t+1}}{K_t}\right)} + D_{t+1} \right) \tag{8}$$

Comparing back to the consumer's first-order conditions and noting that x equals unity in equilibrium, we see that the stock market value of this economy will be given by:

$$V_t = \frac{P_t K_t}{f'\left(\frac{I_t}{K_{t-1}}\right)} \tag{9}$$

2.4 Market-clearing conditions

In this model all markets clear at all times. Thus, we have the following market-clearing conditions for the goods market, the market for loans and the money market, respectively:

$$C_t + I_t = Y_t \tag{10}$$

$$M_t - S_t = W_t h_t J_t \tag{11}$$

$$S_t + \frac{M_t - S_t}{J_t} = P_t C_t \tag{12}$$

2.5 Monetary policy

Monetary policy can be described in one of two ways. Our benchmark is to assume that the money stock evolves in response to a monetary shock in the same way as was estimated in Dhar, Pain and Thomas (1998). This is the benchmark, not because we believe the Bank of England targets money — clearly it does not. We do so for an unashamedly practical reason — namely that the model is much easier to solve under a monetary rule. It is well known that interest rate rules can, for many parameterisations, lead to indeterminacy or, worse, explosiveness.(8)

(8) Of course, indeterminacy is interesting in itself, since it allows an exogenous change in inflation expectations to affect inflation itself. Standard IS-LM models with an inflation target typically have no nominal anchor, other than the target itself. So this model might be seen as a more rigorous example of those: determining the conditions under which sunspots can occur and emphasising the importance of where inflation expectations come from. See Christiano and Gust (1998) for more on both indeterminacy and explosiveness.

However, we also use a technical argument to justify our use of a monetary rule *as a modelling strategy*, even though we know that the central bank does not target money in reality. The argument (made originally, and more forcefully, in Christiano, Eichenbaum and Evans (1998)) runs as follows: in order to implement a rule that makes interest rates depend on endogenous variables, the growth rate of money has to respond to the fundamental shocks hitting the economy in a particular way. Provided we ensure that the response of the money supply to particular shocks is correct, then the responses of the other endogenous variables to these shocks will be identical to those obtained from a model in which the central bank was operating the endogenous monetary policy rule actually applied in the data period. In fact, this gives us a way of dealing with the indeterminacy problem as it picks out the particular (implicit) money growth rule that was actually followed in order to support the endogenous interest rate rule (which otherwise would imply a choice of possible money growth rules).

Given that we are able to identify within a VAR the response of variables to a 'monetary policy' shock, we feel happy in using a money growth rule that responds in the correct way to this shock. We stress that the model will not produce the correct responses of endogenous variables to other shocks, nor will it be able to reproduce the variances and correlations of endogenous variables that we see in the data; to address these issues, we would need to use an independent estimate of the policy rule within the model.(9) (For an example of where this approach has been taken, see Ferré and Millard (1998).) Having said this, provided we can match the responses of variables to a monetary policy shock, we will be able to use the model to run forecasts conditional on particular assumptions about policy (such as 'unchanged interest rates') with the assumption that there are no other shocks during the forecast period. This is how we would propose using this model for policy analysis.

Consequently, we use the following empirical money rule as our benchmark:

$$\Delta m_t = m_t = m + 0.0052 e_t + 0.00168 e_{t-1} + 0.00111 e_{t-2} + 0.00052 e_{t-3} \quad \textbf{(13)}$$

where, e is our monetary policy shock and, as noted before, this univariate process is taken from a structural VAR describing our 'best' estimate of the

(9) We are thankful to Andrew Scott for drawing this point to our attention in the course of a series of exchanges on these issues.

UK monetary transmission mechanism (Dhar, Pain and Thomas (1998)). Note also that the average money growth rate, *m*, is exogenous; when calibrating this model we assume that it is set such that the central bank achieves an inflation target of 2.5% on average. This assumes that agents in the economy are fully aware of the inflation target and that it is totally credible; in other words, for the purposes of this paper we are simply ignoring the important issues surrounding the credibility of central banks.[10] Solving the model subject to this specification of monetary policy throws out decision rules for all the endogenous variables, one of which is the nominal interest rate. This interest rate decision rule will be a function of all the shocks in the model but in no sense can be thought of as a 'monetary policy rule'.

We also examined the model under interest rate targeting. We looked at the following baseline rule — a Taylor rule of the form used by Clarida, Gali and Gertler (1997):

$$\begin{aligned} R_t &= (1-r)R_t^* + rR_{t-1} + e_{m,t} \\ R_t^* &= R^* + a_1 \ln\left(\frac{Y_t}{Y}\right) + a_2 E_t \ln\left(\frac{P_{t+1}}{P_t(1+p^*)}\right) \end{aligned} \qquad \textbf{(14)}$$

Here e_m is the monetary policy shock, R^* is the steady-state nominal interest rate, Y is 'trend' output, and p^* is the inflation target. We found, however, that for values of a_1 and a_2 similar to those estimated elsewhere for the United Kingdom (Nelson (1998)) that the model became indeterminate; this suggests, again, that it is sensible for our current purposes to work with a money growth rule. Christiano and Gust (1998) use a similar model to investigate what parameterisations of the Taylor rule imply indeterminate or explosive solutions. They find that Taylor rules that put high weight on output stabilisation tend to lead to indeterminacy or explosiveness. The logic is that a higher coefficient on output makes the central bank less likely to raise interest rates in response to an exogenous increase in inflation expectations; such an increase is then more likely to

(10) This point is discussed further in Section 3, below.

become self-fulfilling leading to multiple equilibria.[11] For the rest of this paper we report the results that are obtained when the exogenous money growth rule is used — readers interested in the issues surrounding the use of interest rate rules in these models are referred to Christiano and Gust (1998) and Ferré and Millard (1998).

