

Forecasting with measurement errors in dynamic models

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Contents

Abstract	5
Summary	7
1 Introduction	9
2 Optimal choice of the data frontier for forecasting	13
3 Optimising over the choice of data frontier and the projection parameters	18
4 A general approach to forecasting with dynamic models under data revisions	20
5 Empirical illustration	24
6 Summary and conclusion	29
References	30

Abstract

This paper explores the effects of measurement error on dynamic forecasting models. It illustrates a trade-off that confronts forecasters and policymakers when they use data that are measured with error. On the one hand, observations on recent data give valuable clues as to the shocks that are hitting the system and that will be propagated into the variables to be forecast. But on the other, those recent observations are likely to be those least well measured. The paper studies two classes of forecasting problem. The first class includes cases where the forecaster takes the coefficients in the data-generating process as given, and has to choose how much of the historical time series of data to use to form a forecast. We show that if recent data are sufficiently badly measured, relative to older data, it can be optimal not to use recent data at all. The second class of problems we study is more general. We show that for a general class of linear autoregressive forecasting models, the optimal weight to place on a data observation of some age, relative to the weight in the true data-generating process, will depend on the measurement error in that observation. We illustrate the gains in forecasting performance using a model of UK business investment growth.

JEL classification: C32, C53

Summary

This paper explores a trade-off that confronts forecasters and monetary policy makers when they use data that are measured with error (as surely, in reality, they are forced to do). On the one hand, observations on recent data give valuable clues as to the shocks that are hitting the system and that will be propagated into the variables to be forecast (and that will ultimately inform monetary policy). But on the other, those recent observations are likely to be those least well measured. Older data may have been revised a number of times, as more survey returns on the data or other kinds of corroborative evidence have been collected by the statistics agency. We begin by illustrating and proving how, faced with a choice between either using or not using the most recent observations in forecasting, if measurement error is sufficiently large it can be optimal not to use them. We move on to consider a case where measurement error is larger the more recent the data observation: this captures the idea that older data will have been revised and corroborated with information that came after they were first released. We derive conditions under which a many step ahead forecast, (based on older data) will be a better forecast (in fact, the optimal forecast) than a one step ahead forecast. The noisier recent data are, the more likely this is to be true. And the more persistent the data-generating process, the more likely this is to be true: more persistence means that shocks contain more information for the values of variables in the immediate future. We generalise these results further, by allowing the forecaster to ‘aim off’ the coefficients in the true model, to improve forecasts still further. Finally, we derive the optimal forecasting model from the class of linear autoregressive models. These forecasts are optimal subject to the constraint that the forecaster uses as many lags to forecast as are present in the data-generating process itself. It is not surprising that the optimal weighting scheme differs from the weighting scheme that characterises the data-generating process. The greater the signal about the future in a particular data point, the greater the weight in the optimal forecasting model. More recent, and therefore more imprecisely measured, data have a smaller weight. This effect is greater, the more persistent is the data-generating process. We conclude with an application to UK business investment growth, and illustrate the improvement in forecasting performance that can be obtained using our procedure, an improvement that turns out to be statistically significant.

1 Introduction

This paper explores a trade-off that confronts forecasters and monetary policy makers when they use data that are measured with error (as surely, in reality, they are forced to). On the one hand, observations on recent data give valuable clues as to the shocks that are hitting the system and that will be propagated into the variables to be forecast. But on the other, those recent observations, the ones likely to contain the most information about the future profile of the data, contain measurement error that will induce errors in the forecast. Moreover, the more recent a data observation is, the more measurement error it is likely to contain. The most recent data may be a first release. Older data may have been revised a number of times as more survey returns on the data or other kinds of corroborative evidence were collected by the statistics agency.⁽¹⁾ The best forecast in this (very real) situation will balance the information about shocks contained in recent data against the contaminating noise of the measurement error. One option, of course, is to wait for data observations to improve; ie to wait for corroborative evidence to accumulate. However, this is not an available option in one prominent example where forecasting is important: monetary policy making. Monetary policy takes time to have an effect. And the economy may be such that it is appropriate to respond to shocks sooner rather than later to avoid larger fluctuations in (say) inflation and/or output which are themselves undesirable. So there is an advantage to using the signal contained in the most recent data for forecasting and policy.

This is a familiar problem, and there is a large literature that attempts to examine aspects of forecasting and monetary policy making in ‘real time’, when data are likely to be of poorer quality than at some future date. We do not attempt to do any justice to the depth or diversity of this literature here, but to put our work in context it is worth mentioning a few important strands of research.⁽²⁾ Real-time data sets that enable economists to study the properties of different vintages of data relevant to policy making have been compiled by Croushore and Stark (2001) for the United States, and by Eggington, Pick and Vahey (2002), Castle and Ellis (2002) and Patterson and Hervai (1991) for the United Kingdom. Others (for example Mankiw, Runkle and Shapiro (1984), Faust, Rogers and Wright (2001), though this literature is very large indeed) have studied whether the statistics agency

(1) See (Castle and Ellis, 2002, page 44) for a discussion of the reasons why data are revised in the United Kingdom.

(2) A helpful bibliography can be found at <http://phil.frb.org/econ/forecast/reabib.html>.

behaves like a ‘rational’ forecaster by examining whether early releases of data predict later ones. Still others have studied the implications for monetary policy and inflation forecasts of having to use real-time measures of important indicators like the output gap (Orphanides (2003) and Orphanides and Van-Norden (2001)).

Within this broad literature are papers that study the properties of forecast models in the presence of measurement error, and these are the closest intellectual antecedents of our own analysis. One line of enquiry has been to study a problem of joint model estimation and signal extraction/forecasting. Optimal filters/forecasts are studied in a line of work such as, for example, Howrey (1978) and Harvey, McKenzie, Blake and Desai (1983). Koenig, Dolmas and Piger (2001) present informal experiments that reveal the advantages for forecasting of using real-time data for model estimation. Another focus for study has been the idea of evaluating the properties of combinations of forecasts (see, for example, Bates and Granger (1969) and discussions in Hendry and Clements (2003)). Observations on time series at dates leading up to time t are ‘forecasts’ of sorts of data at time t , so the problem of how best to make use of these data is a problem of combining forecasts.⁽³⁾

This paper puts to one side the problem of model estimation. We assume that the forecaster/policymaker knows the true model. Taken at face value, this looks like a very unrealistic assumption. But it has two advantages. First, it enables us to isolate the forecasting problem, without any loss of generality. The second advantage is that it also emphasises an aspect of forecasting and policy that is realistic. The policymaker may have a noisy information source that is contaminated with measurement error, but also contains an important signal about shocks. The policymaker may also have an information source that is not contaminated by (at least that source of) measurement error – an economic prior – but that does not contain the same high-frequency diagnosis of the state of the economy. The set-up we use is just an extreme version of this. We assume that the policymaker’s prior about the structure of the economy (the data-generating process) is correct.

