Direct effects of base money on aggregate demand: theory and evidence

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Abstract

Meltzer (1999a) shows that real monetary base growth is a significant determinant of consumption growth in the United States, controlling for the short-term real interest rate. In this paper, I show that the same property of base money holds for total output (relative to trend or potential) in both the United States and the United Kingdom. The standard optimising IS-LM model cannot account for this result, but I show that it can once the long-term nominal interest rate is included in the money demand function. Because the long-term real rate matters for aggregate demand, the presence of the long-term nominal rate in the money demand function increases the effect of nominal money stock changes on real aggregate demand when prices are sticky.

1. Introduction

Much recent research on monetary policy rules uses small-scale macroeconomic models which include an ‘IS function’, analogous to the IS segment of a traditional IS-LM model. These IS functions range from the purely backward-looking specifications in Fuhrer and Moore (1995) and Rudebusch and Svensson (1999, 2000), to the forward-looking, theory-based ‘optimising IS equations’ in Kerr and King (1996), Rotemberg and Woodford (1997), and McCallum and Nelson (1999a). A common feature of these equations is that they specify the demand for output as a function of the real interest rate. The real money stock (or its growth rate) does not appear in the IS equation. These models therefore limit the influence of monetary policy on output and inflation to its effect on the real interest rate.(1)

In a recent paper, Meltzer (1999a) has challenged these specifications, arguing that they neglect important channels of monetary effects. Open market operations by a central bank affect both the nominal interest rate and the central bank’s balance sheet (the liabilities side of which includes the monetary base). If prices are sufficiently sticky in the short run, these operations also affect both the short-term real interest rate and the real monetary base. Meltzer argues that the short-term real interest rate fails to summarise completely the effect of monetary policy actions on the economy, and that the changes in real monetary base exert separate, or direct, effects on aggregate demand. He presents empirical evidence for this proposition using quarterly US data.

This paper examines the theoretical and empirical grounds for these direct effects of base money on aggregate demand. Throughout this paper, ‘direct’ or ‘separate’ effects of money will refer to the explanatory power for aggregate demand contained in the real money stock (or its growth rate) that is not captured by the short-term real interest rate. This definition of ‘direct effects’ allows for the possibility that money is serving as an index or proxy for yields or relative prices that are relevant for aggregate demand,

(1) In the backward-looking model of Rudebusch and Svensson, the short-term real interest rate typically enters the equation with a lag, while in the forward-looking models, it is typically current and expected future short-term real rates that matter for current aggregate demand. In open-economy versions, the real exchange rate typically also appears in the IS equation. Smets (1995) reports that larger models used by policy institutions have the same limited role for money.

which is contemplated in the discussions by Meltzer (1999a) and Friedman and Schwartz (1963, 1982).[2]

The paper proceeds as follows. In Section 2, I estimate backward-looking IS specifications for both the United States and the United Kingdom. The results provide strong evidence similar to the type presented by Meltzer for the United States—namely, statistically significant and economically sizeable positive coefficients on real monetary base growth on aggregate demand, even after conditioning on the short-term real interest rate. Section 3 discusses some alternative possible explanations for these results that a theoretically rigorous macroeconomic model could provide, and settles on one possibility, namely the inclusion of the long-term interest rate in the money demand function. Section 4 examines this extension in more detail, using a calibrated small-scale macroeconomic model with the short-term nominal interest rate as the monetary authorities' policy instrument. The resulting model is capable of generating regressions that are similar to those reported for the United States and United Kingdom in Section 2. This section also presents some results using the money stock as the monetary policy instrument. Section 5 concludes.

(2) I find the terms 'direct money channel' or 'separate money channel' more useful than 'real balance effect', because the latter is very closely associated with the Pigou (1947)-Patinkin (1965) effect, ie the inclusion of real money balances as a wealth variable in traditional Keynesian consumption functions. This effect is not the one investigated in this paper.

2. Empirical evidence

Meltzer (1999a) reports updated estimates of a specification for US consumption used by Koenig (1990). This regression specifies quarterly real consumption growth as a function of its own lag, the current change in the real interest rate, and the current growth of real money balances (defined as the monetary base). Meltzer reaffirms Koenig's result that real money growth enters positively and significantly.

Meltzer's empirical results open the question of whether they hold for wider measures of real economic activity, such as detrended output, and for less restricted specifications that allow the *level* of the real interest rate to appear as a regressor. Typical empirical 'IS equations' specify aggregate demand (measured by detrended output or an output gap series) as a negative function of the short-term real interest rate, with no separate channel through which money balances can affect economic activity. Rudebusch and Svensson (2000), for example, present the following IS equation for the United States, estimated over the period 1961 Q1–1996 Q4:

$$\tilde{y}_t = \underset{(0.079)}{1.161}\,\tilde{y}_{t-1} - \underset{(0.077)}{0.259}\,\tilde{y}_{t-2} - \underset{(0.032)}{0.088}\left(\left(\Sigma_{j=0}^{3} R_{t-1-j}\right) - \Delta_4 p_{t-1}\right)$$

SEE = 0.823%, DW = 2.08,

where $\tilde{y}_t$ is the Congressional Budget Office's estimate of the output gap (expressed as a fraction),[3] R_t is the Federal funds rate expressed as a quarterly fraction, $\Sigma_{j=0}^{3} R_{t-1-j}$ is a four-quarter sum of lagged funds rates, p_t is the log of the GDP deflator, and Δ_4 is the fourth-difference operator (so that $\Delta_4 p_t = p_t - p_{t-4}$).

Rudebusch and Svensson (2000) report that 'lags of money (in levels or growth rates) were invariably insignificant when added' as regressors to their IS function. Their tests, however, used M2 as the measure of money, whereas Meltzer used the real monetary base. In this section, I report the

[3] Rudebusch and Svensson (1999) report that similar results are obtained using quadratically detrended log GDP as the measure of aggregate demand.

effect of adding lagged real monetary base growth to Rudebusch and Svensson's specification. I use the monetary base series constructed by Anderson and Rasche (2000). Anderson and Rasche's series is the Federal Reserve Bank of St Louis' monetary base series (which makes allowance for the impact of changes in reserve requirements), but, importantly, they adjust the series to exclude the estimated amount of currency held outside the United States.[(4)] I use the fixed-weight GDP deflator to deflate the monetary base series and to compute inflation rates.

In the first regression in Table A, I re-estimate Rudebusch and Svensson's specification over 1961 Q1–1999 Q2, and closely match their results. The second regression adds four lags of real quarterly monetary base growth ($\Delta(m{-}p)$). The sum of these terms is sizeable (0.34, implying a long-run coefficient of 3.40), and statistically significant (with a t-ratio of 4.09).[(5)] A formal test rejects the exclusion of the money terms from the equation ($F(4, 146) = 5.45$ [p value = 0.00]). The real funds rate continues to enter with a significantly negative coefficient, so the money terms evidently contain explanatory power largely separate from that in the funds rate.

The first two regressions in Table A can both be regarded as restrictions on a regression of $\tilde{y}_t$ on lags 1-4 of itself, R, $\Delta(m{-}p)$, and $\Delta_4 p$. The restrictions on this general specification to obtain the first regression in Table A (the Rudebusch-Svensson model) are strongly rejected ($F(13,137) = 3.08$ [p value = 0.00]). By contrast, the restrictions to obtain the second regression, Rudebusch-Svensson augmented with money, are more acceptable, with a test statistic of $F(9,137) = 1.90$ [p value = 0.06].

The velocities of narrow US monetary aggregates such as the monetary base underwent a major shift around early 1982. As Anderson and Rasche (2000) observe, this shift changed domestic base velocity from a series well described as a random walk with drift, to a stationary series. This is

(4) Anderson and Rasche's series covers January 1965–August 1999; I use quarterly averages of the monthly data. I spliced this into the St Louis Adjusted Monetary Base series to generate pre-1965 observations.

