The role of corporate balance sheets and bank lending policies in a financial accelerator framework

Simon Hall
and
Anne Vila Wetherilt

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E-mail: simon.hall@bankofengland.co.uk; anne.vila-wetherilt@bankofengland.co.uk

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Abstract

This paper uses the popular Bernanke, Gertler and Gilchrist (BGG) model to explore links between the financial health of the non-financial corporate sector and bank lending behaviour on the one hand, and the effectiveness of monetary policy on the other. We use the model’s microeconomic contracting framework to generate specific financial scenarios, defined in terms of steady-state credit spreads, bank lending policies and corporate sector financial health. We embed these scenarios in the macroeconomic BGG model and investigate how they affect dynamic responses of the real economy to monetary and real shocks.

Our simulations show that in the context of the BGG model, the balance sheet positions of the financial and non-financial corporate sectors can affect the monetary transmission mechanism. We illustrate that in certain financial scenarios in the model the financial accelerator mechanism is very potent, whereas in others it has little incremental impact. This implies that, for a given shock, monetary policy can be less or more proactive, respectively. In addition, the model simulation results suggest that certain parameters may merit particular attention. For example, the sensitivity of bank lending policy to news about corporate financial health has an especially marked impact in the model’s dynamics. And as illustrated in previous work, corporate leverage also plays an important role in amplifying and propagating shocks.
Summary

Recent financial crises have illustrated that the financial positions of borrowers and lenders—financial stability considerations—can influence the way in which official interest rate changes affect spending and inflation—monetary stability considerations. A substantial academic literature has developed considering potential macroeconomic impacts of financing decisions by borrowers and lenders. Among these so-called ‘credit channel’ models, the recent financial accelerator approach of Bernanke, Gertler and Gilchrist (BGG) seems particularly suited to an analysis of how corporate sector balance sheets and the behaviour of banks can affect the monetary transmission mechanism.

In credit channel models, firms often find it more costly to finance investment projects with external funds rather than with internally generated resources. This ‘external finance premium’ may arise because lenders face costs from observing and/or controlling the risks involved in supplying funds to borrowers. These agency costs, and the external finance premium, may vary with borrowers’ financial health. For example, the stake of a borrower in an investment project (measured by the degree to which it is able to finance a project using internal funds) may provide a signal of the unobserved risk of lending. It may also affect the borrower’s incentive to act diligently and to report project outcomes truthfully.

The financial accelerator model used in this paper embeds a similar imperfect information problem in the supply of external finance in a standard macroeconomic framework. Our paper examines how a range of interesting financial scenarios can arise out of this model and in turn, how these scenarios affect the dynamic response of the model economy to alternative shocks (for example monetary news or productivity shocks). These scenarios are defined in terms of steady-state credit spreads, bank lending policies and corporate financial health. The main objective is to examine how the strength of the monetary transmission mechanism might vary across such scenarios.

Our simulations of the model show how balance sheet positions of the financial and non-financial corporate sectors can affect the monetary transmission mechanism. We show that in certain financial scenarios the financial accelerator mechanism is very potent, whereas in others it has little incremental impact. This implies that, for a given shock in the model economy, monetary policy can be less or more proactive, respectively.
In addition, the model simulation results suggest that certain parameters may merit particular attention. For example, the sensitivity of bank lending to news about corporate financial health has an especially marked impact on the model’s dynamics. And as illustrated in previous work, leverage also plays an important role in amplifying and propagating shocks. But we also show that the strength of the financial accelerator cannot be attributed to a single variable. For example, we observe that the financial accelerator can be weak, both when leverage is low and banks are relatively restrictive in their lending, and when leverage is high and banks are very accommodative.

These theoretical results are consistent with real-world experience that bank and non-financial corporate balance sheets can, at times, have a marked impact on the effectiveness of monetary policy. But while the specific model used in this paper provides an attractive analytical framework for thinking about potential qualitative effects of changes in financial conditions on real variables, we think its quantitative results need to be interpreted with caution, as in all calibrated simulated models. Moreover, although the model can be used to analyse certain important interactions between financial imperfections and the monetary transmission mechanism, it leaves out several features that one might want to incorporate in a more general model of financial stability. For example, the model has a relatively restricted financial structure with a focus on debt finance. Financial institutions are sparsely modelled, with limited potential for effects from the bank lending channel. That suggests that further work to develop quantitative models that incorporate these features may provide further insights into interactions between monetary and financial stability.
1 Introduction

Economic models often assume that the impact of changes in financial conditions on the wider economy can be summarised by a relatively limited set of financial variables, such as short-term risk-free interest rates and long-term government bond rates. But, at times, financial developments can have important effects on the economy which these variables would not necessarily indicate. For example, the experiences of several East Asian emerging market economies following the 1997/98 crisis and, more recently, of Japan suggest that the financial position of borrowers and lenders—financial stability considerations—can influence how official interest rate changes affect spending and inflation—monetary stability considerations.

A substantial literature has developed considering how financing decisions by borrowers and lenders can affect the monetary transmission mechanism. Among these so-called ‘credit channel’ models, a recent approach by Bernanke, Gertler and Gilchrist (1999) (BGG) seems particularly suited to an examination of the potential role of corporate sector balance sheets and the behaviour of banks. A particular attraction of the model is its ability to quantitatively model how financial imperfections affect monetary policy in the familiar setting of a sticky-price dynamic general equilibrium model.

Previous applications of the model for the United States by BGG (1999) and for the United Kingdom by Hall (2001b) have focused on the incremental impact on business cycle behaviour of financial mechanisms in the model. This paper looks more specifically at the sensitivity of the model’s quantitative results to variations in the parameters generating the financial effects in the model. In particular, we examine how alternative financial scenarios can arise out of the model’s financial problem and in turn, how these scenarios affect the dynamic response of the model economy to alternative shocks (for example, monetary news or productivity disturbances). A particular aim is to examine how the monetary transmission mechanism might vary across financial scenarios. This has obvious potential policy implications both for the domestic economy and internationally.

The paper argues that (in the context of the specific model considered) financial conditions can play an important role in the monetary transmission mechanism. In particular, the model suggests that the impact of monetary policy depends on initial conditions in the corporate and

(1) Hall (2001a) provides a short summary of some of the main approaches in this literature.
financial sectors. Moreover, the factors creating these initial conditions in the model are complex, and matter for understanding its transmission mechanisms. For example, we examine how structural shifts in bank behaviour and/or changes in corporate leverage, can alter the sensitivity of the economy to shifts in interest rates.

The paper is organised as follows. In Section 2, we locate the BGG model within the credit channel literature, highlighting why it is a suitable vehicle for exploring the impact of balance sheet conditions and bank lending policies on the transmission mechanism. In Section 3 we set out some of the key features of the BGG model’s microeconomic contracting problem which underpin its financial effects. Section 4 uses the model to generate a set of financial scenarios, based on various combinations of the underlying deep financial parameters. These scenarios form the basis of our model simulations which are presented in Section 5. Section 6 offers some conclusions and ideas for future research.

