The structure of credit risk: spread volatility and ratings transitions

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Contents

Abstract

Knowing the relative riskiness of different types of credit exposure is important for policy-makers designing regulatory capital requirements and for firms allocating economic capital. This paper analyses the risk structure of credit exposures with different maturities and credit qualities. We focus particularly on risks associated with (i) ratings transitions and (ii) spread changes for given ratings. We show that, for high-quality debt, most risk stems from spread changes. This is significant because several recently proposed credit risk models assume no spread risk.

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Summary

Regulators designing capital requirements for loan portfolios or senior bankers deciding levels of economic capital for their institutions need to know the magnitudes of the risks involved in holding portfolios of credit exposure of different types. This paper quantifies the risks involved in holding large portfolios of different credit qualities and times to maturity. To accomplish this, we formulate a ratings-based credit risk model and simulate it for large portfolios of credit exposures.

The model we develop generalises a widely-employed model, namely the Creditmetrics approach popularised by JP Morgan, to include risks not just of rating transition and random recoveries but also random shocks to spreads in non-default states. Spread risk is important since it is highly correlated across different exposures and hence does not diversify away in large portfolios.

Incorporating spread risk is a difficult task not because of the added complexity of the credit risk model which is relatively slight, but more because of the difficulty of estimating a joint distribution of spread changes over the long horizons typically employed in credit risk modelling. We propose a non-parametric approach to estimating moments of spread changes over one year or more and argue that over such long periods, spread changes appear approximately Gaussian.

Our non-parametric approach may be thought of as filtering out high frequency components of spread volatility, leaving volatility associated with permanent shocks to spreads. Since a relatively large proportion of the volatility of high credit quality spreads is made up of short term shocks that are reversed by subsequent, off-setting spread changes, filtering in this way increases the gap between the volatility of changes in high and low credit quality spreads.

Basing our discussion on Value at Risk (VaR) measures of risk, we show that spread volatility contributes much the largest fraction of total risk for investment quality portfolios. For reasonable confidence levels, portfolios of credit exposures that possess the same rating profile as that of an average large US bank turn out to have VaRs similar in magnitude to the capital charges required by the 1988 Basel Accord. Lastly, we document

the fact that credit risk has an important maturity dimension in that we show portfolios with similar ratings profiles but longer maturity possess substantially larger VaRs.

1 Introduction

Understanding the relative riskiness of different types of credit exposure is an important objective for regulators designing regulatory capital requirements and for banks themselves engaged in the allocation of economic capital. The June 1999 discussion document on bank capital issued by the Basel Committee[1] contained illustrative capital charges for differently-rated credit exposures. How appropriate these charges are depends on one's assessment of the relative risks involved in holding different credit exposures. Banks themselves in allocating their economic capital employ risk weights ranging from less than 1% to over 30%.[2] Again, whether these are appropriate depends on one's view of how risk varies across different credit exposures.

Relatively few studies have examined the relative riskiness of different credit exposures. Carey (1998) looks at the default experience and loss distribution for a large group of privately-placed US bonds. Carey (2000) calculates risk measures for portfolios of public bonds by applying bootstrap methods to a large dataset of Moody's rating histories. Both papers yield many insights into the structure of credit risk but the approach they take is 'default-mode' in that they abstract from changes in portfolio value attributable to rating migration to non-default rating categories.

In this paper, we investigate the relative riskiness of credit exposures by calculating Values at Risk (VaRs) for portfolios of credit exposures on a mark-to-market basis.[3] To do this, we formulate a portfolio risk management model which generalises JP Morgan's Creditmetrics techniques[4] by incorporating spread risk. We argue that including stochastically varying spreads is particularly important for the measurement of the risk of portfolios since spread risk is strongly correlated across different exposures and hence will not be diversified away in large portfolios. In Creditmetrics, spreads are deterministic and

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[1]See Basel Committee on Banking Supervision (1999).

[2]See Greenspan (1998).

[3]For a given portfolio, the VaR with a confidence level α is the loss that will be exceeded on a fraction α of occasions by an investor holding the portfolio. For a small α, VaR is a statistic that sheds light on the risk of catastrophic losses and hence is widely used in financial institutions in the context of decisions about appropriate capital levels. The Creditmetrics model employed in this study is a framework for estimating the return distribution of portfolios of credit sensitive investments. In principle, Creditmetrics could be used to generate many different statistics that would shed light on the riskiness of portfolios. In practice, it is commonly used to generate VaRs.

[4]See JP Morgan (1997).

the only source of correlation is the dependence in the distributions of transitions.

An important step in formulating our model is to estimate the joint distribution of credit spreads for long horizons. A common industry practice is to implement credit risk models for one-year holding periods and we follow this approach in our analysis below. At such horizons, mean-reverting components of spread volatility have largely disappeared and only random walk innovations in credit spreads remain. Using non-parametric methods, we estimate moments of long-horizon spread changes and show that spreads over such horizons are approximately normally distributed.

Using the estimated spread distributions and different ratings transition matrices, we then simulate returns on portfolios of credit-sensitive exposures with various credit quality profiles. We study volatilities and VaRs for these portfolios under different assumptions about the degree of correlation between individual credit exposures, the ratings transition matrix employed and the credit quality and maturity of the portfolios in question.

Our main results are that spread risk contributes a significant fraction of credit risk for a portfolio with the same credit ratings profile as the loan book of the average large US bank. Spread risk is most important for high credit quality portfolios. Risk increases with the average maturity of the exposures in the portfolio much more for high than for low credit quality portfolios. It is interesting to note that, according to our simulations, the appropriate level of capital (based on a VaR calculation with a 0.3% confidence level) for a bank with a portfolio of average credit quality just exceeds the 8% capital charge specified by the 1988 Basel Accord.

While several empirical studies of credit risk have been concerned with the distribution of ratings transitions (see Lucas and Lonski (1992) , Carty and Fons (1993), Carty (1997), Standard and Poor's (1998), Altman and Kao (1992a,1992b), and Nickell, Perraudin and Varotto (2000)), and other papers have studied default premia (see, for example, Jones, Mason and Rosenfeld (1984), Sarig and Warga (1989), Duffee (1998,1999)), the contribution of spread risk to credit risk has been neglected. The existing literature on spreads including Pedrosa and Roll (1998) and Collin-Dufresne, Goldstein and Martin (1999) only investigates short-term, market-risk aspects of spreads or their empirical

properties.

Finally, several recent papers have looked at the riskiness of credit exposures in the context of evaluating credit risk models proposed by the finance industry. Gordy (2000) and Crouhy, Galai and Mark (2000) implement credit risk models and calculate levels of capital for particular portfolios, Nickell, Perraudin and Varotto (1999) evaluate the performance of credit risk models out of sample, while Lopez and Saidenberg (2000) discuss cross-sectional evaluation of credit risk models.

Our paper is arranged as follows. In Section 2 we set out the framework of our analysis in which ratings follow a Markov chain and average spreads for different ratings categories are random and correlated between categories although independent of transitions. We describe how one may calculate variances and VaR's of bond prices and portfolio values. In Section 3 we describe our non-parametric approach to estimating spread distributions for long investment horizons from data measured at a higher frequency. In Section 4 we report results for bond and portfolio volatilities and VaRs. Section 5 concludes.

2 Measuring credit risk

2.1 Decomposing credit risk

The risks associated with the future value of a credit-sensitive exposure may be partitioned into: (i) the risk that the firm's ratings changes, (ii) the risk that changes occur in the average spread of credit exposures with the same final rating as the firm, and (iii) the risk that the gap between the firm's idiosyncratic spread and the average spread of credit exposures with the same ratings changes.[5] Since we are primarily interested in credit risk measures for portfolios, we concentrate on (i) and (ii), assuming that (iii) is diversified away.

To describe our framework formally, suppose there are $i = 1, 2, \ldots, N$ different ratings categories of which the Nth corresponds to the default state. Assume that the probability

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[5]To see that all credit risk may be decomposed into these three categories, note that knowing (i), (ii) and (iii), rating, average spread and the gap between average spread and the firm's idiosyncratic spread, one may calculate the level of the firm's spread.

of changes from rating i to rating j over one time period is a constant π_{ij} and let $\Pi \equiv [\pi_{ij}]$ denote the matrix of which the ijth element is π_{ij}. We will assume that the default state is absorbing so $\pi_{Nj} = 0$ for all $j = 1, 2, \ldots, N-1$ and $\pi_{NN} = 1$. Π is then the transition matrix of a Markov chain followed by ratings changes.

Since our primary concern is with the riskiness of portfolios of credit exposures, we are interested not just in the distribution of individual exposures but in their joint distributions. To allow for dependencies between the distributions of transitions for different exposures, we follow the ordered probit approach employed in JP Morgan's (1997) Creditmetrics methodology and used for the econometric study of ratings transitions by Nickell, Perraudin and Varotto (2000).

This consists of assuming that transitions for a single exposure over a given time horizon are driven by an unobserved random factor, which has a standard normal distribution. Conditional on starting in rating i, let Z_{ij} $j = 1, 2, \ldots, N-1$ represent cut-off points such that $\Phi(Z_{i1}) = \pi_{i1}$, $\Phi(Z_{ij}) - \Phi(Z_{ij-1}) = \pi_{ij}$ for $j = 2, 3, \ldots, N-1$ and $1 - \Phi(Z_{iN-1}) = \pi_{iN}$. Given the $\pi_{i1}, \pi_{i2}, \ldots, \pi_{iN-1}$, it is simple to derive the cut-off points, Z_{ij}, by solving the above equations recursively.

To introduce dependencies between the distributions of ratings transitions for *different* credit exposures, one may suppose that the latent normal random variables driving transitions for each exposure are correlated. JP Morgan (1997) suggest that one employ the correlation coefficients of obligors' equity returns.[6] For the simulations we perform below, we experiment with different correlation coefficients in the range suggested by the examples in JP Morgan (1997).

Knowing the distribution of ratings at the end of a given period does not, of course, imply that one knows the distribution of values. The additional information required[7] is the distribution of spreads for different ratings.

JP Morgan (1997) suggest that one use currently observed forward spreads. This amounts

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[6] In fact, rather than employing actual equity return correlations, JP Morgan (1997) suggest one construct pseudo-equity returns based on weighted averages of national and sector equity indices where the weights are selected judgmentally. This has the advantage of allowing one to deduce correlations even when the obligor does not have quoted equity.

