The real interest rate gap as an inflation indicator

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Abstract

A long-standing area of research and policy interest is the construction of a measure of monetary policy stance. One measure that has been proposed—as an alternative to indices that employ monetary aggregates or exchange rates—is the spread between the actual real interest rate and its flexible-price, or natural-rate, counterpart. We examine the properties of the natural real interest rate and 'real interest rate gap' using a dynamic stochastic general equilibrium model. Issues we investigate include: (1) the response of the gap and its components to fundamental economic shocks; and (2) the indicator and forecasting properties of the real interest rate gap for inflation, both in the model and in the data. Our results suggest that the real interest rate gap has value as an inflation indicator, supporting the 'neo-Wicksellian framework' advocated by Woodford (2000).

Summary

In this paper, we investigate the business cycle properties of the real interest rate relative to its natural value. Our investigation into the natural real interest rate is motivated by the possibility of constructing a measure of monetary policy stance based exclusively on interest rates. Recent work by Michael Woodford has revived the ideas of Knut Wicksell by focusing on 'the gap between the current level of the "natural rate" of interest and the interest rate controlled by the central bank' as the key variable for the analysis of 'inflationary or deflationary pressures'. In line with this terminology, we describe the spread between actual and natural real interest rates as the *real interest rate gap*.

This paper examines a number of questions involving the real interest rate gap, including:

- Does the real interest rate gap provide a useful tool for monetary policy analysis?

- Is the real interest rate gap more difficult to measure than the output gap?

- How do empirical measures of the real interest rate gap perform in forecasting UK inflation?

We develop a dynamic stochastic general equilibrium model with sticky prices in order to examine the behaviour of the natural real interest rate and the real interest rate gap. In our model, household spending and asset accumulation, and the prices that firms set, are based on optimising behaviour. We build on the existing literature by including capital formation (subject to adjustment costs), habit persistence in consumption, technology and demand shocks, and two alternative models of price stickiness. The baseline model of price stickiness that we use is Calvo price-setting, which can be interpreted as a system of staggered contracts for nominal prices.

We calibrate the model to the UK economy, and examine the response of the natural real interest rate to shocks to both technology and demand. Our focus is mainly on the *indicator* properties of the real interest rate gap, and so we examine how well the real interest rate gap does in signalling future inflation—both in response to specific shocks (which we examine using impulse response functions) and when all shocks are hitting the economy simultaneously (which we examine using stochastic simulations).

Using our model as a guide, we also construct empirical estimates of the natural real rate and the real interest rate gap from UK data.

Our key results include:

- The response of the natural real interest rate to a technology shock depends on whether or not capital is included in the model and, if so, whether or not there are capital adjustment costs. We find

that with capital adjustment costs, the natural real interest rate can *fall* in response to a technology shock. For a given actual real interest rate, this leads to a rise in the real interest rate gap.

- Conversely, the natural real interest rate rises in response to a demand shock. For a given actual real interest rate, this leads to a decline in the real interest rate gap.

- The less firms and households are willing to adjust their quantities, the more the natural real rate needs to adjust to maintain equilibrium.

- Stochastic simulations indicate that the real interest rate gap and output gap do equally well in forecasting inflation. In addition, the behaviour of the real interest rate is a reasonable approximation for the behaviour of the real interest rate gap. By contrast, output (or detrended output) is not a good indication of the behaviour of the output gap. This suggests the value of constructing measures of both gaps instead of concentrating only on output gap measures.

Finally, we test the predictive power of the real interest rate gap for UK inflation. On quarterly UK data, the real interest rate gap is closely related to future inflation, whether the relationship is judged by correlations or by the marginal predictive content of the gap for inflation in regressions. Our results suggest that constructing a real interest rate gap series, using theory as a guide, can have value for evaluating the stance of monetary policy and the prospects for future inflation, in keeping with the neo-Wicksellian framework of Woodford (2000).

1 Introduction

In evaluating the consequences of monetary policy actions for the future behaviour of inflation, it is often useful to construct a measure of monetary policy stance. Typically, such a measure will use some indicator of monetary or demand conditions, and express that measure relative to a baseline value (such as the value consistent with price stability).

There are several candidates for variables to include in a measure of monetary stance. Monetary aggregates (or their rates of growth) have historically been prominent candidates. A common criticism of their use for this purpose is that they can be distorted by innovations to the financial system.[(1)] In addition, monetary aggregates have been viewed as being too far removed from the typical instrument of monetary policy, namely the short-term nominal interest rate. This is particularly so for broad monetary aggregates.

One measure of the stance of monetary policy that does not rely on monetary aggregates is the monetary conditions index (MCI). An MCI consists of a weighted index of the interest rate and the exchange rate.[(2)] A weakness of this approach is that it relates changes in the exchange rate to inflation without identifying the shock responsible for the change in the exchange rate. For example, MCIs tend to interpret exchange rate depreciations as producing monetary ease, but it is possible for the equilibrium real exchange rate to depreciate under conditions of price stability (see King (1997)). More generally, the weight given to exchange rates in the index may implicitly assume a much closer and more mechanical link between exchange rate depreciations and inflation than can be justified empirically (see McCallum and Nelson (2000) and Stock and Watson (2000)).[(3)]

In light of the arguments against including monetary aggregates or exchange rates in measures of monetary policy stance, one potentially fruitful option is to construct a measure of the monetary policy stance based exclusively on interest rates. In this vein, Woodford (1999a, 2000) has advanced the use of the natural or 'equilibrium real rate of interest' (the real interest rate that prevails under price flexibility) in the analysis of price level determination. Woodford's analysis revives the ideas of Wicksell (1898, 1906) within a dynamic stochastic general equilibrium model. He argues that an analysis of price level determination that is based on the difference between actual and natural real interest rates is preferable to traditional analysis, which is based on the interaction of money demand and supply. In models in which the instrument of monetary policy is a nominal interest rate, this 'neo-Wicksellian' analysis of price level determination avoids the cumbersome procedure of solving for the implied money supply function.

In Woodford's framework, the key variable for the analysis of 'inflationary or deflationary pressures' is 'the gap between the current level of the "natural rate" of interest and the interest rate controlled

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(1) See Svensson (1999) for a recent articulation of this and other criticisms of monetary aggregates.
(2) Ericsson and Kerbeshian (1997) provide a detailed bibliography of the MCI literature.
(3) One part of the problem is that the relative weights on interest rates and exchange rates in MCIs are typically based on the estimated relative effects of the two variables on aggregate demand, rather than inflation *per se*. See Eika, Ericsson and Nymoen (1996) for a discussion of MCI weights.

by the central bank'(Woodford (1999a, page 35)). In line with this terminology, we define the *real interest rate gap* as the spread between the actual and 'natural' real interest rates, and this paper studies this gap concept.

The position of actual real interest rates relative to their 'natural', 'neutral', or 'equilibrium' value, has been considered by policy-makers as well. For example, at the 9–10 December 1998 meeting of the Bank of England's Monetary Policy Committee, '[t]he Committee discussed whether it was helpful to think about the appropriate level of nominal interest rates by reference to the concept of a 'neutral' level, which provides neither stimulus nor restraint to the economy'. The outcome of this discussion was that '[w]hile some members of the Committee found the concept of the neutral rate useful in deciding on interest rate policy, other members found the uncertainty surrounding its level so large that the concept was of little use as a practical guide to policy' (Monetary Policy Committee (1999, page 67)).

Thus it is probably accurate to characterise the attitude by many policy-makers to the *practical* usefulness of the real interest rate gap concept as sceptical. It is also very likely that the real interest rate gap is regarded as less easy to measure empirically than an 'output gap' concept. Nevertheless, we see three key practical reasons for reconsidering the importance of looking at real interest gaps:

1. Understanding the behaviour of the natural real rate appears to be important for understanding the empirical relationship between the real interest rate and output. Consider Table A below, which gives correlations between the level of detrended log GDP (y_t) and the real interest rate (current and lagged) for quarterly US and UK data:

Table A: Correlations between detrended GDP and the real interest rate

	Corr(y_t, r_{t-k})	
	United States 1980 Q1–1999 Q4	United Kingdom 1980 Q1–1999 Q4
$k = 0$	–0.21	0.10
$k = 1$	–0.28	0.08
$k = 2$	–0.27	0.07
$k = 3$	–0.23	0.07
$k = 4$	–0.21	0.11

Note: For both the United States and the United Kingdom, y_t is obtained from the residuals of a regression of log real GDP (seasonally adjusted) on a quadratic trend, 1976 Q1–1999 Q4, and the nominal interest rate used in the calculation of r_t is the quarterly average of the Treasury bill rate. For the United States, r_t is then measured by the *ex post* real interest rate. For the United Kingdom, r_t is (as in Chart 7 and Section 6 below) the Treasury bill rate minus the forecasts of next-period annualised inflation from a VAR(8). The inflation series used in these calculations is the seasonally adjusted quarterly log change in the CPI (US) or the RPIX (UK).

For the United States, the correlation between the two variables is negative and, therefore, perhaps interpretable as reflecting the negative effects of the real interest rate on aggregate demand. But for the United Kingdom, the correlation is generally positive, indicating that the real interest rate is procyclical.[4] To identify the negative effects of the real rate on aggregate demand suggested by economic theory, it may be necessary to control for the fluctuations of the natural levels of output and the real rate in response to real shocks.

2. The real interest rate gap potentially has stronger leading-indicator properties than the output gap. In models where the output gap responds to the real interest rate gap with a lag and inflation reacts to the output gap gradually, the real interest rate gap should give advance information about both the output gap and inflation.

3. It is possible that the real interest rate gap may actually be measured with *less* uncertainty than the output gap. While measures of the output gap are very commonly used in policy analysis and forecasting, it is worth remembering that many of these measures are based on the assumption that potential output evolves according to a deterministic trend (such as a linear, quadratic, or broken-linear trend). Economic theory suggests, instead, that potential output, while certainly containing a trend component, also fluctuates over the business cycle in response to all real shocks. The validity of many standard measures of the output gap therefore rests on the hypothesis that the response of potential output to these shocks is relatively flat, so that detrended output provides a good approximation of the output gap.[5] This hypothesis is difficult to justify in light of the results from many dynamic general equilibrium models (including ours below) that suggest that potential output fluctuates considerably. On the other hand, it is possible that the response of the *natural real rate* to shocks may be relatively flat, which would make it relatively straightforward to construct reliable measures of the real interest rate gap even though data are not available on the natural rate.

The above considerations suggest that there are benefits from further study of the real interest rate gap concept. To this end, this paper develops a sticky-price, dynamic stochastic general equilibrium (DSGE) model in order to examine the behaviour of the natural real interest rate and the real interest rate gap. Any general equilibrium model, whether sticky or flexible-price, implicitly provides a model of the natural real interest rate and the real interest rate gap. In flexible-price models, the behaviour of the real interest rate gap is trivial—it is zero each period by definition, since the real interest rate *is* the natural real rate.[6] In sticky-price models,[7] the real interest rate gap is zero on average (provided that the Phillips curve is vertical in the long run), but will not be zero every period

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[4] In terms of Boldrin, Christiano and Fisher (1999), the real interest rate does not exhibit an 'inverted leading-indicator' property for real GDP in the United Kingdom.
[5] Similarly, production function based approaches to measuring potential output typically do not keep track adequately of the distinction between actual and flexible-price values of capital and labour inputs. For example, measuring flexible-price labour supply by the total labour force involves the assumption that labour supply is inelastic—and tends to generate an overly smooth potential output series. A general equilibrium model such as ours provides a way of keeping track of flexible-price values of variables and of their response to real shocks.
[6] This is so even for models, such as those in Beaudry and Guay (1996), that enrich the dynamics of real business cycle models, but maintain the assumption of price flexibility.
[7] Here we presume that the stickiness of prices lasts more than one period.