2 Credit effects in the monetary transmission mechanism

The credit channel literature has discussed two distinct (but complementary) ways that financial market imperfections might modify traditional monetary transmission mechanisms. The bank lending channel focuses on the impact of monetary policy on the cost and/or availability of intermediated finance.\(^{(2)}\) Here developments in the financial position of lenders can affect loan supply, the cost of finance and investment by bank borrowers. For example, if sharp loan losses deplete bank capital and banks cannot easily access replacement finance, they may tighten loan supply. Aggregate spending may be affected if borrowers are unable to readily substitute other forms of finance for these bank loans, perhaps because imperfect information makes other (non-bank) lenders uncertain about their credit quality and less willing to supply finance.

The balance sheet channel takes a broader view of the impact of capital market imperfections, suggesting that interactions between the financial position of borrowers and their external financing costs might amplify and/or propagate the effects of monetary shocks.\(^{(3)}\) Here it is the financial position of borrowers, rather than lenders, which matters for finance supply. For example, more heavily indebted firms may find it harder or more costly to access finance than firms with identical investment projects but better financial positions.

\(^{(2)}\) For a discussion of the lending view, see Kashyap and Stein (1994).
\(^{(3)}\) Oliner and Rudebusch (1996) discuss the balance sheet channel.
In practice there have been a wide range of approaches to modelling these two generic channels. However, a common feature is the assumption of a financial friction arising from a failure of one (or more) of the perfect capital market assumptions used by Modigliani-Miller (1958). Most commonly, models introduce an agency cost in the supply of finance (although in principle transaction costs might generate simple credit effects) which can arise in either a symmetric or asymmetric information environment.

In a symmetric information setting, Hart and Moore (1994) assume an imperfection in the resale market for corporate assets. Specifically, the value of a project as an on-going concern exceeds its recoverable value in default because of a complementarity between entrepreneur-specific human capital and physical capital. Contracts are incomplete or imperfectly enforceable and producers can credibly threaten to withdraw labour from projects before completion unless the terms of loans are renegotiated in their favour. Lenders are aware of the possibility of *ex post* strategic default and *ex ante* structure financial contracts accordingly. As a result, lenders may only supply finance up to a limit defined by attachable collateral. In this setting, borrower balance sheets and asset prices can play a key role in determining finance supply.

Agency costs have also been modelled in asymmetric information settings. Three well-known generic approaches have been used in the literature:

(i) **Adverse selection**: lenders cannot directly observe the *ex ante* quality of investment projects and are aware that their loan rates may influence the average riskiness of their pool of loan applicants. Loan rates may initially rise to compensate lenders for increased default risk. But in some cases lenders may quantitatively deny credit to borrowers observationally equivalent to those receiving credit.

(ii) **Moral hazard**: lenders cannot perfectly monitor the use of borrowed funds. Contrary to the wishes of lenders, as interest rates rise some borrowers may have an incentive to engage in riskier projects that raise the probability of default.\(^{(4)}\)

(iii) **Costly state verification (CSV)**: lenders must pay a monitoring (‘bankruptcy’) cost to verify the outcome of borrowers’ investment projects. Borrowers have an incentive to under-report project outcomes. Townsend (1979) shows that the optimal contractual form which minimises these verification costs, whilst ensuring incentive-compatibility, is a standard debt contract.

\(^{(4)}\) Stiglitz and Weiss (1981) use adverse selection and moral hazard arguments to model quantitative credit rationing.
There are relatively few macroeconomic credit channel models which formally embed and endogenise these microfoundations. In terms of the bank lending channel, Fisher (1999) adopts a fully-flexible real business cycle framework and incorporates a limited participation assumption (along the lines of Lucas (1990)) to ensure that monetary shocks can affect the real supply of loanable funds. He then assumes that an (exogenous) proportion of firms are dependent on bank finance due to a CSV problem arising from asymmetric information about firm outputs. Other firms can costlessly reveal their outputs and so can obtain finance more cheaply using state-contingent contracts. He qualitatively replicates some elements of the bank lending channel—such as amplified responses to shocks of constrained firms relative to unconstrained firms—although he finds that implausible parameter values are needed to replicate empirical evidence on heterogeneous responses of different types of firm to monetary shocks.

For the balance sheet channel several models have adopted the Hart-Moore specification of agency costs. For example, Kiyotaki and Moore (1997) show how lending may be tied closely to asset prices and the value of borrower collateral. More recently, Cooley, Marrimon and Quadrini (2001) also adopt the Hart-Moore approach in a model illustrating how imperfect contract enforceability can create links between borrower cash flow, firm size and investment. A number of models have adopted the CSV foundation of Townsend (1979). For example, in a stochastic overlapping generations model, Bernanke and Gertler (1989) show how CSV can lead to an external finance premium which is inversely related to borrower cash flow. Carlstrom and Fuerst (1997) embed a similar relationship between finance costs and borrower financial health in a flexible-price general equilibrium model.

A recent paper by Bernanke, Gertler and Gilchrist (1999) develops a dynamic general equilibrium sticky-price model where interactions between borrower financial health and loan supply decisions can result in pronounced amplification and propagation of shocks—so-called ‘financial accelerator’ effects. In the model, households work, consume and invest their savings in a financial intermediary. The financial intermediary pools savings and lends to companies. Companies produce in competitive markets using labour and capital, with capital financed from retained internal (collateralised) funds (net worth) or external (uncollateralised) borrowing. Retailers purchase production and then differentiate goods and sell in monopolistically competitive markets. That permits price stickiness and provides a role for a monetary authority to stabilise inflation.
The key novelty of the model lies in its investment system. Here the cost of investment finance (cost of capital) term is augmented by a wedge (the external finance premium) which responds endogenously and negatively to the level of corporate internal funds (net worth) relative to total financing requirements. When a substantial portion of corporate investment is funded internally (companies have low gearing), the external finance premium is small (tending to zero for investment which is fully internally funded or collateralised). When corporate investment is mainly funded externally (high gearing), the premium is high. The intuition is that the intermediary’s participation constraint in the optimal contract between lenders and borrowers requires a premium sufficient to offset the greater incentive for borrowers to declare default on their investment projects when their financial stake is low. This added element provides for greater amplitude and persistence in response to shocks, and for inter-relationships between real and financial variables unavailable in many standard macroeconomic models.

The BGG (1999) model has some appeal to policy-makers. A key attraction is its ability to quantitatively model interactions between financial imperfections and monetary policy in the familiar economic setting of a sticky-price dynamic general equilibrium model (essentially a model featuring IS-LM type relationships in a dynamic setting). In addition, although essentially a model of the balance sheet channel (with no role for bank lending effects), the BGG approach also synthesises several of the alternative modelling strategies and balance sheet effects described above. For example, it includes both potential for shocks to cash flow and asset prices to affect finance supply, drawing together the effects identified in the Bernanke and Gertler (1989) and Kiyotaki and Moore (1997) models.

In the remainder of this paper we consider whether the model can be used for a deeper analysis of links between the balance sheets of the non-financial corporate sector and bank lending behaviour on the one hand, and the real economy and effectiveness of monetary policy on the other.

3 Financial effects in the BGG model

Financial effects in the BGG model arise from the inability of lenders to costlessly assess the outcome of borrower investment projects. In the model, lenders structure debt contracts to minimise the agency costs arising from this capital market imperfection. This section sets out some of the key features of this contracting problem.\(^{(5)}\) Closely following the exposition in BGG (1999), we show how this contracting problem provides microeconomic underpinnings for

\(^{(5)}\) A more detailed description of the contracting problem is assigned to Appendix 1.
the financial mechanisms in the BGG model—specifically for the existence of an external finance premium and its link to corporate financial conditions.