We begin (in Section 2.1) by illustrating and proving how, faced with a choice between either using or not using most recent observations in forecasting, once measurement error is sufficiently large, it can be optimal not to use them. Specifically, we assume that the

(3) This observation is made in Buseti (2001).

policymaker has access to data for periods $t = 0, \dots, T$ and makes forecasts for date $T + 1$ using data available at date $T + 1 - n$ and the true model. The policymaker's choice variable is n . Conventional forecasting analysis usually assumes that the forecaster/policymaker sets $n = 0$ which means using all available data. Here we relax that assumption. In the illustration we offer, of a mean-reverting dynamic process, the structure of which is known to policy makers, it is optimal to resort to using the mean as the forecast, rather than (in this simple example, any of) the noisy data. In effect this amounts to choosing $n = \infty$.

We move on (in Section 2.2) to consider a case when measurement error is larger the more recent the data observation: this captures the idea that recent data, more likely to be a first release, will be noisier than older data, which may have been revised and corroborated with information that came later. We derive conditions under which a many step ahead forecast, (based on older data) will be a better forecast (in fact the optimal forecast) than a one step ahead forecast. The noisier are recent data, the more likely this is to be true. And the more persistent the data-generating process, the more likely this is to be true (because old shocks contain more information for the values of variables in the immediate future).⁽⁴⁾ By assuming that the variance of measurement error declines as a variable is revised, we therefore create a trade-off between noisier, but more recent data and less noisy, older data. In general, this means that there is some finite n that minimises the mean squared error of the forecasts.

In Section 3 we generalise these results a little further. We begin by deriving optimal forecasts when forecasters take the model parameters as a given, and have to find the optimal point at which to end the data frontier for the forecast. In Section 3, the problem is characterised still as one in which the forecaster has to choose the optimal point (n) at which to end the data frontier for a forecast of a variable at time $T + 1$. But now we assume that the policymaker can also choose the parameters of the forecast model. We analyse the case in which the policymaker knows that the data are generated by a first-order autoregressive process with parameter a . To generate the forecast, we allow the policymaker to use some other parameter \tilde{a} that may differ from the true model parameter, a . We find that the optimal point at which to end the data frontier does not necessarily

(4) These points are illustrated too in Buseti (2001).

imply using all the most recent data, and that that optimal forecast does not imply a parameter equal to the ‘true’ one. (Except, of course, in the limiting case when there is no measurement error.)

Section 4 generalises the results by deriving the optimal forecasting model from the class of linear autoregressive models. We can view this as describing how the policymaker decides on the optimal weights to place on past data. This structure therefore allows the policymaker to include many lags of the data to construct the forecast and place different weights (coefficients) on different lags. This is clearly more general than the analysis in previous sections which constrained the forecaster to either use or not use at all some particular data. It is not surprising that the optimal weighting scheme differs from the weighting scheme that characterises the data-generating process. The greater the signal about the future in a data point, the greater the weight in the optimal forecasting model. More recent and therefore more imprecisely measured data have a smaller weight. The greater the persistence in the data-generating process, the greater the signal in older data for the future, and the more extra measurement error in recent data relative to old data makes it optimal to rely on those older data.

In Section 5, we present an application of the results in Section 4 to a single equation forecasting model for the quarterly growth of business investment spending in the United Kingdom, though we think the theoretical results have a more general implication for linear forecasting models. We use real-time data on revisions to national accounts from Castle and Ellis (2002) to estimate how the variance of measurement error declines as we move back in time from the data frontier at T to some $T - n$. We find, not surprisingly, that indeed the optimal forecasting model differs significantly from the weights put on data implied by the underlying estimated model, suggesting that the problem we study here may well be quantitatively important.

2 Optimal choice of the data frontier for forecasting

2.1 Age-invariant measurement error

We begin, as we described in the introduction, by illustrating how it may be optimal not to use recent (in fact in this example, any) data for forecasting, but instead to rely on the model, which we assume is known. In this section we use a very simple model, but we will relax some of our assumptions in later sections.

Assume that the true model is:

$$y_t^* = ay_{t-1}^* + e_t \tag{1}$$

where $|a| < 1$ and y_t^* denotes the true series. Data are measured with error, and the relationship between the true and observed series is given by:

$$y_t = y_t^* + v_t \tag{2}$$

For this section, we make the following assumptions about the processes for e and v :

$$e_t \sim i.i.d.(0, \sigma_e^2) \tag{3}$$

and

$$v_t \sim i.i.d.(0, \sigma_v^2) \tag{4}$$

which encompasses the assumption that the measured data are unbiased estimates of the true data.⁽⁵⁾

Importantly, at this stage we are assuming that the variance of the distribution of the measurement error does not depend on how recently the data observation was released, or that the measurement error is, as the title of this section dubs it, ‘age-invariant’. We will relax this assumption later. We assume that we have a sample from period $t = 1$ to period $t = T$ and we wish to forecast some future realisation y_{T+1}^* . The standard forecast, (when there is no measurement error) for y_{T+1}^* is denoted by $\hat{y}_{T+1}^{(0)}$ and given by $\hat{y}_{T+1}^{(0)} = ay_T$: this is the forecast that simply projects the most recent observation of y_t using the true model coefficient a . We investigate the mean squared properties of this forecast compared with the

(5) The analysis in Castle and Ellis (2002) focuses on the first-moment properties of revisions and finds some evidence of bias. But we abstract from that issue here.

general forecast $\hat{y}_{T+1}^{(n)} = a^{n+1}y_{T-n}$, a class of forecasts that project using data that are older than the most recent outturn.

We begin by finding an expression for the forecast error, and then computing the mean squared error for different forecasts among the general class described above. The (true) forecast error (which of course we never observe) is given by $\hat{u}_{T+1}^{(n)} = y_{T+1}^* - \hat{y}_{T+1}^{(n)}$. We know that from (1) we can write:

$$y_{T+1}^* = a^{n+1}y_{T-n}^* + \sum_{i=0}^n a^i e_{T+1-i} \quad (5)$$

and from (2) we have:

$$\hat{y}_{T+1}^{(n)} = a^{n+1}y_{T-n} \quad (6)$$

$$= a^{n+1}y_{T-n}^* + a^{n+1}v_{T-n} \quad (7)$$

So:

$$\hat{u}_{T+1}^{(n)} = a^{n+1}y_{T-n}^* + \sum_{i=0}^n a^i e_{T+1-i} - a^{n+1}y_{T-n}^* - a^{n+1}v_{T-n} \quad (8)$$

$$= \sum_{i=0}^n a^i e_{T+1-i} - a^{n+1}v_{T-n} \quad (9)$$

Therefore, the mean squared error is simply given by:⁽⁶⁾

$$MSE(n) = a^{2(n+1)}\sigma_v^2 + \left(1 + \frac{a^2 - a^{2(n+1)}}{1 - a^2}\right)\sigma_e^2 \quad (10)$$

The next step is to explore the condition that the mean squared error from a forecast using the most recent data is less than the mean squared error that uses some other, more restricted information set, or $MSE(0) < MSE(n)$ for some $n > 0$. This will tell us whether there are circumstances under which it is worth forecasting without using the latest data. Doing this gives us:

$$MSE(0) < MSE(n) \Rightarrow a^2\sigma_v^2 + \sigma_e^2 < a^{2(n+1)}\sigma_v^2 + \left(1 + \frac{a^2 - a^{2(n+1)}}{1 - a^2}\right)\sigma_e^2 \quad (11)$$

which can be written as:

$$(a^2 - a^{2(n+1)})\sigma_v^2 < \frac{a^2 - a^{2(n+1)}}{1 - a^2}\sigma_e^2; \quad (12)$$

which in turn reduces to:

$$\sigma_v^2 < \frac{\sigma_e^2}{1 - a^2} \quad (13)$$

(6) Notice that this expression requires that the revision errors in (2) are uncorrelated with future shocks to the model (1). This seems a reasonable assumption.