(except in the very specific case in which the monetary authority runs a policy that eliminates completely the real effects of price stickiness). In addition to the Woodford papers mentioned above, King and Watson (1996), Rotemberg and Woodford (1997), Clarida, Gali and Gertler (1999), and Giannoni (2000) have focused on the behaviour of real rates relative to their natural values in general equilibrium sticky price models. Of these, only King and Watson have explicit capital formation.[(8)] Their model, however, does not include preference shocks, which in theory (and in our own numerical results below) have a major influence on the natural rate. Our model builds on the existing literature by including both capital formation and preference shocks, as well as elements absent from the aforementioned papers, such as non time separable preferences. Unlike the previous work, we also present results for more than one model of price stickiness.

An advantage of our use of a fully specified general equilibrium model is that we can trace the determination of the natural real rate—which we denote r_t*—back to the model's underlying economic shocks. Woodford (1999a, 1999b) assumes that r_t* is a univariate AR(1) exogenous process. By making this high-level assumption, such an analysis does not permit decomposition of the underlying real shocks that determine r_t*. And analysis of the underlying shocks affecting r_t* could aid in reducing the uncertainty surrounding its level, which, as noted above, has inhibited the usefulness of the concept to policy-makers.

The framework we adopt also enables us to examine the effect of particular shocks or model elements on the variance of the natural real rate. This is important in light of the diverging estimates of the variance of the natural real rate in recent papers. King and Watson (1996) find that the natural real rate in a calibrated DSGE model has a variance well *below* that observed in US data for the actual (*ex post*) real rate. The estimates in Rotemberg and Woodford (1997), however, suggest a more volatile natural real interest rate series, with a standard deviation of 3.9 percentage points (annualised) on post-1979 US quarterly data[(9)]—around 1 percentage point greater than that observed empirically for the actual real rate in US and UK data over that period.

Our paper is organised as follows. Section 2 describes the model. Section 3 analyses the response of the natural real interest rate and the real interest rate gap in the model to shocks. Section 4 compares the model's dynamics to those in the data, and Section 5 examines the properties of the real interest gap in the model. Our focus is mainly on the *indicator* properties of the real interest rate gap, and so we examine how well the real interest rate gap does in signalling future inflation when all shocks are hitting the economy simultaneously (which we examine using stochastic simulations). Using the theoretical model as a guide, Section 6 then constructs estimates of the real interest rate gap series from UK data and evaluates the series' forecasting ability for inflation. Section 7 concludes, and an appendix describes how we measure output and real interest rate gaps in the model.

[(8)] Woodford (2000) adds endogenous capital formation to his model. One difference between his model and ours below is that our calibration of preference parameters, which strongly affect the interest elasticity of aggregate demand, is more in keeping with those suggested by econometric studies such as Fuhrer (2000) and Ireland (2000).
[(9)] See Woodford (1999b, footnote 50).

2 The model

This section describes the DSGE model we employ in the paper.

2.1 Households

The economy is inhabited by a large number of households, each of which has preferences defined over a composite consumption good (denoted C_t in period t), leisure (l_t), and real money balances (M_t / P_t). Money enters the utility function directly to capture the idea that real balances provide a transactions-facilitating service. The representative household chooses a sequence of consumption, leisure, nominal money and one-period bond (B_{t+1}) holdings, and capital (K_{t+1}), to maximise lifetime utility:

$$\mathrm{E}_t \sum_{j=0}^{\infty} b^j \left[l_t \frac{s}{s-1} \left(\frac{C_{t+j}}{C^h_{t+j-1}} \right)^{\frac{s-1}{s}} + b l_{t+j} + \frac{g}{1-e} \left(\frac{M_{t+j}}{P_{t+j}} \right)^{1-e} \right] \qquad \textbf{(1)}$$

subject to a series of real period budget constraints:

$$C_{t+j} + \frac{M_{t+j}}{P_{t+j}} + \frac{B_{t+j+1}}{P_{t+j}} + X_{t+j} = w_{t+j} N_{t+j} + z_{t+j} K_{t+j} + R^G_{t+j-1} \frac{B_{t+j}}{P_{t+j}} + \frac{M_{t+j-1}}{P_{t+j}} - \frac{t_{t+j}}{P_{t+j}} - f X^h_{t+j},$$
$$\forall j = 0,1,\ldots,\infty \qquad \textbf{(2)}$$

where $b > 0$, $g > 0$, $e > 0$, and $b \in (0,1)$. X_t in equation **(2)** is related to K_t by:

$$X_{t+j} = K_{t+j+1} - (1-d) K_{t+j} \qquad \textbf{(3)}$$

where $d \in [0,1)$. In equation **(2)**, R_t^G denotes the gross nominal interest rate, N_t denotes labour supplied, t_t denotes government transfers, and z_t the return on capital. In the numerical results of Section 5, we consider two versions of the model: a simplified stripped-down version of the model with no capital variation ($K_t = K^{ss}$ and $X_t = d\, K^{ss}$ for all t), and one with capital accumulation but with adjustment costs operative. If the model had no capital adjustment costs, the variable X_t could be interpreted as investment expenditure; with capital adjustment costs, we refer to X_t as quasi-investment. The size of capital adjustment costs is determined by the parameters j ($j > 0$) and h ($1 < h < \infty$).[10] When capital accumulation is subject to adjustment costs there is a wedge between the return on capital and the real interest rate, beyond the constant wedge implied by the rate of depreciation of capital (d).

[10] Our specification of the capital adjustment cost function follows Abel (1983) and Casares and McCallum (2000).

The parameter s in the consumer's utility function indexes the curvature of the utility function: the larger s, the more the household is willing to shift consumption across periods in response to interest rate changes. $1/(se)$ is the steady-state consumption elasticity of money demand, and g determines the steady-state consumption/money ratio.

Preferences over consumption take on a non time separable form to capture the idea that households may exhibit habit formation in their consumption patterns. The parameter $h \in [0,1)$ indexes the degree of habit formation. If h = 0, the household exhibits no habit formation in consumption, and preferences are time-separable. For 0 < h < 1, utility from current consumption depends partially on consumption in adjacent periods. We work with strictly positive h in this paper in light of evidence that doing so reduces some of the empirical shortcomings of quantitative business cycle models (Boldrin, Christiano and Fisher (1999); Fuhrer (2000)). Finally, the utility function is augmented by a disturbance (l_t) to consumption preferences, which we interpret as an 'IS' or 'real demand' shock.

The time allocation constraint of the household is normalised so that labour and leisure sum to one:

$$N_t + l_t = 1 \qquad \textbf{(4)}$$

The composite consumption good C_t is a Dixit-Stiglitz aggregate of a multiplicity of differentiated goods, indexed by $i \in [0,1]$. Under this scheme, the composite consumption good and price index are defined as:

$$C_t = \left[\int_0^1 C_t(i)^r \, di \right]^{\frac{1}{r}} \qquad \textbf{(5)}$$

$$P_t = \left[\int_0^1 p_t(i)^{\frac{r}{1-r}} di \right]^{\frac{1-r}{r}} \qquad \textbf{(6)}$$

which can be derived from two-stage budgeting. The parameter $r \in (0,1)$ determines the degree of substitutability of differentiated goods in consumption. For example, if r is close to 1, goods are perfectly substitutable in consumption and firms are perfectly competitive. The inverse of r is therefore the (gross) steady-state mark-up of price over marginal cost.

Letting y_t denote the Lagrange multiplier on **(2)**, optimal household behaviour yields the following conditions:

$$C_t\text{:} \quad l_t \left(\frac{C_t}{C_{t-1}^h} \right)^{\frac{s-1}{s}} \frac{1}{C_t} - b h \mathrm{E}_t l_{t+1} \left(\frac{C_{t+1}}{C_t^h} \right)^{\frac{s-1}{s}} \frac{1}{C_t} = y_t \qquad \textbf{(7)}$$

l_t: $b = w_t y_t$ **(8)**

$$M_t:\quad g\left(\frac{M_t}{P_t}\right)^{-e} = y_t - b\,\mathrm{E}_t \frac{y_{t+1}}{P_{t+1}/P_t} \quad \textbf{(9)}$$

$$B_{t+1}:\quad 0 = y_t - R_t^G \mathrm{E}_t \frac{ß y_{t+1}}{P_{t+1}/P_t} \quad \textbf{(10)}$$

$$K_{t+1}:\quad 0 = y_t\left(1 + \mathrm{f}\, h X_t^{h-1}\right) - b\,\mathrm{E}_t y_{t+1}\left[(1-d)\left(1 + f h X_{t+1}^{h-1}\right) + z_{t+1}\right] \quad \textbf{(11)}$$

$$y_t:\quad C_t + \frac{M_t}{P_t} + \frac{B_{t+1}}{P_t} + X_t = w_t N_t + z_t K_t + R_{t-1}^G \frac{B_t}{P_t} + \frac{M_{t-1}}{P_t} - \frac{t_t}{P_t} - f X_t^h \quad \textbf{(12)}$$

2.2 Firms

There is a continuum of monopolistically competitive firms, indexed by $j \in [0,1]$. Each firm *j* chooses price (p_{jt}), labour (N_{jt}), and capital (K_{jt+1}) to maximise its profits:

$$p_{jt}\frac{Y_{jt}}{P_t} - w_t N_{jt} - z_t K_{jt} \quad \textbf{(13)}$$

subject to a demand function for its good, written as:

$$\frac{p_{jt}}{P_t} = \left(\frac{Y_{jt}}{Y_t}\right)^{-(1-r)} \quad \textbf{(14)}$$

and a production function:

$$Y_{jt} = A_t N_{jt}^a K_{jt}^{1-a} \quad \textbf{(15)}$$

where P_t and Y_t are the aggregate price and output levels, A_t is a technology shock, and $a \in (0,1)$. In a symmetric equilibrium (under which we can drop the subscript *j*), the first-order conditions for the representative firm are given by:

$$l_t:\quad a\frac{Y_t}{N_t} = m_t^G w_t \quad \textbf{(16)}$$

K_t: $(1-a)\frac{Y_t}{K_t} = m_t^G z_t$ **(17)**

Here, m_t^G is the gross mark-up of price over marginal cost; in steady state this variable equals $1/r$, as noted above.

2.3 Market-clearing

Finally, the economy is subject to the following resource constraint:

$$Y_t = C_t + X_t + f X_t^h \qquad \textbf{(18)}$$

which differs from the usual closed-economy constraint due to the presence of capital adjustment costs.