In particular, we show how variations in some key inputs into the contracting problem—assumptions about the cost of monitoring borrowers (bankruptcy costs), the degree of uncertainty about investment returns and the natural turnover rate of firms—can lead to different financial steady-states for the macroeconomic model. Figure A provides a schematic description of the relationship between the contracting problem and the BGG macroeconomic model.(6) In subsequent sections we show how these financial scenarios can affect the overall behavioural response of the BGG macroeconomy following shocks.

Figure A
The relationship between the contracting problem and the macro model

<table>
<thead>
<tr>
<th>Deep parameters</th>
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<th>Steady-state financial scenarios, described by values for:</th>
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Key aspects of the investment problem in the BGG model

Risk-neutral firms acquire physical capital $K$ at a price $Q$ at the end of period $t$, for use in production in period $t+1$. They finance capital with internal funds (net worth $N$, consisting of wages and of revenue from previous capital investments) and when this is insufficient by external borrowing $B$ from a financial intermediary. (7) Higher levels of net worth reduce the need to borrow thereby mitigating agency problems in external financing. Greater self-financing of capital will therefore be associated with a lower external finance premium—defined as the difference between the rate the firm pays for external funds and the risk-free rate. Firms combine capital with labour in production. They sell their output to retailers (who differentiate and sell on to consumers), repay borrowing, and then consume and accumulate net worth with any surplus. In any given period, there is a natural turnover of firms (birth/death).

(6) For more detail on the macroeconomic aspects of the model, see BGG (1999). Appendix 2 reports the full model in log-linearised format as simulated using the Winsolve package.

(7) The financial intermediary is most readily interpretable as a bank. However, in principle it could be any external lender that faces imperfect information about project outturns.
Specifically, each firm has a constant probability $γ$ of surviving to the next period, separate from any risk of default.$^{(8)}$

Intermediaries hold a diversified portfolio (as agents for risk-averse households) so their opportunity cost is equal to the risk-free gross rate of return $R$. Firms earn a return on capital that is a combination of an idiosyncratic shock $σ_j$ and the average economy-wide return on capital $R^k$. Adopting the CSV approach, an agency problem arises in the model because intermediaries cannot observe $σ_j$ and need to pay an monitoring cost if they wish to observe outcomes. The financial contract between lender and borrower is designed to minimise these potential agency costs given assumptions about key ‘primitives’ (the size of monitoring costs, the distribution of potential idiosyncratic shocks to firms’ rates of return and natural turnover of firms). This contract (effectively a standard debt contract) includes the following bankruptcy clause:

- If $σ_j ≥ \sigma^J$, then the firm can pay its loan in full out of its revenues and keeps the residual. The lender receives $σ^j R^k_{r+1} Q_j K^j_{r+1} = Z^j_{r+1} B^j_{r+1}$, where $Z$ is the non-default loan rate.

- If $σ_j < \sigma^J$, then the firm defaults on its loan; the lender pays a monitoring cost $μ$ and receives what she finds, namely $(1-μ)σ^j R^k_{r+1} Q_j K^j_{r+1}$. The firm receives nothing.

Consequently, the lenders’ expected return is a function of $σ^J$, the default trigger. Higher levels of $σ^J$ raise $Z$, the non-default pay-off to the lender, but also raise the probability of default ($F(σ^J)$) itself (this will be important in Section 4 in understanding the effects of changes in the primitive parameters). Finally, we define the external finance premium as $R^k/R$.

The firm’s optimisation problem and its role in the macro model

The firm chooses capital $K^J_{r+1}$ and $σ^J$ to maximise its expected return subject to the constraint that lenders earn their opportunity cost. Consequently, the firm absorbs all the risk:

$$\text{MAX} \quad E \int_{σ^J}^{∞} \omega R^k_{r+1} Q_j K^j_{r+1} dF(ω) - (1 - F(σ^J))σ^J R^k_{r+1} Q_j K^j_{r+1}$$

(1)

where $F(σ^J)$ is the probability of default.

(8) Note that $γ$ is distinct from the probability of bankruptcy.
subject to

\[ ((1 - F(\sigma^i))\sigma^i) + (1 - \mu) \int_0^{\sigma^i} \omega \, dF(\omega) \right) R^i_{t+1} Q_t K_t^j = R^i_{t+1} (Q_t K_t^j - N_t^j) \]

The firm’s expected utility consists of two parts: expected revenues when \( \omega^j \) is high enough such that the firm keeps some of its revenues after paying its debtors minus the agreed payment to its debtors. The participation constraint states that the cash flows to the intermediary (the agreed upon payment when \( \omega^j \) is high enough or the available funds minus the monitoring costs if \( \omega^j \) is too low) need to be equal to her opportunity cost. In this form, it is assumed that there is no aggregate risk (\( R^k_{t+1} \) is known): BGG show that model can easily be modified to allow for aggregate uncertainty.

In Appendix 1, we show that the first-order conditions yield the following key relationship:

\[ Q_t K_t^j = \psi(s^i) N_t^j, \psi^j(\cdot) > 0 \]  \hspace{1cm} (2)

where \( s^i = \mathbb{E}[R^k_{t+1} / R_{t+1}] \)

Equation (2) shows that the equilibrium demand for capital by the firm is a function of the firm’s net worth \( N_{t+1} \) and of the expected external finance premium \( s^i \). Equation (2) can be inverted to obtain:

\[ \mathbb{E}[R^k_{t+1}] = s^i [N_{t+1} / Q_t K_t^j] R_{t+1}, \psi^j(\cdot) < 0 \]  \hspace{1cm} (3)

Equation (3) shows how the firm’s required return on capital depends inversely on the share of the firm’s capital investment financed by its own net worth. This can change as a result of either the firm’s decision to increase or decrease its level of capital expenditures \( K_t \), or from changes in asset prices \( Q_t \) and/or net worth \( N_t \). If the firm does not need any external financing, then the equilibrium return to capital is equal to the risk-free rate. When the firm needs to borrow, the required return on capital will be higher reflecting expected agency costs faced by the financial intermediary.

The role for the financial accelerator mechanism in the model can be seen from aggregating equation (3) across firms:

\[ \mathbb{E}[R^k_{t+1}] = s^i [N_{t+1} / Q_t K_t^j] R_{t+1}, \psi^j(\cdot) < 0 \]  \hspace{1cm} (4)

and from the definition of aggregate entrepreneurial net worth:
\[ N_{r+1} = \gamma V_{r+1} + W_{r+1} \]  

Equation (5) indicates that aggregate net worth is the sum of the equity stakes of the \( \gamma \) entrepreneurs that have survived in period \( t+1 \) and of their wages. Equity equals earnings on capital employed (including capital gains) from \( t \) to \( t+1 \) minus the loan repayment.

\[ V_{r+1} = R_k^r Q, K_{r+1} - (R_{t+1} + EFP_t)(Q, K_{r+1} - N_t) \]  

with
\[ EFP_t = \frac{\int_0^\sigma \omega R_k^r Q, K_{r+1} dF(\omega)}{Q, K_{r+1} - N_t} \]

where \( EFP_t \) is the ratio of default costs to the amount borrowed and reflects the external financing premium.