[7] Here, we are abstracting from changes in the gap between idiosyncratic spreads and the average spreads of exposures with the same rating.

to assuming that the evolution of average credit spreads for different ratings categories is deterministic. In the present study, we allow explicitly for stochastically changing spreads and in this regard our approach generalises

JP Morgan's Creditmetrics.

2.2 Credit risk measures and market risk

Let $S_{jt}(k)$ denote the jth credit spread with a maturity k at time t. Below, we omit the dependence on maturity, k, where this does not create ambiguity. The value of a k-maturity pure discount bond rated i at time t, denoted $P_{it}(k)$, may be written as:

$$P_{it}(k) = \exp\left[-(R_t(k) + S_{it}(k))k\right] \tag{1}$$

where $R_t(k)$ is the default-free k-maturity interest rate at time t. For rating N, ie the default state, we define the spread to be $S_{Nt} \equiv -\log(\xi_t)/k$ where ξ_t is the recovery rate in the event of default.

Our interest is in the distributions of changes in individual $P_{it}(k)$'s and changes in weighted sums of bond prices with different i's and k's (ie changes in portfolio values). To simplify matters, we assume zero correlations between transition and spread risks. It might be interesting to incorporate correlation. One can see in our framework how such correlations could be added by allowing the latent variables driving transitions to be correlated with spreads and then solving the model using Monte Carlos. But adding spread risk is in itself a significant step forward. Furthermore, parameterising the correlation between ratings transitions and spread changes in a convincing manner based on data would be difficult to achieve. We therefore prefer to leave the modelling of spread-transition risk correlations for future research.

We further suppose that future interest rates, $R_t(k)$, are non-stochastic. In their modelling of credit risk, JP Morgan (1997) and the specialist, credit risk consulting firm KMV follow the same approach, replacing future interest rates with the currently-observed forward rates observed at time 0. While this approach is obviously counter-factual, it may be justified in practice since, in determining appropriate capital levels, banks often calculate the VaR for their market risk separately from the VaR for

their credit risks then sum the two. Hence, supposing interest rates are non-stochastic in the credit risk calculation does not imply that the bank is ignoring this risk.

Indeed, calculating the VaRs for market and credit risks separately may be a sensible, conservative approach. If two risks are perfectly correlated, the volatility on their sum equals the sum of the two volatilities on the individual risks. If risks are normally distributed, then VaRs are proportional to volatilities and hence the VaR for two perfectly correlated risks combined equals the sum of their respective VaRs. Knowledge of the true correlation between market and credit risks is still rudimentary. Duffee (1999) suggests that spreads and interest rates are negatively correlated, but Morris, Neal and Rolph (1999) find positive correlations for longer holding periods. Hence, supposing risks are perfectly correlated, or equivalently (when returns are normal) that the VaRs must be calculated separately and then summed, seems sensible.

To calculate bond VaR's and volatilities, we may, therefore, work with the distribution of $\exp\left[-S_{it}k\right]$.[8] Let $\psi_{jt}(S|S_0)$ be the marginal density for S_{jt} conditional on the current vector of credit spreads, $S_0 \equiv (S_{10}, S_{20}, \ldots, S_{N0})'$. Given our assumption that transitions and spread changes are independently distributed, the conditional density of the spread one year ahead for an exposure which is initially rated i is:

$$\Psi_{i1}(S|S_0) \quad = \quad \sum_{j=1}^{N} \pi_{ij}\psi_{j1}(S|S_0) \tag{2}$$

The conditional density of $\exp\left[-S_{i1}k\right]$, which we need for calculating individual bond VaRs and volatilities, may then be obtained by an obvious change of variable.

The VaRs and volatilities for portfolios are more difficult to obtain than the corresponding quantities for spreads or pure discount bond prices since they depend on the joint distribution of transitions for the different exposures in the portfolio and the joint conditional distribution of future spreads. Below, we will show that the joint distribution of spread changes is approximately normal. We employ Monte Carlos to obtain the VaRs and volatilities, simulating the model described above for multiple exposures. The steps involved in these Monte Carlos are:

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[8]Using $\exp\left[-S_{it}k\right]$ rather than $\exp\left[-(R_t + S_{it})k\right]$ makes no difference if the R_t are non-stochastic since the volatility and VaRs we calculate are divided by mean values and we only consider portfolios in which all the individual exposures have the same maturity.

(i) Simulate the correlated latent normals which determine the ratings transitions of the different exposures in the portfolio M times.

(ii) For each realisation, $m = 1, 2, \ldots, M$, of the latent normals, calculate the different numbers of units of exposure which are in each of the $n = 1, 2, \ldots, N$ ratings categories, denoted a_{nm}. We employ $M = 40,000$ in our calculations.[9]

(iii) Simulate $l = 1, 2, \ldots, L$ realisations of the correlated spread variables and evaluate:

$$\sum_{n=1}^{N} a_{nm} \exp\left[-S_{itl}k\right] \quad . \tag{3}$$

for $l = 1, 2, \ldots, L$ and $m = 1, 2, \ldots, M$. We set $L = 5,000$ for all our VaR calculations.

(iv) Order the realisations of the expression in equation **(3)** to form a Monte Carlo estimate of the distribution function of the portfolio value.

(v) Use this distribution function to obtain moments of the portfolio value and quantiles for VaR calculations.

For these calculations, we require: (i) a ratings transition matrix, (ii) an assumption about the correlation coefficients of the latent variables driving transitions, and (iii) estimates of the joint distribution of future spreads. In the next section, we focus on the estimation of the joint distribution of spreads.

3   Spread risk

3.1   Descriptive statistics

In estimating spread distributions, we employ daily Bloomberg spread data covering the period April 1991 to November 1998.[10] The spreads equal yields on notional zero

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[9]We checked the sensitivity of the results to this value by calculating jackknife standard errors of the Monte Carlo estimates and concluded that the results were accurate up to the two decimal places reported in the tables.

[10]Bloomberg calculate yield curves for different ratings categories by applying cubic splines on a daily basis to large cross-sections of similarly-rated corporate bonds. Call and convertibility features are allowed for using approximate adjustments for the implied option values.

coupon bond with different credit ratings and maturities issued by US industrials minus the yields of US Treasury strips of the same maturity. The Moody's ratings for which data was available were AAA, AA12, AA3, A1, A2, A3, BBB1, BBB2, BBB3, BB1, BB2, BB3, B1, B2, and B3. We focused on AAA, AA12, A2, BBB2, BB2, and B2, taking these to be representative of spreads for the coarser, non-numbered ratings categories AAA, AA, A, BBB, BB, and B. For each ratings category, we employed credit spreads for maturities of 2, 5 and 10 years. Our data series comprise 1,640 daily observations.

Descriptive statistics for daily yields over Treasury on 2, 5 and 10-year maturity zeros of the coarser ratings categories are shown in Table A. The right-hand columns show annualized, one-day spread volatilities. At first sight, it is remarkable that the volatilities vary so little across different ratings categories, especially for investment grade bonds (BBB and above). As we shall see below, this finding reflects the fact that high-quality spreads exhibit considerable mean-reverting volatility (presumably reflecting liquidity shocks) which disappears over longer investment horizons.[11]

The left-hand columns in Table A show sample means of the levels of different spreads. The term structure of credit spreads evident in these means is, on average, upward-sloping. This is consistent with remarks in Litterman and Iben (1991) although inconsistent with the findings of Sarig and Warga (1989) who examined spreads for US corporates over the period February 1985 to September 1987. (They found that high and low-quality spread curves were respectively upward and downward-sloping and argued that this agreed with the predictions of Merton (1974) and Pitts and Selby (1983).)

3.2   Spread volatility and investment horizon

Our main interest is in credit risk over long periods such as one year or more. Sample volatilities calculated from daily data tell one little about relative riskiness over long horizons. To see this, consider return volatility over different horizons when spreads are generated by either (i) a random walk or (ii) a mean-reverting process. A typical random walk process commonly employed in finance applications is a Brownian motion with drift,

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[11]The relatively slight dependence of daily volatilities on maturity does not, of course, imply that the return volatilities of the corresponding zero bonds are insensitive to maturity since bond prices depend on spreads *multiplied by* maturity (see equation (**1**)).

X_t. The process and its conditional variance are

$$dX_\tau = \mu d\tau + \sigma dW_\tau \qquad \text{and} \qquad \text{Var}_0(X_t) = \sigma^2 t \qquad (4)$$

So volatility increases in proportion to the square root of time. In contrast, consider mean reverting processes of which an example commonly used in finance is the Ornstein-Uhlenbeck process. Such a process and its conditional variance are

$$dY_\tau = \alpha(\theta - Y_\tau)d\tau + \sigma dW_\tau \qquad \text{and} \qquad \text{Var}_0(Y_t) = \frac{\sigma^2}{2\alpha}(1 - \exp[-2\alpha t]) \qquad (5)$$

Thus, the conditional variance of this process approaches a constant as the horizon, t, increases.

In general, one might expect that a credit spread will be neither a pure random walk nor a stationary series, but some combination of the two.[12] To illustrate this, in Figure 1, we provide measures of the conditional volatility of changes in 5-year, AAA-rated discount bond spreads against Treasury strips. The top and bottom lines in the figure show the conditional volatilities obtained by fitting Brownian motion and Ornstein-Uhlenbeck models to daily data on credit spread changes using Maximum Likelihood techniques. The middle line shows non-parametric volatility estimates for different time horizons calculated using overlapping observations.[13]

The fact that the non-parametric volatility estimates lie for most investment horizons between the volatilities implied by the random walk (Brownian motion) model and the mean-reverting (Ornstein-Uhlenbeck) model suggests that the true AAA spread process is first-difference stationary. Although it is not completely obvious from the plot, it is plausible at least that the non-parametric volatility estimates settle down to a line that is proportional to the square root of time, which is what one would expect if the unit root component came to dominate for long investment horizons.

3.3 Unit root and stationarity tests

As the last subsection demonstrated, the presence or otherwise of unit roots in our daily or weekly spread series has important implications for their long-run statistical properties.

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[12]An alternative to our assumption that spreads are driven by sums of stationary and random walk components is to suppose they are fractionally integrated processes. See, for example, Lo (1991).
[13]We discuss below how these are calculated.

We, therefore, performed unit root tests on 2, 5 and 10-year maturities for ratings categories AAA, A12, A2, BBB2, BB2, and B2, and, additionally, on 0.25, 0.5, 1, 4, 7, and 20-year maturities for the investment grade ratings categories. Augmented Dickey-Fuller and Phillips-Perron tests did not reject (at a 10% level) the presence of units roots in all the maturities and ratings categories we examined when we used either daily or weekly data. For some cases, notably higher-rated bond spreads of around 5 years maturity, the differences in the rejection levels were marginal.