So if $\sigma_v^2 > \frac{\sigma_e^2}{1-a^2}$ it is better in terms of MSE *not* to use the most recent data. The intuition is simply that, if the variance of the measurement error σ_v^2 is very large relative to the shocks that hit the data-generating process, (σ_e^2), then it is not worth using the data to forecast: the more so the smaller is the parameter that propagates those shocks (a). In fact, it follows that, if $\sigma_v^2 > \frac{\sigma_e^2}{1-a^2}$, then $MSE(n-1) > MSE(n)$ for all n and therefore we are better off using the unconditional mean of the model to forecast the true series rather than any other data.

There are two alternative ways to describe this result. The first uses the signal to noise ratio, defined as $\sigma^2 = \sigma_e^2/\sigma_v^2$. Then, if $\sigma^2 < (1-a^2)$, it is optimal not to use the most recent data. Under this interpretation, the lower the signal to noise ratio, the more likely it is that using the mean of the process will provide the best forecast. The critical value of the signal to noise ratio is a decreasing function of the persistence of the process – as the persistence of the process increases, the strength of past signals in later data increases. The second interpretation involves the observation that $\sigma_e^2/(1-a^2)$ corresponds to the unconditional variance of the true data, y^* . If we denote the variance of the data as $\sigma_{y^*}^2$, then it is optimal not to use recent data when $\sigma_v^2 > \sigma_{y^*}^2$. In words: it is only worth using recent data if the unconditional variance of the data measurement errors is smaller than the unconditional variance of the true data, which is an intuitive result. The practical relevance of this result may be questionable as it seems likely that there are few data series in practice that are so badly measured. But the underlying assumptions in this section are somewhat oversimplified and the analysis of the subsequent sections is intended to have more general applicability.

The analysis above concentrated on a simple AR(1) model. However, the intuition is clear and is valid for general AR models and more general dynamic models. The increasing cost of the cumulative sum of the structural errors (e_t) is balanced against the falling cost of smaller measurement errors ($a^i v_t$) when older data are used. In the case of this simple model there is no solution where some old data are useful. Either the most recent data should be used or no data at all, once – and this is crucial – the mean of the series is known. This is because older data are as well measured as more recent data.

2.2 Age-dependent measurement error

We now investigate a slightly more complex case where the variance of the data measurement error v_t is assumed to tail off over time. This assumption reflects the observation that, in practice, we observe that statistics agencies often revise data many times after the first release. If we assume that successive estimates of a particular data point are subject to less uncertainty (since they are based on more information), then it seems reasonable to assume that the variance of the revision error embodied in the estimate of a particular data point diminishes over time.

The specific assumption we make here is that:

$$\text{Var}(v_{T-i}) = \begin{cases} b^i \sigma_v^2, & i = 0, 1, \dots, N \\ 0, & i = N + 1, \dots \end{cases} \quad (14)$$

for a parameter $0 < b < 1$. We therefore assume that after a finite number of periods $N + 1$, there are no further revisions to the data. But for the first $N + 1$ periods, the variance of the revision error declines geometrically over time at a constant rate measured by b . This is a fairly specific assumption which we make here for simplicity and tractability (again the analysis of later sections is more general). Indeed, we know that data are revised for reasons other than new information specific to that series (for example rebasing and methodology changes) so the specification of revision error variance may be more complicated than assumed here. But the purpose of the assumption is to be more realistic than the homoskedastic case considered in Section 2.1.

Under our assumptions, the MSE as a function of n is given by:

$$MSE(n) = a^{2n+2} b^n \sigma_v^2 + \sum_{i=0}^n a^{2i} \sigma_e^2, \quad n = 0, 1, \dots, N \quad (15)$$

$$MSE(n) = \sum_{i=0}^n a^{2i} \sigma_e^2 \quad (16)$$

$$= \sum_{i=0}^N a^{2i} \sigma_e^2 + \sum_{i=N+1}^n a^{2i} \sigma_e^2, \quad n = N + 1, \dots \quad (17)$$

We want to examine when $MSE(n) > MSE(N + 1)$, $n = 0, 1, \dots, N$. It is clear that $MSE(n) > MSE(N + 1)$, $n = N + 2, \dots$. So, for $n = 0, 1, \dots, N$

$$MSE(n) > MSE(N + 1) \Rightarrow a^{2n+2} b^n \sigma_v^2 + \sum_{i=0}^n a^{2i} \sigma_e^2 > \sum_{i=0}^n a^{2i} \sigma_e^2 + \sum_{i=n+1}^{N+1} a^{2i} \sigma_e^2 \quad (18)$$

Chart 1: b=0.9

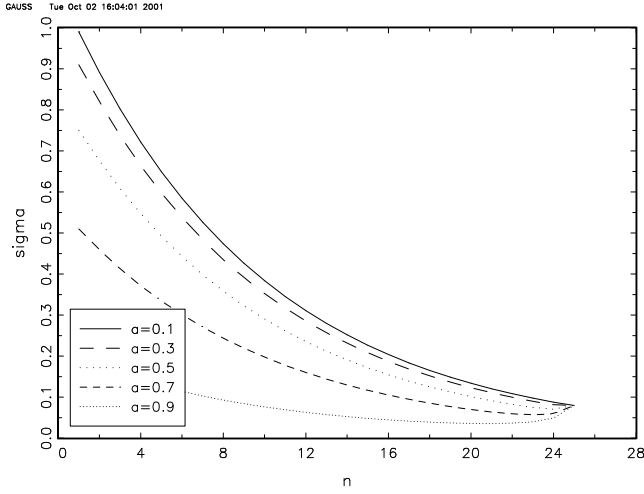
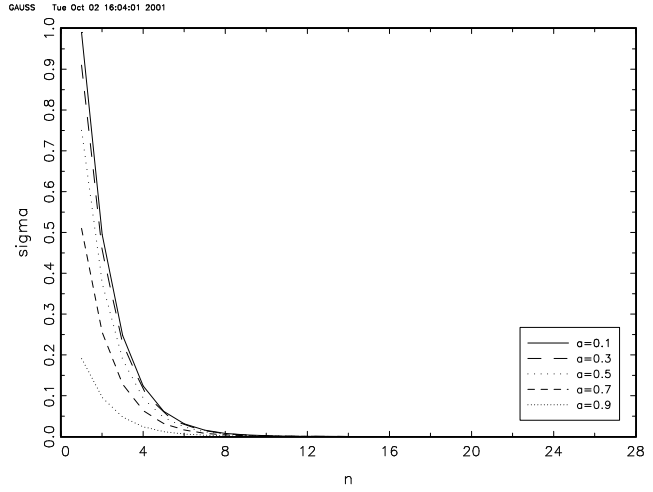


