Persistence and volatility in short-term interest rates

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Abstract

It is important for monetary policy makers to know how closely money market rates follow the policy rates they set. This paper looks at the volatility and persistence of divergences between short-term market interest rates away from policy rates. This may also offer insights into the effectiveness of various approaches that central banks employ to smooth interest rate volatility, such as requiring minimum reserves. Using data for Germany, Italy and the United Kingdom, we find that in all three countries there are significant temporary divergences, although the average divergence is close to zero.

1 Introduction

How much control can central banks exert over short-term market interest rates? Observing how far short-term market rates deviate from policy rates for different countries may give clues about the influence that authorities have over market rates. And since differences in how central banks conduct money market operations remain, such analysis may also offer insights into the effectiveness of various approaches that central banks employ to smooth interest rate volatility, such as requiring minimum reserves.

There is a large literature in which the time series properties of short-term market interest rates are estimated (see, for example, Brennan and Schwartz (1982), Dietrich-Campbell and Schwartz (1986), Sanders and Unal (1988), Chan *et al* (1991) and Nowman (1997)). And a smaller body of work considers the different operating systems central banks use to provide marginal liquidity to the banking system (see, for example, Borio (1997)).

But with the exception of Hamilton (1996) in the United States, there has been little attention paid to the time series profile of divergences in short-term market rates from the policy rates used by the central bank to provide marginal liquidity. In this paper we use a simple reduced-form model to answer some simple questions about the volatility and persistence of divergences of short-term money market rates (overnight, one week and one month) from policy rates across three European countries which had quite different systems of money market operations before the introduction of the euro: Germany, Italy and the United Kingdom. In particular, we consider four questions. Did short-term market rates revert to policy rates in the long run over the sample period? Did the speed with which they reverted to the long-run mean following a shock differ between countries? Did the volatility of short-term market rates differ between countries? And did it differ over time in response to changes in operating systems?

Our objective is to characterise the data rather than explicitly model the behaviour of the interbank market. But this is an important step in helping to evaluate the performance of operating systems. And analysis of these questions is especially pertinent now, given the recent large change in central bank operating systems in Europe associated with the introduction of the euro. We structure the remainder of the paper as follows. In the first section, we outline a simple model of short-term market rates. In Section 3, we briefly compare the operating systems used by central banks to steer short-term market rates in Germany, Italy and the United Kingdom over the sample period. In Section 4, we describe the data we use. This helps to motivate the more detailed empirical work we conduct in Sections 5 and 6, in which we model the time series behaviour of short-term market rates using general method of moments techniques. In the final section we conclude.

2 A simple model

A number of studies in the empirical finance literature have estimated the parameters of a discrete time process in which the conditional mean and variance of the short-term interest rate depend on the level of the interest rate (see Brenner *et al* (1996), Brennan and Schwartz (1982), Dietrich-Campbell and Schwartz (1986), Sanders and Unal (1988) and Chan *et al* (1991)). Our model is based on that of Brenner *et al* (1996). But unlike these earlier studies, our focus is the degree of control that central banks exercise over short-term money market rates: ie the divergence of the market rate from the policy rate (ie the interest rate on the main monetary instrument used by the central bank to provide marginal liquidity to the market). Its specification is:

$$d_t - d_{t-1} = \alpha + \beta d_{t-1} + \varepsilon_t \tag{1}$$

$$E(\varepsilon_t \mid \Psi_{t-1}) = 0 \tag{2}$$

$$E(\varepsilon_t^2 | \Psi_{t-1}) = \sigma_t^2 = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + b \sigma_{t-1}^2 + \alpha_3 d_{t-1}^{2\gamma}$$
(3)

where d_t is the divergence of the market rate from the policy rate and $E(\bullet | \Psi_{t-1})$ denotes the expectation conditional on the information set at time *t*-1. This process allows us considerable flexibility in estimating the persistence and volatility of the short-term interest rate process around the policy rate. The parameters of the model yield some simple insights into central banks' control over short-term market rates.

First, the long-run mean of the divergence between the market rate and the policy rate is given by the expression $(-\alpha/\beta)$.⁽¹⁾ By testing whether this is significantly different from zero in each market, it is possible to infer the

⁽¹⁾ The term 'long run' is used to describe the completion of the adjustment process described by equation (1), which may occur within days.

effectiveness of market operations in achieving a short-term market interest rate that coincides with the policy rate.

Second, the speed of reversion to this long-run mean is given by $-\beta$. Assuming a central bank does have control over long-run market rates, this measures the speed with which its operations drive rates back to their long-run values following shocks to the money market.⁽²⁾ Persistence in the divergence may be particularly important for propagating effects from shocks to short-term rates further up the term structure, since predictability in short-term rates will generate expectations of future divergences, and hence—via the expectations hypothesis—movements in longer rates.

Third, the volatility of short-term money market rates around the policy rate is given by σ_t . The underlying causes of volatility in short-term market rates may be unrelated to the system of money market operations. These may include market speculation over future changes in policy rates, the presence of errors by the central bank or its counterparties in forecasting the amount of funds the market needs to meet its reserve requirements, or non-competitive behaviour among participants in the market for central bank funds. But, as we discuss in more detail below, it is sometimes claimed that the level and autocorrelation of volatility is affected by the operating system adopted by the central bank. And it is certainly possible to test whether a change in a central bank's operating system affects short-term market interest rate volatility.

The model we use is fairly general, nesting two commonly used empirical models: the GARCH specification and the LEVELS specification. Both models include equations (1) and (2), but impose different restrictions on the conditional volatility (equation (3)). The GARCH specification parameterises conditional volatility as a function of unexpected shocks in the previous period to the interest rate and the conditional volatility in the previous period. That is:

(4)

$$\sigma_t^2 = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + b \sigma_{t-1}^2$$

⁽²⁾ We are grateful to an anonymous referee for pointing out that there may be occasions on which the central bank may not wish to drive market rates back to policy rates as soon as possible, and this may bias our estimate of β .

This captures the possibility of 'volatility clustering': high volatility is followed by high volatility, and low volatility by low volatility. The persistence in the variance is measured by $\alpha_1 + b$. If this is large, but less than one, then following a large deviation in interest rates the shocks to rates will have a large variance for a considerable time thereafter. A value greater than one means that the variance is explosive, tending to infinity over time.

Assuming the variance is not explosive, we can calculate an unconditional variance. This is the expected variance as the forecast horizon becomes very large. In the limit, as time (s) tends to infinity, information at time (t) (Ψ_t) ceases to provide useful information about the expected variance of \mathcal{E}_{t+s}^2 . Instead, the expectation for this (unconditional) variance is given by: $\lim_{s\to\infty} E\left[\mathcal{E}_{t+s}^2 | \Psi_t\right] = \frac{\alpha_0}{1-(\alpha_1+b)}$

This equation illustrates the connection between the persistence in variance and the unconditional volatility. Holding α_0 constant, an increase in the persistence of volatility increases the unconditional volatility. But because α_0 may vary across systems, it is not always the case that countries with operating systems that display greater persistence in volatility will also have a larger unconditional volatility.

In contrast to the GARCH specification, the LEVELS specification, analysed in Chan *et al* (1991), parameterises volatility as a function of interest rate levels only.⁽³⁾

$$\sigma_t^2 = \alpha_3 (d_{t-1}^2)^{\gamma} \tag{5}$$

Under this specification, volatility is large following periods when the divergence, rather than volatility, is high. Consequently, large shocks do not necessarily lead to a change in the conditional variance.

⁽³⁾ More precisely, the volatility is modelled as a function of the difference between the interest rate and the policy rate.

