Imperfect competition and the dynamics of mark-ups

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Abstract

In a dynamic general equilibrium (DGE) model where goods markets are imperfectly competitive, we characterise the dynamics of the mark-up of prices over marginal costs under two different sets of assumptions about market structure. In the customer market model, firms lower their mark-up when current output is low relative to future profits, foregoing current profits in order to capture future market share. In markets characterised by implicit collusion, firms lower their mark-up when current output is high relative to future profits in order to lower the incentives to undercut the implicit cartel. We characterise the dynamics by analysing the response of the mark-up, employment and output to shocks to demand (identified by innovations in government expenditure), TFP growth, and to prices of imported materials. Though the two mark-up models have quantitatively similar properties in terms of the dynamics of output and employment, they differ qualitatively—the implicit collusion model increases the output response to shocks to government expenditure and to the price of imported materials, while the customer market model dampens fluctuations in response to these shocks. Only the customer market model generates mark-up dynamics that conform with the empirical evidence on mark-up pricing in the United Kingdom, such as Small (1997), that finds procyclical mark-ups at the sectoral level, and with the priors embedded in typical macroeconometric models, such as the one used at Bank of England. The empirical evidence is at odds with the evidence for the United States and our theoretical investigation suggests that at a macroeconomic level, the only way to reconcile these two facts is to assume different models of pricing behaviour.

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1 Introduction

This paper addresses the question of how firms set prices in response to shocks in an imperfectly competitive environment. This matters because pricing behaviour affects not only the inflation rate but also the output and employment responses to a given shock. The main contribution of the paper is to develop a theoretical framework in which different assumptions about pricing behaviour and their macroeconomic implications can be analysed, and to examine aggregate UK data within this framework. The model—drawn from a sequence of papers by Rotemberg and Woodford—is a dynamic general equilibrium model of an economy where the output market is characterised as imperfectly competitive. Within this model we accommodate two different assumptions about the dynamics of the firms' mark-up of prices over marginal costs—each with different implications for how firms set prices in response to shocks. In the *customer market* model, firms lower their mark-up when current output is low relative to future profits, foregoing current profits in order to capture future market share. In markets characterised by *implicit collusion*, firms lower their mark-up when current output is high relative to future profits in order to lower the incentives to undercut the implicit cartel. We characterise the dynamics by analysing the response of the endogenous variables—primarily the mark-up, employment and output—to exogenous shocks—shocks to demand shocks identified by innovations in government expenditure, shocks to TFP growth and to prices of imported materials.

The model allows us to make general statements about firms' pricing behaviour in response to different shocks; confronting the model with the UK data allows us to provide a specific interpretation of mark-up behaviour at the aggregate level. This means that the model can be used to address questions such as: what happens to the mark-up component of inflation in response to an oil price shock? How do prices change relative to costs when demand increases? How do prices change relative to costs when productivity increases? The underlying philosophy of this paper is to make sense of the diverse pricing behaviour of UK firms in an optimising, general equilibrium framework, and, by doing so, to contribute to the monetary policy debate.

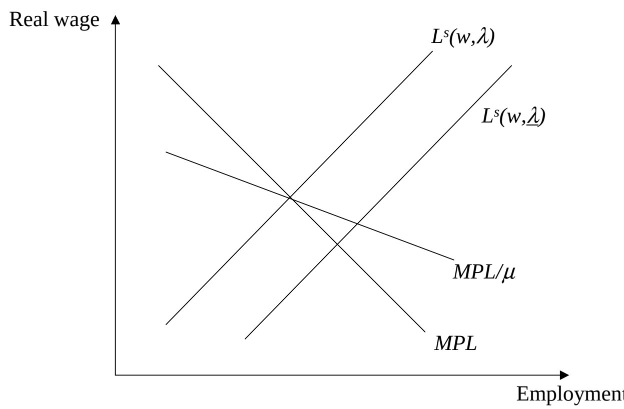
The assumption about pricing behaviour affects output and employment dynamics: implicit collusion increases the output response to shocks to government expenditure and to the price of imported materials, while customer markets dampen fluctuations in response to these shocks. With a model calibrated on UK data, we find that the correlation between output and the mark-up is unambiguously positive for the customer market model and unambiguously negative for the implicit collusion model. This conclusion is insensitive to a wide range of parameter values and to the particular sequence of shocks that characterises the UK economy. Only the customer market model generates mark-up dynamics that conform with the empirical evidence on mark-up pricing in the United Kingdom, such as Small (1997), that find procyclical mark-ups at the sectoral level, and with the priors embedded in typical macroeconometric models fitted on macro data, see for instance Bank of England (1999). The empirical evidence is at odds with the evidence for the United States, see eg Bils (1987) or Rotemberg and Woodford (1991), and our theoretical investigation suggests that at a macroeconomic level, the only way to reconcile these two results is to assume different models of pricing behaviour.

1.1 How does the model work?

The key relationships in the model are the behaviour of labour demand and supply in response to shocks and to variations in the mark-up. Figure 1 provides a static 'snapshot' of these relationships in a real wage/employment space. Labour supply is a function of the real wage, w, and of the marginal utility of wealth, λ. An increase in the marginal utility of wealth, $\underline{\lambda} > \lambda$, implies a shift to the right of the labour supply curve, as workers increase their supply of labour for a given wage to increase income. The labour demand curve is the ratio of the marginal product of labour, MPL, to the mark-up of prices over marginal costs, μ. In a perfectly competitive model $\mu = 1$, while imperfect competition implies that $\mu > 1$.

A positive demand shock, modelled as a surprise increase in government expenditure, increases the marginal utility of wealth and shifts the labour supply curve to the right. The presence of a mark-up *alone* affects the quantitative response to such a shock: a mark-up greater than one effectively reduces the slope of the labour demand curve and hence increases the responsiveness of employment to a change in wages. The fact that the mark-up varies *cyclically* affects the

Figure 1: The effects of procyclical and countercyclical mark-ups



position of the labour demand curve. If the mark-up is countercyclical in response to a demand shock, then the output multiplier is magnified because the labour demand curve shifts to the right when the mark-up falls. A procyclical mark-up will dampen the output response to such a shock. A prior belief that the government expenditure multiplier is large would tend to lead to a prior that the mark-up is countercyclical. On the other hand, macroeconometric models such as those of the Bank of England or the National Institute of Economic and Social Research (NIESR) typically estimate a procyclical mark-up component of inflation. The obvious question is whether these different views can be reconciled within a single model that is subject to shocks other than demand shocks, or whether this in effect requires different models of mark-up dynamics.

We consider two other types of shock in addition to the demand shocks. The first is *technology shocks*. A positive technology shock increases output for a given level of labour and capital input, and increases the marginal product of labour. That shifts the labour demand curve to the right, increasing employment in the short run. In the long run, however, the increase in wealth will reduce the marginal utility of wealth, leading to a fall in labour supply—technology shocks have no long-run effect on employment.

Technology shocks are modelled as permanent, reflecting our prior that such innovations are irreversible. Innovations in technology have a positive mean, so while shocks relative to the mean are frequently negative, the innovations are rarely negative in an absolute sense. The issue of whether shocks to technology are permanent or transitory is important for the dynamic properties of the model because output dynamics under the two assumptions differ substantially.[1] The issue is further complicated by the fact that the technology shock in the model will act as a 'catch-all' when measured in data, and may include features that are not explicitly modelled, such as labour hoarding or cyclical capital utilisation. Neither of these features is specific to this model and both would also apply to a standard real business cycle model, such as Hansen (1985).

The second shock we consider in addition to demand shocks is to the *price of imported materials*, which is assumed to be exogenous and stationary. A positive shock to the price of imported materials causes a shift in the mix of factor inputs away from imported materials—for a given level of output, demand for other factors, including labour, increases. But a fall in aggregate output dominates this substitution effect, so that labour demand decreases.

Despite this shock, the model is essentially a closed-economy model and the resources used to paid for these materials are simply deducted from the resource constraint. Firms are obliged to pay for their imports on a period-by-period basis, so trade is balanced in all periods and the current account is always zero. Open-economy considerations are important for policy-making, but are by no means straightforward when discussing an issue like this—it is not obvious how to model pricing decisions of foreign producers in the domestic economy, or to model the effects of the pricing decisions of domestic producers when trading abroad. This further highlights the fact that the shock structure may be inefficient; with a closed economy we effectively disregard the effects of international demand shocks (however these may be identified) on domestic price-setting.

(1) For a discussion of transitory vs permanent shocks, see Ravn (1997) or, in a UK context, Holland and Scott (1998).

Imperfect competition in product markets raises a number of measurement and calibration issues. In addition to providing a careful calibration of parameters that are standard in the dynamic general equilibrium literature, we also provide estimates of the production function and output elasticities. In terms of analysing the shocks, our main contributions are to provide different measures of TFP shocks that are compatible with imperfect competition and a more extensive description of the stochastic processes governing the real price of imported materials. The TFP measures include a measure based on the dual approach where TFP is derived from the cost function, following Roeger (1995). We model the innovations in the real price of imported materials as innovations to the nominal price of imported materials, allowing empirically for interaction between the domestic price level and the nominal price of imports.

The remainder of the paper is organised as follows. The next section describes the model, focusing on a comparison between the different market structures. This is followed by an extensive description of the calibration—to establish the model as a credible instrument, we need as precise a measurement of the parameters in the model as possible. The fourth section provides qualitative and quantitative answers to the questions posed in this introduction. It also includes a general discussion of the model's dynamic properties; an understanding of the model dynamics and of the model's sensitivity to parameter changes are important when the model is used for practical analysis. This is followed by a brief conclusion.

2 The model

The model is based on a standard dynamic general equilibrium business cycle model with the main changes being in production functions and market structures. Accordingly, the description of the model in this section concentrates these two elements of the model. This is followed by a brief outline of the demand side of the model and a summary of equilibrium conditions.

2.1 Technology

The economy consists of many symmetric firms, indexed by i, organised in I_t industries, indexed by j, with n firms in each industry. Assume, for now, that there is one firm in each industry, ie $n = 1$. Firms produce differentiated goods by operating identical production technologies, characterised by a production function Y,

$$Y_t^i = Y\left(V_t^i, Q_t^i, \phi\right) = F\left(V_t^i, Q_t^i\right) - \phi \tag{1}$$

where Y_t^i is firm i's gross output, while V_t^i and Q_t^i are input indices and ϕ is a fixed cost of production, exogenous to the firm. The index Q_t^i is a materials index that aggregates inputs of materials, $Q_t^i = Q\left(E_t^i, M_t^i\right)$, where E_t^i is input of imported materials and M_t^i denotes input of domestically produced materials. The index V_t^i is a function of capital and labour inputs $V_t^i = V\left(K_t^i, z_t H_t^i\right)$, where K_t^i and H_t^i are capital and labour inputs, with labour being measured in total hours, and z_t is labour-augmenting technological progress, identical for all firms. Technological progress is assumed to be stochastic, and is both a source of fluctuations at the business cycle frequency and of trend growth in output. Finally, the function F aggregates Q and V into gross output. These three aggregators are assumed to have constant returns to scale, and to be increasing and concave in inputs. The advantage of operating with three aggregators is that it restricts the number of parameters in the production function. In the calibration exercise, this limits the estimation requirements to three elasticities of substitution rather than a full set of six. For similar reasons, gross output is measured after the fixed costs have been incurred—in effect, the notation in (**1**) treats fixed costs as a 'quasi input'. With this assumption, we can construct a measure of total factor productivity growth, and in addition can tie down the fixed cost in the calibration exercise in a straightforward manner by restricting steady-state profits.

Although firms produce differentiated goods, all final demand is in terms of a single composite good. The aggregator that turns the differentiated goods into a composite is increasing, concave and homogeneous of degree one in inputs. In addition, it is symmetric in the sense that the quantity of final goods depends on the distribution of individual goods, not on the identity of individual goods.[(2)] An

(2) As an example with a discrete distribution, say two differentiated goods are

agent with a demand for a certain quantity of the composite good will then choose the distribution of individually produced goods as a function of their relative prices only. This could be a CES aggregator as in Dixit and Stiglitz (1977).(3)

Domestically and foreign produced materials are substitutes as inputs in production, but differ in the sense that only the domestically produced good is a consumption and investment good. This assumption captures the fact that a (domestic) firm's output may be another firm's input and gives rise to a double marginalisation effect because these inputs are not priced at marginal cost. We shall assume that the price of imported materials, p_t^E, measured relative to that of domestically produced materials, is exogenous.

Under these assumptions, marginal costs at the firm level are independent of scale, while average costs are falling. At the aggregate level, the gross output function Y_t, measured in terms of the composite good has constant returns to scale in inputs, provided that the fixed costs are accounted for as a factor input. Letting no superscripts indicate aggregate rather than firm-level variables, aggregate gross output, Y_t, is

$$\begin{aligned} Y_t &= I_t Y\left(V\left(K_t^i, z_t H_t^i\right), Q\left(E_t^i, M_t^i\right), \phi\right) \\ &= Y\left(V\left(K_t, z_t H_t\right), Q\left(E_t, M_t\right), \phi I_t\right) \end{aligned} \tag{2}$$

where ϕI_t is the fixed cost 'factor input'. Aggregate value added, ie the resources available to satisfy final demand, is then

$$Y_t - p_t^E E_t - M_t \tag{3}$$

where the price of (gross) output and hence domestically produced materials is normalised at 1.

produced in quantities g_1 and g_2 with a distribution characterised by a vector (g_1, g_2). The quantity of the composite good generated by $(1, 2)$ is the same as the quantity generated by $(2, 1)$, while the distribution $\left(\frac{3}{2}, \frac{3}{2}\right)$ generates more of the composite than $(1, 2)$.

(3) Alternatively, we could describe this as an additional output sector that is perfectly competitive and which operates a production technology with the same properties as the aggregator.

2.2 Market structure

If the firms face no adjustment costs, factor demands are determined by the following marginal conditions:

$$\begin{aligned} Y_V\left(V_t^i, Q_t^i\right) V_K\left(K_t^i, z_t H_t^i\right) &= \mu_t^i r_t \\ Y_V\left(V_t^i, Q_t^i\right) V_H\left(K_t^i, z_t H_t^j\right) z_t &= \mu_t^i w_t \\ Y_Q\left(V_t^i, Q_t^i\right) Q_E\left(E_t^i, M_t^i\right) &= \mu_t^i p_t^E \\ Y_Q\left(V_t^i, Q_t^i\right) Q_M\left(E_t^i, M_t^i\right) &= \mu_t^i \end{aligned} \quad (4)$$

where subscripts indicate derivatives, and where r_t is the real interest rate and w_t the real (hourly) wage. These conditions state that the marginal product of each factor is equal to a mark-up over the respective factor price, with μ_t^i being the wedge between the marginal factor product and the factor price. The dynamics of this mark-up under various assumptions about competition is the main issue of the paper, and so is discussed in some detail.

Although not analysed quantitatively, the case of fixed mark-ups provides a useful analytical reference point. Write demand for goods in industry/firm i, d_t^i, as a function of the relative price it charges

$$d_t^i = \frac{Y_t^D}{I_t} D\left(\frac{p_t^i}{p_t}\right) \Rightarrow \frac{p_t^i}{p_t} = D^{-1}\left(d_t^i \frac{I_t}{Y_t^D}\right) \quad (5)$$

where Y_t^D is total demand for the composite good and D is a time-invariant demand function that gives firm i's share of aggregate demand given relative prices. Assume that $D(1) = 1$ so that for equal prices, firms get an equal share of aggregate demand. Throughout, we shall restrict ourselves to symmetric equilibria where all firms charge identical prices and produce identical quantities. Under these assumptions, it is straightforward to show that the wedge μ_t is time invariant and that:

$$\mu_t = \mu = \frac{1}{(D'(1)^{-1} + 1)} \quad (6)$$

With this analytical benchmark in mind, we proceed to investigate two different models of the mark-up, customer markets and implicit collusion, based on different assumptions about the function D. These models predict different dynamic mark-up responses following demand and technology shocks, a feature which we will use in applying the model to UK data.

2.2.1 Customer markets

In a customer market model, it is assumed that a firm that lowers its current price not only increases current sales, but also expands its customer base—an increase in the customer base raises future demand for any given price. This notion is captured by writing the demand function facing firm i as:

$$d_t^i = m_t^i \frac{Y_t^D}{I_t} D^m \left(\frac{\mu_t^i}{\mu_t} \right) \quad (\mathbf{7})$$

where, as in the static case, $D^m(1) = 1$, and superscript m indicates 'customer markets'. The main difference relative to the static model is the term m_t^i, which is the share of demand that would go to firm i if it charges the same price (or mark-up) as all other firms. A firm's share of aggregate demand may then differ according to history, where m_t^i is a state variable with law of motion

$$m_{t+1}^i = g\left(\frac{\mu_t^i}{\mu_t} \right) m_t^i \quad (\mathbf{8})$$

where it is assumed that $g' < 0$ and $g(1) = 1$. If a firm charges a lower mark-up than its competitors, then that will not only increase current sales, through D^m, but will also increase the firm's market share for given mark-ups in future periods, through the law of motion for market share, (**8**). One rationale for the existence of such a state variable is the existence of customer switching costs, where buyers must engage in costly search if they want to change supplier and may thus be reluctant to change supplier, as suggested by Phelps and Winter (1970) and recently analysed in a macroeconomic set-up by Ireland (1998). Unlike the implicit collusion model, to be described immediately below, this is not based on explicit aggregation from the micro level—without precise assumptions about the character of these switching costs, such aggregation is not possible.