Chart 1: b=0.5



OR

$$a^{2n+2}b^n\sigma_v^2 > \sum_{i=n+1}^{N+1} a^{2i}\sigma_e^2 \quad (19)$$

OR

$$a^{2n+2}b^n\sigma_v^2 > a^{2n+2} \frac{1 - a^{2(N-n+1)}}{1 - a^2} \sigma_e^2 \quad (20)$$

or, in terms of the signal to noise ratio, σ :

$$\frac{b^n(1 - a^2)}{1 - a^{2(N-n+1)}} > \sigma^2 \quad (21)$$

So if $\sigma^2 < \frac{b^n(1-a^2)}{1-a^{2(N-n+1)}}$ for all n then the best forecast for y_{t+1} is $\hat{y}_{t+1}^{(N+1)}$. To clarify the range

of relevant values for σ we graph the quantity $\frac{b^n(1-a^2)}{1-a^{2(N-n+1)}}$ over n for $N = 24$,

$b = 0.99, 0.95, 0.9, 0.5$ and $a = 0.1, 0.3, 0.5, 0.7, 0.9$ in Charts 1 and 2. If each period

corresponds to one quarter, then our assumption $N = 24$ corresponds to the situation in which data are unrevised after six years. While this is naturally an approximation (since rebasing and methodological changes can imply changes to official data over the entire length of the data series) it seems a plausible one.

Clearly, the more persistent the process is (the larger the a) the lower σ^2 has to be for $\hat{y}_{t+1}^{(N+1)}$ to be the best forecast. Also, the more slowly the revision error dies out (the larger the b), the lower σ^2 has to be for $\hat{y}_{t+1}^{(N+1)}$ to be the best forecast. Note that some of the curves in the charts are not monotonic. This indicates that, although $\hat{y}_{t+1}^{(N+1)}$ is a better forecast than $\hat{y}_{t+1}^{(0)}$, there exists some $N + 1 > n > 0$ such that $\hat{y}_{t+1}^{(n)}$ is better than $\hat{y}_{t+1}^{(N+1)}$.

Chart 2: b=0.99

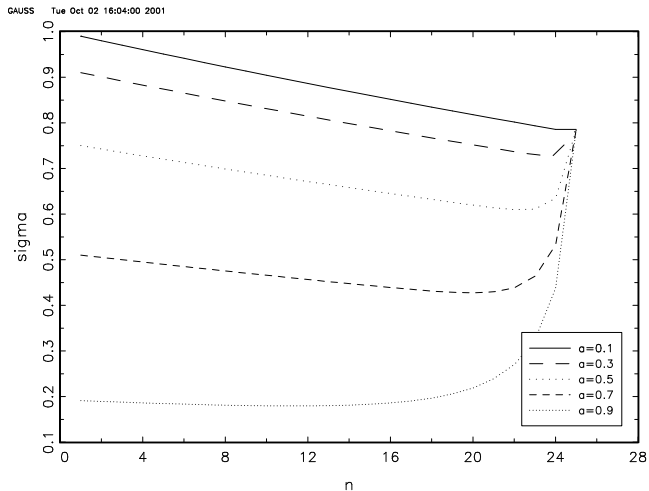
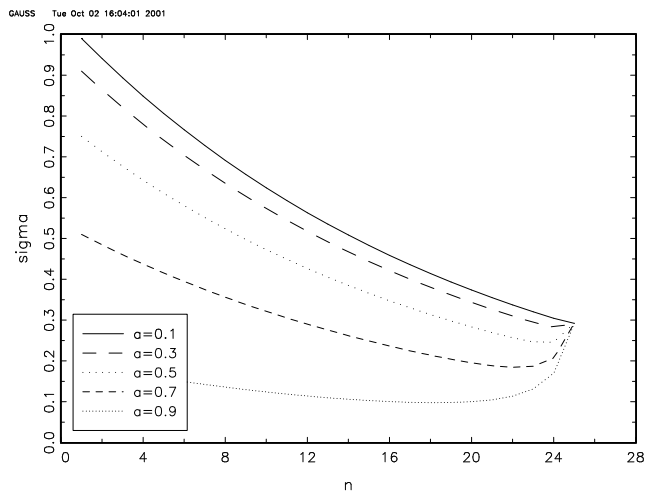


Chart 2: b=0.95



3 Optimising over the choice of data frontier and the projection parameters

The analysis in the previous section constrained the policymaker/forecaster to use the true model when forecasting future outturns. The only choice variable was therefore the horizon n upon which to base the forecast $\hat{y}_{T+1}^{(n)} = a^{n+1}y_{T-n}$. This section generalises the problem of the policymaker/forecaster so that it is possible to construct a forecast that does not use the true model parameter. Specifically, we allow the policymaker/forecaster to use the forecast $\hat{y}_{t+1}^{(n)}(\tilde{a}) = \tilde{a}^{n+1}y_{t-n}$ where \tilde{a} may differ from a . In this setting, there are two choice variables (\tilde{a} and n) and so it might be more appropriate to use a different parameter value, \tilde{a} , and the most recent data as opposed to older data and the true model parameter, a .

This section therefore extends the set-up and views the mean squared error as a function of n and \tilde{a} where the forecast is given by $\hat{y}_{t+1}^{(n)}(\tilde{a}) = \tilde{a}^{n+1}y_{t-n}$ and \tilde{a} , n are to be jointly determined given the structural parameters a , σ_v^2 and σ_e^2 . We extend the analysis along these lines assuming that the revision error variance is given by:

$$Var(v_{t-i}) = b^i \sigma_v^2, i = 0, 1, \dots, N \text{ and } Var(v_{t-i}) = 0, i = N + 1, \dots \text{ as before.}$$

Now the mean squared error is a joint function of n and \tilde{a} given by:

$$MSE(n, \tilde{a}) = (a^{n+1} - \tilde{a}^{n+1})^2 \frac{\sigma_e^2}{1 - a^2} + \frac{(1 - a^{2(n+1)})\sigma_e^2}{1 - a^2} + \tilde{a}^{2(n+1)} b^n \sigma_v^2, \quad n = 0, 1, \dots, N \quad (22)$$

$$MSE(n, \tilde{a}) = (a^{n+1} - \tilde{a}^{n+1})^2 \frac{\sigma_e^2}{1 - a^2} + \frac{(1 - a^{2(n+1)})\sigma_e^2}{1 - a^2}, \quad n = N + 1, \dots \quad (23)$$

and we wish to find the optimal values for n and \tilde{a} . To do so, we analyse a two-step minimisation problem. First, we will minimise the mean squared error with respect to the forecasting parameter \tilde{a} . This allows us to write down a set of mean squared errors that use the optimal forecasting parameters as n changes. To find the best forecast simply requires choosing the n that gives the overall smallest mean squared error.

