# Investment-specific technological change and growth accounting

*Nicholas Oulton**

Working Paper no. 213

* Centre for Economic Performance, London School of Economics, Houghton Street, London WC2A 2AE.
E-mail: n.oulton@lse.ac.uk

The views expressed in this paper are those of the author, and not necessarily those of the Bank of England. This paper was written while I was employed at the Bank of England. I would like to thank, without implicating, Hasan Bakhshi, Dale Jorgenson, Jens Larsen, Simon Price and two anonymous referees for helpful comments and suggestions.

Copies of working papers may be obtained from Publications Group, Bank of England, Threadneedle Street, London, EC2R 8AH; telephone 020 7601 4030, fax 020 7601 3298, e-mail mapublications@bankofengland.co.uk

Working papers are also available at www.bankofengland.co.uk/wp/index.html

The Bank of England's working paper series is externally refereed.

© Bank of England 2004
ISSN 1368-5562

**Contents**

**Abstract**

Greenwood, Hercowitz and Krusell have claimed that the Jorgenson form of growth accounting is conceptually flawed and severely understates the role of technological progress embodied in new capital goods (‘embodiment’) in explaining US growth. To the contrary, this paper shows that in its technology aspects their model is a special case of the Jorgensonian growth-accounting model. What they call investment-specific technological change is shown to be closely related to the more familiar concept of TFP growth: statements about the one can be translated into statements about the other. Empirically, they claim that the proportion of US growth accounted for by embodiment is about twice as large as estimated by conventional growth accounting. But the difference between these estimates is found to be due more to data than to methodology.

Key words: Investment-specific technological change, embodiment, TFP, growth accounting.

*JEL* classification: O47, O41, O51.

**Summary**

In a set of related and influential papers, Greenwood, Hercowitz and Krusell, hereafter GHK, have claimed that the growth-accounting framework that they ascribe to Jorgenson is flawed. They also claim that the methodology of the national accounts is flawed, at least for the purposes of productivity analysis. They develop an alternative framework centred round the concepts of 'neutral technological change' and 'investment-specific technological change'. They use their framework as the basis for determining what proportion of growth is due to investment-specific technological change, ie what is the quantitative importance of 'embodiment'. Embodiment means (roughly) the extent to which in the long run productivity growth is due to improvements in the quality of machinery and equipment, rather than (say) greater efficiency in the way in which production of consumption goods is carried out. GHK claim that Jorgensonian growth accounting severely understates the role of embodiment.

Contrary to their claim, this paper shows that their model can be analysed as a special case of the more general Jorgensonian approach. Consequently, as is also shown, their criticisms of the Jorgenson framework are incorrect. The equations of the GHK model can be derived from a two-sector model in which the production functions are the same up to a scalar multiple (total factor productivity (TFP)). Investment-specific technological change (ISTC) is then found to be closely related to the more familiar concept of TFP growth. In fact, in this special case of the two-sector model, ISTC equals the difference between TFP growth in the investment good sector and TFP growth in the consumption good sector. Neutral technological change is found to equal the growth rate of TFP in the consumption sector.

The two-sector model from which the GHK approach can be derived is consistent with Jorgensonian growth accounting. Jorgenson's approach does not employ the particular aggregate production function that they attribute to him. In his approach, the growth of aggregate output is measured by weighted averages of the growth rates of output in the various sectors, where the weights are the time-varying shares of each sector in the value of output: there is no need to assume that the relative price of investment goods is constant.

GHK criticise the methodology behind the US (and other countries') national accounts, arguing that expenditure on investment goods should be deflated by the price of consumption goods, not the price of investment goods. This argument must also be rejected. The two-sector model that lies behind GHK's results is itself consistent with standard national accounting principles. However, if our interest is in measuring *welfare* rather than output, there is a case for deflating all types of expenditure by the price of consumption. But then it is *net*, not *gross*, domestic product that we should be looking at.

In the empirical section of the paper, we compare two studies of the importance of technical progress in the equipment-producing sectors in explaining US growth, the first by GHK, the second a growth-accounting study by Jorgenson and Stiroh. GHK find embodiment to be twice as important as do Jorgenson and Stiroh. The main reason for this difference is found to be data, not methodology. GHK use a deflator for equipment that falls much more rapidly than the official one. Methodology does provide a subsidiary reason. GHK quantify the role of technical

progress in the equipment-producing sector by asking by how much the steady-state growth rate of consumption would be reduced if ISTC were the *only* source of technical progress. By contrast, the growth-accounting tradition estimates the contribution of TFP growth in a particular sector to aggregate TFP growth. This is measured by TFP growth in the sector in question, weighted by the ratio of the sector's gross output to GDP.

## 1 Introduction

In a set of related papers, Greenwood, Hercowitz and Krusell (1997) and (2000) and Hercowitz (1998), hereafter GHK, have claimed that the growth accounting framework which they ascribe to Jorgenson is flawed. They develop an alternative framework centred round the concepts of 'neutral technological change' and 'investment-specific technological change'. Greenwood *et al* (1997) use their framework as the basis for determining what proportion of growth is due to investment-specific technological change, ie what is the quantitative importance of 'embodiment'. They claim that Jorgensonian growth accounting severely understates the role of embodiment. Greenwood *et al* (2000) use the framework to determine how important investment-specific shocks are to business cycles.

In what follows, I show that, contrary to their claim, their model can be analysed as a special case of the more general Jorgensonian approach. Consequently, as I also show, their criticisms of the Jorgenson framework are incorrect. In the next section I show that their concept of investment-specific technological change (hereafter ISTC) is closely related to the more familiar concept of total factor productivity (TFP). In Section 3, I show why their criticism of the Jorgensonian approach to growth accounting is not correct. Section 3 also discusses how to measure the contribution of embodied technological progress in the GHK and Jorgenson frameworks. Section 4 considers the GHK critique of national accounts methodology and asks whether it is correct to deflate the whole of output by the price of consumption, as GHK advocate. Turning to empirics, Section 5 compares and (as far as possible) reconciles two very different estimates of the contribution of embodied technological progress to US growth, those of Greenwood *et al* (1997) and of Jorgenson and Stiroh (2000a) respectively. Section 6 concludes.