2.6 Calibrating, linearising and solving the model

2.6.i The stationary representation

Before we can calibrate and solve the model, we transform it so that all variables follow stationary processes around the non-stochastic steady state. When we come to compare the analytical model's predictions with those of an estimated econometric model such as a vector autoregression, we will want it, in repeated sampling, to imply distributions for the endogenous variables that do not vary over time.

Notice that all real variables, except for employment and the time cost of adjusting financial portfolios, have a 'common trend': they all grow at the same rate as output. Hence, they can be converted to stationary equivalents by dividing by *exp(gt)*. All nominal variables, excluding the price level, share a 'common trend' with the nominal money stock and so can be made stationary by dividing by M_t. Multiplying the price level by $exp(gt)/M_t$ and the shadow value of money, L_t, by M_t completes our description of the stationary representations of all our variables. Putting this all together leads to the following set of stationary first-order conditions and market-clearing conditions (where lowercase letters represent the stationary equivalents of their uppercase counterparts); notice the resulting appearance of terms involving e^g and m which reflect the two 'common trends'.

$$\frac{w_t}{p_t} = q_0 (h_t + H_t)^q \qquad \textbf{(15)}$$

(11) An alternative approach is to choose a 'minimum state variable' solution that solves the model. (See McCallum (1999).) This procedure will always produce a unique solution to these models which may or may not be explosive. Ferré and Millard (1998) investigate alternative interest rate rules in this model using such an approach.

$$\frac{w_t(1+R_t J_t)}{p_t} = (1-a)A_t e^{-ag}\left(\frac{k_{t-1}}{h_t}\right)^a \qquad \textbf{(16)}$$

$$E_{t-1}\, l_t(1+m_t) = bE_{t-1}(1+R_t)\, l_{t+1} \qquad \textbf{(17)}$$

$$E_t\, \frac{p_t e^g}{p_{t+1}(1+m_t)} = bE_t\, \frac{l_{t+2}}{l_{t+1}(1+m_{t+1})}\, f'\left(\frac{i_t e^g}{k_{t-1}}\right)*$$
$$\left(aA_{t+1}e^{(1-a)g}\left(\frac{k_t}{h_{t+1}}\right)^{a-1} + \frac{1-d+f\left(\frac{i_{t+1}e^g}{k_t}\right)}{f'\left(\frac{i_{t+1}e^g}{k_t}\right)} - \frac{i_{t+1}e^g}{k_t}\right) \qquad \textbf{(18)}$$

$$k_t e^g = k_{t-1}\left(1-d+f\left(\frac{i_t e^g}{k_{t-1}}\right)\right) \qquad \textbf{(19)}$$

$$c_t + i_t = A_t e^{-ag} k_{t-1}^a h_t^{1-a} \qquad \textbf{(20)}$$

$$1 - s_t + m_t = w_t h_t J_t \qquad \textbf{(21)}$$

$$p_t c_t = s_t + \frac{1-s_t+m_t}{J_t} \qquad \textbf{(22)}$$

These eight equations solve for the eight endogenous variables — consumption (c), the capital stock (k), investment (i), spending money (s), total hours worked (h), wages (w), prices (p) and the shadow price of money (l)— given initial conditions k_{t-1}, n_{t-1} and m_{t-1}, the monetary policy rule in force and the stochastic processes generating A and J.

Note that equation **(22)** corresponds to the empirical *money demand* equation. In particular, consumption velocity in the model will be given by:

$$Vel_t = \frac{J_t s_t + 1 - s_t + m_t}{J_t (1 + m_t)} \tag{23}$$

As we can see, in this model velocity will depend on the money demand shock as well as the proportion of money set aside for spending and the money growth rate. Clearly, for *J* at its steady-state value of unity, *Vel* will also equal unity. Also a positive money demand shock (rise in *J*) will lead to a fall in velocity and *vice versa*. In this model, interest rates only affect velocity indirectly through their effects on the proportion of money consumers set aside for spending and the money growth rate (if an interest rate rule is being followed). They will, however, affect the demand for loans.

2.6.ii The steady state and calibration

The model is calibrated by ensuring its non-stochastic steady state matches key features of the UK economy (although in future work we aim to use a maximum likelihood technique to estimate these). Suppressing all time subscripts, the model's steady state can be written:

$$\frac{w}{p} = q_0 h^q \tag{24}$$

$$\frac{w(1+R)}{p} = (1-a)e^{-ag}\left(\frac{k}{h}\right)^a \tag{25}$$

$$1 + m = b(1+R) \tag{26}$$

$$1 = be^{-g}\left(ae^{(1-a)g}\left(\frac{k}{h}\right)^{a-1} + 1 - d\right) \tag{27}$$

$$c + (1 - e^{-g}(1-d))k = e^{-ag} k^a l^{1-a} \tag{28}$$

$$1 + m - s = wl \tag{29}$$

$$pc = 1 + m \tag{30}$$

In order to calibrate the model as closely as possible to real UK data, we first had to construct series that bore some relationship with those in the model. In particular ‘output’ in the model is private domestic demand (as there is no government or foreign sector). For ‘consumption’ we used the ONS data on *Consumers’ Expenditure* (ONS Code: ABJR+HAYO). For ‘investment’ we used the ONS series for *Gross Domestic Fixed Capital Formation* (ONS Code: NPQT) but subtracted from that *Government Investment* (ONS Code: DLWF). ‘Output’ was then calculated as ‘consumption’ plus ‘investment’. For the ‘Price Level’ we used the implicit deflator for this output series. All data ran from 1964 Q1 to 1998 Q2.

The steady-state real interest rate (equal to the ‘risk-free’ rate in the model) was set equal to 3.8% per annum, the average ten-year forward real interest rate calculated by the Bank of England since index-linked gilts were first issued in the United Kingdom.[12] The growth rate, *g*, was set equal to 0.6% per quarter, the average growth rate of our output series. In order to calculate the steady-state nominal interest rate, *R*, and the associated steady-state money growth rate, *m*, we assumed that the central bank met an inflation target of 2.5%. The implied value of *R* is then equal to 6.3% per annum and *m* is 4.9% per annum. In order to calculate $a = 1 - \frac{wh(1+R)}{py}$ we used *Wages and Salaries* (ONS Code: ROYJ) for *wh* and our calculated steady-state value of *R*. Finally, d was calculated such that the capital/output ratio in our model economy matched that in our data (18.56), where the capital stock was calculated by assuming a constant rate of depreciation between the annual ONS observations. Table A shows the implied quarterly values of our parameters.