3 How money market operating systems differ

How do central banks try to influence short-term market rates? Over recent years, the techniques used by central banks in advanced economies have tended to converge. But significant differences still exist between countries, reflecting both historical factors and different priorities and objectives between central banks (see Borio (1997)). Table A summarises many of the main features of the three systems employed in Germany, Italy and the United Kingdom over the sample period of this study, January 1993 to April 1998.⁽⁴⁾

⁽⁴⁾ Our brief review relies heavily upon Borio's (1997) excellent summary of the structure of money market operations across a range of countries. For the United Kingdom we also draw upon a number of other papers, principally Schnadt (1994) and Bank of England (1997).

	United Kingdom	Italy	Germany
Target rate	short term	overnight	overnight
— maturity (days)	30-90	1	1
Corridor (basis points)	no	150	200
Working balances	yes	no	no
Reserve requirements	no	yes	yes
- maintenance period	1 day	1 month	1 month
Main operations	outright and repo	repo	repo
- maturity of policy rate (days)	1-33 (a)	≤ 30	14
— regular intervals	yes	every 4 days, on average	yes
- frequency	≦ 3 per day	≥ 1 per week	1 per week
Overall frequency	> 1 per day	> 1 per week	≥ 1 per week

Table A: Key features of money market systems inGermany, Italy and the United Kingdom

Source: Borio (1997).

(a) Since the introduction of gilt repo in March 1997, the average maturity is less than two weeks.

All three central banks provided the bulk of the liquidity they made available to the market ('refinancing') via open market operations at the policy rate. We highlight a number of key differences. A major distinction was that the United Kingdom had (almost) zero reserve requirements and a one-day maintenance period. Italy and Germany, by contrast, maintained positive reserve requirements. In Germany, the range of reserve ratios fell from 4.15%–12.1% of eligible liabilities in 1991 to 1.5%–2.0% in 1996. In Italy, reserve requirements were among the highest in Europe, at 15.0% in 1996.⁽⁵⁾ In both countries, the main objective for requiring reserves was to provide a buffer to stabilise the overnight rate. Consistent with this objective, the Bundesbank and Banca d'Italia required that reserve requirements be met, on average, over periods of one month.⁽⁶⁾ Hence banks were able to draw down reserve positions held at the central bank in the face of unforeseen liquidity shocks at any time during the maintenance period. In contrast, the Bank of England relied upon fine-tuning the liquidity of the sterling money market more regularly to stabilise short-term rates, holding up to three rounds of open market operations in a day.⁽⁷⁾

⁽⁵⁾ These were reduced from a range of 22.5%-25.0% of eligible liabilities in 1990.

⁽⁶⁾ Positive reserve requirements are not necessary for such smoothing. As discussed in Davies (1998), interest rate smoothing could be achieved with a buffer stock of reserves provided to banks through fully collateralised overdrafts from the central bank. In this case smoothing would be achieved through averaging around a zero reserve requirement.

⁽⁷⁾ This was reduced to two rounds a day in June 1998.

One claim that has been advanced (see, for example, the discussion in Borio (1997)) is that the liquidity buffer provided by averaging reserves offers the central bank that uses them tighter control over short-term market rates than one that does not. To investigate this claim, we examine whether: (i) the long-run mean of the divergence between short-term market rates and the policy rate was lower in Germany and Italy than in the United Kingdom over the sample period; (ii) the speed of reversion to the long-run mean of the divergence was lower in the United Kingdom than in Germany or Italy and; (iii) the volatility of short-term market rates was lower in Germany and Italy than in the United Kingdom.

Another distinction concerns the maturity of the interest rates that the monetary authorities were keenest to influence (denoted the 'target' rate in Table A). In Italy and Germany, the overnight interest rate was — explicitly — the main focus or reference for policy. In contrast, the United Kingdom attached less importance to controlling the overnight rate for the implementation of monetary policy. The authorities set the two-week gilt repo rate and focused on influencing those interest rates at longer maturities that have a more direct effect on real economic activity (ie maturities in the one to three-month range).⁽⁸⁾ Hence it has been claimed (see, for example, Borio (1997)) that overnight market rates in the United Kingdom were more volatile than in Germany and Italy, but relatively less volatile at slightly longer maturities (such as one week and one month).

Over the sample period there were two important changes to the structure of the money markets in the United Kingdom, which may have affected the Bank of England's ability to steer short-term market rates through its open market operations. First, the establishment of the gilt repo market in January 1996 removed a constraint on banks' ability to arbitrage in the interbank market, increased the financing options available to banks, and hence may have reduced the divergence between short-term market rates and the Bank's policy rate. We examine below whether the introduction of gilt repo did reduce the volatility of divergences in short-term market rates. Second, the introduction of two-week gilt repo as the main daily instrument for providing liquidity to the market in March 1997 increased the pool of collateral available to the banks for obtaining funds from the Bank of England, which may have reduced further the divergence between

⁽⁸⁾ See Bank of England (1997, page 205).

short-term market rates and the policy rate. We also examine whether this reform affected the volatility of divergences in short-term market rates.

4 Data description

For short-term market rates in the United Kingdom we use Libor. The value selected was the lowest offer quoted on the screens at 8:30am each morning for the three maturities we examined. Caveats apply: since the quotes were merely indicative, it is unclear how much trade was conducted at these rates; and time-of-day effects may mean that quotes at 8:30am were unrepresentative of rates throughout the day. Despite these reservations, Table B below shows that from January to August 1997, overnight Libor rates offered a reasonable approximation to the rates at which trades were actually transacted.⁽⁹⁾

Table B: Correlation between UK overnight rates and the base rate (Jan. – Aug. 1997)

	SONIA	Libor	Base rate
SONIA	1.000	0.905	0.826
Libor		1.000	0.912
Base rate			1.000

Except for the overnight rates, we use eurocurrency rates taken from Datastream for German and Italian interest rates. These rates represent the average mid-price quoted by a sample of banks active in the appropriate money market. For the overnight rates, we use data provided by the BIS.⁽¹⁰⁾ We use daily data for Germany, Italy and the United Kingdom detailing the market rates for overnight, one-week, one-month, three-month, six-month, and one-year maturities. Our dataset covers the period from January 1993 to April 1998 and contains more than 1,300 observations for each country at each maturity.

Charts 1–3 plot divergences in the overnight rate from the policy rate in the three countries for the part of our sample period since 1996. Sizable

⁽⁹⁾ Sterling overnight interbank average (SONIA) is the average overnight interest rate, weighted by trade volume, on unsecured overnight interbank lending arranged by seven London brokers. The series is not long enough for our purposes, only dating back to April 1997. Moreover, we do not have access to equivalent German or Italian series.
⁽¹⁰⁾ We are grateful to Claudio Borio and his staff for their assistance.

divergences occured in all countries.⁽¹¹⁾ But it is also clear that the pattern of volatility was different across countries. Over the sample period the standard deviation of the divergence in the overnight rate from the policy rate (in basis points) was 27, 30 and 45 in Germany, Italy and the United Kingdom respectively. In all three countries standard deviations were lower for the slightly longer maturities. They fell most rapidly in the United Kingdom. The standard deviation for the weekly (monthly) rates were, in the same order: 15(14), 28(24) and 30(18). Systems that rely on averaging of reserve requirements over a maintenance period can generate regular and quite sharp spiking of very short-term rates around the end of the maintenance period. The regular spikes in the charts for Italy, and especially Germany, illustrate this point. However, in Germany large deviations were rare other than at the end of the maintenance period, whereas they were more frequent in the United Kingdom. On 35 occasions the German overnight rate deviated from the policy rate by 100 basis points, and in a further 42 instances it deviated by more than 50 basis points. In the United Kingdom the overnight rate deviated from the official rate by 100 basis points on 59 occasions, and by 100 points a further 198 times. The numbers were 17 and 59 respectively for Italy.