Equation (6) indicates that net worth would be affected by unexpected changes in the return on capital, changes in the price of capital, in leverage and in default costs. These changes in net worth will in turn affect spreads (equation (4)). In particular, the impact of changes will be magnified in firms with higher leverage. Higher spreads will have a further effect on the demand for capital and investment, which in turn will affect the return on capital, and so on. This is the accelerator effect.

Equation (4) can be rewritten in log-linearised form (with lower-case variables indicating deviations from the steady state):

\[ E_{r+1} r^k - r_{r+1} = - \psi [ n_{r+1} - (q_t + k_{r+1}) ] \]

In the financial accelerator model, changes in corporate leverage—either as a result of shifts in net worth, desired capital holdings or asset prices—affect the premium, with the sensitivity summarised by the elasticity parameter \( \psi \) in equation (7). This coefficient, which is held fixed in the model dynamics, is determined endogenously in the steady-state contracting problem, described in Appendix 1. In particular, we show that the coefficient \( \psi \) is a function of three exogenous parameters: (i) the variability of idiosyncratic shocks to the firm’s investment project; (ii) the initial business failure rate; and (iii) the cost of bankruptcy. A change in any of these variables affects the expected pay-off to the lender, and causes her to revise lending policy (i.e., steady-state spreads and/or the elasticity of the spread with respect to leverage). Variations in the parameter \( \psi \) potentially allow us to illustrate revisions in behaviour in reaction to an
increase in perceived risk such as that observed among major financial intermediaries following the Russia and LTCM crises in 1998.

The dynamic behaviour of leverage and net worth can be affected by both shocks to the real sector of the economy (ultimately affecting output and inflation) and the financial sector (affecting asset prices) and by the initial financial structure (the steady-state external finance premium and leverage) (see equation (8) which is the log-linearised form of equation (5), again written as a deviation from steady state):

\[ n_{r,t} = \frac{\gamma RK}{N} (r^K - r_t) + r_t + n_t + \frac{(R^K - 1)K}{N} (r^k_t + q_{t-1} + k_t) + (1 - \alpha)(1 - \Omega) \frac{Y_t}{N} y_t \]  

where variables without subscripts refer to steady-state variables.

Equation (8) shows that the behaviour of net worth (and its sensitivity to shocks to the return on capital/asset prices)—a key influence on the evolution of the external finance premium—depends on financial structure variables. In particular the steady-state leverage of the corporate sector is a key determinant. Net worth depends on the return to entrepreneurs on their equity stake in capital, with the weighting determined by \( \frac{\gamma RK}{N} \). Net worth of a corporate sector with high initial leverage is much more sensitive to equivalent shocks than a sector with low initial leverage.

4 Financial scenarios

The previous section outlined how the financial mechanisms within the model depend on steady-state values for corporate leverage, the external finance premium and the sensitivity of the finance premium with respect to leverage. As Appendix 1 outlines, these values are derived from certain primitive (‘deep’) parameters inputting into the contract solution to the model’s imperfect information problem:

1. The variance of the idiosyncratic shock to the firm’s investment project \( \sigma^2 \) (a measure of uncertainty).
2. The monitoring/bankruptcy cost \( \mu \) (a measure of the efficiency of bankruptcy/debt recovery).
3. Firms’ rate of survival \( \gamma \) (natural turnover of firms).
Of the three primitive parameters, the variance $\sigma^2$ and the cost of bankruptcy $\mu$ enter the contracting problem directly (see equation (A1)), whereas the survival rate $\gamma$ affects net worth directly (see equation (A10)) and the contracting problem indirectly.

The objective of this section is to identify a set of useful steady-state financial scenarios based on various combinations of these primitives. These steady-state scenarios form the basis for simulations of the full financial accelerator model in the next section. We are particularly interested in the implications of alternative values for $\psi$, the sensitivity of the external finance premium to the corporate sector’s financial position. We view this sensitivity parameter as a potential summary indicator of bank lending behaviour. For example, a higher value of $\psi$ could be interpreted as a reduced willingness of the banking system to extend credit. In the limit, extreme values could be indicative of a credit crunch.

Chart B shows how changes in uncertainty ($\sigma^2$) in the contracting problem affect steady-state outcomes for the model’s key steady-state financial variables. First, increasing uncertainty about firms’ investment projects causes the steady-state external finance premium to rise. As uncertainty increases, the optimal contract specifies a lower cut-off value for $\sigma$, the trigger level for the idiosyncratic shock at which firms can repay debt, hence reducing the probability that insufficient investment revenues will cause the firm to declare bankruptcy. Yet, in spite of the lower cut-off value, the wider dispersion of project outcomes does lead to some increase in the probability of default. Banks, facing both a higher probability of corporate default and a lower expected non-default pay-off, charge a higher premium. Higher uncertainty, and resultant higher steady-state spreads, means that firms opt for lower steady-state leverage. Finally, the sensitivity parameter $\psi$ rises, albeit quite slowly. That suggests that increased uncertainty over project outcomes may affect financial effects in the model more via leverage, than through a change in the elasticity of the external finance premium.
Chart C shows that when monitoring costs $\mu$ are increased (i.e., when the financial sector is less able to recover value from defaulted loans), lenders are less eager to monitor at higher cost, and therefore specify a lower cut-off value $\sigma$ in the debt contract. So for a given variance of the idiosyncratic shock, bankruptcy becomes less likely and the probability of default falls slightly. At the same time, banks require a slightly higher premium in order to cover their increased monitoring costs and the lower expected non-default pay-off. Leverage reacts in the same direction as when varying uncertainty. Firms lower their leverage somewhat as financing costs rise. The sensitivity parameter increases, indicating that in the face of higher monitoring costs, banks write contracts that are more sensitive to changes in corporate leverage.

Finally, Chart D shows that steady-state outcomes for the financial variables fall as firms’ survival rate $\gamma$ rises. Banks are content with lower cut-off values $\sigma$, lower steady-state finance premia and lower sensitivity $\psi$ since firms’ higher survival rate implies that fewer firms will be
leaving the scene without repaying their debt. The lower cost of finance, together with a decline in the cut-off value $\sigma$, reduces the probability of bankruptcy. Interestingly, despite more favourable financing conditions, firms choose lower leverage. This follows from the effect of a higher $\gamma$ on their net worth (see equation (8)), as net worth is now more sensitive to shocks to rates of return.

