The role of expectations in estimates of the NAIRU in the United States and the United Kingdom

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Abstract

During the second half of the 1990s the US economy was characterised as the Goldilocks economy: not too hot, nor too cold, but just right. It was argued that this represented a new paradigm, enabling unemployment to remain low without igniting inflationary pressure. In this paper the evidence for a change in the relationship between inflation and unemployment is examined and the US experience compared with that of the United Kingdom within a common analytical framework. To that end, Phillips-curve models are employed based on estimates of time-varying NAIRUs, obtained using the Kalman filter. The impact of including explicit inflation expectations is also considered. This channel has not been explored in previous work based on Kalman filter estimates of the NAIRU for the United States and United Kingdom. Inflation expectations are found to play a particularly important role in the United States. When expectations are included there is still evidence that the NAIRU steadily declined during the late 1990s, although this decline in the US NAIRU is not found solely in the 1990s.

Key words: Phillips curve, inflation expectations, Kalman filter.

JEL classification: E24, E31.

Summary

During the second half of the 1990s there were similarities in the performance of the US and UK economies. In particular relative to their recent past, both economies were characterised by stronger growth, falling unemployment and both low and more stable inflation. This combination led commentators to label the United States (where these developments were more pronounced) as the Goldilocks economy: one which was neither too hot, nor too cold, but just right. This paper examines the evidence for a change in the relationship between inflation and unemployment in the United States and United Kingdom between the 1990s and earlier periods. The paper contains a potentially important innovation by incorporating an explicit role for inflation expectations derived from survey measures.

All the results for the United States suggest that the non-accelerating inflation rate of unemployment (NAIRU) was steadily declining during the second half of the 1990s. However, inflation expectations are found to play a particularly important role in the United States and when expectations are included our results show that a declining NAIRU is not found solely in the 1990s. On the same basis, our results suggest that the timing of any change in the United Kingdom occurs somewhat later than in the United States. As our modelling strategy uses a reduced-form estimate of the NAIRU, we cannot identify exactly which factors trigger any changes. In addition, one important caveat is that there is typically a high degree of uncertainty surrounding NAIRU estimates. For this reason our results should be seen as illustrative rather than precise point estimates. Furthermore, these estimates should not be interpreted as MPC estimates.

There are two types of explanation for the combination of low inflation and stronger growth witnessed in the United States and the United Kingdom in the second half of the 1990s: favourable shocks or structural shifts. If it was the result of favourable external supply-side shocks, such as falling energy prices, supply potential as well as the relationship between inflation and excess demand factors will be unchanged. The alternative involves a lasting change in the relationship between inflation and excess demand, now known by the collective label of new paradigm economics. Here, such manifestations can take two main forms. The first involves changes in an economy's supply potential, such as an increase in potential growth or a fall in the NAIRU, which alters the level of excess demand for a given level of actual demand. Alternatively, it could imply a change in the relationship between inflation and a given level of excess

demand, for example due to changes in the behaviour of margins resulting from competition. Of course, the true explanation may well be a combination of both favourable supply shocks and structural changes; the paper allows a role for both.

The framework used here relates inflation to a combination of inflation inertia, demand-side factors (provided by the gap between unemployment and the NAIRU) and external exogenous supply shocks (provided by real oil prices and real import prices). Most of the underlying models for this framework assume that price-setters are forward looking, so expectations will be an important determinant of behaviour. However, explicit measures of expectations are not normally included in estimation. When the regime is stable, agents' inflation expectations can be modelled using the actual and lagged values of the variables in their information set, so including an explicit measure of expectations should not matter. However, when the regime has changed (either because of a policy shift or change in the competitive environment), it will be important to include inflation expectations explicitly. This channel has been largely ignored and we try to assess how important this omission may have been.

The evidence is obtained using Kalman filter techniques, which allow the joint estimation of the Phillips curve and a time-varying measure of the NAIRU. As well as representing one of the tools in the policy-makers' tool kit, the use of the Kalman filter has the advantage of providing some direct evidence on whether the NAIRU had in fact fallen in the second half of the 1990s. Such statistical estimates are independent of the correct identification and estimation of the structural factors underlying a fall. The latter are important for pinning down exactly how a fall may differ from past observed falls in the NAIRU. Of course, time-varying NAIRUs are one of several indicators that can be used to interpret movements in the labour market.

1 Introduction

During the second half of the 1990s there were some similarities in the performance of the US and UK economies. In particular, relative to their recent past, both economies were characterised by strong growth, falling unemployment and both low and more stable inflation. This combination led commentators in the US (where these developments were more pronounced) to label it as the Goldilocks economy: one which was neither too hot, nor too cold, but just right (Gordon (1998)). From a policy perspective the late 1990s combination of strong growth, and low inflation and unemployment is obviously an attractive one, if it is sustainable.

There are two main classes of explanation for the combination of low inflation and strong growth witnessed in the UK and US. The first of these focuses solely on the impact of external supply-side shocks, such as falling energy prices (or more generally import prices), but with an unchanged path for supply potential and an unchanged relationship between inflation and excess demand factors. The second class of explanation, while acknowledging the importance of the first mechanism, suggests that there are additional factors at work which may have changed the relationship between inflation and output or unemployment. Such arguments came to be known under the collective label of new paradigm economics, although the meaning and even the timing associated with this label varied quite substantially among commentators. In this context, manifestations of a new paradigm can take two main forms. The first of these involves changes in an economy's potential, for example through an increase in potential growth, or a fall in the non-accelerating inflation rate of unemployment (or NAIRU).[(1)] Such changes will alter the level of excess demand for a given level of actual demand. Alternatively, the effects can manifest themselves as changes in the relationship between inflation and a given level of excess demand, for example because of changes in the behaviour of margins due to competition effects. These could be indicated either by persistent overprediction of inflation for a given model, or by shifts in the estimated relationships as the sample period for estimation changes.[(2)]

[(1)] See King (1999) for a discussion of the non-accelerating inflation rate of unemployment, or NAIRU.
[(2)] Stock (1998) suggests that an even more radical interpretation of the new paradigm is that the traditional Phillips curve is no longer relevant, so that there is no longer any relationship between inflation and the unemployment gap.

The aim of this paper is to determine to what extent (if at all) the above explanations applied in the US and UK.[3] In examining the evidence, the paper concentrates on the relationship between inflation and unemployment. This may seem surprising, given that much of the discussion of the causes of the new paradigm were couched in terms of the impact of factors such as IT on total factor productivity and therefore growth. However, both Gordon (1998) and Stock (1998) found that the biggest puzzles in explaining inflation for the US occur in connection with the labour market, rather than in the relationship between inflation and output gaps (measured using capacity utilisation). The evidence is obtained using Kalman filter techniques which allow the joint estimation of the Phillips curve and a time-varying measure of the NAIRU. As well as representing a common tool in the policy-makers' tool kit, see Bank of England (1999), the use of the Kalman filter has a key advantage, namely to provide some direct evidence on whether the NAIRU was in fact falling over the second half of the 1990s. Such statistical estimates are independent of the correct identification and estimation of the structural factors underlying any fall, which are of course important for pinning down exactly how it may differ from past observed falls in the NAIRU.[4] Of course given the high degree of uncertainty surrounding any NAIRU estimates our results should be seen as purely indicative.

In using the Kalman filter Phillips curve to examine the evidence for a changing inflation-unemployment trade-off this paper is closely related to Gordon (1998). We also adopted the so-called triangle model used in Gordon (1998) where inflation is explained by a combination of inflation inertia, demand-side factors (given by the gap between unemployment and the NAIRU) and external exogenous supply shocks (in this case given by real oil prices and real import prices). Gordon (1998) and Rich and Rissmiller (2000) find this latter type of factor important in eliminating a sizable proportion of any apparent puzzle in the inflation data for the US.[5] This paper, however, contains a potentially important innovation relative to the modelling approach taken in Gordon (1998). This involves incorporating an explicit role for inflation expectations derived from survey measures. Underlying most of the New Keynesian literature on the Phillips curve is the assumption that the existence of nominal inertia will force price setters to be

[3] Previous comparisons of Phillips curve models for the UK and the US include Haldane and Quah (1999) and Laubach (1999).
[4] See Cassino and Thornton (2002) for an alternative approach to estimating NAIRUs.
[5] Gordon (1998) uses food and energy, and import prices to capture traditional supply-side shocks.

forward looking. In cases where the regime is stable then agents' inflation expectations can be modelled using the actual and lagged values of the variables in the system which form their information set. Then assuming that important variables have not been omitted from the model, it should not matter whether an explicit measure of expectations is used or not. However, in cases where the regime has changed (be that as a result of policy shifts or changes to the competitive environment), it will be important to include explicit measures of inflation expectations. Although the role of inflation expectations has been analysed using more traditional econometric techniques, this channel has been largely ignored within the Kalman filter framework.[6] We therefore try to assess how important this omission may have been in the analysis of inflation puzzles.

