Must the growth rate decline?
Baumol's unbalanced growth revisited

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Abstract

According to Baumol's model of unbalanced growth, if resources are shifting towards industries where productivity is growing relatively slowly, the aggregate productivity growth rate will slow down. This conclusion is often applied to the advanced industrial economies, where resources are indeed shifting towards the relatively stagnant service industries. However, I show that Baumol's conclusion only follows if the stagnant industries produce final products. This is important empirically, since the most rapidly expanding service industries are those such as financial and business services, which are large producers of intermediate products. Even if such industries are stagnant, I show that a movement of resources into them may be associated with rising, not falling, aggregate productivity growth.

JEL codes: O40, O47, D24, O52.
Key words: Unbalanced growth, services, productivity.

1 Introduction[1]

1.1 The stagnationist argument and some facts about services

Suppose that productivity is growing at different rates in different industries. Suppose too that resources are gradually shifting towards the industries where productivity is growing more slowly (the technologically stagnant industries). Then the aggregate growth rate of productivity will steadily fall to the rate prevailing in the stagnant industries. This is one conclusion of the unbalanced growth model first set out by Baumol (1967) and further developed in Baumol (1985) and Baumol *et al* (1989).[2]

This stagnationist argument is thought to be particularly applicable to the advanced industrial countries. Productivity growth in services (at least as conventionally measured) is typically lower than in the rest of the economy. The proportions of output in constant prices and of employment accounted for by services have been steadily rising in these countries (Julius and Butler (1998)). So it might seem that the growth rate of the advanced countries is fated to decline.

The response to this gloomy prognostication has usually been to argue that not all service industries have low productivity growth, that even if they do now it may rise in the future, and that anyway measurement errors mean that growth is underestimated (Rowthorn and Ramaswamy (1997)). It is likely that errors in measuring service sector output are quite large (see eg Griliches (1992) and (1994)). But since fixing the errors will not be easy, the stagnationist argument still implies that the growth of *measured* output will decline, unless new technology can be relied on to raise future service sector growth, something it has so far failed to do (Stiroh (1998)).

Whatever the merits of these responses, they all accept the stagnationist argument as correct. This paper argues, to the contrary, that this conclusion of the unbalanced growth model may be incorrect as applied to the advanced countries. The reason is that the argument is logically correct

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(1) This paper has been produced as part of the Bank of England's research programme on the service sector. The 1998 Bank of England Act requires the Bank to have regard to the development of different sectors of the economy. This paper is offered as a small contribution towards meeting that requirement. Oulton (1999b), a companion paper to the present one, discusses some of the same issues from a growth theoretic viewpoint.

(2) The term 'technologically stagnant' and its opposite 'technologically progressive', are due to Baumol.

only if all industries produce final goods. Quite a different conclusion results if some of the industries produce intermediate goods. And this could be the relevant case in practice, since the service industries that have been expanding particularly rapidly are those such as financial and business services, which are large producers of intermediate inputs.

To set the scene for what follows, Table 1 shows growth rates of output by sector from 1973-96 in five leading countries: the United Kingdom, the United States, France, Germany and Japan. We see immediately that manufacturing has been a relatively slow-growing sector in four out of the five, the exception being Japan. The four service sectors—transport and communications, the distributive trades, finance and business services, and miscellaneous personal services—have all been growing more rapidly than manufacturing in the first four countries. In Japan, these sectors have been growing at about the same rate as manufacturing.

Table 2 shows the corresponding rates of growth of labour productivity (value-added in constant prices per hour worked). In the United Kingdom, the United States and France, labour productivity has been growing more slowly than in manufacturing in three out of the four market services (the exception is transport and communications). In Japan, productivity has been growing more slowly in two out of the four (finance and business services and non-market services). In Germany, it has been growing more slowly in the distributive trades.

What has been the consequence of these disparate growth rates for the allocation of resources between sectors? Chart 1 shows the proportion of total hours worked that has been absorbed by market services (the aggregate of the four service sectors in Table 1) over the period 1950-96. In each of the five countries, there has been a strong upward trend. More than half of total hours worked is now absorbed by market services in the United Kingdom, the United States and Japan, with somewhat lower proportions in France and Germany. In 1950, as now, the United States had the highest share but then it was only 40%.

In the present context, particular interest attaches to the share of resources devoted to intermediate production. Within market services, this may be proxied by finance and business services.[3] Chart 2 shows that there has

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[3] The share of finance and business services is only a proxy for intermediate sales, because some finance and business services are sold to final demand, including exports.

Table 1
Growth rates of output in selected sectors, 1973-96
(per cent per annum)

Sector	UK	US	France	Germany	Japan
Agriculture, forestry, and fishing	1.35	1.60	1.27	0.52	-0.51
Manufacturing	0.40	1.98	1.28	0.90	3.92
Utilities	2.47	2.25	4.91	2.82	4.13
Construction	0.54	0.77	-0.13	-0.53	1.82
Transport & communications	2.70	3.24	4.08	3.62	3.01
Distributive trades	1.63	3.52	1.55	1.80	3.83
Financial & business services	5.13	3.76	2.22	5.04	4.09
Miscellaneous personal services	3.55	3.20	2.75	5.59	4.21

Note: Output is value-added in constant prices. Omitted sectors are mining and non-market services (health, education, defence and public administration). Germany is Western Germany.

Source: Calculated from O'Mahony (1999, Table A).

Table 2
Growth rates of labour productivity in selected sectors, 1973-96
(per cent per annum)

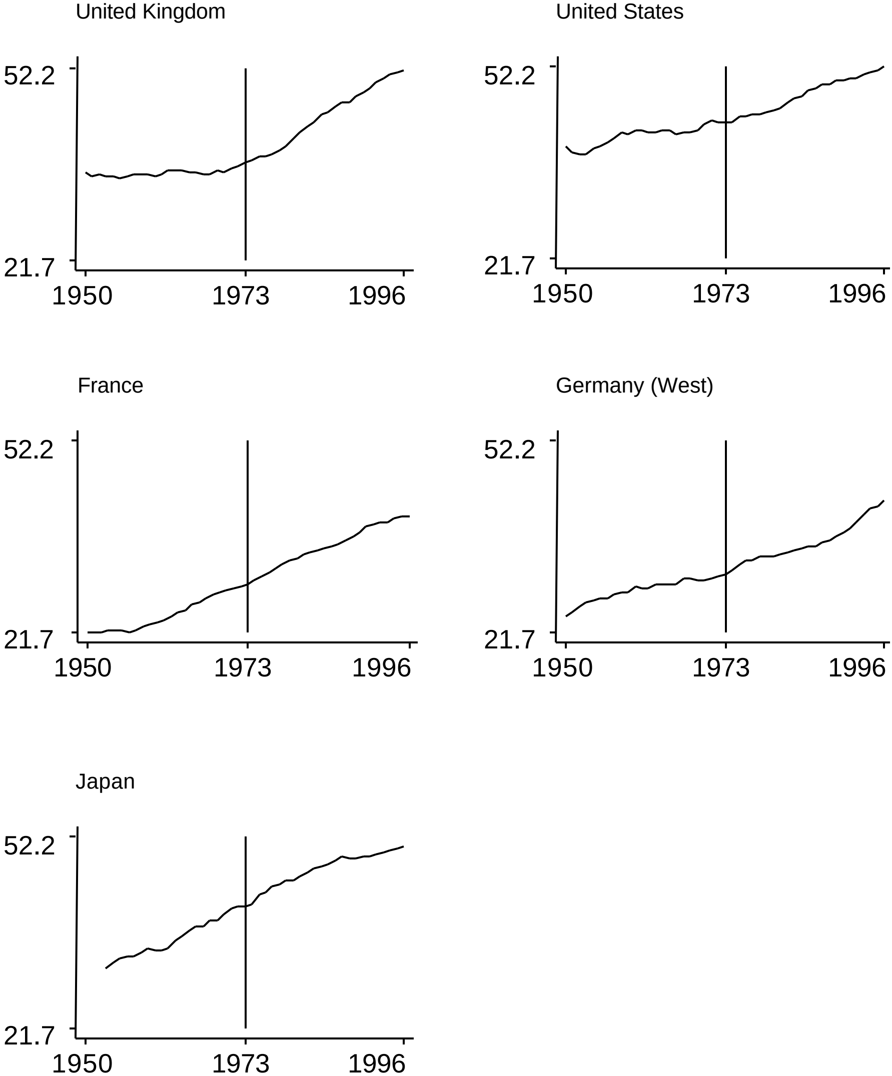
Sector	UK	US	France	Germany	Japan
Agriculture, forestry, and fishing	3.66	3.93	6.03	5.05	2.87
Manufacturing	4.96	1.39	4.22	2.95	2.93
Utilities	2.60	-0.77	2.37	1.04	1.07
Construction	3.14	2.21	3.65	2.93	4.47
Transport & communications	3.88	2.01	3.95	4.23	2.66
Distributive trades	1.52	1.79	1.62	1.74	3.87
Financial & business services	2.07	-0.08	0.26	3.06	1.87
Miscellaneous personal services	1.57	0.46	0.65	3.00	1.60

Note: Labour productivity is value-added in constant prices per hour worked. Omitted sectors are mining and non-market services (health, education, defence and public administration). Germany is Western Germany.

Source: Calculated from O'Mahony (1999, Tables A, B and C).