We also carried out the stationarity test proposed by Kwiatkowski, Phillips, Schmidt and Shin (1992). These led to a 5% rejection of stationarity except for a single series, the 5-year maturity A12 spread. The overall picture suggested by the tests is that the series include unit roots but that a fraction of the volatility is mean-reverting. For the first differences of the spreads we obtain the opposite result: every series rejects the unit root hypothesis at a 1% level in Augmented Dickey-Fuller and Phillips-Perron tests, while the KPSS-test doesn't reject at the 10% level. Therefore we conclude that the series of daily spreads is first-difference stationary. This is in line with the findings of Pedrosa and Roll (1998) who report a sole rejection (at the 10% level) out of 60 comparable spread time series.

3.4 Spread volatility and small-sample adjustment

Although we are interested in the joint distribution of spreads, we initially focus on the term structure of spread volatility, defined as

$$\sigma^2(k) \equiv \operatorname{Var}_n(S_{n+k} - S_n) \tag{6}$$

where, for simplicity, we omit the subscript for rating. To see how one may estimate this term structure and what the properties of the estimator are likely to be, consider a discrete time spread process S_i for $i = 0, 1, \ldots$. Suppose that the first difference in S_i equals

$$S_i - S_{i-1} = \mu + B(L)\epsilon_i \tag{7}$$

where $B(L)$ is the lag operator, $\text{Var}(\epsilon_i) = \sigma_\epsilon^2$ and $\text{Cov}(\epsilon_i, \epsilon_{i-j}) = \rho_j \sigma_\epsilon^2$ for all i and j. It
follows that for any positive integer k

$$\sigma^2(k) \;\; = \;\; k \left(1 + 2 \sum_{j=1}^{k-1} \frac{k-j}{k} \rho_j \right) \sigma_\epsilon^2 \tag{8}$$

Taking a limit as $k \to \infty$ yields

$$\sigma^2(\infty) \;\; \equiv \;\; \lim_{k \to \infty} \sigma^2(k)/k \;\; = \;\; \left(1 + 2 \sum_{j=1}^{\infty} \rho_j \right) \sigma_\epsilon^2 \tag{9}$$

which, in fact, is the spectral density of ΔS_n at frequency zero. To estimate $\sigma^2(k)$, we
may either substitute sample autocorrelation coefficients, $\hat{\rho}_j$, wherever the population
equivalents appear in equation (**8**), or we may calculate a scaled sample variance using
overlapping observations:

$$\hat{\sigma}^2(k) \;\; = \;\; \frac{1}{T-k} \sum_{n=k}^{T} \left(S_n - S_{n-k} - \frac{k}{T}(S_T - S_0) \right)^2 \tag{10}$$

where T is the number of observations in the sample. In large samples, these two
estimators are equivalent but in small samples they will differ and may exhibit significant
bias.

To adjust for the small-sample bias, we follow the approach of Cochrane (1988).[14] His
approach works as follows. Beveridge and Nelson (1981) show that any first-difference
stationary process like S_n may be decomposed into the sum of a unit root component Z_n
and a stationary process C_n. As Cochrane (1988) notes, the variance of innovations in the
unit root component Z_n is the same as the spectral density of ΔS_n at frequency zero
which, as we have already indicated, equals $\sigma^2(\infty)$. In other words, the long-run variance
of spreads divided by the horizon, k, approaches that of the unit root component of the
series.

To devise a theoretical small-sample adjustment that will be approximately valid for large
k, one may therefore calculate the bias one would obtain if one calculated a sample
variance with overlapping observations as in equation (**10**) *in the case of a pure unit root
process.* Cochrane (1988) derives a bias-adjusted estimator for $\sigma^2(k)$, denoted $\hat{\sigma}^2(k)$, using

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[14] Note that Cochrane's interest was in second moments. We extend his approach to third and fourth
moments.

this method. His bias-adjusted estimator is simply the estimator given in equation ($\mathbf{10}$) multiplied by $T/(T - k + 1)$ where there are $T + 1$ observations in the sample and k is the horizon.

3.5 The term structure of spread volatility

Figure 2 shows the term structure of volatility risk (ie $\hat{\sigma}^2(k)$ plotted against maturity) for four different horizons, 1, 5, 125 and 250 days. To estimate the volatility, for example, for a 4-year A-rated spread, we (i) multiply the 4-year maturity spread by 4 and then (ii) calculate the standard deviation based on overlapping observations as described above (ie as in equation ($\mathbf{10}$) multiplied by $T/(T - k + 1)$).[15] To ensure that volatilities are comparable for the different investment horizons, we express them in annualized terms, ie multiplying the 1, 5, 125 and 250-day investment horizon volatilities by the square root of 250, 50, 2 and 1. Finally, we multiplied the volatilities by 100 so they are expressed as percentages.

For the shortest investment horizon of 1 day, the investment grade spread volatilities for AAA, AA, A, and BBB ratings categories are strikingly similar. For example, for a maturity of 5 years, the different investment grade volatilities vary by only a handful of basis points. Sub-investment grade spread volatilities are somewhat greater, suggesting that there is a distinct increase in the relative degree of spread risk when one descends into sub-investment grade exposures. As one extends the investment horizon towards 250 working days (ie to one year), however, the relative volatilities of spreads for different ratings becomes much greater.

The reason is that mean-reverting components contribute a larger proportion of the total volatility of the higher credit quality spreads than of low credit quality spreads. As the investment horizon increases, the mean-reverting components in the high credit quality spreads die out so total volatility grows much less than in proportion to the length of the investment horizon. On the other hand, low credit quality spreads look like random walks even for short investment horizons, so, as the investment horizon increases, volatility rises

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[15]Multiplying by the maturity (4 in our example) is appropriate because we are ultimately interested in the riskiness of bond values. The fact that we multiply spreads by maturity explains why in Figure 2 volatilities are so strongly upward-sloping in maturity.

proportionately.

The economic implication is that spread volatility varies quite considerably across different ratings categories for the holding periods that are of interest for credit risk modelling purposes, ie one year or more. Volatilities for BBB spreads are more than double those for AAA spreads for 10-year maturity exposures held over 1 year. The sub-investment grade exposure volatilities diverge even more from their investment grade equivalents for 1-year investment horizons than they do for short investment horizons of 1 or 5 days.

Another way to study these effects is to calculate the ratio of the variance of spread changes over a investment horizon of k days to that over a 1-day investment horizon multiplied by k.[16] If the spreads were random walks, the ratio should be unity. Figure 3 shows such ratios calculated for 5-year maturity spreads and plotted against the number of holding-period days, k (starting at k equal to 10). The ratios for the AAA and AA spread changes are clearly downward-sloping for short holding periods as the impact of the mean-reverting component quickly dies off. In contrast, the ratios for BB and B are sharply upward-sloping for short maturities, flattening out for k greater than 60 or so. If the variance ratio flattens out, this indicates that the unit root component has come to dominate the total volatility.

Table B shows volatility estimates for changes in 5-year maturity spreads over a 1-year horizon estimated using overlapping observations as described above and adjusted using Cochrane-style small-sample adjustments. Volatility shows a strong negative dependence upon credit quality. We provide asymptotic estimates of standard errors for the volatilities in parentheses.[17] These standard errors are very conservative. The reason is that if one is willing to assume that the unit root component has come to dominate earlier (say at four months) so that $\sigma^2(k)/(\sigma^2(1) \cdot k)$ has settled down to a constant, the asymptotic standard error would be based on the shorter period rather than one year. This would yield a standard error around a half as large.

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[16]The small-sample adjustments for ratios of sample variances calculated with different numbers of overlapping observations is more complicated than simply for a sample variance alone. The reason is that the numerator and denominator in the ratio are likely to be correlated. Cochrane (1988) ignores the problem despite the fact that much of his discussion concerns such variance ratios. We discuss this issue further in Section 3.6.

[17]To calculate these, we use the fact that the asymptotic variance of $\sigma^2(\infty)$ is $4k\sigma^4(\infty)/(3T)$ (see Anderson (1971)).

The table also includes estimates of correlation coefficients for the different 1-year spread changes. It is apparent that the spread changes are very closely correlated especially for spreads from adjacent ratings categories. This correlation is important since it will introduce a source of correlation between different exposures over and above correlations in ratings transitions when we come to calculate risk measures on portfolios below.

3.6   Higher moments and spread normality

We now turn to the distribution of spread changes for long horizons. Since spread changes probably exhibit considerable independence over time, one might expect that the distribution of $S_{n+k} - S_n$ would converge to a normal distribution for large k.[18] To assess this, we calculate skewness and kurtosis coefficients for changes in 5-year spreads for different ratings categories and over a range of horizons, k.

Again, since we used overlapping observations to calculate the skewness and kurtosis, small-sample biases were an issue. Relying once more on the fact that for large k the spread process may be approximated by random walk, we calculated bias corrections for estimators of the third and fourth central moments of the spread changes, namely the expression in equation (**10**) except with the power 2 replaced by 3 and 4 respectively. Using methods similar to Cochrane, we derive in the Appendix small-sample bias adjustments for the third and fourth central moments.

A complication (already referred to in footnote 15) is that the skewness and kurtosis coefficients represent *ratios* of central moments. Possible correlation between the numerators and denominators in these ratios may induce small-sample biases that are not eliminated by Cochrane-style random-walk adjustments. Deriving a small-sample adjustment that allows for the correlation is difficult analytically. We therefore calculated the additional bias stemming from the 'ratio effects' using Monte Carlos. The values discussed below are adjusted in this additional way for ratio effects.

Figures 4 and 5 present estimates of skewness and kurtosis coefficients for the 5-year

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[18]Given that spreads are first-difference stationary (see Section 3.3) standard results from time series analysis, eg chapter 6 of Fuller (1996), imply the validity of a version of the Central Limit Theorem for $S_{n+k} - S_n$ under appropriate moment conditions on the innovations.

maturity spread changes for different ratings categories, plotted against length of investment horizon.[19] The issue here is the degree to which the distributions of spread changes appear to converge to normal distributions. An indication of this is the extent to which kurtosis approaches three and skewness converges to zero. Figure 5 suggests that the very high kurtosis evident in spread changes over short periods substantially dies off as the period lengthens. For spread changes over 1 year, the kurtosis of spread changes for all six ratings categories that we consider is close to or just over 3, the value kurtosis takes for a normally distributed random variable. On the other hand, the skewness coefficients for spread changes appear to turn from positive to negative as the length of the period increases.