## 2 The relationship between ISTC and TFP

I begin by setting out the GHK model and then go on to show how it can be derived as a special case of a two-sector, neo-classical model. In its simplest form the GHK model can be written as follows (using as far as possible their notation):[(1)]

$$y = c + i = z \cdot l \cdot f(k/l) \qquad \textbf{(1)}$$
$$i^* = iq \qquad \textbf{(2)}$$
$$\dot{k} = i^* - \delta k \qquad \textbf{(3)}$$

Equation **(1)** is the aggregate production function, where constant returns to scale have been imposed. Output ($y$), which can be used for either consumption ($c$) or investment ($i$), is produced

---

(1) The model of Greenwood *et al* (1997) and (2000) is actually more complicated since it has two capital goods sectors, structures and equipment, and they also assume that investment incurs adjustment costs. But these complications have no relevance for the issues discussed in Sections 2 and 3. In the empirical analysis of Section 4 and in the appendix, I extend the model to incorporate two capital goods sectors. The model in Hercowitz (1998) is identical to the one considered here except in three unimportant respects. First, constant returns to scale (which GHK assume) are explicitly imposed in equation **(1)**. Second, Hercowitz adds a resource cost term to equation **(1)**, which however is later dropped from his analysis. Third, equation **(3)** in Hercowitz is in discrete time. Growth accounting theory is most easily developed in continuous time, so I employ the continuous analogue of his equation.

with the aid of capital ($k$) and labour ($l$). The output of investment goods is measured in consumption units. $z$ is a productivity shock which GHK refer to as 'neutral technological change'. Equation **(2)** shows investment in efficiency units ($i^*$), that is, the number of new 'machines' produced ($i$), converted to units of constant quality. The new feature here is $q$ which measures 'investment-specific technological change' or ISTC. The growth rate of $q$ can be thought of as the rate of quality improvement in investment goods. Empirically, $q$ is measured by the ratio of consumption goods prices to investment goods prices. The extent to which output growth is due to increased quality of the capital stock is called the extent of embodiment by GHK. Equation **(3)** describes the evolution of the capital stock when measured in efficiency units.

At first sight, this model seems to offer a radically different approach to that of conventional growth accounting. It also offers a challenge to conventional national income accounting. In the GHK model, output is measured in consumption units, ie nominal output of the investment good is deflated by the consumption price index. In national income accounting, nominal expenditure on investment is deflated by price indices for investment goods. However, contrary to appearances, I show in what follows that the model of equations **(1)-(3)** can be derived as a special case of a conventional two-sector model, where TFP is growing at different rates in the two sectors. As a result, everything that GHK say about ISTC and neutral technological change can be translated into statements about TFP growth. Also, there is no need to reformulate national income accounting.

Implicit in the GHK model above are two final goods industries, one producing consumption goods, the other investment goods.(2) Label these two sectors by subscript $c$ and $i$. Measure the output and the price of investment goods in units of constant quality (efficiency units), ie output of investment goods is $i^*$: this is in accordance with standard production theory and national income accounting. Assume that each sector's technology can be described by a production function exhibiting constant returns to scale and unbiased technical progress:

$$c = A_c \cdot l_c \cdot f^c(k_c / l_c), \;\; A_c > 0$$

$$i^* = A_i \cdot l_i \cdot f^i(k_i / l_i), \;\; A_i > 0 \qquad \textbf{(4)}$$

Here $k_x, l_x$ are capital and labour inputs in sector $x \, (x = c, i)$, and $A_x(t)$ is the level of TFP in sector $x$. Sector inputs must add to the economy-wide totals:

$$k_c + k_i = k$$

$$l_c + l_i = l$$

(2) The two-sector interpretation of the GHK model has been noted by Whelan (2001). Ho and Stiroh (2001) also seek to critique GHK by reference to two-sector models.

Denote TFP growth in the two sectors by $\hat{A}_c, \hat{A}_i$, where a 'hat' (^) denotes a growth rate: that is, $\hat{A}_x = d \ln A_x(t)/dt,\ x = c, i$, where *t* is time. Assuming perfect competition, TFP growth (in continuous time) in the two sectors can be measured by

$$\hat{A}_c = \hat{c} - \alpha_c \hat{k}_c - (1-\alpha_c)\hat{l}_c$$
$$\hat{A}_i = \hat{i}^* - \alpha_i \hat{k}_i - (1-\alpha_i)\hat{l}_i \qquad \textbf{(5)}$$

Here $\alpha_x$ is the (possibly time-varying) capital share in industry *x*:

$$\alpha_x = rk_x/(wl_x + rk_x) \qquad x = c, i \qquad \textbf{(6)}$$

where *r* is the rental on capital and *w* is the wage. Competition ensures these input prices are the same in both sectors.

Equations **(5)** measure TFP growth from the 'quantity' side, but TFP growth can also be measured from the 'price' side. Under constant returns, there exists a unit cost function in each sector, which is dual to the production function. Using the dual approach, the growth of TFP equals the rate at which unit cost would fall over time, if input prices were held constant (Jorgenson and Griliches (1967), Barro (1999)). Under the assumptions of constant returns to scale and perfect competition, price equals unit cost. The unit cost functions can then be thought of also as price functions, relating output prices to input prices when costs are minimised. With appropriate normalisation these price functions can be written in the present case as:

$$p_c = A_c^{-1} \cdot w \cdot g^c(r/w)$$
$$p_i = A_i^{-1} \cdot w \cdot g^i(r/w) \qquad \textbf{(7)}$$

where $p_c, p_i$ are the prices of consumption and investment respectively. Note that $q = p_c / p_i$.

Now consider a special case: assume that the unit cost functions, or, equivalently, the production functions, are identical in the two industries up to a time-varying scale factor (TFP): that is $g^c(\cdot) = g^i(\cdot) = g(\cdot)$ say and $f^c(\cdot) = f^i(\cdot) = f(\cdot)$ say. Then input shares and input ratios will be the same in the two sectors, ie $\alpha_c = \alpha_i$ and $k_c/l_c = k_i/l_i = k/l$ at all input prices. And from **(7)** by division,

$$q = p_c / p_i = A_i / A_c \qquad \textbf{(8)}$$

Differentiating with respect to time:

$$\hat{q} = \hat{p}_c - \hat{p}_i = \hat{A}_i - \hat{A}_c \qquad \textbf{(9)}$$

So ISTC turns out to equal just the excess of TFP growth in the investment good sector over TFP growth in the consumption good sector.