Chart 1

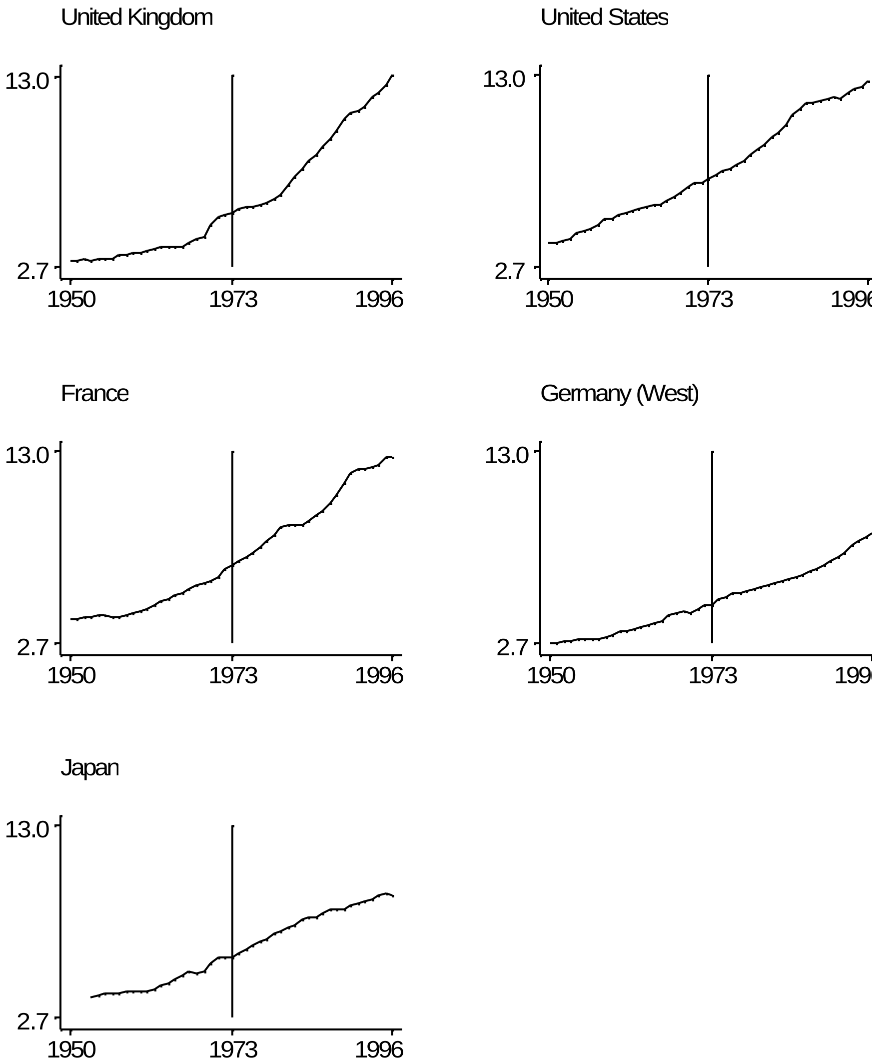
Proportion of aggregate hours worked in market services, 1950-96 (per cent)



Note: Market services comprise transport and communications, the distributive trades, finance and business services, and miscellaneous personal services

Source: Calculated from O'Mahony (1999, Tables B and C).

Chart 2
Proportion of aggregate hours worked in finance and business services, 1950-96 (per cent)



Source: Calculated from O'Mahony (1999, Tables B and C).

been a particularly sharp increase in the share of this sector in total hours worked. The share is now around 13% in the United States, the United Kingdom and France, and around 9% in Germany and Japan. The share was around 3%-4% in 1950, and has been rising steadily for nearly 50 years.(4)

1.2 The plan of the paper

Section 2 of the paper sets out the unbalanced growth model when all goods are final. Following Baumol, it shows the conditions under which, if output in all industries is growing at the same rate, resources will shift to the stagnant industries, with a consequent slowing of the aggregate productivity growth rate.

Section 3 explains how matters are changed when some industries produce intermediate products. A general result is derived, and illustrated by means of a simple, two-industry example. Central to this section is the distinction between two concepts of productivity growth at the industry level, one based on gross output, the other on value-added. The main result is that, under the assumption of perfect competition, a rise in the share of resources absorbed by an industry producing intermediate products raises the aggregate growth rate, provided only that total factor productivity (TFP) growth is positive in this industry. This result has a paradoxical corollary: a shift of resources towards even a stagnant industry that produces intermediate inputs raises the growth rate.

Section 4 addresses the question 'when will a shift in resources towards industries producing intermediate products actually occur?' A simple, two-industry model is developed, in which the first industry supplies an input to the second; perfect competition is assumed. The answer is shown to depend on the elasticity of substitution in the second industry between the intermediate input and the primary input. If this elasticity exceeds one, resources shift towards the first industry. This result requires only that TFP growth in the first industry be positive; it may be slower than TFP growth in the other industry.

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(4) The same source shows that the proportion of aggregate hours worked absorbed by transport and communications has been flat or falling in four of the five countries (France is the exception). The share absorbed by the distributive trades has been rising except in Japan; the share absorbed by other market services has been rising in all five countries. The share absorbed by non-market services has also been rising in all five countries, but output measures are meaningless for this sector.

Section 5 quantifies the effect of structural change on the UK growth rate over the period 1973-95. It demonstrates that the shift to finance and business services could have raised the UK growth rate, even though this sector has low TFP growth. It also discusses the effect of measurement error on the estimates. Section 6 concludes.

1.3 An intuitive argument

Much of the argument to follow is rather technical. So to assist intuition, the central point of the paper will be illustrated by the following, non-rigorous argument. Consider first the intuition behind the unbalanced growth model. For purposes of illustration, suppose there are only two industries which for concreteness we label cars and haircuts. Suppose that labour is the only input. Assume that productivity is rising in cars but not in haircuts. Incomes are rising over time because there is productivity growth in one sector even if not in the other. Suppose that people's demand for the two products rises at an equal rate. (We shall see in a moment that this common growth rate must be declining over time.) Assume that total employment is constant. Then since people want to have their hair cut more frequently as they grow richer, more hairdressers will be employed. Since total employment is fixed, this means that fewer car workers will be employed. This is possible since the productivity of car workers is rising: the growing demand for cars can be satisfied by progressively fewer car workers.

As long as the assumptions continue to apply, the proportion of the workforce employed in hairdressing will go on rising, approaching one asymptotically. Given that total resources are fixed, the overall growth rate of the economy must slow down. This is because aggregate productivity growth is a weighted average of productivity growth in the two sectors, where the weights are shares in total employment.(5) We have already seen that the employment share of haircuts is rising over time. So the sector with zero productivity growth gets an ever-increasing weight and the overall productivity growth rate must therefore decline. Because total employment is fixed, the growth rate of aggregate output must decline too.

What is happening to costs and prices? Assume that wages in the two industries move in step with each other. Then since it always requires the same amount of labour to cut someone's hair but progressively less labour

(5) This is true because we are assuming equal growth rates of output in the two sectors: see Section 2 below.

to produce a car, the relative price of a haircut must be rising. It follows that the proportion of consumers' expenditure which falls on haircuts must also be growing, approaching one asymptotically. Since the product which forms an ever larger share of expenditure is subject to zero productivity growth, the rate at which the standard of living is rising must be declining. More precisely, the growth rate of the standard of living is falling asymptotically to zero.

Having set out the argument in its most basic form, we can now see a simple generalisation. Suppose that productivity growth in haircuts is lower than in cars but greater than zero. Then the overall growth rate of productivity and output will still slow down, approaching now the low rate found in haircuts.

Now imagine another economy where as before one industry produces cars but the other industry supplies an intermediate input, say business services. That is, cars are produced by combining labour and business services. Business services require only labour. Total employment is constant as before. Productivity growth in cars is higher than in business services, but since the car industry has two inputs we must understand productivity growth there to mean growth in total factor productivity. Since business services use only labour, labour productivity and TFP are identical in that industry. At first sight, this change of assumption seems to make no difference and the same argument as above applies. But in fact things are very different. At the aggregate level, we care only about the output of cars since this is the only product demanded by consumers. So the issue is whether a rising share of employment in business services will be accompanied by a rising or falling growth rate of car output.

There are two ways in which the economy can obtain more cars, given that total employment is fixed. One is if TFP rises in the car industry, the other is if TFP rises in business services. TFP growth in the car industry raises the productivity of both the inputs, labour and business services: more cars can be produced for a given amount of labour directly employed in the car industry and indirectly employed in business services. In addition, TFP growth in the business services industry raises the productivity of labour employed there. This means that more business services can be produced for a given amount of labour. Hence TFP growth in the business services industry causes higher car output, since the car industry buys in business services. The higher the proportion of the labour force employed in business services, the bigger the impact on car output of TFP growth in business services. Hence even if productivity growth is low in business services, a shift of resources into this industry will be accompanied by rising, not falling, growth of car output. The reason is that such a shift will raise the contribution to the aggregate coming from business services without reducing the contribution coming from the car industry.

So for the cars/business services economy we reach exactly the opposite conclusion to the one for the cars/haircuts economy. The argument just stated provides support for the more general proposition, that a shift in resources into intermediate-producing industries raises the aggregate growth rate. As the paper will make clear, the argument depends on the shift being market-induced, as a result of profit-maximising behaviour by producers, in circumstances where externalities and other market failures do not unduly influence outcomes.

2 The stagnationist argument

2.1 The Baumol model when all goods are final

Assume that all goods and services are final, ie there are no intermediate inputs. Let y_i denote the gross output of the ith product (i = 1,...,n), and let y denote aggregate output. The growth rate of aggregate output can be defined as a weighted average of the growth rates of the industry outputs, where the weights are the value shares (s_i) of the industries in the total value of output:

$$\hat{y} = \sum_{i=1}^{n} s_i \hat{y}_i, \qquad s_i \equiv \frac{p_i y_i}{\sum_{i=1}^{n} p_i y_i}, \quad \sum_{i=1}^{n} s_i = 1 \qquad \textbf{(1)}$$

Here, the p_i are the prices and a hat (^) denotes a growth rate (logarithmic derivative with respect to time, t). Also

$$\sum_{i=1}^{n} p_i y_i \equiv py$$

which implicitly defines p as the aggregate price index.

Let x_i be an index of total input into the ith industry, and let x be aggregate input in the whole economy. All inputs are primary. We can think of the x_i either as a single input, say labour, or as a bundle of primary inputs whose composition may well vary across industries. The growth of aggregate input may be defined as a weighted average of the growth rates of industry inputs:

$$\hat{x} = \sum_{i=1}^{n} r_i \hat{x}_i , \qquad \sum_{i=1}^{n} r_i = 1$$

where r_i is the proportion of aggregate input employed in the ith industry. These proportions will be defined more precisely below.