Using Monte Carlos, we calculated 95% confidence intervals for skewness and kurtosis estimates for a 1-year horizon with overlapping observations assuming that the true process was a random walk with drift and based on the same number of observations as in our sample. For kurtosis, the confidence interval was 2.45 to 5.16, while for skewness it was -1.40 to 1.40. This suggests that the estimated kurtosis and skewness for a 1-year horizon were within reasonable sampling errors of the levels appropriate for normally distributed processes.

4 Risk measures

4.1 Distributional assumptions

The results of the previous section imply it is reasonable to suppose that the distribution of spreads over a 1-year horizon, S_{jt} for $j = 1, 2, \ldots, N-1$, is approximately joint normal with variances and covariances based on our small-sample-adjusted estimates. We assume that the distribution of the 'spread' in the default state, $S_{Nt} \equiv -\log(\xi_t)/k$, is such that the recovery rate, ξ_t, is beta distributed. This is consistent with the assumption of JP Morgan (1997). We select the parameters of the beta distribution for ξ_t so that recoveries have a mean and standard deviation of 51.13% and 25.45%. These were the moments estimated by Carty and Lieberman (1996) in their study of recoveries on Moody's rated

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[19]The skewness and kurtosis plots are adjusted for small-sample bias using Monte Carlos.

defaulting senior unsecured bonds.[20] Finally, we suppose that recovery-rate risk is independent across individual exposures and independent of other sources of risk. This is the standard approach in credit risk modelling although it might be questioned.[21]

We experimented with different transition matrices. The baseline matrix we employed was the transition matrix for US industrials estimated by Nickell, Perraudin and Varotto (2000) using senior, unsecured ratings for all Moody's-rated obligors from January 1970 to December 1997, see Table C. This matrix is consistent with those reported in Carty and Fons (1993) and Carty (1997) except for our use of more up-to-date data and the restriction to industrials.[22] It made sense to employ transition matrices for industrials since the Bloomberg spread data we employed was for that sector.[23]

Rather than employing a transition matrix based simply on the historical experience of ratings transitions, Creditmetrics recommend that one use matrices for which the long-run behaviour accords with the long-run, steady-state distribution of ratings. The lower part of Table C contains a matrix of this type in which transition probabilities have been adjusted to fit a given set of steady-state probabilities recommended by the Creditmetrics manual. The Creditmetrics matrix differs significantly from our baseline transition matrix estimated from Moody's data. It implies higher volatility especially for lower-rated obligors and has small probabilities of very large rating category changes for highly-rated bond issuers.

4.2   Single rating grade VaRs

In Table D, we report single rating grade, 1-year holding-period VaRs for portfolios of 500 exposures, each having the same initial rating. Each exposure in the portfolio consists of a 5-year maturity,[24] pure discount bond so its price has the form $\exp[-S_{jt}(k)k]$ multiplied

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[20]Carey (1998) provides some evidence that recoveries on private placements is higher at about 60%.

[21]In a credit-derivative pricing model, Das and Tufano (1996) suppose that interest-rate and recovery-rate risk are correlated.

[22]It is also very similar to the one based on Moody's data, which appears in Chapter 6 of the Creditmetrics manual.

[23]We do not address the issue within this study of the stability of ratings transition matrices, either in the time series dimension looked at by Blume, Lim and MacKinlay (1998) or the cross-sectional stability studied by Nickell, Perraudin and Varotto (2000).

[24]When we calculate a VaR for say an n-year maturity bond, we mean that the bond will have a maturity of n years at the end of the 1-year horizon of the VaR calculation.

by the price of a default-free pure discount bond.[25] We are only interested in the credit risk component of total risk, so we only consider changes in the spread factor $\exp[-S_{jt}(k)k]$ in our VaR calculation.

The baseline transition matrix employed in the VaR calculations is the matrix for US industrials estimated by Nickell, Perraudin and Varotto (2000). This matrix differs from those reported by Moody's themselves in, for example, Carty (1997), only because of differences in sample period. We report VaRs for 1% and 0.3% confidence levels. Many banks use confidence levels like 0.3% (or even smaller) but 1% VaR calculations are more statistically reliable since only 30 years of data are used in the estimation of ratings transition matrices.

We also calculate VaRs using a transition matrix suggested by the Creditmetrics manual which is designed to capture the steady-state behaviour of ratings changes (ie the distribution of ratings changes over long horizons). This matrix has larger probabilities of substantial ratings changes (including transitions into default) for highly-rated exposures.[26] Throughout, we suppose that the correlation coefficient for the credit-quality latent variables, ρ, equals 0.2.

We calculate VaRs in three ways: (i) default-mode (labelled 'D'), (ii) mark-to-market mode allowing for ratings transition and recovery-rate risk (labeled 'D+T'), and (iii) mark-to-market mode allowing for ratings transition, recovery rate and spread risk (labelled 'D+T+S'). Default-mode VaRs (D) effectively only count losses in value associated with exposures that actually default. Mark-to-market mode calculations (D+T) correspond to standard implementations of Creditmetrics.[27] In all cases, the VaRs are scaled by the initial value of the portfolio and multiplied by 100. This scaling means that they are of the same order as standard percentage capital charges like those

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[25] We choose a 5-year maturity since this is the (value-weighted) average maturity for one bank with which we have had discussions. The effective maturity of a credit may be longer than the contractually-specified one, since banks are often reluctant to raise their lending rates when a particular customer's credit standing deteriorates for fear of spoiling a long-term banking relationship. Hence, 5 years may be a reasonable assumption even when the average maturity of a bank's book is shorter.

[26] In the Moody's data employed by Nickell, Perraudin and Varotto (2000) which comprised all non-municipals rated by Moody's in the period January 1970 to December 1997, not a single obligor rated A or above defaulted within a one-year horizon; but the Creditmetrics transition matrix includes a significant default probability even for AAA-rated borrowers.

[27] When we omit spread risk, we set spreads equal to the sample averages of the Bloomberg data.

suggested by Basel Committee on Banking Supervision (1999).

As expected, the single rating grade VaRs reported in Table D increase sharply as credit quality declines from AAA to CCC. The rate of increase is greatest for the default-mode VaRs (D) and least for the mark-to-market VaRs with spread risk (D+T+S). In general, D+T+S VaRs exceed those for D+T which in turn exceed those for D. The differences are greatest for high credit quality exposures and largely disappear for CCC-rated exposures since default risk dominates for low credit quality exposures.

The degree to which spread risk increases VaRs especially for high credit quality bonds is striking. For AA and A-rated exposures, VaRs with spread risk are three times higher when spread risk is included (and the calculation is performed using the Moody's data transition matrix). The increase is less considerable if the calculations employ the Creditmetrics transition matrix because, for high-rated bonds, the transition risk is much greater.

One may compare our results with those of Carey (2000). For single-grade portfolios comprising BBB, BB, and B-rated exposures, his baseline 1% VaRs (equal to the difference between the 99% loss percentile and the means in his Table C) equal 0.9%, 2.4% and 5.3%. These may be compared with our default-mode VaRs for BBB, BB and B portfolios of 0.5%, 3.7%, and 8.6% respectively. The fact that our results are slightly higher for BB and B (although not for BBB) probably reflects our more conservative assumptions about loss given default.

An interesting issue is whether credit risk varies across industries. In particular, are exposures to financials including banks more or less risky than those to industrials?[28] The upper part of Table E contains VaRs for single rating grade portfolios calculated using the ratings transition matrix for banks and other financials estimated by Nickell, Perraudin and Varotto (2000).

For high credit quality portfolios, the bank-other-financial VaRs in the upper part of Table E are very similar to the corresponding VaRs for industrials in Table D. When

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[28]It is often presumed that banks of a given rating are less risky counterparties than industrials. This is why they receive favourable treatment in the Basel Accord and in the capital weight proposals in Basel Committee on Banking Supervision (1999).

credit quality is low, however, the VaRs based on a bank and other financial transition matrix are higher. This reflects the fact, noted by Nickell, Perraudin and Varotto (2000), that when a bank's credit standing declines below BBB, the likelihood that it will survive is much smaller than for a comparably-rated industrial.

Another important issue is the sensitivity of our results to assumptions about recovery rates. Carey (1998) argues that recoveries on private placements (which in some ways resemble loans) is higher than those on bonds. The lower part of Table E reports VaRs assuming a mean recovery rate of 60% rather than the 51% rate used in all our other VaR calculations. The VaRs with the higher recovery rates are broadly unchanged from those reported in Table D for investment quality (BBB and above) ratings categories. For very low quality credits, the VaRs with the higher recovery rate are about a third lower in the B and CCC categories.

Lastly, it is important to consider the sensitivity of our calculations to the degree of diversification. Table F contains 0.3% VaRs for single rating grade portfolios satisfying our baseline assumptions (five-year maturity, industrial transition matrix, spread and transition risks included), but containing 5, 10, 50, and 100 equal-sized exposures rather than the 500 exposures assumed in the VaRs of Tables D and E. The most striking feature of the results in Table F is that, in the case of low credit quality portfolios, the VaRs only become substantially larger than those in Table D when the number of exposures drops to 50 or below. This suggests that diversification begins to influence VaR calculations rather early as the number of exposures in the portfolio increases. In part, this reflects the fact that spread risk, which dominates for high credit quality exposures, is highly correlated.

4.3   Value at Risk for actual bank portfolios

Single rating grade VaRs reveal much about the structure of credit risk, but it is also important to consider portfolios like those of actual banks made up of differently-rated exposures. Table G reports 1% and 0.3% VaRs for three such realistic portfolios under different assumptions about the correlation between the latent variables that generate ratings transitions for different obligors. The ratings distributions of the three portfolios (each of which contains 500 equal-sized exposures) are based on data in Gordy (2000).

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The 'average quality portfolio' has the same ratings distribution as the average of a large sample of US banks surveyed by the Federal Reserve Board (see Gordy (2000)). The 'high-quality' portfolio has the average distribution of a subset of banks surveyed which had relatively low-risk portfolios. The 'investment quality' portfolio has the same shares invested in different ratings categories as that part of the average portfolio which was rated BBB or above.[29]

We calculate VaRs under different assumptions about the correlation coefficient of the latent variables, denoted ρ, namely that ρ equals 0.1, 0.2, or 0.3. The Creditmetrics manual reports a matrix of typical correlation coefficients for a sample of bonds. The average of the off-diagonal entries in their correlation matrix is 0.198, to some extent justifying our choice of 0.2 as a baseline value.