(5) Following Rudebusch and Svensson, I treat the dependent variable (the CBO output gap series) as an $I(0)$ time series, in which case standard t and F tests are valid. The true output gap series should certainly be $I(0)$; the danger that the empirical gap series could be $I(1)$ arises mainly from the possibility that it is obtained largely by deterministic detrending of a series, log GDP, that may contain a stochastic trend. Table D below addresses the problem by presenting results for the growth rate of GDP.

visually apparent in Chart 1, which plots base velocity for the United States (the log of the ratio of quarterly nominal GDP to the quarterly average of the Anderson-Rasche series). An important question is whether the significant money coefficients in the Table A regressions are due merely to the inclusion of pre-1982 observations. To test this, the final regression in Table A re-estimates the specification over the sample period 1982 Q1–1999 Q2. This does not change inferences about the importance of real money base growth; it continues to enter significantly at lag four, and the coefficient sum on money growth is on the borderline of significance. This coefficient sum is estimated to be 0.23 (compared to 0.34 for the full sample) and implies a long-run coefficient on real money growth of 3.05 (close to the full-sample estimate of 3.40). If anything, the estimates based on the more recent sample cast doubt on the significance of the real interest rate term, rather than that of real monetary base growth; the coefficient on the real funds rate is now positive and insignificant. Overall, then, there are empirical grounds for adding real monetary base growth to Rudebusch and Svensson's baseline specification.

I now turn to evidence for the United Kingdom. Table B provides estimates for the United Kingdom of a backward-looking IS specification. The dependent variable is detrended output (denoted y_t), defined as the deviations of log seasonally adjusted real UK GDP from a quadratic trend, where the trend is estimated over 1961 Q1–1999 Q2. The right-hand-side variables in these equations are lags of the dependent variable, real monetary base growth (the log-difference of $M0_t$ / P_t, where $M0_t$ is the quarterly average of seasonally adjusted base money and P_t is the quarterly average of the RPIX index), and a real interest rate variable.[(6)] I follow Rudebusch and Svensson by measuring the real interest rate as $(\Sigma_{j=0}^{3} R_{t-j}) - \Delta_4 p_t$, a smoothed version of the 'pseudo-real' interest rate (in the terminology of Svensson (1999)). Here, R_t is the quarterly average of the nominal Treasury bill rate (expressed as a quarterly fraction). Four lags of each regressor and of the dependent variable are included.

(6) The official version of the $M0$ series begins in July 1969; pre-1969 observations were spliced in using quarterly averages of the data in Capie and Webber (1985). The RPIX series is not seasonally adjusted. Therefore, the $\Delta(m-p)_t$ series used in Tables B and C is defined as the residuals from a regression of $\Delta \log (M0_t / RPIX_t)$ on seasonal dummies, with the mean of the unadjusted series restored.

The first regression in Table B does not include real money growth terms. In this case, the estimated sum of coefficients on the real interest rate has a negative sign, as in the US regression for the same sample period, but in contrast to the US case, this sum is not statistically significant. Correspondingly, the long-run coefficient on the real interest rate, –0.155, has a relatively large standard error of 0.270.

Adding four lags of real base growth (the second regression in Table B) improves the fit of the regression significantly: the F-statistic testing the exclusion of these terms is $F(4, 141) = 2.71$ [p value = 0.03]. Furthermore, the sum of coefficients on real money growth is significantly positive, and implies a long-run coefficient of 3.42 (s.e. 1.21), similar to that found for the United States. In addition, the real interest rate terms move toward statistical significance: the long-run coefficient on the real interest rate is –0.39 (0.21). The growth rate of real base money is therefore a significant variable for explaining the behaviour of detrended UK output, and this is the case even if one controls for the effect of real interest rates on GDP.

The third regression in Table B follows Table A by removing pre-1982 observations from the sample period.(7) Just as I found for the United States, this has the effect of making the interest rate terms positively signed and insignificant, while the significance of the money terms is maintained.(8) The regressions in Table B do not allow open-economy factors to exert separate effects on aggregate demand. To relax this restriction, in Table C I add four lags of the UK real exchange rate to the specification. This series (whose log I denote q_t) only starts in 1979, so the sample period for the regression in Table C is 1980 Q1–1999 Q2. The real exchange rate is measured in units such that a fall implies a depreciation.

(7) In contrast to the US regressions, where the non-normality of the estimated residuals disappeared once pre-1982 observations were dropped, Table B indicates that residual non-normality is present in the UK regression even when the estimation period starts in 1982 Q1. However, this non-normality is due to a single outlier—a large negative residual in 1984 Q2 associated with the peak effect of the 1984–85 coal-mining strike. If the last regression in Table B is re-estimated with a dummy variable for this observation, the coefficient sum on the money terms is 0.322 (s.e. 0.118), their estimated long-run effect is 4.015 (s.e. 1.261), and a test for their exclusion yields $F(4,56) = 3.68$ [p value = 0.009]. The p-value for the χ^2 test of non-normality is only 0.296 in this case.

(8) Monetary base velocity in the United Kingdom has undergone a similar trend-break to that observed in Chart 1 for the United States, but the break occurred in the 1990s rather than the 1980s. If I include only 1990s observations in the UK regression, the money terms retain significance and a positive coefficient sum, while the sum of the interest rate coefficients is positive.

The exchange rate terms are insignificant in Table C and their sum is very close to zero. In addition, the coefficient sum on the real interest rate is of the 'wrong' (positive) sign—indeed, significantly positive.

By contrast, the money growth coefficients are more interpretable, as their sum is positive and (in the restricted regression) statistically significant. The implied long-run coefficient on money growth is 3.84.

I now report two robustness checks. First, as a check on the sensitivity of the results to the procedure used to filter output, Table D presents regressions for the United States and the United Kingdom in which the dependent variable and its lags are replaced by Δy_t, the first difference of log real GDP. As the table indicates, direct effects of base money remain positive and highly significant under this alternative specification.

Secondly, it is of interest to see how sensitive are the results to the exclusion of contemporaneous real money growth from the estimated equations. If $\Delta(m\text{–}p)_t$ is added to the main US regression (the second equation in Table A), then it is insignificant (coefficient 0.077, s.e. 0.097). If, however, $\Delta(m\text{–}p)_t$ is added to the UK regression (the second equation in Table B), it is significant (coefficient 0.235, s.e. 0.058). But if one re-estimates this equation using instrumental variables (with $\Delta(m\text{–}p)_{t-5}$ and with $\Delta(m\text{–}p)_{t-6}$ serving as additional instruments), then the coefficient on $\Delta(m\text{–}p)_t$ becomes wrongly signed and insignificant (–0.333, s.e. 0.603). The coefficients on lags of real money growth, on the other hand, are little affected by the inclusion of $\Delta(m\text{–}p)_t$ or by the estimation procedure. Therefore, I focus on the regressions above that exclude contemporaneous real money growth.

The common feature of the regressions in this section is that for the United States and the United Kingdom, real money growth enters output regressions sizeably, positively, and significantly. The real interest rate generally enters with a negative sign, though both the sign and the significance of the real interest rate terms appear to be less consistent across sub-samples than those of the money growth terms.

How can a theoretically rigorous macroeconomic model rationalise these findings? The next section discusses some of what theory has to say about the direct money channel.

3. The direct money channel: theoretical issues

This section discusses alternative theoretical rationalisations for the appearance of separate money terms in aggregate demand functions.