The aims of the paper are therefore to explore whether there was a change in the inflation-unemployment relationship in the US and UK, and what role expectations may have played. The structure of the paper is as follows. Section 2 discusses the theoretical background. Section 3 provides a description of the methodology used. The results themselves are presented in Section 4. Finally, Section 5 concludes.

2 Theoretical background

Stories of new paradigm economics have rested on the assumption (explicit or implicit) that there was a breakdown in the relationship between inflation and demand. This could have occurred either because the underlying potential changed, or because the relationship between inflation and a given level of excess demand changed. Whatever its cause, the breakdown would take the form of a reduced level of inflation for a given level of actual demand. Many explanations were advanced for the appearance of a benign inflation outlook in the US, see in particular the discussion in Gordon (1998). These arguments included whether the IT revolution raised the rate of trend productivity growth for the whole economy, reducing the inflationary pressure associated with given levels of growth and unemployment;[7] the impact of globalisation, which may increase competition, so reducing margins; and whether changes in labour market conditions such

[6] One exception to this is Gruen *et al* (1999) who examine the role of inflation expectations derived from bond markets in the behaviour of the Australian Phillips curve.
[7] See Oliner and Sichel (2000). This type of factor is most closely associated with the so-called new economy.

as heightened job insecurity played a role.[8] Although the causes of the new paradigm are extremely important, rather than investigate these directly the aim of this paper is somewhat simpler, namely to shed light on to whether there was any change in the trade-off in the second half of the 1990s.

The starting point for the investigation is the triangle model of Gordon (1997 and 1998) where inflation depends on a combination of inertia in inflation, demand pressures, and exogenous supply shocks.[9] In other words a simple, backward-looking Phillips curve model.[10] The measure for demand pressure is provided by the deviation of unemployment from a time-varying NAIRU, which is estimated simultaneously with the rest of the model using Kalman filter techniques.[11]

Theory suggests, however, that in models which exhibit price stickiness inflation expectations will play an important role, as agents will have to take explicit account of their expectations about future prices when making decisions.[12] This will induce forward-looking dynamics in the Phillips curve. Such models have been christened New Keynesian Phillips curves. In cases where both the structure of the economy and policy regimes have remained constant, inflation expectations can be represented as a function of existing historical relationships. In such cases, assuming the model incorporates all the variables that agents use when forming their expectations, it is perfectly possible to use backward-looking data to model expectations. If, however, agents perceive that either there has been a structural change in the economy, or that the policy regime has shifted, then they will adjust how much weight they place on different pieces of information

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[8] Brinner (1999) for example discusses the impact of changes in the demographic (sex-age) composition of the labour force in this context. Katz and Krueger (1999) consider additional demographic influences, including for example the impact of the rise in the US prison population. These are more traditional arguments for changes in the NAIRU.

[9] A single measure of demand pressure is used in the estimation in this paper. For a discussion of the implications of multiple gaps see Astley and Yates (1999).

[10] The term Phillips curve originally arose from the relationship between money wage growth and unemployment documented in Phillips (1958).

[11] An alternative formulation of the Phillips curve would have been in terms of the output gap, or the difference between actual and potential output, see for example Kuttner (1994). However, Gordon (1998) and Stock (1998) both find that the biggest puzzles in explaining inflation in the US occur in connection with the unemployment gap, rather than the output gap (measured using a time-varying non-accelerating inflation rate of capacity utilisation). For this reason this paper concentrates on the former.

[12] Of course, the inclusion of expectations within the Phillips curve is not solely the province of New Keynesian economics. For example, Friedman (1968) also emphasised the role of expectations, although of contemporaneous not future prices.

accordingly. In such cases it will be important to include explicit measures of inflation expectations. This paper therefore investigates the implications of an explicit role for inflation expectations, which has not been explored in previous work using the Kalman filter for the US and the UK. Existing work investigating this type of channel using alternative econometric techniques tends to find that it can be important. See for example Roberts (1995 and 1997) and Galí and Gertler (1999) for the US.

Simple New Keynesian Phillips curves can be derived from a number of models of wage and price behaviour, see for example Roberts (1995 and 1997).[13] In these models, the inflation dynamics consist entirely of forward-looking elements. In practice, however, inflation dynamics are likely to be composed of both forward and backward-looking elements, so that inflation inertia exists and shocks have more protracted effects. One such model is given by Fuhrer and Moore (1995) where agents set nominal wages to deliver a desired real wage and where wage contracts are of the form given by Taylor (1980). The sum of the parameters on the backward-looking terms provides a measure of inflation stickiness. The evidence on whether inflation is sticky is mixed. On the one hand, Roberts (1997) finds that in the US it is not inflation, but prices, which are sticky. On the other hand, Fuhrer (1997) finds that the coefficient on expected inflation in the US is insignificant in a regression which includes both forward and backward-looking inflation terms, but the inclusion of expected inflation significantly improves the model simulation properties.

3 Methodological approach

This paper compares the inflation performance of the UK and the US and the impact of including explicit measures of inflation expectations within a Phillips curve framework. In each case the results are obtained using a Kalman filter to estimate the unobservable level of the time-varying NAIRU (TV-NAIRU) at the same time as estimating the coefficients associated with the Phillips curve model.[14] The models tested in this paper are therefore of the form:

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[13] The examples of price stickiness considered include the existence of menu costs (as in Rotemberg (1982)), or fixed contracts (as in Taylor (1979 and 1980) and Calvo (1983)). Although the arguments for menu costs, which result in state dependent pricing decisions, are persuasive, the aggregate implications of this type of set-up are in fact unclear, see for example Caplin and Spulber (1987).

[14] Appendix A contains a technical description of Kalman filter estimation.

$$\pi_t = g(L)\pi_{t-1} + \alpha\pi^e{}_{t+4} + b(L)(U_t - U_t^N) + c(L)z_t + \varepsilon_t$$
$$U_t^N = U_{t-1}^N + \eta_t,$$
$$\varepsilon_t \sim N(0,\sigma_\varepsilon^2), \qquad \eta_t \sim N(0,\sigma_\eta^2), \qquad \text{cov}(\varepsilon_t,\eta_t) = 0.$$

where: $g(L)$, $b(L)$ and $c(L)$ are lag polynomials; annual price inflation is given by π, U is unemployment and U^N is the time-varying NAIRU; z represent exogenous supply-side shocks (namely the log of real oil prices and the log of real import prices).

The Kalman filter works by estimating the equations for inflation and the NAIRU simultaneously, where the NAIRU is allowed to vary over time. It works by specifying hypothesised behaviour of the unobservable level of the NAIRU, in this case a simple random walk over time, at the same time as allowing the behaviour of the gap between unemployment and the NAIRU to help explain inflation.[15] In the terminology of Gordon (1998) the TV-NAIRU calculated here can be thought of as a no-shock NAIRU, or the level of unemployment which would have been consistent with stable inflation in the absence of exogenous supply-side shocks.[16] We consider two possible sources of exogenous external supply-side shocks: real oil prices and real import prices. The shocks are presented in real terms in order to preserve dynamic homogeneity. In addition, the shocks have been normalised so that on average their impact over the sample period is zero. Once the impact of these shocks on inflation has been accounted for the NAIRU is then calculated to be the level of unemployment which is compatible with stable inflation.

There is no universally agreed rule as to how to choose the optimal signal-to-noise ratio. Here the variability of the estimated NAIRU is restricted so that the resulting NAIRU is relatively smooth over time.[17] Gordon (1998) argues that this is important because

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[15] In terms of the discussion in Appendix A, the equation for inflation can be thought of as the measurement equation, while that for the unobservable level of the NAIRU is the transition equation.
[16] We consider the estimates derived in this paper to be of the long-run NAIRU since our models are reduced-form and do not explicitly include structural variables (such as the replacement ratio or union density) that determine the natural rate. The reason these measures are long-run rather than short-run NAIRUs is that we allow for temporary supply shocks. The estimates of the NAIRU that we show in this paper are the smoothed NAIRU estimates (which use the information set for the whole sample) rather than the filtered estimates (which only use the information available at the time the forecast was made).
[17] Stock and Watson (1998) do provide some suggestions as to how this parameter could be estimated. However, in part to allow for greater comparability with earlier results, the values chosen here reflect parameters commonly used in the literature. Another alternative would be to estimate freely, although this would substantially increase the volatility of the estimated NAIRU.

economic theory would suggest that it is implausible to expect the underlying potential of the economy to jump around over time. Within the Kalman filter framework this restriction is imposed by restricting the relative variance of the error terms $(\sigma_\eta^2 / \sigma_\varepsilon^2)$.[18] Increasing the relative variance acts to increase the volatility of the calculated NAIRU. We impose the relative variance of the models to be 0.16 in the case of the UK and 0.04 for the US. A variance of 0.04 corresponds to the majority of the results for the US presented in Gordon (1997). A value of 0.16 is also investigated in Gordon (1997) and was used for the UK based on the analysis on Greenslade *et al* (2003).[19] Of course different assumptions will yield different estimates of the NAIRU, which is another reason why the estimates contained in this paper should not be considered as point estimates.