To find the firm's optimal mark-up and derive the dynamic equation for mark-ups, we find the optimality condition for the mark-up decision. Profits in firm i at t are given as:

$$\frac{\mu_t^i - 1}{\mu_t} d_t^i = \frac{\mu_t^i - 1}{\mu_t} m_t^i \frac{Y_t^D}{I_t} D^m \left(\frac{\mu_t^i}{\mu_t} \right) \quad (\mathbf{9})$$

Note that we can write m_{t+j} for any $j > 0$ as a function of mark-ups from t to $t+j$:

$$m^i_{t+j} = m^i_t \prod_{z=0}^{j} g\left(\frac{\mu^i_{t+z}}{\mu_{t+z}}\right) \qquad (\mathbf{10})$$

We assume that there is a fixed probability $1-\alpha$ that the firm will receive a market share next period which is independent of the firm's pricing history.[4] Then firm i's expected present value of future profits at time t can be expressed as:

$$E_t \sum_{j=0}^{\infty} \alpha^j \beta^j \frac{\lambda_{t+j}}{\lambda_t} \frac{\mu^i_{t+j}-1}{\mu_{t+j}} D^m \left(\frac{\mu^i_{t+j}}{\mu_{t+j}}\right) \frac{Y^D_{t+j}}{I_{t+j}} m^i_t \prod_{z=0}^{j-1} g\left(\frac{\mu^i_{t+z}}{\mu_{t+z}}\right) \qquad (\mathbf{11})$$

where λ_{t+j} is the marginal utility of wealth at time t and β is a fixed discount factor. In effect the terms $\alpha^j \beta^j \lambda_{t+j}/\lambda_t$ act as a discount factor, α being the 'survival' probability, λ_{t+j}/λ_t giving the future marginal utility of an additional unit of profit relative to the current, and β discounting it to time t.[5] Firms now choose μ^i_t to maximise (**11**), with a first-order condition:

$$\begin{aligned} & E_t \sum_{j=1}^{\infty} \alpha^j \beta^j \frac{\lambda_{t+j}}{\lambda_t} \frac{\mu^i_{t+j}-1}{\mu_{t+j}} D^m \left(\frac{\mu^i_{t+j}}{\mu_{t+j}}\right) \frac{Y^D_{t+j}}{I_{t+j}} m^i_t \prod_{z=1}^{j} g\left(\frac{\mu^i_{t+j}}{\mu_{t+j}}\right) \\ = \ & g'\left(\frac{\mu^i_t}{\mu_t}\right)^{-1} \frac{1-\mu^i_t}{\mu_t} D^m_\mu \left(\frac{\mu^i_t}{\mu_t}\right) \frac{Y^D_t}{I_t} - D^m\left(\frac{\mu^i_t}{\mu_t}\right) \frac{Y^D_t}{I_t} \end{aligned} \qquad (\mathbf{12})$$

where D^m_μ indicates derivative of $D^m(.)$ with respect to μ^i_t. To ease exposition, define a variable X^i_t as expected future profits at time t for firm i, the expectational term in (**12**):

$$X^i_t = E_t \sum_{j=1}^{\infty} \alpha^j \beta^j \frac{\lambda_{t+j}}{\lambda_t} \frac{\mu^i_{t+j}-1}{\mu_{t+j}} D^m \left(\frac{\mu^i_{t+j}}{\mu_{t+j}}\right) \frac{Y^D_{t+j}}{I_{t+j}} m^i_t \prod_{z=1}^{j} g\left(\frac{\mu^i_{t+j}}{\mu_{t+j}}\right) \qquad (\mathbf{13})$$

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[4] This assumption is only needed here to make the expression for future profits equivalent to the one used for the implicit collusion model, where α plays an important role.

[5] By using marginal utility, we are in effect anticipating equilibrium: more correctly, we would assume a set of prices for contingent securities, and then in equilibrium show that the prices are equal to λ_{t+j}/λ_t.

To define a similar aggregate measure, symmetry is imposed, and aggregate expected future profits X_t are

$$\frac{X_t}{I_t} = E_t \sum_{j=1}^{\infty} \alpha^j \beta^j \frac{\lambda_{t+j}}{\lambda_t} \frac{\mu^i_{t+j} - 1}{\mu_{t+j}} \frac{Y^D_{t+j}}{I_{t+j}}. \qquad (\mathbf{14})$$

For later purposes, we derive the recursive form of this variable as

$$\left.\begin{array}{l} \frac{X_t}{I_t} = E_t\alpha\beta\frac{\lambda_{t+1}}{\lambda_t}\left(\frac{\mu_{t+1}-1}{\mu_{t+1}}\right)\frac{Y_{t+1}}{I_{t+1}} + E_t\alpha^2\beta^2\frac{\lambda_{t+2}}{\lambda_t}\left(\frac{\mu_{t+2}-1}{\mu_{t+2}}\right)\frac{Y_{t+2}}{I_{t+2}} + ... \\ \frac{X_{t+1}}{I_{t+1}} = I_{t+1}E_{t+1}\alpha\beta\frac{\lambda_{t+2}}{\lambda_{t+1}}\left(\frac{\mu_{t+2}-1}{\mu_{t+2}}\right)\frac{Y_{t+2}}{I_{t+2}} + ... \end{array}\right\} \Rightarrow$$

$$\frac{X_t}{I_t} = E_t\alpha\beta\frac{\lambda_{t+1}}{\lambda_t}\left(\frac{\mu_{t+1}-1}{\mu_{t+1}}\right)\frac{Y_{t+1}}{I_{t+1}} + E_t\alpha\beta\frac{\lambda_{t+1}}{\lambda_t}\frac{X_{t+1}}{I_{t+1}} \qquad (\mathbf{15})$$

Imposing symmetry on (**12**), the relation between μ_t and $\frac{X_t}{Y_t}$ can be found as

$$1 + D^m_\mu(1)\left(1 - \frac{1}{\mu_t}\right) + g'(1)\frac{X_t}{Y_t} = 0$$

$$\mu_t = \frac{D^m_\mu(1)}{1 + D^m_\mu(1) + g'(1)\frac{X_t}{Y_t}} \qquad (\mathbf{16})$$

To characterise the cyclical properties of the mark-up, differentiate (**16**) with respect to $\frac{X_t}{Y_t}$:

$$\frac{\partial \mu_t}{\partial\left(\frac{X_t}{Y_t}\right)} = -\frac{g'(1)}{D^m_\mu(1)}\mu_t^2 \qquad (\mathbf{17})$$

As both D^m_μ and g' are negative, this derivative is negative, implying that the mark-up is procyclical ie that the mark-up is high when current output, and hence current profits, are high relative to expected future profits. The intuition is the following. When future profits are high relative to current output, firms will lower their current price to gain future market share. Because current output is low, the (negative) revenue effect of lowering prices is relatively low while the future gains in terms of market share are high. When current profits are high relative to future profits, the future market share is relatively less important, and firms will raise prices to increase current revenues.

2.2.2 Implicit collusion

To model implicit collusion, we assume that each of the I_t industries consists of $n > 1$ identical firms, with industries as previously being indexed by j while firms are indexed by i.[6] The goods produced within an industry are close substitutes, so without some form of collusion, prices would collapse to marginal costs. We assume that explicit price agreements are not legally enforceable, so any agreement between the firms must be implicit. To sustain a collusive equilibrium, collusion must then be privately incentive-compatible: each firm finds a strategy of sticking to the agreement optimal relative to the best alternative.

If a firm existed for one period only, or was myopic for some reason, then a firm would choose its mark-up to maximise current profits given the mark-ups of all other firms—ie Bertrand competition. As is well-known from the 'folk theorem literature', in a dynamic infinite horizon setting where firms care about the expected present value of current and future profits, strategies that involve setting higher prices can be sustained as a subgame perfect equilibrium, provided that firms that undercut are punished sufficiently and the discount rate is sufficiently high. With repeated games of this type, there are many equilibria with different pricing and punishment strategies. Choosing from these equilibria, we assume that firms manage to implement the symmetric equilibrium that is jointly best for them, ie the equilibrium that maximises profits in industry i given mark-ups of other industries, asset prices and demand. It can be shown (Abreu (1986)) that this requires that the deviators face the most severe punishment possible. If firms can leave an industry, then a possible worst punishment is expected present value of future profits of zero, corresponding to exit if there are no fixed costs to leaving the industry. Subgame perfect equilibrium also requires that firms are willing to implement such a punishment strategy and that it is feasible. These conditions are described in detail in Rotemberg and Woodford (1992).

We assume that demand for firm i's goods depends on the mark-up charged by the firm, μ_t^{ij}, and the mark-up charged by other firms in the same industry, μ_t^j, relative to the mark-up charged by all other

[6] This can be seen simply as an additional aggregator over differentiated goods within an industry.

firms in all other industries, μ_t. We can then write demand and profits for firm i in industry j[7] as

$$d_t^{ij} = D^{ci}\left(\frac{\mu_t^{ij}}{\mu_t}, \frac{\mu_t^{j}}{\mu_t}\right)\frac{Y_t^D}{I_t}, \quad D^{ci}(1,1) = \frac{1}{n}$$

$$\pi_t^{ij} = \frac{\mu_t^{ij}-1}{\mu_t} d_t^{ij} = \frac{\mu_t^{ij}-1}{\mu_t} D^{ci}\left(\frac{\mu_t^{ij}}{\mu_t}, \frac{\mu_t^{j}}{\mu_t}\right)\frac{Y_t^D}{I_t} \qquad \textbf{(18)}$$

Because the punishment to deviators is zero future profits, a deviating firm will choose the mark-up, μ_t^{ij}, that maximises current profits only. By analogy to the customer market model, we can let X_t^j denote expected future profits in period t for each firm in industry j, now under the provision that collusion is sustained in the future. The incentive compatibility constraint can then be written as

$$\left\{\max_{\mu_t^{ij}} \pi_t^{ij}\right\} \leq \pi_t^j + X_t^j \qquad \textbf{(19)}$$

with the left-hand side accounting for the return to undercutting and gaining no future profits, and the right-hand side being the profits in period t if collusion is sustained, π_t^j, and the expected future profits from collusion.

The parameter α plays a key role in the implicit collusion model. As before, $1-\alpha$ is the probability that pricing is independent of history, but in this case it is interpreted as the probability that a collusive agreement will be re-established such that previous deviators can enter again. The extent to which the incentive-compatibility constraint (**19**) binds will then depend on α: if α is considerably smaller than one, then the relative returns from future collusion are small, because the probability of re-entry into a collusive agreement after having cheated is large. Here it is assumed that α is sufficiently small to ensure that the incentive-compatibility constraint always binds—the precise conditions are outlined in Rotemberg and Woodford (1992).

Following the line of argument from above, we can write the condition in symmetric equilibrium as

$$\max_{\rho}\left(\rho - \frac{1}{\mu_t}\right) D^c(\rho,1)\frac{Y_t}{I_t} = \left(1-\frac{1}{\mu_t}\right)\frac{Y_t}{nI_t} + \frac{X_t}{nI_t} \qquad \textbf{(20)}$$

[7] Where superscript c denotes 'collusion'.

where ρ is the firm's mark-up ratio relative to the mark-up of all other firms, μ_t. Solving the maximisation problem for the optimal mark-up ratio $\hat{\rho}$, we find

$$\begin{aligned}
\frac{Y_t}{nI_t} + \left(\hat{\rho} - \frac{1}{\mu_t}\right) D^c_\rho(\hat{\rho},1)\frac{Y_t}{I_t} &= \left(1 - \frac{1}{\mu_t}\right)\frac{Y_t}{nI_t} + \frac{X_t}{nI_t} \\
1 + \left(\hat{\rho} - \frac{1}{\mu_t}\right) D^c_\rho(\hat{\rho},1)n &= \left(1 - \frac{1}{\mu_t}\right) + \frac{X_t}{Y_t} \\
\hat{\rho} D^c_\rho(\hat{\rho},1)n - \frac{X_t}{Y_t} &= \frac{1}{\mu_t}\left(D^c_\rho(\hat{\rho},1)n - 1\right) \qquad \textbf{(21)} \\
\mu_t &= \frac{D^c_\rho(\hat{\rho},1)n - 1}{\hat{\rho} D^c_\rho(\hat{\rho},1)n - \frac{X_t}{Y_t}} \qquad \textbf{(22)}
\end{aligned}$$

As previously, we will need to determine the sign of the partial derivative:

$$\frac{\partial \mu_t}{\partial \frac{X_t}{Y_t}} = \left(1 - D^c_\rho(\rho,1)n\right)\mu_t^2 \qquad \textbf{(23)}$$

The derivative is positive provided that $D^c_\rho(\rho,1)$ is negative. In contrast to the customer market model, the mark-up is countercyclical, ie the mark-up is low when current profits are high relative to future profits. The intuition is the following: high future profits relative to current output make the collusive agreement easier to sustain, because the gains to undercutting are small and the gains from future collusion high. Under such circumstances, the implicitly agreed mark-up will be high. If, on the other hand, current output is high, then the incentive to undercut is high, and the cartel must cut its mark-ups in order to sustain collusion. By cutting mark-ups, the share of current output that the deviator can pick up for a given price cut will decrease, thus lowering profits and the incentive to cheat. The key to understanding the argument is that we have ensured that equilibrium is sustained, ie there is no undercutting: to sustain equilibrium, the incentive to cheat must thus be lowered.

2.3 Households

Having described the market structure, we now turn to the households in the economy. The households solve a standard dynamic optimisation problem, determining the path of current and future consumption of the composite good, c_t, and supply of labour, h_t. From standard dynamic optimisation theory, the first-order conditions for the household will lead to a consumption demand function and a labour supply function that can be written as functions of real wages, w_t, and the expected marginal utility of wealth, λ_t:

$$\begin{aligned} c_t &= c(w_t, \lambda_t), \; c_w > 0, \; c_\lambda < 0 && \mathbf{(24)} \\ h_t &= h(w_t, \lambda_t), \; h_w > 0, \; h_\lambda > 0 && \mathbf{(25)} \end{aligned}$$

The households are the capitalists of this economy, owning both the physical capital stock and the firms. Capital is rented to firms on a one-period basis and this gives rise to rental income. Any profits from production are distributed to the shareholders on a period-by-period basis. Because the households are identical and risk averse, it is easy to show that each household will hold a fully diversified portfolio of shares, hence eliminating idiosyncratic risk, and that no trade in shares is taking place.

The Euler equation that relates the marginal utility of consumption now to the marginal utility of consumption in the future is important, as it appear as a discount factor when the firms calculate expected future profits. The asset price Euler equation is

$$1 = \beta E_t \left[\frac{\lambda_{t+1}}{\lambda_t} \left(r_{t+1} + (1 - \delta) \right) \right] \qquad \mathbf{(26)}$$

where r_{t+1} is the expected real interest rate in period $t+1$ and δ is the depreciation rate on capital.

To ensure the existence of a balanced steady-state growth path we impose restrictions on the functional form of the utility function. These restrictions, discussed in detail in King, Plosser and Rebelo (1988a), require only that there exists a $\sigma > 0$ such that $c(.)$ is homogeneous of degree one and $h(.)$ is homogeneous of degree zero in $\left(w, \lambda^{-\frac{1}{\sigma}}\right)$, where $1/\sigma$ is the intertemporal elasticity of substitution of consumption with hours held constant. This assumption ensures that along the steady-state growth path hours worked per capita are constant, because the income effect, which tends to lower hours as income increases, and the substitution effect, which tends to increase hours as wages increase, exactly cancel out. Also along this path, the consumption–output ratio is constant.

2.4 Demand

We have previously described how demand for individual firms' goods is affected by relative prices through firms' decisions about their mark-up. This section describes demand for final goods, ie the composite good consisting of an aggregate of the differentiated goods. As the model is designed to focus on imperfect competition and pricing, the demand for final goods is kept as simple as possible. Aggregate demand for value added, ie gross output minus costs of imported and domestic materials, consists of aggregate consumption and investment demand by households, C_t and χ_t respectively, and government expenditure, G_t. The resource constraint requires that demand for final goods equals value added, ie that:

$$C_t + \chi_t + G_t = Y_t - p_t^E E_t - M_t \quad \mathbf{(27)}$$

Aggregate private consumption demand is simply the product of the population at t, N_t, and individual household demand $c_t(\lambda_t, w_t)$. Government expenditure has the same structure over the I_t goods as households' and firms' demands and is assumed to be an exogenous stochastic variable; shocks to government consumption constitute the demand shocks in this economy. We will return to the specification of the stochastic process for this variable later. Finally, aggregate investment is related to the capital stock by the following law of motion:

$$\chi_t = K_{t+1} - (1 - \delta)K_t, \quad \mathbf{(28)}$$

which assumes no adjustment costs or irreversibility constraints on investment.