We now show how the other concept of the GHK model, neutral technological change, is related to TFP. Continuing with the special case, output measured in consumption units is

$$\begin{aligned} y &= c + i = c + i^* / q = c + \left( A_c / A_i \right) \cdot i^* \\ &= A_c \cdot l \cdot f(k/l) \end{aligned} \qquad \textbf{(10)}$$

using **(8)**, **(1)**, **(2)** and **(4)**. Now comparing equation **(10)** with equation **(1)** of the GHK model, we find that

$$z = A_c \qquad \textbf{(11)}$$

In other words, 'neutral technological change' is equivalent to TFP growth in the consumption good sector. Hence, as asserted, the GHK model can be derived from a special case of the two-sector model.

In general, relative prices may change either because TFP is growing at different rates or because one sector is more capital intensive than the other. For example, suppose the investment good sector is the more capital intensive, that wages are rising over time relative to the rental on capital (as would be the case in a steady state), and that TFP growth rates are the same in the two sectors. Then the relative price of the investment good will be falling: ie *q* will be rising. But we would hesitate to describe this fall in relative price as ISTC, since by assumption TFP growth is the same in both sectors. The contrary assumption, that price and production functions are identical up to a time-varying scale factor, leaves differences in TFP growth as the sole cause of relative price change. And only by making this crucial assumption can GHK derive an aggregate production function like **(1)**.[3] [4]

## 3 Jorgensonian growth accounting

The previous section showed the relationship between the GHK concepts of neutral technological change and of ISTC on the one hand and sectoral TFP growth rates on the other. Section 3 derives the relationship between the GHK concepts and the *aggregate* TFP growth rate. In so doing, it shows that GHK's criticisms of Jorgensonian growth accounting are incorrect.

---

(3) This is acknowledged in their (1997) paper (see pages 356-58) and in their (2000) paper (see pages 108-09).
(4) The first draft of this paper was written about the same time as, but independently of, Ho and Stiroh (2001), which is also a critique of the GHK approach. In some respects their paper has a wider focus, complementary to that of the present one. For example, they argue that a broader theoretical framework is needed to capture key features of the data and they quantify some of these concerns. In other respects our approaches and conclusions differ. Like the present paper, Ho and Stiroh seek to interpret the GHK model in terms of a two-sector model. As their title and various comments within the body of their paper suggest, they argue that the GHK framework is logically and conceptually inadequate for the task at hand. For example, they state: 'Our primary point is that a one-sector model should not be used to explain the movement in the relative prices of two goods, and cannot be used to distinguish between embodied and disembodied technical change' (page 1). By contrast this paper argues that the GHK model (though apparently one-sector) can be derived as a special case of the standard neo-classical two-sector model (which Ho and Stiroh refer to as the 'non-joint two-sector model').

Hercowitz (1998) and Greenwood *et al* (1997) draw a contrast between what they call the 'Solow' view and the 'Domar-Jorgenson' or just 'Jorgenson' model of growth accounting. In aid of their interpretation they cite Jorgenson's (1966) article on embodiment and also Hulten (1992). They claim that Jorgenson is advocating a particular form of the aggregate production function given by

$$y = c + qi = z \cdot l \cdot f(k/l) \qquad \textbf{(12)}$$

This equation should be compared with equation **(1)**: the only difference is the presence of $q$. Now if equation **(1)**, together with **(2)** and **(3)**, constitute the correct model, then equation **(12)** is just a mistake, unless $q$ happens to be constant, in which case these two models are equivalent. But if $q$ is constant, then ISTC must necessarily be zero. Empirically, however, GHK find ISTC to be large. So this leads them to conclude that the Jorgenson approach is wrong and underestimates the role of technological change embodied in new equipment.

There is only one problem with this argument. Neither in the cited 1966 article nor anywhere else in his extensive empirical work does Jorgenson rely on an aggregate production function of the form of **(12)**.[(5)] His starting point is always the accounting relationship that the value of all types of output in current prices must equal the value of all types of inputs, also in current prices. Specialised to the present model of two industries where each produces only final goods, the accounting relationship is

$$p_c c + p_i i^* = wl + rk \qquad \textbf{(13)}$$

Like all good accounting relationships, this equation also embodies a model, namely that competitive payments to inputs exhaust the value of output. By totalling differentiating this relationship with respect to time, we can derive Divisia quantity and price indices for aggregate output and aggregate input, in the spirit of Jorgenson (1966). Define $s$ as the current price share of investment in the value of output:

$$s = p_i i^* / (p_c c + p_i i^*) = i/y \qquad \textbf{(14)}$$

Then a Divisia index of output (GDP), denoted by $Y$, is:

$$\hat{Y} = (1-s)\hat{c} + s\hat{i}^* \qquad \textbf{(15)}$$

Note that a different symbol is used for output here, $Y$ not $y$, since this is not the same concept as the output of GHK's aggregate production function, equation **(1)**.

---

(5) It is true that Hulten (1992) appears to attribute to Jorgenson an aggregate production function of the form **(12)**: this is Hulten's equation (7). But Hulten just uses this to derive Divisia indices of output and input (his equation (8)), for which the aggregate production function is not necessary. So the aggregate production function is not an essential part of Hulten's argument. (Apart from this point, nothing in the present paper is in disagreement with Hulten (1992).) Nor does the aggregate production function **(12)** appear in Domar (1963), which is cited by Greenwood *et al* (1997).