**Chart D**

**Steady-state values varying firm rate of survival**

In Table A below, we present five steady-state scenarios described by various combinations of key financial variables. We generate these scenarios from alternative assumptions about values for primitive parameters in the contracting problem. In Scenario 1, steady-state leverage and corporate default are relatively high, whereas the sensitivity parameter and the external finance premium are low. Scenario 2 is a mirror image with low leverage, very low bankruptcy, a high sensitivity parameter and a relatively higher premium. Scenarios 1 and 2 are obtained by varying the cost of monitoring $\mu$ and the uncertainty parameter $\sigma^2$, but leaving the survival rate $\gamma$ constant. In Scenario 3, the survival rate is decreased, leading to a marked rise in steady-state leverage, premia, the sensitivity parameter and default rates. Finally, in Scenarios 4 and 5, only the uncertainty parameter is varied. This leads to pronounced reactions in leverage, as well as changes in the sensitivity parameter and finance premia. Default, the premium and lending sensitivity are either all low (Scenario 4) or all high (Scenario 5). Leverage is high in Scenario 4, and low in Scenario 5. In each case, ‘high’ and ‘low’ are measured with respect to the benchmark scenarios used in BGG (1999).
Table A
Selected financial scenarios

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<thead>
<tr>
<th></th>
<th>Scenario 1</th>
<th>Scenario 2</th>
<th>Scenario 3</th>
<th>Scenario 4</th>
<th>Scenario 5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Steady-state leverage</td>
<td>High</td>
<td>Medium/ Low</td>
<td>High</td>
<td>High</td>
<td>Low</td>
</tr>
<tr>
<td>Steady-state premium</td>
<td>Medium/ Low</td>
<td>Medium/ High</td>
<td>High</td>
<td>Low</td>
<td>High</td>
</tr>
<tr>
<td>Bank lending ($\psi$)</td>
<td>Low</td>
<td>High</td>
<td>High</td>
<td>Low</td>
<td>High</td>
</tr>
<tr>
<td>Default probability</td>
<td>High</td>
<td>Low</td>
<td>High</td>
<td>Low</td>
<td>High</td>
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</table>

<table>
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<th>Scenario 2</th>
<th>Scenario 3</th>
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<td>Steady-state leverage</td>
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<td>350</td>
<td>1200</td>
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<td>480</td>
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<tr>
<td>Bank lending ($\psi$)</td>
<td>0.053</td>
<td>0.029</td>
<td>0.08</td>
<td>0.076</td>
<td>0.038</td>
<td>0.089</td>
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<tr>
<td>Default probability (%)</td>
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<td>13.4</td>
<td>1.9</td>
<td>31.7</td>
<td>3.1</td>
<td>13.0</td>
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</table>

Note: Premia and default probability are expressed in annualised terms.

Table B
Underlying primitive parameters generating scenarios

<table>
<thead>
<tr>
<th></th>
<th>Scenario 1</th>
<th>Scenario 2</th>
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<tr>
<td>$\sigma$</td>
<td>Low</td>
<td>Base</td>
<td>Base</td>
<td>Low</td>
<td>High</td>
</tr>
<tr>
<td>$\mu$</td>
<td>Low</td>
<td>High</td>
<td>Base</td>
<td>Base</td>
<td>Base</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>Base</td>
<td>Base</td>
<td>Low</td>
<td>Base</td>
<td>Base</td>
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<table>
<thead>
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<th>Base</th>
<th>Scenario 1</th>
<th>Scenario 2</th>
<th>Scenario 3</th>
<th>Scenario 4</th>
<th>Scenario 5</th>
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<tbody>
<tr>
<td>$\sigma$</td>
<td>0.28</td>
<td>0.26</td>
<td>0.28</td>
<td>0.28</td>
<td>0.18</td>
<td>0.56</td>
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<tr>
<td>$\mu$</td>
<td>0.12</td>
<td>0.05</td>
<td>0.3</td>
<td>0.12</td>
<td>0.12</td>
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<tr>
<td>$\gamma$</td>
<td>0.973</td>
<td>0.973</td>
<td>0.973</td>
<td>0.923</td>
<td>0.973</td>
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</table>

Scenario 1 describes an economy where corporate sector borrowing (leverage) and default rates are relatively high but where (because of low monitoring costs) the banking sector is relatively accommodative, charging low spreads in steady state and with a muted response of spreads to unexpected shifts in corporate financial health. In contrast, Scenario 2 shows an economy with a (less-efficient) banking sector that is more sensitive to news about corporate financial conditions, despite lower steady-state corporate leverage. In Scenario 3, firms are highly levered, bank lending is restrictive, and borrowing costs and corporate default are high. Scenario 4 has a highly levered corporate sector (firms are more optimistic, knowing the risk on investments is lower) but the financial system is accommodative, with low spreads, low corporate default and relatively low bank sensitivity to leverage. Finally, in Scenario 5, where returns are more uncertain, steady-state spreads are high, the banking sector is sensitive to shocks and firms rely on internal finance rather than external borrowing.

Finally, we should note here that these financial scenarios are far from exhaustive and additional scenarios may be worth exploring in further work. However, our initial findings suggest that we cannot choose financial scenarios at will in the BGG model. For example, the macro model
appears to be less stable when simulated under extreme steady-state values of bank sensitivity or corporate leverage. This is not unreasonable: and in reality, shocks generating large movements in real variables in fragile economies might have feedback effects on steady-state conditions, invalidating our maintained assumption that steady states are fixed within scenarios. As such, it may be best to view the BGG model as a helpful tool to understand the qualitative implications of certain financial conditions, rather than a model to simulate the management of financial crises.

5 Model simulations

In this section, we present simulations of shocks to model economies defined by the five steady-state financial scenarios selected above. In each set of simulations these steady-state values remain fixed and the economy eventually returns to its starting position. We consider two types of shocks: a temporary rise in interest rates, and a permanent shock to productivity. Our discussion focuses on the paths of four endogenous variables: on the real side, investment and output; and on the financial side, the external finance premium and asset prices. In each simulation, we show results of the model economy with the financial accelerator mechanism operating and with it switched off (i.e., the premium remains fixed at its steady-state value). Finally, note these simulations incorporate a simple monetary policy feedback rule used by BGG whereby the authorities increase interest rates when inflation is above target and vice versa.

Standard monetary transmission mechanisms are augmented in the BGG model by effects on firms’ financial positions and the subsequent impact on the cost of finance. This mechanism works as follows. Consider an unexpected rise in interest rates. In the first instance, this will cause investment demand to fall, and with it the price of capital. The leveraged firm will face a fall in net worth, mainly due to the unexpected drop in investment returns, lower investment and a fall in the price of capital (see equation (6)). At the same time, the lender faces a potential increase in its expected loss, as a firm with a lower equity stake in projects may now have more of an incentive to declare bankruptcy. The lender will therefore reconsider the previously described trade-off between the non-default pay-off and the probability of default and will require a higher external finance premium. Both the higher external finance premium and the lower net worth reinforce the downward impact of higher interest rates on investment.

Technology shocks will affect the external finance premium through an alternative channel. A fall in productivity caused by such a shock will lower the demand for, and the price of, capital.
But the impact on net worth in a leveraged firm will be greater than the fall in the demand for capital, and that will tend to boost the external finance premium. That in turn reinforces the initial impact on investment and output.

Charts E1 and E2 show results for our base case scenario, which adopts the calibrations used by BGG (1999). As shown in Table A above, primitive variables in this base scenario are selected to imply steady-state leverage ($QK/N$) of about 2 (equivalent to capital gearing of 50%); an annualised external finance premium (spread) of just over 300 basis points; and an annualised business failure rate of about 5%. We shock the model economy by a 25 basis point rise in nominal interest rates (on a quarterly basis) lasting one period (Chart E1); and a 1 percentage point permanent fall in productivity (Chart E2). The charts clearly show the incremental effect of the financial accelerator mechanism in response to shocks. The initial impact of monetary and productivity shocks are magnified and prolonged.