In the long run the Phillips curve is assumed to be vertical, so that there is no long-run trade-off between inflation and real activity. To achieve this long-run neutrality, the sum of the coefficients on the inflation terms, α and $g(L)$, must equal unity. In cases where inflation expectations are omitted α is set to zero. In view of the mixed conclusions about whether it is inflation or prices which display inertia, the results here follow the more general specification and allow for inflation inertia. There is no theoretical restriction on the timing of the forward-looking term in inflation expectations (π^e), since it will be determined by factors such as the length of contract. However, because both measures of inflation expectations used in estimation are for inflation one year ahead, the timing of the inflation expectations term has been set to (t+4). Where inflation expectations are included, these expectations are derived from surveys and the details of these measures can be found in Appendix B.[20] Roberts (1995 and 1997) discusses some of the issues which arise when survey data is used to proxy inflation expectations. These include assessing their forecast performance and whether these expectations are rational. Croushore (1993) presents evidence of the forecast performance of the Survey of Professional Forecasters. Bank research by Bakhshi and Yates (1998) on the monthly Gallup series for inflation expectations one year ahead has clearly demonstrated that

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[18] The variance of the error in the inflation equation (σ_ε^2) is normalised to unity for convenience.

[19] Greenslade *et al* (2003) show that when the signal-to-noise ratio for the UK is freely estimated the value of 0.16 used in this paper is smaller than the point estimate, but lies within the confidence interval. As the resulting NAIRUs display large variation we maintain the lower figure.

[20] Even when inflation expectations are included it should be realised that observed changes in the NAIRU may still reflect expectational errors.

individuals' expectations of prices do not behave in a 'rational' fashion. Although individuals' forecasts of future inflation do not permanently diverge from true inflation, they can over or underpredict it systematically and persistently by a constant amount. This persistence can be clearly seen in Charts 1 and 2 which show the difference between expected inflation and the actual outturns and is clearly even more of an issue in the US than in the UK.[21] It should be realised that measures of expectations from different sets of agents may present a different picture. This could be one source of the differences between the results for the US and UK. The reason the measures used in the paper are chosen is because they have the longest available sample period.

In order not to prejudice the conclusions on the importance of expectations, two testing strategies are pursued. The first strategy starts with a model which includes inflation expectations and then tests down using a general-to-specific modelling strategy. The results from this baseline are then compared to what happens if inflation expectations are excluded from the model (and the parameters are reestimated). The second strategy is to start from a model without expectations and to test down using the general-to-specific methodology. Inflation expectations are then added to the model and the parameters reestimated. This exercise is conducted to ensure that the results of the models presented are not biased by the choice of the initial model.

One important issue is the lag structure of the model. For both countries it proves important to include several lags of inflation, whether or not inflation expectations are included in the model. This is consistent with earlier findings using the Kalman filter to estimate backward-looking models. For example Gordon (1998) and Brayton *et al* (1999) both consider lag lengths of up to 24 quarters and restrict the number of estimated coefficients by placing restrictions on how these lags behave. However, in view of the shorter sample period considered here (the size of the sample is restricted by the availability of inflation expectations data), this paper uses shorter lag lengths but the estimated coefficients are left unrestricted as in Staiger *et al* (1997b). Another issue is

[21] Particularly for the US, the persistent overprediction of inflation suggests that it is not possible to explain the new paradigm in terms of increased credibility of monetary policy. A possible caveat to this would be if expectations were always determined by the inflation target, so that the central bank was always perfectly credible, but that actual inflation continually undershot the target. The question in those circumstances would be why do expectations not adjust.

whether lags of the unemployment gaps should be included. The theory underlying the New Keynesian Phillips curve is somewhat agnostic on this point, as the dynamics depend on the type of rigidities assumed. Gordon (1997) argues that lagged gaps should be included, as the rate of change of the gap may also be important. Greenslade *et al* (2003) show that the UK NAIRUs based upon various lag structures for the unemployment gap are virtually unchanged. The results in this paper therefore do not include lagged gaps for simplicity.

4 Results

The next two subsections present detailed results for the US and UK respectively. Essentially in each case three questions are being asked. The first is simply does the empirical evidence suggest that there is a role for inflation expectations within a Phillips curve relating inflation to unemployment. The second question is does there appear to be evidence that the degree of inflationary pressure has changed for a given level of actual unemployment, which for the late 1990s was often taken as a symptom of the new paradigm.[(22)] A final question is to what extent is the answer to the second question affected by the inclusion of expectations. In addition to this the sensitivity of the results to different assumptions is also examined. Details of the variables used in estimation can be found in Appendix B. The charts and tables from the estimation are contained in Appendix C.

4.1 The US

The first thing to notice from the results for the US, which are presented in Table A, is that they provide a clear indication of the importance of the inflation expectations channel. Inflation expectations are strongly significant and enter with a coefficient of almost 0.3. This is true both for model 1E, which tests from a general-to-specific model including expectations from the start, and model 2E where expectations are inserted into a model obtained without including expectations initially. The log likelihoods are also significantly greater in the models with expectations than those where expectations are

[(22)] The actual timing of the start of the new paradigm in the US is somewhat vague. One possibility would be the start of the recent strong productivity growth in 1996. However, Stock (1998) finds evidence of a breakdown in the relationship between inflation and unemployment occurring as early as 1992 Q4. An alternative explanation might therefore relate to the increased importance of globalisation.

excluded. This is particularly true in the case of the choice between models 1E and 1. The inclusion of expectations also leads to a significant reduction in the size of the coefficient on the unemployment gap, though it remains statistically significant, see Table A.

Given the size of the coefficient on expectations and in particular of the increase in the likelihood associated with including expectations, it seems helpful to explore the role of expectations somewhat further. To do this we reestimated all four models (1, 1E, 2 and 2E) without imposing the restriction that the sum of the coefficients on the inflation terms must equal unity (or dynamic homogeneity). The results from this exercise are presented in Table B and prove to be quite revealing. The sum of the coefficients on the inflation terms is presented in the first row. In both models where inflation expectations are included the sum of the coefficients is very close to unity (0.98 and 0.99 respectively). In the models without expectations the sum of the coefficients is less than 0.9. As the model with dynamic homogeneity imposed is much closer to the unrestricted model in the case of expectations, imposing the constraint has very little impact on the log likelihood.

The next question is therefore to what extent these models provide evidence of a changing relationship between unemployment and inflation and whether that evidence is affected by the inclusion of expectations. A change in the trade-off could take several forms: a fall in the calculated NAIRU; persistent overprediction of inflation compared to actual outturns,[23] and evidence of a breakdown in the estimated Phillips curve. One issue is going to be the timing associated with any changes. In the context of the relationship between inflation and unemployment Gordon (1998) speculates that the important changes occurred in 1994 when unemployment fell below 6% without an associated rise in inflation. This is because 6% was the level which many, including Gordon himself, had thought represented the level of the (constant) NAIRU for the US. Stock (1998) finds that there is some evidence of a structural break in the inflation-unemployment relationship as early as 1992 Q4. We examine each of these aspects in turn.

Charts 3 and 4 present the estimated NAIRUs associated with the two modelling strategies. In all cases the estimated NAIRUs were falling for much of the 1990s and

[23] As the statistical technique used will effectively translate any persistent negative residuals into a fall in the estimated NAIRU this test may well not yield any clear answers. The extent of any deviations will in part be determined by the signal-to-noise ratio. Increasing the variance reduces the persistence of any deviations.

would be consistent with a new paradigm story.[24] The estimated NAIRU is a statistical construct, rather than something estimated directly from structural factors, and in that sense its fall is unexplained and suggests that inflationary pressure within the US has declined for a given level of actual unemployment. What is noticeable from these charts however, is the impact of including expectations on the estimated profile of the NAIRU. When expectations are excluded, the NAIRU is fairly flat, and if anything rising slightly in the early part of the 1990s and only starts to fall in late 1993, early 1994. This coincides with the timing of Goldilocks' appearance in Gordon (1998). In contrast, when expectations are included, the fall in the NAIRU in the recent past began as early as 1990, long before most economists would point to a new paradigm. Indeed, it would be possible to look at the NAIRU estimates which are obtained when expectations are included and suggest that on average the NAIRU has been falling (albeit very gradually) since at least the mid-1970s, suggesting that the question might be why should the new paradigm have been seen as new?[25]

One factor affecting the shape of the NAIRU is of course the nature of expectations. As Chart 1 shows agents (in this case professional forecasters) have persistently overpredicted inflation in the US, but the extent of this overprediction has been falling. Indeed by the end of the sample period agents are underpredicting inflation. The implication of overpredicting inflation is that the level of actual unemployment, for a given level of the NAIRU, will be temporarily higher. If this overprediction of inflation is reduced therefore, the associated decline in the gap between actual unemployment and the NAIRU could be long-lasting.[26]