Non-separability in utility: The optimising IS function used by Kerr and King (1996), Rotemberg and Woodford (1997), and McCallum and Nelson (1999a) can be obtained from the household's first-order condition for consumption in a dynamic stochastic general equilibrium model. This condition is of the form $u_c(C_t, M_t / P_t) = (1 + r_t)\mathrm{E}_t\, u_c(C_{t+1}, M_{t+1} / P_{t+1})$, where $u(C_t, M_t / P_t)$ is a utility function increasing in consumption (C_t) and real money balances (M_t / P_t), r_t is the real interest rate, and u_c is the marginal utility of consumption. If, as these papers assume, utility is separable in consumption and real balances, then a log-linear approximation of u_c will be a function of log C_t (denoted c_t) alone. Along with certain assumptions about the capital stock, log-linearisation of the first-order condition for consumption delivers the optimising IS equation for log output (y_t):

$$y_t = -b_1 r_t + \mathrm{E}_t y_{t+1}, \qquad \textbf{(1)}$$

with $b_1 > 0$. But if utility is non-separable in consumption and real money holdings, then $\log u_c \approx a_1\, c_t + a_2 \log (M_t / P_t)$ where $a_1 < 0$, $a_2 < 0$, and log-linearisation of the first-order condition for consumption instead leads to an IS equation of the form

$$y_t = -b_1 r_t + b_2[\log (M_t / P_t) - \mathrm{E}_t \log(M_{t+1} / P_{t+1})] + \mathrm{E}_t y_{t+1},\ \ b_2 = a_2/a_1. \qquad \textbf{(2)}$$

A real money term in the IS function can therefore be justified by optimising behaviour. Koenig (1990) found that money balances entered consumption regressions significantly, a result he interpreted as supportive of non-separable utility. Woodford (1999) and McCallum (2000), however, argue that reasonable parameterisations of the utility function lead to very small coefficients on money in the IS equation, and Ireland (2000), using M2 as the measure of money, finds little econometric support in US data for the importance of non-separable utility. In light of their results, non-separability of utility is *not* pursued in this paper.

Money as a superior index of monetary policy effects: An alternative argument for why money terms enter aggregate demand equations is advanced by Meltzer (1999a). Meltzer contends that changes in real monetary base exert effects on real aggregate demand not summarised by the real interest rate on short-term securities. He argues that these results reflect not 'real balance effects'[9] but the fact that there are many real interest rates, implicit and explicit, relevant for economic activity, and that the real interest rate on short-term securities can be an inadequate stand-in for these yields (or relative prices). This argument has also appeared in much of Meltzer's collaborated work with Karl Brunner[10] as well as in Friedman and Schwartz (1963, 1982). Some representative quotes are:

Meltzer (1999b): 'Monetary policy works by changing relative prices. There are many, many such prices. Some economists erroneously believe... monetary policy works only by changing a single short-term interest rate.'

Friedman and Schwartz (1982, pages 57, 58): 'Keynesians regard a change in the quantity of money as affecting in the first instance 'the' interest rate, interpreted as a market rate on a fairly narrow class of financial liabilities... We insist that a far wider range of marketable assets and interest rates must be taken into account… [We] interpret the transmission mechanism in terms of relative price adjustment over a broad area rather than in terms of narrowly defined interest rates.'

These critiques of standard analysis appear to focus on the number of interest rates and relative prices considered, rather than the treatment of money *per se*. But Meltzer (1999c) notes the role that money balances might play under this broad interpretation of the transmission mechanism: he 'views the gap between desired and actual real balances as a measure of the relative price adjustment required to restore full equilibrium'. A measure of monetary conditions based on the real money stock might serve as a better summary of the various changes in yields than a measure based on a specific real interest rate. One reason Meltzer offers for this is that

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(9) Real balance effects here refer to Pigou-Patinkin effects—the presence of real money balances as a wealth term in the consumption function. Such effects are widely agreed to be unlikely to be of importance for moderate changes in the monetary base, because empirically the base is a negligible fraction of total wealth.

(10) See, for example, Brunner and Meltzer (1993) and the papers collected in Brunner and Meltzer (1989).

money demand might, like aggregate demand, be a function of many interest rates, as in Friedman (1956). [11]

The dilemma from a modelling perspective is how to capture, in a small-scale macroeconomic model such as the optimising IS-LM specification described above, the idea that there are many interest rates relevant for aggregate demand. A model that would do full justice to the broad relative price adjustment process envisaged by Brunner and Meltzer and Friedman and Schwartz would incorporate multiple assets and imperfect substitutability between these assets into the optimising IS-LM framework. Such features would break the mechanical relationships that tend to hold between different asset yields in standard optimising models. In these circumstances, the short-term interest rate could become less adequate as an indicator of monetary pressure, and the money stock could provide auxiliary information.

In this paper, I instead make only a single modification to the standard optimising IS-LM model, namely a change to the money demand function. Prior to introducing this modification, it is worth noting that even in the most basic optimising IS-LM framework, the current short-term real interest rate is not the crucial interest rate for aggregate demand. Iterations on the IS function **(1)** produce:

$$\begin{aligned} y_t &= -b_1 r_t + E_t y_{t+1} \\ &= -b_1 r_t - b_1 E_t r_{t+1} + E_t y_{t+2} \\ &= \ldots \\ &= -b_1 r^l_t \end{aligned} \qquad \textbf{(3)}$$

where $r^l_t \equiv E_t \Sigma_{j=0}^{\infty} r_{t+j}$ is a long-term real interest rate, applying the expectations theory of the term structure.[12] Formulation **(3)** of the

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[11] To uncover the effects of these yields on the quantity of money demanded, one would need first to condition on the own rate on money. For the monetary base in the United States and the United Kingdom, this is a trivial exercise since the own rate is constant (zero), in contrast to broader monetary aggregates, on which the own rate fluctuates. For broader measures of money, a less trivial exercise would be required, such as constructing a Divisia version of the aggregate.

[12] The variables r^l_t and r_t are not measured in comparable units, because the former is not divided by the length of the maturity. In Section 4 below, I use a measure of the long-term interest rate that is divided by the maturity—see equation **(24)**—and which can be regarded as a finite-maturity approximation of r^l_t.

forward-looking IS function indicates that, as Rotemberg and Woodford (1997, 1999) stress, it is the long-term real interest rate that matters for aggregate demand.[(13)]

With this result in mind, it is worth returning to the hypothesis advanced by Friedman and Schwartz and Brunner and Meltzer that the short-term nominal interest rate is not the only yield relevant for money demand.[(14)] A special case of this position is the contention that long-term nominal interest rates, rather than (or in addition to) short-term nominal interest rates, enter the money demand function. Meltzer (1963, 1998), for example, shows that the long-run behaviour of US monetary base velocity is accounted for by long-term nominal interest rate behaviour. To apply this finding to the present discussion, suppose we assume a semi-logarithmic long-run money demand function and a partial-adjustment formulation of dynamic adjustment:

$$m_t - p_t = c_1 y_t - c_2 R^l_t + c_3(m_{t-1} - p_{t-1}), \qquad \textbf{(4)}$$

where lower cases denote logs, $c_1 > 0$, $c_2 > 0$, $0 \leq c_3 < 1$, and $R^l_t \equiv E_t \Sigma_{j=0}^{\infty} (\Delta p_{t+j+1} + r_{t+j})$ is the nominal long-term rate. Empirical estimates of partial adjustment money demand specifications typically produce estimates of c_3 near unity. Taylor (1993), for example, finds $c_3 = 0.95$ on US quarterly data.[(15)] If we employ—temporarily and for ease of exposition—the approximation $c_3 \approx 1.0$ for the coefficient on the lagged money term in equation **(4)**, and use equation **(3)**, the result is:

$$\Delta(m - p)_t \approx -b_1 c_1 r^l_t - c_2 R^l_t \, . \qquad \textbf{(5)}$$

The change in real money balances then depends negatively on both the real and the nominal long-term interest rate. If inflation persistence makes r^l_t and R^l_t highly correlated, then $\Delta(m - p)_t$ will be a good indicator of the real

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(13) Rotemberg and Woodford (1999) show that, consequently, policy actions on the short-term nominal interest rate can better control aggregate demand if they are designed to have an effect on the long-term rate in the same direction.
(14) See Friedman (1956), Friedman and Schwartz (1982), and Brunner and Meltzer (1989, 1993), *inter alia*.
(15) In the case of error-correction models of money demand, which permit more general dynamics than partial adjustment models, the parameter that corresponds most closely to c_3 is one plus the coefficient on the error-correction term. This parameter is also typically estimated to be very close to unity. For example, for the quarterly demand for base money in the United Kingdom, Janssen (1998) finds an estimate of this parameter equal to 0.94.

long-term yield r^l_t, which, as we have seen, is the crucial interest rate for aggregate demand in the optimising IS-LM model.