The portfolio VaRs in Table G again suggest that spread risk is important, especially when the correlation parameter ρ is relatively small. In this case, transition risk is diversified away in large portfolios but spread risk is correlated across different exposures and hence is not diversified. Even when $\rho = 0.2$, the high-quality portfolio has a VaR which is a third higher when spread risk is included.

For the average quality portfolio, the VaRs are about a fifth higher with spread risk for $\rho = 0.2$ and a third higher for $\rho = 0.1$. These and other results appear less sensitive to the choice of transition matrix when one calculates portfolio VaRs rather than VaRs for individual bond values. It is noticeable that, with the baseline correlation coefficient of $\rho = 0.2$, the average portfolio VaR (for our assumed confidence levels) is between 5.5% and 7%, only a little lower than the 8% level built into the 1988 Basel Accord. Using the Creditmetrics transition matrix, the figures are slightly higher at 6% to 7.5%.

Again, one may compare the default-mode VaRs reported in Table G with the results of Carey (2000) since he calculates quantiles of the loss distribution for the same average quality portfolio. The 1% VaR implied by the results in his Table 2 is 2.1%. Once again,

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[29]More specifically, the average portfolio contains 15 AAA, 25 AA, 67 A, 156 BBB, 162 BB, 56 B, and 20 CCC-rated credits each of the same face value and the same maturity of 5 years. The high-quality portfolio which is another portfolio considered by Gordy (2000) contains 19 AAA, 30 AA, 146 A, 190 BBB, 95 BB, 13 B, and 6 CCC-rated credits. The investment quality portfolio contains 28 AAA, 48 AA, 128 A, 297 BBB-rated credits.

this is probably slightly lower than our baseline ($\rho = 0.2$) default-mode VaR for the average portfolio of 3.2% because of our more conservative loss given default assumptions.

4.4 The effects of maturity

A dimension of credit risk that one might expect would be important is maturity. Table H shows 0.3% VaRs calculated on a default-mode basis (D) and on a mark-to-market basis with and without spread risk (D+T+S and D+T, respectively) for our three portfolios and for different maturities. The holding period for the VaR in all cases is 1 year and the maturities of the portfolios at the end of that year are 2, 5 and 10 years.

At short maturities spread risk matters very little, reflecting the fact that spreads are multiplied by maturity when they enter bond pricing formulae. On the other hand, recovery rates are independent of maturity, so these come to matter substantially as maturity shrinks. These observations also explain why high-quality debt shows a much more marked dependence on maturity than the average portfolio which contains a substantial proportion of non investment grade exposures.

4.5 Sensitivity analysis

Given the multi-stage nature of the estimations and calculations we perform in this paper, it is difficult to put standard errors on the VaRs we report. (To do so would involve repeated simulation of computationally demanding Monte Carlo estimates.) In Table I, we therefore perform a sensitivity analysis, perturbing the estimates employed in our calculations by reasonable amounts and then seeing how this affects VaRs. We are particularly interested in the robustness of our finding that spread risk is important especially for high credit quality portfolios. Hence, in Table I, we report portfolio VaRs under the assumption that all the spread volatilities and covariances are reduced by one standard deviation. The 1% VaR for the average portfolio falls from 5.6% to 5.1% while that for the investment quality portfolio drops from 2.8% to 2.2%. We conclude from this that our results are reasonably robust to changes in the spread volatility estimates.

5 Conclusion

This paper attempts to quantify the riskiness of different kinds of credit exposure, looking both at credit quality and maturity dimensions. We also examine the composition of credit risk, in particular the relative importance of risks associated with ratings transitions, recovery rates and changes in spreads for different ratings categories.

Our conclusions are, first, that spread risk is important for relatively high-quality debt, and models such as Creditmetrics that assume zero spread risk may underestimate the riskiness of highly-rated portfolios.

Second, under reasonable assumptions about correlations of ratings transitions, the total VaRs we obtain including spread risk are of the order of 7% for a portfolio of 5-year maturity exposures with a credit quality distribution equal to the average of a large sample of US banks. It is interesting that this is close to the 8% required by the 1988 Basel Accord. Similar investment quality portfolios, on the other hand, yield VaRs of the order of 3%-4%.

Third, the dependence of VaRs on maturity depends very much on whether the exposures are low or high credit quality. Low credit quality exposures are quite insensitive to maturity because recovery risk is a substantial fraction of total risk. For high credit quality exposures, spread risk is more important and this leads to a strong positive dependence on maturity since spreads are scaled up by maturity when they enter bond values expressions.

An important question is to what degree are our conclusions applicable to bank loan books as well as to bond portfolios?[30] Typically, banks adjust their lending rates quite infrequently so some of our conclusions about the importance of spread risk might appear to be less relevant for loan portfolios. Furthermore, the impact of fluctuations in value due to liquidity shocks might affect bond portfolios differently from portfolios of loans.

In our view, the smoothness of loan spreads reflects the fact that banks are conducting long-term relationships with borrowers and not any corresponding smoothness in the

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[30]Carey (1995) considers the consistency of bond and loan pricing while Altman and Suggitt (2000) compare the transition probabilities of loans and bonds.

credit standing of borrowers. Furthermore, our procedures for estimating spread risk over long horizons specifically filters out high-frequency, mean-reverting components of risk such as liquidity shocks. It is plausible, therefore, to argue that a reasonable fraction of the spread risk we measure is applicable to loan as well as bond portfolios.

Table A: **Descriptive statistics for daily spread data**

	Means			Volatilities		
Maturity (years)	2	5	10	2	5	10
AAA	27.7	31.2	30.7	2.3	2.1	2.8
	(0.2)	(0.1)	(0.2)	(0.1)	(0.1)	(0.3)
AA	34.8	35.2	36.3	2.5	2.1	2.8
	(0.2)	(0.1)	(0.2)	(0.0)	(0.1)	(0.3)
A	46.4	54.1	58.7	2.3	2.2	2.8
	(0.4)	(0.3)	(0.3)	(0.2)	(0.1)	(0.3)
BBB	70.1	74.4	80.5	2.1	2.1	2.8
	(0.7)	(0.5)	(0.4)	(0.1)	(0.1)	(0.3)
BB	163.1	179.6	216.9	3.1	3.0	3.8
	(1.9)	(1.1)	(0.9)	(0.2)	(0.2)	(0.4)
B	307.0	334.5	350.9	3.8	4.0	4.3
	(2.2)	(1.5)	(1.1)	(0.3)	(0.3)	(0.4)

Note: Means are sample means of spread levels in basis
points. Volatilities are sample standard deviations of daily
spread changes in basis points. Newey-West standard
errors appear in parentheses.

Table B: **Five-year spread correlations at a one-year horizon**

	Volatilities					
	AAA	AA	A	BBB	BB	B
	0.41	0.45	0.95	1.27	2.42	4.77
	(0.09)	(0.10)	(0.21)	(0.27)	(0.53)	(1.05)
	Correlation matrix					
	AAA	AA	A	BBB	BB	B
AAA	1.00	0.75	0.63	0.67	0.51	0.53
AA	0.75	1.00	0.76	0.73	0.49	0.57
A	0.63	0.76	1.00	0.91	0.62	0.59
BBB	0.67	0.73	0.91	1.00	0.71	0.57
BB	0.51	0.49	0.62	0.71	1.00	0.62
B	0.53	0.57	0.59	0.57	0.62	1.00

Note: Asymptotic standard errors appear in brackets.
Volatilities are standard deviations of one-year changes
in spreads multiplied by maturity (5) and by 100.

Table C: **Transition matrices**

	AAA	AA	A	BBB	BB	B	CCC	D
Moody's data †								
AAA	91.60	7.80	0.70	0.00	0.00	0.00	0.00	0.00
AA	1.10	89.30	9.10	0.30	0.20	0.00	0.00	0.00
A	0.10	1.90	92.40	4.80	0.60	0.20	0.00	0.00
BBB	0.00	0.10	3.90	89.90	4.90	0.80	0.10	0.20
BB	0.00	0.10	0.40	3.40	87.00	7.40	0.20	1.50
B	0.00	0.10	0.20	0.50	6.20	84.00	2.30	6.80
CCC	0.00	0.00	0.00	0.60	1.90	7.30	73.10	17.00
D	0.00	0.00	0.00	0.00	0.00	0.00	0.00	100.00
Creditmetrics ‡								
AAA	87.74	10.93	0.45	0.63	0.12	0.10	0.02	0.02
AA	0.84	88.23	7.47	2.16	1.11	0.13	0.05	0.02
A	0.27	1.59	89.05	7.40	1.48	0.13	0.06	0.03
BBB	1.84	1.89	5.00	84.21	6.51	0.32	0.16	0.07
BB	0.08	2.91	3.29	5.53	74.68	8.05	4.14	1.32
B	0.21	0.36	9.25	8.29	2.31	63.89	10.13	5.58
CCC	0.06	0.25	1.85	2.06	12.34	24.86	39.97	18.60
D	0.00	0.00	0.00	0.00	0.00	0.00	0.00	100.00

Note: Transition matrices (in per cent) are for one year.

† Source: Nickell, Perraudin and Varotto (2000).

‡ Source: JP Morgan (1997).

Table D: **Single grade VaRs**

Using Moody's data transition matrix (Nickell, Perraudin and Varotto (2000))						
	1% VaR			0.3% VaR		
Rating	D+T+S	D+T	D	D+T+S	D+T	D
AAA	0.95	0.10	0.00	1.13	0.16	0.00
AA	1.20	0.40	0.00	1.52	0.57	0.00
A	2.31	0.67	0.00	2.74	1.03	0.00
BBB	3.65	2.42	0.95	4.68	3.54	1.69
BB	7.43	5.80	4.12	9.40	7.72	5.88
B	13.11	10.67	9.77	15.76	13.58	13.80
CCC	17.00	15.50	15.00	19.87	18.23	18.22
Using JP Morgan transition matrix (JP Morgan (1997))						
	1% VaR			0.3% VaR		
Rating	D+T+S	D+T	D	D+T+S	D+T	D
AAA	1.04	0.54	0.19	1.31	0.86	0.33
AA	1.58	1.09	0.18	2.16	1.58	0.33
A	2.47	1.12	0.26	3.03	1.66	0.45
BBB	3.46	2.01	0.45	4.30	2.81	0.79
BB	8.56	6.58	3.70	10.88	8.85	5.54
B	14.14	11.67	8.58	17.10	14.80	11.50
CCC	18.85	17.68	15.55	22.15	21.26	19.05

Notes: Portfolios consist of 500 exposures of equal face value.
VaRs are measured in per cent of the expected value. The correlation coefficient of the latent transitions, ρ, equals 0.2.
D indicates default-mode VaR calculation.
D+T corresponds to standard Creditmetrics.
D+T+S corresponds to Creditmetrics including spread risk.