Define $q_i \equiv y_i / x_i$ as productivity in the ith industry and $q \equiv y / x$ as aggregate productivity. Productivity growth in the ith industry is then

$$\hat{q}_i = \hat{y}_i - \hat{x}_i \qquad \textbf{(2)}$$

The industry-level productivity growth rates are assumed to be exogenous. The growth rate of aggregate productivity is:

$$\hat{q} = \hat{y} - \hat{x} = \sum_{i=1}^{n} s_i \hat{y}_i - \sum_{i=1}^{n} r_i \hat{x}_i$$

$$= \sum_{i=1}^{n} r_i \hat{q}_i + \sum_{i=1}^{n} (s_i - r_i) \hat{y}_i \qquad \textbf{(3)}$$

We see that aggregate productivity growth is not simply a weighted average of industry productivity growth rates, the first term in the equation, because of the presence of the second term. The latter measures the effect of shifts in the composition of output.

There are two cases where the second term in equation **(3)** will be zero. The first case is the benchmark case considered by Baumol *et al* (1989, Appendix to Chapter 6), where the composition of output is assumed constant, ie output is growing at the same rate in all industries: $\hat{y}_i = \hat{y}$, all i. Since the output and input shares both add to one, the second term in equation **(3)** is zero, and so in this special case

$$\hat{q} = \sum_{i=1}^{n} r_i \hat{q}_i \qquad \textbf{(4)}$$

The second case in which **(3)** reduces to **(4)** is when there are constant returns to scale, and perfect competition prevails in all markets, including the markets for inputs. Under these conditions, the price of a given input is the same in all industries, and measures its social marginal product. It is then appropriate to aggregate the industry inputs using input prices, ie we can define the weights as the value shares:

$$r_i \equiv w_i x_i / \sum_{i=1}^{n} w_i x_i, \quad \sum_{i=1}^{n} r_i = 1 \qquad \textbf{(5)}$$

where w_i is the price of the primary input bundle in industry i. Also, the price of aggregate input w is implicitly defined by the accounting relationship:

$$wx = \sum_{i=1}^{n} w_i x_i$$

Under perfect competition, in long-run equilibrium, the value of output equals the cost of the inputs (including a normal return on capital):

$$p_i y_i = w_i x_i, \qquad i = 1, \ldots, n$$

$$py = wx$$

Hence, dividing,

$$s_i = r_i, \qquad \text{all } i \qquad \textbf{(6)}$$

and again **(3)** reduces to **(4).** These accounting relationships imply that under perfect competition the level of productivity in current prices is the same in all industries: $p_i y_i / w_i x_i = 1, \quad \text{all } i$. So the levels effect in the aggregate productivity growth equation **(3)** disappears, irrespective of the growth rates of industry outputs. Since all industries have the same productivity level in value terms, there is no gain to reallocation.(6)

Suppose that either of these two cases applies, so that the aggregate growth rate is given by **(4)**. Then if resources are shifting to industries with comparatively low productivity growth, the aggregate growth rate will clearly decline. We may note in passing that if **(4)** holds, the growth rate of GDP is given by

$$\hat{y} = \hat{x} + \sum_{i=1}^{n} r_i \hat{q}_i$$

So if the growth rate of aggregate input is taken to be constant, falling productivity growth implies falling GDP growth too.

In fact, with some additional assumptions, a stronger proposition applies. We show in the next sub-section that if output is growing at the same rate in all industries, then the lower a sector's productivity growth rate, the faster its share of total input is rising. Then equation **(4)** implies that the aggregate productivity growth rate will fall monotonically, converging on the growth rate of the most stagnant sector:

$$\hat{q} \to \min\{\hat{q}_i\} \text{ as } t \to \infty$$

This is the strong version of the stagnationist result.

2.2 Relative prices and input shares

So far in deriving the stagnationist result we have made two assumptions: first, that output grows at the same rate in all industries, or alternatively, that the economy is competitive; and second, that the share of resources going to the stagnant industries is rising. But if the economy is competitive, the second assumption can be derived as an implication of the assumption of equal output growth rates, provided that we make the further simplifying assumption that the price of the primary input bundle is the same in all

(6) Even in a competitive economy, levels effects would still arise if we chose to measure productivity growth using a fixed weight (eg Laspeyres) index. But it is better to use a Divisia (chain) index, as here. Levels effects can also arise in a competitive economy, if we are considering labour productivity and labour is not the only primary input.

industries. This last assumption means that there is in effect only one primary input.[7]

Let us assume that $w_i = w$, all i. Then we have

$$r_i = x_i / x, \quad \sum_{i=1}^{n} r_i = 1$$

and

$$x = \sum_{i=1}^{n} x_i$$

The accounting relationship that the value of output equals the cost of the inputs now becomes:

$$p_i y_i = w x_i, \qquad i = 1,...,n \qquad \textbf{(7)}$$

Rearranging this relationship,

$$\frac{p_i y_i}{x_i} = p_i q_i = w, \qquad i = 1,..., n \qquad \textbf{(8)}$$

ie the level of productivity in current prices (the current-price value of output per physical unit of input) is the same in all industries, even though the growth rate of productivity may differ between industries. This is because in a competitive economy, prices adjust to make this so, as we can see by logarithmically differentiating the accounting relationships with respect to time:

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(7) In general, the composition of the primary input bundle varies between industries. Hence, even if input markets are undistorted, so that the price of a given primary input is the same in all industries, the w_i will not in general be equal across industries. It can be shown that the w_i will be equal only under restrictive assumptions: either if there is only one primary input, or if relative input prices are always the same, or finally if input intensities are the same in all industries, at given input prices. The last condition amounts to assuming that all industries have the same production function, up to a multiplicative factor.

$$\hat{p}_i + \hat{q}_i = \hat{w}$$

$$\hat{p} + \hat{q} = \hat{w}$$

whence

$$\hat{p}_i - \hat{p} = \hat{q} - \hat{q}_i \qquad \textbf{(9)}$$

ie relative to the general price level, the price of the *i*th product rises by the difference between productivity growth in *i* and aggregate productivity growth.[8] Note that this result depends crucially on the assumption of only one (possibly composite) primary input. If there were more than one primary input, then the evolution of relative product prices would also be influenced by the evolution of relative input prices. For example, if the price of labour is rising relative to that of capital, the price of a labour-intensive product may be rising too, even if the industry is technologically progressive (Oulton (1999a)).

The time-paths of the output and resource shares come from logarithmically differentiating the equation defining the output share **(1)** with respect to time, and using **(6)** and **(8)**:

$$\hat{s}_i = \hat{r}_i = \hat{q} - \hat{q}_i + \hat{y}_i - \hat{y}$$

Now consider the benchmark case in which output grows at the same rate in all industries (ie $\hat{y}_i = \hat{y}$, all i). Then

$$\hat{s}_i = \hat{r}_i = \hat{q} - \hat{q}_i \qquad \textbf{(10)}$$

That is, resources shift continuously towards the relatively stagnant industries. The slower productivity growth is, the more rapid the shift. The stagnationist argument is therefore strengthened. Under the conditions assumed here, if output grows at the same rate in all industries, then aggregate productivity growth will slow down asymptotically to that of the

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(8) The equality of productivity levels in different sectors under competition has been discussed by Baumol and Wolff (1984).

slowest industry. If output growth is faster in more stagnant industries, the slowdown will be more rapid.

3 The stagnationist argument when some products are intermediate

3.1 A value-added model of productivity growth

Suppose now that some industries produce products that are consumed by other industries. It might be thought that this would make little difference, if we adopt the usual value-added approach. Let real value-added in industry *i* be v_i, and let nominal value-added be V_i. The growth of aggregate value-added (v) or GDP is a weighted average of the growth rates of industry-level value-added, where the weights are each industry's share (u_i) in aggregate nominal value added:

$$\hat{v} = \sum_{i=1}^{n} u_i \hat{v}_i , \qquad u_i \equiv \frac{V_i}{\sum_{i=1}^{n} V_i}, \quad \sum_{i=1}^{n} u_i = 1 \qquad \textbf{(11)}$$

We can define an industry-level price of value-added p_i^v and an aggregate price of value-added p^v from the accounting relationships:

$$V_i = p_i^v v_i$$

$$\sum_{i=1}^{n} V_i = p^v v$$

Industry-level productivity growth can be defined as:

$$\hat{q}_i^v = \hat{v}_i - \hat{x}_i \qquad \textbf{(12)}$$

Note that we use a different symbol here for productivity growth from that used in Section 2.1 — $\hat{q}_i^v$, not $\hat{q}_i$ — since these two symbols refer to different concepts. Previously, output was taken to be gross output (compare equation **(2)**), now it is value-added. We discuss below the relationship between these two concepts of productivity growth at the industry level.

Aggregate productivity growth can now be written

$$\begin{aligned}\hat{q} = \hat{v} - \hat{x} &= \sum_{i=1}^{n} u_i \hat{v}_i - \sum_{i=1}^{n} r_i \hat{x}_i \\ &= \sum_{i=1}^{n} r_i q_i^v + \sum_{i=1}^{n} (u_i - r_i) \hat{v}_i \end{aligned} \qquad \textbf{(13)}$$

At the aggregate level, we use the same symbol for productivity growth as in Section 2.1, $\hat{q}$, since the growth of GDP viewed as a sum of value-added must be the same in principle as the growth of GDP viewed as a sum of final expenditures.

Now consider the special case where value-added is growing at the same rate in all industries: $\hat{v}_i = \hat{v}$, all *i*. Now the levels effect is zero and **(13)** becomes:

$$\hat{q} = \sum_{i=1}^{n} r_i \hat{q}_i^v \qquad \textbf{(14)}$$

Alternatively, by assuming a competitive economy and employing an exactly analogous argument to that of Section 2.2, we can prove that the output and resource shares are equal: $u_i = r_i$, all *i*. So **(14)** applies irrespective of the value-added growth rates. Given competition and equal value-added growth rates, we can also prove that the share of primary inputs devoted to an industry will rise faster, the slower its productivity growth rate ($\hat{q}_i^v$):

$$\hat{u}_i = \hat{r}_i = \hat{q} - \hat{q}_i^v \qquad \textbf{(15)}$$

The stagnationist argument would thus seem to go through as before. If the share of inputs going to the stagnant industries is rising, then the aggregate productivity growth rate must fall monotonically to that of the most stagnant industry—or at any rate, so equation **(14)** seems to be saying.