In accounting for the gross output and value added, notice that final goods are used as domestically produced materials in production and as payment for imported materials. For domestically produced materials, firms must purchase production goods of other firms to obtain materials input—the assumption that firms use the composite good as materials and do not distort their input of materials in favour of the good they produce themselves, despite the goods that other firms produce being priced above marginal costs, is reasonable because we have restricted the equilibrium to be one where all firms charge identical mark-ups and produce identical quantities. The resource constraint implies that imported materials must be paid for period by period; in aggregate the trade balance clears in every period.

2.5 Equilibrium

Equilibrium is characterised by the demand functions for consumption and leisure (with consumption of leisure being $(1-h_t)$), the (factor) demand functions of the firms, market-clearing conditions for goods and the stochastic processes for exogenous variables. Define aggregate labour supply as $H_t = N_t h(w_t, \lambda_t)$. In equilibrium, the following five conditions, summarising equilibrium in markets for capital ((**29**) and (**30**)), labour (**31**), imported materials (**32**), domestically produced materials (**33**), and market clearing in the goods market (**34**), must hold:

$$\beta E_t \left\{ \frac{\lambda_{t+1}}{\lambda_t} (r_{t+1} + 1 - \delta) \right\} = 1 \quad (\mathbf{29})$$

$$Y_V(V_t, Q_t) V_K(K_t, z_t H_t) = \mu_t r_t \quad (\mathbf{30})$$

$$Y_V(V_t, Q_t) V_H(K_t, z_t N_t h(w_t, \lambda_t)) z_t = \mu_t w_t \quad (\mathbf{31})$$

$$Y_Q(V_t, Q_t) Q_M(E_t, M_t) = \mu_t p_t^E \quad (\mathbf{32})$$

$$Y_Q(V_t, Q_t) Q_M(E_t, M_t) = \mu_t \quad (\mathbf{33})$$

$$C_t + [K_{t+1} - (1-\delta)K_t] + G_t + p_t^E E_t + M_t = Y_t \quad (\mathbf{34})$$

The dynamics of the mark-up are characterised by

$$\mu_t = E_t \mu \left(\frac{X_t}{Y_t} \right) \quad (\mathbf{35})$$

$$X_t = I_t E_t \sum_{j=1}^{\inf} \alpha^j \beta^j \frac{\lambda_{t+j}}{\lambda_t} \left(\frac{\mu_{t+j} - 1}{\mu_{t+j}} \right) \frac{Y_{t+j}}{I_{t+j}} \quad (\mathbf{36})$$

referring to the market structures discussed in detail above, ie customer markets (**16**) and implicit collusion (**22**).

These conditions describe the economy at any point in time. In the following, we characterise this equilibrium as small fluctuations around a steady-state growth path, characterised by growth in technology z_t and in population N_t. Define the stochastic growth rate of labour-augmenting technological progress as $\gamma_t^z = z_t / z_{t-1}$, with an average growth rate γ^z, and a deterministic growth rate of the population, $\gamma^N = N_t / N_{t-1}$. In steady state, aggregate expenditure components and factor demand (except demand for labour) grow at the rate $\gamma^z + \gamma^N$, while aggregate employment grows at the rate γ^N and average real wages grow at the rate γ^z.

Shocks to labour-augmenting technological progress, z_t, are assumed to be non-stationary and hence shift the steady-state path, while shocks to government expenditure, G_t, and the price of imported materials, p_t^E, push the equilibrium off the steady-state path, but do not change the path itself. These shocks, whether temporary or permanent, are propagated exogenously through persistence in the shocks, and internally through changes in the desired capital stock. There are two internal propagation mechanisms in the model, wealth effects and intertemporal substitution of labour. First, shocks may affect the marginal utility of wealth, λ_t, corresponding to a shift to the right of the labour supply curve in Figure 1. If the marginal utility of wealth increases, agents will lower current consumption and increase current labour supply to increase current wealth. Such an increase in labour supply in turn increases the marginal product of capital and the real interest rate above steady-state levels, leading to an increase in investment and hence in the capital stock. The second mechanism is intertemporal substitution in labour supply. If the real wage is currently above its steady-state level, then workers will substitute current for future leisure, and increase labour supply, effectively a movement along the labour supply curve. Part of the increase in income will be directed towards increasing the capital stock—an increase in the labour supply increases the marginal product of capital and this increase in the real interest rate results in an increase in the capital stock.[(8)]

2.6 Solution technique

As mentioned in the previous section, we focus on (small) business cycle fluctuations around a steady-state growth path, allowing for non-stationary, stochastic labour-augmenting technical progress and for deterministic population growth. We apply the methodology described by King, Plosser, and Rebelo (1988b). This involves first transforming the non-stationary equilibrium conditions into stationary ones by normalising the variables on the size of the population and the level of technology, and then making a log–linear approximation around the

[(8)] Notice that the mark-up is not currently a source of propagation: the mark-up is determined on the basis of information available at time t, and is a 'jump' rather than a predetermined variable. It is possible to describe the model with a predetermined mark-up, in which case (**35**) should be written as $\mu_{t+1} = E_t\mu\left(X_{t+1}/Y_{t+1}\right)$.

steady state of this stationary system. The outcome is a system of linear difference equations, written in variables defined in deviations from steady state, where the coefficients are derivatives evaluated at steady state. These derivatives, which are effectively own and cross-elasticities, are the parameters that are calibrated—the details of this process is described in the following section. Details of the methodology and solution method are provided in the appendix.

3 Calibration

The coefficients of the log-linearised equilibrium conditions are functions of the parameters listed in the first column of Table A. Some of these variables are standard dynamic general equilibrium model parameters, for example, the intertemporal elasticity of labour supply and the real rate of return. Others, however, only arise because of our specification of a gross output production function, for example the elasticity of substitution between value-added and intermediate inputs, or because we take account of imperfect competition, for example the mark-up of price over marginal cost. The discussion in this section focuses on how we calibrate these latter parameters.

Recall that we have approximated the equilibrium around its steady-state path: the parameters in the system are therefore evaluated at steady state. The equilibrium conditions impose restrictions on the parameters, and our calibration procedure must ensure that these relations hold. These parameter restrictions are discussed in the appendix.

The majority of the parameters are estimated using data for the UK economy, and for comparative purposes the corresponding values from Rotemberg and Woodford (1995) are included in the third column of Table A. The remainder, for which we have no UK evidence, are set at the values they take in Rotemberg and Woodford (1995).

The properties of the gross output production function can be summarised by the following seven parameters—the output elasticities of labour (s_H), capital (s_K), imported materials (s_E) and domestically produced materials (s_M), and the elasticities of substitution between the value-added aggregator and the materials aggregator (ϵ_{VQ}), labour and capital (ϵ_{KH}) and imported and domestically produced materials (ϵ_{EM}).

The output elasticities (s_H, s_K, s_E, s_M) equal their respective share of total costs: to see this, note that the assumption that firms earn zero profits in the steady state implies that

$$\phi\tilde{I} = (\mu - 1)\,\tilde{Y} \tag{37}$$

where as before no subscript indicates steady-state values and a tilde indicates normalised values. Given (**37**) and (**1**), it follows that $\mu\tilde{Y} = F\left(V\left(\tilde{K}, \tilde{H}\right), Q\left(\tilde{E}, \tilde{M}\right)\right)$. Therefore, in steady state the output elasticity of the various inputs *also* equals its share in the value of gross output.(9) The main difficulty in estimating each input's share of gross output is the lack of data on gross output, domestically produced and imported materials. The only source of consistent data on these variables are the Input-Output Tables, but these are only calculated for specific years, so we only have cross-sectional data. Therefore, we set the steady-state cost shares of labour, imported intermediate inputs and domestically produced intermediate inputs equal to their respective weighted average of industry costs shares in the 1974, 1979 and 1984 Input-Output Tables. The steady-state share of capital is calculated as a residual, as the four gross output shares must sum to unity.

Estimates of the elasticity of substitution between labour and capital, ϵ_H, are available for the United Kingdom. For example, in the NIESR macro model this elasticity equals 0.5. There are, however, very few estimates of the elasticity of substitution between the aggregators, ϵ_{VQ}, and no estimates for the elasticity of substitution between domestically produced and imported intermediate inputs, ϵ_{EM}. We obtain estimates from running three cross-sectional regressions, derived from combining the production function (**1**) with first-order conditions for labour, materials and the split between the materials composite (Q_t) and value added composite (V_t):

$$\ln\left(\frac{VA_j}{L_j}\right) = \alpha^1 + \epsilon_{KL}\ln(P_j^L/P_j^{VA}) + u_j^1 \tag{38}$$

$$\ln\left(\frac{MA_j}{M_j}\right) = \alpha^2 + \epsilon_{EM}\ln(P_j^M/P_j^{MA}) + u_j^2 \tag{39}$$

(9) Recall that gross output is measured net of fixed costs.

Table A: Calibration of the benchmark

Parameter	Value	RW 95	Description
s_C	0.607	0.697	Consumption share of value-added
s_G	0.226	0.117	Government share of value-added
s_I	0.167	0.186	Investment share of value-added
s_E	0.120	0.5	Imp. mat. share in total costs
s_M	0.388		Dom. prod. mat. share in total costs
s_H	0.313	0.36	Labour share in total costs
s_K	0.178	0.12	Capital share in total costs
ϵ_{KH}	0.970	1	Elas. of subst., labour and capital
ϵ_{VQ}	0.463	0.69, 0.0001	Elas. of subst., value-added and intermediate inputs aggregator
ϵ_{EM}	0.492		Elasticity of substitution between imp. and dom. prod. intermediate inputs
r	1.0187	1.014	Steady-state real rate of return (quarter)
δ	1.014	1.013	Rate of depreciation per quarter
$1/\sigma$	1	0.5	Intertemporal elasticity of substitution of consumption (h constant)
ϵ_{wH}	4	4	Intertemporal elasticity of labour supply
ϵ_{μ}	-0.3, 0.1	-1, 0.15	Elas. of mark-up with respect to X/Y
μ	1.11	1.2	Steady-state mark-up
α	0.89	0.89	Expected rate of growth of market share

Note: 'RW 95' refers to values assumed in Rotemberg and Woodford (1995).

$$\ln\left(\frac{Y_j}{MA_j}\right) \quad = \quad \alpha^3 + \epsilon_{VQ}\ln(P_j^{MA}/P_j^Y) + u_j^3 \qquad \textbf{(40)}$$

where α^i is a constant term, u_j^i a residual term, P_j^i price of factor i, P_j^Y the price of gross output and P_j^V the price of value added in sector j. VA_j is value added in sector j, proxying V_j, while MA_j is a materials composite, proxying Q_j. We assume that $P_j^{VA} = P_j^Y$. The constant terms, α_i, are non-linear functions of the parameters in the production function and the mark-up in each industry. To estimate these regressions, we use cross-sectional data on quantities in 77 manufacturing industries from the 1984 Input-Output Tables. We construct MA_j using a quantity index defined on the most appropriate two-digit industry and a price index for domestic and imported materials, constructed on three-digit industry data. The remaining price data are also measured at three-digit level. Based on these estimates we set $\epsilon_{HK} = 0.98$—this is statistically insignificantly different from the standard assumption in the real business cycle literature that the elasticity of substitution between labour and capital is unity. We set $\epsilon_{VQ} = 0.46$, slightly lower than the estimate obtained by Rotemberg and Woodford (1996) using time series data on two-digit US manufacturing industries, and $\epsilon_{EM} = 0.459$.

The equilibrium behaviour of the mark-up is described by three parameters. The first is the steady-state value of the mark-up, μ. A number of authors have used Hall's approach to estimate the mark-up. Small (1997) estimates the average mark-up in 16 two-digit manufacturing and service industries. The weighted average of these estimates is 1.392. Bean and Symons (1989) and Haskel, Martin, and Small (1995) also estimate the average mark-up using two-digit data. Their average estimates are 2.00 and 1.52 respectively. However, neither of these estimates of the mark-up is consistent with the assumption of constant returns to scale as they both imply that labour's output elasticity is greater than unity, and hence that capital's output elasticity is negative. So we use the estimates of Small (1997). But because he estimates the mark-up of price over the marginal cost of *value-added* rather than *gross output*, his figure needs to be adjusted using the share of value-added in gross output to get the mark-up of price over the marginal cost of gross output, and we set $\mu = 1.11$.[10]

[10] The relationship between the two mark-ups is $\mu = \mu^{VA}/\left(1 + s_M\left(\mu^{VA} - 1\right)\right)$, see Rotemberg and Woodford (1992).

This is slightly lower than the value used by Rotemberg and Woodford (1996). It implies that for the typical firm price is 11% higher than marginal cost and fixed costs account for 10% of total costs.

The other two parameters that determine the behaviour of the mark-up are the elasticity of the mark-up with respect to expected future profits, ϵ_μ, and the parameter α. The first parameter distinguishes between the customer market model and the implicit collusion model. The theory outlined above suggests that in the customer market model the mark-up is negatively related to this ratio, but suggests no specific value. We set $\epsilon_\mu = -0.3$ and provide a sensitivity analysis of the implications of this choice. In the case of the implicit collusion model theory restricts ϵ_μ to be less than $\mu - 1$, and we set it to 0.1, close to the maximum of 0.11. There is little independent basis for setting the value of the second variable, so we follow Rotemberg and Woodford (1995) and set it equal to 0.89. This ensures that the incentive-compatibility constraint always binds.

All aspects of the individual's preferences can be summarised by just two parameters, the elasticity of consumption growth with respect to changes in the real rate of return holding hours constant, and the intertemporal elasticity of labour supply. While these parameters are indeed important for the model, they are not specific to this study and we follow Rotemberg and Woodford (1992).

The remaining parameters which enter the log-linearised equilibrium conditions are the steady-state share of consumption, investment and government expenditure in value-added, and the steady-state values of depreciation and the real rate of return. These parameters are relatively straightforward to calibrate using data in the UK National Accounts. The steady-state shares of consumption, investment and government expenditure are set equal to their respective average shares of GDP over the period 1960 Q1–1998 Q3. This gives a consumption share of 0.607, a government expenditure share of 0.236 and an investment share of 0.167. The share of investment in income implies that the steady-state rate of depreciation equals 0.0154. Finally, the real rate of return is set equal to the average real rate of return on equities in the United Kingdom over the period 1919 to 1998.

To complete the calibration, we need estimates for the parameters in the stochastic processes for the control variables. We could in principle estimate a joint VAR(1) directly without imposing any conditions, but in the spirit of Rotemberg and Woodford (1996), we will to focus analytically on independent shocks and how these shocks feed through via the endogenous propagation mechanism: we assume that shocks are independent and that there is no cross-propagation, ie that we can estimate the processes separately.[11] In the following, we look at these processes in turn.

3.1 Imported materials

In the model, we treat the real price of imported materials, p_t^E, as exogenous. Importantly, what matters to the allocation of resources and to the dynamics of the mark-up is the real price of imported materials, defined as:

$$p_t^E = \frac{P_t^E}{P_t} \tag{41}$$

where P_t^E is the nominal price of imported materials and P_t is the price of final goods, ie gross output.[12] Having constructed a series for the real price of imports, the simplest approach to model and calibrate a stochastic process for p_t^E is to assume that $\hat{p}_t^E$ follows an AR(1) and that the shocks are iid, eg

$$\hat{p}_t^E = \rho^E \hat{p}_{t-1}^E + \varepsilon_t^p \tag{42}$$

This would not identify exogenous shocks to the economy however: if P_t is endogenous, then shocks to p_t^E would be correlated with other exogenous shocks or indeed with endogenous variables. We identify shocks to p_t^E by assuming that innovations in p_t^E are caused entirely by innovations in P_t^E. This is modelled by assuming that the process for $\hat{p}_t^E$ can be written as:

$$\hat{p}_t^E = \rho(L) \begin{bmatrix} \hat{P}_{t-1}^E \\ \hat{p}_{t-1}^E \end{bmatrix} + \rho_0 \hat{P}_t^E \tag{43}$$

[11] A full VAR estimation may be statistically more appropriate, but would involve estimating a large number of nuisance parameters. As we do not provide any formal statistical analysis, the gains in terms of ease of exposition outweigh the costs from failing to obtain the 'correct' covariance matrix.

[12] While conceptually straightforward, this presents some measurement problems for the calibration exercise. Lacking an appropriate series for imported materials, we use a general price index for imports. To measure P_t we need a price series for gross output. No such time series is available, and instead we use the GDP deflator. We have experimented with constructing price indicies; these alternative measures do not appear to affect the results materially.