Table E: **Single grade VaRs**

	1% VaR			0.3% VaR		
	Using bank transition matrix					
	(Nickell, Perraudin and Varotto (2000))					
Rating	D+T+S	D+T	D	D+T+S	D+T	D
AAA	0.94	0.10	0.00	1.12	0.13	0.00
AA	1.24	0.33	0.00	1.50	0.43	0.00
A	2.29	0.74	0.00	2.76	1.08	0.00
BBB	3.68	2.21	0.99	4.61	2.95	1.61
BB	9.25	8.08	4.19	11.90	10.83	5.86
B	12.21	9.02	9.85	14.72	11.52	12.95
CCC	21.88	21.81	17.51	24.05	24.01	20.53
	Using industrial transition matrix 60% recovery rate					
	(JP Morgan (1997))					
	1% VaR			0.3% VaR		
Rating	D+T+S	D+T	D	D+T+S	D+T	D
AAA	0.95	0.10	0.00	1.13	0.15	0.00
AA	1.24	0.41	0.00	1.50	0.59	0.00
A	2.27	0.68	0.00	2.72	0.98	0.00
BBB	3.57	2.18	0.76	4.52	3.23	1.28
BB	6.92	4.89	3.25	8.68	6.66	4.78
B	11.58	7.97	7.07	13.58	10.29	9.62
CCC	13.42	11.25	10.56	15.50	13.59	13.20

See notes to Table D.

Table F: **Diversification: 1% VaRs**

	Using Moody's data transition matrix			
	(Nickell, Perraudin and Varotto (2000))			
Rating	# Bonds			
	100.00	50.00	10.00	5.00
AAA	1.11	1.06	1.13	1.15
AA	1.46	1.53	1.67	2.08
A	2.86	2.82	3.15	3.80
BBB	4.91	5.15	8.88	13.38
BB	10.10	10.69	15.72	21.12
B	16.73	17.84	24.68	33.99
CCC	21.70	22.75	32.88	42.74

See notes to Table D.

Table G: **Portfolio VARs**

Portfolio	ρ	1% VaR			0.3% VaR		
		D+T+S	D+T	D	D+T+S	D+T	D
Average	0.10	4.69	2.69	1.95	5.65	3.51	2.68
	0.20	5.58	4.26	3.18	7.05	5.90	4.55
	0.30	6.76	5.80	4.33	9.05	8.47	6.25
High	0.10	3.31	1.63	1.03	3.98	2.20	1.37
	0.20	3.76	2.63	1.52	4.76	3.60	2.33
	0.30	4.41	3.58	2.11	6.10	5.33	3.13
Investment	0.10	2.55	1.01	0.41	3.04	1.43	0.60
	0.20	2.84	1.65	0.60	3.56	2.48	0.97
	0.30	3.17	2.27	0.81	4.31	3.49	1.51

Using industrial transition matrix (Nickell, Perraudin and Varotto (2000))

Portfolio	ρ	1% VaR			0.3% VaR		
		D+T+S	D+T	D	D+T+S	D+T	D
Average	0.10	4.98	3.14	1.79	6.03	4.07	2.36
	0.20	6.03	4.74	2.85	7.62	6.41	4.06
	0.30	7.27	6.29	3.89	9.66	8.86	5.51
High	0.10	3.42	1.82	0.87	4.13	2.36	1.18
	0.20	4.01	2.87	1.33	5.08	3.88	1.95
	0.30	4.70	3.74	1.81	6.27	5.32	2.85
Investment	0.10	2.60	0.98	0.27	3.14	1.32	0.40
	0.20	2.80	1.61	0.36	3.50	2.34	0.62
	0.30	3.19	2.19	0.43	4.22	3.15	0.87

Using Creditmetrics transition matrix (JP Morgan (1997))

Notes: Portfolios consist of 500 exposures of equal face value.

The ratings compositions of portfolios is given in the text.

VaRs are measured in per cent of the expected value.

D indicates default-mode VaR calculation.

D+T corresponds to standard Creditmetrics.

D+T+S corresponds to Creditmetrics including spread risk.

Table H: **Effect of maturity on 0.3% VaRs**

Portfolio	2yr bonds		5yr bonds		10yr bonds	
	D+T+S	D+T	D+T+S	D+T	D+T+S	D+T
Average quality	5.97	5.83	7.05	5.90	11.62	6.43
High quality	3.18	2.94	4.76	3.60	8.79	4.73
Investment quality	1.79	1.49	3.56	2.48	7.06	4.00

See notes to Table D. $\rho = 0.2$.

Table I: **Sensitivity analysis 1% VaR**

	Using Moody's data transition matrix (Nickell, Perraudin and Varotto (2000))		
Portfolio	D+T+S	D+T	D
Average	5.08	4.26	3.18
High	3.18	2.63	1.52
Investment quality	2.24	1.65	0.60

Notes: Calculations are same as in Table G except that
spread variances are reduced by one standard deviation.
The composition of the portfolios is explained in the text.
Simulations use transition matrix based on Moody's data and
$\rho = 0.2$. For definitions of D, D+T
and D+T+S, see notes to Table D.

Notes to Figures

Figure 1

The figure shows conditional volatilities for different horizons estimated on daily spread data assuming they follow (i) an Onstein-Uhlenbeck, (ii) an arithmetic Brownian motion with drift, and (iii) a discrete-time, first-difference stationary process. Maximum Likelihood estimation is employed in cases (i) and (ii) while in case (iii) the volatilities are estimated using a non-parametric technique described in the text.

Figure 2

Volatilities for different ratings are plotted against maturity for four different investment horizons. Volatilities are divided by the square root of the investment horizon and multiplied by 100 so they are in per cent. For the one-day horizon, investment grade volatilities (BBB2 and above) show almost no variation for a given maturity. Spreads for longer horizons vary substantially across ratings category for given maturities.

Figure 3

The ratio of estimated spread variances for different investment horizons (k) to k times the one-day variance is shown plotted against k. For a random walk, the (population) ratio equals unity. For first difference stationary series, the ratio converges asymtotically to a constant for large k.

Figure 4

The skewness coefficient (defined as the third central moment divided by the cubed standard deviation) is shown for spread changes over different time horizons and ratings categories.

Figure 5

The kurtosis coefficient (defined as the fourth central moment divided by the squared variance) is shown for spread changes over different time horizons and ratings categories.