However, this argument contains a hidden assumption, namely that TFP growth rates in the value-added sense ($\hat{q}_i^v$) are parameters. In Section 2.1 we assumed that TFP growth rates in the gross output sense ($\hat{q}_i$) were parameters. We now show that these two assumptions are inconsistent: if the $\hat{q}_i$ are constants and if the share of intermediate inputs is changing, then the $\hat{q}_i^v$ must be changing too.

3.2 Two concepts of sectoral TFP growth

Let the gross output production function be

$$y_i = f_i(x_i, m_i, t) \qquad \textbf{(16)}$$

where m_i is an index of intermediate input (purchases from other industries) in industry *i*. The accounting relationship is now

$$p_i y_i = w_i x_i + p_i^m m_i$$

where p_i^m is the price of intermediate input. Then, assuming competition so that input shares can be equated to the elasticity of output with respect to each input, TFP growth in the gross output sense in industry *i* is

$$\hat{q}_i = \hat{y}_i - \left(\frac{w_i x_i}{p_i y_i}\right)\hat{x}_i - \left(\frac{p_i^m m_i}{p_i y_i}\right)\hat{m}_i \qquad \textbf{(17)}$$

We want to find the relationship between $\hat{q}_i$ and $\hat{q}_i^v = \hat{v}_i - \hat{x}_i$ (see equation **(12)**). Assuming that the gross output production function **(16)** is separable, we can write it in the form:

$$y_i = f_i(v_i, m_i) \qquad \textbf{(17¢)}$$

where

$$v_i = g_i(x_i, t) \qquad \textbf{(18)}$$

and $g_i(\cdot)$ is the value-added production function. Differentiating **(17¢)** with respect to time and still assuming competition, we obtain the growth rate of real value-added:(9)

$$\hat{v}_i = \left(\frac{p_i y_i}{w_i x_i}\right)\hat{y}_i - \left(\frac{p_i^m m_i}{w_i x_i}\right)\hat{m}_i \qquad \textbf{(19)}$$

Substituting this into equation **(12)** and using **(16),** we obtain:

$$\hat{q}_i^v = \left(\frac{p_i y_i}{w_i x_i}\right)\hat{q}_i \qquad \textbf{(20)}$$

Or in words,

$$\text{TFP growth in value-added sense} = \frac{\text{TFP growth in gross output sense}}{\text{Share of value-added in gross output}}$$

Clearly, TFP growth in the value-added sense can never be less than and will usually be larger than TFP growth in the gross output sense.(10) Also if we take the $\hat{q}_i$ as parameters, then the $\hat{q}_i^v$ become variables, since the value-added share is determined by the relative prices of primary and intermediate inputs, and will vary over time.

Having established this relationship between the two concepts of productivity growth at the industry level, equation **(20),** we can now substitute from this into the equation for aggregate productivity growth **(14)** to get:

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(9) Equation **(19)** is a continuous-time, Divisia-index form of double deflation: see Appendix B for a comparison of double with single deflation. An alternative way of deriving **(19)** is to start with the definition of nominal value-added in industry i: $p_i^v v_i = p_i y_i - p_i^m m_i$. We can then obtain **(19)** by differentiating this equation with respect to time, while holding prices constant. The treatment in the text has the advantage of showing how double deflation is consistent with production theory.

(10) The relationship between these two concepts of TFP growth was discussed in Oulton and O'Mahony (1994), Chapters 1 and 6.

$$\hat{q} = \sum_{i=1}^{n} \left(\frac{p_i y_i}{p^v v} \right) q_i \qquad \textbf{(21)}$$

This last result makes use of the fact that

$$r_i = w_i x_i / \sum_{i=1}^{n} w_i x_i = w_i x_i / p^v v \; .$$

Equation **(21)** exemplifies what has been called Domar aggregation (Oulton and O'Mahony (1994), Chapter 5, following Domar (1961)). The general principle is that aggregate TFP growth is a weighted sum of the industry-level TFP growth rates. The Domar weights are the ratio of gross output in each industry to aggregate value-added (total final output).(11) Note that these weights sum to more than one.

Domar aggregation was given a theoretical justification by Hulten (1978): on the assumption of constant returns to scale and competitive markets, it measures the rate at which the social production possibility frontier is shifting out over time. The intuition behind Domar aggregation is that productivity growth in an industry contributes directly to aggregate productivity growth (via its final output), but also indirectly when it supplies other industries. Costs fall in the purchasing industries, and this effect is obviously bigger the larger such purchases are.

It is useful to split the Domar weight into two, gross output for final use and gross output for intermediate use, both expressed as a proportion of aggregate final output:

$$\text{Domar weight of sector } i = \frac{\text{Intermediate sales of } i}{\text{Total final output}} + \frac{\text{Final sales of } i}{\text{Total final output}}$$

Note that the sum across industries of the second fraction, final sales of industry *i*/total final output, is one. So if this fraction rises for one industry, it must fall by a corresponding amount for one or more other industries. But the same is not true of a rise in the intermediate part of the Domar weight. This first fraction can rise for one industry without a corresponding fall for any other sector. For example, suppose that industry 1 has only

(11) In the most general case, the denominator of the Domar weights is total final output. In a closed economy, total final output equals nominal GDP. In an open economy, it exceeds the latter by the amount of intermediate imports, which should also be considered a primary input (Gollop 1983).

intermediate sales. Suppose that there are other industries that sell only to final demand, and that these now purchase more of industry 1's product, substituting it for primary input. Then the Domar weight for industry 1 will rise without any corresponding fall in any other sector's weight. It follows that the overall productivity growth rate must rise too: see equation **(20)**.

To clarify the argument, recall that we have derived two equations for aggregate productivity growth, repeated here for convenience. We also repeat the relationship between the two concepts of productivity growth:

$$\hat{q} = \sum_{i=1}^{n} r_i \hat{q}_i^v, \qquad r_i = w_i x_i / p^v v \qquad \textbf{(14)}$$

$$\hat{q} = \sum_{i=1}^{n} \left(\frac{p_i y_i}{p^v v} \right) q_i \qquad \textbf{(21)}$$

$$\hat{q}_i^v = \left(\frac{p_i y_i}{w_i x_i} \right) \hat{q}_i \qquad \textbf{(20)}$$

From the first of these equations, **(14)**, it appears that a rise in the resource share (r_i) of a stagnant industry, counterbalanced by a fall in the share of a progressive industry, will lower the aggregate productivity growth rate, irrespective of whether the stagnant industry supplies intermediate or final goods. From the second, **(21)**, a quite different conclusion emerges. If there is a rise in the Domar weight of an industry supplying an intermediate product, the aggregate productivity growth rate will rise, even if the industry in question has lower-than-average productivity growth. More precisely, aggregate productivity growth will rise provided only that TFP growth in the industry is positive.

The resolution of this apparent contradiction comes from taking account of the third equation, **(20)**. As the industries supplying final goods purchase more from the one supplying intermediate goods, so the formers' value-added shares decline. Consequently, their TFP growth rates in the value-added sense ($\hat{q}_i^v$) *rise.* This 'gain' more than outweighs the 'loss' from the reallocation of resources in favour of the stagnant industry.

The argument may also be illustrated by a simple case of a two-industry economy. Suppose that industry 1 produces an intermediate product, which it supplies to industry 2. Industry 2 supplies a final product only. In this economy, total final output equals the output of industry 2 $(p_2 q_2 = p^v v)$. Hence from **(14), (20)** and **(21)**

$$\hat{q} = r_1 \hat{q}_1^v + (1 - r_1)\hat{q}_2^v = r_1 \hat{q}_1 + \hat{q}_2 \qquad \textbf{(22)}$$

where

$$\hat{q}_2^v = \frac{\hat{q}_2}{(1 - r_1)} \text{ and } \hat{q}_1^v = \hat{q}_1$$

This simple case makes clear that a shift towards industry 1 (a rise in r_1) raises the overall productivity growth rate provided only that $\hat{q}_1$ is positive.

A possible riposte, reinstating the stagnationist conclusion, is to turn the argument around and assume that the *value-added* TFP growth rates $(\hat{q}_i^v)$ are parameters, and hence that the *gross output* TFP growth rates $(\hat{q}_i)$ will vary with value-added shares. This seems an implausible move. The gross output production function is the fundamental concept. The value-added production function is derived from it as an intellectual construct. Indeed, the latter does not even exist unless the gross output production function is separable. For example, if the gross output production function is a CES function of capital, labour and materials, no value-added production function exists.

In summary, we have shown that if resources are shifting to industries producing intermediate inputs, the aggregate productivity growth rate will rise, however low the TFP growth rates (in the gross output sense) are in those industries, provided only that they are positive. We have also shown, with some additional assumptions, that a shift of resources towards stagnant industries will occur in Baumol's benchmark case of equal growth rates of real value-added in all industries. But if these industries produce for intermediate use, there will be no slowdown in aggregate growth, but rather a speed-up (provided again that productivity growth in the stagnant industries is positive).

The benchmark case of equal growth rates if anything understates the observed shift to services, as we have seen (see Table 1). Even so, it might be criticised as resting on an arbitrary assumption. In the next section, therefore, we develop the simple two-industry model just introduced, in order to explore the conditions under which the stagnant industry will indeed absorb an increasing share of resources, while simultaneously the aggregate growth rate rises.

4 A simple model of endogenous structural change

4.1 The model

Consider an economy with two industries. As in Section 3, industry 1 supplies an intermediate input to industry 2 but produces no final output. Industry 2 produces only final output. We assume that there is only one primary input, labour, that there are constant returns to scale, and that technical progress is unbiased (Hicks neutral).[12] The gross output production functions are:

$$y_2 = f(y_1, x_2)\exp(\hat{q}_2 t) \tag{23}$$

$$y_1 = x_1 \exp(\hat{q}_1 t) \tag{24}$$

where, as before, $\hat{q}_1$ and $\hat{q}_2$ are the growth rates of TFP.