The above example illustrates the general point that, when yields beside the short-term rate enter both the IS and LM relations, it is possible that real money growth might be a valuable summary statistic for these yields, and might therefore contain information about GDP not present in short-term interest rates.[16] The next section demonstrates this point more rigorously in a general equilibrium model.

[16] Below I also contemplate a case where the money stock is the policy instrument, so that money serves a role not just as an indicator of yield behaviour but as a driving force behind those yields.

4. An extended optimising IS-LM model

The model used in this section has many similarities to previous work on sticky-price general equilibrium models, especially King and Watson (1996), Woodford (1996), Ireland (1997), Fuhrer (2000), and McCallum and Nelson (1999b). Apart from the specification of how money enters the utility function, it is identical to the model used by Neiss and Nelson (2001) to investigate the indicator properties of the gap between actual and equilibrium real interest rates. Consequently, the derivations of first-order conditions are for the most part omitted as they can be found in several of the aforementioned papers.

In the model, a typical household makes optimal decisions regarding consumption as well as accumulation of money, capital, and bonds. The household also provides labour—choosing an amount of its time allocation each period to make available for employment—and produces and sells a good, over which it has market power. Production of this good takes place with the function $y_t = A_t K_t^{\alpha} N_t^{1-\alpha}$, $\alpha \in (0,1)$. Here A_t is a technology shock in each household's production function, K_t is the amount of capital the household has accumulated, and N_t is the amount of labour it hires from the labour market in period t. Capital accumulation is subject to the adjustment costs specified in Abel (1983) and introduced to the optimising IS-LM specification by Casares and McCallum (2000). Price-setting decisions follow the Calvo (1983) model of gradual price adjustment.

The household's expected lifetime utility is given by $E_t S_{j=0}^{\infty} b^j u(C_{t+j}, C_{t+j-1}, M_{t+j} / P_{t+j}, M_{t+j-1} / P_{t+j-1}), (1-N_{t+j})),\ b \in (0,1)$, where the instantaneous utility function $u(\bullet)$ is given by

$$u(C_t, C_{t-1}, M_t / P_t, M_{t-1} / P_{t-1}), (1-N_t))$$
$$= \exp(v_t)(s/(s-1))(C_t / C^h_{t-1})^{(\sigma-1)/\sigma} + \exp(w_t)(1-\varepsilon_m)^{-1}(M_t / P_t)^{(1-e_m)} \quad \textbf{(7)}$$
$$+ h_N(1 - N_t) + G(M_t / P_t, M_{t-1} / P_{t-1}),$$

where s, e_m, and h_N are all positive parameters, v_t and w_t are preference shocks, and $h \in [0,1)$. Preferences over consumption in **(7)** exhibit habit formation as specified in Fuhrer (2000).

The last term in equation **(7)** represents portfolio adjustment costs. The cost function $G(\bullet)$ takes the form:

$$G(M_t / P_t, M_{t-1} / P_{t-1}) = d\,\{\exp[c(M_t / P_t)/(M_{t-1} / P_{t-1}) - 1] + \exp[-c(M_t / P_t)/(M_{t-1} / P_{t-1}) - 1] - 2\} \quad \textbf{(8)}$$

where $c > 0$, $d > 0$. This specification is based on that in Christiano and Gust (1999), and has the advantage that costs that are quite small nevertheless have important effects on aggregate dynamics.(17) The only difference between **(8)** and Christiano and Gust's function is that **(8)** is specified in terms of real money, M_t / P_t, rather than nominal money. For a given set of economic conditions, the function implies that households would prefer to maintain the amount of purchasing power they hold in the form of money relatively stable over time.

The first-order condition for money holding, after log-linearising and suppressing constants, is:

$$m_t - p_t + (1/\varepsilon_m)\, j_t + (1/(\varepsilon_m{*}R^{ss}))R_t - h_t + a_m\Delta(m_t - p_t) - ba_m \mathrm{E}_t\, \Delta(m_{t+1} - p_{t+1}) = 0, \quad \textbf{(9)}$$

where $h_t \equiv (1/e_m)\, w_t$ is a money demand shock, j_t is the log marginal utility of consumption, R^{ss} is the steady-state value of R_t, and a_m is a positive parameter that is increasing in c and d.(18) Solving this Euler equation as in Rotemberg (1982, page 1,191) or Hendry (1995, page 258) produces the forward-looking partial adjustment equation:

$$m_t - p_t = (1 - c_3)(1 - \beta c_3)\mathrm{E}_t \mathrm{S}_{j=0}{}^{\infty}(\beta c_3)^j\{(-1/\varepsilon_m)\, \varphi_{t+j} + (-1/(\varepsilon_m{*}R^{ss}))R_{t+j} + \eta_{t+j}\} + c_3(m_{t-1} - p_{t-1}), \quad \textbf{(10)}$$

(17) These authors used this cost function to prolong the effects of monetary shocks in 'limited participation' models. In the present paper, the cost function does not serve that function, since my model does not have limited participation features—households choose their money holdings for period t in period t, not in period t–1.

(18) The relationship between a_m and the model parameters is $a_m = -2(1/e_m)(V_c^{ss})^{1+em}c^2 d > 0$. V_c^{ss} is the steady-state value of quarterly consumption velocity, and steady-state consumption has been normalised to unity. By way of comparison with the values of c and d implied by my calibration, Christiano and Gust set $c = d = 2$.

where $c_3 \in (0,1)$ is a nonlinear function of a_m and b. Aside from the money demand shock term and the forward-looking terms in the scale variable j_t, equation **(10)** is similar in form to equation **(4)**: real money balances depend positively on their lagged value, and negatively on $E_t S_{j=0}^{\infty} (\beta c_3)^j R_{t+j}$, which is a discounted version of the long-term interest rate $R^l_t \equiv E_t S_{j=0}^{\infty} R_{t+j}$. The adjustment costs have had the effect of rendering money demand forward-looking and so making the long rate the relevant opportunity cost variable. [19]

The complete model may be written in log-linearised form as:

$$j_t = g_1 E_t c_{t+1} + g_2 c_t + g_3 c_{t-1} + g_4 v_t \quad \textbf{(11)}$$

$$j_t = E_t\ j_{t+1} + r_t \quad \textbf{(12)}$$

$$m_t - p_t = (1/e_m)\ j_t + a_m \Delta(m-p)_t + \beta a_m E_t\ \Delta(m-p)_{t+1} \quad \textbf{(13)}$$
$$+ (1/(e_m{*}R^{ss}))R_t + \eta_t$$

$$r_t = R_t - E_t \Delta p_{t+1} \quad \textbf{(14)}$$

$$\Delta p_t = b\ E_t \Delta p_{t+1} - a_m m_t \quad \textbf{(15)}$$

$$j_t + y_t - n_t - m_t = 0 \quad \textbf{(16)}$$

$$y_t = a_t + a k_t + (1-a) n_t \quad \textbf{(17)}$$

$$y_t = s_c c_t + (1 - s_c) x_t \quad \textbf{(18)}$$

$$d x_t = k_{t+1} - (1-d)\ k_t \quad \textbf{(19)}$$

$$k_1\ E_t(y_{t+1} - k_{t+1} - m_{t+1}) + (1-d)\kappa_2\ E_t x_{t+1} = k_2\ x_t + r_t, \quad \textbf{(20)}$$

where $g_1 = -bh(s-1)/(s(1-bh))$, $g_2 = (bh^2 s + bh\,s - bh^2 - 1)/(s(1-bh))$, $g_3 = -g_1/b$, $g_4 = ((1-bh\,r_v)/(1-bh))$, and r_v is the AR(1) parameter for the IS shock process (v_t). All variables except interest rates are in logs, and constant terms are suppressed. Equation **(11)** defines marginal utility (j_t) when the utility function is given by **(7)**. Equation **(12)** is the standard Euler equation connecting marginal utility to its expected future value and

[19] Hendry (1995) discusses work by himself and others that finds evidence against forward-looking behaviour in money demand functions. But if valid, this critique would preclude the terms in $E_t \varphi_{t+j}$ from appearing in equation **(10)**; it would not rule out the long-term interest rate as an important factor affecting money demand, which is the crucial feature of **(10)** for this paper.

the real interest rate. Equation **(13)** is the forward-looking money demand function, and equation **(14)** defines the real interest rate. Equation **(15)** is the Calvo price-setting equation, with the forcing process for inflation expressed in terms of the log mark-up, m_t. Equation **(16)** is obtained by combining the first-order conditions for labour demand and supply in the model.