(12) There is an issue here about how good a measure of the ‘risk-free one-period (quarter) real rate’ this is, but that is outside the scope of the current paper.

Table A: Calibrating parameters

Variable	*Parameter*	*Assumed value*	*Comment*
Real interest rate	r	0.010	Average IG 10 Yr. Fwd Yield
Money growth rate	m	0.012	Meets inflation target
Capital's share	a	0.384	To match Wh/Y ratio
Depreciation rate	d	0.011	To match K/Y ratio
Trend productivity growth	g	0.006	Average for our data
Hours supplied	h	1	Normalisation
Labour supply elasticity	$(1/q)$	2.5	Christiano-Gust (1998)
Adjustment cost scale	d	2	Christiano-Gust (1998)
Adjustment cost elasticity	c	2	Christiano-Gust (1998)
Productivity persistence	r_A	0.91	To match its value in our data.

Given these parameters, the equations above solve for the following values of variables in steady state:

Table B: Steady state calculations

Variable	*Parameter*	*Value*
Discount rate	b	0.997
Nominal interest rate	R	0.016
Capital stock	k	114.225
Output	y	6.154
Investment	i	1.968
Consumption	c	4.186
Price level	p	0.242
Nominal wage level	w	0.902
Utility weight of consumption	q_0	3.732
Spending share of money	s	0.110

2.6.iii Linearised decision rules

Having solved for the (non-stochastic) steady state, we can now derive linear decision rules for the endogenous variables in the neighbourhood of this steady state using the method of undetermined coefficients.[13] Here we simply present the linearised decision rules, which are used in all the empirical simulations shown in the next section. They can be written as follows:

$$\begin{aligned}
\hat{k}_t &= 0.99\hat{k}_{t-1} - 0.00\hat{s}_{t-1} + 0.03\ln(a_{t-1}) + 0.00\hat{M}_{t-1} - 0.00\ln(J_{t-1}) + 0.03e_{A,t} + 0.00e_{m,t} - 0.00e_{J,t} \\
\hat{s}_t &= -0.04\hat{k}_{t-1} + 0.95\hat{s}_{t-1} - 0.08\ln(a_{t-1}) + 0.05\hat{M}_{t-1} - 0.04\ln(J_{t-1}) \\
\hat{R}_t &= 0.01\hat{k}_{t-1} + 0.01\hat{s}_{t-1} + 0.120\ln(a_{t-1}) - 0.01\hat{M}_{t-1} + 0.01\ln(J_{t-1}) + 0.13e_{A,t} - 0.00e_{m,t} + 0.01e_{J,t} \\
\hat{I}_t &= 0.01\hat{k}_{t-1} - 0.00\hat{s}_{t-1} + 0.03\ln(a_{t-1}) + 0.00\hat{M}_{t-1} - 0.00\ln(J_{t-1}) + 0.03e_{A,t} + 0.00e_{m,t} - 0.00e_{J,t} \\
\hat{h}_t &= 0.48\hat{k}_{t-1} - 0.02\hat{s}_{t-1} + 1.01\ln(a_{t-1}) + 0.02\hat{M}_{t-1} - 0.03\ln(J_{t-1}) + 1.11e_{A,t} + 0.00e_{m,t} - 0.03e_{J,t}
\end{aligned} \quad \textbf{(31)}$$

where a ˆ over variables denotes log-deviations from trend, eg

$\hat{k}_t = \ln\left(\frac{k_t}{k}\right)$, where k is the trend level of the capital stock. In the case of the nominal interest rate, R, the ˆ denotes the absolute deviation from steady state.

With these rules we can simulate the response of variables within the analytical model to a monetary policy shock, and compare these responses with those implied by an empirical counterpart (eg a VAR). We go on to do this in the next sections of the paper.

(13) To solve the model we used software kindly provided by Professor Robert King. The solution algorithm is described in King and Watson (1998). A replication diskette is available on request.

3. Validating the model: impulse responses

In this section, we see how well the responses of endogenous variables within our model to a monetary policy shock can match the responses estimated within a small identified structural vector autoregression (VAR) model. (The estimation of this SVAR is described in Dhar, Pain and Thomas (1998).) Of course, we do not consider these shocks to be capturing 'monetary policy', since we normally think of this as the endogenous reaction of the monetary authority to economic events in order to achieve its policy goals; rather they represent policy 'errors' due to, say, mistakes in the data or political factors. We carry out this exercise because we believe that a good model should be able to reproduce these responses to an innovation in the policy instrument; this is an important test of the empirical validity of the model's transmission mechanism, particularly if we are to be confident in making forecasts for the economy conditional on a particular path of policy. We should note that the result of Christiano, Eichenbaum and Evans (1998) tells us that these responses should be unaffected by the assumption that we are following an exogenous money rule. Once we have assessed how well our model can reproduce the properties of the data, we can judge how confident we can be about using it to conduct policy analysis.

The paper of Dhar, Pain and Thomas (1998) involved estimating a structural model of the UK monetary transmission mechanism. Using techniques developed by Blanchard and Quah (1989), Shapiro and Watson (1988) and Gali (1997), amongst (many) others, they sought to identify the primitive structural disturbances hitting the economy by placing economically defensible restrictions on a reduced-form representation of the transmission mechanism. In what follows, we summarise what is done in that work and use the impulse response functions to a temporary monetary policy shock in the SVAR as a key test of the validity of using our model to carry out policy analysis on UK data.