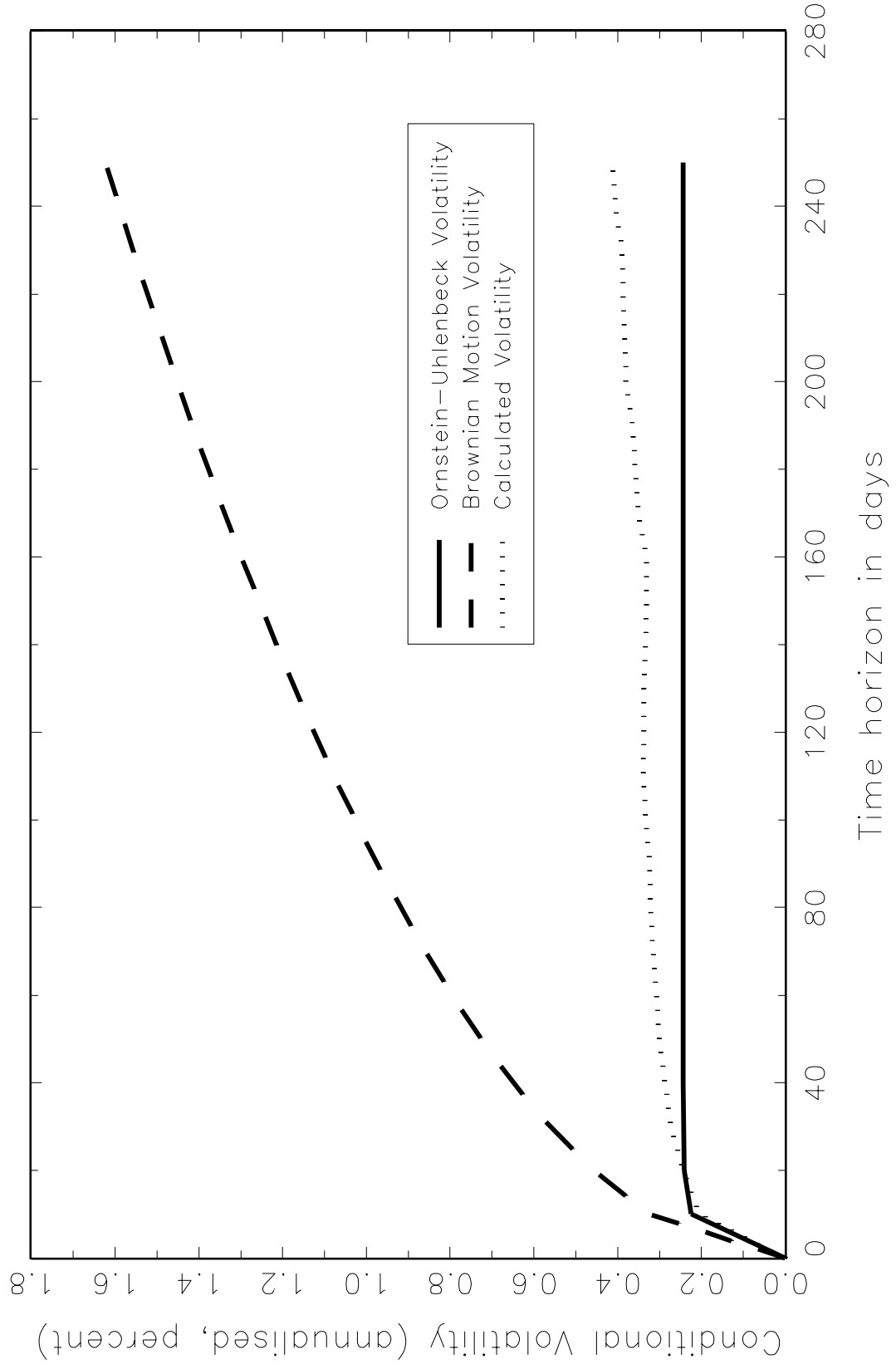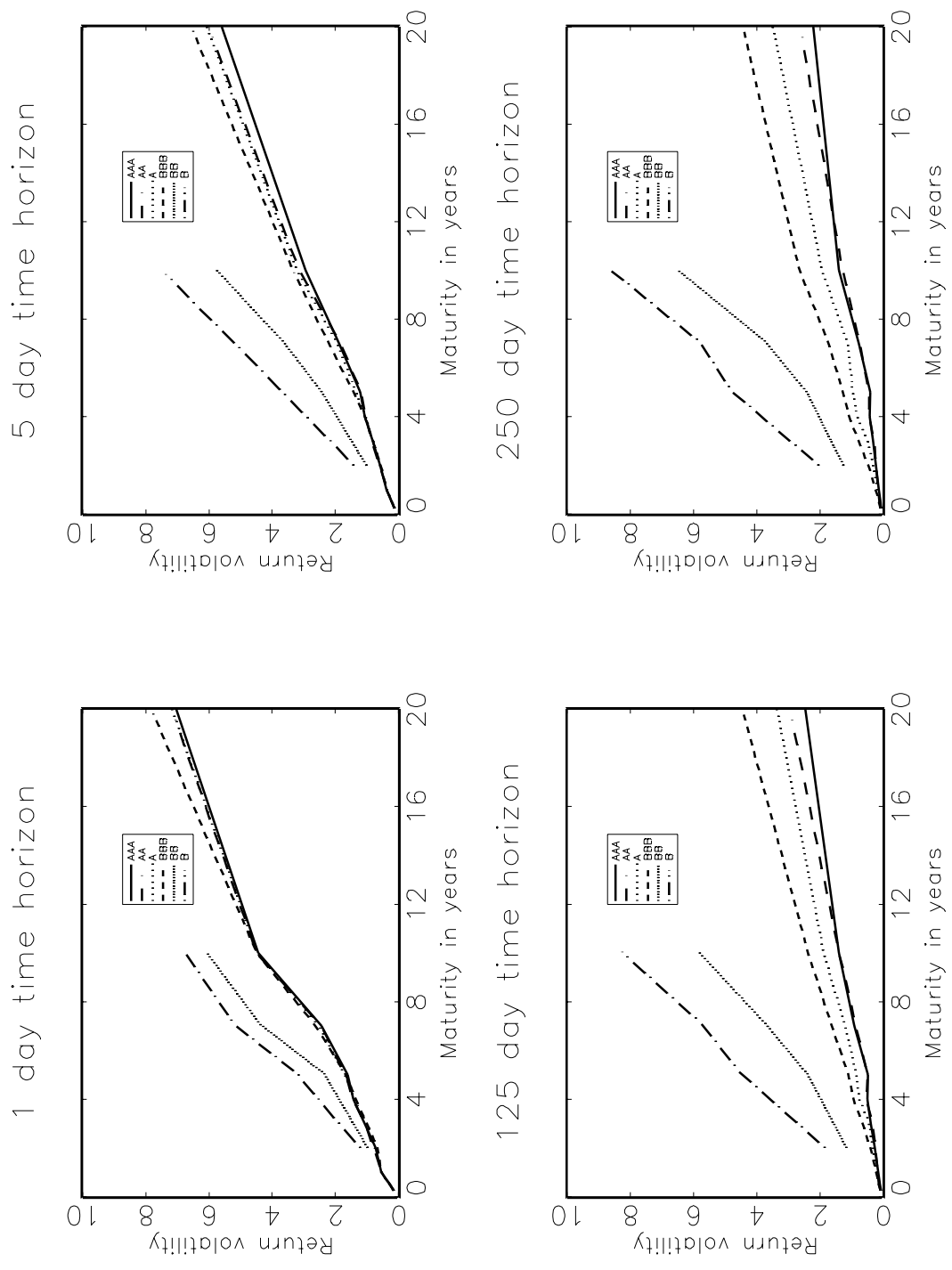Figure 1: Conditional Return Volatility for 5-Year AAA Spread

GAUSS   Sun Aug 22 19:25:54 1999

Figure 2: Return Volatilities (Annualised)

Figure 3: Variance Ratio of spread changes
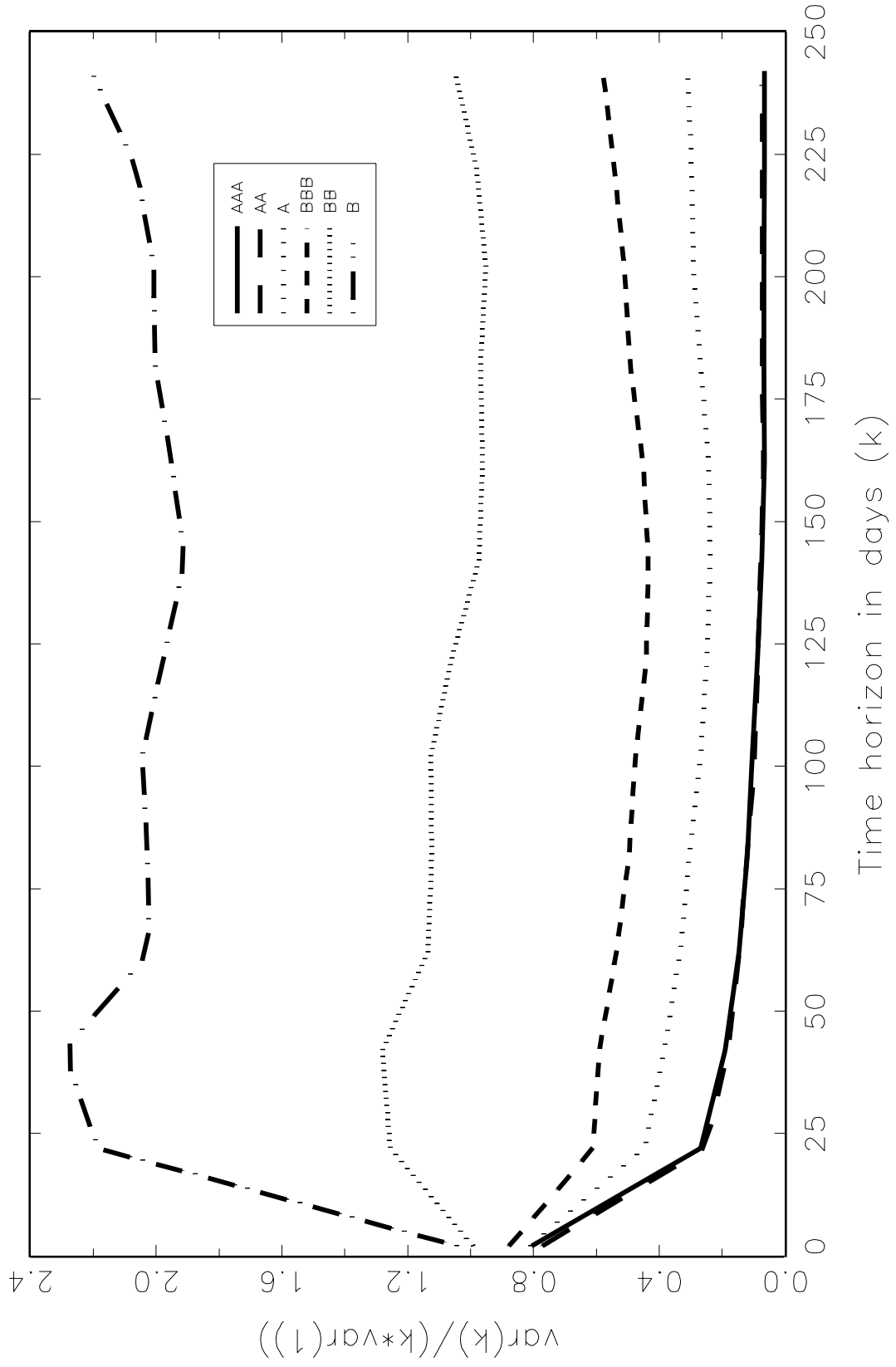
GAUSS   Tue Aug 24 11:36:40 1999