Equations **(17)** and **(18)** are log-linearisations of the production function and resource constraint (with s_c denoting the steady-state consumption-income ratio, C^{ss}/Y^{ss}). Equations **(19)** and **(20)** pertain to capital accumulation—**(19)** being the law of motion for capital and **(20)** a condition for optimal capital choice. The underlying model of capital accumulation is one where capital adjustment costs each period take the form $q_K(x_t)^2$, $q_K > 0$. x_t is 'quasi-investment', so-called because (as **(19)** indicates) it would be identical to investment if there were no capital adjustment. Here $k_1 = (1-a)(Y^{ss}/K^{ss})$, $k_2 = 2q_K(X^{ss})$. I set the capital adjustment cost parameter q_K to 0.25, capital's share of income (a) to 0.36, and the depreciation rate d to 0.025.

For the monetary policy rule, I use estimated interest rate equations that are intended to describe US or UK monetary policy behaviour. To represent US monetary policy, I use Rotemberg and Woodford's (1997) specification, which I re-estimate on an updated sample period (1980 Q1–1999 Q4):[(20)]

$$R_t = \underset{(0.115)}{-0.008}\,\Delta p_t + \underset{(0.135)}{0.117}\,\Delta p_{t-1} + \underset{(0.129)}{0.516}\,\Delta p_{t-2} + \underset{(0.038)}{0.147}\,y_t + \underset{(0.066)}{0.008}\,y_{t-1} - \underset{(0.040)}{0.112}\,y_{t-2}$$

$$+ \underset{(0.104)}{0.536}\,R_{t-1} - \underset{(0.127)}{0.058}\,R_{t-2} + \underset{(0.094)}{0.198}\,R_{t-3} + e_{Rt}, \qquad \textbf{(21)}$$

$R^2 = 0.951$,

where y_t is detrended output (the deviations of US log real GDP from its 1980 Q1–1999 Q4 linear trend).

(20) In addition to the coefficient estimates reported, this equation has an estimated intercept of 0.0005 (s.e. 0.0006).

For the United Kingdom, the approximation that the monetary policy regime was constant from 1980 to 1999 is less tenable than it is for the United States, so I do not use a policy rule estimated on that entire sample. Instead, I use the following policy rule, estimated in Nelson (2000) for the United Kingdom for 1992 Q4–1997 Q1:

$$R_t = 0.29R_{t-1} + 0.90(\tfrac{1}{4}E_t \Delta_4 p_{t+1}) + 0.08y_t + e_{Rt}. \qquad \textbf{(22)}$$

I solve the model subject to either the US policy rule **(21)** or the UK policy rule **(22)**.

To represent longer-term securities, 'ten-year bond rates', both nominal and real, are introduced into the model. These rates are simply defined using the expectations theory of the term structure as:

$$R^l_{10t} = (1/40)E_t\Sigma_{i=0}^{39} R_{t+i} \qquad \textbf{(23)}$$

for the nominal bond rate, and

$$r^l_{10t} = (1/40)E_t\Sigma_{i=0}^{39} r_{t+i} \qquad \textbf{(24)}$$

for the real bond rate.(21) In the calculation of moments from the above model, these series will proxy for the infinite-maturity long-term rates (r^l_t and R^l_t) that appear in **(3)** and **(9)**.

For the quantitative experiments, a value of the intertemporal elasticity of substitution in consumption, σ in **(7)**, of 0.2 is chosen, while the habit persistence parameter h is set to 0.8. Both these values are close to Fuhrer's (2000) estimates. The steady-state fraction of time devoted to work is 1/3. A unitary long-run consumption elasticity of money demand is imposed, which implies e_m = 5 in **(7)**.(22)

(21) These equations (as well as analogous relationships for securities at other maturities) can be derived from the current model by including markets for these securities explicitly. See Sargent (1987, pages 102–105) for details. His derivation also makes it clear that the linearisations implicit in definitions **(23)** and **(24)** suppress the covariance terms that appear in the exact (nonlinear) expectations theory of the term structure.

(22) In the no habit persistence, no portfolio adjustment cost case, this would imply a steady-state value of the short-term interest rate elasticity of money demand of –0.2.

The value of the money demand adjustment cost term a_m in **(9)** should be consistent with empirical studies of money demand. As noted earlier, these studies suggest that the lagged dependent variable coefficient (c_3 in equation **(10)**) is close to unity. I choose a value of a_m = 10. This choice implies a value of c_3 = 0.7 in **(10)**, which is on the low side of estimates of lagged dependent variable coefficients in money demand studies using quarterly data.[(23)] A value of a_m = 10 is therefore a relatively conservative choice given the available empirical evidence.[(24)] The parameter values in the cost function **(8)** associated with a_m = 10 are approximately $c = d$ = 0.43.[(25)]

The coefficient a_m in the Calvo price-setting equation **(15)** is set to 0.086, following Bernanke and Gertler (1999); this value is consistent with 25% of firms changing price each quarter.

The IS shock v_t is assumed to be an AR(1) process with AR coefficient 0.33 and innovation standard deviation 0.01, in line with estimates in McCallum and Nelson (1999b). I assume that the technology shock a_t is AR(1) with coefficient 0.95 and innovation standard deviation 0.007, and that the money demand shock h_t is white noise with standard deviation 0.01. For both policy rules, I assume a shock standard deviation of 0.2%, which is the estimated residual standard deviation from the estimated US policy rule **(21)**.[(26)] I assume that all shocks are mutually orthogonal.

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(23) These estimates have typically not used the forward-looking money demand equation **(10)**, but instead have come from regressions in which no expectational terms appear. These estimates may therefore be more comparable to the *decision rule* for $(m_t - p_t)$ that comes out of the present model than to equation **(10)**. A value of a_m = 10 remains an appropriate choice in that case, for it is also consistent with delivering a value of about 0.7 on $(m_{t-1} - p_{t-1})$ in the decision rule for $(m_t - p_t)$.

(24) Goodfriend (1985) argues that measurement error tends to bias the estimated lagged dependent variable coefficient in money demand studies toward unity. This is an additional reason for a conservative choice of a_m. Goodfriend's argument does not preclude the existence of substantial portfolio adjustment costs and, indeed, Goodfriend (2000) argues in favour of an 'important role played by portfolio adjustment costs' in the analysis of monetary policy.

(25) These are the values of c and d consistent with a_m = 10 when the steady-state ratio of nominal consumption to the monetary base is 4.75 (its approximate value in UK data) and R^{ss} is 0.016. By way of comparison, Christiano and Gust (1999) set $c = d = 2$.

(26) The estimated UK policy rule would suggest a smaller standard deviation for the policy shock, but as this rule was estimated on only five years of observations, the estimated standard deviation may be spuriously low. Therefore I use the value suggested by the US estimates.

Table E reports moments from each version of the model, obtained from analytical formulae for the covariance matrix of the model variables (see Hamilton (1994, page 265)). Results for two policy rule settings are presented, the US policy rule **(21)** and the UK policy rule **(22)**. Due to the absence of trends in real variables in the models, the standard deviations of output, consumption, and real balances reported in Table E should be compared with the behaviour of their detrended counterparts in the data.(27)

As the tables indicate, the standard deviations of all variables beside real balances are invariant to the inclusion of portfolio adjustment costs. This reflects the fact that, as in prior work with the optimising IS-LM model in which money does not appear in the policy rule, the money stock is not a state variable, so the paths of all other variables may be obtained without reference to money.