Another feature of the data illustrated by Charts 1–3 was the persistence of divergences in the overnight rate from the policy rate and the persistence of shocks to volatility itself. In Germany, however, the divergence of rates from the policy rate appeared to be both more regular and less persistent. This reflected the sharp but temporary effect on money market rates of the end of the monthly averaging period.

5 Results

In this section, we estimate the parameters for the model outlined in Section 2. We use a system of moment equations within the Generalised-Method-of-Moments (GMM) framework⁽¹²⁾ developed by Hansen (1982) for the following reasons. First, we do not need to assume that the distribution of interest rate changes is normal: the asymptotic justification for the GMM procedure requires only that the joint distribution

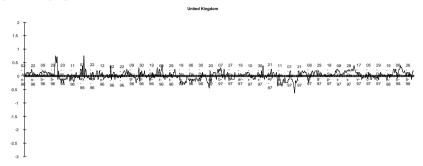
⁽¹¹⁾ Note that the difference between the overnight rate and the policy rate includes a term-structure element associated with market speculation over future policy changes.

⁽¹²⁾ This is the same technique used by Harvey (1988), Longstaff (1989), Chan *et al* (1991), Gibbons and Ramaswamy (1993), and Murphy (1994). An alternative approach, maximum likelihood, is used by Brenner *et al* (1996), who assume a t-distribution.

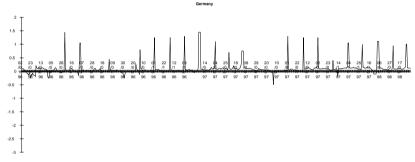
of interest rate changes be stationary and ergodic, and that the relevant expectations exist. Second, the GMM estimators and their standard errors are consistent even if the disturbances are conditionally heteroskedastic and serially correlated. Although maximum likelihood (ML) is more efficient than GMM for values of $\gamma > 1$, ML has the disadvantage that it maximises the likelihood of some observations too much relative to other observations (see Hull and White (1993)). However, we do use ML estimators as starting-values for the GMM estimators. We also impose inequality constraints requiring that the coefficients α_0 , α_1 , *b*, α_3 and γ be positive.

Charts 1–3: Overnight rates minus the 'policy' rate (Jan. 1996 – Mar. 1998)

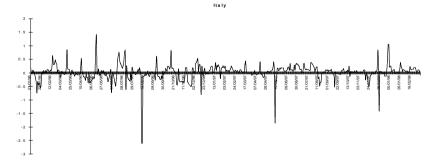
percentage points



percentage points



percentage points



5.1 General and nested specifications

The estimation results for the unrestricted model are presented in full in Annex 1. The three markets display some differences in the degree of time variation of volatility (see equation (3)). But in all three, the estimated coefficients on the volatility parameters (α_0 , α_1 , b, α_3 and γ) for divergences in the overnight rates are individually insignificant, although as a group they are jointly significant. So it is possible to impose parameter restrictions. Consequently we estimate the nested GARCH and LEVELS models outlined in equations (4) and (5).⁽¹³⁾

For the overnight rate in both Italy and the United Kingdom we are unable to reject either the GARCH model or the LEVELS model. However goodness-of-fit tests suggest the GARCH model better described the UK data, while the LEVELS model is our preferred model for Italy (see Table C). In Germany, for overnight rates we can reject the restrictions implied by the LEVELS model. Consequently we choose the GARCH model as our preferred specification for German overnight rates. But once we strip out end-of-month maintenance-period effects (shown in Column 3 of Table C) the fit is reduced.

⁽¹³⁾ The full results are presented in Annex 2 (GARCH) and Annex 3 (LEVELS).

	Germany	Germany (filtered)	Italy	UK
	GARCH	GARCH	LEVELS	GARCH
Reversion rate $(-\alpha/\beta)$	0.000853	0.000418	0.00011	-0.00113
	(5.33)	(4.17)	(0.77)	(-4.97)
Speed of reversion (- β)	0.5824	0.2792	0.4505	0.4752
	(7.88)	(7.22)	(5.72)	(12.88)
$lpha_0$	6.63E-07	3.88E-07	-	2.43E-21
	(0.37)	(3.52)		(0.00)
$lpha_{ m l}$	0.5385	0.2711	-	0.9987
	(0.87)	(3.24)		(3.74)
b	0.3452	1.84E-15	-	4.56E-14
	(0.44)	(0.00)		(0.00)
α_3	-		0.0422	-
			(3.34)	
γ	-		0.7315	-
			(25.95)	
p-value	0.97	0.01	0.48	0.40
Т	1364	1284	1364	1324

Table C: GMM estimates of the *preferred* model for the policy-adjusted overnight rate

Note: t-statistics in parentheses.

5.2 Long-run mean of the divergence

Using the coefficient estimates from our preferred specifications in Table C, we estimate the long-run mean of the divergence between short-term money market rates and the policy rate $(-\alpha/\beta)$ for each country. In Germany, all three short-term market rates diverged from the policy rate in the long run by a positive and significant amount. However, the magnitude of the mean divergence was small (ie 6–8 basis points). In the United Kingdom, the long-run divergence in both the overnight and one-week interest rates were also significantly different from zero.⁽¹⁴⁾ The estimated long-run means were -11 basis points in both cases, ie market rates tended to be slightly lower than the policy rate. However, the one-month market rate is not significantly different from the policy rate. This contrasts with the results for Italy, where we find no significantly negative (-11 basis points) long-run mean for one-month rates. Taken together, the results provide no evidence

⁽¹⁴⁾ All levels of significance are measured at 95% significance level.

to support the claim that control over short-term market rates in Germany or Italy was greater than in the United Kingdom.

5.3 The speed of mean reversion

In all three countries, the value of the coefficient $(-\beta)$ was significantly different from zero for the overnight rate. These results supported the view that overnight interest rate shocks were persistent. For example, approximately 53% of a shock to the UK overnight rate, on average, remained the next day and nearly 8% remained a week later. Our estimates of $-\beta$ for Italy and the United Kingdom (0.45 and 0.47 respectively) did not differ in a statistically significant way: we cannot reject the null hypothesis that the degree of persistence in shocks to overnight market rates was the same in both countries.

The evidence for Germany suggested that the speed of reversion in overnight rates differed from those found for Italy and the United Kingdom. Unfortunately a case can be made for this difference being in either direction! Using the whole dataset, we calculated a speed of mean reversion of 0.58. However, this reflected the pronounced end-of-month spiking of interest rates in Germany associated with the end of the monthly maintenance period. Re-estimating our model using the 'filtered' series that excluded the end-months from the sample, we found that the speed of mean reversion was only 0.28, significantly less statistically than in the other two countries. Hence the persistence of shocks, unassociated with the end of the maintenance period, was higher in Germany than in Italy or the United Kingdom.

In all three countries the speed of mean reversion falls for the longer maturities; ie shocks were more persistent. The fall was greatest in the United Kingdom, with the speed of reversion falling to 0.17 for one-week rates, and 0.1 for one-month rates. This latter estimate was significantly lower than the corresponding figure for Italy (0.2). It was also lower than the German estimate (0.14), but not sufficiently for us to reject the null hypothesis that the persistence in shocks to German one-month rates was the same as the persistence in UK rates.

5.4 Volatility across countries

The estimated coefficients for the GARCH model can be used to compare the unconditional volatility across the three countries for the three maturities we

consider, which we present in Table D. The qualitative result that German and UK one-month rates were less volatile than Italian one-month rates remained true when comparing unconditional, rather than conditional, standard deviations.