**Chart E1**
**Base case (monetary shock)**
Charts F to J show the behaviour of alternative model economies, subject to identical initial shocks, but with their steady states calibrated to match our new financial stability scenarios.

Charts F1 and F2 show results for simulations of Scenario 1 which represents an economy in which steady-state leverage is relatively high (suggesting that the corporate sector is optimistic about returns to investment). In addition, the financial sector is relatively willing to extend credit due to low monitoring costs—the steady-state external finance premium is slightly lower than in the base case and more importantly, the sensitivity of the premium to shifts in the leverage ratio (following a shock) is low. High leverage by itself might be expected to raise the potency of the financial accelerator mechanism by heightening the sensitivity of corporate financial health to an unexpected shock. However, financial sector lending behaviour is relatively more accommodative in response to news about corporate financial health (the sensitivity of lending spreads to an unexpected shock to net worth is almost half that in the base case). So unexpected interest rate rises, while having a relatively pronounced affect on firms’ net worth, do not lead to marked changes in banks’ lending behaviour. The impact of the latter effect outweighs the former, and the overall response to a monetary shock is slightly more muted than in our base case. A qualitatively similar response is seen in Chart F2 that tracks the impact of a productivity shock.
Charts G1 and G2 show results for Scenario 2. Unlike Scenario 1, leverage is relatively low. But the financial sector is more sensitive to news about corporate financial health. This scenario might represent an economy in which the corporate sector is relatively optimistic about prospects, but the financial sector is cautious in its lending behaviour. The simulations show that this credit policy makes the economy more sensitive to shocks, despite lower leverage. In response to good or bad news about returns to investment projects, the banking sector does much of the work in the transmission mechanism. So in the context of this model, a monetary
authority interested in stabilising an economy in this position needs to do less given the implied response of the corporate sector.

**Chart G1**
**Scenario 2 (monetary shock)**

Scenario 3 illustrates the behaviour of an economy with a similar sensitivity of the banking sector to shocks as in Scenario 2, but now also with very high leverage. The interaction of these steady-state financial parameters generates an economy that is highly sensitive to shocks to returns. Not only does corporate financial health change markedly following a shock due to the high initial leverage of the sector, but banks also respond more to a given change in corporate
financial health. Charts H1 and H2 show sharp falls in output, investment and asset prices, and a marked rise in external finance spreads, following negative monetary and productivity shocks.

Chart H1
Scenario 3 (monetary shock)

Chart H2
Scenario 3 (productivity shock)

Our fourth scenario models an economy with similarly high leverage as in Scenario 3, but with a much more accommodative banking sector: steady-state finance spreads and bank sensitivity are relatively low. Although the high leverage of this economy means that financial accelerator effects are fairly marked, the banking sector is able to offer a partial offset. As a result, the
quantitative impact of shocks is more muted than in Scenario 3, although still large relative to our base case (see Charts I1 and I2).

Chart I1
Scenario 4 (monetary shock)

Chart I2
Scenario 4 (productivity shock)
Finally, Scenario 5 (Charts J1 and J2) models an economy where financial sector credit supply is relatively restrictive: steady-state spreads and sensitivity to news about corporate financial health are both higher than in the base case. But in contrast to Scenario 3, corporate sector leverage is low. Interestingly, in this world the financial accelerator mechanism has little purchase. Other things equal, the relatively high sensitivity of lending spreads to corporate health would be expected to make the economy sensitive to shocks. But because firms tend to finance much of their investment from internal funds, shocks to rates of return have little impact on lending spreads. But that does not imply that policy-makers should not worry about the financial accelerator in this environment. We view the financial accelerator mechanism as a normal feature of the transmission mechanism in a developed monetary system. In Scenario 5, part of that mechanism is inoperative and policy-makers may need to do more to stabilise the economy than might be suggested by average relationships between the policy instrument and output in other economies.

**Chart J1**
**Scenario 5 (monetary shock)**
Chart J2
Scenario 5 (productivity shock)

Taken together these illustrative scenarios show the importance to BGG model dynamics of the initial financial position of the corporate sector as well as the financial sector’s sensitivity to corporate financial health. The relative importance of the different financial structure parameters in determining the strength of the financial accelerator effect can be seen from Charts K1 and K2. They show that the financial accelerator effect is strongest in Scenario 3 and weakest in Scenario 5.
Chart K1
All scenarios (monetary shock)

Output

Asset prices

Investment

Premium

Chart K2
All scenarios (productivity shock)
At the same time, the charts show that one cannot attribute the strength of the financial accelerator to a single variable. For example, in Scenario 1, the muted impact of the accelerator is primarily a result of the weak responsiveness of the banking sector to changes in corporate balance sheets (net worth). By contrast, in Scenario 5, the financial accelerator is very weak because firms choose not to rely too heavily on external funding. Hence, the unfavourable credit conditions that characterise this scenario have less impact on the firm’s investment decisions. Put differently, we observe that the financial accelerator can be weak both when leverage is low and banks are restrictive in their lending, and when leverage is high and banks are very accommodative.

6 Conclusions

This paper uses the well-known Bernanke, Gertler and Gilchrist (1999) model to explore potential links between the financial health of the corporate sector and bank lending behaviour on the one hand, and the effectiveness of the monetary transmission mechanism on the other. We use the model’s micro-contracting framework to generate specific financial scenarios defined in terms of credit spreads, bank lending policies and corporate financial health. We embed these scenarios in the macroeconomic BGG model and investigate how they affect dynamic responses of the real economy to monetary and real shocks.

Our simulations show that in the context of the BGG model, the monetary transmission mechanism can be affected by the health of the banking and corporate sectors. We illustrate that in certain financial scenarios the financial accelerator mechanism can be very potent, whereas in others it has little incremental impact. Hence for a given shock, monetary policymakers in the model economy can be less or more proactive respectively. In addition, the model simulation results suggest that certain parameters may merit particular attention. For example, the sensitivity of bank lending policy to news about corporate financial health has an especially marked impact on the model’s dynamics. And as illustrated in previous work, leverage also plays an important role in amplifying and propagating shocks. These theoretical results are consistent with real-world experience that bank and non-financial corporate balance sheets can, at times, have a marked impact on the effectiveness of monetary policy.

While the model provides a useful analytical framework for thinking about potential qualitative effects of changes in financial conditions on real variables, as for all calibrated simulated models, its quantitative results need to be interpreted with caution. In fact, we think that there
are reasons to be particularly wary about over interpretation of the results from our exercise on
this model. First, our findings are dependent on the precise microeconomic financial contracting
problem proposed by BGG. Our examination of this part of the BGG model has revealed that
small shifts in the underlying primitive parameters can sometimes lead to large shifts in
scenarios. Second, the real side of the model in our simulations is only stable for a restricted
range of financial scenarios. We think this is not unreasonable: and in reality, shocks
generating large movements in real variables in fragile economies might have feedback effects
on steady-state conditions, invalidating our maintained assumption that steady states are fixed
within scenarios.