By assuming that the real price deviations from trend $\hat{p}_t^E$ are affected by past values of the variable itself and by *current* and past values of $\hat{P}_t$, we effectively treat innovations in P_t^E as innovations in $\hat{p}_t^E$. To calibrate the process, we estimate the following linear AR models

$$\bar{P}_t^E = \delta(L)\begin{bmatrix} \bar{P}_{t-1}^E \\ \bar{p}_{t-1}^E \end{bmatrix} + \varepsilon_t^P \quad (\mathbf{44})$$

$$\bar{p}_t^E = \rho(L)\begin{bmatrix} \bar{P}_{t-1}^E \\ \bar{p}_{t-1}^E \end{bmatrix} + \rho_0 \bar{P}_t^E + \varepsilon_t^p \quad (\mathbf{45})$$

where bar denotes detrended variables. Given that the errors are orthogonal by construction, we can estimate this by OLS on the equations separately. These equations are then combined into a law of motion for p_t^E as a function of lagged variables and a single contemporaneous innovation by assuming that $\varepsilon_t^p \equiv 0$.

$$\hat{p}_t^E = \left(\check{\rho}_0\check{\delta}(L) + \check{\rho}(L)\right)\begin{bmatrix} \hat{P}_{t-1}^E \\ \hat{p}_{t-1}^E \end{bmatrix} + \check{\rho}_0\varepsilon_t^P \quad (\mathbf{46})$$

where $\left(\check{\rho}_0, \check{\delta}(L), \check{\rho}(L)\right)$ are the OLS estimates obtained from (**44**) and (**45**). The residuals from (**44**), $\check{\varepsilon}_t^P$, are treated as the sequence of exogenous shocks. These shocks can be interpreted as terms of trade shocks so long as the nature of trade is kept in mind: in this economy, only final goods are exported and only materials imported.

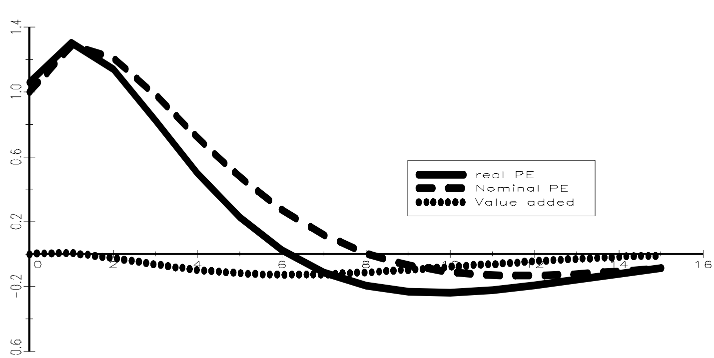
By using this procedure, we exclude immediate effects from the domestic price level P_t to the nominal price of imported materials P_t^E. However, we do not exclude feedback of this kind in general—in principle, the nominal price of imported materials could be affected by domestic market conditions, but we do not assert any theory of why and how such effects may occur. Explicitly excluding feedback effects from the domestic price level to the nominal price of imported materials amounts to assuming that the second row in $\delta(L)$ is zero.

Table B gives the estimation results for (**44**) and (**45**). The variables are detrended using a Hodrick-Prescott filter ($\lambda = 1600$). We test the filtered variables for stationarity and find them all stationary (results not reported). In addition, we report results for estimating an output equation:

$$\bar{Y}_t = \eta_0^E \bar{P}_t^E + \eta^E(L) \begin{bmatrix} \bar{P}_{t-1}^E \\ \bar{p}_{t-1}^E \\ \bar{Y}_{t-1} \end{bmatrix} + \varepsilon_t^Y \qquad (\mathbf{47})$$

in order to provide an empirical measure of the effect of an innovation in $\bar{P}_t^E$. The lag length is set at two, based on simple significance tests. In the real price equation all parameter estimates are significant, and the coefficient on the structural shock, ie the coefficient to $\bar{P}_t^E$, is greater than one. The assertion that the domestic price level affects the price of imports is not confirmed, with the coefficients on lagged $\bar{p}_t^E$ being insignificant. The coefficient on $\bar{P}_t^E$ is negative and just insignificant at a 5% significance level, so, as expected, a positive price shock affects output negatively. Consistent with our assertions, the nominal price affects output only on impact, as the coefficients on lagged values of $\bar{P}_t^E$ are insignificant, whereas the lagged values of $\bar{p}_t^E$ are significant.(13)

Figure 2: Empirical impulse response. Price of imported materials



By inserting (**44**) into (**45**) and (**47**), empirical impulse responses for these two variables can be drawn. This is done in Figure 2. In response

(13) An alternative way of estimating these empirical impulse response would be to estimate a VAR, imposing identifying order restriction, ie order the variables with $\bar{P}_t^E$, $\bar{p}_t^E$ and $\bar{Y}_t$ and use the Cholesky decomposition; considering that the purpose of the exercise is to calibrate the process for p_t^E, this would involve estimating nuisance parameters. In any case, the empirical impulse responses from estimating this VAR are not qualitatively different from the ones reported above.

to a 1% shock to the nominal price of imported materials, $\bar{P}_t^E$, the real price of imported materials increases by slightly more than a percent on impact, and the effect peaks one period after the shock. The real price is almost as persistent as the nominal price, suggesting that the persistence in the nominal price is the driving force behind the persistence in the real price. Output contracts sharply, levelling out after five quarters at 0.12 percent below the initial level, and the effect persists for close to 18 quarters.

Table B: Estimation results. Price of imported materials

Independent variable	Dependent variable: Nom. price P_t^E		Real price p_t^E		Output VA_t	
P_{t-1}^E	1.20	*0.17*	-1.18	*0.14*	-0.29	*0.13*
P_{t-2}^E	-0.45	*0.17*	0.32	*0.11*	0.10	*0.11*
p_{t-1}^E	0.01	*0.15*	1.20	*0.10*	0.38	*0.11*
p_{t-1}^E	0.01	*0.14*	-0.36	*0.09*	-0.24	*0.10*
P_t^E			0.95	*0.14*	0.01	*0.06*
VA_{t-1}					0.51	*0.12*
VA_{t-2}					0.28	*0.12*
Adj. R^2	0.76		0.93		0.78	
S.E.R.	0.75		0.01		0.01	
F-stat.	95.14		292.34		57.59	

Notes: All variables are in logs and deviations from trend. Standard errors in italics. S.E.R. is standard error of regression, while F-stat is the F-statistic on the hypothesis that all parameters are insignificant.

In making comparisons between the empirical impulse responses and the impulse response functions generated by the model, it is important to keep in mind that the detrending method differs—in the model, we can correctly detrend the variables, given our knowledge about the properties of the exogenous variables. In terms of the empirical estimates, we have to choose a detrending method. Hence we cannot provide a formal comparison of the empirical and model generated impulse response functions, and the empirical impulse response function should be used only as an informal assessment of the extent to which the model's response is plausible.

3.2 Government expenditure shocks

Shocks to demand are modelled as innovations in government expenditure. Here we use the simplest possible model, assuming that government expenditure follows an AR process where

$$\hat{g}_t = \nu(L)\hat{g}_{t-1}. \tag{48}$$

As before, this is estimated running an OLS regression on the following equation

$$\bar{g}_t = \nu(L)\bar{g}_{t-1} + \varepsilon_t^g \tag{49}$$

where government expenditure has been detrended using a Hodrick-Prescott filter. For the purpose of comparison, we also estimate an output equation

$$\bar{Y}_t = \eta_0^G \bar{g}_t + \eta^G(L) \begin{bmatrix} \bar{g}_{t-1} \\ \bar{Y}_{t-1} \end{bmatrix} + \varepsilon_t^{GY} \tag{50}$$

treating shocks to government expenditure, ε_t^g as the exogenous shock.

The estimation results are reported in Table C. The most interesting results reported in this table relate to the effects on output of a surprise increase in government expenditure: while such an increase is expansionary on impact, it is contractionary even in the short run. The coefficients on contemporaneous and lagged $\bar{g}_t$ are however all insignificant, and a joint F-test on the significance of the three parameters suggests that they can all be dropped ($F(3,110)=2.19$, p-value 0.09). This is illustrated further in Figure 3; a 1% increase in government expenditure has expansionary effects only in the first period.

3.3 Shocks to technology

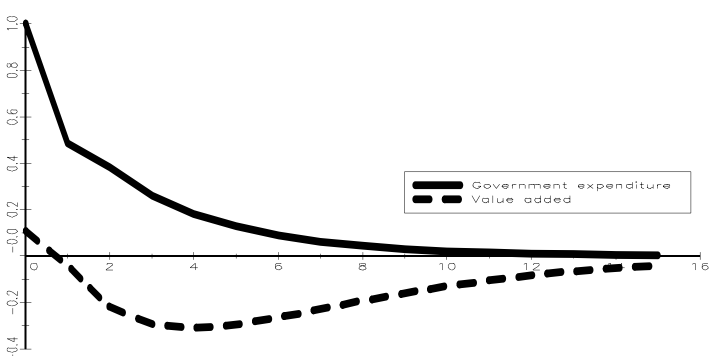
Estimating the shocks to TFP growth raises two issues. First, because labour-augmenting technological progress is subject to permanent

Table C: Estimation results. Government expenditure

Independent variable	Dependent variable: Gov't expenditure g_t		GDP VA_t	
g_{t-1}	0.49	*0.09*	-0.19	*0.12*
g_{t-2}	0.15	*0.09*	-0.17	*0.11*
VA_{t-1}			0.86	*0.10*
VA_{t-2}			-0.08	*0.10*
g_t			0.11	*0.11*
Adj. R^2	0.34		0.69	
S.E.R.	0.01		0.01	
F.Stat	57.63		62.15	

Notes: All variables are in logs and deviations from trend. Standard errors in italics. S.E.R. is standard error of regression, while F-stat is the F-statistic on the hypothesis that all parameters are insignificant.

Figure 3: Empirical impulse response. Government expenditure



shocks, we need to make an assumption about the entry and exit of firms; without this assumption, a permanent technology shock would imply positive/negative profits in the new steady state. The second issue is that because we have assumed a gross output production function and imperfect competition, we lack the data to construct a series for the technology shocks which is consistent with the assumption of our model. Instead we construct three proxy series for the shocks to TFP growth.

Like Rotemberg and Woodford (1995) we assume that labour-augmenting technological progress is subject to permanent shocks, ie that z_t follows a random walk.[14] Then the growth rate of technological progress $\hat{\gamma}_t^z$ is an independently distributed variable

$$\hat{\gamma}_t = \xi_t^{\gamma},\ \xi_t^{\gamma} \sim N\left(0, \sigma^{\gamma}\right) \tag{51}$$

and the only parameters we need to calibrate are the steady-state value γ^z and the standard deviation of the shocks, σ^{γ}. When shocks to z_t are permanent, we need to address the issue of profits in the steady state. Arguably, any profits would be competed away in the long run by either entry or exit, ie a change in the number of firms or industries and hence in the fixed costs. We do not attempt to model entry and exit specifically, but instead assume, in line with Rotemberg and Woodford (1995), that entry follows the following error correction process of the form:

$$\ln I_t = \kappa \ln\left(I z_t N_t\right) + (1-\kappa) \ln I_{t-1} \tag{52}$$

reflecting that entry and exit take time. This implies a reduced-form law of motion:

$$\hat{I}_t = (1-\kappa)\, \hat{I}_{t-1} \tag{53}$$

The key feature of this process for the model is the assumption that the number of firms/industries is highly persistent. We follow Rotemberg and Woodford (1995) and set $\kappa = 0.02$. Notice that this adjustment process is consistent with relatively high entry and exit

[14] This assumption is by no means innocuous—indeed, Ravn (1997) shows that temporary productivity shocks are important if the standard RBC model is to account for the business cycle moments.

rates—the assumption does not preclude an adjustment in I_t but prevents high-frequency movement in I_t from being the main business cycle determinant in the model.

Differentiating the gross output production function and rearranging gives the following expression for total factor productivity growth:

$$\Delta \ln z_t = \frac{1}{\mu s_H} \begin{array}{l} (\Delta y_t - \mu s_K \Delta k_t - \mu s_H \Delta h_t \\ \quad -\mu s_E \Delta e_t - \mu s_M \Delta m_t - (\mu - 1)\, \Delta I_t) \end{array} \tag{54}$$

where lower-case letters are natural logs of variables and μ and s_i are the steady-state mark-up and revenue shares respectively. Implicit in this derivation is the assumption that all variations in output can be ascribed to changes in factor inputs; this essentially is an approximation around a steady-state path where the mark-up and factor shares can reasonably be assumed constant. This does however mean that we are ascribing variations in output caused by variations in the mark-up and factor shares to changes in TFP.

Removing the means from (**54**) and substituting the expression for the entry of new firms gives the following expression for the shocks to gross output TFP growth:

$$\Delta \ln z_t = \Omega(L) \frac{1}{\mu s_H} (\Delta y_t - \mu s_K \Delta k_t - \mu s_H \Delta h_t - \mu s_E \Delta e_t - \mu s_M \Delta m_t) \tag{55}$$

where $\Omega(L)$ is a lag polynomial.[15] This expression cannot, however, be used to calculate the shocks to gross output TFP growth as there are no time series data for gross output and the two types of materials available for the United Kingdom. In a perfectly competitive model, this is not an issue: factor shares are constant, and (**55**) can be reduced to include only value added, capital and labour inputs. Assuming no substitution between V and Q would have similar effects. With imperfect competition and substitution between inputs, the production function cannot be captured by a 'reduced' value-added production function that does not include either materials or materials prices. The reason why (**55**) cannot be rewritten in terms of just value-added output, labour, capital and the price of intermediate

[15] There is a substantial amount of algebra involved in going from (**54**) to (**55**). Details of this step and the precise composition of $\Omega(L)$ are available on request.

inputs is because the contribution of intermediate inputs to gross output exceeds their revenue share.

To illustrate this problem, we use the definition of real value added, V_t^A:

$$V_t^A = Y_t - p_t^E E_t - M_t \qquad (\mathbf{56})$$

Differentiating and rearranging we get:

$$s_V \Delta v_t^A = y_t - s_E \Delta p_t^E - s_E \Delta e_t - s_M \Delta m_t \qquad (\mathbf{57})$$

where s_V is the share of value added in revenues and small letters signify natural logs. Using this expression to substitute the growth rate of gross output out of (**54**) gives

$$\begin{aligned} \Delta \ln z_t &= \frac{1}{\mu s_H} \left\{ s_V \Delta v_t^A - \mu s_K \Delta k_t - \mu s_H \Delta h_t - (\mu - 1) \Delta I_t \right. \\ & \left. + (1 - \mu) s_E \Delta e_t + (1 - \mu) s_M \Delta m_t + s_E \Delta p_t^E \right\} \qquad (\mathbf{58}) \end{aligned}$$

Rotemberg and Woodford (1995) construct the shocks to TFP growth using the following expression:

$$TFP_1 = \frac{1}{\mu s_H} \left(s_V \Delta v_t^A - \mu s_K \Delta k_t - \mu s_H \Delta h_t - (\mu - 1) \Delta I_t \right) \qquad (\mathbf{59})$$

Although they also assume a gross output production function and imperfect competition, they avoid the previously mentioned problem by assuming that there is no substitutability between value added and the intermediate inputs. This means that despite the assumption of imperfect competition, the contribution of the materials to gross output equals their revenue share, and TFP growth can be estimated just using data on value-added output, labour and capital. However, the elasticity of substitution obtained from estimating (**39**) suggests that value added and the intermediate inputs aggregator are indeed substitutes in production. So measuring the shocks using (**59**) generates technology shocks which are inconsistent with our model assumptions, because it ignores changes in prices and volumes of materials.

Given that time series data on the price of imported intermediate inputs is available, it is possible to ‘improve’ the estimates obtained from (**59**) by using the following expression to estimate the shocks to TFP growth:

$$\begin{aligned} TFP_2 &= \Delta \ln z_t - \frac{1}{\mu s_H}\left((1-\mu) s_E \Delta e_t + (1-\mu) s_M \Delta m_t\right) \qquad (\mathbf{60}) \\ &= \frac{1}{\mu s_H}\left(s_V \Delta v_t^A - \mu s_K \Delta k_t - \mu s_H \Delta h_t - s_E \Delta p_t^E - (\mu - 1)\Delta I_t\right) \end{aligned}$$

where $\Delta \ln z_t$ is taken from (**58**). These estimates take account of changes in the price of imported materials. However, the estimates are still inconsistent with the assumptions of our model, as the shocks estimated in (**60**) still do not take account of changes in materials.