Table D: Estimates of the unconditional standard deviationfor the policy-adjusted short-term rates, from the estimatesfor the GARCH specification(1993-98)

	Germany	Italy	UK
Overnight	23.89	22.11	Persistence in
			variance
Overnight (filtered)	7.30	-	-
One week	8.01	Explosive	16.49
One month	7.06	12.50	8.39

Comparing unconditional volatility across all three countries for overnight and one-week rates was complicated by the results for the UK overnight and Italian one-week rates. These results suggested the presence of either outliers or misspecification of the process (for a fuller discussion, see Lamoureux and Lastrapes (1990)). We discuss the possibility of a misspecification of the process for UK overnight rates in Section 5.5 when we consider the possibility of structural breaks.

We cannot estimate an unconditional volatility for Italian one-week rates since the series is explosive ($\alpha_1 + b > 1$): we suspect outliers.

Consequently we re-ran the GARCH model for the divergence in the Italian one-week rate, excluding four observations that we classify as outliers.⁽¹⁵⁾ The results are presented in Table A2.3 of Annex 2. The omission of the outliers reduced the persistence in volatility to 0.59, allowing us to calculate an unconditional standard deviation of just over twelve basis points, in between the values calculated for German and UK one-month rates.

5.5 Changes in volatility over time: testing for structural breaks in the United Kingdom

The sample period included two major changes to the structure of the money market in the United Kingdom. Did these changes have any effect

⁽¹⁵⁾ These four consecutive outliers run from 10-13 August 1993.

on the unconditional volatility of short-term market rates around the policy rate? If so, which had the largest effect?

First, we allow for a structural break in January 1996 following the introduction of the gilt repo market in the United Kingdom (see Section 3 for a fuller discussion). We estimate the unrestricted and GARCH models for the two sample periods either side of that date. This removes the persistence in the variance of the divergence of the overnight rate previously discussed: for both sub-samples we obtain unconditional standard deviations. Table E presents these standard deviations, along with those for the one-week and one-month rates.⁽¹⁶⁾

Our major finding was that the introduction of the repo market led to a sharp reduction in the degree of unconditional volatility, particularly for divergences in the overnight rate. After 1996 unconditional volatility was lower in the United Kingdom than for either Italy or Germany over the same period.

⁽¹⁶⁾ Annexes 4 and 5 present more detailed results.

 Table E: Estimates of the unconditional standard deviation

 for UK policy-adjusted short-term rates, from the estimates

 for the GARCH specification (basis points)

•	1993-95	1996-98
Overnight	48.85	11.74
One week	20.86	6.23
One month	9.26	3.73

The second structural break we consider relates to the changes to the Bank's official operating system introduced in March 1997 (see Section 3). We estimate the unrestricted model for sub-samples covering the periods approximately one year before and after the changes. The results are shown in Annexes 6 and 7. As Table F shows, the structural break had no significant effect on the unconditional variation of the one-week or one-month rates.

Table F: Estimates of the unconditional standard deviationfor UK policy-adjusted short-term rates, from the estimatesfor the GARCH specification (basis points)

26.2.96-28.2.97	3.3.97-6.3.98
13.37	Persistence in variance
6.37	5.79
2.63	2.05
	13.37 6.37

6 Conclusion

A number of empirical studies have estimated the time series properties of short-term market rates. But few have examined divergences in short-term market rates from the policy rates that central banks use to provide marginal liquidity to the banking system. In the light of this, this paper makes use of a simple reduced-form model to answer some empirical questions about the time series behaviour of divergences of short-term market rates from policy rates in three European countries with different central bank operating systems, prior to the introduction of the euro.

First, we considered to what extent there were differences in the long-run divergence between market rates and policy rates. Taken together, our results provide no evidence to support the view that the degree of control over short-term market rates was lower in the United Kingdom than in either Italy or Germany.

Second, we considered whether there were differences in the speed with which rates reverted back to policy rates following a shock to the money market. Using the entire dataset, our results suggested that short-term rates in Germany reverted significantly faster than in either Italy or the United Kingdom. But after allowing for interest rate spikes associated with the end of the monthly maintenance period, shocks to German rates reverted more slowly than in the other two countries.

Third, we found some evidence to support the view that the volatility of overnight rates was higher in the United Kingdom than in Italy or Germany. But at slightly longer maturities, volatility was little different from that in Germany and was lower than in Italy. And we found that after the introduction of the gilt repo market in 1996, volatility was lower in the United Kingdom than in either of the other countries for all the maturities considered.

Fourth, our tests for structural breaks in the United Kingdom suggested that the introduction of the gilt repo market had a substantially larger effect on reducing volatility than did the introduction of gilt repo as the main daily instrument for providing marginal liquidity to the banking system.

Annex 1: GMM estimates of the general model for the policy-adjusted short rate (1993-98)

Dependent variable: overnight rate	;		Coefficient	t-statistics
Long-run mean of the divergence:		-α/β	-0.00114	-4.83084
Speed of mean reversion:		-β	0.47562	12.62795
		a_0	2.76E-21	0.00000
		a ₁	0.94722	0.30445
		b	7.78E-07	0.00000
		a_3	0.00132	0.00200
		γ	0.67229	0.01114
Goodness of fit (χ^2)	1.86795	Observations	1324	

Table A1.1: Results for the United Kingdom

Dependent variable: one-week ra	ate		Coefficient	t-statistics
Long-run mean of the divergence:		-α/β	-0.00119	-3.81199
Speed of mean reversion:		-β	0.16687	5.13839
		a_0	2.09E-06	6.94781
		a ₁	0.07007	0.51052
		b	3.77E-16	0.00000
		a ₃	976.9187	0.01538
		γ	2.03835	0.28663
Goodness of fit (χ^2)	0.82899	Observations	1324	

Dependent variable: one-month rat	e		Coefficient	t-statistics
Long-run mean of the divergence:		-α/β	-0.00021	-0.84090
Speed of mean reversion:		-β	0.09705	3.02324
		a_0	3.94E-07	9.58702
		a ₁	1.82E-14	0.00000
		b	0.09359	0.15550
		a ₃	5456.941	0.25787
		γ	2.09631	5.98151
Goodness of fit (χ^2)	0.03343	Observations	1324	

Coefficients are estimated from the following model:

$$\begin{aligned} &(1) \ d_t - d_{t-1} = \alpha + \beta d_{t-1} + \varepsilon_t , \ &(2) \ E(\varepsilon_t + \Psi_{t-1}) = 0 , \\ &(3) \ E(\varepsilon_t^2 + \Psi_{t-1}) = \sigma_t^2 = \alpha_o + \alpha_1 \varepsilon_{t-1}^2 + \delta \sigma_{t-1}^2 + \alpha_3 d_{t-1}^{2\gamma} \end{aligned}$$

Dependent variable: overnight rate			Coefficient	t-statistics
Long-run mean of the divergence:		-α/β	0.00085	4.96639
Speed of mean reversion:		-β	0.58250	7.20706
		a_0	6.61E-07	0.20275
		a_1	0.53774	0.21896
		b	0.34634	0.15808
		a_3	4.37E-07	0.00000
		γ	1.38647	0.00000
Goodness of fit (χ^2)	0.05195	Observations	1364	

Table A1.2: Results for Germany

Dependent variable: filtered overnig	ht rate		Coefficient	t-statistics
Long-run mean of the divergence:		-α/β	0.00042	3.91857
Speed of mean reversion:		-β	0.27903	6.89313
		a_0	3.89E-07	0.09112
		a ₁	0.27045	2.48802
		b	1.68E-08	0.00000
		a_3	1.98E-22	0.00000
		γ	2.12E-07	0.00000
Goodness of fit (χ^2)	8.84132	Observations	1284	

Dependent variable: one week ra	ite		Coefficient	t-statistics
Long-run mean of the divergence:		-α/β	0.00075	5.47764
Speed of mean reversion:		-β	0.19932	6.22750
		a_0	1.52E-07	0.53899
		a ₁	0.02433	0.06901
		b	0.73083	1.75346
		a ₃	9.38E+09	0.00125
		γ	3.72889	0.02651
Goodness of fit (χ^2)	0.07267	Observations	1364	