In addition, although the BGG model is well suited to an analysis of some important interactions
between financial imperfections and the monetary transmission mechanism, it is by no means
the only way of doing so. It is quite possible that its key quantitative and/or qualitative results
may not be robust to alternative model specifications. Moreover, the model leaves out several
features that one might want to incorporate in a more general model of financial stability. For
example, the model has a relatively limited financial structure with a focus on debt finance.
Firm heterogeneity is also suppressed. Perhaps most importantly, financial institutions are
sparsely modelled with little potential for bank lending channel effects.

Our work has suggested a number of avenues for potential future research. Building on the
existing model, there would be scope for further sensitivity analysis on alternative calibrations,
both in the contracting and real sides of the model. However, the sensitivity of the model
simulations to parameter specification suggests that empirical work—which would need to
consider the potential for non-linearities in financial-monetary interactions—is needed to
examine the robustness of the quantitative and qualitative model findings. Finally, at a
theoretical level, an additional check on robustness would be the development of further
quantitative macroeconomic models, using alternative specifications of the information problem
between borrowers and lenders.
Appendix 1: The firm’s steady-state contracting problem

Closely following the exposition by BGG (1999), we set out the lender’s optimisation problem to provide expressions for the steady-state external finance premium. To do so, we first consider an idiosyncratic shock $\omega$ with a continuous probability distribution $F(\omega)$ over a support $(0, \infty)$ with an expected value of 1. As explained in Section 2, the optimal contract specifies a cut-off value $\sigma$ that determines the division of expected profits between the firm and the lender. In what follows, we show how to determine $\sigma$.

First, define the expected gross share of profits going to the lender as:

$$
\Gamma(\sigma) = \int_{0}^{\infty} \sigma f(\omega) d\omega + \int_{\sigma}^{\infty} f(\omega) d\omega
$$

with $\Gamma'(\sigma) = 1-F(\sigma)$ and $\Gamma''(\sigma) = -f(\sigma)$.

Recall that the lender incurs monitoring costs $\mu$ if the firm defaults. Hence, the lender’s expected net share of profits is given by $\Gamma(\sigma) - \mu G(\sigma)$, where

$$
G(\sigma) = \int_{0}^{\infty} \sigma f(\omega) d\omega
$$

and $\mu G'(\sigma) = \mu \sigma f(\sigma)$. The firm expects to receive $1-\Gamma(\sigma)$, and its optimisation problem can now be rewritten as:

$$
MAX[1 - \Gamma(\sigma)] R^k QK
$$

subject to:

$$
[\Gamma(\sigma) - \mu G(\sigma)] R^k QK = R(QK - N)
$$

Next, define $k = QK/N$ and $s = R^k/R$, and rewrite (A1) (using the assumption of constant returns to scale in the firm’s production function):

$$
MAX[1 - \Gamma(\sigma)] s k
$$

subject to:

$$
[\Gamma(\sigma) - \mu G(\sigma)] s k = k - 1
$$

The first-order conditions for this problem are:

$$
\Gamma'(\sigma) - \lambda [\Gamma'(\sigma) - \mu G'(\sigma)] = 0
$$

$$
[(1 - \Gamma(\sigma)) + \lambda (\Gamma(\sigma) - \mu G(\sigma))] s - \lambda = 0
$$

$$
[\Gamma(\sigma) - \mu G(\sigma)] s k - (k - 1) = 0
$$
where $\lambda$ is the Lagrange multiplier. Assuming an interior solution (see BGG (1999) for the necessary conditions), (A4) can be rearranged to obtain:

$$s = \frac{\lambda}{(1 - \Gamma(\sigma)) + \lambda(\Gamma(\sigma) - \mu G(\sigma))}$$  \hfill (A6)

From (A3), we obtain:

$$\lambda = \frac{\Gamma'(\sigma)}{\Gamma'(\sigma) - \mu G'(\sigma)}$$  \hfill (A7)

Note here that if monitoring costs $\mu$ are zero, then $\lambda = 1$ and $s = 1$. In other words, there is no external finance premium if monitoring is costless.

From the first-order conditions, we also obtain an expression for $k$:

$$k = 1 + \frac{\lambda[\Gamma(\sigma) - \mu G(\sigma)]}{1 - \Gamma(\sigma)}$$  \hfill (A8)

Combining (A6) and (A8), we can express $k$ as a function of $s$:

$$k = 1 + \frac{[\Gamma(\sigma) - \mu G(\sigma)]}{[1 - (\Gamma(\sigma) - \mu G(\sigma))]s}$$  \hfill (A9)

or $k = \psi(s)$ with $\psi'(s) > 0$.

Having derived functional forms for $k$ and $s$, all that remains to be done is to determine the cut-off point $\sigma$. This involves two further steps. First, we define a distance function, being the difference between the optimal $k$ (equation (A9)) and a ‘definitional’ $k$, derived below. Second, this distance is minimised using numerical procedures, which yields a unique value for $\sigma$.

In order to derive the ‘definitional’ $k$, we start with the expression for net worth, evaluated in steady state:

$$N = \gamma V + (1 - \alpha)(1 - \Omega)Y,$$  \hfill (A10)

which is equivalent to equation (5), with $W^e$ replaced by $(1 - \alpha)(1 - \Omega)Y$, the entrepreneur’s wage from participating in the labour market (with $(1 - \alpha)(1 - \Omega)$ being entrepreneurs’ labour share in production). Dividing (A10) through by $K$, (and with $Q = 1$ in steady state), will give the desired expression for $k$ (which is equal to $QK/N$). Consequently, (steady-state) expressions for $V/K$ and $Y/K$ are required.

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$V/K$ can be derived as the entrepreneurs’ share, $1-\Gamma(\sigma)$, of gross profits, $R^kQK$:

$$\frac{V}{K} = (1 - \Gamma(\sigma))\frac{s}{\beta}$$  \hspace{1cm} (A11)

with $\beta = 1/R$.

To obtain $Y/K$, we start by defining the steady-state gross expected return to holding one unit of capital:

$$E(R^k) = E(\frac{MRP_k + Q(1-\delta)}{Q})$$  \hspace{1cm} (A12)

The first term, $MRP$, is the product of the marginal product and the sale price of production, with $\delta$ being the rate of depreciation. Assuming a Cobb-Douglas production function $Y = AK^{\alpha}L^{1-\alpha}$, where $K$ is capital employed, $L$ is labour input and $A$ an exogenous technology parameter, and given a price $X$ for output sold by entrepreneurs, $MRP$ can be written as:

$$MRP = \frac{\alpha Y}{XK}$$  \hspace{1cm} (A13)

Combining (A12) and (A13) (and assuming $Q=L=1$) gives us the following expression, which can then be solved for $Y/K$:

$$E(R^k) = E(\frac{\alpha Y}{KX} + (1-\delta))$$  \hspace{1cm} (A14)

$$\frac{Y}{K} = 1 - \alpha \frac{s}{\beta} - (1-\delta)$$  \hspace{1cm} (A15)

Hence, the ‘definitional’ $k$ can be defined as:

$$k_{def} = \gamma[1 - \Gamma(\sigma)]\frac{s}{\beta} + \frac{(1-\alpha)(1-\Omega)}{\alpha} \frac{s}{\beta} - (1-\delta)$$  \hspace{1cm} (A16)

The distance function is defined as the difference between equations (A9) and (A16). By assuming that $\omega$ is distributed log-normally with mean $-1/2\sigma^2$ and variance $\sigma^2$, we obtain explicit solutions for $\Gamma(\omega)$ and $G(\omega)$ that will enter the distance function. The reader can see from the above that the distance function, and hence the numerical solution for $\omega$, will be a function of a number of primitive parameters that related to both the real side ($\alpha$, $\delta$ and $\Omega$) and the financial side of the economy ($\beta$, $\mu$, $\sigma$ and $\gamma$).