In a system involving real GDP, inflation, real aggregate M4, the spread of deposit rates over the base rate, real equity prices and the real exchange rate, they found four common stochastic trends, shocks to which they interpret as:

- A technology or aggregate supply shock.

- A relative foreign demand shock.

- A shock to the 'implicit inflation target' in the monetary policy rule (a 'nominal anchor' shock).

- A financial intermediation shock.

In our model, by contrast, there are two deterministic common trends: a real (aggregate supply) trend and a nominal (inflation target) trend.

Next, Dhar, Pain and Thomas (1998) identified four temporary shocks:

- A money demand or velocity shock.

- A domestic demand shock.

- A foreign exchange risk premium shock.

- A monetary policy shock (a temporary shock to interest rates given the monetary policy rule in force).

The responses of variables to the last of these shocks is what we compare to the responses of variables in our model to a monetary policy shock in the model.

The nominal anchor shock differs from the monetary policy shocks typically discussed in the identified VAR literature (Sims and Zha (1995), Christiano *et al* (1996, 1998)) which generally refer to short-term monetary surprises, rather than to the conduct of long-run monetary policy. We consider an understanding of the effects of such regime changes to be important for explaining past monetary data and as a guide to what to expect should such a regime change occur in the future; however, until such a regime change happens, we feel confident in using a model in which the regime is taken as given.

An alternative way of highlighting the difference between these shocks is to consider a simple Taylor rule of the form:

$$R = R^{*} + a_1(y - y^{*}) + a_2(p - p^{*}) + e \qquad \textbf{(32)}$$

where R refers to the short-term interest rate set by the monetary authority, p is the inflation rate, y is (the log of) real GDP, stars denote natural or target values, and e is a disturbance. The distinction between the two monetary shocks referred to above amounts to drawing a distinction between a shock to p^*, the target inflation rate and e. In particular, we want to allow for the possibility that the 'liquidity effect' highlighted by much of this literature, and for which there is evidence in the United Kingdom (eg Millard (1998)), is associated with draws of e rather than lower-frequency (and possibly discrete) changes in the monetary policy framework.

Notice that the steady-state inflation rate in the model, p^*, is held constant at 2.5%, so that the 'nominal anchor' shock is not present. As we would expect the steady-state inflation rate to be on target (since we are fully committed to the target) 2.5% seems a sensible value to use. Now, it is possible that agents in the economy do not fully believe in our commitment to the target and so expect a higher rate of inflation. In the structural VAR this would be picked up by the 'nominal anchor' shock. Within the model, an exogenous increase in inflation expectations would lead agents to deposit less money with the banks and keep more back as 'spending money'. This would create a liquidity shortage at current interest rates, leaving the central bank with the dilemma of either having to allow interest rates to rise dramatically or injecting the extra liquidity (an action that would cause the agents inflation expectations to become self-fulfilling). In specifications of the model in which there is a unique equilibrium (such as the one we have been using throughout the paper), this exogenous rise in inflation expectations cannot happen. If we allowed inflation expectations to be based on some sort of 'learning' rather than being fully rational, we may be able to capture the fact that inflation expectations in the data are stickier than would be suggested by the model.

Charts 3 to 5, below describe the response of nominal variables to a temporary monetary policy shock as defined above. In each case we plot the responses of the variables in the data (of course, using a particular model of the data — the SVECM of Dhar, Pain and Thomas (1998) — to identify the 'policy shock') and the equivalent responses of the variables in the model. Note that this is not the same as simulating monetary policy, the largest components of which are the endogenous response to deviations of output and inflation (say) from target and changes in the inflation target itself, and is affected by changes in the monetary regime. But we use these responses as another diagnostic with which to assess the model's fit.

A surprise monetary tightening is associated with a fall in money supply growth and a (small) rise in nominal interest rates: the liquidity effect. In the model, this is the result of the agent heterogeneity we described above influencing outcomes in the period of the shock. Specifically, households have deposited their money at the bank at the beginning of the period, and then observe the monetary contraction (equivalently they see interest rates go up). They are prevented from depositing more money in the bank to take advantage of the higher interest rate, whereas firms already have a known demand for loans equal to the proportion of the wage bill they choose to finance this way. In order to convince firms to borrow less, short-term nominal interest rates have to rise temporarily, even though all agents know that this will mean lower inflation and nominal interest rates in the future. After the first period, real and nominal variables adjust sluggishly. Note that the model predicts a small liquidity effect: something that seems to be the case for UK data suggesting that the model may well be useful in a UK context.

Chart 3: Response of money growth to a monetary policy shock

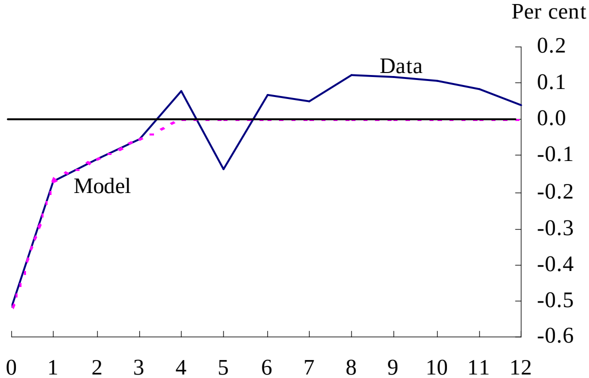
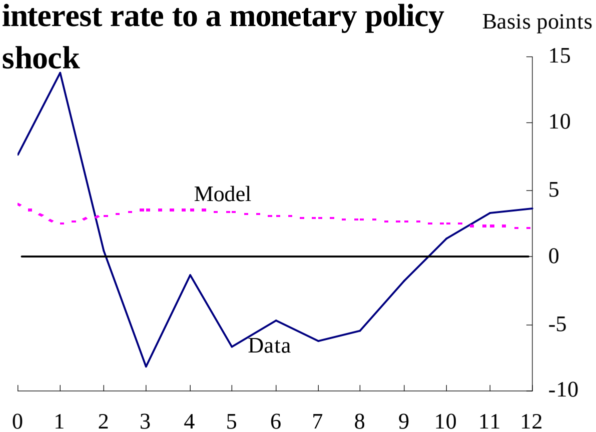


Chart 4: Response of nominal interest rate to a monetary policy shock



The peak response in prices is reached after four quarters. As the response is not immediate, the model can be seen to be displaying endogenous price stickiness: endogenous in the sense that the Phillips curve is not imposed from the outside. In this model, the price stickiness emerges as an equilibrium phenomenon: a product of the credit market frictions (portfolio adjustment costs). In the data, the price level takes a much longer time to move to its new trend.