Dependent variable: one-month rat	e		Coefficient	t-statistics
Long-run mean of the divergence:		-α/β	0.00061	4.36221
Speed of mean reversion:		-β	0.14116	5.80908
		a_0	3.32E-07	1.21197
		a1	1.51E-12	1.96E-12
		b	0.24417	0.60954
		a3	2.74E+13	0.15518
		γ	4.33709	100.17500
Goodness of fit (χ^2)	0.00337	Observations	1364	

Coefficients are estimated from the following model:

 $\begin{aligned} & (1) d_t - d_{t-1} = \alpha + \beta d_{t-1} + \varepsilon_t , \quad (2) \, E(\varepsilon_t \mid \Psi_{t-1}) = 0 , \\ & (3) \, E(\varepsilon_t^2 \mid \Psi_{t-1}) = \sigma_t^2 = \alpha_o + \alpha_1 \varepsilon_{t-1}^2 + \delta \sigma_{t-1}^2 + \alpha_3 d_{t-1}^{2\gamma} \end{aligned}$

Dependent variable: overnigh	t rate		Coefficient	t-statistics
Long-run mean of the divergence	e:	-α/β	0.00011	0.72255
Speed of mean reversion:		-β	0.45369	5.47070
		a_0	4.79E-07	0.02581
		a 1	3.88E-08	0.00000
		b	6.94E-06	0.00001
		a3	0.05112	0.05949
		γ	0.75844	0.40550
Goodness of fit (χ^2)	2.41383	Observations	1364	

Table A1.3: Results for Italy

Dependent variable: one-wee	k rate		Coefficient	t-statistics
Long-run mean of the divergence	e:	-α/β	-0.00064	-1.74032
Speed of mean reversion:		-β	0.24468	2.85302
		a_0	3.79E-09	0.00000
		a ₁	0.43281	0.29682
		b	0.59186	0.19355
		a_3	3.46E-09	0.00000
		γ	0.01480	0.00000
Goodness of fit (χ^2)	0.72119	Observations	1364	

Dependent variable: one-month r	ate		Coefficient	t-statistics
Long-run mean of the divergence:		-α/β	-0.00105	-3.98055
Speed of mean reversion:		-β	0.20034	7.10477
		a_0	5.55E-11	0.00000
		a 1	0.04939	0.02584
		b	0.94646	0.27805
		a3	5.11E-06	0.00006
		γ	0.54473	0.00033
Goodness of fit (χ^2)	1.43353	Observations	1364	

Coefficients are estimated from the following model:

$$\begin{aligned} & (1) d_t - d_{t-1} = \alpha + \beta d_{t-1} + \varepsilon_t , \quad (2) \, E(\varepsilon_t \mid \Psi_{t-1}) = 0 , \\ & (3) \ E(\varepsilon_t^2 \mid \Psi_{t-1}) = \sigma_t^2 = \alpha_o + \alpha_1 \varepsilon_{t-1}^2 + b \sigma_{t-1}^2 + \alpha_3 d_{t-1}^{2\gamma} \end{aligned}$$

Annex 2: GMM estimates of the GARCH model for the policy-adjusted short rate (1993-98)

Table A2.1: Results for the United Kingdom

Dependent variable: overnight rate			Coefficient	t-statistics
Long-run mean of the divergence:		-α/β	-0.00113	-4.96872
Speed of mean reversion:		-β	0.47522	12.87827
		a_0	2.43E-21	0.00000
		a ₁	0.99874	3.73687
		b	4.56E-14	0.00000
Goodness of fit (χ^2 ,2 df)	1.82368	Observations	1324	

Memo item: persistence in variance

Dependent variable:	one-week rate		Coefficient	t-statistics
Long-run mean of the	divergence:	-α/β	-0.00120	-3.55400
Speed of mean rever	sion:	-β	0.14990	5.23018
		a_0	1.24E-06	2.05306
		a ₁	0.54538	2.58018
		b	1.52E-10	0.00000
Goodness of fit (χ^2 , 2	df) 3.3734	5 Observations	1324	
Memo item long-run unconditional standard deviation (bp):			16.49	

Dependent va	riable: one-month	rate		Coefficient	t-statistics
Long-run mear	n of the divergence:		-α/β	-0.00025	-1.30062
Speed of mear	n reversion:		-β	0.10917	4.50939
			a_0	1.82E-07	2.92173
			a ₁	0.74131	7.29233
			b	1.64E-13	0.00000
Goodness of fi	t (χ ² , 2 df)	1.15324	Observations	1324	
Memo item	emo item long-run unconditional standard deviation (bp):			8.39	

Coefficients are estimated from the following model:

$$\begin{split} & \left(1\right)d_t - d_{t-1} = \alpha + \beta d_{t-1} + \varepsilon_t \ , \ \left(2\right) E(\varepsilon_t \mid \Psi_{t-1}) = 0 \ , \\ & \left(3\right) E(\varepsilon_t^2 \mid \Psi_{t-1}) = \sigma_t^2 = \alpha_o + \alpha_1 \varepsilon_{t-1}^2 + b \sigma_{t-1}^2 \end{split}$$

Dependent variable: overnight rate		Coefficient	t-statistics
Long-run mean of the divergence:	-α/β	0.00085	5.33117
Speed of mean reversion:	-β	0.58243	7.88244
	a_0	6.64E-07	0.36989
	a ₁	0.53851	0.87360
	b	0.34529	0.44002
Goodness of fit (χ^2 , 2 df) 0.056	58 Observations	1384	
Memo item: long-run unconditional stand	23.89		

Table A2.2: Results for Germany

Dependent varia	ble: filtered overnig	ght rate		Coefficient	t-statistics
Long-run mean of	the divergence:		-α/β	0.00042	4.17427
Speed of mean re	version:		-β	0.27918	7.21948
			a_0	3.88E-07	3.52323
			a 1	0.27109	3.24494
			b	1.84E-15	0.00000
Goodness of fit (χ	² , 2 df)	8.84771	Observations	1284	
Memo item: long-run unconditional standard deviation (bp):			7.30		

Dependent va	riable: one-week rate	9		Coefficient	t-statistics
Long-run mear	of the divergence:		-α/β	0.00075	5.08155
Speed of mear	reversion:		-β	0.17739	5.93693
			a_0	3.8E-07	4.71359
			a ₁	0.40801	3.61898
			b	5.28E-12	0.00000
Goodness of fit	t (χ², 2 df)	4.67726	Observations	1364	
Memo item: long-run unconditional standard deviation (bp):			8.01		

Dependent variable: one-month rate	9		Coefficient	t-statistics
Long-run mean of the divergence:		-α/β	0.00061	4.3992
Speed of mean reversion:		-β	0.14114	5.98084
		a_0	7.14E-08	1.64617
		a1	0.60093	4.17277
		b	0.25585	1.36059
Goodness of fit (χ^2 , 2 df)	0.40756	Observations	1364	
Memo item: long-run unconditional standard deviation (bp):			7.06	

Coefficients are estimated from the following model:

(1) $d_t - d_{t-1} = \alpha + \beta d_{t-1} + \varepsilon_t$, (2) $E(\varepsilon_t | \Psi_{t-1}) = 0$, (3) $E(\varepsilon_t^2 + W_{t-1}) = \frac{2}{2} + \frac{2}{2} + \frac{2}{2} + \frac{2}{2}$

$$(3) E(\varepsilon_t^2 \mid \Psi_{t-1}) = \sigma_t^2 = \alpha_o + \alpha_1 \varepsilon_{t-1}^2 + b \sigma_{t-1}^2$$