We therefore begin by minimising $MSE(n, \tilde{a})$ with respect to \tilde{a} . The first-order necessary conditions are:

$$\frac{\partial MSE(n, \tilde{a})}{\partial \tilde{a}} = \begin{cases} -2(a^{n+1} - \tilde{a}^{n+1})(n+1)\tilde{a}^n \frac{\sigma_e^2}{1-\tilde{a}^2} + 2(n+1)\tilde{a}^{2n+1}b^n\sigma_v^2 = 0 & n = 0, 1, \dots, N \\ -2(a^{n+1} - \tilde{a}^{n+1})^2(n+1)\tilde{a}^n \frac{\sigma_e^2}{1-\tilde{a}^2} = 0 & n = N+1, \dots \end{cases} \quad (24)$$

Rearranging gives:

$$-\tilde{a}^n \left[a^{n+1} - \left(1 - \frac{b^n(1-a^2)}{\sigma^2} \right) \tilde{a}^{n+1} \right] = 0, \quad n = 1, \dots, N \quad (25)$$

$$-\tilde{a}^n (a^{n+1} - \tilde{a}^{n+1}) = 0, \quad n = N+1, \dots \quad (26)$$

For (25), disregarding complex roots and under the convention of square roots being positive numbers, the solutions are $\tilde{a} = 0$ and $\tilde{a} = \sqrt[n+1]{\theta}$ where $\theta = \frac{\sigma^2 a^{n+1}}{\sigma^2 + b^n(1-a^2)}$. For (26), they are, intuitively, $\tilde{a} = 0$ and $\tilde{a} = a$. Note that for positive a , $\theta \geq 0$, making sure that the second solution of (25) is real. Just to verify that the solutions we have are proper minima we compute the second derivatives. These are given by:

$$-n\tilde{a}^{n-1} \left[a^{n+1} - \left(1 - \frac{b^n(1-a^2)}{\sigma^2} \right) \tilde{a}^{n+1} \right] + \tilde{a}^n \left(1 - \frac{b^n(1-a^2)}{\sigma^2} \right) (n+1)\tilde{a}^n \quad n = 0, 1, \dots, N \quad (27)$$

$$-n\tilde{a}^{n-1} (a^{n+1} - \tilde{a}^{n+1}) + (n+1)\tilde{a}^{2n} \quad n = N+1, \dots \quad (28)$$

In both cases the second derivatives are positive for the non-zero solution and zero for the zero solution. The non-zero solutions are therefore minima.

We can now incorporate the solutions of this minimisation into the expression for the mean squared error. We define $\hat{MSE}(n) = \min_{\tilde{a}} MSE(n, \tilde{a})$:

$$\hat{MSE}(n) = [(a^{n+1} - \theta)^2 + (1 - a^{2(n+1)})] \frac{\sigma_e^2}{1-a^2} + \theta^2 b^n \sigma_v^2 \quad n = 0, 1, \dots, N \quad (29)$$

$$\hat{MSE}(n) = \frac{(1 - a^{2(n+1)})\sigma_e^2}{1-a^2} \quad n = N+1, \dots \quad (30)$$

which has to be minimised over n . Standard methods do not apply as n takes only discrete values.⁽⁷⁾ However, it is easily seen that $M\hat{S}E(n)$ is not necessarily minimised at 1, thereby leading to the same conclusion as before.

4 A general approach to forecasting with dynamic models under data revisions

The analysis of previous sections has gradually increased the generality of the problem under consideration. To recap, we began by considering the choice of the lag, n , of data that the policymaker/forecaster would use with the true model parameter in the face of (both age-invariant and age-dependent) measurement error. We then allowed the policymaker/forecaster to choose both the lag, n , and the parameter with which to project the lagged data, \tilde{a} .

In this section we propose a general method of forecasting in autoregressive models under a general known form of data revisions. The extension from the previous sections is that we optimise the forecasting model from within the linear class of models. Specifically, we allow the policymaker/forecaster to choose the optimal weights and lags on all past data. So, in the example of the previous sections the policymaker/forecaster knew that the data-generating process was an AR(1) with parameter a . But even so, this section allows the policymaker/forecaster to choose from all linear AR models and it may be the case that the best forecasting model is (say) an AR(2) with coefficients b and c . The method described here can be easily extended to multivariate models. In particular, VAR models could easily be accommodated (including of course vector error correction models).

We assume a univariate $AR(p)$ model to illustrate the method. The true process is given by:

$$y_t^* = \sum_{i=1}^p a_i y_{t-i}^* + e_t \quad (31)$$

This can be written as a VAR(1) model of the form:

$$\mathbf{y}_t^* = \mathbf{A}\mathbf{y}_{t-1}^* + \boldsymbol{\epsilon}_t \quad (32)$$

(7) As the minimum of any function is also the minimum of a positive monotonic transformation of that function, we can take logs of the mean squared error expressions to show that, for real n , the minimum of (29) is obtained for:

$$n = \frac{\log \left[\frac{(1-a^2)\theta \log(b)}{2\sigma^2 a \log(a)} \right]}{\log(a/b)}$$

where $\mathbf{y}_t^* = (y_t^*, y_{t-1}^*, \dots, y_{t-p+1}^*)'$, $\mathbf{A} = (\mathbf{a}, \mathbf{e}_1, \dots, \mathbf{e}_{p-1})'$; \mathbf{e}_i is a $p \times 1$ vector with an element of 1 at the i -th place and zeroes everywhere else; \mathbf{a} is a $p \times 1$ vector of the autoregressive coefficients a_i ; $\epsilon_t = (e_t, 0 \dots, 0)'$. Now the observed data are given by:

$$\mathbf{y}_t = \mathbf{y}_t^* + \mathbf{v}_t \quad (33)$$

where $\mathbf{v}_t = (v_t, v_{t-1}, \dots, v_{t-p+1})$. At time T we wish to determine the optimal forecast for y_{T+1}^* . We assume that the revision error \mathbf{v}_T has a variance matrix which is given by Σ_v^T . Our aim is to determine the optimal forecasting model of the form $\hat{y}_{T+1} = \tilde{\mathbf{A}}_1 \mathbf{y}_T$ in terms of mean squared error, where $\tilde{\mathbf{A}}_1$ is a $1 \times p$ vector. Note that the restriction on the dimension of $\tilde{\mathbf{A}}_1$ to be the same as that of the order of the true process is not problematic because we can simply increase the order of the process by setting the higher order a 's equal to zero. This means that the true data-generating process might be an AR(1) even though we can write it as an AR(p) with the coefficients on lags $2, \dots, p$ set equal to zero.