2.4 Equilibrium

In order to investigate the dynamics of the model, we log-linearise the above optimality conditions and technological constraints around the steady state. The resulting equations are as follows (with the superscript *ss* denoting steady-state values):

Consumption

$$\frac{bh(s-1)}{s(1-bh)} \mathrm{E}_t c_{t+1} = \frac{bh^2 s - bh^2 + bhs - 1}{s(1-bh)} c_t - \frac{h(s-1)}{s(1-bh)} c_{t-1} - y_t + \frac{1-bhr_l}{1-bh} l_t \qquad \textbf{(19)}$$

Labour market equilibrium

$$0 = y_t - n_t + y_t - m_t \qquad \textbf{(20)}$$

Money demand

$$0 = -\frac{1}{e} y_t - rm_t - \frac{1}{e}\frac{1}{R^{ss}} R_t \qquad \textbf{(21)}$$

Euler equation

$$E_t y_{t+1} = y_t - r_t \qquad \textbf{(22)}$$

Fisher equation

$$E_t p_{t+1} = R_t - r_t \tag{23}$$

Quasi-investment

$$(1-d)\,\mathrm{E}_t x_{t+1} + \frac{(1-a)\, r\, Y^{ss} \big/ K^{ss}}{(h-1) f h \left(X^{ss}\right)^{h-1}} \mathrm{E}_t\left(y_{t+1} - k_{t+1} - m_{t+1}\right) = x_t + \frac{1}{(h-1) f h \left(X^{ss}\right)^{h-1}} r_t \tag{24}$$

Law of motion for capital

$$k_{t+1} = d\, x_t + (1-d) k_t \tag{25}$$

Resource constraint

$$y_t = \frac{C^{ss}}{Y^{ss}} c_t + \frac{X^{ss} + f h X^{ss^h}}{Y^{ss}} x_t \tag{26}$$

Production function

$$y_t = a_t + a\, n_t + (1-a) k_t \tag{27}$$

where y_t, c_t, k_t, n_t, rm_t, m_t, and a_t are the log deviations of Y_t, C_t, K_t, N_t, (M_t / P_t), m_t^G and A_t, respectively, from their steady-state values. Similarly, the Lagrange multipliers should now be interpreted as log-deviations from the steady state of the corresponding variables in the original non-linear model. p_t is the de-meaned quarterly net inflation rate, and R_t and r_t denote the de-meaned net nominal and real interest rates respectively.

Thus we have a log-linear model describing the path of eleven endogenous variables: output (y_t), capital (k_t), consumption (c_t), quasi-investment (x_t), the nominal interest rate (R_t), the real interest rate (r_t), the marginal utility of consumption (y_t), real balances (rm_t), labour input (n_t), the mark-up (m_t), and inflation (p_t). The system so far consists of nine equations. To complete the model, we need two more equations: a price-setting equation and a monetary policy rule. We turn now to the first of these.

2.5 Price-setting

This paper is concerned with the real interest rate gap, defined as the spread between the actual and 'natural' real interest rates. We therefore need to specify the equilibria associated with each of the two interest rate concepts. The natural real interest rate is the real rate that prevails in the case of fully flexible prices, whereas the actual real interest rate is the real rate that prevails under sticky prices.[11]

Our procedure for obtaining a sticky-price equilibrium consists of two steps:

(*a*) First, we solve for natural output (in logs, y_t*) and the natural real interest rate (r_t*) by obtaining the flexible-price equilibrium of the log-linear model above.[12] This flexible-price equilibrium is characterised by a constant mark-up, so that $m_t = 0$ (see for example Ireland (1997)). The flexible-price equilibrium can therefore be obtained by imposing this condition.

(*b*) We specify a model of gradual price adjustment, describing how the price level responds to the output gap ($y_t - y_t$*). Our baseline specification of gradual price adjustment will be the Rotemberg (1982)–Calvo (1983) model, which, following Roberts (1995), we write as the New Keynesian Phillips curve:

$$b\,\mathrm{E}_t p_{t+1} = p_t + a_m m_t \quad \textbf{(28)}$$

where $a_m > 0$. To examine the sensitivity of our results to the price-setting specification, we report supplementary results using a different model of price-setting, that of Fuhrer and Moore (1995):

$$0.5\mathrm{E}_t p_{t+1} = p_t - 0.5 p_{t-1} + a_m m_t \quad \textbf{(29)}$$

In both models, the size of a_m governs the degree to which prices are sticky. The larger is a_m, the more flexible are goods prices.[13]

2.6 Shocks

There are two types of real shocks in this model: a technology and a demand (IS) shock. Each of the two shocks is assumed to follow an AR(1) process:

$$a_t = r_a a_{t-1} + e_{at} \quad \textbf{(30)}$$

$$l_t = r_? l_{t-1} + e_{?t} \quad \textbf{(31)}$$

where r_a and r_l lie in [0,1], and e_{at} and e_{lt} are white noise innovations.

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(11) As in other sticky-price DSGE models, the assumption of monopolistic competition among firms provides groundwork for the assumption of gradual price-setting.
(12) A monetary policy rule needs to be appended to this system to complete the model, but due to monetary neutrality under price flexibility, the solution for potential output will be invariant to the rule selected.
(13) For the Calvo model, the flexible-price equilibrium is reached as a_m approaches infinity.

2.7 Policy rule

In an environment of flexible prices, the actual real interest rate will always equal the natural rate regardless of the monetary authority's policy rule. On the other hand, monetary policy has real effects in an environment of sticky prices. Therefore, the specification of the monetary policy rule will have implications for the real interest rate gap. Taking this into account, we have examined the properties of the real interest rate gap under a variety of different rules. Our results below will focus on a baseline policy rule estimated on UK data and described in Section 3.

2.8 Open-economy considerations

Our model has no explicit open-economy elements. It is therefore worthwhile to discuss in what sense the model's explanation of real interest rate determination may be useful for small open economies in a world of integrated capital markets.

One interpretation of our model is that it refers to the determination of the world interest rate, or the interest rate in a large country. We believe, however, that our analysis has some value as a description of interest rate determination in a small open economy. The conditions under which a small open economy's real interest rate is determined completely by global factors are actually quite stringent. One consequence of this may be that in small open economies, domestic rather than solely global factors play a significant role in the determination of real interest rates.

This point is particularly relevant for a model intended for monetary policy analysis. Inflation-targeting central banks typically operate with a short-term nominal interest rate instrument, which, combined with some inertia in inflation, implies considerable short-run influence by the domestic monetary authority over the domestic short-term real interest rate, even though these economies are highly open and are part of a global capital market. Indeed, many observers would consider the following model features essential for a realistic analysis of monetary policy: (i) central bank control of nominal interest rates and short-run influence over real rates; (ii) a considerable amount of persistence in inflation; and (iii) investment in physical capital being very important for cyclical fluctuations and being a major channel through which monetary policy affects aggregate demand. Our closed-economy model can (under certain settings) satisfy all three criteria; yet very few existing open-economy DSGE models can. In the Obstfeld-Rogoff (1995) model, for example, real interest rates are not affected by domestic monetary policy and are equal to the foreign real rate every period. The failure of standard open-economy models to meet criterion (ii) is documented in McCallum and Nelson (2000); and introducing endogenous physical capital is often problematic in open-economy models.

We therefore proceed with our use of a closed-economy model, but take the openness of the UK economy into account in our calibration of the model.

In addition to their effect on capital markets, open-economy elements alter goods market behaviour by adding net export demand, which is a function of foreign and domestic demand, to total domestic aggregate demand. We can take these factors into account in our closed-economy model by modifying the calibration of the aggregate demand side of the economy. Consider the Euler equation for consumption in our model, for simplicity abstracting from habit formation (h = 0). In this case, $c_t = \mathrm{E}_t c_{t+1} - \sigma r_t + \sigma(1-\rho_l)\lambda_t$. Iterations on this condition produce:

$$c_t = -\sigma \mathrm{E}_t \Sigma_{j=0}^{\infty} r_{t+j} + \lambda_t \qquad \textbf{(32)}$$

so consumption depends on current and expected future real short rates. Now suppose that net foreign demand for domestic output is given by the log-linear equation:

$$nx_t = b_1 q_t + k_t \qquad \textbf{(33)}$$

where q_t is the real exchange rate (an increase signifying a depreciation), k_t is a shock to foreign demand, and $b_1 > 0$. Supplementing the model with the real interest parity condition:

$$q_t = \mathrm{E}_t q_{t+1} - r_t + u_t \qquad \textbf{(34)}$$

where the shock u_t includes both the foreign real interest rate and the foreign exchange risk premium, one can write **(33)** as:

$$nx_t = -b_1 \mathrm{E}_t \Sigma_{j=0}^{\infty} r_{t+j} + b_1 \mathrm{E}_t \Sigma_{j=0}^{\infty} u_{t+j} + k_t \qquad \textbf{(35)}$$

Thus, aggregate non-investment demand is given by:

$$s_c c_t + s_{NX}\, nx_t = -(s_c \sigma + s_{NX} b_1) \mathrm{E}_t \Sigma_{j=0}^{\infty} r_{t+j} + \text{exogenous shocks} \qquad \textbf{(36)}$$

where s_c and s_{NX} are the steady-state shares of consumption and net exports in GDP respectively. The effect of adding open-economy influences is therefore to raise the interest elasticity of (non-investment) aggregate demand from $s_c\sigma$ to $(s_c\sigma + s_{NX}b_1)$. In our calibration, we take these influences into account by calibrating σ to a higher value than would be suggested by the interest responsiveness of consumption alone.

Finally, we discuss the relevance for our analysis of two other aspects of open-economy analysis. First, the consumption Euler equation **(19)** in our model would still hold if we made the model open. However, some open-economy models assume either finite horizons for consumers or an endogenous discount factor, with the effect of making the external asset position relevant for consumption behaviour.[14] The evidence suggests, however, that the business cycle frequency

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[14] See Kollmann (1991), Kim and Kose (2000), and Smets and Wouters (2000) for applications.

dynamics of endogenous variables—the frequency with which the present paper is concerned—are little changed by the introduction of these features (see Kollmann (1991), and Kim and Kose (2000)).

Second, openness makes imports a component of the consumer price index; this may create an extra channel through which shocks are transmitted to inflation—the 'direct exchange rate' channel. This would appear to justify the inclusion of separate exchange rate terms in the Phillips curve. However, as we discussed in the introduction, the empirical relationship between exchange rates and inflation is weak. There is little empirical case for including exchange rate terms in the Phillips curve (see Stock and Watson (2000)), possibly because of slow or incomplete pass-through of exchange rate movements to import prices. This suggests that our use of a Phillips curve with no explicit open-economy terms is a reasonable approximation.

3 Model calibration and properties

In this section we describe the responses of the natural real rate and the real interest rate gap in a calibrated version of our model. We first turn to our calibration.

3.1 Calibration

The parameter values assigned to our model correspond to those commonly found in the literature and are similar to those found in earlier work on sticky-price general equilibrium models, including King and Watson (1996) and McCallum and Nelson (1999). These are presented in Table B, and we now discuss some of the key choices.

The parameter s indexes the curvature of the utility function, and also determines the interest-elasticity of the non-investment component of aggregate demand. Due to the habit formation in our utility function, our calibrated value for this parameter must be between 0 and 1. While many business cycle studies select values of s near unity, the estimates of Euler and optimising IS equations on US data in Hall (1988), McCallum and Nelson (1999), Fuhrer (2000) and Ireland (2000) suggest a lower value, of around 0.2.[15] On the other hand, as we discussed in Section 2.8, the openness of the UK economy justifies a higher value. We therefore choose $s = 0.6$.[16]

The habit formation parameter is set to $h = 0.8$, in line with Fuhrer's (2000) estimate. The capital adjustment cost parameters are calibrated so that (quasi-)investment is considerably more interest-elastic than consumption, but not implausibly large. The capital adjustment cost settings in

[15] Higher values of s have been reported by Rotemberg and Woodford (1997) and Amato and Laubach (1999), but, unlike the studies mentioned in the text, these are not based on conventional econometric estimation procedures such as instrumental variables or maximum likelihood.
[16] By comparison, Boldrin, Christiano and Fisher take logarithmic preferences (the limit as $s \rightarrow 1$) and set $h = 0.9$. Beaudry and Guay (1996) impose logarithmic preferences and estimate h to be in the 0.3–0.5 range. Woodford (2000) sets $s \rightarrow 1$ and $h = 0$. All of these parameter settings seem to us to impose an unrealistically high interest elasticity of consumption, compared with the estimates cited above.