An alternative way of estimating the shocks to TFP growth is to use the dual approach. This involves using the cost function dual to the assumed production function to derive an expression for the growth rate of marginal cost, substituting marginal cost out using the output price and the mark-up and then rearranging this to give an expression for TFP growth. The total variable cost function is:

$$C_t = F_t c\left(\frac{W_t}{z_t}, R_t, P_t^E, P_t^M\right) \qquad (\mathbf{61})$$

The marginal costs are

$$MC_t = \frac{\partial C_t}{\partial F_t} = c\left(\frac{W_t}{z_t}, R_t, P_t^E, P_t^M\right) \qquad (\mathbf{62})$$

Differentiating, we get the marginal variable cost function as

$$\Delta mc_t = \frac{z c_W}{c}\Delta w_t + \frac{c_R}{c}\Delta r_t + \frac{c_{P^E}}{c}\Delta p_t^E + \frac{c_{P^M}}{c}\Delta p_t^M - \frac{W c_W}{c}\Delta \ln z_t \quad (\mathbf{63})$$

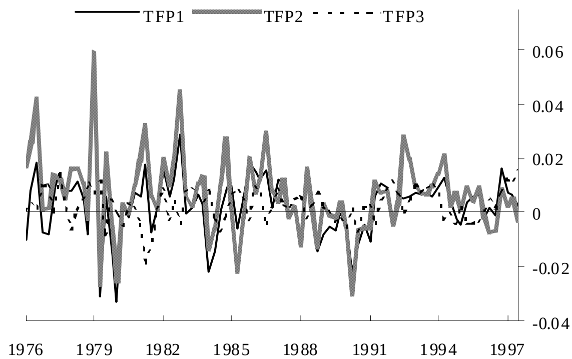
Prices are set as a mark-up on marginal variable costs (in steady state the mark-up equals fixed costs per unit of output), so the growth rate of marginal cost equals the growth rate of price minus the growth in the mark-up:

$$\Delta mc_t = \Delta p_t - \Delta \ln \mu_t \qquad (\mathbf{64})$$

Substituting (**64**) into (**63**) and using Shephard's lemma gives the following expression for TFP growth:

$$TFP_3 = \frac{1}{s_H}\left(\Delta p_t + \Delta \ln \mu_t - s_H \Delta w_t - s_K \Delta r_t - s_E \Delta p_t^E - s_M \Delta p_t^M\right) \qquad (\mathbf{65})$$

Figure 4: Alternative measures of TFP growth



The advantages of using this expression to estimate the shocks to TFP growth are that time series data are available or can be relatively easily constructed for all the price data, and all the inputs in the production function are taken account of. The disadvantage is that it requires data on the movements in the mark-up, which is endogenous to our model. If we assume that $\Delta \ln \mu_t$ can be ignored, our measure of TFP will also reflect cyclical changes in the mark-up.

Given that we cannot construct estimates of the TFP shocks which are consistent with our model assumptions using either the primal or the dual approach, we construct proxies for the technology shocks using equations (**59**), (**60**) and (**65**). Figure 4 compares the properties of the three proxies of TFP growth. The two primal measures of TFP growth are highly correlated. The main difference between them is that the measure which partially allows for the substitutability of value-added and intermediate materials, TFP_2, is more volatile than the measure that assumes non-substitutability, TFP_1. This reflects high volatility in prices of imported materials. The dual measure of TFP growth, TFP_3, is not highly correlated with the two primal measures, in particular it is less volatile. This could be either because the dual measure takes account of all the inputs in the production function, or because firms absorb some of the shocks to input prices in the mark-up.

4 Dynamic properties of the model

This section explores the dynamic properties of the model by analysing the response to exogenous shocks. This analysis has two parts: first, we analyse impulse response functions, tracing the response of the endogenous variables to a one off-shock to one of the exogenous variables. Second, we investigate the model's stochastic properties using a simulation approach, calculating standard statistics for a sequence of endogenous variables generated as a response to a sequence of random exogenous shocks with stochastic properties similar to those observed in the UK economy. In this part of the exercise, we also run the model using the sequences of shocks estimated previously—the question of interest is whether the model is able to replicate observed behaviour and to assess the relative importance of the shocks in explaining output fluctuations.

4.1 Demand shocks

In the literature on the cyclicality of mark-ups, attention has focused on the effects of demand shocks—indeed a main point of the research programme is to show that the interaction between mark-ups and demand shocks is important, and that a countercyclical mark-up will amplify the output response of a demand shock. In the model analysed here, a demand shock increases value added by a negative wealth effect: an increase in government expenditure increases the marginal utility of wealth, thereby increasing labour supply. Summarising the discussion in the introduction, the presence of a mark-up *alone* affects the quantitative response: a mark-up greater than unity effectively reduces the slope of the labour demand curve and hence increases the responsiveness of employment to a change in wages, as discussed in the introduction. The fact that the mark-up varies *cyclically* affects the position of the labour demand curve.

The key issue is how the demand shock affects expected future profits relative to current output, ie the ratio X_t/Y_t. An increase in government expenditure increases the marginal utility of wealth λ_t, and the marginal utility is on a decreasing path back towards steady state, ie $\hat{\lambda}_t > \hat{\lambda}_{t+1}$. This increases the real interest rate and lowers the discount factor, tending to lower X_t for a given output path. Provided a government expenditure shock expands current output and that the dynamic response of output follows a decreasing path, the ratio X_t/Y_t will unambiguously decrease. In the customer market model, the assumption is that the mark-up responds positively to such a decrease ($\epsilon_\mu < 0$), leading to a procyclical mark-up response. The fact that the mark-up moves procyclically reduces the positive output and employment effects of such a demand shock: an increase in the mark-up shifts the labour demand curve to the left, and this offsets the employment response but magnifies the wage response. In contrast, the implicit collusion model predicts that the cyclical movements in the mark-up will amplify the output response: the mark-up responds countercyclically ($\epsilon_\mu > 0$), shifting the labour demand schedule to the right, hence amplifying the output multiplier. The impulse response functions under the two different assumptions about mark-up dynamics are illustrated in Figure 5. The decrease in wages in the customer market model is quantitatively larger than the decrease in the implicit collusion model, reflecting the shift leftward of the labour demand curve; the hours and output response in the implicit collusion model is thus larger and closer to the empirical impulse response function, estimated in Section 3.2. Still, the output multiplier is smaller than the empirically estimated: with a government expenditure share of value added of around 22%, the empirical multiplier is close to 0.45, while the multiplier in the implicit collusion model is approximately 0.16 and in the customer market model only 0.04. As before, the propagation of the shock largely reflects the persistence of the exogenous shocks rather than internal propagation.

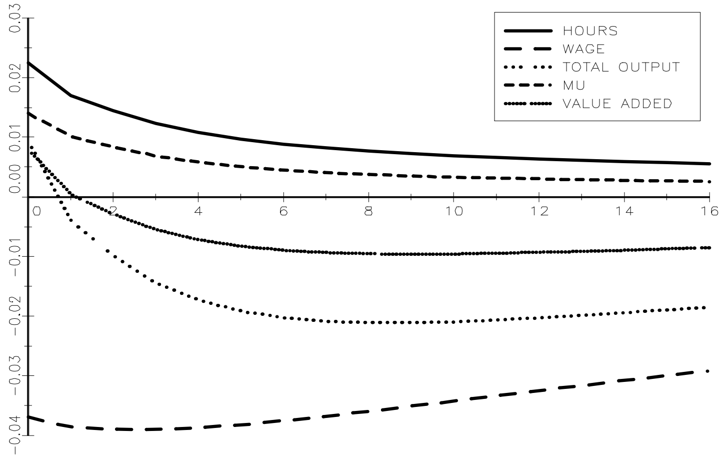
Rather than interpreting this as evidence in favour of the implicit collusion model, we interpret this as evidence that the model does not capture the effects of shocks to government expenditure well—even in the implicit collusion model, the elasticity of the mark-up, ϵ_μ, would have to increase by a factor of three to account for the size of the empirical multiplier; and the propagation of this initial effect would largely reflect that persistence in government expenditure.

4.2 A shock to the price of imported materials

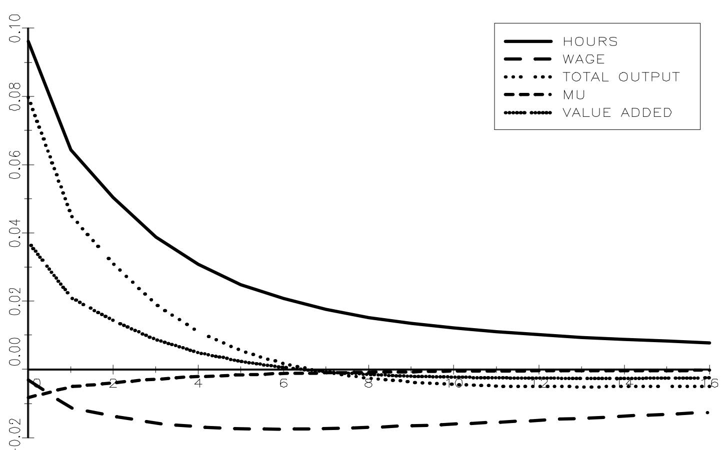
This section considers the response to a shock to p_t^E, the real price of imported materials. Recall that these shocks are identified as shocks to

Figure 5: Model impulse response. Government expenditure

(A) Customer markets



(B) Implicit collusion



the *nominal* price of imports, and that the stochastic process used here allows for interaction between lagged values of real and nominal prices. This is important, because the exogenous variables themselves are persistent—a shock to $\hat{p}_t^E$ is propagated exogenously, and this affects the response of the endogenous variables. Without such persistence, a surprise increase in $\hat{p}_t^E$ may have substantial immediate impact on output but is not propagated strongly, so that the effects of a shock die out quickly.

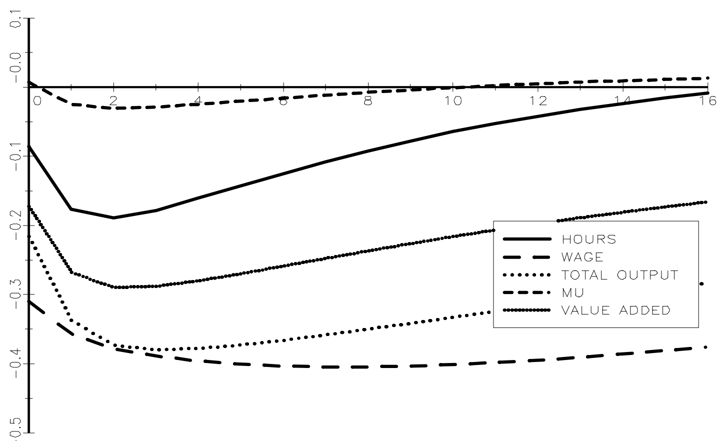
This is reflected in the impulse response functions generated by the model, reported in Figure 6: in both the customer market model and the implicit collusion model, the paths of output, value added, employment and wages all follow the path of the real price, described in Section 3.1. Independent of the choice of mark-up behaviour, a nominal price shock is propagated in the real variables over 6 to 8 quarters, where the empirical response indicates a stronger propagation. The peak effect differs sharply across the two models: in the implicit collusion model, the countercyclical behaviour of the mark-up amplifies the response, by magnifying the labour demand response. In the customer market, where the mark-up moves procyclically, the output response is dampened, bringing it closer into line with the magnitude of the response observed in the empirical impulse response function.

Notice that the customer market indirectly supports the idea of a gradual pass-through from import prices to domestic prices.[16] Even in a model such as the current one where prices are fully flexible, procyclical moves in the mark-up can lead to gradual pass-through because an increase in the price of imported materials effectively triggers a recession. This result supports the conclusions reached by McCallum and Nelson (1998) who also treat innovations in the price of imported materials as an exogenous 'supply shock'. In their model domestic inflation and real output are positively correlated, so although a positive price shock will raise the consumer price index directly, it will lower inflation because it is in effect a negative supply shock. A mark-up effect like the one described above will work *in addition* to this effect: not only would pass-through be delayed directly through an inflation effect, but also diminished by a decrease in the mark-up.

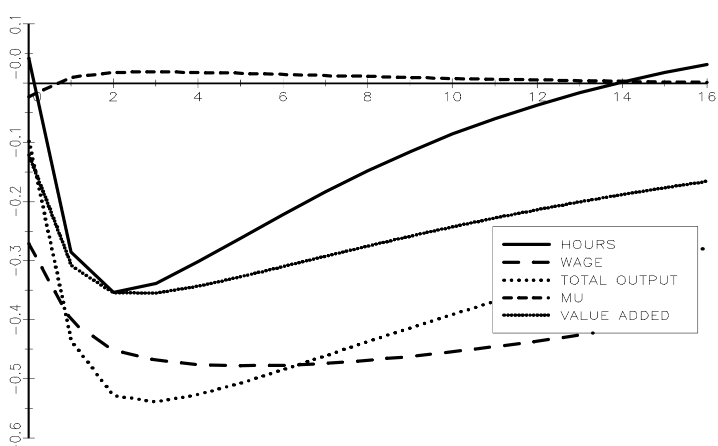
[16] In the empirical literature, the puzzle is formulated as a lack of high correlation between innovations in the real exchange rate and innovations in inflation—without nominal variables and a more elaborate model of trade, we cannot address the puzzle directly.

Figure 6: Model impulse response. Price of imported materials.

(A) Customer markets



(B) Implicit collusion



4.3 Technology

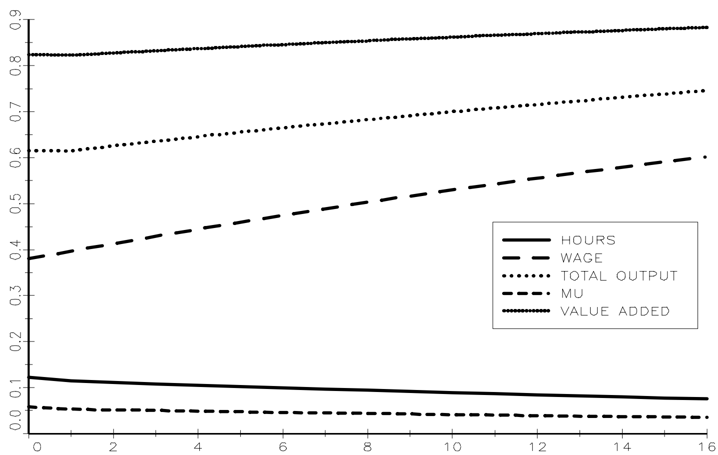
Recall that technology shocks are permanent—a shock to γ_t^z is a permanent increase in the level of labour-augmenting technological progress, and hence a permanent shift in output and its components to a new trend path. In Figure 1, a positive shock to the level of technology corresponds to a permanent shift to the right of the labour demand curve, where the magnitude of the shift depends on the size of the shock and the elasticities of substitution between the factors. As before, the presence of a mark-up alone affects the magnitude of the shock: a permanent increase in the marginal product of labour will, for any given level of capital, shift the labour demand curve relatively less in the presence of a mark-up. In addition, a technology shock increases the present value of future wealth, and the presence of a mark-up tends to increase this wealth effect. If the wealth effect is sufficiently large and dominates the intertemporal substitution effect, then hours may actually decrease in response to an increase in technology.

The response of the model under the two different assumptions about mark-up dynamics are illustrated in Figure 7. A shock to technology changes the steady-state path, and the dynamic adjustment after the shock is governed by the fact that a permanent shock to technology leaves the capital stock below its desired level—the marginal utility of wealth is above its new (and higher) long-run level, and this in effect increases the real interest rate above the long-run level; resources are shifted towards capital accumulation and investment overshoots its new long run-level (not illustrated). This additional accumulation can be brought about through several channels. First, for a given level of inputs, output has increased due to the increase in the level of technology. Second, labour supply can increase: there is a movement along the labour supply curve, following the shift in labour demand, and possibly a shift in the labour supply curve, if the marginal utility of wealth increases following this shock, ie if the incentive to accumulate capital is sufficiently strong. Both these effects tend to increase the total amount of available resources. Third, consumption can be lowered below its (new) long-run level, increasing resources available for investment for a given level of employment and output.