Having solved for the cut-off point $\omega$, (A6) and (A9) can be solved to obtain steady-state values for the external finance premium $s$ and the capital wealth ratio $k$. 

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Appendix 2: The dynamic BGG model

**Aggregate demand**

**Resource constraint**

\[ y_t = \frac{C}{Y} c_t + \frac{I}{Y} \ln a_{t-1} + \frac{G}{Y} g_t + \frac{C^e}{Y} c^e_t \]  \hfill (B1)

**Euler relation for consumption**

\[ c_t = -\sigma r_t + E_t c_{t+1} \]  \hfill (B2)

**Relation between price of external funds and interest rate**

\[ E_t f_{t+1} = r_t - \psi[n_t - (q_t + k_t)] \]  \hfill (B3)

**External finance premium**

\[ s_t = E_t f_{t+1} - r_t \]  \hfill (B4)

**Ex post price of external funds**

\[ f_t = (1 - \varepsilon)(a_t + y_t - k_{t-1}) + \alpha q_t - q_{t-1} \]  \hfill (B5)

**Relation between asset valuations and investment**

\[ E_t q_{t+1} = \varphi(\ln a_t - k_t) \]  \hfill (B6)

**Aggregate supply**

**Production function**

\[ y_t = a_t + 1 - \alpha k_{t-1} + (1 - \alpha) l_t \]  \hfill (B7)

**Labour market equilibrium**

\[ y_t = \gamma l_t - x_t + \gamma c_t \]  \hfill (B8)

**Phillips curve**

\[ E_t \pi_{t+1} = \lambda E_t x_{t+1} + \gamma f E_t \pi_{t+2} + \gamma \pi_t \]  \hfill (B9)


**Evolution of state variables**

**Capital accumulation**

\[ k_t = \delta \text{inv}_t + (1 - \delta)k_{t-1} \]  \hspace{1cm} (B10)

**Investment delay**

\[ \text{inv}_t = \text{ina}_{t-1} \]  \hspace{1cm} (B11)

**Net worth accumulation process**

\[ n_t = \lambda_t - \chi (1 - nk) r_{t-1} - \chi (1 - nk) p_k r_{t-1} - \chi (1 - nk) p_q r_{t-1} + \chi ((1 - nk) p_n + nk) n_{t-1} + \frac{\chi (1 - \gamma r_k) nk}{\gamma} y_t \]  \hspace{1cm} (B12)

**Entrepreneurial consumption**

\[ c^e_t = kn^e_t - kn (1 - nk) r_{t-1} - kn (1 - nk) p_k r_{t-1} - kn (1 - nk) p_q r_{t-1} + kn ((1 - nk) p_n + nk) n_{t-1} \]  \hspace{1cm} (B13)

**Monetary policy rule and shock processes**

**Real interest rate**

\[ r_t = r^n_t - E_t \pi_{t+1} \]  \hspace{1cm} (B14)

**Monetary policy rule**

\[ r^n_t = \rho r^n_{t-1} + (1 - \rho) \gamma_{\pi} \pi_{t+1} + \eta_{t} \]  \hspace{1cm} (B15)

**Government spending process**

\[ g_t = \rho_g g_{t-1} + \eta_{g_t} \]  \hspace{1cm} (B16)

**Technology process**

\[ a_t = \rho_a a_{t-1} + \eta_{a_t} \]  \hspace{1cm} (B17)
### Variables

| y  | output       | k  | capital             |
| c  | consumption  | x  | marginal cost       |
| ina | investment   | l  | labour              |
| g  | government spending | π  | inflation            |
| r  | real interest rate | r^n | nominal interest rate |
| n  | net worth     | inv | investment delay variable |
| f  | expected price of capital | a  | technology |
| e  | entrepreneurial consumption | q  | Tobin’s Q (price of capital) |
| s  | external finance premium | η'  | Additive shock to r^n |
| η^g | additive shock to g | η^a | Additive shock to a |

### Selected parameter values and features of steady state (SS)

**‘Standard’**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Baseline value</th>
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<td>C/Y</td>
<td>SS consumption-output share</td>
<td>Varied in simulations</td>
</tr>
<tr>
<td>I/Y</td>
<td>SS investment-output share</td>
<td>Varied in simulations</td>
</tr>
<tr>
<td>G/Y</td>
<td>SS government-output share</td>
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<tr>
<td>C'/Y</td>
<td>SS entrepreneurs’ consumption-output share</td>
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</tr>
<tr>
<td>σ</td>
<td>Consumption elasticity of substitution</td>
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<tr>
<td>ε</td>
<td>Parameter on marginal product in investment demand</td>
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<tr>
<td>φ</td>
<td>Elasticity of price of capital to investment/capital ratio</td>
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<tr>
<td>α</td>
<td>Capital share</td>
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<td>(1-α)</td>
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<tr>
<td>γ_l</td>
<td>Coefficient on labour (1+1/labour supply elasticity)</td>
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<td>γ_c</td>
<td>Coefficient on consumption in labour market equation (1/σ)</td>
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<tr>
<td>λ</td>
<td>Parameter on marginal cost in Phillips curve</td>
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<td>γ_f</td>
<td>Coefficient on forward-looking inflation in Phillips curve</td>
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<tr>
<td>γ_b</td>
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<tr>
<td>δ</td>
<td>Quarterly depreciation rate</td>
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<tr>
<td>ρ</td>
<td>SC parameter for interest rate in policy rule</td>
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<td>(1-ρ)γ_f</td>
<td>Coefficient on inflation in policy rule</td>
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<tr>
<td>ρ_g</td>
<td>SC parameter for government spending shock</td>
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<td>ρ_a</td>
<td>SC parameter for technology shock</td>
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<td>β</td>
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<td>θ</td>
<td>Probability of price change</td>
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**‘Non-standard’**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Baseline value</th>
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<tr>
<td>R'-R</td>
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<td>Varied in simulations</td>
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<tr>
<td>F(w)</td>
<td>SS business failure rate</td>
<td>Varied in simulations</td>
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<td>nk</td>
<td>SS leverage ratio</td>
<td>Varied in simulations</td>
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<tr>
<td>kn</td>
<td>1/nk</td>
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<tr>
<td>X</td>
<td>Parameter on net wealth accumulation</td>
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<tr>
<td>1-γ</td>
<td>Death rate of entrepreneurs</td>
<td>Varied in simulations</td>
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<td>Log(w)</td>
<td>Productivity log normally distributed</td>
<td>Varied in simulations</td>
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<tr>
<td>m</td>
<td>Share of payoffs lost in bankruptcy costs</td>
<td>Varied in simulations</td>
</tr>
<tr>
<td>ψ</td>
<td>Elasticity of external finance premium to leverage</td>
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References