The forecast error for the forecast of the above form is given by:

$$y_{T+1}^* - \hat{y}_{T+1} = \mathbf{A}_1 y_T^* + \epsilon_T - \tilde{\mathbf{A}}_1 \mathbf{y}_T^* + \tilde{\mathbf{A}}_1 \mathbf{v}_T = (\mathbf{A}_1 - \tilde{\mathbf{A}}_1) \mathbf{y}_T^* + \tilde{\mathbf{A}}_1 \mathbf{v}_T + \epsilon_{T+1} \quad (34)$$

where \mathbf{A}_1 is the first row of \mathbf{A} . The mean squared error is given by:

$$(\mathbf{A}_1 - \tilde{\mathbf{A}}_1) \mathbf{\Gamma} (\mathbf{A}_1 - \tilde{\mathbf{A}}_1)' + \tilde{\mathbf{A}}_1 \Sigma_v^T \tilde{\mathbf{A}}_1' + \sigma_\epsilon^2 \quad (35)$$

where $\mathbf{\Gamma} = E(\mathbf{y}_T^* \mathbf{y}_T^{*'})$. The covariances of an AR(p) process are given by the first p elements of the first column of the matrix $\sigma_\epsilon^2 [\mathbf{I}_{p^2} - \mathbf{A} \otimes \mathbf{A}]^{-1}$. We have assumed that the error process is uncorrelated with the true process of the data. In the data revision literature this is referred to as the error-in-variables model. This assumption is not crucial to our analysis and could be relaxed as long as the covariances between the true process and the data revision errors could be estimated. We want to minimise the mean squared error in terms of $\tilde{\mathbf{A}}_1$. We will use matrix optimisation calculus to solve this problem. We rewrite the expression for the mean squared error using only terms involving $\tilde{\mathbf{A}}_1$ since the rest of the terms will not affect the minimisation. We have that the mean squared error is given by:

$$\tilde{\mathbf{A}}_1 \mathbf{\Gamma} \tilde{\mathbf{A}}_1' + \tilde{\mathbf{A}}_1 \Sigma_v^T \tilde{\mathbf{A}}_1' - \mathbf{A}_1 \mathbf{\Gamma} \tilde{\mathbf{A}}_1' - \tilde{\mathbf{A}}_1 \mathbf{\Gamma} \mathbf{A}_1' = \tilde{\mathbf{A}}_1 (\mathbf{\Gamma} + \Sigma_v^T) \tilde{\mathbf{A}}_1' - 2 \tilde{\mathbf{A}}_1 \mathbf{\Gamma} \mathbf{A}_1' \quad (36)$$

We differentiate with respect to $\tilde{\mathbf{A}}_1$ and set to zero to get:

$$(\mathbf{\Gamma} + \Sigma_v^T) \tilde{\mathbf{A}}_1' - \mathbf{\Gamma} \mathbf{A}_1' = 0; \quad (37)$$

which gives

$$\tilde{\mathbf{A}}_1^{opt'} = (\mathbf{\Gamma} + \Sigma_v^T)^{-1} \mathbf{\Gamma} \mathbf{A}_1' \quad (38)$$

The second derivative is given by $(\mathbf{\Gamma} + \mathbf{\Sigma}_v^T)$ and by the positive definiteness of this matrix the second-order condition for minimisation of the mean squared error is satisfied. This result is of some interest because it may be viewed as analogous to similar results in other literatures. Note first the similarity between this result and the standard signal extraction result which says that the optimal filter for distinguishing between signal and noise is equal to the autocovariance of the signal ($\mathbf{\Gamma}$ in our case) divided by the sum of the signal and noise autocovariances. Note that if $p = 1$ then $\tilde{\mathbf{A}}_1^2 \leq \mathbf{A}_1^2$. By the positive-definiteness of $\mathbf{\Gamma}$ and $\mathbf{\Sigma}_v^T$ one might conjecture that this result would extend to the multivariate case where $\tilde{\mathbf{A}}_1 \tilde{\mathbf{A}}_1' \leq \mathbf{A}_1 \mathbf{A}_1'$. Unfortunately, this is not the case. Although it is likely that this result will hold it is by no means certain. Another interesting corollary of the above result is that the method applies equally to measurement error. The only assumption we have made is that there exists an error in the measurement of the true data whose covariance is given by $\mathbf{\Sigma}_v^T$. This clearly covers cases of data measurement error.

The above analysis concentrated on one step ahead forecasts. The general problem of h step ahead forecasting can be dealt with similarly by minimising the sum of the 1 to h step ahead forecast errors with respect to a suitably defined set of coefficients $\tilde{\mathbf{A}}$, just as we did above. We analyse this case in what follows: we want to minimise the variance of the forecast errors of the 1 step to n step ahead forecasts. As we need to minimise a scalar function we choose to minimise the trace of the forecast error variance covariance matrix of the 1 to h step forecasts. We assume for simplicity that $p > h$. If this is not case it can always be made the case by increasing p . Using the previous notation we know that:

$$\mathbf{y}_{T+h}^* = \mathbf{A}^n \mathbf{y}_T^* + \mathbf{A}^{h-1} \boldsymbol{\epsilon}_T + \dots + \boldsymbol{\epsilon}_{T+n} \quad (39)$$

So this means that the following holds:

$$\mathbf{y}_{T+h,h}^* = (y_{T+h,1}^*, \dots, y_{T+h,h}^*) = \mathbf{A}^{(n)} \mathbf{y}_T^* + \mathbf{A}^{(h-1)} \boldsymbol{\epsilon}_T + \dots + \boldsymbol{\epsilon}_{T+h,h} \quad (40)$$

where $\mathbf{A}^{(h)}$ denote the first h rows of \mathbf{A}^h and $\boldsymbol{\epsilon}_{T+h,h}$ is a vector of the first h of the vector $\boldsymbol{\epsilon}_{T+h}$. The forecast error is therefore given by:

$$\mathbf{y}_{T+h,h}^* - \hat{\mathbf{y}}_{T+h,h} = \mathbf{A}^{(h)} \mathbf{y}_T^* + \mathbf{A}^{(h-1)} \boldsymbol{\epsilon}_T + \dots + \boldsymbol{\epsilon}_{T+h,h} - \tilde{\mathbf{A}} \mathbf{y}_T^* - \tilde{\mathbf{A}} \mathbf{v}_T \quad (41)$$

The part of the variance of the forecast error, depending on $\tilde{\mathbf{A}}$, that is relevant for the minimisation problem, is given as before by:

$$\tilde{\mathbf{A}}(\mathbf{\Gamma} + \mathbf{\Sigma}_v^T) \tilde{\mathbf{A}}' - 2\tilde{\mathbf{A}}\mathbf{\Gamma}\mathbf{A}^{(h)'} \quad (42)$$

Differentiating and noting that the derivative of the trace of the above matrix is the trace

of the derivative gives:

$$tr((\mathbf{\Gamma} + \mathbf{\Sigma}_v^T)\tilde{\mathbf{A}}' - \mathbf{\Gamma}\mathbf{A}^{(h)'}) = 0 \quad (43)$$

If the matrix is equal to zero then the trace is equal to zero and so the following holds:

$$(\mathbf{\Gamma} + \mathbf{\Sigma}_v^T)\tilde{\mathbf{A}}' - \mathbf{\Gamma}\mathbf{A}^{(h)'}) = 0 \quad (44)$$

An equivalent statement is:

$$\tilde{\mathbf{A}}^{opt'} = (\mathbf{\Gamma} + \mathbf{\Sigma}_v^T)^{-1}\mathbf{\Gamma}\mathbf{A}^{(h)'} \quad (45)$$

As we see from the above exposition, our method essentially adjusts the coefficients of the AR model to reflect the existence of the measurement error. Of course, using the ‘wrong’ coefficients to forecast introduces bias. Often, we restrict attention to best linear unbiased (BLU) estimates. But this means that if we have two models, one unbiased but with high variance, and the other with a small bias but low variance, we always pick the former. This would be true even if the low-variance estimator had a lower forecast MSE than the unbiased estimator. So the reduction in variance induced by ‘aiming off’ the true coefficients outweighs the loss from bias.