Notice that although output and wages are *below* their new steady-state levels, both will grow at any point in time after the shock. For the dynamics of the mark-up, it is again the ratio X_t/Y_t that

Figure 7: (A) Technology shocks. Customer markets

(i)



(ii)

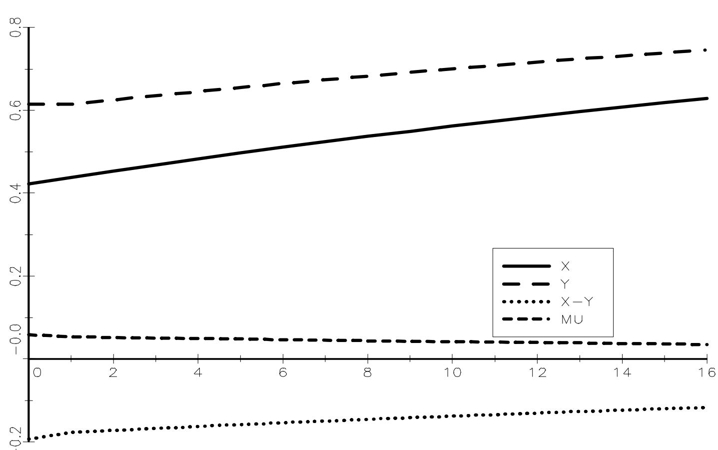
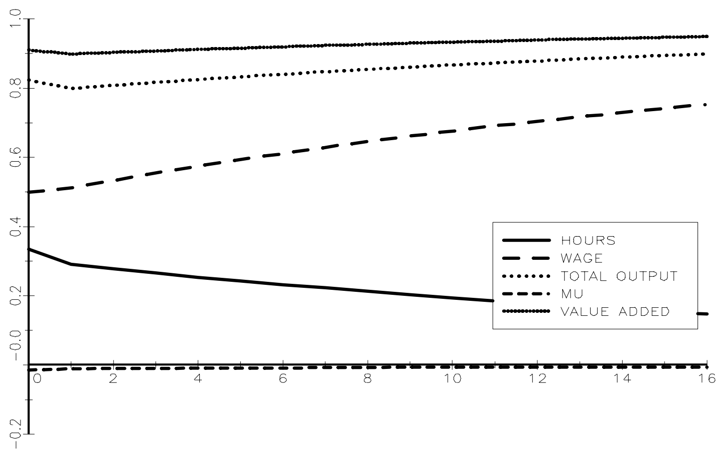
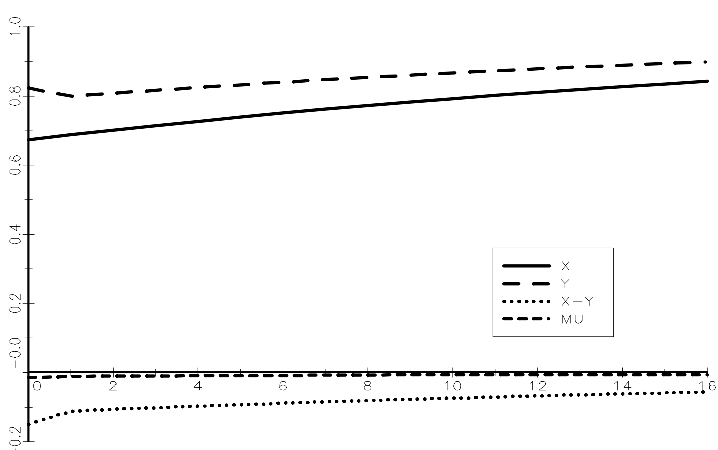


Figure 7: (B) Technology shocks. Implicit collusion

(i)



(ii)



matters; this ratio is, of course, stationary. The important point is that output is below its new trend level and is increasing: for a given discount factor and sequence of mark-ups, this will require that X_t undershoots its new long-run level more than Y_t, and that the ratio of the two decreases. In addition, the marginal utility of wealth exceeds its new long-run level $\left(\hat{\lambda}_t > \hat{\lambda}_{t+1}\right)$. This implies that real interest rates are below steady-state levels: the discount factor in calculating the expected present value of future profits decreases, leading to a decrease in X_t.

It should be emphasised that although output is below its new trend level, output is growing above the old trend level at all points. This is important in the simulation exercise: if the trend is measured correctly, ie detrended by the level of z_t, then the dynamic properties of the mark-up may differ—output is below trend and so is the ratio X_t/Y_t, so in the customer market model, the mark-up will be above steady state, and below in the implicit collusion model. In the simulation exercise, however, the detrending method is necessarily the same as the one applied to the data, ie a Hodrick-Prescott filter. If the filter smoothes output sufficiently, then the initial response of output may be interpreted as an above-trend increase; in the benchmark case, this materialises and the mark-up preserves its cyclical properties.

Apart from the mark-up, the dynamic responses of the endogenous variables are similar in terms of their patterns, but differ in magnitude—the customer market model amplifies the response to a technology shock while the implicit collusion model dampens it, in contrast to the demand shock. Unsurprisingly, in both cases the output response to a percentage change in γ_t^z is larger than the response to demand shocks.

4.4 Simulation properties

This section further characterises the quantitative properties of the model in a simulation exercise. The purpose of the exercise is to establish how well the model performs in the particular dimensions of interest, and whether the model is stable for small pertubations in the underlying parameters. Following standard dynamic general equilibrium methodology, the technique is to subject the model to sequences of exogenous random shocks, calculate the endogenous response and characterise this response by a set of standard statistics. These statistics are then compared with a set of statistics calculated on aggregate UK data. In the sensitivity analysis, selected parameters are varied to see the extent to which this affects the model's properties.

We use three statistical time series in this comparison. To compare with the model's value-added, we use the GDP (output) series, and for average wages the revised average earnings index. Both series are seasonally adjusted and taken directly from the Office for National Statistics. The measure of total hours worked is the series used in the Bank of England's forecasting model (see Bank of England (1999)). This series is based on the Labour Force Survey (LFS) measure of total hours worked in all industries from 1992-97. Prior to this, from 1984 to 1992, the corresponding series from the old LFS is used. This is adjusted to take into account the fact that the old LFS only covers Great Britain rather than the entire United Kingdom. Prior to 1984, no whole-economy measure exists: the historical relation between manufacturing and economy-wide total hours post–1984 is used together with the series for manufacturing hours to construct a longer run of data. All series are quarterly and seasonally adjusted, and the sample runs from 1963 Q1 to 1998 Q3.

The benchmark model is calibrated as described above. The length of the simulation is set at the sample length of the aggregate series, ie 140 quarters. For the statistics to be comparable, the series generated by the model and the UK aggregates are subject to the same detrending method: the series generated by the model are converted from deviations around a stochastic trend into levels, and then, like the UK data, filtered with a Hodrick-Prescott filter. The data used for this analysis correspond to the data used in the Bank's macroeconometric model (for details see Bank of England (1999)).

Table D reports standard deviations relative to output, output correlation and first-order autocorrelation of output, employment, and average wages from UK data and from the two models of mark-ups. For illustrative purposes, the table also reports statistics for imported and domestically produced materials—although we do not have equivalent time series evidence, the properties of these series are of interest.

As expected, the model largely replicates the properties of the standard dynamic general equilibrium models, such as Hansen (1985) or

Table D: Standard statistics. UK data and model

UK data

Variable	σ_x	$\frac{\sigma_x}{\sigma_{VA}}$	ρ_x	$\frac{\sigma_{xy}}{\sigma_x \sigma_{VA}}$
Hours worked	1.83	1.18	0.87	0.39
Average wages	1.75	1.12	0.72	0.48
GDP	1.56	1.00	0.80	1.00

Implicit collusion model

Variable	σ_x		$\frac{\sigma_x}{\sigma_{VA}}$		ρ_x		$\frac{\sigma_{xy}}{\sigma_x \sigma_{VA}}$	
Hours	1.27	*0.15*	0.55	*0.06*	0.77	*0.04*	0.80	*0.04*
Imp. mat.	2.85	*0.30*	1.23	*0.11*	0.76	*0.04*	0.87	*0.03*
Dom. mat.	2.29	*0.22*	0.98	*0.04*	0.73	*0.05*	0.98	*0.01*
Av. wage	1.36	*0.14*	0.58	*0.02*	0.74	*0.05*	0.98	*0.01*
Gross output	2.27	*0.22*	0.97	*0.04*	0.72	*0.05*	0.98	*0.01*
Mark-up	0.09	*0.01*	0.04	*0.01*	0.81	*0.03*	-0.60	*0.08*
Value added	2.34	*0.24*	1.00	*0.00*	0.71	*0.06*	1.00	*0.00*

Customer market model

Variable	σ_x		$\frac{\sigma_x}{\sigma_{VA}}$		ρ_x		$\frac{\sigma_{xy}}{\sigma_x \sigma_{VA}}$	
Hours	0.47	*0.05*	0.23	*0.03*	0.76	*0.04*	0.77	*0.05*
Imp. mat.	1.99	*0.20*	0.96	*0.09*	0.75	*0.04*	0.84	*0.04*
Dom. mat.	1.51	*0.15*	0.73	*0.01*	0.71	*0.06*	0.99	*0.00*
Av. wage	0.99	*0.10*	0.48	*0.01*	0.72	*0.05*	0.98	*0.01*
Gross output	1.59	*0.16*	0.76	*0.02*	0.71	*0.06*	0.99	*0.00*
Mark-up	0.18	*0.02*	0.09	*0.01*	0.73	*0.05*	0.89	*0.03*
Value added	2.09	*0.22*	1.00	*0.00*	0.71	*0.06*	1.00	*0.00*

Notes: σ_x is the standard deviation of the variable X, $\frac{\sigma_x}{\sigma_{VA}}$ the standard deviation of X relative to GDP, ρ_x the autocorrelation coefficient and $\frac{\sigma_{xy}}{\sigma_x \sigma_{VA}}$ the correlation coefficient with GDP. Standard errors in italics.

Danthine and Donaldson (1993). Like the standard models, the model generates insufficient relative variability in employment, and average wages that are too highly correlated with output. While these are indeed problematic features of the models, they are not a particular feature of this imperfectly competitive model or of the dynamic properties of the mark-up. Unlike the standard models, the models with imperfect competition generate excessive variability in output given the standard deviations of the shocks. This is caused by a much higher volatility in the shocks to $\hat{\gamma}_t^z$; we will return to this in the following section.

Comparing the two models of mark-ups in terms of their standard statistics, the most notable difference is that for given standard deviations of the shocks, the implicit collusion model generates larger absolute fluctuations in output and larger relative variability in employment and wages.

The important issue is the dynamics of the mark-up, and in particular the cross-correlation with output. Table E reports cross-correlations at various horizons. The signs of the contemporaneous correlations confirm the theoretical priors—given the stochastic properties of the shocks and the particular calibration, the customer market model generates procyclical mark-ups, while the implicit collusion model generates countercyclical mark-ups. The cross-correlation structure preserves the sign in both lags and leads, but in both cases the contemporaneous correlation is the highest. This is because the mark-up is a contemporaneous control variable, with μ_t being set after the shocks of period t have been observed—experiments with a model where μ_t is measured with respect to information at $t-1$, ie the mark-up is a predetermined variable, suggest that this may lead to a much richer lag and lead structure of these cross-correlations. In general, this complicates the dynamics of the model considerably, taking it further away from a standard dynamic general equilibrium model.

In terms of the calibration exercise, this is the important result—the customer market model and the implicit collusion model generate cross-correlations between the mark-up and output that are unambiguously signed at short leads and lags. With the empirical evidence for the United Kingdom suggesting a (weakly) procyclical mark-up, the simulation exercise suggests that the customer market

Table E: Cross-correlation between mark-up and output

Model \ j	-3	-2	-1	0	1	2	3
Implicit collusion							
$\frac{\sigma(\mu_{t+j}, VA_t)}{\sigma_\mu \sigma_{VA}}$	-0.11	-0.27	-0.42	-0.60	-0.45	-0.29	-0.15
standard error	*0.15*	*0.14*	*0.11*	*0.09*	*0.10*	*0.13*	*0.15*
Customer market							
$\frac{\sigma(\mu_{t+j}, VA_t)}{\sigma_\mu \sigma_{VA}}$	0.16	0.36	0.60	0.89	0.66	0.45	0.27
standard error	*0.12*	*0.10*	*0.07*	*0.03*	*0.07*	*0.11*	*0.13*

model is the 'correct' model in the sense that it replicates the particular feature that we are interested in. In addition, the table suggests that we cannot accommodate the differing views on the empirical cyclical properties of the mark-up within one theoretical model—only the customer market model delivers the procyclical features we claim to observe empirically in the United Kingdom.

Given this conclusion, it is interesting to establish the extent to which this result is affected by changes in parameters. Rather than carrying out a full-scale sensitivity analysis, we focus on the parameters for which the calibration exercise suggested the greatest uncertainty.

Unlike in the implicit collusion model, no parameter restrictions tie down the elasticity of the mark-up with respect to the ratio of expected future profits to current output. The benchmark is set at -0.3 and the standard statistics, including the cross-correlation between output and the mark-up, are not significantly affected for small changes of this value. Large mark-up elasticities can however affect the dynamics significantly, as revealed by Table F. The table reports results from experiments where the elasticity of the mark-up is increased (in absolute value) to -0.7 and -1.

An increase in the elasticity of the mark-up dampens the volatility in output, employment and wages; for a sufficient (absolute) increase in the elasticity, the mark-up becomes close to acyclical, with a negative correlation at all lags. To shed some light at this results, Figure 8 plots the impulse response of the expected present value of future profits X_t and current output Y_t, the main determinants of the mark-up, in

Table F: Sensitivity analysis. Cross-correlation between mark-up and output in the customer market model

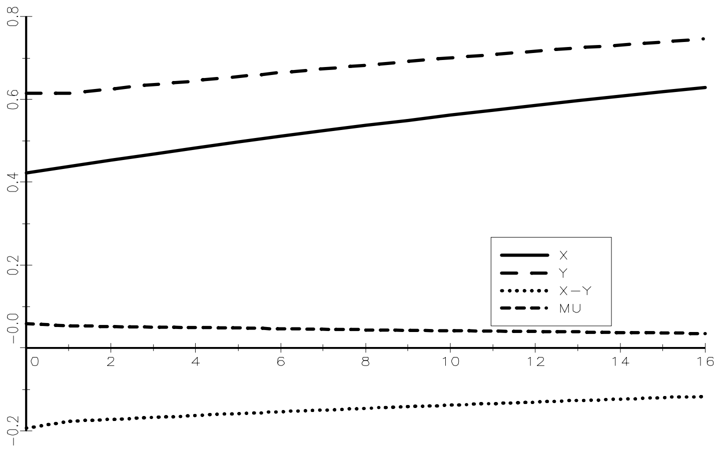
ϵ_μ	j	-3	-2	-1	0	1	2	3
-0.3	$\frac{\sigma(\mu_{t+j},VA_t)}{\sigma_\mu \sigma_{VA}}$	0.16	0.36	0.60	0.89	0.66	0.45	0.27
	standard error	*0.12*	*0.10*	*0.07*	*0.03*	*0.07*	*0.11*	*0.13*
-0.7	$\frac{\sigma(\mu_{t+j},VA_t)}{\sigma_\mu \sigma_{VA}}$	-0.02	-0.23	-0.50	-0.85	-0.63	-0.46	-0.32
	standard error	*0.10*	*0.10*	*0.09*	*0.05*	*0.08*	*0.11*	*0.13*
-1	$\frac{\sigma(\mu_{t+j},VA_t)}{\sigma_\mu \sigma_{VA}}$	0.08	-0.13	-0.45	-0.89	-0.64	-0.45	-0.30
	standard error	*0.08*	*0.09*	*0.08*	*0.03*	*0.07*	*0.10*	*0.12*

response to a government expenditure shock for $\epsilon_\mu = \{-0.3, -1\}$ where -0.3 corresponds to the benchmark value. A large response in the mark-up (when $\epsilon_\mu = -1$) implies a *reversal* in the sign of the correlation between the mark-up and output. The explanation for this is simply that with a sufficient mark-up response, output will exceed its new steady-state level in response to a technology shock; this will force the ratio X_t/Y_t to increase rather than decrease. This is illustrated in Figure 8: an increase in output pushed the expected present value of profits further up than current output, causing a fall in the mark-up. It requires a large elasticity to overshoot the new long-run level, and the undershooting path may appear more plausible: if output overshoots its new long-run level, then in all but the first period output growth would be falling. This implies a counterfactual sharp expansion and prolonged contraction. Note also that the correlation is not linear in the parameter ϵ_μ: in the interval between 0.35 and 0.65, the equations in the system becomes linearly dependent and no solution can be found.

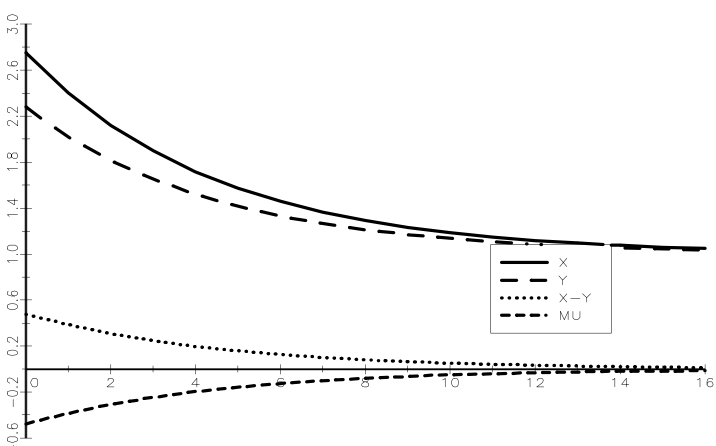
This also implies that the split between temporary and permanent technology shocks matters: with a positive but temporary technology shock, output will exceed its steady-state level, and the mark-up in the customer market model may become countercyclical. Here we have chosen to model technology shocks as permanent and hence shut down that potential channel of countercyclicality in the mark-up. It is not

Figure 8: Sensitivity analysis. Cross-correlation between mark-up and output

(A) Customer markets. $\epsilon_\mu = -0.3$



(B) Customer markets. $\epsilon_\mu = -1$



obvious that this is the right choice—the issue is at the heart of the controversy of permanent vs temporary and demand vs supply shocks; it does however re-emphasise the importance of technology shocks in determining the properties of the model.[17]

The production function parameters estimated previously suggest a high degree of substitutability in the three elasticities in the production function. Recall that the functional form has been restricted to allow substitution between capital and labour, ϵ_{KH}, between imported and domestically produced materials, ϵ_{EM}, and between the value added and materials aggregators V_t and Q_t, ϵ_{VQ}. Table G suggests that the benchmark results with the customer markets model are fairly robust when lowering these elasticities, reporting results from four simulations where the elasticities of substitution are varied. The results in the first two columns are obtained by lowering the elasticity of substitution between labour and capital to 0.6 and to 0.2 respectively. Reducing this elasticity reduces the absolute variability in output and the relative variability in employment relative to the benchmark, but the autocorrelation patterns remains largely unchanged. When the elasticity of substitution between imported and domestically produced materials is lowered, the relative standard deviation of the larger component of Q_t, M_t, increases while that of the smaller, E_t, decreases. Similarly, when the elasticity of substitution between V_t and Q_t decreases, the relative volatility in the bigger component, V_t, increases while that of Q_t decreases.