Using now the more familiar quantity approach, and writing the aggregate share of capital as $\alpha = rk/(rk + wl)$, TFP growth at the aggregate level ( $\mu$ ) is found to be

$$\mu \equiv \partial \ln Y / \partial t = \hat{Y} - \alpha\hat{k} - (1-\alpha)\hat{l}$$
$$= (1-s)\hat{A}_c + s\hat{A}_i \qquad \textbf{(16)}$$

where use is made of **(5)**, **(6)** and **(15)**. In other words, aggregate TFP growth is a weighted average of TFP growth rates in the two sectors, where the weights are shares in final output. This is a special case of Domar aggregation (Domar (1961)).[(6)]

We saw in the previous section that neutral and investment-specific technological change can be interpreted in terms of sectoral TFP. So it is no surprise that the aggregate TFP growth rate can also be translated into GHK terminology. Substituting **(9)** and **(11)** into **(16)**,

$$\mu = \hat{z} + s\hat{q} \qquad \textbf{(17)}$$

In the terminology of GHK, the aggregate TFP growth rate equals neutral technological change plus ISTC, the latter weighted by the investment share. Of course, this interpretation is subject to their (implicit) assumption of identical production functions in the two sectors: empirically, **(16)** and **(17)** will yield different results if this assumption is not a good approximation.

Now let us consider the different ways to answer the question of interest to GHK: what is the proportion of growth which is attributable to embodiment? The traditional answer of growth accounting is to express the contribution of TFP growth in the investment goods industry as a proportion of aggregate TFP growth: from **(16)** this is

$$\frac{s\hat{A}_i}{\mu} = \frac{s\hat{A}_i}{(1-s)\hat{A}_c + s\hat{A}_i} \qquad \textbf{(18)}$$

Alternatively, and more in the spirit if GHK, we could calculate the proportional contribution of ISTC to aggregate TFP growth:

$$\frac{s\hat{q}}{\mu} = \frac{s\hat{q}}{\hat{z} + s\hat{q}} = \frac{s(\hat{A}_i - \hat{A}_c)}{(1-s)\hat{A}_c + s\hat{A}_i} \qquad \textbf{(19)}$$

GHK however ask a different question: what is the contribution of embodiment (ISTC) to the growth of consumption per hour in balanced (steady-state) growth? As shown in the appendix,

---

(6) In the more general case where industries make intermediate sales as well as final ones, the Domar weights are the ratios of gross output to aggregate final output; aggregate TFP growth is then a weighted sum, not a weighted average. The more general result is derived in Hulten (1978) and Jorgenson *et al* (1987) and is discussed in Oulton (2000). Jorgenson *et al* (1987) also show how the result is modified if some of the assumptions are relaxed, eg if any of the input prices differ between industries. Domar aggregation has been applied recently to study the impact of productivity growth in ICT industries and investment in ICT on aggregate output and productivity (Jorgenson and Stiroh (2000b), Oliner and Sichel (2000)).

the balanced growth rate of consumption per hour worked, denoted by *h*, in the two-sector model is

$$h = \frac{(1-\alpha)\hat{A}_c + \alpha \hat{A}_e}{1-\alpha} = \frac{\hat{z} + \alpha \hat{q}}{1-\alpha} \quad \textbf{(20)}$$

The capital share $\alpha$ is now assumed constant, implying that the underlying production functions are Cobb-Douglas, since with two or more sectors this is required for the existence of balanced growth. The contribution of ISTC is now defined by GHK to be what balanced growth would be if sector neutral technological progress ( $\hat{z}$ ) were zero.[(7)] Expressed as a proportion of the actual balanced growth rate, this is

$$\frac{\alpha \hat{q}/(1-\alpha)}{h} = \frac{\alpha \hat{q}}{\hat{z} + \alpha \hat{q}} = \frac{\alpha(\hat{A}_i - \hat{A}_c)}{(1-\alpha)\hat{A}_c + \alpha \hat{A}_i} \quad \textbf{(21)}$$

In summary, we can calculate the contribution of ISTC to aggregate TFP growth, as in **(19)**, or its contribution to balanced growth of consumption, as in **(21)**. The results will differ, since **(19)** weights by the investment ratio ($s$) while **(21)** weights by the capital share ($\alpha$). Empirically, the capital share is likely to exceed the investment ratio, so the GHK measure is likely to be the higher of the two.

## 4 Should the whole of output be deflated by the price of consumption?

In their 1997 article (Greenwood *et al* (1997), GHK argue that the US (and implicitly, every other country's) national income and product accounts (NIPA) are flawed as tools for productivity analysis. Their reason is that in the NIPA, just as in Jorgensonian growth accounting, nominal output of investment goods is deflated by price indices that adjust for quality change (at least in principle), and not by the price of consumption goods. But in the previous sections, their results have been derived as special cases of a two- (or three-) sector model that is itself consistent with (a simplified form of) the NIPA. Hence their claim must be rejected.

Index number theory leaves us in no doubt that, for an index of output like GDP, expenditure on each final good should be deflated by its own price (Diewert (1976) and (1987)). But the same is not necessarily true of an index of *welfare*. For the latter, the leading candidate is the measure suggested by Weitzman (1976), which I call Weitzman's net domestic product, denoted by *WNDP*. This is defined as net domestic product,[(8)] measured in consumption units.

---

(7) The use of the balanced growth rate as the reference point is consistent with Hulten (1979); he argued that the contribution of TFP to economic growth should be measured as inclusive of the contribution of the capital accumulation induced by a positive rate of growth of TFP. In a one-sector model, the balanced growth rate is $\mu/(1-\alpha)$ and Hulten argues that the proportional contribution of TFP should be measured as the balanced growth rate divided by the actual growth rate, with the remainder being ascribed to capital accumulation.

(8) Net national product is Weitzman's term, though net national *income* might be more appropriate. I use the term 'net domestic product' to retain the link with the national accounts and because net income from abroad is ignored here. See Sefton and Weale (1996) for extensions of Weitzman's analysis to an open economy and to trade in natural resources.

Alternatively, it is nominal NDP deflated by the price index for consumption. In symbols, using the same notation as before,

$$
\begin{aligned}
WNDP(t) &= c(t) + [\dot{k}(t) / q(t)] \\
&= c(t) + [i^{*}(t) / q(t)] - [\delta k(t) / q(t)] \\
&= c(t) + i(t) - [\delta k(t) / q(t)]
\end{aligned}
\qquad \textbf{(22)}
$$

Here $\dot{k}$ is real *net* investment measured in its own (efficiency) units. (The definition is easily extended to encompass many investment goods.) *WNDP* has some similarity to the GHK concept of output defined in equation **(1)** $(y = c + i)$. The difference is that in the GHK measure it is nominal GDP, not NDP, that is deflated by the price of consumption.