We assume perfectly competitive conditions, so the value of output must equal the cost of the inputs. Since the wage must be the same in both industries, as we assume homogeneous labour and competitive labour markets,

$$p_2 y_2 = p_1 y_1 + w x_2 \tag{25}$$

[12] Technical progress is neutral in the Hicksian sense if the marginal rate of substitution between the two inputs (the slope of the isoquants) at a given input ratio is unchanged. That is, TFP growth shifts the isoquants inwards radially.

$$p_1 y_1 = w x_1 \qquad \textbf{(26)}$$

whence

$$p_2 y_2 = w(x_1 + x_2)$$

or, in national accounting terms, final expenditure equals factor income.

The share of industry 1 in the total cost of industry 2 is

$$p_1 y_1 / p_2 y_2 = w x_1 / w(x_1 + x_2) = r_1$$

and the share of primary input x_2 in the total cost of industry 2 is $r_2 = 1 - r_1$. Totally differentiating the production functions **(23)** and **(24)** with respect to time, we obtain the usual growth-accounting expressions for TFP growth:

$$\begin{aligned} \hat{q}_2 &= \hat{y}_2 - (1 - r_1)\hat{x}_2 - r_1 \hat{y}_1 \\ \hat{q}_1 &= \hat{y}_1 - \hat{x}_1 \end{aligned} \qquad \textbf{(27)}$$

Here we rely on the fact that under competition, the elasticity of output with respect to any input is equal to the share of that input in total cost. Alternatively, we can calculate TFP growth using prices rather than quantities. By totally differentiating the accounting relationships **(25)** and **(26)**, and using **(27)**, we obtain [13]

$$\begin{aligned} \hat{q}_2 &= -[\hat{p}_2 - r_2 \hat{w} - r_1 \hat{p}_1] \\ \hat{q}_1 &= -[\hat{p}_1 - \hat{w}] \end{aligned} \qquad \textbf{(28)}$$

Note that we have already derived the aggregate growth rate of productivity: see equation **(22)** above.

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[13] This is the dual approach to growth accounting, which uses prices. It must yield the same results as the primal approach based on quantities, provided that the accounting framework is consistent (Jorgenson and Griliches (1967), Barro (1998)).

For concreteness, let us agree to call industry 1 business services and industry 2 cars. Car producers must choose the profit-maximising input ratio in their industry, the ratio of labour to business services (x_2 / y_1). Since there are only two inputs, they must be substitutes. The profit-maximising input ratio depends on the relative price of the two inputs, w/ p_1. From **(28)**, this relative price is rising, provided that TFP growth in business services is positive: $\hat{w} - \hat{p}_1 = \hat{q}_i$. Hence, it pays to substitute business services for labour directly employed in cars, ie x_2 / y_1 is falling.

The argument is illustrated in Chart 3, which shows the production function for cars. Here the initial equilibrium is at *R*, where *aa* is the unit isoquant, the slope of *AB* shows the initial relative price of the two inputs, and the slope of *OR* shows the input ratio. Then the relative price of labour in terms of business services rises to a new level, given by the slope of *CD*. Simultaneously, TFP growth in cars causes the unit isoquant to shift inwards in a radial fashion, owing due to neutral technical progress. The new equilibrium is at *T* on *bb*, with a lower ratio of labour to business services in the car industry.

More precisely, the evolution of the input ratio in cars is given by:

$$\frac{d \log(x_2 / y_1)}{dt} = \frac{d \log(x_2 / y_1)}{d \log(w / p_1)} \frac{d \log(w / p_1)}{dt}$$

(29)

$$= -s\ (\hat{w} - \hat{p}_1)$$

$$= -\sigma \hat{q}_1$$

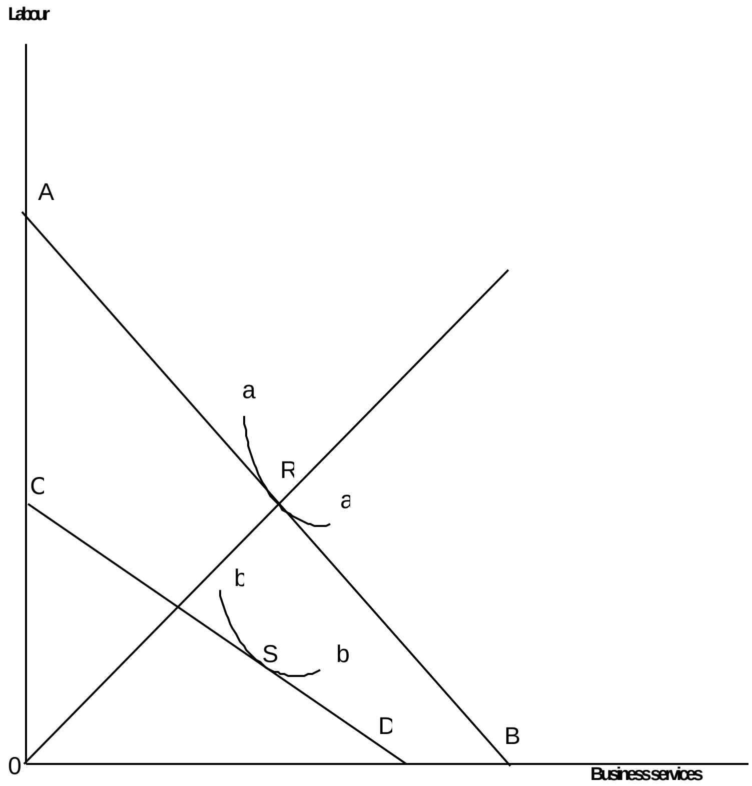
where use is made of **(28)** and *s* is the elasticity of substitution:

$$\sigma \equiv - \frac{d \log(x_2 / y_1)}{d \log(w / p_1)} > 0$$

Hence, the ratio x_2/y_1 is falling at the rate $\sigma \hat{q}_1$.

However, we are more interested in the ratio of labour in the two industries, x_2/x_1. The fact that x_2/y_1 is falling does not necessarily mean that x_2/x_1 is falling, ie that less labour will be employed in cars and more in business

Chart 3
Production function for cars



services, since fewer workers are now required to produce a unit of business services. Using **(27)** and **(29)**, the evolution of the labour ratio is given by:

$$\frac{d\log(x_2/x_1)}{dt} = \frac{d\log(x_2/y_1)}{dt} + \hat{q}_1 \qquad \textbf{(30)}$$
$$= (1-\sigma)\hat{q}_1$$

Finally, note that since $r_i = x_i/(x_1 + x_2)$, then $x_2/x_1 = (1-r_1)/r_1$, so

$$\frac{d\log(x_2/x_1)}{dt} = -\frac{\dot{r}_1}{r_1(1-r_1)}$$

and hence from **(30)**

$$\dot{r}_1 = r_1(1-r_1)(\sigma-1)\hat{q}_1 \qquad \textbf{(31)}$$

Hence $\dot{r}_1 > 0$ if $\sigma > 1$ and $\hat{q}_1 > 0$. The conclusion may be summed up in a proposition:

PROPOSITION 1 In the model of equations **(23)-(26)**, suppose that (a) technical progress in industry 1 (business services) is positive $(\hat{q}_1 > 0)$, and (b) the elasticity of substitution in industry 2 (cars) exceeds one ($\sigma > 1$). Then resources will shift to industry 1 so that r_1 approaches 1 asymptotically and aggregate TFP growth will rise, approaching $\hat{q} = \hat{q}_2 + \hat{q}_1$.

PROOF See the discussion above, and equations **(22)** and **(31)**. Proposition 1 assumes Hicks-neutral technical progress. If technical progress in cars is biased towards labour, the proposition is qualified though not reversed: see Appendix A.

4.2 Discussion

The intuition behind this result is that what matters for this economy is a reduction in the resources required to produce a given level of y_2. This can occur either directly in industry 2 or indirectly, through a reduction in the resources required to produce y_1. For example, suppose that business services are required in the production of cars. A rise in TFP in business services will either allow the same amount of business services to be produced with fewer resources, thus freeing some resources for direct employment in producing cars; or it will allow an expansion of business services without reducing resources directly employed in cars, thus indirectly enabling more cars to be produced; or some optimal combination of these two options can be chosen by profit-maximising producers. The more important y_1 is in the production of y_2 (the higher r_1 is), then for given TFP growth rates in the two industries, the faster aggregate TFP will grow.

This result, that transferring resources into an activity with low productivity growth will raise the aggregate growth rate, may seem very paradoxical. Does it imply that it would be a good idea to transfer as many resources as possible into input-supplying industries even if they have low productivity growth? No. Maximising the current rate of productivity growth is not a sensible policy objective. The result has been derived on the assumption that the economy is competitive and that market prices measure social costs. So transferring more resources than profit maximisation would indicate would reduce the level of output. The correct interpretation is this: if in such an economy we observe a shift of resources into industries supplying inputs, and if TFP growth rates in the gross output sense are constant, then the aggregate rate of productivity growth will be rising, not falling.

Now let us compare the present cars/business services model with the corresponding case where there are two industries producing final goods only, say cars and haircuts. What is the essential difference between the cars/haircuts economy and the cars/business services economy? In the former, the crucial relative price is that of cars for haircuts. Assuming that TFP growth is faster in cars than in haircuts, this relative price falls continuously: see equation **(9)**. Hence, consumers tend to reduce their purchases of haircuts. But incomes are rising over time, owing to aggregate TFP growth, and assuming that the income elasticity of demand for haircuts is positive, this tends to push demand in the opposite direction. So whether aggregate TFP growth approaches asymptotically the rate in cars (high) or that in haircuts (low) depends on the relative strength of these two forces.