Gordon (1998) and Stock (1998) used a persistent overprediction of inflation to identify the existence of an inflation puzzle, by showing the degree to which their models overpredict inflation in the four quarters up to 1998 Q2.[27] While the residuals have been

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[24] When expectations are included the NAIRU stabilises towards the very end of the 1990s, albeit at a lower level.
[25] Results in Ball (1997) using an alternative approach also suggest that the NAIRU in the US may have fallen since the early 1980s.
[26] The Kalman filter works by using the actual level of unemployment to estimate the NAIRU as part of an equation for inflation, instead of estimating the NAIRU directly using structural methods. For this reason, the overprediction of inflation will lead to a fall in the observed NAIRU and is one reason why the NAIRUs estimated including inflation expectations tend to be below more traditional estimates of the NAIRU for much of our sample.
[27] An overprediction of inflation would imply negative residuals.

slightly negative on average since the mid-1990s, the evidence for this is very marginal. Looking at the period used for the contributions exercise in Gordon (1998) the four-quarter average to 1998 Q2 for the residuals in our work is –0.24 for Model 1 which excludes expectations and –0.41 for Model 1E which includes expectations. If the exercise had been done for the four quarter period up to 1999 Q4 then the outcome becomes –0.36 for Model 1 (excluding expectations) and –0.14 for Model 1E (including expectations). However, in the intervening period in both cases there are periods when the four-quarter average for the residuals turns positive.[28] Including expectations does not appear to have a big impact on the evidence for the overprediction of inflation as the results are largely similar whether or not expectations are included.

Another way to demonstrate whether the US showed any evidence of being subject to new paradigm forces would be to establish whether there had been a breakdown in the structure of the estimated Phillips curve. In order to do this we re-estimated Models 1 and 1E for three additional sample periods. In each case the sample period starts in 1975 Q1, while the chosen end-points are 1998 Q2 (to act as a comparison to Gordon (1998)), 1994 Q4 (which is when Gordon (1998) dates the start of the new paradigm) and 1992 Q4 (which is when Stock (1998) identifies a structural break in the Phillips curve). The results from this exercise can be found in Table Ca for the case excluding expectations and Cb for the case including expectations. What is particularly striking is the stability of the estimated coefficients on both the unemployment gap and the inflation terms over time.[29] Only for Model 1 for the sample ending in 1992 Q4 is there any real sense of a shift in the coefficients taking place. Charts 5 and 6 present the associated NAIRUs. For Model 1, without expectations, it is difficult to distinguish the NAIRUs at a given point in time from the estimates obtained when the sample ends in 1992 Q4 or 1994 Q4. However, at each point in time there does seem to have been a downward shift in the calculated NAIRUs from around 1993 if the sample is extended beyond 1994 Q4. The results for Model 1E, including expectations, produce a fall in the estimated NAIRUs for a given period each time the sample period is extended, although these falls are most noticeable when the sample period is extended beyond 1994 Q4. This again potentially suggests differences in timing.

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[28] This is to be expected, given that we are using the Kalman filter approach and may suggest that this test is less useful than the other two approaches that we have used.

[29] The coefficients on the exogenous supply-side shocks have been omitted from Table C in order to save space. However, these coefficients are also remarkably stable over time.

The results for the US only provide support for a new paradigm interpretation to the extent that the NAIRU fell during the 1990s, but they also indicate that a declining NAIRU was not unique to that decade, even though the end-points are consistently lower than at any time during the past 25 years. Including expectations reduces the estimates of the NAIRU almost across the whole period and brings forward the most recent decline to the beginning of the 1990s. The average NAIRU for 1999 for the models shown in Table A is around 4.5% for those which include expectations and 4.7% when expectations are excluded. These estimates are at the bottom end of the range of estimates of between 5.6 and 4.5 for 1999 quoted in Meyer (2000) and suggest there may be less evidence of inflationary pressure from the demand side than suggested by some other estimates. For the two models which exclude expectations (Model 1 and 2) the estimated TV-NAIRU has been above actual unemployment since around mid-1997. In contrast when expectations are included the NAIRU only rises above actual unemployment at the very end of the sample period.[30] Of course one caveat on placing too much emphasis on changes in the estimated NAIRUs is the degree of uncertainty associated with NAIRU estimates. Staiger *et al* (1997a), calculate (using a constant, rather than time-varying NAIRU) that the 95% confidence interval for the NAIRU falls between 4.3% and 7.3%, which clearly suggests a high degree of imprecision. Laubach (1999) and Greenslade *et al* (2003) find evidence that the confidence intervals associated with TV-NAIRUs can be similarly wide. This uncertainty is confirmed by looking at the confidence intervals associated with Model 1E (see Chart 7).[31] However, Gordon (1997) argues that what is important is not the level of the NAIRU *per se*, but the ability of this type of model to say something about inflation in the sense that a consistent estimate over time provides a benchmark to judge relative inflationary pressures irrespective of the level. Nonetheless, while this information may still be useful, the results do suggest not to place too much emphasis on a particular point estimate.

For the US, some additional sensitivity analysis is also conducted. This investigates the impact of changing to a variance of 0.16 from 0.04. A variance of 0.04 corresponds to the majority of the results presented in Gordon (1997). However, a value of 0.16 is also investigated in Gordon (1997) and so we investigate the sensitivity of the results to

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[30] The sample period ends in 2000 Q4.

[31] We calculate the confidence intervals for the NAIRU using only the filter uncertainty that is calculated from the smoothing iteration. This represents the uncertainty that would be present even if true values of the parameters were known.

changing this estimate. The estimation results from this can also be found in Table D. Chart 8 shows the impact of changing the variance on the calculated TV-NAIRU for Model 1 and Chart 9 shows the impact for Model 1E. As can be seen, increasing the variance to 0.16 significantly increases the volatility of the calculated NAIRU, particularly for the model excluding expectations. See Greenslade *et al* (2003) for a similar sensitivity analysis in the case of the UK.

4.2 The UK

The results for the UK are very similar to one another whichever testing strategy is used, see Table E. The lag structures remain similar whether a strategy of testing down from a model including expectations (Model 3E) or without expectations (Model 4) is used. For a given model, excluding expectations (moving from 3E to 3) or including expectations (moving from 4 to 4E) also seems to produce only minor differences in the implications for inflation. The differences in the two sets of time-varying NAIRUs are quite small (see Charts 10 and 11). In both cases (3E and 4E) the coefficient on inflation expectations is only around 0.1 and in the case of Model 4E it is only significant at the 90% level. On balance the results for the UK are therefore borderline as to whether or not inflation expectations should be included. Within these models inflation expectations appear to play a less important role in the UK than in the US.

The evidence for the existence of new paradigm forces at work in the UK is slightly more mixed than in the US. Certainly the timing of any change is likely to have been somewhat different. In both the US and the UK the estimated NAIRUs started to decline in the early part of the 1990s. In the US this decline continued unchecked until around 2000, when the estimates suggest that the NAIRU stabilised. In the UK (particularly in the case of Model 3E and 3) the NAIRU stabilised around 1996, starting to fall again towards the very end of the 1990s.[32] Within the Gordon triangle model the explanation for low inflation in the mid-1990s must lie with the exogenous supply shocks and this result seems stronger than the parallel result for the US. Real import prices were falling and the model suggests this was an important factor in keeping inflation down.[33] The relative

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[32] A falling NAIRU towards the very end of the 1990s is compatible with other estimates of the NAIRU for the UK, see for example HM Treasury (2002).
[33] Real oil prices proved insignificant in the UK. Earlier estimates of TV-NAIRUs for the UK (when estimation ended in 1999 Q2) using this framework found real oil prices were significant.

weights associated with the explanations of structural change and supply shocks therefore vary over time. In terms of the implications for inflation, all four sets of residuals show very little evidence of a systematic breakdown in the relationship between inflation and unemployment in the UK.

To check for any sign of a breakdown in the Phillips curve relationship for the UK we reestimated Models 3 and 3E over different sample periods, see Table Fa and Fb respectively. As with the US, the estimated coefficients for both unemployment and inflation are remarkably stable over time for Model 3. For Model 3E, it is noticeable that the size and significance of the coefficient on inflation expectations declines sharply when the estimation period is curtailed. When the associated NAIRUs are examined, it is almost impossible to distinguish the estimates associated with the different sample periods, see Charts 12 and 13. The exception to this occurs when the sample period is extended from 1998 Q2 to 2000 Q4, as the estimated NAIRU for a given point in time falls slightly as the sample period is extended. This suggests that the experience of the UK has differed from that of the US, at least until the end of the 1990s.