Chart 5: Response of prices to a monetary policy shock

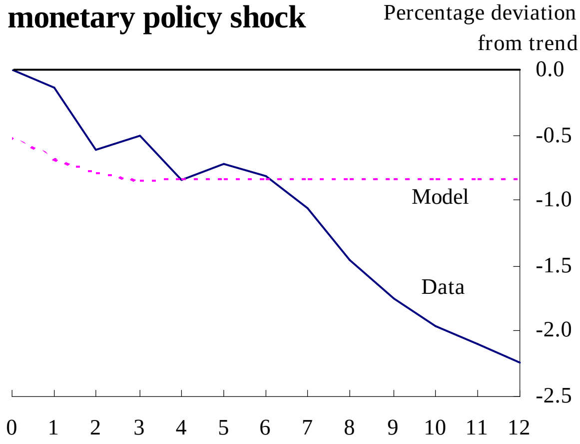
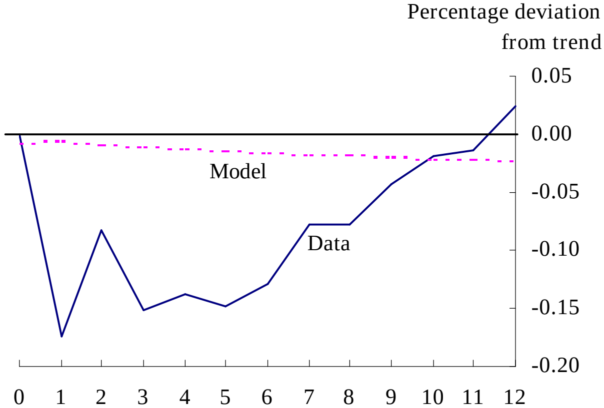


Chart 6 plots the response of output to monetary policy shocks in the data and in the model. The effect of the shock is small in the data and negligible in the model, reflecting the small effect on interest rates of the monetary shock. This analysis suggests that other shocks are likely to be the major determinants of output volatility in the United Kingdom. To the extent that we believe this, it is not so worrying that monetary shocks in our model have such a small effect on output.

Chart 6: Response of output to a monetary policy shock



Finally, Charts 7 and 8 plot the responses of employment and real wages to monetary policy shocks in the data and in the model. In order to do this, we obtained a time series for the 'monetary policy shocks' in the structural VAR and ran regressions of the form:

$$y_t = \sum_{j=0}^{12} b_j e_{t-j} + aX_t \quad \textbf{(33)}$$

where y is the variable in which we are interested, e is the monetary policy shock obtained from the structural VAR and X is a vector that includes a constant and a time trend. The coefficients, b's, then give us the response over time of variable y to a monetary policy shock.(14)

(14) If we are convinced that the monetary policy shocks identified in the SVAR are truly exogenous then this procedure will give us consistent, though not efficient, estimates of the impulse response functions. As a check, we found that for those variables in the SVAR, this method did not produce impulse responses that were statistically different from those obtained directly from the SVAR.

Chart 7: Response of employment to a monetary policy shock

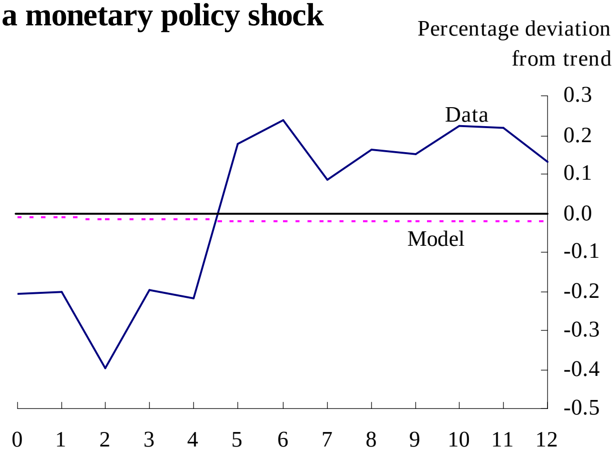
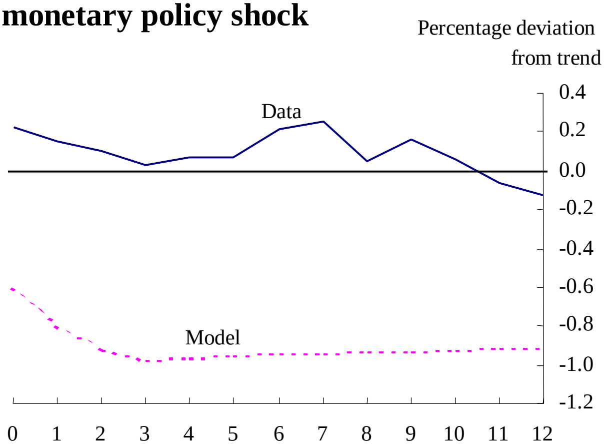