The intuition behind WNDP is that only consumption matters for welfare. So the current level of consumption must obviously be part of the measure. In addition, net investment increases future consumption. The second term in **(22)** can be interpreted as the present value of the future stream of consumption generated by this period's addition to the capital stock.

Weitzman (1976) gave a more formal justification (see also Weitzman (1997) and (2003)). He showed that *WNDP* is proportional to the yield on wealth (assuming perfect competition and no externalities). That is, it is equivalent to permanent income. His argument was as follows. Consider an economy that behaves as if it is governed by a social planner who seeks to maximise wealth (*W*), defined as the present value of the stream of real consumption:

$$\max_{c(s)} W(t) = \int_t^{\infty} c(s) \cdot e^{-\rho(s-t)} \cdot ds \qquad \textbf{(23)}$$

subject to given initial capital stocks, the production set and the capital accumulation equations (eg in the present case equations **(4)** and **(3)**); here $\rho$ is the rate of discount (real rate of interest). Along the optimal path, Weitzman showed that

$$WNDP(t) = \rho \cdot \int_t^{\infty} c(s) \cdot e^{-\rho(s-t)} \cdot ds = \rho \cdot W(t) \qquad \textbf{(24)}$$

So *WNDP* is the yield on wealth (the present value of consumption), ie it is equivalent to permanent income. *WNDP* can therefore be considered a cardinal measure of welfare.(9)
In a steady state the current price shares of net investment in NDP and of gross investment in GDP are both constant, so consumption, the GHK measure of output (*y)* and *WNDP* will all grow

(9) Others, eg Usher (1980) and Scott (1990), have also suggested that, when measuring real income, investment should be deflated by the price of consumption goods. The original idea goes back to Hicks (1939, chapter XIV) and (1940). But Weitzman was the first to put this suggestion on a clear theoretical footing. Note that *WNDP* as defined in equation **(22)** is the Hamiltonian associated with the problem of equation **(23)**, provided that we can equate actual market prices with shadow prices.

at the same rate. Under the assumptions of the GHK model, the output of investment goods is rising more rapidly than that of consumption goods. So in steady state GDP as conventionally measured will be rising more rapidly than *WNDP* or GHK output. But out of steady state this need not be the case: *WNDP* could even rise more rapidly than GDP (Oulton (2002)). In general, GHK output does not grow at the same rate as *WNDP*. So the GHK concept does not measure correctly the growth rate of welfare. Also, by including depreciation, the GHK measure overstates the level of welfare at a point in time.[10]

## 5 What proportion of growth is due to embodiment (ISTC)?

Empirically, the issue of interest to Greenwood *et al* (1997) is the extent to which technological progress is embodied, ie caused by ISTC. They find that ISTC accounted for 58% of the growth in consumption per hour over the period 1954-90. As we have argued, this issue can also be addressed using the more general growth accounting framework. Jorgenson and Stiroh (2000a, Table 1) provide data on Domar weights and TFP growth rates in the equipment-producing sectors of the US economy. It can then be calculated (using equations **(16)** and **(18)**) that TFP growth in these sectors accounts for 33% of aggregate TFP growth over 1958-96.[11] This large difference is due to some extent to methodology but mostly to data. These will be discussed in turn.

Greenwood *et al* (1997) actually work with a slightly more complicated model than that of Sections 2 and 3. They assume two capital good sectors, structures and equipment, but that only equipment enjoys ISTC; in our terms this means that TFP growth in structures is the same as in the consumption sector. The properties of this model are derived in the appendix. As shown there, in this extended model

$$\hat{q} = \hat{A}_e - \hat{A}_c \qquad \textbf{(25)}$$

where $\hat{A}_e$ is the growth rate of TFP in the equipment sector (compare **(9)**). The balanced growth rate of consumption per hour is now

$$h = \frac{(1-\varepsilon)\hat{A}_c + \varepsilon\hat{A}_e}{1-\varepsilon-\sigma} = \frac{z+\varepsilon\hat{q}}{1-\varepsilon-\sigma} \qquad \textbf{(26)}$$

where $\varepsilon$ is the elasticity of output (whether of consumption or investment goods) to equipment and $\sigma$ is the elasticity of output with respect to structures. (This equation can be compared to

---

(10) See Oulton (2002) for further analysis of *WNDP*. This paper considers the empirical relationship between *WNDP* and GDP in the United States. It also shows that growth accounting can be done for *WNDP* in a way analogous to the way it is done for GDP.

(11) TFP growth in the equipment-producing sectors averaged 1.59% per annum and these sectors had a Domar weight of 0.101. The overall TFP growth rate was 0.48% per annum. Hence the contribution of the equipment-producing sectors was 0.101*1.59/0.48 = 0.33. The figure of 1.59% per annum for TFP growth in the equipment-producing industries is derived as a weighted average of the TFP growth rates in three industries: Industrial Machinery and Equipment, Electronic and Electric Equipment, and Instruments; the weights are the Domar weights. These data are from Table 1 of Jorgenson and Stiroh (2000a).

**(20)**: note that $\alpha = \varepsilon + \sigma$ where (as before) $\alpha$ is the aggregate share of capital in income.) These elasticities are assumed constant, implying that the underlying production functions are Cobb-Douglas, as is required for the existence of balanced growth.

The aggregate TFP growth rate in the extended model is

$$\mu = (1 - s_e)\hat{A}_c + s_e\hat{A}_e = \hat{z} + s_e\hat{q} \qquad \textbf{(27)}$$

where $s_e$ is the current price share of equipment investment in aggregate output (compare **(17)**). Hence the contribution of ISTC to aggregate TFP growth is

$$\frac{s_e\hat{q}}{\mu} = \frac{s_e\hat{q}}{\hat{z} + s_e\hat{q}} = \frac{s_e(\hat{A}_e - \hat{A}_c)}{(1 - s_e)\hat{A}_c + s_e\hat{A}_e} \qquad \textbf{(28)}$$

which can be compared to **(19)**, while the contribution of ISTC to balanced growth is now

$$\frac{\varepsilon\hat{q}/(1-\alpha)}{h} = \frac{\varepsilon\hat{q}}{\hat{z} + \varepsilon\hat{q}} = \frac{\varepsilon(\hat{A}_e - \hat{A}_c)}{(1-\varepsilon)\hat{A}_c + \varepsilon\hat{A}_i} \qquad \textbf{(29)}$$

which can be compared to **(21)**.