Table B imply a semi-elasticity of investment with respect to the short-term real interest rate of about 3.2%.(17)

Table B: Model calibration		
Parameter	*Description*	*Quarterly value*
a	Labour share	0.64
b	Discount factor	0.99
s	Parameter indexing the curvature of the utility function	0.6
h	Habit formation parameter	0.8
d	Rate of depreciation	0.025
$1/se$	Scale elasticity of money demand	1
j	Capital adjustment cost parameter	0.75
h	Capital adjustment cost parameter	2
$1/r, m$	Steady state gross mark-up	1.25
r_l	AR(1) parameter, IS shock	0.33
r_a	AR(1) parameter, technology shock	0.95
$\text{Var}(e_a)$	Variance of technology innovations	$(0.007)^2$
$\text{Var}(e_l)$	Variance of IS innovations	$(0.01)^2$
N^{ss}	Steady-state fraction of time in employment	0.33
a_m	Degree of nominal rigidity under sticky prices. Calibrated value corresponds to a 75% probability that firms will not be able to change their price.	0.086

3.2 Model properties

3.2.1 Response of the natural real rate to shocks

Charts 1a and 1b plot the responses of the natural real interest rate to technology and real demand shocks for the calibrated model.(18) Three cases are considered: a setting of the model with no capital; the model with capital formation but with adjustment costs; and the model with capital that can be adjusted costlessly. The no-capital and costless capital adjustment cases are presented to indicate the effect our specification of capital formation has on the behaviour of the natural real rate.

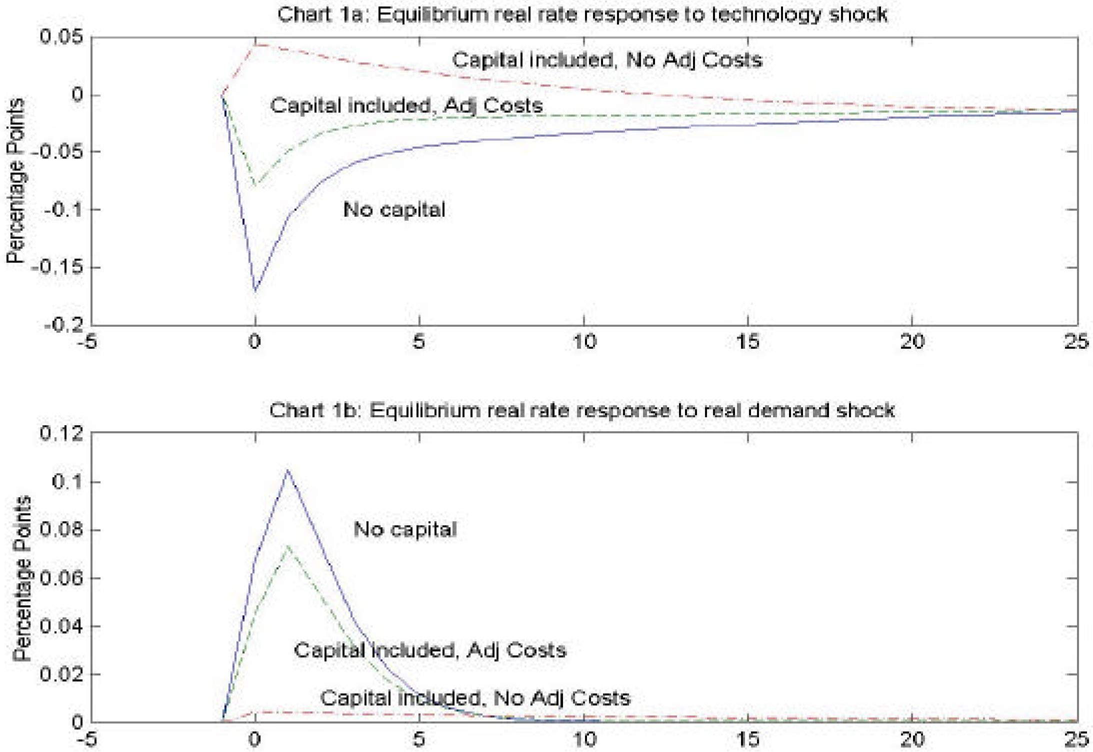
Chart 1a indicates that a temporary 1% shock to technology raises the natural real rate by about 5 basis points when capital adjustment is costless, but that it reduces the rate in the cases of no capital and capital with adjustment costs. In the no-capital case, a technology shock raises output and

(17) Capital adjustment costs are also important for generating realistic output behaviour under sticky prices. Without capital adjustment costs, output exhibits an extremely elastic response to monetary policy shocks (see Casares and McCallum (2000)).

(18) Details on how we computed these impulse responses are presented in the appendix.

consumption today by more than in future periods. Output and consumption therefore jump today and return later to their original levels. Households would like to smooth their consumption

Chart 1: Equilibrium real interest rate responses to shocks



(especially given the habit formation in their preferences), and attempt to shift resources away from the current period to future periods. But, in equilibrium, all output must be consumed today. The natural real interest rate declines to ensure that this occurs.(19)

When capital formation is present, investment demand goes up because the technology shock has raised the profitability of production. By itself, this would push up the real interest rate. Offsetting pressure comes from households' desire to save some of their increased income, and the disincentive to rapid investment provided by adjustment costs. The net effect of these pressures is again to reduce the real interest rate, though by less than in the no-capital case.

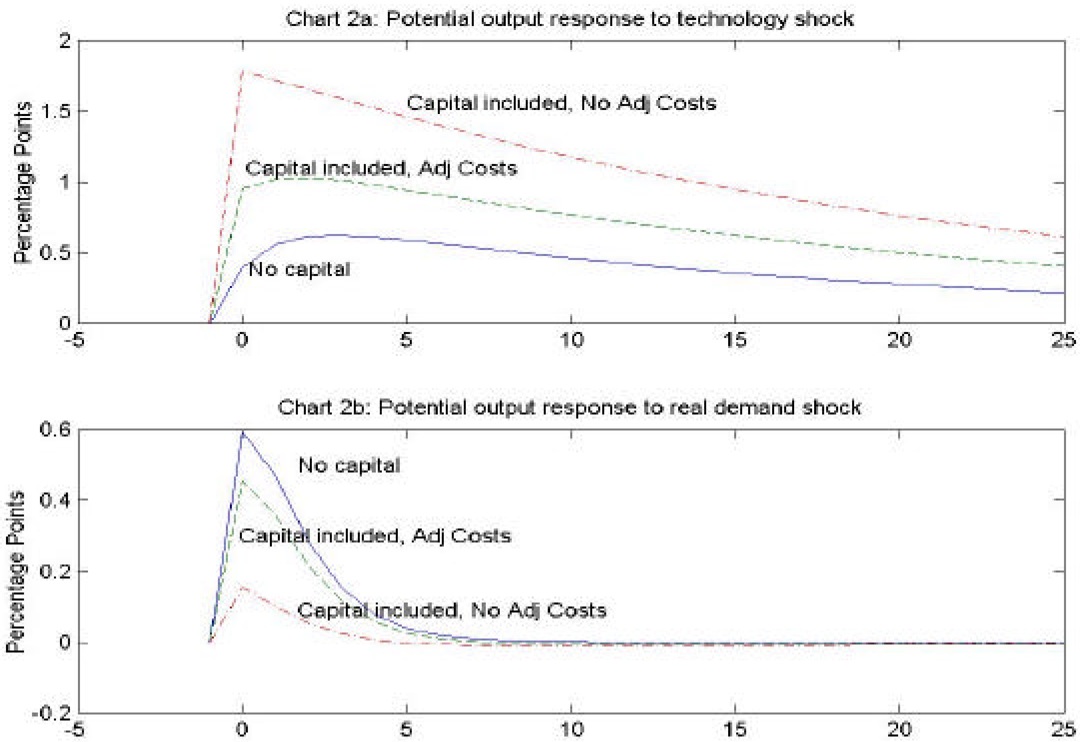
Chart 1b depicts the effect of a temporary 1% shock to real demand (a shock to households' preferences). An 'IS' shock of this type, unlike a monetary policy shock, affect the values of real variables such as the real interest rate, even when prices are flexible. This shock raises households'

(19) Chart 1a matches Woodford's (2001) result that temporary productivity shocks (ie $0 \leq r_a < 1$) lead to a counter-cyclical response of the equilibrium real interest rate. If the technology shock were completely persistent (ie had a unit root), and there were no habit formation in preferences, the technology shock would leave the equilibrium real rate unchanged, as in Clarida, Gali and Gertler (1999).

consumption demand on impact. With fully flexible capital, firms are willing to adjust their investment in response to higher aggregate demand without a large change in the real interest rate; hence the limited response of the natural rate depicted for that case in Chart 1b. If there is no capital or if firms face large adjustment costs, then production does not meet the higher demand, and so real interest rates must rise by a relatively large amount to dampen the rise in consumption.

A common feature of Charts 1a and 1b is that the natural real interest rate responds by more when the capital stock cannot adjust costlessly.[20] A flexible-price, flexible-capital model would imply almost no variation in the natural real interest rate since quantities bear the bulk of the adjustment.

Chart 2: Potential output responses to shocks



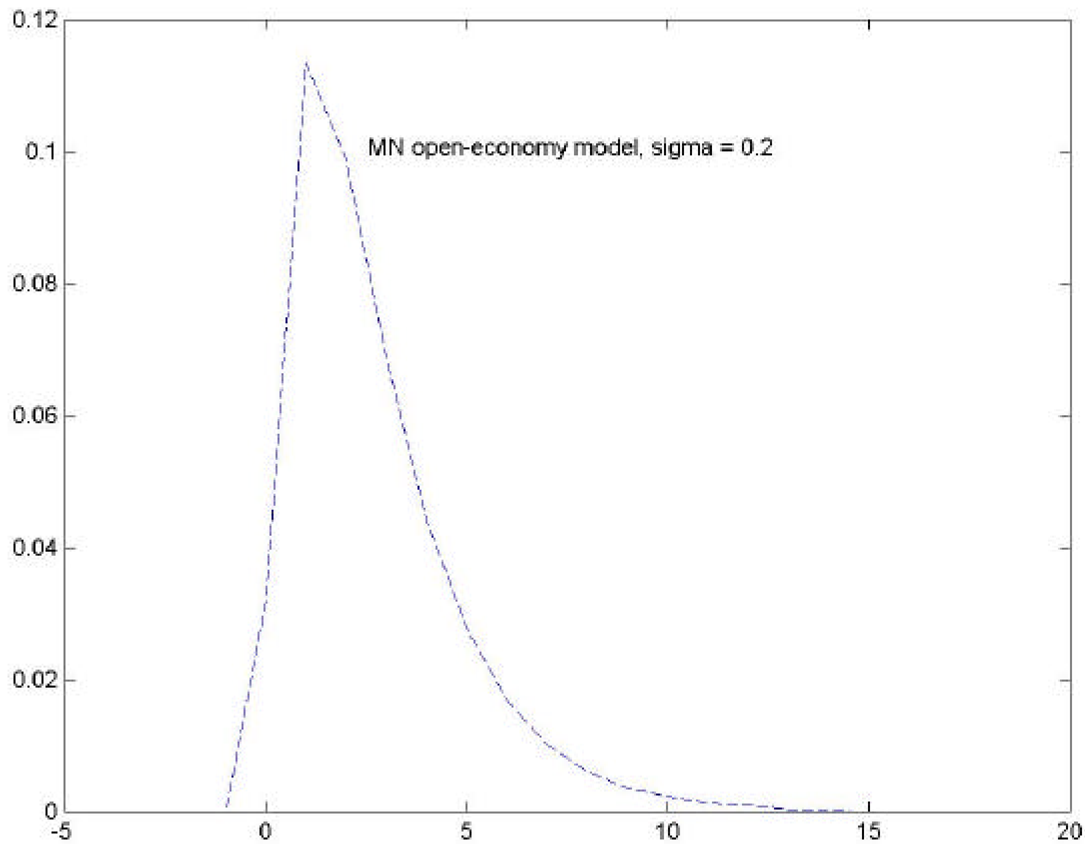
The responses for potential output (Charts 2a and 2b) are the mirror image of the real interest rate responses. Although the capital adjustment cost case continues to hold an intermediate position, potential output responds *less* to a technology shock when there is no capital in the model and *more* in response to a real demand shock. Impediments to adjusting capital inhibit the rise in investment that a technology shock would normally induce. And when the capital stock cannot be varied at all, it is not possible for higher consumption demand to be satisfied partly by a rise in consumption's share of income (which a rise in real rates would normally induce, as investment is more

(20) This is true even if preferences do not exhibit habit formation, although habit formation magnifies the response of the real rate to both technology and real demand shocks.

interest-elastic than consumption). So the real demand shock leads to sharp increases in both real interest rates and output.