The model reproduces the persistence of inflation and detrended output quite well.(28) The inflation persistence in the models implies that nominal short and long-term rates are quite strongly correlated with their corresponding real rates. In addition, short and long rates are strongly though not perfectly correlated. And, of the two interest rates, aggregate demand seems more tightly related with the long-term rate; $Corr(y_t, r_t)$ is much less negative than $Corr(y_t, r^l_{10t})$.(29) This reflects the forward-looking nature of aggregate demand.

The moments reported involving the growth rate of real balances, $\Delta(m-p)_t$, are sensitive to the inclusion of portfolio adjustment costs. Without portfolio adjustment costs, the real money growth rate is more closely

(27) For 1961 Q1–1999 Q2, the empirical standard deviations of quadratically detrended GDP, private consumption expenditure on non-durables, and the real monetary base are:

	US	*UK*
GDP	2.83%	3.00%
Consumption	1.74%	3.07%
Real balances	8.07%	7.16%

The standard deviations of annualised quarterly inflation and nominal short-term interest rates are 2.44 and 3.23 percentage points for the United States; 5.47 and 3.15 percentage points for the United Kingdom.

(28) The first-order serial correlation of quarterly inflation is 0.90 for the United States, 0.76 for the United Kingdom; of quadratically detrended output, 0.93 for the United States, 0.94 for the United Kingdom.

(29) That this is true in an economy without capital largely follows from equation **(3)**. In the models of Table E, which do have capital formation, quasi-investment depends negatively on both the short-term and the long-term interest rate, though more strongly on the latter.

correlated with the short-term nominal interest rate than with the long-term interest rate. This is especially so for the results using the US policy rule.

Including portfolio adjustment costs changes the picture sharply. Real money demand now depends on the long-term expectation of short rates. Consequently, the growth rate of real balances now has a stronger correlation with the nominal long-term interest rate (correlations of –0.31 or –0.35, depending on the policy rule) than with the nominal short-term rate (correlations of –0.30 or –0.26). Furthermore, in the no-portfolio adjustment costs cases, $\Delta(m - p)_t$ was negatively correlated with both the short real rate and the long real rate; now the correlation with the long real rate becomes more negative, but the correlation with the real short rate turns positive.(30) All in all, portfolio adjustment costs transform real money balances into a much better indicator of long-term interest rates, both nominal and real, than of short-term rates.

Since, in this model, the real long rate is the key determinant of aggregate demand, it is of interest to see what effects portfolio adjustment costs have on regressions estimated using data for these economies. I now estimate two ‘backward-looking IS equations’ on data generated from the model. These regressions are simplified analogues of the specifications estimated in Section 2 above:

‘Regression 1’:

$$y_t = b_0 + b_1 y_{t-1} + b_2 4^* r_t.$$

‘Regression 2’:

$$y_t = b_0 + b_1 y_{t-1} + b_2 4^* r_t + b_3 \Delta(m - p)_{t-1}.$$

‘Regression 1’ is a backward-looking regression for output with the short-term real interest rate as the explanatory variable, in the spirit of Rudebusch and Svensson (1999, 2000). ‘Regression 2’ adds the lagged change in real balances, $\Delta(m - p)_{t-1}$, as a regressor. This term will be

(30) Consistent with these results, the partial correlation between the real long-term rate and real money growth, given the real short rate, becomes more strongly negative when portfolio adjustment costs are included: from +0.00 to –0.17 when the US policy rule is used, and from –0.06 to –0.27 when the UK policy rule is used.

statistically significant only if the prior change in real balances contains information about next period's output not already present in lagged output and current short-term real interest rates.

For each version of the model, these regressions were estimated on each of 100 simulation runs, and the averages of the resulting parameter estimates (along with averages of their standard errors) are reported in Table F (using the US policy rule) and Table G (using the UK policy rule).

Consider first the results with no portfolio adjustment costs. Qualitatively, the estimates of 'Regression 1' in Tables F and G tell a similar story: the short-term real interest rate enters the regression with a significantly negative sign. The estimates of 'Regression 2' with no portfolio adjustment costs indicate that lagged real money growth provides no explanatory power when the UK policy rule is used, and has a perverse (negative) sign under the US policy rule.

Addition of portfolio adjustment costs to the model cannot change the results of 'Regression 1'. But for 'Regression 2', these costs greatly improve the explanatory power of real balances: the coefficient on real money growth becomes strongly significant with a coefficient of 0.547 (s.e. 0.149) under the US policy rule, and of 0.245 (s.e. 0.039) under the UK policy rule. The short-term real rates remain statistically significant (and negative) in these regressions, so money growth is supplying auxiliary information separate from that in the short rate.[31]

An interesting feature of the 'Regression 2' results under adjustment costs is that the 'long-run coefficient' on real money growth on aggregate demand is found to be 1.99 under the UK policy rule, 4.36 under the US policy rule. These values bracket the estimated long-run coefficient of around 3.4 found to hold empirically in Section 2 for both the United States and the United Kingdom. The results in this section therefore provide a rationalisation for the significance of the money growth terms in those output regressions—namely, the relevance of the long-term interest rate for both money and aggregate demand.

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[31] If one replaces r_t in these regressions with r_{t-1}, the coefficients on lagged real money growth remain positive and significant, but now the interest rate coefficient is generally positive.

In the present model, conditioning on the real long rate would be sufficient to remove the incremental information contained in money growth about economic activity. But it is conceivable that in more general cases, where many yields enter both the aggregate demand and money demand functions, the information in money about aggregate demand would be beyond that contained in securities market interest rates, both short-term and long-term.

Finally, one respect in which the exercises in this section are not directly comparable to the discussions in Brunner and Meltzer (1989, 1993) and Friedman and Schwartz (1963, 1982), is that my model does not allow for exogenous money supply shocks. The model is one where the nominal interest rate is the policy instrument and money is endogenous; when short- or long-term interest rates change, the nominal quantity of money tends to adjust in response. By contrast, the authors whose work motivated the discussion in Section 3 were largely concerned with the adjustment of interest rates, prices, and output to exogenous changes in money. I have examined a case closer to the one that these authors contemplated by re-solving the model subject to the exogenous money rule,

$$\Delta m_t = 0.5\Delta m_{t-1} + e_{Mt}, \quad E_t e_{Mt+1} = 0. \qquad \textbf{(25)}$$

Impulse response functions for real interest rates and output, in response to a unit increase in e_{Mt} in period 1, are presented in Chart 2. Interest rate responses are expressed in annualised units. The chart makes it clear that portfolio adjustment costs magnify the impact of injections of money on real interest rates (both short and long-term) and on aggregate demand.

5. Conclusion

In this paper, I presented empirical evidence for the United States and the United Kingdom that reaffirmed and extended Meltzer's (1999a) evidence that real money base growth matters for real economic activity, for a given short-term real interest rate. Standard optimising models provide little rationale for this finding. But they can provide a rationale if the money demand function is generalised to include extra yields. The model used in this paper incorporates several features stressed in Friedman and Schwartz's and Brunner and Meltzer's work: (i) Yields beside the short-term interest rate enter both the aggregate demand and the money demand function.

(ii) Consequently, the money stock provides information about economic activity not present in short-term real interest rates. (iii) The more general specification of money demand also increases the effect on aggregate demand of shocks to monetary growth.

Further work could extend the optimising IS-LM model to incorporate many of the features emphasised in Brunner and Meltzer's (1989, 1993) models, including more financial assets and the distinction between permanent and transitory shocks. It would be interesting to see if these features further improve the value of the money stock, *vis a vis* money market interest rates, as an indicator of monetary conditions.[32]

There are three issues closely related to the subject matter of this paper which I have not covered. One is the issue of whether monetary policy can significantly affect aggregate demand when nominal short-term interest rates are (near) zero. Meltzer (1999c) argues that monetary base expansion can stimulate the economy even at near-zero interest rates, ie there is no 'liquidity trap'. He argues that the key reason why base expansion can stimulate activity is that short-term nominal securities are not the only substitute for money. Further work is needed on the optimising IS-LM model to explore the case where nominal interest rates are zero, and money demand functions are closer to being as general as those envisaged by Meltzer. An ambitious element of this work would be to model the relationship between monetary policy changes and risk premia. Rudebusch and Svensson (1999), Meltzer (1999a), and McCallum (2000) argue that monetary authorities can use the monetary base to engineer exchange rate depreciation even when short-term nominal interest rates are zero. Implicitly, they view the risk premium in the uncovered interest parity relationship as subject to monetary policy manipulation in the short run. As King (1999) puts it, 'A full explanation of the transmission mechanism of monetary policy at zero interest rates will require a general equilibrium theory of risk premia and how those risk premia are affected by monetary policy.'