We can now quantify these various concepts using data from Greenwood *et al* (1997) on the one hand and from Jorgenson and Stiroh (2000a) on the other. Table A puts the relevant data from each of these articles side by side. It also uses each article's data to estimate the other article's growth concepts. For example, we can use the Jorgenson-Stiroh data to estimate ISTC and the GHK data to estimate TFP. Using the GHK data, the aggregate TFP growth rate was 0.62% per annum while using Jorgenson and Stiroh's it was only 0.48% per annum. The (literal) bottom line is that, on the GHK data, 58% of balanced growth is due to ISTC, while using the Jorgenson-Stiroh data this proportion is only 37%.

The main reason for this disparity is that the estimate of ISTC from the Jorgenson-Stiroh data is only 1.23% per annum while that from the GHK data is 3.21% per annum. Another way of making the same point is that TFP growth in the equipment-producing industries is 1.59% per annum using Jorgenson-Stiroh data, but 3.60% per annum using GHK data. The Jorgenson-Stiroh estimates are based ultimately on the US NIPA, including the official price indices. The GHK estimate of $\hat{q}$ is the difference between the growth rates of a deflator for consumption (excluding housing and durables) and one for producers' durable equipment. The deflator for durable equipment, which derives from Gordon (1990), falls much more rapidly than the official deflator: see Figure 1 of Hercowitz (1998). In fact, in extending the Gordon series, which ends in 1983, to 1990, GHK state that they cut about 1.5 percentage points per annum from the growth of the official deflator (Greenwood *et al* (1997), footnote 20).

**Table A**
**GHK versus growth accounting: a comparison of estimates**

| | *Greenwood et al (1997)* | *Jorgenson and Stiroh (2000a)* |
|---|---|---|
| *Period* | *1954-90* | *1958-96* |
| $\hat{z}$ | ***0.39*** | 0.36 |
| $\hat{q}$ | ***3.21*** | 1.23 |
| $\varepsilon$ | ***0.17*** | n.a. |
| $\sigma$ | ***0.13*** | n.a. |
| $s_e$ or Domar weight[(a)] | ***0.073*** | ***0.101*** |
| $\hat{A}_c$ | 0.39 | ***0.36*** |
| $\hat{A}_e$ | 3.60 | ***1.59*** |
| $\mu$ | 0.62 | ***0.48*** |
| $s_e \hat{A}_i / \mu$ | 0.42 | ***0.33*** |
| $h$ | ***1.34*** | 0.81 |
| *Embodiment (ISTC)* | | |
| Contribution to $\mu$ [= $s_e \hat{q}/(\hat{z}+s_e \hat{q})$, from **(28)**] | 0.38 | 0.26 |
| Contribution to $h$ [= $\varepsilon \hat{q}/(\hat{z}+\varepsilon \hat{q})$, from **(29)**] | ***0.58*** | 0.37 |

*Note:* Growth rates (denoted by 'hats') are in per cent per annum. Other quantities are ratios. Figures in ***bold italic*** are those given in, or directly derived from, the article at the head of the column. Other figures are derived by me from the figures in bold italic, using equations **(25)-(29)**. $\varepsilon, \sigma$ are the elasticities of output with respect to equipment and structures respectively. Estimates involving balanced growth use the GHK estimates of $\varepsilon$ and $\sigma$.

(a) In an open economy, the Domar weight need not equal the investment ratio.

In summary, the main reason for the different results yielded by the GHK and Jorgenson-Stiroh data is that GHK use a deflator for durable equipment which falls much more rapidly than the official one which is implicit in Jorgenson-Stiroh. Deciding which deflator is nearer the truth is beyond the scope of this paper.

A second reason relates to GHK's use of balanced consumption growth as the reference. When embodiment is measured relative to TFP growth, the two estimates of the embodiment contribution are much closer: from the penultimate line of Table A, the contributions are 38% according to GHK versus 26% according to Jorgenson-Stiroh. The formula for the contribution to TFP, equation **(28)**, involves the share of equipment investment in GDP ($s_e$) while the formula for the contribution to balanced growth, equation **(29)**, involves the elasticity of output with respect to equipment ($\varepsilon$). According to GHK, the actual share of equipment investment in output ($s_e$) averaged 0.073, whereas their calibration yields an elasticity of output with respect to equipment ($\varepsilon$) which is more than twice as high, 0.17. So GHK's higher estimate of ISTC gets a bigger weight in calculating the contribution to balanced growth and this accentuates the gap between the GHK and Jorgenson-Stiroh results.

There is certainly theoretical support for using balanced growth as the benchmark (Hulten (1979)), but the fact that GHK are asking a different question should be borne in mind when comparing their results with those of growth-accounting studies.

## 6 Conclusion

The GHK model has been shown to be a special case of the framework developed and applied by Jorgenson and others to the study of productivity growth. The equations of the GHK model can be derived from a two-sector model in which the production functions are the same up to a scalar multiple (TFP). Investment-specific technological change (ISTC) is then found to be closely related to the more familiar concept of TFP growth. In fact, in the special case of the two-sector model it equals the difference between TFP growth in the investment good sector and TFP growth in the consumption good sector. Neutral technological change is found to equal the growth rate of TFP in the consumption sector. The criticism that GHK make of Jorgenson's approach has been shown to be incorrect: Jorgenson's approach does not employ the particular aggregate production function which they attribute to him.

There is a large difference between GHK and the growth-accounting study of Jorgenson and Stiroh (2000a) over the importance of technical progress in the equipment-producing sectors in explaining US growth. But the main reason for this difference is data, not methodology. GHK use a deflator for equipment that falls much more rapidly than the official one. Methodology does provide a subsidiary reason. GHK quantify the role of technical progress in the equipment-producing sectors by asking by how much the steady-state growth rate of consumption would be reduced if ISTC were the *only* source of technical progress. By contrast the growth-accounting tradition asks, what is the contribution of TFP growth in different sectors to aggregate TFP growth?