In some senses the relative importance of inflation expectations in the US compared to the UK seems strange. In particular there have been more shifts in policy regimes in the UK compared to the US, which would suggest that expectations should play a larger role in the UK. One factor explaining this puzzle might be the accuracy of the different measures of inflation expectations used here. In the UK inflation expectations are not drawn directly from consumers, but are estimated from a qualitative survey, where respondents are not required to put a number on their inflation expectations.[(34)] The measure of inflation expectations in the US is drawn from a survey of professional forecasters, who provide an explicit inflation forecast. In addition inflation expectations in the US may be important not because policy regimes have changed, but because the structure of the economy itself is changing.[(35)]

[(34)] The Gallup and GFK surveys were chosen because information is available for a longer sample period.
[(35)] One way to account for survey differences would be to use the actual outturn for inflation four quarters ahead, essentially combining rational expectations with perfect foresight. However, doing so does not improve the estimated significance of inflation expectations for the UK and if anything makes it worse. In a similar exercise for the US, both the size and significance of the expectations term is also reduced, although one can still easily accept its significance at the 5% level. Roberts (1995 and 1997) also notes that the estimates of the coefficient on inflation expectations for the US becomes less precise when actual outturns are used to replace survey measures.

Finally, it is again worth emphasising the uncertainty surrounding NAIRU estimates, which, in the case of the UK, can be seen from the size of the confidence intervals associated with Model 3E, see Chart 14. The size of these confidence intervals may be related to many factors, including for example the signal-to-noise restriction. Greenslade *et al* (2003) find that increasing the signal-to-noise restriction leads to a narrower confidence interval.[36]

5 Conclusions

This paper compares the behaviour for the US and the UK of Phillips curve models based on calculations of time-varying NAIRUs. The framework chosen is similar to the triangle model of Gordon (1997 and 1998) where inflation depends on a combination of inertia, demand pressures, and exogenous supply shocks. The models considered here all investigate the behaviour of consumer price inflation, with exogenous supply shocks given by real oil prices and real import prices. The measure for demand pressure is provided by the deviation of unemployment from a time-varying NAIRU, where the unobservable level of the NAIRU is estimated simultaneously with the rest of the model using Kalman filter techniques. Within this framework, the paper investigates the role of explicit measures of inflation expectations, which has not been pursued in previous work for the US and the UK using the Kalman filter. It should, of course, be remembered that there is a high degree of uncertainty associated with any NAIRU estimates. The results should therefore be seen as indicative, rather than an exact point estimate.

The estimation allows us to ask three questions. First, is there a role for inflation expectations within Phillips curve models, as would be predicted by theory. The second set of questions ask to what extent is there any evidence for the existence of a shift in the behaviour of inflation for a given level of actual demand, particularly in the second half of the 1990s. The paper provides three potential indicators. The first of these are provided by the estimated NAIRU themselves, as a fall in the NAIRU will reduce the level of excess demand for a given level of actual demand. Although we would attach less weight to this, a second potential indicator, which has been used in the previous literature, is whether there has been persistent overprediction of inflation. Finally a breakdown in the

[36] This will be one factor accounting for the relative size of the confidence intervals in the US and the UK cases presented here as the signal-to-noise restriction is 0.16 for the UK compared to 0.04 for the US.

previous relationship between inflation and demand pressures could be indicated either by changes in the estimated NAIRUs or shifts in the coefficients themselves when these are estimated over different sample periods. The final question this paper addresses is whether the inclusion of explicit measures of inflation expectations affects our conclusions.

For each of the questions discussed above the answers for the US and UK are subtly different. In the case of the US, inflation expectations play a more important role than for the UK, particularly in the cases where estimation ends before 2000 Q4. In addition, while the results provide some tentative support for the existence of a new paradigm, the timing of these changes are very different in the US compared to the UK.[37] In particular in all cases the estimated NAIRUs in the US fell from at least the mid-1990s. The models for the US also suggest that there may have been a small degree of overprediction for inflation in the late 1990s, which is how Gordon (1998) identifies a puzzle in the relationship between inflation and unemployment. Finally, the size of the fall in the estimated NAIRUs increases as the sample period is extended, although the estimated coefficients associated with the models remain remarkably similar. What is noticeable about these results is that including inflation expectations changes the profile of the estimated NAIRUs and also the timing of their decline, but not the conclusion that there has been a fall in inflationary pressure for a given level of actual demand. In contrast for the UK the estimated NAIRUs fall in the early 1990s, before flattening out around 1996. Only in the late 1990s do the estimated NAIRUs start to decline again. Calculations of the NAIRU for a given point in time are also largely unaffected by the choice of sample period, although there is a slight downwards shift in the estimated NAIRUs for a given point in time when estimation is extended from 1998 Q4 to 2000 Q4. In addition there is no evidence for a systematic trend to over (or under) predict inflation for any of the models considered.

In conclusion therefore, if a new paradigm exists the evidence suggests that it first took place in the US. That may not seem surprising – no one disputes that if the new paradigm exists it first appeared in the US. However, this does not invalidate the comparison, as

[37] The estimated models do not directly measure the impact of the causes of the new paradigm, but instead assess whether the symptoms are compatible. As these symptoms could potentially also have been caused by other factors any results can only be interpreted as tentative support.

one of the motives for this paper was to construct an analytical framework which over time could be used to monitor whether these developments are taking place in the UK. With that in mind, the jury is still out on the role of expectations in this model. Inflation expectations are found to be highly significant in the US and clearly improve the performance of the models. Expectations also influence the profile of the NAIRU for the US, which is consistent with the persistent overprediction of inflation outturns observed in the data.

Finally, it is worth mentioning that one of the puzzles that Gordon (1998) identifies is the divergence in behaviour between goods and labour markets. One possible extension is therefore to compare the behaviour of this type of model when a time-varying non-accelerating rate of capacity utilisation is used instead of a TV-NAIRU. Additional work could also be done to experiment with alternative measures of inflation expectations.

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Appendix A: The Kalman filter

The Kalman filter of Kalman (1960) and Kalman and Bucy (1961) is an algorithm for generating minimum mean square error forecasts in a state-space model. If Gaussian errors are assumed, the filter allows the computation of the log-likelihood function of the model. This enables the parameters to be estimated easily using maximum likelihood methods. [38]

The state-space form comprises two equations: a measurement equation and a transition equation. The *measurement equation* specifies how the vector of *n* observed variables, y_t, is related to a vector of *m* unobserved state variables, α_t (the *state vector*), and is given by:

$$y_t = Z_t\alpha_t + X_t d + \varepsilon_t \qquad t = 1,\ldots\ldots,T \qquad \textbf{(1)}$$

where Z_t is an *n* x *m* matrix and X_t is an *n* x *k* matrix of exogenous variables and where ε_t is an observational error with

$$\text{var}(\varepsilon_t) = \sigma^2 H_t.$$

The *transition equation* specifies the time series process generating the unobservable state variables and is given by:

$$\alpha_t = T_t\alpha_{t-1} + c_t + R_t\eta_t \qquad t = 1,\ldots\ldots,T \qquad \textbf{(2)}$$

where T_t is an *m* x *m* matrix, c_t is an *m* x 1 vector, R_t is an *m* x *g* matrix and η_t is a *g* x 1 vector of serially uncorrelated disturbances with

$$\text{var}(\eta_t) = \sigma^2 Q_t.$$

The matrices Z_t, H_t, T_t, R_t, Q_t and G_t are known as the *system matrices*. Most of the elements of these matrices will be fixed values, mainly ones and zeros. However, they will also contain elements corresponding to the underlying parameters of the system,

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[38] This appendix treats the general case. In the application in this paper, we have $n = m = 1$ so that the variance matrices H_t and Q_t are scalars and by assumption are time-invariant.

known as the system *hyper-parameters*. The vectors c_t and d may also contain parameters but these do not affect the stochastic properties of the model.

Let a_{t-1} be the minimum mean square linear estimator (*MMSLE*) of the state vector α_{t-1} based on information available at time t-1 and let P_{t-1} be the m x m, covariance matrix of the estimation error defined by

$$P_{t-1} = E(a_{t-1} - \alpha_{t-1})(a_{t-1} - \alpha_{t-1})'$$

Then the Kalman Filter comprises two sets of recursive equations: the prediction equations and the updating equations.

The prediction equations give the optimal predictors of the state vector α_t and its covariance matrix based on information available at time t-1.

$$a_{t|t-1} = T_t a_{t-1} + c_t \quad \textbf{(3)}$$

$$P_{t|t-1} = T_t P_{t-1} T_t^{'} + R_t Q_t R_t^{'} \quad \textbf{(4)}$$

The updating equations update this predictor using new information available at time t embodied in the prediction error

$$v_t = y_t - Z_t a_{t|t-1} - X_t d \quad \textbf{(5)}$$

The updating equations are given by:

$$a_t = a_{t|t-1} + P_{t|t-1} Z_t^{'} F_t^{-1} v_t \quad \textbf{(6)}$$

and

$$P_t = P_{t|t-1} - P_{t|t-1} Z_t^{'} F_t^{-1} Z_t P_{t|t-1} \quad \textbf{(7)}$$

where

$$F_t = Z_t P_{t|t-1} Z_t^{'} + H_t$$

Thus, the Kalman filter is a recursive process for calculating the optimal estimator of the state vector given the information set at that time. The repeated process of optimal

prediction, getting the prediction errors and updating the predictions are the essence of the Kalman filter algorithm.