[32] This is the implication of Friedman and Schwartz's (1963, 1982) and Brunner and Meltzer's discussions. By contrast, Tobin (1974, pages 88–89) expresses scepticism that enriching the asset structure of standard IS-LM models would make the money stock a more central indicator.

A second issue not explored in this paper is the possible presence of money terms in the aggregate supply equations of a macroeconomic model. In an econometric study of euro-area inflation, Gerlach and Svensson (1999) find that real money stock terms enter very significantly. I have focused on explaining the presence of money terms in output equations, given the short-term interest rate; but it is important also to investigate whether the presence of money in inflation equations, given the output gap, can be rigorously justified.

Finally, I have not explored the relationship between monetary growth and the difference between actual and equilibrium real interest rates. Woodford (1999) argues that the spread between the actual and natural real interest rate provides an alternative, neo-Wicksellian basis for the analysis of price level determination, in place of analysing the process of money supply and demand interaction. But from a different perspective, a framework where the 'real interest rate gap' matters might enhance the value of the monetary base as an indicator. For suppose that the natural real rate rose while actual nominal and real interest rates remained unchanged. To offset the likely upward pressure on nominal interest rates arising from the shock that raised the natural rate, the central bank creates more base money. Judged by actual real and nominal interest rates, policy does not appear to have eased; but judged by the 'real interest rate gap', it has eased. The increase in base money also serves as a useful indicator of the easing in monetary policy. Investigation of this conjecture could form part of future work on the role of money in optimising models of the business cycle.

Table A: Output gap regressions: United States

Dependent variable: $\tilde{y}_t$ (CBO output gap series)			
	Sample period		
	1961 Q1–1999 Q2	1961 Q1–1999 Q2	1982 Q1–1999 Q2
Constant	0.002 (0.001)	0.001 (0.001)	–0.002 (0.002)
$\tilde{y}_{t-1}$	1.176 (0.076)	1.079 (0.077)	1.342 (0.098)
$\tilde{y}_{t-2}$	–0.265 (0.075)	–0.178 (0.076)	–0.416 (0.099)
$(S_{j=0}^{3} R_{t-1-j}) - \Delta_4 p_{t-1}$	–0.075 (0.029)	–0.095 (0.028)	0.013 (0.039)
$\Delta(m-p)_{t-1}$	–	0.216 (0.093)	0.000 (0.091)
$\Delta(m-p)_{t-2}$	–	–0.036 (0.098)	0.121 (0.098)
$\Delta(m-p)_{t-3}$	–	–0.030 (0.097)	–0.219 (0.096)
$\Delta(m-p)_{t-4}$	–	0.186 (0.095)	0.325 (0.081)
Sum of real money growth coefficients	–	0.336 (0.082)	0.227 (0.109)
R^2	0.907	0.919	0.962
SEE	0.791%	0.748%	0.487%
DW	2.10	2.06	2.12
p values for χ^2 tests for:			
Residual serial correlation 1–4	0.016	0.156	0.915
Normality	0.004	0.008	0.246
ARCH(1)	0.318	0.727	0.074

Table B: Regressions for detrended output: United Kingdom

Dependent variable: y_t (quadratically detrended log GDP)			
	Sample period		
	1961 Q1–1999 Q2	1961 Q1–1999 Q2	1982 Q1–1999 Q2
Constant	0.001 (0.001)	0.002 (0.001)	–0.002 (0.003)
y_{t-1}	0.975 (0.083)	0.867 (0.089)	1.181 (0.129)
y_{t-2}	0.002 (0.117)	–0.004 (0.115)	–0.373 (0.189)
y_{t-3}	0.157 (0.114)	0.141 (0.112)	0.455 (0.182)
y_{t-4}	–0.216 (0.082)	–0.115 (0.086)	–0.330 (0.115)
$\Delta(m-p)_{t-1}$	—	0.140 (0.070)	0.200 (0.079)
$\Delta(m-p)_{t-2}$	—	0.108 (0.070)	0.079 (0.084)
$\Delta(m-p)_{t-3}$	—	0.122 (0.067)	–0.031 (0.081)
$\Delta(m-p)_{t-4}$	—	0.011 (0.063)	0.044 (0.077)
Sum of coefficients on real money growth	—	0.381 (0.129)	0.292 (0.129)
$(S_{j=0}^{3} R_{t-1-j}) - \Delta_4 p_{t-1}$	–0.149 (0.084)	–0.187 (0.091)	0.060 (0.121)
$(S_{j=0}^{3} R_{t-2-j}) - \Delta_4 p_{t-2}$	0.280 (0.143)	0.254 (0.154)	0.046 (0.187)
$(S_{j=0}^{3} R_{t-3-j}) - \Delta_4 p_{t-3}$	–0.098 (0.145)	–0.078 (0.155)	–0.116 (0.191)
$(S_{j=0}^{3} R_{t-4-j}) - \Delta_4 p_{t-4}$	–0.050 (0.084)	–0.034 (0.088)	0.053 (0.118)
Sum of coefficients on real rate	–0.013 (0.023)	–0.044 (0.025)	0.043 (0.051)
R^2	0.893	0.901	0.979
SEE	1.013%	0.990 %	0.501%
DW	2.01	1.98	2.03
p values for χ^2 tests for:			
Residual serial correlation 1–4	0.782	0.692	0.825
Normality	0.000	0.000	0.034
ARCH(1)	0.033	0.091	0.554

Table C: Regressions for detrended output: United Kingdom	
Dependent variable: y_t (quadratically detrended log GDP)	
	Sample period 1980 Q1–1999 Q2
Constant	0.031 (0.051)
y_{t-1}	1.067 (0.135)
y_{t-2}	–0.202 (0.207)
y_{t-3}	0.195 (0.190)
y_{t-4}	–0.156 (0.109)
$\Delta(m-p)_{t-1}$	0.147 (0.081)
$\Delta(m-p)_{t-2}$	0.169 (0.083)
$\Delta(m-p)_{t-3}$	0.057 (0.080)
$\Delta(m-p)_{t-4}$	0.001 (0.072)
Sum of coefficients on real money growth	0.374 (0.123)
$(S_{j=0}^{3} R_{t-1-j}) - \Delta_4 p_{t-1}$	0.003 (0.094)
$(S_{j=0}^{3} R_{t-2-j}) - \Delta_4 p_{t-2}$	0.154 (0.141)
$(S_{j=0}^{3} R_{t-3-j}) - \Delta_4 p_{t-3}$	–0.044 (0.134)
$(S_{j=0}^{3} R_{t-4-j}) - \Delta_4 p_{t-4}$	–0.033 (0.085)
Sum of coefficients on real rate	0.079 (0.038)
q_{t-1}	–0.003 (0.022)
q_{t-2}	–0.029 (0.035)
q_{t-3}	0.042 (0.036)
q_{t-4}	–0.003 (0.023)
Sum of coefficients on real exchange rate	0.008 (0.011)
R^2	0.978
SEE	0.542%
DW	1.88
p value for χ^2 tests for:	
Residual serial correlation 1–4	0.429
Normality	0.008
ARCH(1)	0.913