Chart 8: Response of real wages to a monetary policy shock



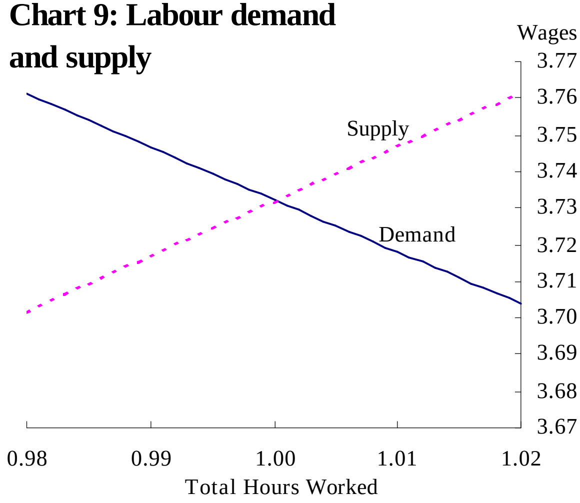
We can see that the model is not able to match the response of labour market variables to a monetary shock. The short-run response of employment in the model is, again, negligible, reflecting the small movement in interest rates that results from the monetary policy shock. The response of real wages in the model is in the opposite direction to that in the data. In order to understand what is going on here, recall the (detrended) labour supply and labour demand equations:

$$\frac{w_t}{p_t} = q_0 \left(h_t + H_t\right)^q \quad \textbf{(34)}$$

$$\frac{w_t}{p_t}(1 + R_t J_t) = (1 - a)e^{-ag} A_t \left(\frac{k_{t-1}}{h_t}\right)^a \quad \textbf{(35)}$$

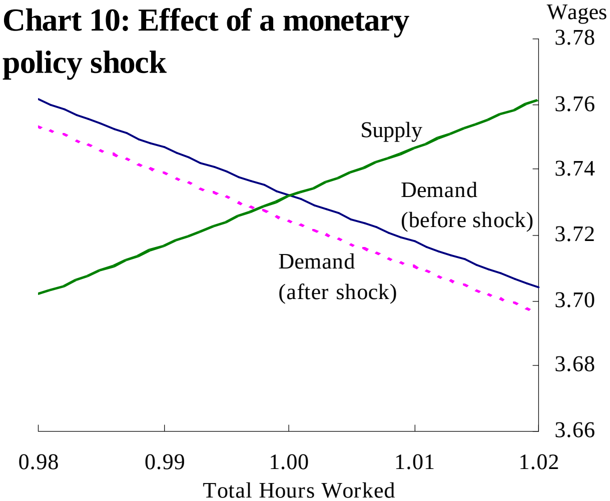
Equation **(34)** is the labour supply curve and equation **(35)** is the labour demand curve. For the values of the parameters in our baseline calibration, these curves are plotted in Chart 9, below, in employment-real wage space:

Chart 9: Labour demand and supply



A monetary policy shock leads to a fall in labour demand without any effect on labour supply. This results in an immediate fall in employment and real wages, as shown in Chart 10, below.

Chart 10: Effect of a monetary policy shock



Notice that there is no effect on labour supply. This is a result of the functional form that we have assumed for utility, since it implies that the wealth elasticity of labour supply is zero.[(15)] This in turn means that employment and real wages will always move in the same direction in our model. This disagrees with the data that suggest they move in opposite directions in response to a policy shock. If modelling the labour market were our primary concern, this would suggest the need to introduce another shock — such as a preference or distortionary income tax shock — to shift the labour supply curve around. Alternatively, the introduction of 'sticky' nominal wages would solve the problem. In this case, a monetary contraction will lead (as before) to lower prices but, with nominal wages 'sticky', this would imply higher real wages.

Given the response of employment to the policy shock in the model, it is straightforward to calculate the response of output from the production function:

$$y_t = e^{-ag} q_t k_{t-1}^{a} h_t^{1-a} \qquad \textbf{(36)}$$

(15) In the data, this assumption would appear to be invalid as leisure is clearly a normal good. However, this result that the response of real wages and hours worked to shocks is in the same direction seems to be robust to specifications of the utility function where there is some impact of wealth on labour supply and is a common problem of many DSGE model. (See, Millard, Scott and Sensier (1997) for a survey.)

In particular, for a money demand or monetary policy shock the response will be $(1-a)$ times (ie less than) the response of employment. In the data, we find that the output response is about the same as the employment response (after an initial period during which output was constrained to have no response). The addition of 'labour hoarding' considerations into the model might be able to sort out this problem. However, these labour market phenomena are not our priorities, so we leave them aside for now.

In all, we take the results of this section to confirm that the analytical model is capable of generating predictions that are reasonably qualitatively consistent with the economy's response to monetary surprises, and that this suggests it might be a useful test-bed on which to conduct monetary policy experiments, and maybe even to construct forecasts conditional on particular draws of monetary policy shocks (as embodied by, say, a constant interest rate assumption). Furthermore, for these purposes it is fine to assume an exogenous process for money that has money growth reacting in an appropriate way to monetary policy shocks, as the responses and forecasts will be identical to those in a model in which policy were reacting endogenously.

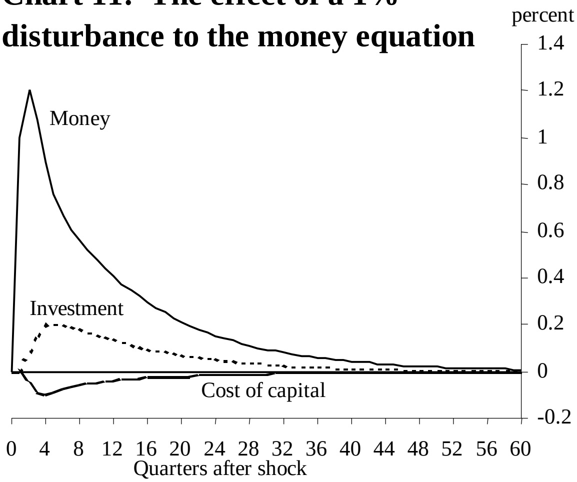
4. Using the LP model to understand *M-M**

In this section, we revisit some earlier results — the sectoral analysis of the demand for M4 by Thomas (1997b). In particular, we explore what light our model can shed on the relationship between ICCs' *M-M** and investment as estimated in Thomas (1997b). A potential shortcoming of the *M-M** approach derives from the econometrics — in that structural identification is achieved by means of within and cross-equation restrictions in the dynamic system of equations. When the question being asked is 'what is our best estimate of desired money demand given estimates of wealth, income and interest rates' this modelling strategy is both highly efficient and appropriate. However, it is less so when the question is 'what type of shock most likely generated the recent pattern of distribution of *M-M**?'. This is because the identification strategy employed does not uniquely identify structural disturbances; in other words, the residual in the money demand equation cannot be identified as a money demand shock, nor that in the sectoral demand equation as a 'preference shock' or some other primitive.