Assuming that the disturbances are normally distributed, the log-likelihood function for the model can be computed from the prediction errors v_t and their associated covariance matrix F_t and is defined by:

$$L = -\frac{nT}{2}\log(2\pi\sigma^2) - \frac{1}{2}\sum_{t=1}^{T}\log|F_t| - \frac{1}{2\sigma^2}\sum_{t=1}^{T} v_t' F_t^{-1} v_t \qquad \textbf{(8)}$$

The Kalman filter predictors $a_{t|t-1}$ and $P_{t|t-1}$ give the optimal predictors of the state vector α_t and its covariance matrix based on information available at time *t*-1. So this procedure of obtaining *filtered* estimates of unobserved state variable (in this case the NAIRU) does not use all the available information. The Kalman filter allows a backward recursion known as *smoothing*. The *smoothed* estimators $a_{t|T}$ and $P_{t|T}$ give the optimal predictors of α_t and $\text{var}(\alpha_t)$ based on *all* the information in the sample. These smoothed estimators can be generated from the backward recursions

$$a_{t|T} = a_t + P_t^*(a_{t+1|T} - T_{t+1}a_t - c_t) \qquad \textbf{(9)}$$

and

$$P_{t|T} = P_t + P_t^*(P_{t+1|T} - P_{t+1|t})P_t^{*\prime} \qquad \textbf{(10)}$$

where

$$P_t^* = P_t T_{t+1} P_{t+1|t}^{-1}$$

If $P_{t+1|t}$ is singular, its inverse can be replaced by a generalised inverse $P_{t+1|t}^-$.

Appendix B: Data appendix

The chief aims of this paper are to conduct a comparison both of the behaviour of the two economies and of the impact of including inflation expectations. In each case, therefore, care has been taken to ensure the maximum possible degree of comparability in the series used. Furthermore, as Kalman filter estimates are likely to be sensitive to the choice of sample period, if the early part of the period is one characterised by large changes, the sample has been extended as far back as possible. Inevitably, however, this has involved compromises.

The sample period for both the US and the UK runs from 1975 Q1 until 2000 Q4.

Inflation: for the UK is taken to be RPIX inflation; for the US is total CPI inflation.
Unemployment: for the UK is LFS unemployment until 1992 and OECD measure prior to 1992; for the US is unemployment based on survey measures.
Real import prices: in both cases import prices are deflated using the appropriate price index (RPIX or CPI) and nominal import prices are given by the implicit import price deflator.
Real oil prices: takes Brent oil prices in US dollars. For the UK these are converted into pounds sterling. The series are again deflated using the appropriate price index (RPIX or CPI).
Inflation expectations:
US: Are taken from the Survey of Professional Forecasters (which was known as the ASA/NBER Economic Outlook Survey prior to 1990), see Croushore (1993) for details. In each case the median forecast for annual inflation is used. From 1981 Q3 until 1999 Q4 the forecasts correspond directly to CPI inflation. Prior to 1981 Q3 the survey did not ask about CPI inflation, but forecasters were asked to provide inflation expectations for the implicit price deflator for GDP (PGDP). Our strategy is therefore to use expectations for PGDP inflation as a proxy for CPI inflation expectations in the earlier part of the sample period. There is a relatively close correspondence between the behaviour of both the actual levels and the expected levels of these two series from 1981 Q3. However, there is a much larger divergence between actual CPI and PGDP inflation prior to 1981, which is mostly accounted for by terms of trade shocks. This suggests that we need to take account of this difference in estimation. Several possible strategies present themselves. One alternative would have been to use actual PGDP expectations and simply to include a step dummy which is one for the period up until 1981 Q3 and

zero thereafter. If this is done, the dummy proves to be insignificant, which suggests that the switching survey measures will not generate significant problems for estimation. However, it seems unlikely that forecasters would take no notice of changes in the difference between actual PGDP inflation and CPI inflation when making their predictions. Another alternative is therefore to adjust the series for PGDP expectations by the difference between CPI inflation and PGDP inflation in the period for which the expectations relate (ie t+4). This effectively implies that forecasters perfectly predict the level of the terms of trade shocks, which again seems unlikely. A final alternative is to adjust the series for PGDP expectations by the discrepancy between actual CPI and PGDP inflation in the period when the predictions are made (t), so that terms of trade shocks are unanticipated. It is this latter strategy which we employ in this paper. Using the strategy where terms of trade shocks are perfectly anticipated does not diminish the role of inflation expectations, as they remain strongly significant, although there is a slightly flatter profile for the NAIRU.

UK: Inflation expectations for the UK are constructed from the Gallup and GFK consumer confidence surveys. These surveys ask a sample of consumers to rank a) how high they think inflation has been over the past twelve months and b) how high they think inflation will be over the next twelve months. Responses to the question b) about future inflation are classified by whether they expect, in relation to now, that: there will be a more rapid increase in prices; prices will increase at the same rate; prices will increase at a slower rate; prices will be stable; prices will fall slightly; or that they do not know. The responses are quantified by summing the weighted proportions of the sample for the question about future inflation to obtain a series for expected future annual inflation one year ahead. The surveyed responses are far more volatile than actual inflation and we assume that this is because there are short-run biases in the surveyed consumers' reporting of their expectations. The weights are therefore estimated as the long-run coefficients in a dynamic time-series regression of actual inflation on the response proportions to the question about future inflation (see Pesaran (1989).[(39)] The question on expected inflation has changed twice since the survey has been constructed. Prior to 1981 Q1, the questions related to prices rather than inflation. To overcome the resulting

[(39)] For earlier work on quantifying inflation expectations from surveys see Carlson and Parkin (1975) and Batchelor and Orr (1988).

volatility in quantified inflation expectations we estimated inflation expectations using rolling coefficients for the early period.

Appendix C: Charts and tables

Chart 1: US expected minus actual inflation

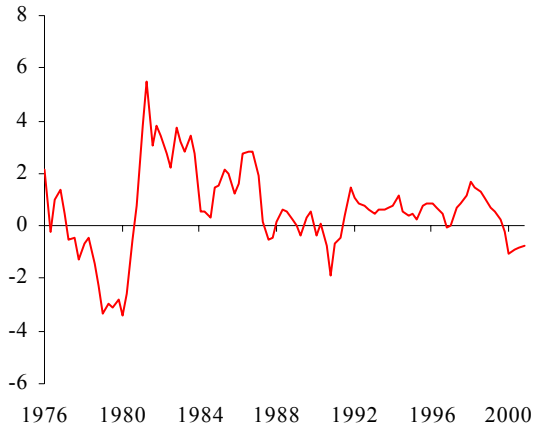

Chart 2: UK expected minus actual inflation

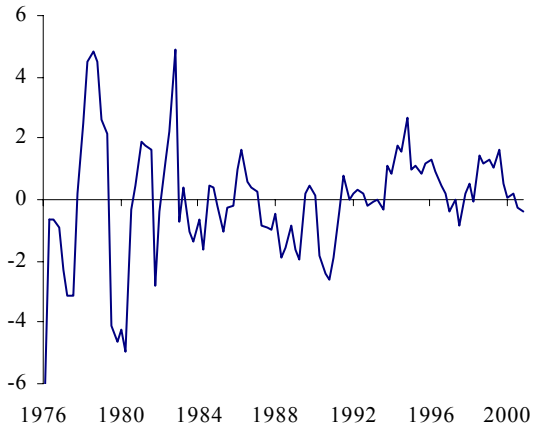

Table A: Estimating US CPI inflation using the Kalman filter, 1975 Q1-2000 Q4

	Π_t	Π_t	Π_t	Π_t
	(Model 1E)	(Model 1)	(Model 2)	(Model 2E)
u-u*	-0.23	-0.38	-0.35	-0.22
	[-3.86]	[-3.53]	[-3.37]	[-3.41]
Π^e	0.28	-	-	0.27
	[7.54]			[7.05]
Π_{t-1}	0.67	0.95	1.03	0.75
	[9.56]	[11.25]	[13.68]	[11.27]
Π_{t-3}	0.23	0.18	-	-
	[2.80]	[1.84]		
Π_{t-4}	-0.50	-0.58	-0.44	-0.32
	[-4.94]	[-4.66]	[-4.47]	[-3.95]
Π_{t-5}	0.32	0.45	0.41	0.30
	[-]	[-]	[-]	[-]
ΔRELPM$_{t-1}$	0.17	0.20	0.21	0.18
	[2.43]	[2.19]	[2.22]	[2.61]
Δ*RELPOIL*$_{t-1}$	0.28	0.28	0.26	0.26
	[4.43]	[3.62]	[3.29]	[3.89]
ΔRELPOIL$_{t-2}$	-0.26	-0.34	-0.34	-0.25
	[-3.80]	[-4.16]	[-4.06]	[-3.56]
LL	46.77	25.17	23.54	43.03

t-statistics are in parentheses.