Clearly the method we suggest is optimal in terms of mean squared forecasting error, conditional on being restricted to use p periods of past data, where $p = T$ is a possibility. It is therefore equivalent to using the Kalman filter on a state space model once $p = T$.⁽⁸⁾ Nevertheless, the method we suggest may have advantages over the Kalman filter in many cases. First, the method we suggest is transparent and easy to interpret structurally. For example, one can say something about the coefficients entering the regression and how they change when revisions occur. Second, it is also possible to carry out inference on the new coefficients. We can obtain the standard errors of the modified coefficients from the standard errors of the original coefficients. So in forecasting one can say something about the importance (weight) of given variables and the statistical significance of those weights. Third, from a practical point of view where a large model with many equations is being used for forecasting, and one that must bear the weight of economic story-telling, one may want to fix the coefficients for a few periods and not re-estimate the whole model. Our method has some advantages over the Kalman Filter in uses of this sort, since it just uses

(8) For more details on the state space representation of the case we consider see Harvey *et al.* (1983). There the authors develop a state space methodology for irregular revisions for univariate models. Multivariate models are briefly discussed as well.

the same coefficients rather than applying a full Kalman filter every period. Finally, the method we have is rather nonparametric as far as variances for the revision error are concerned. We have a $T \times 1$ vector of errors at time T . In the most general case, these errors can have any $T \times T$ covariance matrix that represents all possibilities for how the variance of measurement error varies by vintage, over time, (and, in a multivariate setting, across variables). In other words, our procedure allows for time variation in the covariances, heteroscedasticity and serial correlation. The state space cannot easily attain that sort of generality. In fact, a standard state space imposes rather strict forms of covariance to the errors that are unappealing in the context we are envisaging. These can be relaxed with great difficulty only and by experienced state space modellers. Some of these advantages may become clearer with an empirical illustration.

5 Empirical illustration

We apply the general method of optimising a forecast model to an investment forecasting equation similar to those used in a number of macroeconomic models for the UK economy. A stylised fact from the data revision analysis in the United Kingdom is that some of the investment series are among the most heavily revised series in the national accounts - see, for example Castle and Ellis (2002). This is one motivation for considering investment equations. The general equation we consider is given by:

$$\Delta i_t = a_0 + \sum_{i=1}^p a_i \Delta i_{t-i} + a_{p+1} \Delta g_{t-1} + e_t \quad (46)$$

where i_t is the (log of) business investment and g_t is the (log of) GDP at market prices. Many equations of this general form include an error correction term. These terms however, usually include variables such as the capital stock and the cost of capital. For our analysis, these variables suffer from two key problems: they are difficult to measure in the first place; and no satisfactory analysis of the properties of the revisions in these series exists. Further, there is significant evidence to indicate that error correction terms may not be very helpful in a forecasting context. Evidence presented by Hoffman and Rasche (1996) demonstrates that the forecasting performance of VAR models may be better than that of error correction models over the short forecasting horizons that concern us. Only over long horizons are error correction models shown to have an advantage. Christoffersen and Diebold (1998) cast doubt on the notion that error correction models are better forecasting tools even at long horizons, at least with respect to the standard root mean

squared forecasting error criterion. They also argue that, although unit roots are estimated consistently, modelling non-stationary series in (log) levels is likely to produce forecasts that are suboptimal in finite samples, relative to a procedure that imposes unit roots, such as differencing, a phenomenon exacerbated by small-sample estimation bias.

We use real-time data from 1975 Q1-1995 Q2. We use the revision data available to provide estimates of the revision error variances. We assume that revisions do not occur in general after 24 revision rounds. We estimate the data revision variances as follows: we use real-time data to estimate the variance of the revision error between release i and release $i + 1$, $i = 1, \dots, 24$. Denote these estimates of the variance by ζ_i . Then, the variance of the revision error of the i -th release is equal to $\sum_{j=i}^{24} \zeta_j$. The variances for ζ_i estimated from the data revisions using data from 1980 onwards are given in Table A for business investment and Table B for GDP. These are also plotted in Chart 3.

Table A: Business investment revision error variances

Horizon	Variance
1	0.0003134
2	0.0002032
3	0.0002744
4	0.0001065
5	0.0006326
6	0.0003410
7	0.001036
8	0.0004604
9	0.0003588
10	0.0009119
11	0.001275
12	0.0001918
13	9.580e-005
14	3.423e-005
15	3.339e-005
16	5.245e-005
17	4.742e-005
18	1.286e-005
19	1.354e-005
20	2.295e-005
21	1.454e-005
22	5.166e-005
23	3.239e-006

We want to investigate the out-of-sample performance of the above equation. We consider

Table B: GDP revision error variances

Horizon	Variance
1	2.202e-006
2	9.134e-006
3	2.136e-006
4	1.551e-005
5	6.912e-007
6	2.655e-005
7	1.156e-005
8	1.103e-005
9	9.416e-006
10	8.282e-006
11	1.029e-005
12	7.553e-006
13	2.834e-006
14	4.285e-007
15	4.357e-006
16	4.678e-007
17	3.364e-006
18	4.375e-007
19	1.835e-006
20	3.300e-007
21	3.558e-006
22	5.340e-007
23	3.612e-006

eight variants of it. Four variants do not include GDP in the equation and four do. The four variants reflect the number of lags of investment considered which varies from 1 to 4. We assume that the revision error becomes smaller and eventually disappears after 24 rounds of revisions.

We compare the forecasting performance of the models using optimal parameter estimates, obtained from (38), and standard parameter estimates. The out-of-sample forecast evaluation exercise is carried out as follows. Starting at 1985 Q2 the model is estimated over the period 1975 Q1-1982 Q2 and investment at 1988 Q3 is forecast. The reason for not using the period 1982 Q3-1987 Q4 data for estimating the coefficients is to ensure (within the assumptions of the experiment) that the original parameter estimate reflects the true parameter rather than be contaminated by revision errors in the data. We continue producing investment forecasts until 1998 Q2. So the whole forecast evaluation period is 1988 Q3-1998 Q2 (10 years). The reason we stop at 1998 Q2 is because we need to use the