GHK criticise the methodology behind the US (and other countries') national accounts, arguing that expenditure on investment goods should be deflated by the price of consumption goods, not the price of investment goods. This argument also must be rejected. The two-sector model that lies behind GHK's results is itself consistent with standard national accounting principles. However, if our interest is in measuring welfare rather than output, there is a case for deflating all types of expenditure by the price of consumption. But then it is *net*, not *gross*, domestic product that we should be looking at (the measure advocated by Weitzman (1976)).

While this paper has been critical of GHK, it is only fair to point out that they have gone beyond growth accounting pure and simple by embedding their assumptions about technology in a DSGE model. This enables them to model (for example) the effect on the US business cycle of productivity shocks which hit the equipment-producing sector.[12] The present paper, which aims mainly to clarify the relationship of their work to growth accounting, should not be taken as impugning the value of this type of contribution.

[12] Bakhshi and Larsen (2000) apply their model, with extensions, to study the effect on the UK business cycle of shocks arising from the ICT sector.

## Appendix

### Balanced growth in a three-sector model

This appendix derives the balanced (steady-state) growth rate of a continuous time version of the three-sector model of Greenwood *et al* (1997). The equations of the model are:[13]

$$c = A_c \cdot l_c \cdot (k_e / l)^{\varepsilon} \cdot (k_s / l)^{\sigma}, \quad A_c > 0,\ 0 < \varepsilon, \sigma < 1$$

$$i_s = A_c \cdot l_s \cdot (k_e / l)^{\varepsilon} \cdot (k_s / l)^{\sigma}$$

$$i_e^* = A_e \cdot l_e \cdot (k_e / l)^{\varepsilon} \cdot (k_s / l)^{\sigma},\ A_e > 0$$

$$\dot{k}_s = i_s - \delta_s k_s$$

$$\dot{k}_e = i_e^* - \delta_e k_e$$

Here $i_s$ is investment in structures and $i_e^*$ is investment in equipment, measured in units of constant quality (efficiency units); $k_s$, $k_e$ are the corresponding capital stocks and $\delta_s, \delta_e$ the corresponding depreciation rates. Hours worked in sector *x* are $l_x$, $x = c, s, e$. Total hours worked *l*, where $l = l_c + l_s + l_e$, are taken to be exogenous. The first three equations are the production functions. These are assumed to be the same, up to a time-varying scalar multiple (TFP); consequently, the capital-labour ratios are the same in all three industries and equal to the aggregate capital-labour ratios. The production functions are also assumed to take the Cobb-Douglas form, since it can be shown that, for a steady state to exist, they must be of this form, at least asymptotically. TFP in the consumption and structures industries grows at the constant rate $\hat{A}_c$, while in the equipment industry it grows at the constant rate $\hat{A}_e$.

The three nominal prices are $p_c, p_s, p_e$. We write the relative price of equipment and consumption as

$$q = p_c / p_e$$

By invoking unit cost functions dual to the production functions and assuming perfect competition so that price equals unit cost (as in the text), we can show that

$$\hat{q} = \hat{p}_c - \hat{p}_e = \hat{A}_e - \hat{A}_c$$

Similarly, we can show that (with suitable normalisation) $p_c = p_s$.

(13) The model is obviously incomplete since it lacks a household or a government sector. But it turns out that the balanced growth rate depends only on production-side parameters and not (for example) on consumer preferences, so for present purposes these sectors can be ignored.

By definition of a steady state, output of consumption goods, normalised by aggregate (whole economy) hours, grows at a constant rate, say *h*, and the two current price investment ratios, that of investment in equipment to output and that of investment in structures to output, are constant. These ratios are

$$p_s i_s /(p_c c + p_s i_s + p_e i_e^*) = i_s /(c + i_s + (i_e^* / q))$$

and

$$p_e i_e^* /(p_c c + p_s i_s + p_e i_e^*) = (i_e^* / q)/(c + i_s + (i_e^* / q))$$

If aggregate consumption output is growing at the steady-state rate $h + \hat{l}$, then constancy of these ratios implies that $i_s$ and $i_e^* / q$ are both growing at $h + \hat{l}$ too. This implies that $i_e^*$ is growing at $h + \hat{l} + \hat{q} = h + \hat{l} + \hat{A}_e - \hat{A}_c$. In steady state, the capital stocks must be growing at the same rate as the corresponding investment rates. That is,

$$\hat{c} = \hat{i}_s = \hat{k}_s = h + \hat{l};\ \hat{i}_e^* = \hat{k}_e = h + \hat{l} + \hat{A}_e - \hat{A}_c$$

Using these facts and totally differentiating the production functions with respect to time, we find that in steady state

$$\hat{l}_c = \hat{l}_s = \hat{l}_e = \hat{l}$$

Now focus on the total differential of the production function for consumption goods with respect to time:

$$\hat{c} = \hat{A}_c + \hat{l}_c + \varepsilon \hat{k}_e + \sigma \hat{k}_s - \varepsilon \hat{l} - \sigma \hat{l}$$

Substituting in the steady-state values and solving for *h*,

$$h = \frac{(1-\varepsilon)\hat{A}_c + \varepsilon \hat{A}_e}{1 - \varepsilon - \sigma} = \frac{(1-\varepsilon)\hat{A}_c + \varepsilon \hat{A}_e}{1 - \alpha}$$

where $\alpha = \varepsilon + \sigma$ is the aggregate capital share. Using GHK terminology, this can also be written as

$$h = \frac{\hat{z} + \varepsilon \hat{q}}{1 - \varepsilon - \sigma}$$

where, as in the two-sector model of Sections 2 and 3, $\hat{z} = \hat{A}_c$. The steady-state solution for the two-sector model now follows by setting $\sigma = 0$, so that $\varepsilon = \alpha$.

The aggregate TFP growth rate can be found by applying the principle of Domar aggregation:

$$\mu = (1 - s_e)\hat{A}_c + s_e \hat{A}_e = \hat{z} + s_e \hat{q}$$

where (as in the text) $s_e$ is the value share of equipment investment in output. An interesting feature of the model is that, though the balanced growth rate of consumption is independent of (for example) depreciation rates and time preference, the same is not true of the aggregate TFP growth rate or of Divisia output, even in steady state. If depreciation rates are low, or if consumers are patient, the steady-state level of the equipment capital stock will be comparatively high in relation to output. This means that the steady-state share of equipment investment in output will be high too. Consequently, the steady-state growth rates of aggregate (Divisia) output and of aggregate TFP will be high, since equipment investment will receive a larger weight.