Table A2.3: Results for Italy

Dependent variab	le: overnight rate			Coefficient	t-statistics
Long-run mean of	the divergence:		-α/β	0.00017	1.48560
Speed of mean rev	version:		-β	0.53932	8.42438
			a_0	3.84E-06	10.16176
			a ₁	0.21491	6.50311
			b	2.04E-16	0.00000
Goodness of fit (χ^2	, 2 df)	2.99631	Observations	1364	
Memo item: long-run unconditional standard deviation (bp)			22.11		

Dependent variable: one-week rate			Coefficient	t-statistics	
Long-run mean of the divergence:		-α/β	-0.00064	-1.74906	
Speed of mean reversion:		-β	0.244709	2.86935	
		a_0	1.18E-17	0.00000	
		a ₁	0.431991	2.14645	
		b	0.596658	2.58566	
Goodness of fit (χ^2 , 2 df)	0.74416	Observations	1364		
Memo item: persistence in varian	ce				

Dependent var	Dependent variable: one-week rate (excluding outliers)		outliers)	Coefficient	t-statistics
Long-run mean	of the divergence:		-α/ <u>/</u>	-0.00086	-4.30715
Speed of mean	reversion:		-/-	0.29482	-6.63286
			a	6.06E-07	-2.84027
			a	0.58672	6.34858
			Ł	3.79E-14	0.00000
Goodness of fit	(χ², 2 df)	5.23614	Observations	1360)
Memo item: long-run unconditional standard deviation (bp):			12.11		

Dependent variable: one-month ra	ate	0	Coefficient	t-statistics
Long-run mean of the divergence:		-α/β	-0.00105	-4.71589
Speed of mean reversion:		-β	0.20035	8.01350
		a_0	8.47E-09	0.06977
		a1	0.04804	1.13134
		b	0.94651	9.24830
Goodness of fit (χ^2 , 2 df)	1.43478	Observationns	1364	
Memo item: long-run unconditional standard deviation (bp):			12.50	

Coefficients are estimated from the following model:

(1) $d_t - d_{t-1} = \alpha + \beta d_{t-1} + \varepsilon_t$, (2) $E(\varepsilon_t | \Psi_{t-1}) = 0$,

 $(3) \, E(\varepsilon_t^2 \mid \Psi_{t-1}) = \sigma_t^2 = \alpha_o + \alpha_1 \varepsilon_{t-1}^2 + b \sigma_{t-1}^2$

Annex 3: GMM estimates of the levels model for the policy-adjusted short rate (1993-98)

Table A3.1: Results for the United Kingdom

Dependent variable: overnight rate			Coefficient	t-statistics
Long-run mean of the divergence:		-α/β	-0.00112	-5.04521
Speed of mean reversion:		-β	0.49013	13.24783
		a_3	0.00015	1.92928
		γ	0.18444	4.03747
Goodness of fit (χ^2 , 3 df)	4.30238	Observations	1324	

Dependent variable: one-week rate			Coefficient	t-statistics
Long-run mean of the divergence:		-α/β	-0.00119	-3.84336
Speed of mean reversion:		-β	0.16793	5.73770
		a3	4.70E-05	1.48966
		γ	0.21256	3.76501
Goodness of fit (χ^2 , 3 df)	1.39880	Observations	1324	

Dependent variable: one-month ra		Coefficient	t-statistics	
Long-run mean of the divergence:		-α/β	-0.00019	-0.71906
Speed of mean reversion:		-β	0.07207	3.41411
		a_3	0.00190	0.86062
		γ	0.63728	6.65539
Goodness of fit (χ^2 , 3 df)	2.70442	Observations	1324	

Coefficients are estimated from the following model:

 $\begin{aligned} & \left(1\right) d_t - d_{t-1} = \alpha + \beta d_{t-1} + \varepsilon_t , \quad & \left(2\right) E(\varepsilon_t \mid \Psi_{t-1}) = 0 , \\ & \left(3\right) E(\varepsilon_t^2 \mid \Psi_{t-1}) = \sigma_t^2 = \alpha_3 d_{t-1}^{2\gamma} \end{aligned}$

Table A3.2: Results for Germany

Dependent variable: overnight rate			Coefficient	t-statistics
Long-run mean of the divergence:		-α/β	0.00060	4.14885
Speed of mean reversion:		-β	0.51007	7.19187
		a 3	5.21E-06	0.51920
		γ	2.74E-06	0.00002
Goodness of fit (χ^2 , 3 df)	18.11826	Observations	1364	

Dependent variable: filtered overnight rate			Coefficient	t-statistics
Long-run mean of the divergence:		-α/β	0.00041	4.19494
Speed of mean reversion:		-β	0.29502	7.58613
		a 3	5.74E-07	0.42836
		γ	2.63E-06	0.00001
Goodness of fit (χ^2 , 3 df)	12.65449	Observations	1284	

Dependent variable: one-week rate			Coefficient	t-statistics
Long-run mean of the divergence:		-α/β	0.00073	4.92736
Speed of mean reversion:		-β	0.16700	5.80204
		a_3	2.06E-05	1.21017
		γ	0.24563	4.05736
Goodness of fit (χ^2 , 3 df)	6.49244	Observations	1364	

Dependent variable: one-month rate			Coefficient	t-statistics
Long-run mean of the divergence:		-α/β	0.00062	4.20716
Speed of mean reversion:		-β	0.13101	5.86423
		a3	0.00011	1.88590
		γ	0.38816	9.89689
Goodness of fit (χ^2 , 3 df)	2.16307	Observations	1364	

Coefficients are estimated from the following model: (1) $d_t - d_{t-1} = \alpha + \beta d_{t-1} + \varepsilon_t$, (2) $E(\varepsilon_t | \Psi_{t-1}) = 0$,

 $(3) \, E(\varepsilon_t^2 \mid \Psi_{t-1}) = \sigma_t^2 = \alpha_3 d_{t-1}^{2\gamma}$

Dependent variable: overnight rate			Coefficient	t-statistics	
Long-run mean of the divergence:		-α/β	0.00011	0.76937	
Speed of mean reversion:		-β	0.45053	5.71578	
		a_3	0.04218	3.33941	
		γ	0.73147	25.95553	
Goodness of fit (χ^2 , 3 df)	2.46450	Observations	1364		

Table A3.3: Results for Italy

Dependent variable: one-week rate		Coefficient		t-statistics	
Long-run mean of the divergence:		-α/β	-0.00069	-1.59076	
Speed of mean reversion:		-β	0.20869	2.55624	
		a3	0.07235	1.79206	
		γ	0.83690	16.60714	
Goodness of fit (χ^2 , 3 df)	3.21773	Observations	1364		

Dependent variable: one-month rate			Coefficient	t-statistics
Long-run mean of the divergence:		-α/β	-0.00109	-4.28519
Speed of mean reversion:		-β	0.18707	7.01297
		a3	1.59E-06	1.38032
		γ	1.19E-16	0.00000
goodness of fit (χ^2 , 3 df)	5.73000	Observations	1364	

Coefficients are estimated from the following model:

$$\begin{aligned} & (1) d_t - d_{t-1} = \alpha + \beta d_{t-1} + \varepsilon_t , \quad (2) \, E(\varepsilon_t \mid \Psi_{t-1}) = 0 , \\ & (3) \, E(\varepsilon_t^2 \mid \Psi_{t-1}) = \sigma_t^2 = \alpha_3 d_{t-1}^{2\gamma} \end{aligned}$$

Annex 4: GMM estimates of the UK policy-adjusted short rate (1993-95)