That said, the impulse responses from the Thomas corporate sector model do motivate our interest in answering the 'what is the shock?' question. Chart 11, below, (reproducing a chart in Thomas (1997b)) shows the responses of money, investment and the cost of capital to a one standard error disturbance to the money equation residual. What is interesting is that money rises fairly quickly in response to a 'money shock', and the effects persist, whereas the effect on investment takes time to come through.

We can use our model to construct explanations for this pattern of impulse responses. Thomas argues that the 'money shock' represents an unexpected inflow to ICCs deposits which are then gradually spent. In the model, firms use the increase in their money balances to increase their wage payments and produce more output. In addition, we would expect to see an impulse to money lead to a rise in investment — the effect noted in Thomas's work as described in Chart 11.

Chart 11: The effect of a 1% disturbance to the money equation



In terms of the variables in the model, we can write firms' *M*-*M** as:

$$
\begin{aligned}
(M - M^*)_t &= \frac{1}{1-s}\hat{M}_t - \frac{s}{1-s}\hat{s}_t \\
&= 0.005\hat{k}_{t-1} - 0.117\hat{s}_{t-1} + 0.010\ln(a_{t-1}) + 1.117\hat{M}_{t-1} + 0.005\ln(J_{t-1}) \\
&\quad + 1.124(0.0052\boldsymbol{e}_{m,t} + 0.0017\boldsymbol{e}_{m,t-1} + 0.0011\boldsymbol{e}_{m,t-2} + 0.0005\boldsymbol{e}_{m,t-3})
\end{aligned} \quad \textbf{(37)}
$$

Our investment equation is:

$$
\begin{aligned}
\hat{I}_t &= 0.01\hat{k}_{t-1} - 0.00\hat{s}_{t-1} + 0.03\ln(a_{t-1}) + 0.00\hat{M}_{t-1} - 0.00\ln(J_{t-1}) \\
&\quad + 0.03\boldsymbol{e}_{A,t} + 0.00\boldsymbol{e}_{m,t} - 0.00\boldsymbol{e}_{J,t}
\end{aligned} \quad \textbf{(38)}
$$

From equation **(37)**, we can see that a positive shock to firms' *M*-*M** would be associated with a positive monetary policy shock, $e_{m,t}$. Equation **(38)** shows that such a shock is also associated with a (very small) rise in investment. Hence, we would expect to see the same responses of investment and *M*-*M** for a 'money' shock estimated using Thomas's method on data generated by the model.

The bottom line of this is that our model can potentially be used to match the impulse responses estimated by Thomas. Moreover, the intuition behind what is driving them is similar (Thomas (1997b)). The potential value-added comes from being able to use our model to tell a 'structural' story about what fundamental shocks drive the estimated 'shocks' in his econometric work.

5. Conclusions

There are, of course, still many problems with our model and still much more work that we could do. However, before going into possible extensions, we first summarise what we think we have achieved. We have developed an analytical model of the UK economy that has concentrated on the interaction of money with real variables. The model is a 'dynamic stochastic general equilibrium' model and, as such, answers the critique of Lucas by the fact that its equations are expressed in terms of 'deep' structural parameters and the underlying shocks affecting the economy. This enables it to be used for policy analysis: in particular, producing conditional forecasts such as those in the *Inflation Report* that are immune to the Lucas critique. The particular model we use is of the 'limited participation' variety. This means that it is able to reproduce the 'liquidity effect' seen in the data (the inverse correlation of short-run nominal interest rates and monetary growth in response to monetary policy shocks).

We showed that the responses of money growth, interest rates and prices to a monetary shock were fairly similar in the model to what we observe in the data. The effect of such shocks on output we found to be small in the data and negligible in the model; this suggests that the model is not so good for explaining movements in output. The model's key failing is that it still predicts too large a short-run response in prices to a monetary policy shock. In terms of future work, this suggests that the addition of some kind of nominal rigidity in addition to the portfolio adjustment costs will be necessary if we are to trust the model's conditional forecasts for short-term inflation. A possible answer could be the addition of 'sticky wages' to the model: something that would also allow the model to better match the responses of employment and real wages to a monetary policy shock. (For an example of a paper that takes this approach see Hendry and Zhang (1998).) Of course, we would have to think hard about how to justify the rigidity with microeconomic foundations and ensure that the relevant maximisation problems that led to this rigidity were well specified.

One problem with the model as it currently stands is in matching the 'money demand' shock, J, to the data. Throughout the period that we consider, broad money velocity has been trending downwards in the data. In the model it is stationary and averages $\frac{1}{2-s}$. This suggests that we need to think about allowing our money demand shock to trend. We see sorting this problem out as one of our main priorities.

As discussed earlier in the paper, although the model is capable of generating the reaction we would expect to an increase in inflation expectations, it is not capable of dealing with the issue of credibility. In order to deal with this problem we would need inflation expectations to be imperfectly rational. One way of doing this would be to introduce a learning mechanism. Given this, inflation expectations could feed off inflation outturns while affecting the outturn through the mechanism described earlier that is already present in the model.

Looking further ahead, a fuller understanding of the monetary transmission mechanism in the United Kingdom requires us to introduce (at least) two further channels of monetary transmission: an exchange rate channel and a credit channel. To analyse the exchange rate requires the addition of a foreign sector. A sensible starting-point would be to assume that the United Kingdom is a small open economy and, as such, takes the world real interest rate as given. To analyse 'credit channel' effects requires specifying in a more meaningful way the maximisation problem of the banks in the model (recall that currently they are just pure intermediaries).