In Chart 3, we present the natural real rate response to a domestic demand shock for a *different* model, namely McCallum and Nelson's (2000) open-economy model. We argued in Section 2.8 above that our choice of $\sigma = 0.6$ in the closed-economy model studied in the present paper serves to approximate the effects of openness on real interest rate dynamics. The chart gives the natural rate response in the McCallum-Nelson (MN) model to a shock to households' preferences in the domestic

Chart 3: Equilibrium real rate response to domestic demand shock: open economy



economy, where preferences are characterised by equation **(1)** but with $\sigma = 0.2$.[21] As the MN model has no capital, Chart 3 is an open-economy analogue of the no-capital case in Chart 1a. The size and shape of the response in Chart 3 closely resembles that in Chart 1a. Thus our approximation of open-economy effects appears to be a good one.

[21] The model is identical to that in McCallum and Nelson (2000, Figures 2–4) except that: (i) flexible prices are assumed, to isolate the natural rate response; (ii) the steady-state share of exports in GDP has been raised from 0.11 to 0.25, a more appropriate value for the United Kingdom; and (iii) habit formation (with parameter $h = 0.8$) has been included to make preferences identical (other than regarding the σ value) to those in our model.

3.2.2 Response of the real interest rate gap to shocks

We now turn to the sticky-price case, and examine the response of the real interest rate gap—the spread between actual and natural real interest rates—to shocks.

Examination of the sticky-price case requires that we choose a price-adjustment specification and policy rule to close the model. For price adjustment, we use the Calvo model, although in Section 4 we will also present results with the Fuhrer-Moore model. For the monetary policy rule, an estimated policy rule for the United Kingdom for 1980–99 would be desirable, as we will later be comparing our model with data for that sample period. But regime changes in UK monetary policy over 1980–99 mean that the full-sample estimates of a policy rule are not very reliable, and exhibit some problems of interpretation (for example, they fail to satisfy the condition that the nominal interest rate responds by more than one-for-one to expected inflation). Instead, we use the following policy rule estimated by Nelson (2000) for the United Kingdom for the sub-sample 1992 Q4–1997 Q1:

$$4*R_t = \rho_R\, 4*R_{t-1} + (1-\rho_R)1.267E_t\, \Delta_4 p_{t+1} + (1-\rho_R)0.47y_t + 4*e_{Rt} \qquad \textbf{(37)}$$

where $\rho_R = 0.29$, $\Delta_4 p_t = \Sigma_{j=0}^{3} p_{t-j}$ is the annual inflation rate, and the standard deviation of the policy shock e_{Rt} is 0.001. The policy rule is similar in specification to the Taylor (1993) rules estimated by Clarida, Gali and Gertler (1999). According to equation **(37)**, monetary policy adjusts the nominal interest rate in response to expected future annual inflation (with a greater than one-for-one long-run response), and to detrended output (y_t). Detrended output in the policy rule is meant to proxy for the output gap.

Charts 4 to 6 display the response of the real interest rate gap, detrended output, and the difference between output and potential output to technology, demand, and policy shocks respectively. The policy rule operates according to equation **(37)**, and capital formation is subject to adjustment costs.

Chart 4 depicts the effect of a technology shock. The shock generates an increase in the real interest rate gap—an effective policy tightening—for two reasons. First, the natural real rate falls, so for a given actual real interest rate monetary policy becomes tighter. Second, if policy responds to the level of output, a productivity shock induces policy-makers to raise the nominal interest rate, raising the actual real interest rate in the process.

In the case of a real demand shock, however, both the actual and natural real rate increase (Chart 5). The rise in the actual real rate, due to the monetary policy tightening in response to higher output, is less than the rise in the natural rate. The overall policy stance is therefore looser, once the natural rate movement is taken into account.

Chart 6 simply illustrates that a monetary policy tightening affects only the actual real interest rate. The real rate gap therefore rises one-for-one with the increase in the actual real interest rate. The output gap and output responses are identical because monetary policy cannot affect potential output.

The response of the real rate gap to different shocks illustrates how a policy rule like **(37)**, which responds to the level of output, can yield perverse results. In effect, the rule responds symmetrically to positive supply and demand shocks. This is because it does not take into account that the natural values of both the real interest rate and output have been affected by the real shocks. The natural real interest rate falls in response to the technology shock, and therefore the monetary policy response should be to lower, not raise, interest rates. In the case of a real demand shock, on the other hand, the natural rate increases. A policy aimed at minimising output gap and inflation variations would raise the interest rate in line with the increase in the natural rate. But in fact the policy rule **(37)** generates a smaller increase in the real rate than is necessary to keep the interest rate gap zero. One reason why responding to output appears to lead to counterproductive results, for both technology and real demand shocks, is that detrended output is not a good indicator of output gap behaviour in our model. Charts 4–6 also demonstrate the mirror-image relationship between the output gap and the real interest rate gap. We discuss these issues further in Section 5.

Chart 4: Responses to technology shock

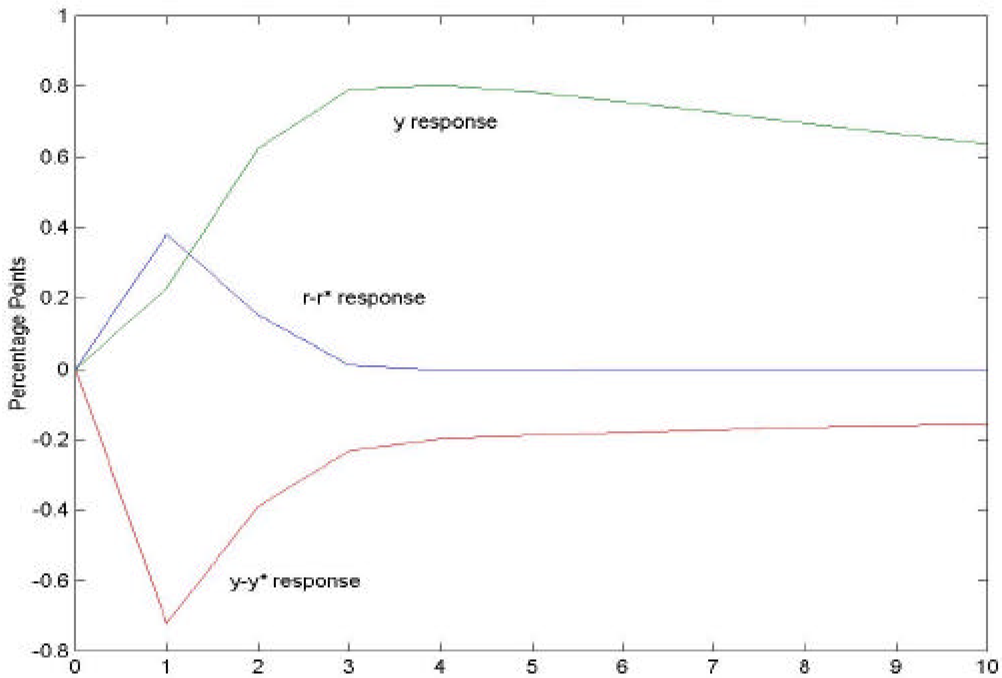


Chart 5: Responses to real demand shock

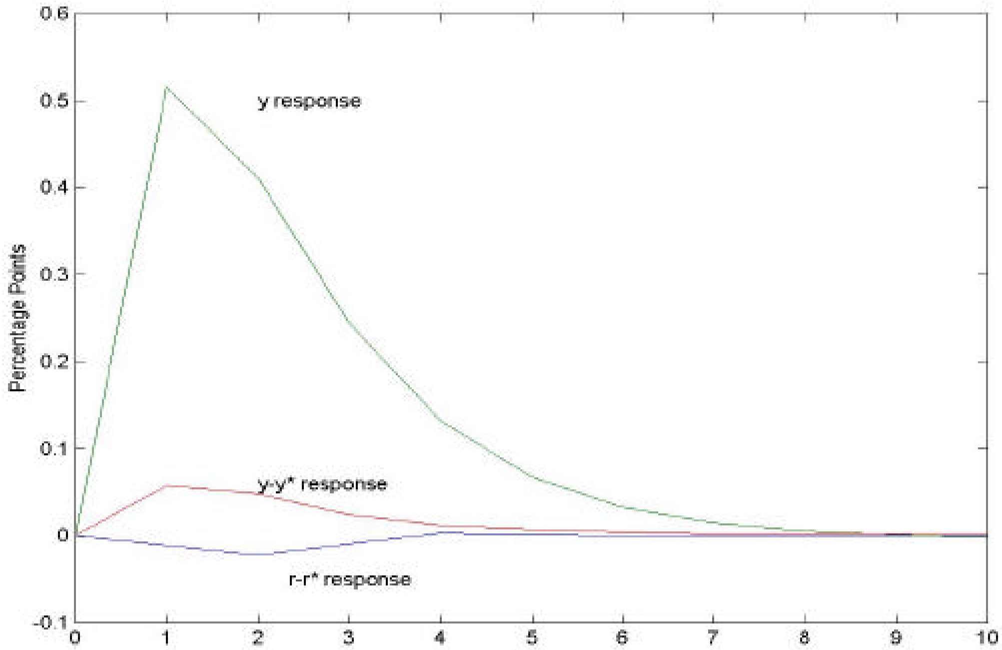
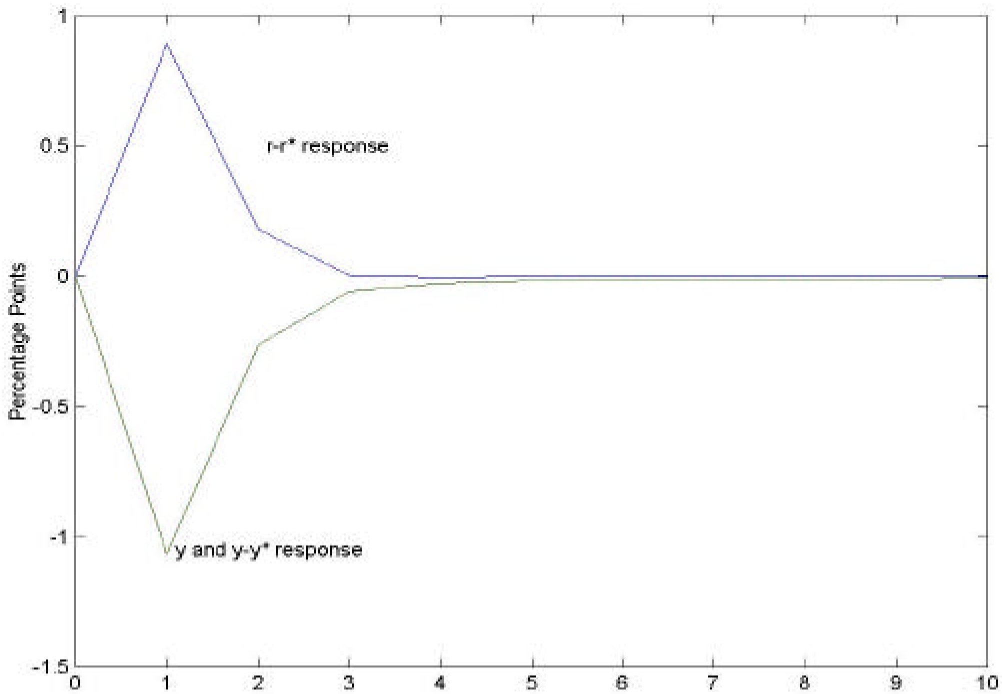