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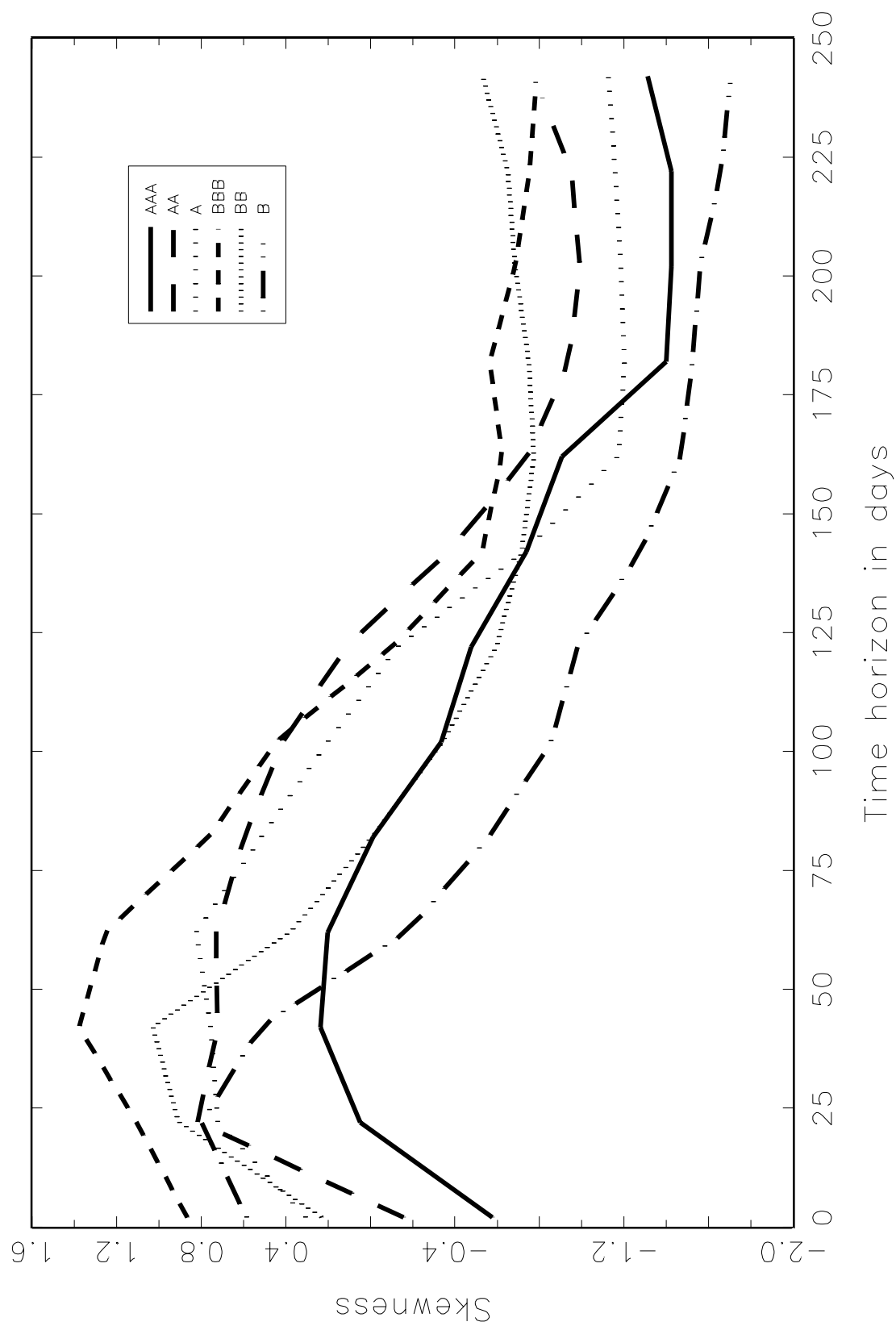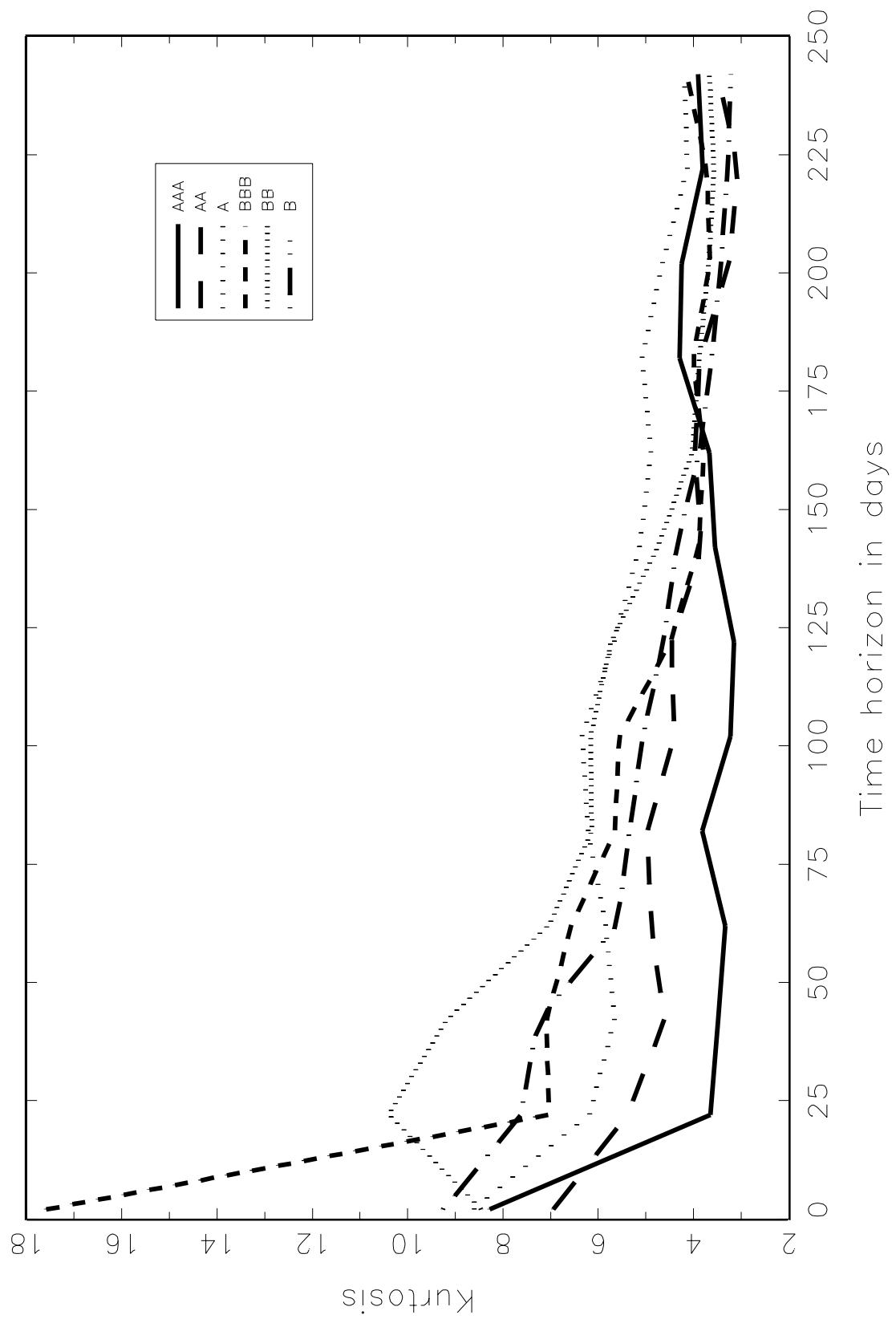Figure 4: Skewness coefficient of spread changes

Figure 5: Kurtosis coefficient of spread changes

Appendix

Cochrane-type bias correction

We assume that the data generating process is a pure unit root process, ie equation (7) becomes

$$S_i - S_{i-1} = \mu + \epsilon_i \tag{A1}$$

with ϵ_i assumed to be i.i.d. with expectation zero and variance σ_ϵ^2. We are interested in constructing an estimator for σ_ϵ^2 using k-lagged differences, ie

$$S_n - S_{n-k} = k\mu + \sum_{j=0}^{k-1} \epsilon_{n-j}$$

Using $k(S_T - S_0)/T$ as an estimator for μ we can write the nominator of the standard sample variance

$$
\begin{aligned}
N_\sigma &= \sum_{n=k}^{T} \left((S_n - S_{n-k}) - \frac{k}{T}(S_T - S_0) \right)^2 \\
&= \sum_{n=k}^{T} \left(k\mu + \sum_{j=0}^{k-1} \epsilon_{n-j} - \frac{k}{T}\left(T\mu + \sum_{j=0}^{T-1} \epsilon_{T-j} \right) \right)^2 \\
&= \sum_{n=k}^{T} \left(\sum_{j=n-k+1}^{n} \epsilon_j - \frac{k}{T}\sum_{j=1}^{T} \epsilon_j \right)^2
\end{aligned}
$$

Defining $Z_n = \sum_{j=n-k+1}^{n} \epsilon_j$ and using the fact that the ϵ_i are i.i.d with zero mean (ie $\mathrm{Var}(\epsilon) = \mathrm{E}(\epsilon^2)$) we get

$$
\begin{aligned}
\mathrm{E}(N_\sigma) &= \sum_{n=k}^{T} \left(\mathrm{E}(Z_n^2) - \frac{2k}{T}\mathrm{E}(Z_n Z_T) + \frac{k^2}{T^2}\mathrm{E}(Z_T^2) \right) \\
&= \mathrm{E}(\epsilon^2) \sum_{n=k}^{T} \left(k - \frac{2k^2}{T} + \frac{k^2}{T} \right) = \sigma_\epsilon^2 (T-k+1)(T-k)\frac{k}{T}
\end{aligned}
$$

So in order to get an unbiased estimator for σ_ϵ^2 using the quantity N_σ we have to multiply it by

$$\frac{T}{k(T-k+1)(T-k)}$$

which is just the Cochrane adjustment. So we define

$$\hat\sigma^2(k) = \frac{T}{(T-k)(T-k+1)} \sum_{j=k}^{T} \left[(S_j - S_{j-k}) - \frac{k}{T}(S_T - S_0) \right]^2 \tag{A2}$$

as the Cochrane estimator of the variance.

We can repeat the above procedure to obtain unbiased versions for the estimation of the correlation between categories, the skewness and the kurtosis.

For estimation of the correlation we assume that the covariance structure between categories is given by

$$\mathrm{Cov}(\epsilon_j^A, \epsilon_i^B) = \mathrm{E}(\epsilon_j^A, \epsilon_i^B) = \delta_{ij}\rho$$

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where $\delta_{ij} = 1$ for $i = j$ and zero elsewhere. An unbiased estimator of the sample covariance $\gamma^{AB}(k)$ in the random walk model is then (proceeding as above)

$$\hat{\gamma}^{AB}(k) = \frac{T}{(T-k)(T-k+1)} \sum_{j=k}^{T} [(S_j^A - S_{j-k}^A) - \hat{\mu}^A][(S_j^B - S_{j-k}^B) - \hat{\mu}^B] \qquad \textbf{(A3)}$$

(use the Cochrane sample means $\hat{\mu} = \frac{k}{T}(S_T - S_0)$). The correlations $\rho^{AB}(k)$ can then be estimated as

$$\hat{\rho}^{AB}(k) = \frac{\hat{\gamma}^{AB}(k)}{\hat{\sigma}(k)^A \hat{\sigma}(k)^B} \qquad \textbf{(A4)}$$

The sample skewness estimator is a quotient with nominator

$$N_S = \sum_{n=k+1}^{T} \left((S_n - S_{n-k}) - \frac{k}{T}(S_T - S_0) \right)^3$$

With the technique above we get

$$\mathrm{E}(N_S) = (T - k + 1)(T - k)(T - 2k)\frac{k}{T^2}\mathrm{E}(\epsilon^3)$$

So the adjusted skewness estimator is given by

$$\frac{T^2}{(T - k + 1)(T - k)(T - 2k)} \frac{N_S}{(\hat{\sigma}(k))^3}$$

Finally, for the sample kurtosis estimator the nominator is

$$N_K = \sum_{n=k+1}^{T} \left((S_n - S_{n-k}) - \frac{k}{T}(S_T - S_0) \right)^4$$

and

$$\mathrm{E}(N_K) = (T - k + 1)(T - k)(T^2 - 3kT + 3k^2)\frac{k}{T^3}\mathrm{E}(\epsilon^4)$$

So the adjusted kurtosis estimator is given by

$$\frac{T^3}{(T - k + 1)(T - k)(T^2 - 3kT + 3k^2)} \frac{N_K}{(\hat{\sigma}(k))^4}$$

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