Chart 3: Revision error variances for business investment

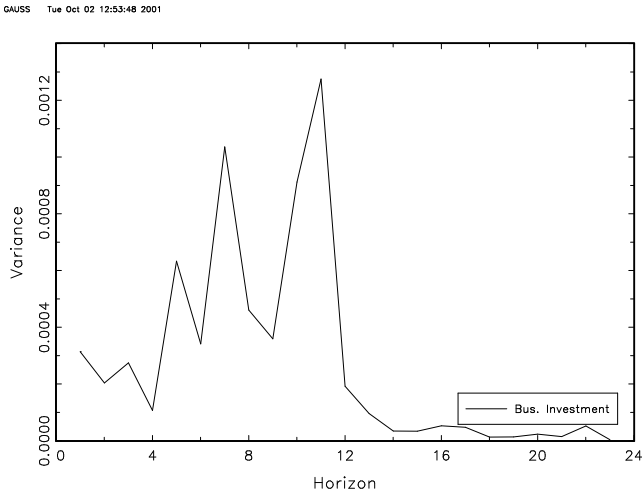
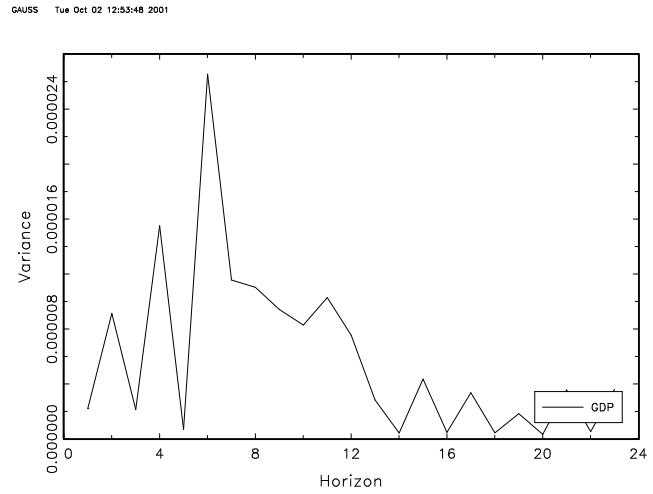


Chart 3: Revision error variances for GDP



most recently available data for the evaluation period as proxies for the true data (uncontaminated by noise).

We compute the ratios of the RMSEs from optimal and standard, unadjusted parameter models. We use the Diebold-Mariano tests (see Diebold and Mariano (1995)) to assess whether there is a statistically significant difference between the two.⁽⁹⁾ We present figures for the whole period, and the two five-year subperiods. These are shown in Table C. The coefficients of the original and uncertainty-corrected models are presented in Tables D and E.

Clearly, the forecasts using the optimal coefficients significantly outperform the standard forecasts for all models for the whole period and the second subperiod, according to the Diebold-Mariano test. For all models they outperform the standard forecast in the first subperiod as well. For the first subperiod, this result is not statistically significant according to the Diebold-Mariano statistic for all models. Overall, the evidence in favour of the optimal forecast is clear-cut.

(9) Positive test statistics indicate superiority of the forecasts based on the optimal forecasting coefficients and vice versa.

Table C: MSE ratios and Diebold-Mariano tests

Model	Whole period		First subperiod		Second subperiod	
	MSE Ratio	D-M Test	MSE Ratio	D-M Test	MSE Ratio	D-M Test
ARDL(1)	0.813	3.226*	0.867	1.894	0.762	2.611*
ARDL(2)	0.758	3.99*	0.778	2.444*	0.738	3.17*
ARDL(3)	0.781	3.764*	0.791	2.42*	0.77	2.895*
ARDL(4)	0.827	3.166*	0.857	1.843	0.796	2.611*
AR(1)	0.845	3.428*	0.889	2.591*	0.802	2.494*
AR(2)	0.795	3.95*	0.79	2.639*	0.801	2.989*
AR(3)	0.784	3.608*	0.751	2.704*	0.827	2.58*
AR(4)	0.823	3.105*	0.847	1.757	0.795	2.74*

* denotes significance at the 1% level.

Table D: Whole period coefficients in AR forecasting models for UK business investment: standard and uncertainty corrected

	AR(1)	AR(2)	AR(3)	AR(4)
standard	-0.150 _(0.121)			
	-0.144 _(0.122)	-0.042 _(0.123)		
	-0.117 _(0.123)	-0.036 _(0.124)	0.090 _(0.124)	
	-0.165 _(0.120)	-0.007 _(0.121)	0.123 _(0.121)	0.251 _(0.119)
optimal	-0.048 _(0.039)			
	-0.043 _(0.038)	-0.008 _(0.039)		
	-0.035 _(0.037)	-0.011 _(0.038)	0.031 _(0.039)	
	-0.043 _(0.034)	-0.004 _(0.036)	0.030 _(0.036)	0.072 _(0.038)

Table E: Whole period coefficients in ARDL forecasting models for UK business investment: standard and uncertainty corrected

	AR(1)	AR(2)	AR(3)	AR(4)	GDP(1)
standard	-0.149 _(0.121)				0.134 _(0.472)
	-0.144 _(0.122)	-0.041 _(0.128)			-0.007 _(0.516)
	-0.117 _(0.123)	-0.037 _(0.130)	0.090 _(0.125)		0.018 _(0.522)
	-0.170 _(0.120)	-0.025 _(0.125)	0.113 _(0.122)	0.256 _(0.119)	0.267 _(0.515)
optimal	-0.048 _(0.039)				0.079 _(0.267)
	-0.043 _(0.038)	-0.008 _(0.038)			-0.026 _(0.268)
	-0.035 _(0.037)	-0.011 _(0.037)	0.031 _(0.039)		0.008 _(0.267)
	-0.044 _(0.034)	-0.005 _(0.035)	0.029 _(0.036)	0.073 _(0.038)	0.100 _(0.258)

6 Summary and conclusion

A brief summary of our analysis is as follows:

- Section 2.1 assumed a data-generating process $y_t^* = ay_{t-1}^* + e_t$ and a measurement equation $y_t = y_t^* + v_t$ with homoskedastic errors, v_t . We showed that the unconditional mean of the process forms a better forecast of the true data than any forecast $\hat{y}_{T+1}^{(n)} = a^{n+1}y_{T-n}$ when the variance of the measurement errors is greater than the variance of the true data.
- Section 2.2 generalised the analysis to the case in which the variance of v_t increases with t , reflecting the fact that more recent data are measured less reliably. In that case we found that it can be optimal to forecast using $\hat{y}_{T+1}^{(n)} = a^{n+1}y_{T-n}$ for some $n \geq 1$: it is better not to use the most recent data in constructing the forecast.
- Section 3 generalised the problem to allow the forecaster to choose the parameter of the forecasting model as well as the forecasting lag. In that case we found that it can be optimal to forecast using $\hat{y}_{T+1}^{(n)} = \tilde{a}^{n+1}y_{T-n}$ for some $n \geq 1$: it is better to use less recent data and a parameter \tilde{a} that differs from the true model parameter a .
- Section 4 generalised the problem further to allow the forecaster to choose a forecasting model from a general linear specification. Specifically, we allowed the forecaster to choose the coefficients on a general AR process that minimise the mean squared error of the forecast. We found that it can be optimal to forecast with an AR model of higher order than the true data-generating process.
- Section 5 applied the analysis of Section 4 to an equation estimated on UK investment data. We found that the forecasts using the optimal forecast model outperformed forecasts from the standard model when compared using the Diebold-Mariano test.

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