Chart 6: Responses to monetary policy shock

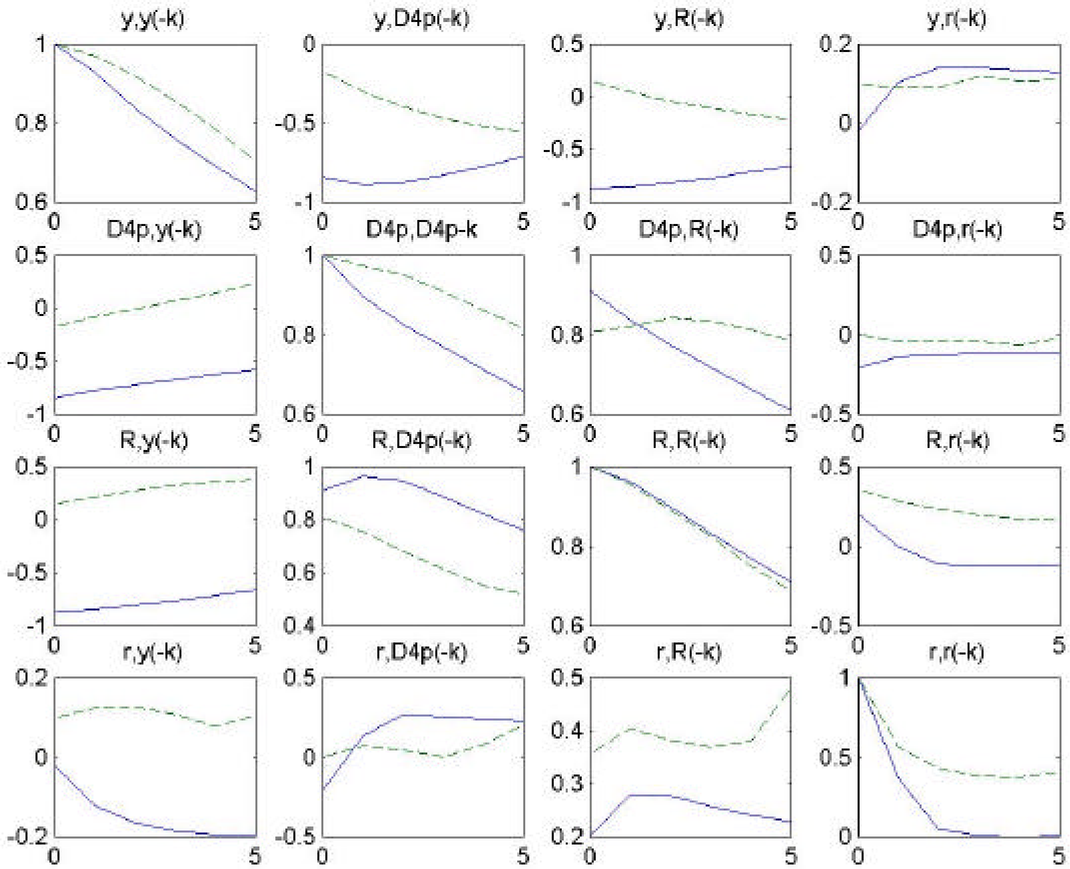


4 Model evaluation

We now present some evidence on the match between our model and the data by examining data and model vector autocorrelations. The version of the model we consider includes Calvo price-setting, capital formation subject to adjustment costs, and the policy rule **(37)** described in the previous section. The model correlations are averages of the output of 100 stochastic simulations of the model (for 200 periods each time). The corresponding data correlations are based on quarterly UK data for the 1980 Q1–1999 Q4 sample period. The data consist of y_t (quadratically detrended log real GDP), R_t (the quarterly average of the nominal Treasury bill rate), r_t (R_t minus the expected value of next quarter's seasonally adjusted RPIX inflation, p_{t+1}),[22] and annual inflation, $\Delta_4 p_t$.

Chart 7 plots the correlations for the model (solid line) and data (broken line). The model succeeds in reproducing the autocorrelations of the four series and, to a lesser extent, matching several other dynamic relationships, including the real/ nominal interest rate relationship, and the close correlation between four-quarter inflation and the nominal interest rate. The correlation between the level of

Chart 7: Empirical and model vector autocorrelations



[22] Expected inflation is calculated from the VAR(8) described in Section 6 below.

output and the lags of the real interest rate is generally positive in the model, which is qualitatively the relationship observed in UK data, as noted earlier in Table A. The main exception is the contemporaneous correlation between the two series, which is slightly negative in the model.

The model's most serious weaknesses are its ability to generate negative correlations between output and future interest rates (both nominal and real). These are at variance with the positive correlations observed in the United Kingdom for 1980–99. Part of this problem may reflect the use in the model of a policy reaction function that is estimated over only a sub-sample of 1980–99: the reaction of nominal interest rates to output movements may have differed substantially in that sub-sample (1992–97) from that observed over the full sample. But the discrepancy between the data and the model also reflects a property of the structure of the model: the model's forward-looking aggregate demand specification implies that aggregate demand (y_t) depends negatively on expected future real interest rates.

The model also does poorly in accounting for the correlation between output and annual inflation. The divergences between data and model in Chart 7 suggest that future work could focus on modifying the aggregate demand specification.[23]

5 Indicator properties of the real interest rate gap

We now examine some dynamic properties of our model, focusing on statistics that describe the relationship between the real interest rate gap, aggregate demand, and inflation. Table C gives selected correlations and standard deviations for four settings of the model (constant or fluctuating capital, with Calvo or Fuhrer-Moore price-setting). The variables focused on are log output (y_t), the real interest rate (r_t), log of natural output (y_t*), the natural real interest rate (r_t*), the output gap ($y_t - y_t$*), and the real interest rate gap ($r_t - r_t$*). A notable feature of Table C is that it reveals that the real interest rate gap and the output gap have quite different properties. The two series have the expected strong negative relationship (the correlation ranges from –0.63 to –0.92 depending on model setting), but differences between the two series emerge when we analyse the contribution of individual components of each gap.

In particular, the behaviour of the real interest rate (r_t) is a reasonable approximation for the behaviour of the real interest rate gap. The correlation between these two series is relatively high, and the two series are of roughly the same degree of volatility. Correspondingly, the natural real rate varies much less than the actual real rate. This contrasts with Rotemberg and Woodford's (1997) dynamic stochastic general equilibrium model, which generates a standard deviation for the natural real rate, of 3.7%, that is higher than that observed empirically for the actual real rate. But it is consistent with King and Watson (1996), whose DSGE model produced low variability in the real interest rate under price flexibility. Rotemberg and Woodford's finding may have arisen partly from larger shock variances than are typically found in other studies.

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[23] Other model specifications we analyse, including the no-capital and Fuhrer and Moore pricing specification, retain many of the weaknesses discussed here.

In addition, the actual and natural real interest rates are not very closely related to each other (the correlation between the two series ranges from +0.02 to –0.17). In other words, the variation in the real interest rate gap is dominated by variation in the 'observable' component of the gap, the real interest rate, rather than the 'unobservable' natural real rate component.

By contrast, the level of y_t is not a good indication of the behaviour of the output gap $y_t - y_t*$. In fact, the two series have an *inverse* relationship, with correlations ranging from –0.06 to –0.68, and the output gap has a standard deviation that is less than half that of output. Due to the absence of steady-state growth in our model, output in our model corresponds to detrended output in the data. Our results therefore question the widespread practice of measuring the output gap by detrended actual output. Rather, in our model variations in detrended output are dominated by variations in

Table C: Model statistics

	Model setting			
	No capital; Calvo	Capital with adjustment costs; Calvo	No capital; Fuhrer-Moore	Capital with adjustment costs; Fuhrer-Moore
s.dev.(y_t)	1.59	2.22	1.76	2.70
s.dev.(y_t*)	1.81	2.82	1.81	2.82
s.dev.($y_t - y_t*$)	0.30	0.80	0.19	0.69
s.dev.(r_t)	1.05	1.06	1.25	1.32
s.dev.(r_t*)	0.96	0.56	0.96	0.57
s.dev.($r_t - r_t*$)	1.41	1.21	1.68	1.53
Corr(y_t, y_t*)	0.99	0.98	0.99	0.97
Corr(y_t, $y_t - y_t*$)	–0.68	–0.68	–0.20	–0.06
Corr(r_t, r_t*)	0.02	–0.03	–0.14	–0.17
Corr(r_t, $r_t - r_t*$)	0.73	0.88	0.82	0.93
Corr(y_t, r_t)	–0.02	–0.05	–0.04	–0.03
Corr(y_t, r_{t-1})	–0.02	0.06	–0.00	0.09
Corr($y_t - y_t*$, $r_t - r_t*$)	–0.58	–0.74	–0.93	–0.97
Corr($y_t - y_t*$, $r_{t-1} - r_{t-1}*$)	–0.54	–0.40	–0.72	–0.45
Corr(Δp_t, $r_t - r_t*$)	–0.50	–0.41	–0.38	–0.33
Corr(Δp_t, $r_{t-1} - r_{t-1}*$)	–0.42	–0.34	–0.44	–0.38
Corr(Δp_t, $r_{t-2} - r_{t-2}*$)	–0.39	–0.31	–0.43	–0.37

Notes: s.dev(•) denotes standard deviation; Corr(•,•) the simple correlation coefficient. Standard deviations of the interest rate variables are annualised. Statistics reported are computed from analytical formulae for the model moments.

potential output—the two series have a correlation ranging from 0.96 to 0.99—so the 'unobservable' potential output variable should not be treated as constant (or growing steadily) when generating an output gap series.

The last statistics in Table C indicate that the real interest rate gap seems to have a reasonably strong negative correlation with quarterly inflation, Δp_t. Fuhrer-Moore price-setting appears to make this relationship a leading relationship. To examine the forecasting properties of the real interest rate gap in a different way, we now report averages across model simulations of estimates of the regression:

$$\Delta_4 p_t = b_{10} + b_{11}(r_{t-1} - r_{t-1}^{*}) + b_{12}\Delta_4 p_{t-1} \qquad \textbf{(38)}$$

which can be thought of as the inflation equation in a one-lag, bivariate VAR system that consists of the annual inflation rate, $\Delta_4 p_t$, and the real interest rate gap. For comparison, we also report results using the output gap:

$$\Delta_4 p_t = b_{20} + b_{21}(y_{t-1} - y_{t-1}^{*}) + b_{22}\Delta_4 p_{t-1} \qquad \textbf{(39)}$$

We supplement these results by reporting regressions that replace the 'gap' variables with the actual real interest rate and actual output.

$$\Delta_4 p_t = b_{30} + b_{31} r_{t-1} + b_{32}\Delta_4 p_{t-1} \qquad \textbf{(40)}$$

$$\Delta_4 p_t = b_{40} + b_{41} y_{t-1} + b_{42}\Delta_4 p_{t-1} \qquad \textbf{(41)}$$

These last two regressions can be thought of as what a researcher might estimate if they approximated the natural real interest rate by a constant in constructing a real interest rate gap series, or potential output by a trend in constructing an output gap series.