Table A4.1: General model

Dependent variable: overnight ra	ate		Coefficient	t-statistics
Long-run mean of the divergence:		-α/β	-0.00231	-6.01037
Speed of mean reversion:		-β	0.53462	12.98377
		a_0	1.03E-05	0.10375
		a 1	0.03330	0.10735
		b	7.78E-07	0.00000
		a3	1.4E-05	0.26107
		γ	2.92E-13	0.00000
Goodness of fit (χ^2)	2.10086	Observations	753	

Dependent variable: one-week rate	;		Coefficient	t-statistics
Long-run mean of the divergence:		-α/β	-0.00242	-4.26922
Speed of mean reversion:		-β	0.20100	5.04817
		a_0	4.04E-06	0.09547
		a 1	0.07221	0.59301
		b	2.68E-12	0.00000
		a3	0.01651	0.00000
		γ	2.50801	0.00000
Goodness of fit (χ^2)	1.49002	Observations	753	

Dependent variable: one-month rat	e		Coefficient	t-statistics
Long-run mean of the divergence:		-α/β	-0.00117	-2.22672
Speed of mean reversion:		-β	0.14915	2.89048
		a_0	7.45E-07	7.87937
		a ₁	5.76E-06	2.84E-06
		b	0.07626	0.04429
		a3	7.25E+22	0.00078
		γ	7.32753	11.10544
Goodness of fit (χ^2)	0.38602	Observations	753	

Coefficients are estimated from the following model:

$$\begin{split} & (1) \, d_t - d_{t-1} = \alpha + \beta d_{t-1} + \varepsilon_t \ , \ \ & (2) \, E(\varepsilon_t \mid \Psi_{t-1}) = 0 \ , \\ & (3) \, E(\varepsilon_t^2 \mid \Psi_{t-1}) = \sigma_t^2 = \alpha_o + \alpha_1 \varepsilon_{t-1}^2 + b \sigma_{t-1}^2 + \alpha_3 d_{t-1}^{2\gamma} \end{split}$$

Table A4.2: GARCH model

Dependent variable: overnight rate			Coefficient	t-statistics	
Long-run mear	of the divergence:		-α/β	-0.00232	-6.13447
Speed of mean	reversion:		-β	0.53359	13.13623
			a_0	1.26E-05	0.05425
			a ₁	0.01243	0.09552
			b	0.45802	0.04683
Goodness of fi	$t(\chi^2, 2 df)$	2.17830	Observations	753	
Memo item:	long-run unconditional standard deviation (bp):			48.85	

Dependent variable: one-week rate		Coefficient	t-statistics
Long-run mean of the divergence:	-α/β	-0.00242	-4.44149
Speed of mean reversion:	-β	0.20099	5.68764
	a_0	4.04E-06	1.05041
	a ₁	0.07220	0.46295
	b	1.02E-14	0.00000
Goodness of fit (χ^2 , 2 df) 1.4	49021 Observations	753	
Memo item: long-run unconditional star	long-run unconditional standard deviation (bp):		

Dependent va	riable: one-month ra	te		Coefficient	t-statistics
Long-run mear	of the divergence:		-α/β	-0.00122	-4.06290
Speed of mear	reversion:		-β	0.18102	5.00714
			a_0	4.69E-07	3.83534
			a ₁	0.45256	5.35801
			b	3.26E-14	0.00000
Goodness of fi	t (χ ² , 2 df)	1.73547	Observations	753	
Memo item: long-run unconditional standard deviation (bp):			9.26		

Coefficients are estimated from the following model:

(1) $d_t - d_{t-1} = \alpha + \beta d_{t-1} + \varepsilon_t$, (2) $E(\varepsilon_t | \Psi_{t-1}) = 0$,

(3)
$$E(\varepsilon_t^2 | \Psi_{t-1}) = \sigma_t^2 = \alpha_o + \alpha_1 \varepsilon_{t-1}^2 + b \sigma_{t-1}^2$$

Annex 5: GMM estimates of the UK policy-adjusted short rate (1996-98)

Table A5.1: General model

Dependent variable: overnight rate)		Coefficient	t-statistics
Long-run mean of the divergence:		-α/β	0.00030	2.42544
Speed of mean reversion:		-β	0.41935	6.94971
		a_0	3.81E-07	0.36515
		a 1	0.72376	0.61777
		b	1.79E-15	0.00000
		a_3	76575.220	0.00000
		γ	4.20334	0.00000
Goodness of fit (χ^2)	1.16289	Observations	567	

Dependent variable: one-week rate			Coefficient	t-statistics
Long-run mean of the divergence:		-α/β	0.00033	2.81253
Speed of mean reversion:		-β	0.24796	6.89956
		a_0	2.33E-07	0.22729
		a 1	0.40011	0.10182
		b	3.98E-13	0.00000
		a3	0.01651	0.00000
		γ	2.50800	0.00000
Goodness of fit (χ^2)	1.28594	Observations	567	

Dependent variable: one-month rate	e		Coefficient	t-statistics
Long-run mean of the divergence:		-α/β	0.00104	4.04649
Speed of mean reversion:		-β	0.12130	3.94270
		a_0	8.84E-25	0.00000
		a ₁	0.01499	0.00851
		b	5.86E-16	0.00000
		a3	1.52E-06	0.48438
		γ	0.15747	1.18978
Goodness of fit (χ^2)	1.57494	Observations	567	

Coefficients are estimated from the following model:

$$\begin{aligned} & (1) d_t - d_{t-1} = \alpha + \beta d_{t-1} + \varepsilon_t , \quad (2) \ E(\varepsilon_t \mid \Psi_{t-1}) = 0 , \\ & (3) \ E(\varepsilon_t^2 \mid \Psi_{t-1}) = \sigma_t^2 = \alpha_o + \alpha_1 \varepsilon_{t-1}^2 + \delta \sigma_{t-1}^2 + \alpha_3 d_{t-1}^{2\gamma} \end{aligned}$$

Table A5.2: GARCH model

Dependent variable: overnight rate		Coefficient	t-statistics		
Long-run mear	of the divergence:		-α/β	0.00030	2.43611
Speed of mean	reversion:		-β	0.41932	7.25670
			a_0	3.81E-07	1.19908
			a ₁	0.72360	3.01857
			b	1.61E-15	0.00000
Goodness of fit	$(\chi^2, 2 df)$	1.16011	Observations	567	
Memo item:	long-run unconditional standard deviation (bp):			11.74	

Dependent var	riable: one-week rate)		Coefficient	t-statistics
Long-run mean	of the divergence:		-α/β	0.00033	2.96410
Speed of mean	reversion:		-β	0.24798	-7.28022
			a_0	2.33E-07	2.22226
			a ₁	0.39893	2.45743
			b	8.37E-13	0.00000
Goodness of fit (χ^2 , 2 df) 1.29122		Observations	567		
Memo item:	emo item: long-run unconditional standard deviation (bp):			6.23	

Dependent va	riable: one-month ra	te		Coefficient	t-statistics
Long-run mear	of the divergence:		-α/β	0.00119	3.83982
Speed of mear	reversion:		-β	0.10131	3.35433
			a_0	1.18E-07	3.61721
			a ₁	0.15434	0.84294
			b	2.19E-14	0.00000
Goodness of fit (χ^2 , 2 df) 4.18969		Observations	567		
Memo item: long-run unconditional standard deviation (bp):			3.73		

Coefficients are estimated from the following model:

(1) $d_t - d_{t-1} = \alpha + \beta d_{t-1} + \varepsilon_t$, (2) $E(\varepsilon_t | \Psi_{t-1}) = 0$,

$$(3) E(\varepsilon_t^2 \mid \Psi_{t-1}) = \sigma_t^2 = \alpha_o + \alpha_1 \varepsilon_{t-1}^2 + b \sigma_{t-1}^2$$

Annex 6: GMM estimates of the UK policy-adjusted short rate (26 February 1996 – 28 February 1997)