In the cars/business services economy, if TFP growth is assumed to be faster in cars than in business services, then here too the relative price of cars will fall over time, as can be deduced from equations **(28)**.[14] But this relative price, p_2/p_1, does not enter into anyone's choices directly. The important relative price is that of the two inputs into car manufacturing, namely labour and business services (w/p_1). Even though business services

[14] Actually, the condition is somewhat weaker than stated: p_2 / p_1 will be falling if $\hat{q}_2 > (1 - r_1)\hat{q}_1$.

are becoming more expensive relative to cars, they are becoming cheaper relative to labour, and this is what drives the result.

5. TFP growth in the United Kingdom: a sectoral analysis

In this section, we consider the empirical importance of structural change for the United Kingdom growth rate since 1973.

5.1 Quantifying the contributions of each sector

In practice, all 123 industries distinguished in the UK input-output tables supply both final and intermediate output. Even business services supply final output, since some of their output is exported. So the extent to which the shift to services has reduced or increased aggregate growth remains an empirical matter. It is not possible to carry out the analysis at the level of detail of the Input-Output tables, because data on inputs are currently lacking. Instead, we illustrate using the estimates of TFP growth derived by O'Mahony (1999) for ten sectors covering the whole economy for the period 1973-95. These TFP estimates employed the value-added approach. TFP growth is measured as the growth rate of real value-added minus the growth rate of hours worked (weighted by labour's share in value-added) minus the growth rate of physical capital (weighted by capital's share). The capital stock estimates distinguish between structures and equipment.

These TFP growth rates appear in Table 3.[(15)] Table 3 also shows the ratios of value-added to gross output in each industry, which enable us to convert TFP growth on the value-added approach to the gross output approach. The latter can then be aggregated, by weighting each growth rate by the ratio of gross output to total final output (Domar aggregation). Total final output is defined as aggregate value-added plus intermediate imports (the latter is 13.8% of the total). These ratios were calculated from the 1995 input-output (IO) tables. Note that each sector consists of a number of IO

(15) O'Mahony cautions against taking the estimated TFP growth rate for non-market services (which is mainly government services) very seriously, because of the well-known deficiencies in the measurement of output in this sector.

Table 3
TFP growth in the United Kingdom, by sector, 1973-95

	(1)	(2)	(3)	(4)	(5)	(6)
	Value-added/ gross output	TFP growth (VA)	TFP growth (GO)	Gross output/ Total final output	Contribution to aggregate TFP growth	
Sector	Ratio	% pa	% pa.	Ratio	% pa	% of total
Agriculture, forestry, and fishing	0.514	2.92	1.50	0.029	0.04	4.6
Mining and oil refining	0.736	-2.15	-1.58	0.028	-0.04	-4.7
Utilities	0.499	2.87	1.43	0.039	0.06	6.0
Manufacturing	0.488	1.85	0.90	0.355	0.32	34.1
Construction	0.490	2.15	1.05	0.085	0.09	9.5
Transport & communications	0.587	3.06	1.80	0.113	0.20	21.5
Distributive trades	0.558	0.43	0.24	0.210	0.05	5.4
Financial & business services	0.692	0.98	0.68	0.198	0.13	14.3
Miscellaneous personal services	0.606	1.21	0.73	0.079	0.06	6.2
Non-market services	0.788	0.17	0.13	0.216	0.03	3.1
Total	—	—	—	1.352	0.94	100.0

Note: Non-market services comprise health, education, public administration and defence.
Sources: Value-added/gross output and gross output/total final output ratios from 1995 input-output tables, with intra-sector sales netted out. Total final output is aggregate gross value-added at basic prices + intermediate imports. TFP growth (VA) from O'Mahony (1999), Table 1.9. TFP growth (GO), column (3) is value- added/gross output ratio x TFP growth (VA) [column (1) x column (2)]. Contribution to aggregate TFP growth, column (5), is column (3) x column (4).

groups or industries. Gross output of a sector is not the sum of gross outputs of the groups, but is net of intra-sector sales.[16]

Some interesting facts emerge. First, TFP growth has indeed been lower in most services than in manufacturing, construction and the utilities. This is the case whether we employ the value-added or the gross output approach. However, transport and communications exhibit the highest growth rate of all. The overall TFP growth rate was 0.94% per annum over 1973-95. Second, financial and business services and the distributive trades both receive a large weight in the calculation of aggregate TFP growth (0.198 and 0.210 respectively), second only to manufacturing (0.355). These weights reflect both their size and the extent of their links with the rest of the economy. By contrast, the employment shares of these two sectors are 13.4% and 21.9% respectively, compared with 18.7% for manufacturing.[17] But the contribution of financial and business services to overall TFP growth was comparatively small, 14.3% of the total compared with 34.1% from manufacturing. Arithmetically, this is because its own TFP growth rate was quite low. The contribution of the distributive trades was still smaller, 5.4%.

5.2 The effects of structural change in the United Kingdom and the United States

How much has the aggregate growth rate been affected by the shift of resources into services? To answer this question, we should ideally compare the Domar weights of 1973 with those of 1995. But the limitations of the input-output tables mean that the longest comparison that can reasonably be made is for 1979 with 1995.[18] Following the analysis of Section 3.2, we split the Domar weights into two, intermediate sales and final sales, both expressed as a proportion of aggregate final output.

Table 4 shows the extent of structural change in the United Kingdom between 1979 and 1995. Overall, there has been a small rise in the degree of inter-relatedness of the economy. The sum of the Domar weights was

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(16) The netting-off was done using an unpublished domestic use table for 1995, supplied by the ONS. O'Mahony's sectors may not correspond exactly to the ones in the IO table, since she used SIC80 and the IO tables use SIC92. Also, in the calculation of aggregate TFP growth, it would be better to use an average of 1973 and 1995 weights, but the former are not available.

(17) O'Mahony (1999), Table I.1.

(18) There are no tables for 1973. There are tables for 1974, but the treatment of services is too aggregated to be useful.

1.352 in 1995, up from 1.288 in 1979, an increase of 5%. The Domar weights of all sectors producing goods fell over this period, whereas the weights rose for all service sectors except transport and communications. The increase for financial and business services was particularly striking, 11 percentage points. If we look specifically at the intermediate part of the Domar weights, two large changes stand out. First, the intermediate importance of manufacturing increased by 7 percentage points. That is, UK manufacturing became a more important source of inputs for the rest of the UK economy. But UK manufacturing became a much less important source of final output, so overall, its Domar weight fell by 3 percentage points.

The second large change on the intermediate side is the 10 percentage point rise in the weight for financial and business services, which accounts for nearly all the rise in its overall Domar weight. If we calculate the aggregate TFP growth rate using the 1979 Domar weights instead of the 1995 ones, we find it to be 0.85% per annum, lower by 0.09% per annum. Arithmetically, the shift to financial and business services raised the growth rate by 0.08% per annum.(19) This may seem a small amount, but recall that, on the conventional argument, the shift to this sector would have reduced the aggregate growth rate.

These ratios can also be calculated for the United States, using the 1977 and 1992 benchmark input-output tables (1992 is the latest available). The results appear in Table 5 for four sectors of interest: trade, finance and

(19) The shift away from mining also raised the growth rate, and by the same amount, 0.08% per annum. This is because TFP growth in mining was negative. The favourable effect of the shift away from mining was counter-balanced by the unfavourable effects of the shift away from the other goods-producing sectors.

Table 4
Structural change in the United Kingdom, 1979-95

	Intermediate sales/Total final output			Gross output/Total final output		
Sector	1979	1995	Change	1979	1995	Change
Agriculture, forestry, and fishing	0.032	0.014	-0.018	0.042	0.029	-0.013
Mining and oil refining	0.041	0.013	-0.027	0.079	0.028	-0.051
Utilities	0.028	0.014	-0.014	0.056	0.039	-0.017
Manufacturing	0.016	0.087	0.072	0.382	0.355	-0.027
Construction	0.007	0.012	0.006	0.100	0.085	-0.015
Transport & communications	0.060	0.055	-0.005	0.116	0.113	-0.003
Distributive trades	0.034	0.011	-0.023	0.200	0.210	0.010
Financial & business services	0.030	0.127	0.097	0.084	0.198	0.114
Miscellaneous personal services	0.040	0.007	-0.033	0.069	0.079	0.010
Non-market services	0.000	0.010	0.010	0.161	0.216	0.055
Total	0.288	0.352	0.064	1.288	1.352	0.064

Note: Gross output and intermediate sales (sales to other UK sectors) are net of intra-sector sales. Total final output is aggregate gross value-added + aggregate intermediate imports. (Gross value-added includes taxes less subsidies and, in 1979, sales by final buyers).

Source: 1979 and 1995 input-output tables. For 1979, all calculations use Table B of the 1979 tables. For 1995, both the combined use and the (unpublished) domestic use matrix were employed.
1979: Gross output is from row 107 of Table B, adjusted for intra-sectoral sales (sales by each of the industries within a sector to other industries in the same sector); intra-sectoral sales are the relevant column elements of Table B. Intermediate sales are derived as a residual: gross output minus sales to final demand. Sales to final demand is from column 107 of Table B of the 1979 tables.
1995: Gross output is from row 130 of the combined use matrix (Table 3), adjusted for intra-sectoral sales. Intermediate sales are from column 125 of the domestic use matrix, adjusted for intra-sectoral sales. Intra-sectoral sales are also from the domestic use matrix.

Table 5
Structural change in the United States, 1977-92

	Intermediate sales/Total final output			*Gross output/Total final output*		
Sector	*1977*	*1992*	*Change*	*1977*	*1992*	*Change*
Trade	0.054	0.048	-0.006	0.193	0.172	-0.020
Finance and insurance	0.017	0.019	0.002	0.052	0.078	0.026
Business services	0.064	0.091	0.028	0.079	0.119	0.040
Real estate and rental	0.031	0.045	0.013	0.132	0.155	0.022
Total (4 sectors)	0.166	0.203	0.037	0.457	0.524	0.067

Note: Trade is wholesale and retail trade (row 69 of the tables). Finance and insurance is row 70. Business services is row 73. Real estate (row 71) includes the imputed rent of owner-occupiers, which was not separately identified in 1977. Gross output for intermediate use is net of intra-sectoral sales. It was not possible to remove imports from the value of sales, but in these four sectors they are small. Total final output is GDP at purchasers' prices.