Regressions on the artificial data sets are summarised in Table D. Comparison of the regressions that use the gap measures indicates that the lagged real interest rate gap and the lagged output gap each is highly significant when added to an autoregression for inflation. Moreover, the signs of the estimated coefficients have economic interpretations—the positive output gap coefficient reflect the effect of 'excess demand' on inflation; and the real interest rate gap coefficient indicates the negative relationship between real interest rates (relative to their natural value) and excess demand.(24) The regressions with the output gap fit better than the regressions with the real interest rate gap, though the difference is large in only one case (Fuhrer-Moore price-setting with no capital).

(24) Beyond this sign interpretation, it would not be sensible to interpret the estimated coefficients further, as they are reduced-form. In particular, if interpreted as structural equations, the estimated inflation regressions appear to suggest that long-run non-zero output and real interest rate gaps can be obtained by altering the long-run inflation rate. But the underlying structural model that generated the data used for the regressions does not have this property.

A more practical question, however, is how much of the fit of these regressions is due to the explanatory power provided by the ‘unobservable’ components of the gaps—the natural values of the real interest rate and output. The answer to this question is provided by the regressions in Table D that use the actual values of r_t and y_t as regressors rather than the corresponding gap variables. For

Table D: Regressions on simulated data: dependent variable $\Delta_4 p_t$ (annual inflation rate)

Model setting	Coefficient on:		SEE	s.dev.($\Delta_4 p_t$)
	$r_{t-1} - r_{t-1}$*	$\Delta_4 p_{t-1}$		
No capital, Calvo	–0.90 (0.08)	0.96 (0.01)	0.40	2.44
Capital with adj. costs, Calvo	–1.16 (0.11)	0.97 (0.01)	0.47	3.03
No capital, FM	–0.95 (0.05)	0.97 (0.01)	0.32	2.80
Capital with adj. costs, FM	–1.19 (0.07)	0.98 (0.01)	0.38	3.59
	$y_{t-1} - y_{t-1}$*	$\Delta_4 p_{t-1}$		
No capital, Calvo	2.80 (0.19)	0.72 (0.02)	0.35	2.44
Capital with adj. costs, Calvo	0.70 (0.06)	0.87 (0.01)	0.45	3.03
No capital, FM	2.49 (0.09)	0.95 (0.01)	0.23	2.80
Capital with adj. costs, FM	0.67 (0.04)	0.98 (0.01)	0.39	3.59
	r_{t-1}	$\Delta_4 p_{t-1}$		
No capital, Calvo	–1.00 (0.13)	1.01 (0.01)	0.44	2.44
Capital with adj. costs, Calvo	–1.20 (0.14)	0.99 (0.01)	0.49	3.03
No capital, FM	–1.17 (0.09)	1.02 (0.01)	0.36	2.80
Capital with adj. costs, FM	–1.34 (0.09)	1.00 (0.01)	0.40	3.59
	y_{t-1}	$\Delta_4 p_{t-1}$		
No capital, Calvo	0.00 (0.04)	0.98 (0.02)	0.40	2.44
Capital with adj. costs, Calvo	–0.05 (0.05)	0.95 (0.03)	0.47	3.03
No capital, FM	–0.02 (0.04)	0.97 (0.02)	0.32	2.80
Capital with adj. costs, FM	–0.08 (0.06)	0.94 (0.03)	0.38	3.59

Note: Numbers reported in columns 2–3 are the means across simulations of parameter estimates and their corresponding standard errors.

the regressions with r_t, the omission of the r_t* term does cost some explanatory power (confirming that $r_t - r_t$* is the more appropriate index of inflationary pressure) but—because of the low variability of r_t* in our model—the loss of fit is minor. In other words, the real interest rate again appears to be a reasonable proxy for the real rate gap for the purpose of forecasting inflation.

On the other hand, replacing $y_{t-1} - y_{t-1}$* by y_{t-1} leads to a drastic loss of fit and to a 'wrongly' signed parameter estimate (if the coefficient on output is to be interpreted as an excess demand coefficient). Thus, in this model, both the real interest rate gap and the output gap are significant leading indicators of inflation—but the leading-indicator properties of the real rate gap largely come from the observable component, while the leading-indicator properties of the output gap are attributable to the unobservable potential output component. Our results suggest that over the business cycle, fluctuations in potential output are great enough to make detrended output an unreliable indicator of the output gap, but that the accompanying fluctuations in the natural real rate are quite small. Under these circumstances, the real interest rate can be quite useful as an indicator of demand pressure and of future inflation.

6 Empirical properties of the real interest rate gap

In this section we construct, for the United Kingdom, empirical counterparts to the natural real interest rate and the real interest rate gap series that appear in our model, and investigate the relationship between the resulting gap series and inflation.

In our model, the natural real rate interest is a function of IS (real demand) and technology shocks. Technology shocks are typically measured by 'Solow residuals', which are obtained by subtracting log values of labour and capital inputs (each respectively weighted by their coefficients in the production function) from the log of total output. We use a Solow residual series constructed from quarterly UK data.[25]

We measure IS shocks as follows. Combining the log-linearised Euler equations for consumption and bonds, as well as the Fisher equation, the IS shock l_t is given by:

$$l_t = (-b g_1 \mathrm{E}_t \Delta c_{t+2} + g_2 \mathrm{E}_t \Delta c_{t+1} + g_3 \mathrm{E}_t \Delta p_{t+1} - g_1 \Delta c_t - g_3 R_t) / g_4 \qquad \textbf{(42)}$$

where $g_1 = (h - sh)$, $g_2 = (1 + bh^2 - sbh^2 - sbh)$, $g_3 = s(1 - bh)$, and $g_4 = -s(1 - r_l + bhr_l^2 - bhr_l)$. We use formula **(42)** to generate IS shocks from quarterly UK data for

[25] The Solow residual is defined empirically by $\log(Y_t) - (1-a)\log(K_t) - a\log(N_t)$, as suggested by equation **(15)**. Y_t is measured by the level of quarterly UK real GDP. K_t is a capital stock series obtained by formula **(3)**, with X_t measured by the investment series in Bank of England (2000, page 62), and the initial (1962 Q1) value of K given by our model's steady-state (K/X) value multiplied by the 1962 average level of X_t. Labour input is measured by employment (*British Labour Statistics* data up to 1978, spliced into the series defined in Batini, Jackson and Nickell (2000) from 1978). As our model does not allow for secular growth, the a_t series in our model corresponds to detrended total factor productivity. We quadratically detrend our empirical Solow residual series to construct an empirical counterpart to the a_t series.

1980 Q1–1999 Q4. In constructing the shock series, we measure R_t by the Treasury bill rate (quarterly average), c_t by the log of quarterly private consumption (less durables), and p_t by the seasonally adjusted log-change in the quarterly average of the RPIX index.

We measure expected values of series by forecasts from an eight-lag VAR estimated over 1976 Q1–1999 Q4. The variables in the VAR are R_t, Δc_t, and p_t, and each equation is supplemented by two intercept-dummy variables, $DERM_t$ and $D924_t$. These dummy variables capture changes in UK monetary policy regime, and take non-zero values during the United Kingdom's membership of the Exchange Rate Mechanism (1990 Q4–1992 Q3) and its period of inflation targeting (1992 Q4 onward). Once expected values of variables have been constructed, the IS shock series is then generated using our calibrated values of b, s, r_l, and h.

In the version of our model with capital formation (subject to adjustment costs), the natural real rate is related to IS and technology shocks according to:

$$\begin{aligned} r_t^* = \ & 0.0452\ l_t + 0.0582\ l_{t-1} + 0.0283\ l_{t-2} + 0.0139\ l_{t-3} \\ & + 0.0070\ l_{t-4} + 0.0036\ l_{t-5} + 0.0019\ l_{t-6} - 0.0781\ a_t \\ & + 0.0264\ a_{t-1} + 0.0125\ a_{t-2} + 0.0057\ a_{t-3} + 0.0023\ a_{t-4} + 0.0007\ a_{t-5} \end{aligned}$$

where we neglect the low coefficients on further lags of l_t and a_t. We use this formula to generate an r_t^* series for the United Kingdom, and a real interest rate gap series ($r_t - r_t^*$).

Table E gives the simple correlations on UK data between annual inflation ($\Delta_4 p_t$) and lags of the real interest rate gap series. For comparison, the correlations between inflation and the real interest rate are also reported. The results indicate that the correlation between the real interest rate gap and inflation is negative, and tends to be more negative than the correlations between inflation and the real rate. These results are in keeping with our model's predictions.

On the other hand, there is a limitation in studying simple correlations calculated over the 1980–99 sample period. The model we have used in this paper was one that described fluctuations around a fixed steady state for a given monetary policy rule. By contrast, the period 1980–99 is characterised by several changes in the United Kingdom's monetary policy regime—most notably the movements into the Exchange Rate Mechanism in 1990 and to inflation targeting in 1992, described above—and by different unconditional means of the real interest rate across regimes (with higher means in the pre-inflation targeting period).

In evaluating the quality of our real rate gap series as an inflation indicator, it is desirable to account for these shifts. Therefore, we also include in Table E the *partial* correlation coefficients, which condition on the regime shifts in 1990 and 1992. Essentially these are correlation coefficients between the real rate, real rate gap series and inflation, once each series has been purged of its correlation with the dummy variables $DERM_t$ and $D924_t$ described above. The partial correlations are uniformly more negative than the simple correlations, confirming that controlling for regime change is important.

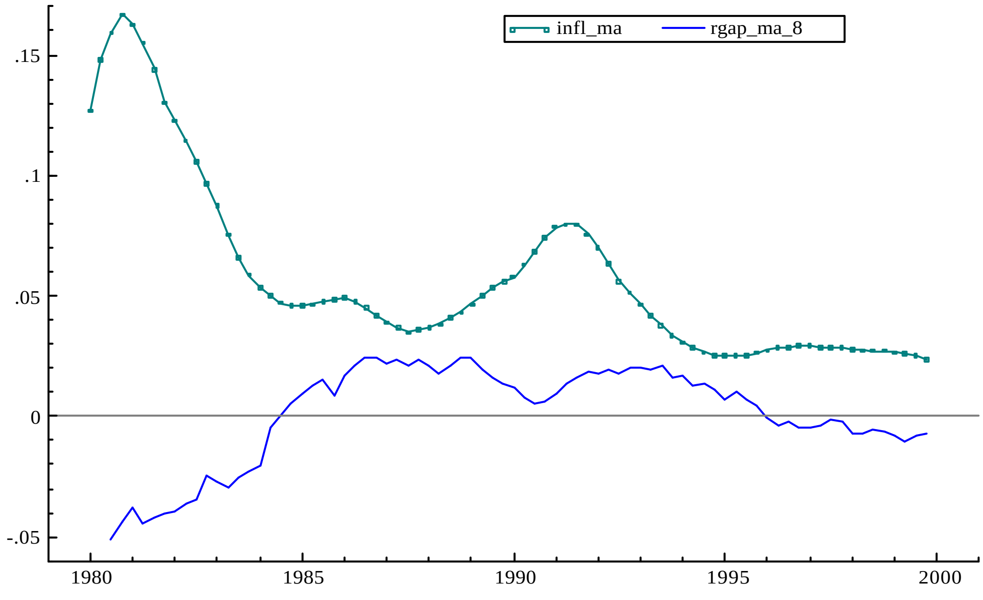
Table E indicates that both the simple and partial correlations between inflation and the real interest rate gap tend to be more negative at lower lags than those between inflation and the real interest rate. Correspondingly, a smoothed version of the real interest rate gap series appears to have a reasonably strong inverse leading relationship with smoothed UK inflation (Chart 8).