In addition to these results, the model's sensitivity to changes in the preference parameters (σ and ϵ_{HW}), the steady-state level of real interest rates and the steady-state level of the mark-up has been assessed. For small changes in these parameters, the simulation properties are not affected materially and we do not report further results from these experiments.

As a final assessment of the model, we simulate the endogenous response to a particular sequence of shocks: we trace the response of output, employment, wages and the mark-up for the sequence of shocks that we have derived for the United Kingdom. The purpose of the exercise is to get an idea of the relative importance of the shocks and the extent to which the model generates cycles that look like the ones that have actually been observed.

(17) For a discussion of permanent vs temporary technology shocks, see Ravn (1997).

Table G: (A) Sensitivity analysis. Changes in elasticity of substitution between labour and capital

Elasticity of substitution = 0.02

Variable	σ_x		$\frac{\sigma_x}{\sigma_{VA}}$		ρ_x		$\frac{\sigma_{xy}}{\sigma_x \sigma_{VA}}$	
Hours	1.68	*0.20*	1.26	*0.06*	0.72	*0.06*	-0.94	*0.02*
Imp. mat.	1.23	*0.17*	0.93	*0.15*	0.80	*0.04*	0.21	*0.13*
Dom. mat.	0.34	*0.05*	0.26	*0.04*	0.83	*0.03*	0.18	*0.15*
Av. wage	1.57	*0.19*	1.17	*0.08*	0.71	*0.06*	-0.91	*0.03*
Gross output	0.37	*0.05*	0.28	*0.05*	0.82	*0.03*	0.27	*0.14*
Mark-up	0.10	*0.01*	0.07	*0.01*	0.69	*0.06*	0.87	*0.03*
Value added	1.34	*0.14*	1.00	*0.00*	0.70	*0.06*	1.00	*0.00*

j	-3	-2	-1	0	1	2	3
$\frac{\sigma(\mu_{t+j}, VA_t)}{\sigma_\mu \sigma_{VA}}$	0.16	0.36	0.60	0.89	0.66	0.45	0.27
standard. error	*0.12*	*0.10*	*0.07*	*0.03*	*0.07*	*0.11*	*0.13*

Elasticity of substitution = 0.6

Variable	σ_x		$\frac{\sigma_x}{\sigma_{VA}}$		ρ_x		$\frac{\sigma_{xy}}{\sigma_x \sigma_{VA}}$	
Hours	0.32	*0.04*	0.17	*0.03*	0.81	*0.03*	-0.24	*0.15*
Imp. mat.	1.71	*0.19*	0.90	*0.11*	0.76	*0.04*	0.76	*0.05*
Dom. mat.	1.15	*0.11*	0.60	*0.02*	0.72	*0.05*	0.98	*0.01*
Av. wage	0.45	*0.05*	0.24	*0.03*	0.81	*0.04*	0.67	*0.07*
Gross output	1.24	*0.12*	0.65	*0.02*	0.72	*0.05*	0.98	*0.01*
Mark-up	0.20	*0.02*	0.10	*0.01*	0.71	*0.05*	0.95	*0.01*
Value added	1.92	*0.20*	1.00	*0.00*	0.71	*0.06*	1.00	*0.00*

j	-3	-2	-1	0	1	2	3
$\frac{\sigma(\mu_{t+j}, VA_t)}{\sigma_\mu \sigma_{VA}}$	0.16	0.34	0.56	0.87	0.63	0.42	0.24
standard error	*0.12*	*0.11*	*0.08*	*0.03*	*0.07*	*0.11*	*0.12*

Table G: (B) Sensitivity analysis. Changes in elasticity of substitution

Elasticity of substitution between
domestic and imported materials = 0.01

Variable	σ_x		$\frac{\sigma_x}{\sigma_{VA}}$		ρ_x		$\frac{\sigma_{xy}}{\sigma_x \sigma_{VA}}$	
Hours	0.47	*0.05*	0.23	*0.03*	0.76	*0.04*	0.77	*0.05*
Imp. mat.	1.59	*0.15*	0.76	*0.03*	0.72	*0.05*	0.97	*0.01*
Dom. mat.	1.58	*0.15*	0.76	*0.03*	0.72	*0.05*	0.97	*0.01*
Av. wage	0.99	*0.10*	0.48	*0.01*	0.72	*0.05*	0.98	*0.01*
Gross output	1.59	*0.16*	0.76	*0.02*	0.71	*0.06*	0.99	*0.00*
Mark-up	0.18	*0.02*	0.09	*0.01*	0.73	*0.05*	0.89	*0.03*
Value added	2.09	*0.22*	1.00	*0.00*	0.71	*0.06*	1.00	*0.00*

j	-3	-2	-1	0	1	2	3
$\frac{\sigma(\mu_{t+j}, VA_t)}{\sigma_\mu \sigma_{VA}}$	0.16	0.36	0.60	0.89	0.66	0.45	0.27
standard error	*0.12*	*0.10*	*0.07*	*0.03*	*0.07*	*0.11*	*0.13*

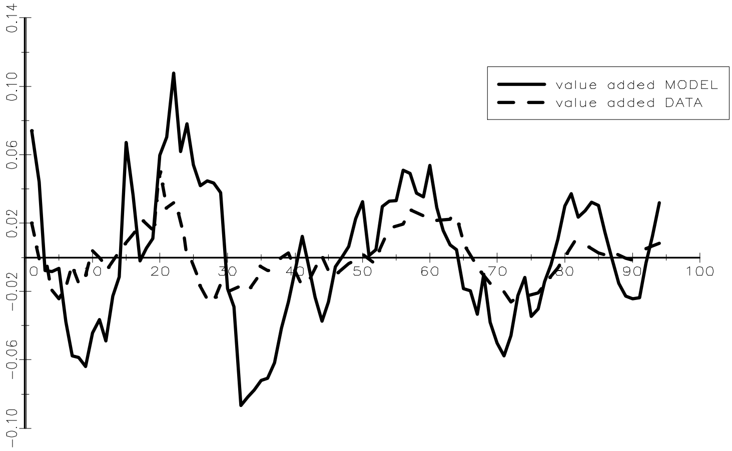
Elasticity of substitution
between V and Q = 0.01

Variable	σ_x		$\frac{\sigma_x}{\sigma_{VA}}$		ρ_x		$\frac{\sigma_{xy}}{\sigma_x \sigma_{VA}}$	
Hours	0.54	*0.07*	0.26	*0.04*	0.78	*0.04*	0.64	*0.07*
Imp. mat.	1.91	*0.18*	0.92	*0.07*	0.74	*0.05*	0.92	*0.02*
Dom. mat.	1.57	*0.17*	0.76	*0.01*	0.70	*0.06*	0.99	*0.00*
Av. wage	0.99	*0.10*	0.48	*0.01*	0.73	*0.05*	0.98	*0.01*
Gross output	1.61	*0.17*	0.77	*0.01*	0.71	*0.06*	1.00	*0.00*
Mark-up	0.17	*0.02*	0.08	*0.00*	0.71	*0.05*	0.95	*0.01*
Value added	2.08	*0.22*	1.00	*0.00*	0.71	*0.06*	1.00	*0.00*

j	-3	-2	-1	0	1	2	3
$\frac{\sigma(\mu_{t+j}, VA_t)}{\sigma_\mu \sigma_{VA}}$	0.17	0.38	0.64	0.95	0.71	0.48	0.29
standard error	*0.11*	*0.09*	*0.06*	*0.01*	*0.06*	*0.10*	*0.12*

Figure 9: Output. Model vs data

(A) Customer markets



(B) Implicit collusion

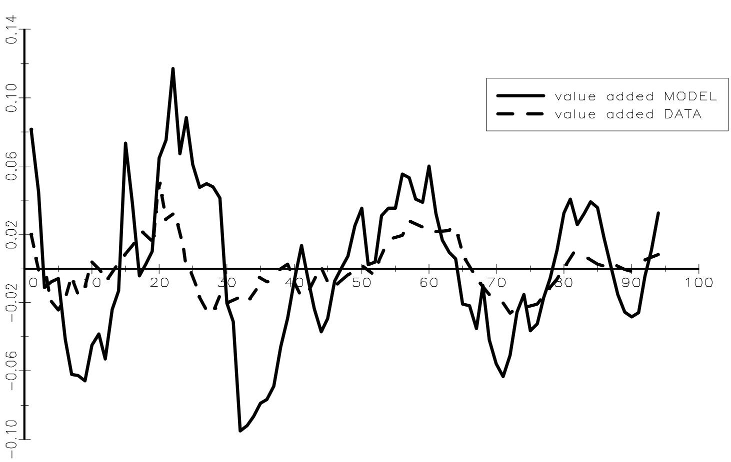


Figure 9 plots the deviations from trend, derived using a Hodrick-Prescott filter, of output generated by the customer market and implicit collusion model, and of actual output. As noted previously, both models generate excessive cyclical fluctuations in output, although the cyclical pattern is broadly right—the correlation between actual and generated output is high. The two competing models generate almost identical cyclical patterns, with the fluctuations in the customer market model being smaller.

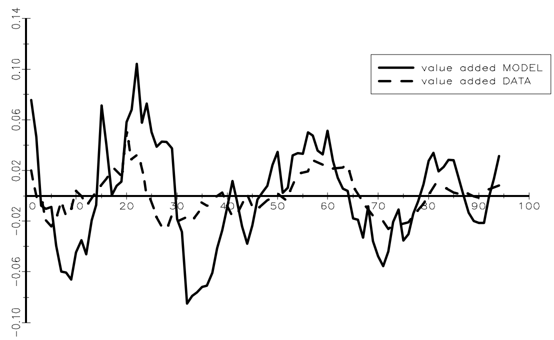
These fluctuations are largely driven by the technology shocks. Figure 10 illustrates the output paths generated by the customer market model when selected shocks are 'turned off' by inserting sequences with no shocks in place of the ones derived from data. In panel (A), where only technology shocks are non-zero, the model mimics the path with all shocks non-zero closely, while in panels (B) and (C), with only shocks to government expenditure and to the price of imported materials respectively, it generates insufficient volatility in output.

This is a particularly important observation because of the sensitivity of the technology measure: recall the previous demonstration that a TFP measure based on a production function that allows for substitution between the materials index and the capital/labour index implies higher volatility in TFP growth, caused by the volatility in the price of imported materials. If we use the measure suggested by Rotemberg and Woodford (1995) and, inconsistent with the model assumptions, assume no substitution between V and Q, the fluctuations in model output are dampened strongly. As illustrated in Figure 11, this also implies that the cyclical pattern accords better with the actual output sequence. This merely underlines that what is important for output dynamics (or in general, the dynamics of real variables) tends to be real shocks, and here mainly shocks to TFP growth, and that the internal propagation mechanism of the model is weak.(18)

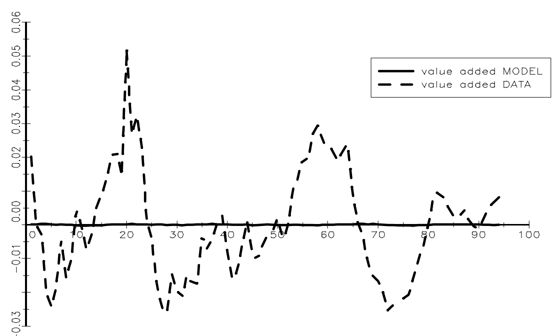
(18) This assumption would also hold if we assumed temporary productivity shocks—these would allow a better cyclical fit, but would not change the conclusion that TFP shocks drive output.

Figure 10: Output path. Model vs data. One shock only

(A) Productivity shocks only



(B) Government expenditure shocks only



(C) Shocks to price of imported materials only

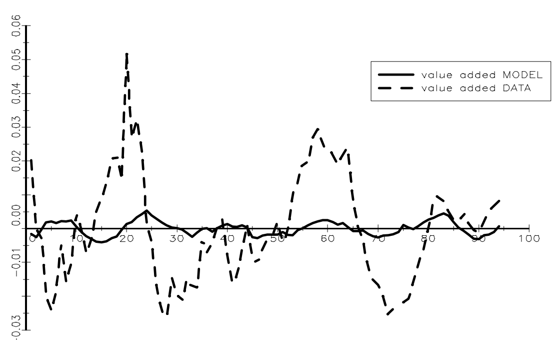
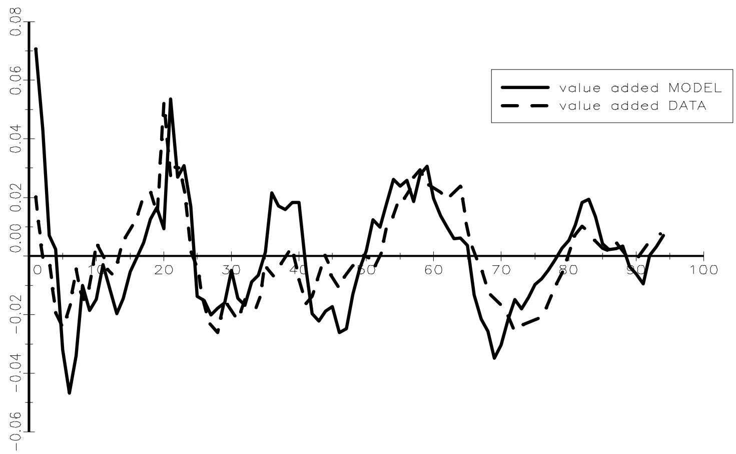


Figure 11: Output, model vs data. Alternative productivity shocks



5 Conclusion

This paper has characterised the dynamics of mark-ups, output and employment in a dynamic general equilibrium model, based on Rotemberg and Woodford (1995). We compare two models of the mark-up calibrated on UK data and find that they result in cross-correlation structures with output that allow us to distinguish them empirically—only the customer market model provides a procyclical mark-up in line with the evidence in Small (1997). We show, in the impulse response analysis, that the customer market model leads to a gradual pass-through of shocks to the price of imported materials, in accordance with empirical evidence on real exchange shocks. In addition, we demonstrate that the conflicting views on the cyclicality of the mark-up cannot be accommodated within one model—the result that the mark-up is procyclical in the customer market and countercyclical in the implicit collusion model stand for a wide range of parameter values and is apparently not affected by the specific sequence of shocks. We conclude that for an aggregate view of mark-up behaviour, the customer market model is the most appropriate.

It is worth recalling that this model inherits some of the weaknesses of standard dynamic general equilibrium models. In particular, the internal propagation mechanism is weak and the dynamics of the model are largely given by the stochastic properties of the exogenous shocks. The dynamics of the mark-up do not affect this: the behaviour of the mark-up affects the immediate response to a shock but does not itself provide a stronger propagation mechanism.

Appendix
Solving the model

This technical appendix provides details of how the model is solved. As mentioned previously, in this analysis we focus on (small) business cycle fluctuations around a steady-state growth path, allowing for non-stationary, stochastic labour-augmenting technical progress and for deterministic population growth.