Chart 3: US unemployment and NAIRU estimates from Models 1E and 1

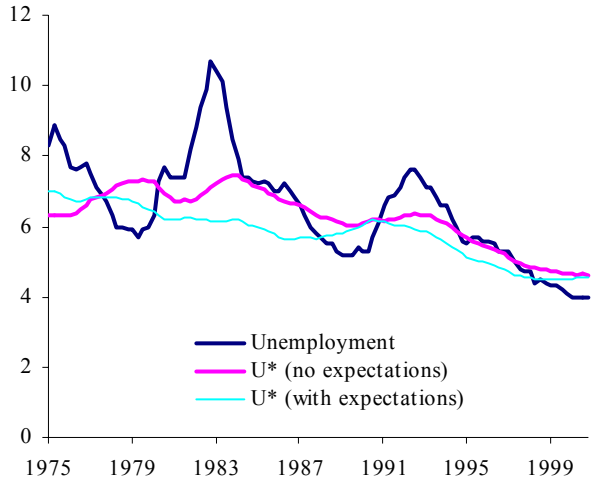

Chart 4: US unemployment and NAIRU estimates from Models 2E and 2

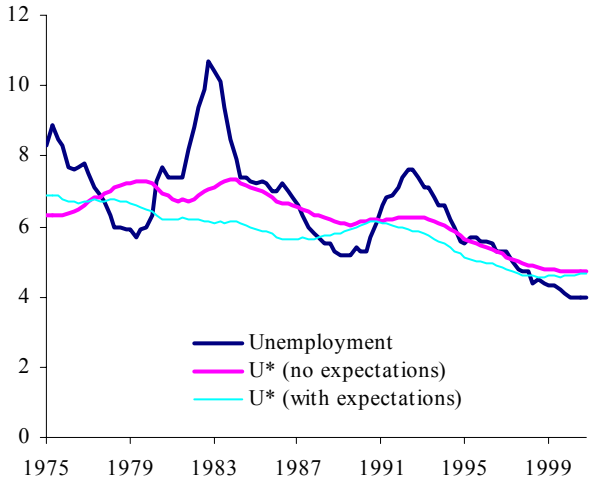

Table B: Estimating US CPI inflation using the Kalman filter, unrestricted coefficients on inflation terms, 1975 Q1-2000 Q4

	Π_t	Π_t	Π_t	Π_t
	Model 1E	Model 1	Model 2	Model 2E
Sum	0.98	0.87	0.88	0.99
Π^e	0.27	-	-	0.26
	[6.52]			[6.18]
Π_{t-1}	0.66	0.78	0.87	0.75
	[9.27]	[9.33]	[10.75]	[10.99]
Π_{t-3}	0.23	0.20	-	-
	[2.83]	[2.32]		
Π_{t-4}	-0.50	-0.49	-0.34	-0.31
	[-4.84]	[-4.37]	[-3.65]	[-3.81]
Π_{t-5}	0.32	0.37	0.35	0.29
	[4.76]	[4.99]	[4.54]	[4.18]
LL	46.93	30.97	28.42	43.12

t-statistics are in parentheses.

Table Ca: Estimating US CPI inflation using the Kalman filter, various sample periods (Model 1)

	Π_t	Π_t	Π_t	Π_t
	To 00q4	To 98q2	To 94q4	To 92q4
u-u*	-0.38	-0.36	-0.39	-0.45
	[-3.53]	[-3.21]	[-3.48]	[-3.64]
Π^e	-	-	-	-
Π_{t-1}	0.95	0.97	0.95	0.91
	[11.28]	[10.20]	[9.87]	[8.95]
Π_{t-3}	0.18	0.17	0.17	0.17
	[1.84]	[1.66]	[1.54]	[1.48]
Π_{t-4}	-0.58	-0.57	-0.57	-0.55
	[-4.66]	[-4.39]	[-3.98]	[-3.75]
Π_{t-5}	0.44	0.43	0.45	0.47
	[-]	[-]	[-]	[-]

t-statistics are in parentheses.

Table Cb: Estimating US CPI inflation using the Kalman filter, various sample periods (Model 1E)

	Π_t	Π_t	Π_t	Π_t
	To 00q4	To 98q2	To 94q4	To 92q4
u-u*	-0.23	-0.23	-0.25	-0.27
	[-3.86]	[-3.51]	[-3.93]	[-3.22]
Π^e	0.28	0.28	0.28	0.28
	[7.53]	[7.19]	[6.77]	[6.48]
Π_{t-1}	0.67	0.68	0.66	0.65
	[9.56]	[8.93]	[8.28]	[7.62]
Π_{t-3}	0.23	0.22	0.22	0.22
	[2.80]	[2.57]	[2.41]	[2.27]
Π_{t-4}	-0.50	-0.50	-0.49	-0.48
	[-4.94]	[-4.66]	[-4.24]	[-4.03]
Π_{t-5}	0.33	0.32	0.33	0.33
	[-]	[-]	[-]	[-]

t-statistics are in parentheses.

Chart 5: US Model 1 (without expectations) for different sample periods

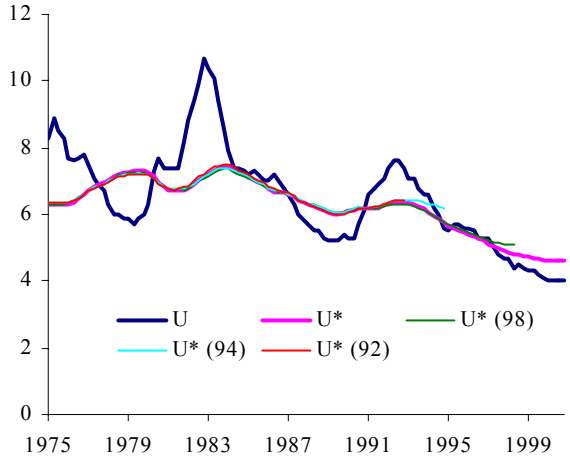

Chart 6: US Model 1E (with expectations) for different sample periods

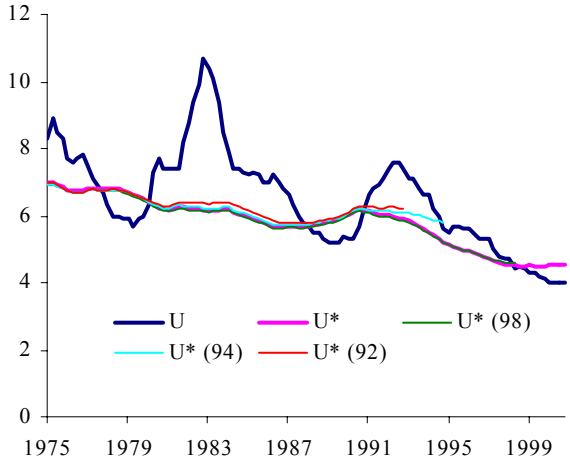

Chart 7: US NAIRU estimates and standard error bands (Model 1E)

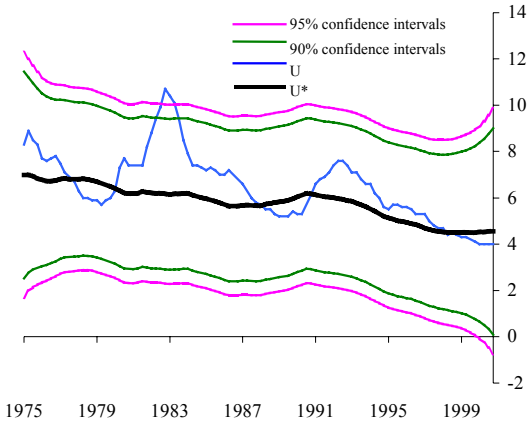

Table D: Estimating US CPI inflation using the Kalman filter, 1975 Q1-2000 Q4

	Π_t	Π_t	Π_t	Π_t
	(Model 1)	(Model 1)	(Model 1E)	(Model 1E)
	variance = 0.04	variance = 0.16	variance = 0.04	variance = 0.16
u-u*	-0.38	-0.56	-0.23	-0.24
	[-3.53]	[-2.55]	[-3.86]	[-3.31]
Π^e	-	-	0.28	0.30
			[7.54]	[7.08]
Π_{t-1}	0.95	0.78	0.67	0.63
	[11.25]	[6.25]	[9.56]	[8.13]
Π_{t-3}	0.18	0.26	0.23	0.24
	[1.84]	[2.70]	[2.80]	[3.00]
Π_{t-4}	-0.58	-0.48	-0.50	-0.50
	[-4.66]	[-4.16]	[-4.94]	[-5.00]
Π_{t-5}	0.45	0.45	0.32	0.33
	[-]	[-]	[-]	[-]
ΔRELPM$_{t-1}$	0.20	0.32	0.17	0.18
	[2.19]	[2.84]	[2.43]	[2.34]
ΔRELPOIL$_{t-1}$	0.28	0.26	0.28	0.27
	[3.62]	[3.32]	[4.43]	[4.21]
ΔRELPOIL$_{t-2}$	-0.34	-0.30	-0.26	-0.26
	[-4.16]	[-3.48]	[-3.80]	[-3.78]
LL	25.17	28.36	46.77	45.95

t-statistics are in parentheses.