Table E: Correlations between inflation and real interest rate gap, UK data
Sample period: 1980 Q1–1999 Q4

	Simple correlations								
	k=0	k=1	k=2	k=3	k=4	k=5	k=6	k=7	k=8
Corr ($\Delta_4 p_t$, $r_{t-k} - r_{t-k}$*)	–0.13	–0.21	–0.26	–0.40	–0.47	–0.40	–0.42	–0.46	–0.48
Corr ($\Delta_4 p_t$, r_{t-k})	0.01	–0.10	–0.18	–0.35	–0.44	–0.38	–0.42	–0.47	–0.52
	Partial correlations, conditional on policy regime breaks								
	k=0	k=1	k=2	k=3	k=4	k=5	k=6	k=7	k=8
Corr ($\Delta_4 p_t$, $r_{t-k} - r_{t-k}$*)	–0.50	–0.54	–0.58	–0.66	–0.73	–0.62	–0.60	–0.60	–0.59
Corr ($\Delta_4 p_t$, r_{t-k})	–0.37	–0.45	–0.52	–0.61	–0.70	–0.61	–0.59	–0.61	–0.61

Note: Construction of r_t and r_t* series is as described in the text.

Chart 8: Annual inflation and real interest rate gap two years earlier; UK data 1980–99, six-quarter moving averages



Finally, we use our interest rate gap series for the United Kingdom to estimate, on actual data for 1979 Q1 to 1999 Q4, the equation that we estimated on artificial data sets in Table D: a regression of $\Delta_4 p_t$ on $\Delta_4 p_{t-1}$ and $(r_{t-1} - r_{t-1}*)$. A constant and the $DERM_t$ and $D924_t$ intercept dummies are also included in the equation. The estimated coefficient on the lagged real interest rate gap is of the expected, negative, sign; and a formal test for the exclusion of the gap from the regression leads to a rejection with a probability higher than 0.001, reinforcing the visual and correlation evidence. Thus there is some tentative evidence that our attempt to control for movements in the natural interest rate has enhanced the value of the real interest rate as an indicator of inflation.

7 Conclusions

This paper has examined what a dynamic stochastic general equilibrium model has to say about the value of the real interest rate gap as an indicator of inflation. A shortcoming of the concept is that it requires the construction of an 'unobservable' natural real rate series. However, the same is true of the output gap concept, and our results suggest that output gap concepts as often constructed (which are based on a deterministic model of potential output) can be seriously misleading. Under these circumstances, a real interest rate gap series could provide useful auxiliary information in evaluating the monetary policy stance and the prospects for future inflation, in keeping with the neo-Wicksellian framework of Woodford (1999a, 2000).

Our model is capable of reproducing several empirical regularities in UK data, notably the procyclical behaviour of real interest rates. In other respects, there is room for improvement in the dynamic specification of the model. In particular, the model predicts that output is negatively correlated with future inflation and interest rates, which is the opposite of the pattern observed in UK data. It would also be desirable to obtain a more gradual inflation response to movements in the real interest rate gap.

Our empirical results in Section 6 suggested that there are some benefits to inflation forecasting from controlling for movements in the natural real rate, and basing the construction of the natural rate series on general equilibrium theory. The quantitative benefits may not be great, however, because the empirical variation in the natural rate appears to be quite small.

Appendix: Calculation of output and interest rate gaps

In this appendix, we describe some of the complications involved in calculating (log) capacity output (y_t*) and the natural real interest rate (r_t*) in our model, and our procedure for overcoming the complications. This allows us then to compute the impulse responses for the natural real rate in Section 3, and to analyse the output gap ($y_t - y_t$*) and the real interest rate gap ($r_t - r_t$*) in Section 5.

The model outlined above features a time-varying, endogenous capital stock, as well as habit formation in consumption. Consequently, both the period t capital stock and the previous period's consumption level are elements of the state vector. Thus there are two endogenous variables in the state vector, in addition to the exogenous technology (a_t), real demand (λ_t), and monetary policy (e_{Rt}) shock variables. As a result, the model's solutions for y_t* and r_t* cannot be written as simple functions of the exogenous variables alone.

To see this, consider output under sticky prices. From the production function

$$y_t = a_t + (1 - \alpha)\, k_t + \alpha\, n_t \qquad \textbf{(43)}$$

Similarly, output under flexible prices, or capacity output, may be written as:[26]

$$Y_t^* = a_t + (1 - \alpha)\, k_t^* + \alpha\, n_t^* \qquad \textbf{(44)}$$

where asterisks denote flexible-price values. Had we assumed inelastic labour supply and an exogenous capital stock, then y_t* would simply be a linear combination of two current-dated exogenous variables, a_t and k_t*, and calculation of y_t - y_t* would be straightforward. But for the more general case the solution to k_t* and n_t* are functions of the whole state vector:

$$k_t^* = f_1 \boldsymbol{s}_{t-1}^* \qquad \textbf{(45)}$$

$$n_t^* = f_2 \boldsymbol{s}_{t-1}^* \qquad \textbf{(46)}$$

where the f_i are 1 x 4 coefficient vectors, and $\boldsymbol{s}_t^* = [k_t^*\ c_{t-1}^*\ a_t\ \lambda_t]'$ is the state vector under flexible prices.[27]

To calculate y_t*, one procedure that would appear illegitimate is to condition on actual capital and consumption k_t and c_{t-1}—in effect replacing the unobserved flexible-price variables in $\boldsymbol{s}_t$* with their sticky-price counterparts, and using equations **(44)**, **(45)** and **(46)** to compute y_t*. This procedure seems illegitimate because the behaviour of the two endogenous state variables, capital and

[26] The technology shock a_t, being exogenous, behaves identically under price flexibility and price stickiness, and therefore requires no asterisk in equation **(44)**.
[27] The monetary policy shock e_{Rt} is omitted from this list due to the neutrality of real variables to monetary shocks under flexible prices.

consumption, will be a function of monetary shocks and the monetary policy rule under sticky prices. However, Woodford (2000) has made a case for this method of defining the natural output and interest rate, and we discuss it further below.

In principle, another method would be to enlarge the model by supplementing the list of endogenous variables in the model with their flexible-price counterparts, and including a flexible-price and a sticky-price version of each behavioural equation. But by effectively doubling the size of the model to be solved, this procedure would be quite cumbersome.

Instead, we employ a more economical method for calculating y_t* and r_t*. In essence, we want to express y_t* and r_t* as functions of the exogenous variables only—effectively 'substituting out' k_t* and c_{t-1}* of the solution equations. To this end, we note that since we are assuming (vector) autoregressive processes for the exogenous elements of $\boldsymbol{s}_t$*, $\boldsymbol{s}_t$* follows the law of motion $\boldsymbol{s}_t^* = \boldsymbol{R}\boldsymbol{s}_{t-1}^* + Y_e e_t$, where e_t is a vector of innovations. $\boldsymbol{s}_t$* thus has a vector moving-average representation giving each element of $\boldsymbol{s}_t$* as a (possibly infinite) distributed lag of the e_t. Specifically, k_t* and c_t* may be written as:

$$k_t^* = f_1(e_{t-1}, e_{t-2}, e_{t-3}, ...) \qquad \textbf{(47)}$$

$$c_t^* = f_2(e_t, e_{t-1}, e_{t-2}, ...) \qquad \textbf{(48)}$$

where the $f_i(\bullet)$ are linear functions. It follows from **(47)** and **(48)** that

$$y_t^* = a_t + f_3(e_t, e_{t-1}, e_{t-2}, ...) \qquad \textbf{(49)}$$

Since the exogenous variables a_t and l_t each are infinite moving averages of the e_t, it follows that:

$$y_t^* = f_4(a_t, a_{t-1}, ..., l_t, l_{t-1}, ...) \qquad \textbf{(50)}$$

By the same argument, r_t* may be written as:

$$r_t^* = f_5(a_t, a_{t-1}, ..., l_t, l_{t-1}, ...) \qquad \textbf{(51)}$$

To obtain the impulse responses and simulation results reported in the text, we approximate the right-hand sides of **(50)** and **(51)** by a long but finite distributed lag of the exogenous variables. Our complete procedure for calculating the gap measures ($y_t - y_t$*) and ($r_t - r_t$*) is as follows:

1. Solve the model of Section 2 under flexible prices.
2. Simulate the model. Using the data generated from these simulations, run a regression on generated data of the form:

$$y_t^* = c_1 a_t + ... + c_j a_{t-j} + d_1 l_t + ... + d_j l_{t-j}$$

$r_t{*}= g_1 a_t + ... + g_j\, a_{t-j} + h_1\, l_t + ... + h_j\, l_{t-j}$

for a finite j that is high enough to generate a good fit.

4. Store the c_i, d_i, g_i and h_i coefficients (actually, averages of each of these coefficients across simulations).

5. Solve the model of Section 2 incorporating sticky prices. Augment the exogenous variable vector with y_t* and r_t*, where these are defined as the indicated linear combinations of current and lagged a_t and l_t.

6. Define the output gap as $y_t - [y_t*$ as defined in Step 4]; the real interest rate gap as $r_t - [r_t*$ as defined in Step 4].

We have found that this procedure generates fitted measures of r_t* and y_t* that are near-perfect approximations for the correct measures given by equations **(50)** and **(51)**.

Woodford (2000) has argued instead for defining the natural rate conditional on the realised (sticky-price) values of endogenous state variables such as the capital stock. He argues that this is preferable to our procedure given above because '[i]t seems odd to define the economy's "natural" level of activity… in a way that makes irrelevant the capital stock that actually exists and the effects of this upon the economy's productive capacity' (Woodford (2000 page 67)). But our procedure does not make the actual capital stock irrelevant: while the flexible-price capital stock concept appears in our definitions of natural levels of variables, it is the actual, existing capital stock that appears in the production function, capital law of motion, capital adjustment cost function, and resource constraint of the sticky-price economy.

One consideration is that conditioning on the actual capital stock in defining potential output, as Woodford recommends, can lead to some curious policy prescriptions, if the monetary policy reaction function is characterised by occasional unavoidable mistakes that can be corrected only in later periods. For then, a policy mistake last period that reduced investment demand reduces the capital stock today; this automatically reduces potential output today, so need not create an output gap, and therefore no compensating policy response today to correct last period's mistake. Under our alternative definition, potential output (but not actual output) is invariant to monetary policy, so last period's mistake creates an output gap today and justifies an easing today to correct the mistake.

Most importantly, our definition readily extends to the case of endogenous state variables beside the capital stock. The physical constraint evoked by Woodford to justify his definition is applicable only to state variables that enter the production function, whereas our model contains an endogenous state variable, lagged consumption, that does not enter the production function.

Nonetheless, we would not over-emphasise the differences between Woodford's natural-rate concept and our own. The impulse response functions for the natural real rate and potential output in Section 3.2.1, for example, would be identical under either definition, since those responses start from a steady state common to both the flexible and sticky-price economies.

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