A.1 Stationary representation

To analyse this economy, we transform the non-stationary equilibrium conditions into stationary ones, and make a log–linear approximation around the steady-state of this stationary system, using the method described by King, Plosser, and Rebelo (1988b). Define:

$$\begin{aligned}\tilde{Y}_t &= \frac{Y_t}{z_t N_t};\ \tilde{V}_t = \frac{V_t}{z_t N_t};\ \tilde{Q}_t = \frac{Q_t}{z_t N_t}; \\ \tilde{K}_t &= \frac{K_t}{z_{t-1} N_t};\ \tilde{H}_t = \frac{H_t}{N_t};\ \tilde{w}_t = \frac{w_t}{z_t};\ \tilde{\lambda}_t = \lambda_t z_t^{\sigma}\end{aligned} \qquad (\mathbf{A1})$$

With the homogeneity assumption on $c(.)$ and $h(.)$, the variables defined in (**A1**) are stationary given stationary process for transformed exogenous variables, defined as:

$$\tilde{G}_t = \frac{G_t}{z_t N_t},\ \tilde{I}_t = \frac{I_t}{z_t N_t},\ \gamma_t^z = \frac{z_t}{z_{t-1}},\ \gamma^N = \frac{N_t}{N_{t-1}} \qquad (\mathbf{A2})$$

The equilibrium conditions (**29**) to (**36**) are rewritten in terms of these variables. In their stationary form, these equilibrium conditions are:

$$\tilde{C}_t = \frac{N_t c\left(w_t, \lambda_t\right)}{z_t N_t} = c\left(\frac{w_t}{z_t}, \lambda_t z_t^{\sigma}\right) = c\left(\tilde{w}_t, \tilde{\lambda}_t\right) \qquad (\mathbf{A3})$$

$$\tilde{H}_t = h\left(w_t, \lambda_t\right) \qquad (\mathbf{A4})$$

$$\begin{aligned}& Y\left(F\left(\left(\gamma_t^z\right)^{-1}\tilde{K}_t, \tilde{H}_t\right) - \tilde{I}_t\phi, Q\left(\tilde{E}_t, \hat{M}_{jt}\right)\right) - p_t^E\tilde{E}_t - \tilde{M}_t \\ =\ & c\left(\tilde{w}_t, \tilde{\lambda}_t\right) + \left(\tilde{K}_{t+1}\gamma^N + (1-\delta)\tilde{K}_t\gamma_t^{z-1}\right) + \tilde{G}_t\end{aligned} \qquad (\mathbf{A5})$$

$$E_t \left\{ \left(\gamma^z_{t+1}\right)^{-\sigma} \frac{\tilde{\lambda}_{t+1}}{\tilde{\lambda}_t} \left(\frac{Y_{Vt+1} V_K \left(\left(\gamma^z_{t+1}\right)^{-1} \tilde{K}_{t+1}, \tilde{H}_{t+1} \right)}{\mu_{t+1}} + 1 - \delta \right) \right\} = \frac{1}{\beta} \tag{A6}$$

$$Y_V \left(\tilde{V}_t, Q_t\right) V_H \left((\gamma^z_t)^{-1} \tilde{K}_t, \tilde{H}_t \right) = \mu_t \tilde{w}_t \tag{A7}$$

$$Y_Q \left(\tilde{V}_t, Q_t\right) Q_E \left(\tilde{E}_t, \tilde{M}_t\right) = \mu_t p^E_t \tag{A8}$$

$$Y_Q \left(\tilde{V}_t, Q_t\right) Q_M \left(\tilde{E}_t, \tilde{M}_t\right) = \mu_t \tag{A9}$$

The state variable accounting for expected future profits can be written as

$$\frac{\tilde{X}_t}{\tilde{I}_t} = E_t \alpha \beta \frac{\tilde{\lambda}_{t+1}}{\tilde{\lambda}_t} \left(\gamma^z_{t+1}\right)^{-\sigma} \left\{ \left(\frac{\mu_{t+1} - 1}{\mu_{t+1}} \right) \frac{\tilde{Y}_{t+1}}{\tilde{I}_{t+1}} + \frac{\tilde{X}_{t+1}}{\tilde{I}_{t+1}} \right\} \tag{A10}$$

Notice that investment is implicit in the resource constraint:

$$\tilde{K}_{t+1} \gamma^N = (1 - \delta) \tilde{K}_t \left(\gamma^z_t\right)^{-1} + \tilde{\chi}_t$$

A.2 Steady state

For the purpose of calibration and simulation, we solve the model for its non-stochastic steady state. Let no subscript t indicate steady-state value. First, we assume that in steady state, there are no pure profits. From the Euler theorem and the assumption that Y, Q and V have constant returns to scale, gross output (ie output after fixed costs have been deducted) can be written as

$$Y = Y_Q Q_E E + Y_Q Q_M M + Y_V V_H z H + Y_V V_K K - Y_V \phi I \tag{A11}$$

We define total factor costs as

$$\begin{aligned} TFC &= p^E E + M + wh + rK \\ &= \frac{Y_Q Q_E}{\mu} E + \frac{Y_Q Q_M}{\mu} M + \frac{Y_V V_H}{\mu} H + \frac{Y_V V_K}{\mu} K \\ &= \frac{Y + Y_V \phi I}{\mu} \end{aligned} \tag{A12}$$

Pure profits are defined as output minus factor costs

$$\begin{aligned}\Pi &= Y - TFC = \left(1 - \frac{1}{\mu}\right) Y - \frac{Y_V \phi I}{\mu} \\ &= \left(1 - \frac{1}{\mu}\right) Y - \frac{Y_V \phi I}{\mu} \end{aligned} \quad \textbf{(A13)}$$

The assumption of no pure profits imposes the condition that $Y = TFC$, ie that:

$$\begin{aligned} Y_V \phi I &= (\mu - 1) Y \\ \frac{Y_V V}{Y} \frac{\phi I}{V} &= (\mu - 1) \end{aligned}$$

The factor shares can be written

$$s_E = \frac{p^E E}{TFC} = \frac{Y_Q Q_E E}{\mu Q} \frac{Q}{Y}; \; s_M = \frac{M}{TFC} = \frac{Y_Q Q_M M}{\mu Q} \frac{Q}{Y}; \quad \textbf{(A14)}$$

$$s_H = \frac{wH}{TFC} = \frac{Y_V V_H H}{\mu F} \frac{F}{Y}; \; s_K = \frac{rK}{TFC} = \frac{Y_V V_K K}{\mu F} \frac{F}{Y} \quad \textbf{(A15)}$$

This also implies that:

$$s_E + s_M + s_H + s_K = 1 \quad \textbf{(A16)}$$

We can then write that

$$\begin{aligned} Y &= Y_V V + Y_Q Q = Y_V V + Y_Q Q_E E + Y_Q Q_M M \;\; \Rightarrow \\ \frac{Y_V V}{Y} &= 1 - \mu \left(s_E + s_M\right) \end{aligned} \quad \textbf{(A17)}$$

The key is that because there are zero profits, $TFC = Y$. Also, the share of value added in total output is

$$\begin{aligned} s_V &= \frac{V}{Y} = 1 - \frac{p^E E + M}{Y} \\ &= 1 - \frac{Y_Q Q_E}{\mu Y} E - \frac{Y_Q Q_M}{\mu Y} M = 1 - \left(s_E + s_M\right) \end{aligned} \quad \textbf{(A18)}$$

Solving for the steady-state capital to investment ratio, we can tie down the capital investment ratio as a function of the model parameters only:

$$(1 - \delta) \tilde{K} \left(\gamma^z\right)^{-1} + \tilde{\chi} = \tilde{K} \gamma^N$$

$$
\begin{aligned}
\tilde{K}(\gamma^N\gamma^z-(1-\delta)) &= \gamma^z\tilde{X} \\
\frac{\tilde{K}}{\tilde{\chi}} &= \frac{\gamma^z}{\gamma^N\gamma^z+(\delta-1)} &\textbf{(A19)} \\
\frac{\tilde{\chi}}{\tilde{Y}} &= \frac{\tilde{\chi}}{\tilde{K}}\frac{1}{r}\frac{rK}{\tilde{Y}} \\
&= \frac{r\gamma^z}{\gamma^N\gamma^z+(\delta-1)}s_K &\textbf{(A20)}
\end{aligned}
$$

Similarly, the steady-state real rate of return after depreciation, $r-\delta$, is tied down as:

$$
\begin{aligned}
\beta\left(\gamma^z\right)^{-\sigma}(r-\delta+1) &= 1 \\
1+r-\delta &= \frac{\left(\gamma^z\right)^{\sigma}}{\beta} &\textbf{(A21)}
\end{aligned}
$$

$$
\begin{aligned}
\frac{\tilde{X}}{\tilde{I}} &= \alpha\beta\left(\gamma^z\right)^{-\sigma}\left\{\left(\frac{\mu-1}{\mu}\right)\frac{\tilde{Y}}{\tilde{I}}+\frac{\tilde{X}}{\tilde{I}}\right\}\Rightarrow \\
\frac{\tilde{X}/\tilde{I}}{\left(\frac{\mu-1}{\mu}\right)\tilde{Y}/\tilde{I}+\tilde{X}/\tilde{I}} &= \frac{\alpha\beta}{\left(\gamma^z\right)^{\sigma}}=\frac{\alpha}{1+r-\delta} &\textbf{(A22)} \\
\frac{\left(\frac{\mu-1}{\mu}\right)\tilde{Y}/\tilde{I}}{\left(\frac{\mu-1}{\mu}\right)\tilde{Y}/\tilde{I}+\tilde{X}/\tilde{I}} &= \left(\frac{\left(\gamma^z\right)^{\sigma}}{\alpha\beta}-1\right)\frac{\tilde{X}}{\tilde{I}} \\
&= 1-\frac{\alpha\beta}{\left(\gamma^z\right)^{\sigma}}=\frac{1+r-\delta-\alpha}{1+r-\delta} &\textbf{(A23)}
\end{aligned}
$$

Also, $c(w,\lambda)$ is hod(1) and $h(w,\lambda)$ is hod(1):

$$
\begin{aligned}
\epsilon_{Hw}-\sigma\epsilon_{H\lambda} &= 0 \\
\epsilon_{Cw}-\sigma\epsilon_{C\lambda} &= 1
\end{aligned}
$$

and:

$$
\begin{aligned}
\frac{\epsilon_{Cw}}{\epsilon_{Hw}} &= \frac{\sigma-1}{\sigma}\frac{\tilde{w}\tilde{H}}{\tilde{C}} \\
&= \frac{\sigma-1}{\sigma}\frac{\tilde{w}\tilde{H}/\tilde{Y}}{\frac{\tilde{C}}{\tilde{V}}\frac{\tilde{V}}{\tilde{Y}}}=\frac{\sigma-1}{\sigma}\frac{s_H}{s_c[1-\mu(s_E+s_M)]}
\end{aligned}
$$

A.3 Fluctuations around steady state

For small deviations around steady state, we can approximate the actual value of a variable Z_t by its deviations from steady state. If we use an approximation to make the system linear, we can then solve and transform back into levels. This is basically a Taylor approximation:

$$\hat{Z}_t = \ln\left(\frac{Z_t}{Z}\right) \approx \ln\left(\frac{Z + \Delta Z_t}{Z}\right) = \ln\left(1 + \frac{\Delta Z_t}{Z}\right) \approx \frac{\Delta Z_t}{Z}$$

For a function of a variable $f(Z_t)$, we find

$$\frac{d \ln f(Z_t)}{dZ_t} \approx \frac{f'(Z_t)}{f(Z_t)}|_Z * \Delta Z_t = \frac{f'(Z_t)\, Z}{f(Z_t)}|_Z * \frac{\Delta Z_t}{Z} = \frac{f'(Z_t)\, Z}{f(Z_t)}|_Z * \hat{Z}_t$$

The term $\frac{f'(Z_t)Z}{f(Z_t)}$ can usually be written as an elasticity or a share, evaluated in steady state. This system of non-linear equations is then approximated around the (non-stochastic) steady state by what is effectively a first-order Taylor approximation. The outcome is a system of linear difference equations, written in variables defined in deviations from steady state, where the coefficients are derivatives evaluated at steady state. Let 'hatted' variables denote percentage deviation from steady state, eg $\hat{Z}_t = \ln\left(\frac{\tilde{Z}_t}{\tilde{Z}}\right)$. The approximations of (**29**) to (**36**) are then

$$\frac{1}{\epsilon_{VQ}}\left(\hat{Y}_t - \hat{Q}_t\right) + \frac{1}{\epsilon_{EM}}\left(\hat{Q}_t - E_t\right) = \hat{p}_t^E + \hat{\mu}_t \qquad \textbf{(A24)}$$

$$\frac{1}{\epsilon_{VQ}}\left(\hat{Y}_t - \hat{Q}_t\right) + \frac{1}{\epsilon_{EM}}\left(\hat{Q}_t - \hat{M}_t\right) = \hat{\mu}_t \qquad \textbf{(A25)}$$

$$\frac{1}{\epsilon_{VQ}}\left(\hat{Y}_t - \hat{V}_t\right) + \frac{1}{\epsilon_{HK}}\left(\hat{V}_t - \hat{H}_t\right) = \hat{w}_t + \hat{\mu}_t \qquad \textbf{(A26)}$$

$$\frac{1}{\epsilon_{VQ}}\left(\hat{Y}_t - \hat{V}_t\right) + \frac{1}{\epsilon_{HK}}\left(\hat{V}_t - \hat{K}_t\right) = \hat{r}_t + \hat{\mu}_t \qquad \textbf{(A27)}$$

$$\begin{aligned}\hat{Y}_t &= s_E\left(\hat{E}_t - \hat{p}_t^E\right) + s_M \hat{M}_t + s_C s_V \hat{c}_t + s_G s_V \hat{G}_t \qquad \textbf{(A28)} \\ &\quad + \frac{s_I s_V}{\gamma^N \gamma^z + (\delta - 1)}\left[\gamma^N \gamma^z E_t \hat{K}_{t+1} - (1-\delta)\left(\hat{K}_t - \hat{\gamma}_t^z\right)\right]\end{aligned}$$

$$\sigma E_t \hat{\gamma}_{t+1} = E_t \hat{\lambda}_{t+1} - \hat{\lambda}_t + \frac{r}{1 + r - \delta} E_t \hat{r}_{t+1} \qquad \textbf{(A29)}$$

$$\begin{aligned}
\mu_t &= \epsilon_\mu \left(\hat{X}_t - \hat{Y}_t\right) && (\mathbf{A30}) \\
\hat{X}_t - \hat{I}_t &= E_t\left\{\hat{\lambda}_{t+1} - \hat{\lambda}_t + \frac{\alpha}{1+r-\delta}\left(\hat{X}_{t+1} - \hat{I}_{t+1}\right)\right. && (\mathbf{A31}) \\
&\left. -\sigma\hat{\gamma}^z_{t+1} + \frac{1+r-\delta-\alpha}{1+r-\delta}\left(\frac{1}{\mu-1}\hat{\mu}_{t+1} + \hat{Y}_{t+1} - \hat{I}_{t+1}\right)\right\}
\end{aligned}$$

where

$$\begin{aligned}
\hat{c}_t &= \epsilon_{c\lambda}\hat{\lambda}_t + \epsilon_{cw}\hat{w}_t && (\mathbf{A32}) \\
\hat{H}_t &= \epsilon_{H\lambda}\hat{\lambda}_t + \epsilon_{Hw}\hat{w}_t && (\mathbf{A33})
\end{aligned}$$

Here, ϵ_j $(j = \{V, Q\}, \{HK\}, \{EM\})$ are the elasticities of substitution in the production function, ϵ_i $(i = \{c\lambda\}, \{cw\}, \{H\lambda\}, \{Hw\})$ are elasticities of substitution in utility terms, and ϵ_μ is the elasticity of the function μ with respect to its argument. The parameters s_j $\{j = C, I, G\}$ are the expenditure shares of value added of consumption, investment, and government expenditure, while s_i $\{i = V, M, E\}$ are shares of total costs of the index V, domestically produced materials M_t and imported materials E_t. The remaining parameters are r, the steady-state real rate of interest before deprecation, and μ, the steady-state mark-up. To complete the system, the following three accounting relations that define the dynamics of the aggregators V_t, Q_t and Y_t are needed:

$$\begin{aligned}
\hat{V}_t &= \frac{\mu s_K\left(\hat{K}_t - \hat{\gamma}_t\right) + \mu s_H\hat{H}_t - (\mu-1)\hat{I}_t}{(1-\mu(s_E+s_M))} && (\mathbf{A34}) \\
\hat{Q}_t &= \frac{\mu s_E\hat{E}_t + \mu s_M\hat{M}_t}{\mu(s_E+s_M)} && (\mathbf{A35}) \\
\hat{Y}_t &= (1-\mu(s_E+s_M))\hat{V}_t + \mu(s_E+s_M)\hat{Q}_t && (\mathbf{A36})
\end{aligned}$$

as well as a specification of the stochastic process for the exogenous variables. These are discussed in detail in Section 2.6.

A.4 Solution method

These equations, (**A24**) to (**A36**), define a system of linear, rational expectations difference equations. In dynamic programming terminology, we define the control variables of the system as $\hat{c}_t$, $\hat{H}_t$ and $\hat{\mu}_t$, the endogenous state variables as $\hat{K}_t$, the co-state variables as $\hat{\lambda}_t$ and $\hat{X}_t$, and the exogenous state variables as $\hat{\gamma}_t^z$, $\hat{p}_t^E$, $\hat{G}_t$ and $\hat{I}_t$. We solve the log-linearised problem for policy rules that give control variables at t and endogenous state variables at $t+1$ as functions of endogenous and exogenous state variables at t. The task is to solve a first order stochastic difference equation of the form

$$
\begin{aligned}
M_{cc}C_t &= M_{cs}S_t + M_{ce}E_t \qquad (\mathbf{A37}) \\
M_{ss0}S_{t+1} + M_{ss1}S_t &= M_{sc0}C_{t+1} + M_{sc1}C_t + M_{se0}E_{t+1} + M_{se1}E_t
\end{aligned}
$$

where C_t is a vector of control variables, S_t is a vector of endogenous state and co-state variables, and E_t is a vector of exogenous state variables. Here we define:

$$
\begin{aligned}
C_t &= \hat{c}_t, \hat{H}_t, \hat{E}_t, \hat{M}_t, \hat{w}_t, \hat{Y}_t, \hat{Q}_t, \hat{V}_t, \mu_t \qquad (\mathbf{A38}) \\
ES_t &= \hat{K}_t \\
L_t &= \hat{\lambda}_t, \hat{X}_t \\
E_t &= \hat{\gamma}_t^z, \hat{G}_t, \hat{p}_t^E, \hat{I}_t.
\end{aligned}
$$

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