Source: US input-output tables, 1977 and 1992 (Survey of Current Business, May 1984 and November 1997).
.

insurance, business services, and real estate and rental.[20] Rather surprisingly, the importance of trade has shrunk since 1977, both in final and intermediate output, though it still gets the largest weight overall (0.172). Finance and insurance has grown in importance over this 15-year period: its Domar weight has grown by 49% to 0.078. A similar increase appears in business services. Its weight, three quarters of which is intermediate, has risen by 50% to 0.119. The intermediate part has risen by 43%, from 0.064 to 0.091. Most of real estate and rental is the imputed rent of owner-occupiers, which cannot be separately identified in 1977. The intermediate part (mostly wholesale trade) has risen by 43%. The total Domar weight of these four sectors has risen by 15% to 0.524, and the intermediate part has risen to 0.203, or by 22%. Thus here too, structural change seems to have been favourable to productivity growth.[21]

5.3 The effect of measurement error

It is plausible that the output of financial and business services is underestimated, because of the rather crude methods of measurement currently used.[22] If output growth in this sector when measured correctly turned out to be appreciably higher, so that TFP growth was also higher, what would be the effect on Table 3? Since most of this sector's output is sold to other domestic sectors, the probable effect would be a corresponding reduction of the TFP growth rates of other sectors, with only a small effect on the overall TFP growth rate. This is because correction of an error in the, growth rate of gross output in an intermediate-producing sector will, under double deflation, cause changes in the growth rates of value-added in other

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(20) We use GDP at purchasers' prices to measure aggregate final output, since imports for intermediate use are not given separately in the United States input-output tables. These are probably a small proportion of GDP in the United States, since total imports of goods and services were only 10% of GDP in 1992. The absence of information on intermediate imports does not matter much for the four sectors in Table 5, since here imports (final plus intermediate) are small. But they may be more important in other sectors, hence no calculations are shown for these.

(21) But there is a qualification to this conclusion. The most sophisticated calculations of TFP growth at the sectoral level are by Jorgenson *et al* (1987). An updated version of the dataset developed there (down-loadable from Jorgenson's web page, linked to Harvard University, www.harvard.edu) shows that TFP growth in the finance, insurance and real estate sector was negative over the period 1977-91. It is possible, however, that output growth is under-estimated in this sector (see below).

(22) Current ONS methods are described by Sharp (1998) and discussed by Oulton (1999c).

purchasing sectors (see equation **(19)**). The other sectors' contribution to the overall rate would be lowered, whereas that of financial and business services would rise.

In the simple example of Section 3.2, correction of an error e in measuring output growth in industry 1 would lead to offsetting changes in equation **(21)**: $\hat{q}_1$ would rise by e, and $\hat{q}_2$ would fall by $r_1 e$, leaving $\hat{q}$ unaffected. However, this assumes that value-added is measured by double deflation. This netting-off of errors will not occur if single deflation is employed, as currently in the UK national accounts. Under single deflation, correction of such measurement errors will lead to a spurious rise in whole-economy productivity growth. See Appendix B for details.

The alternative way to calculate whole-economy TFP growth is to use only aggregate data, since $\hat{q} = \hat{v} - \hat{x}$ (see equation **(13)**). GDP is a sum of final expenditures as well as a sum of value-added, so we can use the expenditure estimate of GDP and thus avoid any errors in measuring output in industries producing intermediate goods. (In the simple example, the growth of GDP is simply the growth of gross output in industry 2, cars, and is independent of any error in measuring output in industry 1, business services.) However, though this gives the correct answer at the aggregate level, it fails to tell us the contribution of each sector.[23]

5.4 Outsourcing and the relabelling of economic activities

There is an alternative view of the rising share of finance and business services. According to this view, economic activities are simply being relabelled. An activity such as cleaning, once done within, say, a manufacturing firm, is now carried out by a contract cleaning firm located in the services sector. Why should such a relabelling of activities have any effect on aggregate productivity growth? This argument may seem superficially plausible. But it leaves unexplained why there should be such a consistent tendency to relocate activities to the services sector. As Chart 2

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(23) This raises the issue of the consistency of the constant price estimates of GDP under the output and expenditure approaches. The expenditure estimates are largely unaffected by errors in sectors such as financial and business services, except insofar as these produce final output. The current practice in the United Kingdom is to bring the annual output estimates into line with the expenditure estimates whenever a serious discrepancy occurs. (For the quarterly growth rate of GDP, the output estimates are considered more reliable, and so it is the expenditure estimates that are adjusted). See Office for National Statistics (1998a), pages 137-38 and (1998b), Chapter 11.

has shown, this tendency is longstanding. If activities are simply being relabelled, why do we not see as many transfers from services to the production sector as in the other direction?

This argument suggests that the relocation of activities is in response to profit opportunities. To continue the example, even if the cleaners are the same people as before, the organisation of their work and the management of their time may now be different. And higher profits may correspond to efficiency gains.

Another factor may also be at work. One reason for outsourcing activities previously done within the firm is that the wages paid in the new firm may be lower. Union or insider power may have pushed wages above the market level in the original firm. This possibility does not arise in the theoretical framework used here, since competition in input markets is assumed. So there cannot be any differences between the wages paid by different firms for the same type of labour. But note that if outsourcing occurs, because the price of the activity is lower outside the firm than within it, there is still scope for the kind of efficiency gains analysed here in a competitive setting. If the price is lower, more of the activity will be demanded and this may lead to an increase in TFP growth for the same reason as in the competitive case.

6. Conclusion

The main conclusion of this paper is an optimistic one. Even if resources are shifting towards industries such as financial and business services, whose productivity is growing slowly, the aggregate growth rate of productivity need not fall. Quite the contrary: if resources shift towards even stagnant intermediate industries, aggregate productivity growth may rise, not fall. Of course, it is better still if resources shift to sectors where productivity growth is high.

Thus, contrary to the stagnationist argument, the shift to service industries in the advanced industrial countries may well be favourable, not harmful, to productivity growth.

Over the period 1973-95, TFP growth in market services in the United Kingdom has mostly been lower than in the production sector. But the financial and business services sector has currently (with distribution) the second-largest weight after manufacturing in the calculation of aggregate productivity growth. Productivity growth in this sector is therefore crucial to Britain's future economic prospects. At the moment, the productivity growth rate appears to be quite low and its contribution to overall growth is also comparatively low. But structural change has favoured this sector both in the United Kingdom and the United States, so that its contribution is calculated to have raised, not lowered, aggregate growth.

It is quite possible that TFP growth is underestimated in financial and business services, owing to error in measuring output. Correction of this error (if it is one) would probably make this sector appear even more important for overall growth, and other sectors such as manufacturing less important. But it would not lead to an appreciable upward revision in the overall productivity growth rate.

References

Barro, R J (1998), 'Notes on growth accounting', *NBER Working Paper No 6654*, Cambridge, MA.

Baumol, W J (1967), 'Macroeconomics of unbalanced growth: the anatomy of urban crisis', *American Economic Review*, 57, pages 415-26.

Baumol, W J (1985), 'Productivity policy and the service sector', in Inman R P (ed), *Managing the Service Economy: Prospects and Problems,* Cambridge: Cambridge University Press.

Baumol, W J, Batey Blackman, S A and Wolff, E N (1989), *Productivity and American Leadership,* Cambridge, MA: The MIT Press.

Baumol, W J and Wolff, E N (1984), 'On interindustry differences in absolute productivity', *Journal of Political Economy,* 92 (December), pages 1,017-34.

Domar, E D (1961), 'On the measurement of technological change', *Economic Journal,* LXXI, pages 709-29.

Gollop, F M (1983), 'Growth accounting in an open economy', in Dogramaci A (ed), *Developments in Econometric Analyses of Productivity,* Boston, MA: Klumer Nijhoff.

Griliches, Z (ed) (1992), 'Output measurement in the service sectors', *NBER Studies in Income and Wealth,* Volume 56, Chicago and London: University of Chicago Press.

Griliches, Z (1994), 'Productivity, R&D, and the data constraint', *American Economic Review,* 84, pages 1-23.

Hill, T P (1971), *The Measurement of Real Product: A Theoretical and Empirical Analysis of the Growth Rates, For Different Industries and Countries,* Paris: OECD.

Hulten, C R (1978), 'Growth accounting with intermediate inputs', *Review of Economic Studies*, 45, pages 511-18.

Jorgenson, D W, Gollop, F M and Fraumeni, B M. (1987), *Productivity and US Economic Growth,* Cambridge, MA: Harvard University Press.

Jorgenson, D W and Griliches, Z (1967), 'The explanation of productivity change', *Review of Economic Studies,* 34, pages 249-83.

Julius, D and Butler, J (1998), 'Inflation and growth in a service economy', *Bank of England Quarterly Bulletin,* Vol 38 (4), pages 338-46.

Office for National Statistics (1998a), *United Kingdom National Accounts 1998,* London.

Office for National Statistics (1998b) , *United Kingdom National Accounts: Concepts, Sources and Methods* (1998 edition), London.

O'Mahony, M (1999), *Britain's Productivity Performance 1950-1996: An International Perspective,* London: National Institute of Economic and Social Research.

Oulton, N (1999a), 'Trends in relative prices: theory and an accounting framework', Bank of England, *mimeo.*

Oulton, N (1999b), 'Rising outsourcing as an engine of growth', Bank of England, *mimeo.*

Oulton, N (1999c), 'The measurement of output and prices in the service sector', Bank of England, *mimeo.*

Oulton, N and O'Mahony, M (1994), *Productivity and Growth: A Study of British Industry 1954-1986,* Cambridge: Cambridge University Press.

Rowthorn, R and Ramaswamy, R (1997), 'Deindustrialization: causes and implications', *Staff Studies for the World Economic Outlook,* Washington, DC: International Monetary Fund.

Sharp, P (1998), *Gross Domestic Product: Output Methodological Guide,* GSS Methodology Series No 5, London: Office for National Statistics.