Chart 8: US unemployment and NAIRU estimates with different variances (Model 1)

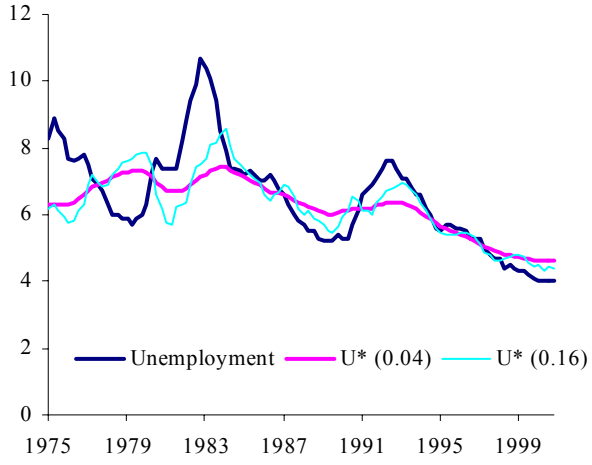

Chart 9: US unemployment and NAIRU estimates with different variances (Model 1E)

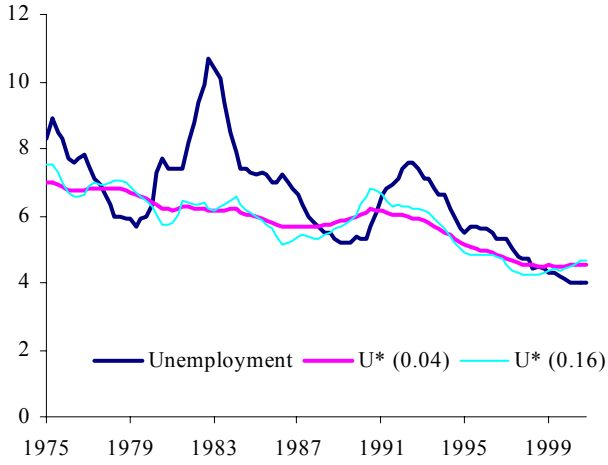

Table E: Estimating UK RPIX inflation using the Kalman filter, 1975 Q1-2000 Q4

	Π_t	Π_t	Π_t	Π_t
	(Model 3E)	(Model 3)	(Model 4)	(Model 4E)
u-u*	-0.78	-0.82	-0.72	-0.68
	[-5.84]	[-6.12]	[-4.45]	[-4.33]
Π^e	0.10	-	-	0.09
	[2.01]			[1.87]
Π_{t-1}	0.84	0.90	0.99	0.93
	[15.53]	[22.16]	[18.07]	[14.81]
Π_{t-3}	-	-	-0.25	-0.25
			[-4.70]	[-4.95]
Π_{t-4}	-0.27	-0.26	-	-
	[-4.18]	[-3.85]		
Π_{t-5}	0.33	0.26	0.26	0.23
	[-]	[-]	[-]	[-]
ΔRELPM$_{t-1}$	0.29	0.36	0.34	0.29
	[3.05]	[3.87]	[3.89]	[3.17]
ΔRELPM$_{t-4}$	0.45	0.48	0.48	0.46
	[4.16]	[4.24]	[4.44]	[4.39]
D79/80	-3.02	-3.17	-3.44	-3.30
	[-6.49]	[-6.64]	[-7.36]	[-7.21]
LL	-27.61	-29.52	-26.13	-24.49

t-statistics are in parentheses.

Chart 10: UK unemployment and NAIRU estimates from Models 3E and 3

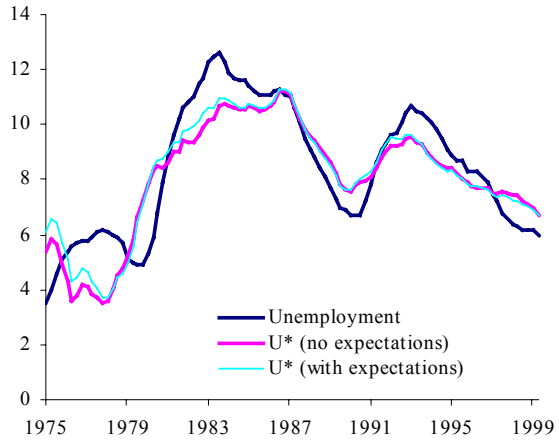

Chart 11: UK unemployment and NAIRU estimates from Models 4E and 4

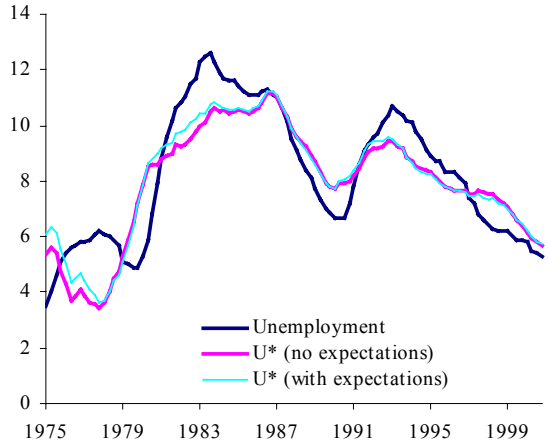

Table Fa: Estimating UK RPIX inflation using the Kalman filter, various sample periods (Model 3)

	Π_t	Π_t	Π_t	Π_t
	To 00q4	To 98q2	To 94q4	To 92q4
u-u*	-0.82	-0.81	-0.83	-0.86
	[-6.12]	[-4.38]	[-2.37]	[-3.60]
Π^e	-	-	-	-
Π_{t-1}	0.90	0.90	0.90	0.90
	[22.16]	[18.63]	[12.84]	[16.05]
Π_{t-4}	-0.26	-0.26	-0.26	-0.26
	[-3.85]	[-3.61]	[-3.32]	[-3.21]
Π_{t-5}	0.36	0.36	0.36	0.36
	[-]	[-]	[-]	[-]

t-statistics are in parentheses.

Table Fb: Estimating UK RPIX inflation using the Kalman filter, various sample periods (Model 3E)

	Π_t	Π_t	Π_t	Π_t
	To 00q4	To 98q2	To 94q4	To 92q4
u-u*	-0.78	-0.78	-0.81	-0.83
	[-5.84]	[-5.00]	[-4.18]	[-3.40]
Π^e	0.10	0.09	0.08	0.08
	[2.01]	[1.73]	[1.32]	[1.37]
Π_{t-1}	0.84	0.85	0.85	0.85
	[15.53]	[14.94]	[13.75]	[12.40]
Π_{t-4}	-0.27	-0.27	-0.27	-0.27
	[-4.18]	[-3.94]	[-3.67]	[-3.45]
Π_{t-5}	0.33	0.33	0.34	0.34
	[-]	[-]	[-]	[-]

t-statistics are in parentheses.

Chart 12: UK Model 3, different sample periods

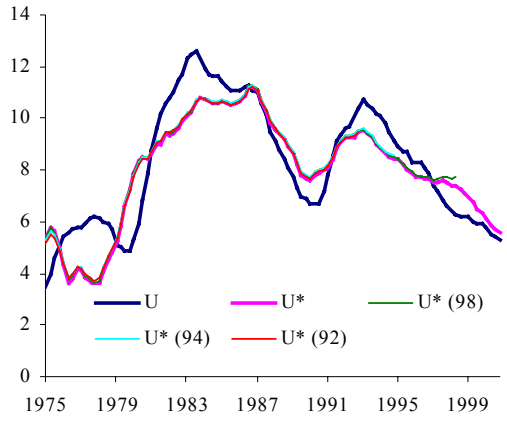

Chart 13: UK Model 3E, different sample periods

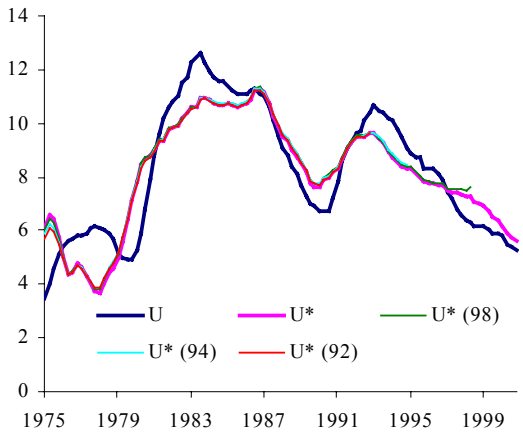

Chart 14: UK NAIRU estimates and standard error bands (Model 3E)