Table D: Regressions for real GDP growth rate		
Dependent variable: Δy_t (log-difference in real GDP)		
	United States 1960 Q3–1999 Q2	United Kingdom 1959 Q4–1999 Q2
Constant	0.007 (0.001)	0.007 (0.001)
Δy_{t-1}	0.165 (0.080)	–0.055 (0.082)
Δy_{t-2}	0.123 (0.079)	–0.041 (0.083)
$(\Sigma_{j=0}^{3} R_{t-1-j}) - \Delta_4 p_{t-1}$	–0.256 (0.131)	–0.116 (0.082)
$(\Sigma_{j=0}^{3} R_{t-2-j}) - \Delta_4 p_{t-2}$	0.179 (0.135)	0.109 (0.080)
Sum of real rate coefficients	–0.077 (0.031)	–0.008 (0.023)
$\Delta(m-p)_{t-1}$	0.280 (0.091)	0.122 (0.067)
$\Delta(m-p)_{t-2}$	–0.056 (0.095)	0.140 (0.062)
Sum of real money growth coefficients	0.224 (0.086)	0.262 (0.085)
R^2	0.217	0.070
SEE	0.80%	1.04%
DW	2.01	2.02
F test for excluding money coefficients	$F(2,149) = 5.69$ [p value = 0.004]	$F(2,152) = 4.84$ [p value = 0.009]
F test for excluding lag 3 of all variables	$F(3,146) = 0.53$ [p value = 0.662]	$F(3,148) = 2.12$ [p value = 0.101]
p values for χ^2 tests for:		
Residual serial correlation 1–4	0.430	0.514
Normality	0.011	0.000
ARCH(1)	0.935	0.228

Table E: Model statistics				
	US policy rule		UK policy rule	
	No portfolio adjustment costs	Portfolio adjustment costs	No portfolio adjustment costs	Portfolio adjustment costs
Standard deviations (%)				
y_t	2.20	2.20	1.98	1.98
c_t	1.48	1.48	1.37	1.37
$4^*\Delta p_t$	1.48	1.48	3.61	3.61
4^*R_t	1.20	1.20	3.55	3.55
$m_t - p_t$	4.33	3.55	11.46	9.98
Correlations				
$Corr(\Delta p_t, \Delta p_{t-1})$	0.76	0.76	0.94	0.94
$Corr(y_t, y_{t-1})$	0.94	0.94	0.95	0.95
$Corr(y_t, r_t)$	–0.27	–0.27	–0.28	–0.28
$Corr(y_t, r^l_{t10})$	–0.93	–0.93	–0.94	–0.94
$Corr(c_t, r_t)$	–0.13	–0.13	–0.20	–0.20
$Corr(c_t, r^l_{10t})$	–0.93	–0.93	–0.93	–0.93
$Corr(R_t, R^l_{10t})$	0.85	0.85	0.97	0.97
$Corr(r_t, r^l_{10t})$	0.27	0.27	0.27	0.27
$Corr(r_t, R_t)$	0.48	0.48	0.28	0.28
$Corr(r^l_{10t}, R^l_{10t})$	0.94	0.94	0.96	0.96
$Corr(R_t, \Delta(m-p)_t)$	–0.26	–0.30	–0.13	–0.26
$Corr(R^l_{10t}, \Delta(m-p)_t)$	–0.06	–0.31	–0.12	–0.35
$Corr(r_t, \Delta(m-p)_t)$	–0.26	0.12	–0.05	0.34
$Corr(r^l_{10t}, \Delta(m-p)_t)$	–0.07	–0.13	–0.07	–0.15

Table F: GDP regressions for model economies with US policy rule

	Regression 1: $y_t = b_0 + b_1 y_{t-1} + b_2 4{*}r_t$		
	Coefficient on:		
	y_{t-1}	$4{*}r_t$	$\Delta(m-p)_{t-1}$
No portfolio adjustment costs	0.901 (0.027)	−0.161 (0.049)	—
Portfolio adjustment costs	0.912 (0.026)	−0.156 (0.048)	—
	Regression 2: $y_t = b_0 + b_1 y_{t-1} + b_2 4{*}r_t + b_3 \Delta(m-p)_{t-1}$		
	Coefficient on:		
	y_{t-1}	$4{*}r_t$	$\Delta(m-p)_{t-1}$
No portfolio adjustment costs	0.909 (0.027)	−0.164 (0.048)	−0.039 (0.018)
Portfolio adjustment costs	0.875 (0.027)	−0.199 (0.048)	0.547 (0.049)

Note: For each version of the model, 100 artificial datasets of 200 observations were generated, and the above regressions run on each dataset. The regression output reported is the average of the relevant parameter estimate and the average of its standard error, across the 100 datasets.

Table G: GDP regressions for model economies with UK policy rule

	Regression 1: $y_t = b_0 + b_1 y_{t-1} + b_2 4{*}r_t$		
	Coefficient on:		
	y_{t-1}	$4{*}r_t$	$\Delta(m-p)_{t-1}$
No portfolio adjustment costs	0.916 (0.026)	–0.093 (0.045)	—
Portfolio adjustment costs	0.928 (0.024)	–0.081 (0.044)	—
	Regression 2: $y_t = b_0 + b_1 y_{t-1} + b_2 4{*}r_t + b_3 \Delta(m-p)_{t-1}$		
	Coefficient on:		
	y_{t-1}	$4{*}r_t$	$\Delta(m-p)_{t-1}$
No portfolio adjustment costs	0.914 (0.027)	–0.093 (0.045)	0.005 (0.012)
Portfolio adjustment costs	0.877 (0.024)	–0.126 (0.042)	0.245 (0.039)

Note: For each version of the model, 100 artificial datasets of 200 observations were generated, and the above regressions run on each dataset. The regression output reported is the average of the relevant parameter estimate and the average of its standard error, across the 100 datasets.

Chart 1: Velocity of US domestic monetary base (logarithm), quarterly

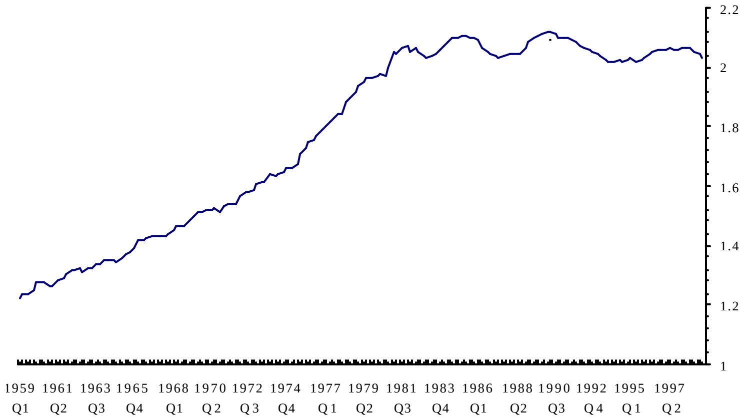
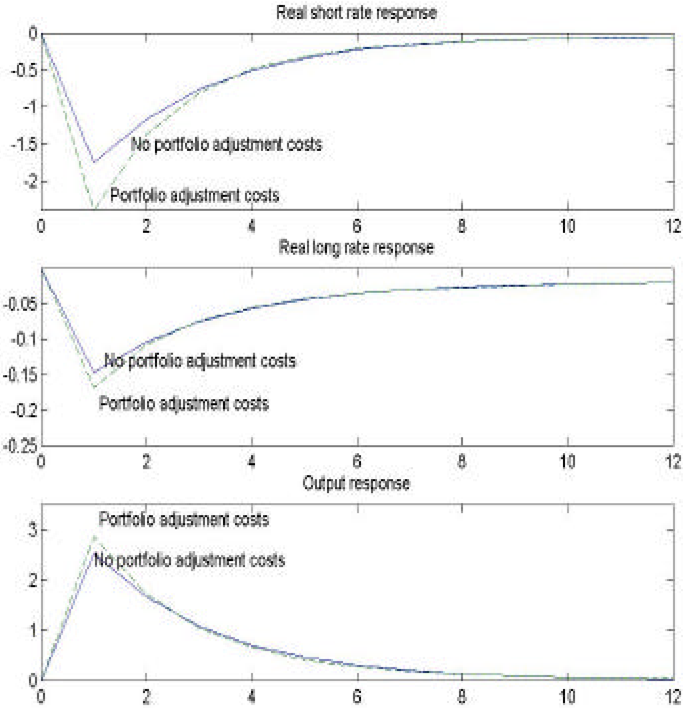


Chart 2: Impulse response functions for money supply shock, rule (25)



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