Stiroh, K J (1998), 'Computers, productivity and input substitution', *Economic Inquiry,* XXXVI (April), pages 175-91.

Stoneman, P and Francis, N (1994), 'Double deflation and the measurement of productivity growth in UK manufacturing 1979-89', *International Journal of the Economics of Business,* pages 1, 423-437.

APPENDIX A: BIASED TECHNICAL PROGRESS

In Section 4 it is assumed that technical progress in industry 2 is unbiased. This appendix considers the effect of relaxing this assumption and allowing for factor augmenting technical progress at possibly different rates. For simplicity, we assume that the production function for industry 2 is CES so equation **(23)** becomes:

$$y_2 = [(y_1 e^{\lambda_1 t})^\rho + (x_2 e^{\lambda_2 t})^\rho]^{1/\rho}, \qquad \rho < 1, \lambda_1 > 0, \lambda_2 > 0$$

which is of identical form to **(23)** when $l_2 = l_1$. Here λ_1 and λ_2 are the rates of factor augmenting technical progress. $\lambda_2 - \lambda_1$ is a measure of the bias of technical progress in favour of labour in the car industry. The elasticity of substitution is $\sigma = 1/(1-\rho)$.

Substituting equation **(24)** into the last equation, we find the aggregate relationship:

$$y_2 = [(x_1 e^{(\lambda_1 + \hat{q}_1)t})^\rho + (x_2 e^{\lambda_2 t})^\rho]^{1/\rho}$$

from which we derive the aggregate growth rate of TFP:

$$\hat{q} = r(\hat{q}_1 - (\lambda_2 - \lambda_1)) + \lambda_2$$

(compare **(22)**). Hence if *r* is rising, $\hat{q}$ will be rising too provided that

$$\hat{q}_1 > (\lambda_2 - \lambda_1) \qquad \textbf{(A.1)}$$

ie TFP growth in business services must exceed the bias of technical progress towards labour in cars.

We can find the conditions under which rising outsourcing will actually occur in the presence of biased technical progress by considering the conditions for profit maximisation. Profit maximisation in the car industry requires that the ratio of the marginal products should equal the ratio of the input prices at all times:

$$e^{\rho(\lambda_1 - \lambda_2)}(x_2 / y_1)^{\rho - 1} = w / p_1$$

We can use this equation to find the evolution over time of the input ratio x_2/y_1. Totally differentiating with respect to time and using **(28)**,

$$\frac{d\log(x_2/y_1)}{dt} = -\sigma\hat{q}_1 + (\sigma-1)(\lambda_2-\lambda_1)$$

which may be compared with **(29)**. We see that a sufficient degree of bias in technical progress towards labour $(\lambda_2 > \lambda_1)$ may result in the labour/business services ratio rising in the car industry if $\sigma > 1$, despite the fact that labour is getting more expensive. The condition for the ratio to *fall* as it does in the unbiased case is:

$$\hat{q}_1 > \frac{\sigma-1}{\sigma}(\lambda_2-\lambda_1)$$

Previously (see equation **(29)**), the condition was just $\hat{q}_1 > 0$ to which this last equation reduces when there is no bias ($\lambda_2 = \lambda_1$).

The relative growth rate of labour in the two industries is (using **(27)**):

$$\frac{d\log(x_2/x_1)}{dt} = -(\sigma-1)[\hat{q}_1-(\lambda_2-\lambda_1)] \qquad \textbf{(A.2)}$$

which may be compared with **(30)**. If $\sigma > 1$, the condition for rising outsourcing to occur is now:

$$\frac{d\log(x_2/x_1)}{dt} < 0 \quad \text{if} \quad \hat{q}_1 > (\lambda_2-\lambda_1) \text{ and } \sigma > 1$$

Of course if technical progress is biased towards business services $(\lambda_2 < \lambda_1)$ then this condition is weaker than in the unbiased case. In the presence of biased technical progress, rising outsourcing can occur even if $\sigma < 1$:

$$\frac{d\log(x_2/x_1)}{dt} < 0 \quad \text{if} \quad \hat{q}_1 < (\lambda_2-\lambda_1) \text{ and } \sigma < 1$$

So a combination of a low elasticity of substitution and a strong bias in technical progress towards labour in cars can lead to a rising share of labour in business services. However in this case rising outsourcing will *reduce* the aggregate TFP growth rate: see equation **(A.1)**.

In summary, biased technical progress qualifies rather than overturns the results of Section 4. Rising outsourcing will occur and will raise the aggregate TFP growth rate if (a) as before, the elasticity of substitution exceeds one and (b) TFP growth in industry 1 exceeds the bias towards labour in industry 2, $\hat{q}_1 > (\lambda_2 - \lambda_1)$, rather than TFP growth in industry 1 just being positive. So if the bias is towards labour, the main result of Section 4 is qualified though not reversed. But if the bias is towards business services, the main result is strengthened.

APPENDIX B: THE MEASUREMENT OF REAL VALUE-ADDED

National income statisticians employ two main methods of measuring value-added in constant prices, single deflation and double-deflation. The double deflation method is as follows. We write nominal value added in sector *i* at time *t*, showing explicitly the dependence on time, as:

$$p_i^v(t)v_i(t) = p_i(t)y_i(t) - p_i^m(t)m_i(t)$$

where $v_i(t), y_i(t), m_i(t)$ are the quantities of value added, gross output and material input respectively at time *t* and $p_i^v(t), p_i(t), p_i^m(t)$ are the corresponding prices. Now deflate nominal gross output, $p_i(t)y_i(t)$, by the price index for gross output, $p_i(t)/p_i(0)$, and nominal intermediate input, $p_i^m(t)m_i(t)$, by the price index for intermediate input, $p_i^m(t)/p_i^m(0)$. The result is double-deflated value-added in constant (time 0) prices:

$$p_i^v(0)v_i(t) = p_i(0)y_i(t) - p_i^m(0)m_i(t) \qquad \textbf{(B.1)}$$

Taking growth rates in **(B.1)**, we get

$$\frac{v_i(t)-v_i(0)}{v_i(0)} = \left[\frac{p_i(0)y_i(0)}{p_i^v(0)v_i(0)}\right]\left[\frac{y_i(t)-y_i(0)}{y_i(0)}\right] - \left[\frac{p_i^m(0)m_i(0)}{p_i^v(0)v_i(0)}\right]\left[\frac{m_i(t)-m_i(0)}{m_i(0)}\right]$$

(B.2)

This equation is analogous to **(19)**. There are two differences. First, **(B.2)** uses a fixed base, time zero, and second, it is in discrete not continuous time.

Single deflation measures the growth rate of real value-added simply by the gross rate of real gross output:

$$\frac{v_i(t)-v_i(0)}{v_i(0)} = \frac{y_i(t)-y_i(0)}{y_i(0)} \qquad \textbf{(B.3)}$$

It can be seen that double deflation and single deflation will produce the same answer only if gross output and intermediate input are growing at the same rate:

$$\frac{y_i(t) - y_i(0)}{y_i(0)} = \frac{m_i(t) - m_i(0)}{m_i(0)} \qquad \textbf{(B.4)}$$

Equivalently, the answers will be the same only if the ratio of intermediate input to gross output in constant prices is constant. This is clearly a highly restrictive condition.

Double deflation has two theoretical advantages over single deflation. First, it is consistent with the theory of production, as shown by the derivation of **(19)**. Second, we want the growth rate of real GDP from the expenditure side to equal the same growth rate measured from the output side. Aside from errors and omissions, double deflation satisfies this condition but single deflation in general does not, as we can illustrate with the model of Section 4.1.

From the expenditure side, the growth rate of real GDP is simply the growth rate of real expenditure on the output of industry 2:

$$\text{Growth of GDP(E)} = \hat{y}_2 \qquad \textbf{(B.5)}$$

From the output side, we would measure GDP growth as a weighted average of the growth rates of double-deflated value-added in the two industries, the weights being each industry's share of aggregate nominal value-added:

$$\text{Growth of GDP(O)}_{\text{Double}} = \left(\frac{wx_1}{p_2 y_2}\right)\hat{v}_1 + \left(\frac{wx_2}{p_2 y_2}\right)\hat{v}_2 \qquad \textbf{(B.6)}$$

Using **(19)** in conjunction with **(25)** and **(26)**, we find the double-deflated value-added growth rates to be:

$$\hat{v}_1 = \hat{y}_1$$

$$\hat{v}_2 = \left(\frac{p_2 y_2}{w x_2}\right)\hat{y}_2 - \left(\frac{p_1 y_1}{w x_2}\right)\hat{y}_1$$

So substituting into **(B.6)**, we see that

$$\text{Growth of GDP(O)}_{\text{Double}} = \hat{y}_2 \qquad \textbf{(B.7)}$$

in agreement with the expenditure-side estimate, **(B.5)**.

Some statistical agencies use single deflation for practical reasons. If the pattern of intermediate input purchases is not known with precision, it may be better to use single deflation, despite its theoretical disadvantages (Hill 1971). In the United Kingdom, the Office for National Statistics (ONS) uses double deflation only for agriculture and electricity. For all other industries, the ONS uses single deflation (Sharp 1998). Stoneman and Francis (1994) have estimated the measurement error due to the use of single deflation in UK manufacturing in the 1980s.

There is another effect of measurement error that also deserves consideration. Suppose there is an error e in measuring the growth rate of the gross output of industry 1. Measured growth is $\hat{y}_1 + e$, whereas true growth is $\hat{y}_1$. Then GDP growth measured using double deflation is unaffected. The error e in industry 1 induces an error of $-(p_1 y_1 / w x_2)e$ in the estimate of value-added growth in industry 2. As can be seen from **(B.6)**, these two errors cancel out at the aggregate level. Suppose, however, that single deflation is employed. Then

$$\text{Growth of GDP(O)}_{\text{Single}} = \left(\frac{w x_1}{p_2 y_2}\right)(\hat{y}_1 + e) + \left(\frac{w x_2}{p_2 y_2}\right)\hat{y}_2$$

So a positive error in measuring output growth in industry 1 will raise the estimated growth rate of GDP, and hence that of aggregate TFP as well.