Table A6.1: General model

Dependent variable: overnight rate	e		Coefficient	t-statistics
Long-run mean of the divergence:		-α/β	4.45E-05	0.24812
Speed of mean reversion:		-β	0.50452	5.09854
		a_0	6.58E-07	0.22815
		a1	0.58860	0.49448
		b	0.04322	0.14874
		a_3	0.00253	0.00000
		γ	3.07910	2.66105
Goodness of fit (χ^2)	1.14540	Observations	253	

Dependent variable: one-week ra	te		Coefficient	t-statistics
Long-run mean of the divergence:		-α/β	2.94E-05	0.22977
Speed of mean reversion:		-β	0.32890	5.62286
		a_0	2.90E-07	0.32891
		a ₁	3.54E-09	0.00000
		b	2.56E-08	0.00000
		a3	2.87E-05	0.01356
		γ	0.36647	0.06204
Goodness of fit (χ^2)	1.50600	Observations	253	

Dependent variable: one-month ra	te		Coefficient	t-statistics
Long-run mean of the divergence:		-α/β	0.00045	1.524509
Speed of mean reversion:		-β	0.13799	2.495758
		a_0	1.51E-18	0.00000
		a ₁	3.26E-09	0.00000
		b	1.27E-09	0.00000
		a_3	9.35E-06	0.014561
		γ	0.30867	0.060171
Goodness of fit (χ^2)	4.37764	Observations	253	

Coefficients are estimated from the following model:

$$\begin{split} &(1) \, d_t - d_{t-1} = \alpha + \beta d_{t-1} + \varepsilon_t \ , \ &(2) \, E(\varepsilon_t \mid \Psi_{t-1}) = 0 \ , \\ &(3) \, E(\varepsilon_t^2 \mid \Psi_{t-1}) = \sigma_t^2 = \alpha_o + \alpha_1 \varepsilon_{t-1}^2 + b \sigma_{t-1}^2 + \alpha_3 d_{t-1}^{2\gamma} \end{split}$$

Table A6.2: GARCH model

Dependent variable: overnight rate			Coefficient	t-statistics	
Long-run mean of the div	vergence:	-α/β	4.43E-05	0.25014	
Speed of mean reversion	n:	-β	0.50458	5.51671	
		a_0	6.59E-07	2.01888	
		a1	0.587881	3.28413	
		b	0.043517	0.20434	
Goodness of fit (χ^2 , 2 df)	1.14888	Observations	253	3	
Memo item: long-run	lemo item: long-run unconditional standard deviation (bp):			,	

Dependent vari	able: one-week rate)		Coefficient	t-statistics
Long-run mean	of the divergence:		-α/β	3.49E-05	0.28003
Speed of mean	reversion:		-β	0.32698	6.17826
			a_0	4.06E-07	0.00011
			a ₁	8.38E-13	0.00000
			b	2.78E-10	0.00000
Goodness of fit	(χ ² , 2 df)	1.58435	Observations	253	
Memo item: long-run unconditional standard deviation (bp):			6.37	,	

Dependent va	riable: one-month ra	te		Coefficient	t-statistics
Long-run mear	n of the divergence:		-α/β	0.00053	1.90424
Speed of mear	n reversion:		-β	0.12812	2.35540
			a_0	6.74E-08	0.00000
			a ₁	2.23E-07	0.00000
			b	0.02381	0.00000
Goodness of fit (χ^2 , 2 df) 6.56251		Observations	253		
Memo item: long-run unconditional standard deviation (bp):			2.63		

Coefficients are estimated from the following model:

(1) $d_t - d_{t-1} = \alpha + \beta d_{t-1} + \varepsilon_t$, (2) $E(\varepsilon_t | \Psi_{t-1}) = 0$,

$$(3) E(\varepsilon_t^2 \mid \Psi_{t-1}) = \sigma_t^2 = \alpha_o + \alpha_1 \varepsilon_{t-1}^2 + b \sigma_{t-1}^2$$

Annex 7: GMM estimates of the UK policy-adjusted short rate (3 March 1997 – 6 March 1998)

Table A7.1: General Model

Dependent variable: overnight rate	9		Coefficient	t-statistics
Long-run mean of the divergence:		-α/β	0.00037	1.61813
Speed of mean reversion:		-β	0.31894	4.89402
		a_0	6.40E-17	0.00000
		a1	0.04468	0.41780
		b	0.94626	1074100570
		a_3	0.27693	0.00000
		γ	2.49500	0.00000
Goodness of fit (χ^2)	4.24375	Observations	253	

Dependent variable: one-week ra	ate		Coefficient	t-statistics
Long-run mean of the divergence:		-α/β	0.00051	2.29868
Speed of mean reversion:		-β	0.19938	4.35874
		a_0	2.25E-07	0.00000
		a ₁	3.55E-09	0.00000
		b	0.32737	0.00000
		a3	8.49E-12	0.00000
		γ	0.94109	0.00000
Goodness of fit (χ^2)	0.22825	Observations	253	

Dependent variable: one-month ra	ite		Coefficient	t-statistics
Long-run mean of the divergence:		-α/β	0.00142	3.69115
Speed of mean reversion:		-β	0.15814	3.17870
		a_0	6.98E-20	0.00000
		a ₁	3.26E-09	0.00000
		b	1.27E-09	0.00000
		a_3	24.97022	0.03869
		γ	1.49188	0.68022
Goodness of fit (χ^2)	2.15240	Observations	253	

Coefficients are estimated from the following model:

(1)
$$d_t - d_{t-1} = \alpha + \beta d_{t-1} + \varepsilon_t$$
, (2) $E(\varepsilon_t | \Psi_{t-1}) = 0$,
(3) $E(\varepsilon_t^2 | \Psi_{t-1}) = \sigma_t^2 = \alpha_o + \alpha_1 \varepsilon_{t-1}^2 + b \sigma_{t-1}^2 + \alpha_3 d_{t-1}^{2\gamma}$

Table A7.2: GARCH Model

Dependent variable: overnight rate	;		Coefficient	t-statistics
Long-run mean of the divergence:		-α/β	0.00036	1.55649
Speed of mean reversion:		-β	0.31585	4.90029
		a_0	6.17E-23	0.00000
		a ₁	0.04649	9.97155
		b	0.94447	2.96E+09
Goodness of fit (χ^2 , 2 df)	4.12886	Observations	253	i

Memo item: persistence in variance

Dependent variable: o	one-week rate	Coefficient		t-statistics
Long-run mean of the d	vergence:	-α/β	0.00051	2.36078
Speed of mean reversion:		-β	0.19937	4.48130
		a_0	3.35E-07	0.00007
		a ₁	2.04E-16	0.00000
		b	2.78E-10	0.00000
Goodness of fit (χ^2 , 2 df) 0.22836	Observations	253	
Memo item: long-run unconditional standard deviation (bp):			5.79	

Dependent variable: one-month rate				Coefficient	t-statistics
Long-run mear	n of the divergence:		-α/β	0.00194	2.97972
Speed of mean reversion:		-β	0.06666	2.38013	
			a_0	3.53E-08	1.54537
			a ₁	0.16125	3.82576
			b	3.87E-13	0.00000
Goodness of fit (χ^2 , 2 df)		6.42742	Observations	253	
Memo item:	em: long-run unconditional standard deviation (bp):			2.05	

Coefficients are estimated from the following model:

(1) $d_t - d_{t-1} = \alpha + \beta d_{t-1} + \varepsilon_t$, (2) $E(\varepsilon_t | \Psi_{t-1}) = 0$,

$$(3) E(\varepsilon_t^2 \mid \Psi_{t-1}) = \sigma_t^2 = \alpha_o + \alpha_1 \varepsilon_{t-1}^2 + b \sigma_{t-1}^2$$

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