Finally, the model above is in continuous time but Greenwood *et al* (1997) derive an analogous result for balanced growth in a discrete time model. Their equation (8) for the steady-state growth rate is, in their notation:

$$g = \gamma_z^{1/(1-\alpha_e-\alpha_s)} \gamma_q^{\alpha_e/(1-\alpha_e-\alpha_s)}$$

where $g, \gamma_z, \gamma_q$ are the *gross* growth rates of output per hour, *z* and *q* respectively, and where a gross growth rate is one plus an ordinary discrete growth rate. Also, translating their notation to mine, $\alpha_e = \varepsilon$ and $\alpha_s = \sigma$. Hence taking logs in the last equation, and using the fact that $\ln(1+x) \approx x$ for small *x*, we find that

$$\ln g = \frac{\ln \gamma_z + \varepsilon \ln \gamma_q}{1-\varepsilon-\sigma} \approx h$$

Thus the discrete and continuous models are essentially equivalent, as is to be expected.

**References**

**Bakhshi, H and Larsen, J (2000)**, 'Investment-specific technological progress and the UK business cycle', *Bank of England Working Paper no. 129.*

**Barro, R J (1999)**, 'Notes on growth accounting', *Journal of Economic Growth*, Vol. 4, pages 119-37.

**Diewert, W E (1976)**, 'Exact and superlative index numbers', *Journal of Econometrics*, Vol. 4, pages 115-46.

**Diewert, W E (1987)**, 'Index numbers', in Eatwell, J, Milgate, M and Newman, P (eds), *The new Palgrave: a dictionary of economics*, London and Basingstoke: Macmillan Press.

**Domar, E D (1961)**, 'On the measurement of technological change', *Economic Journal*, Vol. LXXI, pages 709-29.

**Domar, E D (1963)**, 'Total productivity and the quality of capital', *Journal of Political Economy*, Vol. 71, pages 586-88.

**Gordon, R J (1990)**, *The measurement of durable goods prices*, Chicago: University of Chicago Press.

**Greenwood, J, Hercowitz, Z and Krusell, P (1997)**, 'Long-run implications of investment-specific technological change', *American Economic Review*, Vol. 87, pages 342-62.

**Greenwood, J, Hercowitz, Z and Krusell, P (2000)**, 'The role of investment-specific technological change in the business cycle', *European Economic Review*, Vol. 44, pages 91-115.

**Hercowitz, Z (1998)**, 'The 'embodiment' controversy: a review essay', *Journal of Monetary Economics*, Vol. 41, pages 217-24.

**Hicks, J R (1939)**, *Value and capital: an inquiry into some fundamental principles of economic theory*, Oxford: Clarendon Press.

**Hicks, J R (1940)**, 'The valuation of the social income', *Economica*, Vol. VII, pages 105-24.

**Ho, M S and Stiroh, K J (2001)**, 'The embodiment controversy: you can't have two prices in a one-sector model', *mimeo*.

**Hulten, C R (1978)**, 'Growth accounting with intermediate inputs', *Review of Economic Studies*, Vol. 45, pages 511-18.

**Hulten, C R (1979)**, 'On the 'importance' of productivity change', *American Economic Review*, Vol. 69, pages 126-36.

**Hulten, C R (1992)**, 'Growth accounting when technical change is embodied in capital', *American Economic Review*, Vol. 82, pages 964-80.

**Jorgenson, D W (1966)**, 'The embodiment hypothesis', *Journal of Political Economy*, Vol. 74, pages 1-17. Reprinted in Jorgenson, D W, *Productivity: Volume 1: Postwar U.S. economic growth*, Cambridge, MA: The MIT Press.

**Jorgenson, D W, Gollop, F M and Fraumeni, B M (1987)**, *Productivity and U.S. economic growth*, Cambridge, MA: Harvard University Press.

**Jorgenson, D W and Griliches, Z (1967)**, 'The explanation of productivity change', *Review of Economic Studies*, Vol. 34, pages 249-83. Reprinted in Jorgenson, DW, *Productivity: Volume 1: Postwar U.S. economic growth*, Cambridge, MA: The MIT Press.

**Jorgenson, D W and Stiroh, K J (2000a)**, 'U.S. economic growth at the industry level', *American Economic Review*, Papers and Proceedings, Vol. 90, pages 161-68.

**Jorgenson, D W and Stiroh, K J (2000b)**, 'Raising the speed limit: U.S. economic growth in the information age', *Brookings Papers on Economic Activity*, 1, pages 125-211.

**Oliner, S D and Sichel, D E (2000)**, 'The resurgence of growth in the late 1990s: is information technology the story?', *Journal of Economic Perspectives*, Vol. 14 (Fall), pages 3-22.

**Oulton, N (2000)**, 'Must the growth rate decline? Baumol's unbalanced growth revisited', *Oxford Economic Papers*, Vol. 53, pages 605-27.

**Oulton, N (2002)**, 'Productivity versus welfare: or, GDP versus Weitzman's NDP', *Bank of England Working Paper no. 163*.

**Scott, M (1990)**, 'Extended accounts for national income and product: a comment', *Journal of Economic Literature*, Vol. XXVIII (September), pages 1,172-79.

**Sefton, J A and Weale, M R (1996)**, 'The net national product and exhaustible resources: the effects of foreign trade', *Journal of Public Economics*, Vol. 61, pages 21-47.

**Usher, D (1980)**, *The measurement of economic growth*, Oxford: Basil Blackwell.

**Weitzman, M L (1976)**, 'On the welfare significance of national product in a dynamic economy', *Quarterly Journal of Economics*, Vol. XC, pages 156-62.

**Weitzman, M L (1997)**, 'Sustainability and technical progress', *Scandinavian Journal of Economics*, Vol. 99, pages 1-14.

**Weitzman, M L (2003)**, *Income, wealth, and the maximum principle*, Harvard University Press, Cambridge, MA.

**Whelan, K (2001)**, 'A two-sector approach to modeling U.S. NIPA data', Board of Governors of the Federal Reserve, *Finance and Economics Discussion Series No. 2001-04*.