If we were going to introduce the foreign sector, it would also be sensible to introduce the public sector. This would enable us to examine an additional source of shock that could possible be matched to the 'demand' shock in Dhar, Pain and Thomas (1998). Such a shock may also help to improve the performance of the model in matching the near-zero correlations of wages with employment and wages with output in the data. In addition, we would then be in a position to run conditional forecasts of GDP and inflation, as measured by the GDP deflator.

References

Blanchard, O and Quah, D (1989), 'The dynamic effects of aggregate demand and supply disturbances', *American Economic Review*, pages 655-73.

Chirinko, R S (1993), 'Business fixed investment spending: a critical survey of modelling strategies, empirical results and policy implications', *Journal of Economic Literature*.

Christiano, L J and Eichenbaum, M (1992), ' Liquidity effects and the monetary transmission mechanism', *American Economic Review*, pages 346-53.

Christiano, L J and Eichenbaum, M (1995), 'Liquidity effects, monetary policy and the business cycle', *Journal of Money, Credit and Banking*, pages 1,113-36.

Christiano, L J, Eichenbaum, M and Evans, C (1996), 'The effects of monetary policy shocks: some evidence from the flow of funds', *Review of Economics and Statistics*, pages 16-34.

Christiano, L J, Eichenbaum, M and Evans, C (1997), 'Sticky price and limited participation models: a comparison', *European Economic Review*, pages 1,201-49.

Christiano, L J, Eichenbaum, M and Evans, C (1998), 'Modelling money', *National Bureau of Economic Research Working Paper*, No 6371.

Christiano, L J and Gust, C J (1998), 'Discussion of Levin, A, Wieland, V, and Williams, J C, 'Are simple monetary policy rules robust to model uncertainty?' forthcoming in Taylor, J (ed), *Monetary policy rules*.

Clarida, R, Gali, J and Gertler, M (1997), 'Monetary policy rules and macroeconomic stability: evidence and some theory', *mimeo*.

Cooley, T F (ed) (1995), *Frontiers of business cycle research*.

Cooley, T F and Nam, K (1998), 'Asymmetric information, financial intermediation and business cycles', *Journal of Economic Theory*.

Dhar, S, Pain, D and Thomas, R (1998), 'Money in the forecast part two: a small structural empirical model of the UK monetary transmission mechanism', *mimeo*.

Farmer, R E A (1993), *The macroeconomics of self-fulfilling prophesies*.

Ferré, M and Millard, S P (1998), 'Monetary policy rules in a limited participation model', *mimeo*.

Fuerst, T S (1992), 'Liquidity, loanable funds and real activity', *Journal of Monetary Economics*, pages 3-24.

Gali, J (1997), 'Technology, employment and the business cycle: do technology shocks explain aggregate fluctuations?', *NBER Working Paper*, No 5721.

Hendry, S and Zhang, G-J (1998), 'Liquidity effects and market frictions', *Bank of Canada Working Paper*, No 98-11.

Hoover, K D (1995), 'Facts and artefacts: calibration and empirical assessment of real business cycle models', *Oxford Economic Papers*, pages 24-44.

Johansen, S and Juselius, K (1994), 'Identification of the long-run and short-run structure: an application to the IS-LM model', *Journal of Econometrics*, pages 7-36.

King, R G and Rebelo, S T (1998), 'Resuscitating real business cycles', forthcoming in *Handbook of Macroeconomics*.

King, R G and Watson, M W (1996), 'Money, prices, interest rates and the business cycle', *Review of Economics and Statistics*, pages 35-53.

King, R G and Watson, M W (1998), 'System reduction and solution algorithms for singular linear difference systems under rational expectations', *mimeo*.

Kirman, A P (1992), 'Whom or what does the representative individual represent?', *Journal of Economic Perspectives*, pages 117-36.

Kydland, F E and Prescott, E C (1982), 'Time to build and aggregate fluctuations', *Econometrica*, pages 1,345-70.

Laidler, D (1984), 'The buffer-stock notion in monetary economics', *Economic Journal*, supplement, pages 17-34.

Lucas, R E Jr. (1976), 'Econometric policy evaluation: a critique', in Brunner, K and Meltzer, A (eds), *The Phillips curve and labor markets*.

Lucas, R E Jr. (1990), 'Liquidity and interest rates', *Journal of Economic Theory*, pages 237-64.

McCallum, B T (1999), 'Role of the minimal state variable criterion in rational expectations models', *mimeo*.

Millard, S P (1998), 'A preliminary examination of the monetary transmission mechanism in the United Kingdom', *mimeo*.

Millard, S P, Scott, A J and Sensier, M (1987), 'Business cycles and the labour market: can theory fit the facts?', *Oxford Review of Economic Policy*, Vol 13, No 3, pages 70-91.

Millbourne, R (1988), 'Disequilibrium buffer-stock models: a survey', *Journal of Economic Surveys*, pages 198-208.

Nelson, E (1998), 'Taylor rule estimates for the United Kingdom: 1972-97', *mimeo*.

Shapiro, M and Watson, M (1988), 'Sources of business cycle fluctuations', *NBER Macroeconomics Annual*, pages 111-48.

Sims, C A and Zha, T (1995), 'Does monetary policy generate recessions?', *mimeo*.

Thomas, R S J (1997a), 'The demand for M4: A sectoral analysis. Part 1 - The personal sector', *Bank of England Working Paper*, No 61.

Thomas, R S J (1997b), 'The demand for M4: A sectoral analysis. Part 2 **-** The corporate sector', *Bank of England Working Paper*, No 62.

Whitley, J (1994), *A course in macroeconomic modelling and forecasting*.

Wicksell, K (1935), *Lectures on political economy, Vol II: Money*.