Asset pricing, asymmetric information and rating announcements: does benchmarking on ratings matter?

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Abstract

Using an intertemporal model of asset pricing under asymmetric information, we demonstrate how public ratings about the quality of a risky asset could enhance information efficiency, albeit at a cost of higher asset price volatility. The analysis also draws implications for the use of ratings for benchmarking purposes, in particular, ratings-based capital requirements and an investment/subinvestment grade dichotomy depending on the rating of the asset. In this situation, allowing a class of market participants (eg pension funds) to hold an asset only if its rating exceeds a certain threshold may lead informed traders to overreact to news about fundamentals. In this case, ratings induce lower price efficiency and excessive asset price volatility.

Key words: Asset pricing, ratings, benchmarking.

JEL classification: D82, D84, G12, G14.

Summary

This paper discusses an intertemporal model of asset pricing under asymmetric information, demonstrating how noisy public ratings about the quality of a risky asset could enhance information efficiency, albeit at a cost of higher asset price volatility. The analysis also draws implications for the use of ratings for benchmarking purposes, with most notable example the dichotomy between *investment* and *subinvestment* grade credits. In particular, we consider a stylised version of benchmarking investment decisions to ratings, whereby a residual class of (noise) traders link their net supply of a rated asset to some measure of the probability that the rating next period will fall below a given threshold. Thus, benchmarking to ratings can be rationalised as the result of forced sales by a class of regulated investors (eg pension funds) that are restricted to hold securities whose ratings are above a prespecified threshold, and unload their holdings to the market proportionally to the probability such downgrading will take place.

The main conclusion from the analysis is that, with benchmarking, price efficiency drops while volatility increases. That is because, perceived changes in fundamentals feed into prices not only through changes in perceptions about future income from holding the asset, but also through beliefs about capital gains that depend on the net supply of the asset. Given that benchmarking renders the net supply of traded assets partly forecastable, informed traders are inclined to trade more aggressively on any item of news that could imply a change in fundamentals in order to exploit perceived mispricings. Thus, informed traders become more prone to misinterpret any item of news as information about fundamentals leading to less informative and more volatile prices.

1 Introduction

Credit ratings are summary statistics that reflect a rating agency's opinion, as of a specific date, of the creditworthiness and financial robustness of a particular entity. Rating agencies' assessment is mainly based on fundamental analysis and have traditionally measured creditworthiness in the context of a capital, asset quality, management, earnings and liquidity analysis.

Following a series of high-profile credit events (eg the Enron bankruptcy in 2001), the Sarbanes-Oxley Act of 2002 has required the US Securities and Exchange Commission (SEC) to conduct a study on rating agencies and their role in securities markets.[(1)] In the course of that study, market representatives have suggested, among others, that ratings cause undue volatility in securities markets and have called for more transparency regarding the information relied upon by the rating agencies. A market participant,[(2)] for example, has claimed that

... one of the first things we wonder is what is it that they [the rating agencies] know, and I think that adds unnecessary volatility and uncertainty to the marketplace...

Also the scope and application of credit ratings nowadays stretches beyond the provision of information to market participants. Ratings, for example, have been used to facilitate monitoring the risk of investments by regulated entities and to set capital charges for banks and securities firms. Two notable examples that relate to the use of ratings for capital adequacy purposes are the rules under the New Capital Accord, that have been proposed by the Basel Committee on Banking Supervision and will apply to banks, and the US Net Capital Rule[(3)] that applies to broker-dealers. Both sets of rules provide for the deduction from capital of a certain percentage of the value of security holdings depending on the credit rating of those securities. Moreover, regulators often restrict certain classes of market participants from investing in securities below a rating threshold, with most notable the dichotomy between *investment* and *subinvestment* grade credits.[(4)] Rule 2a-7 of the US Investment Company Act, for example, restricts money market funds from investing in commercial paper below a rating threshold. Similar rules apply to insurance companies and pension funds.

(1) US Securities and Exchange Commission, '*Report on the Role and Function of Credit Rating Agencies in the Operation of the Securities Markets*', January 2003.
(2) Testimony of Cynthia L. Strauss, Director of Taxable Bond Research, Fidelity Investments Money Management Inc., 15 November 2002.
(3) See *Adoption of Alternative Net Capital Requirement for Certain Brokers and Dealers*, Release No. 40 FR 29795 (16 July 1975).
(4) Namely, ratings above or below BBB grade, in Standard & Poor's representation.

From a theoretical perspective, the role of information in asset pricing has been discussed both in a competitive market context and in the presence of strategic interactions among market participants. Kyle (1985) and its extensions,[5] for example, consider an oligopoly of imperfectly informed investors having identical information. They show that, with identical information, there is an intense *pre-emption* phase where informed investors compete very aggressively and, as a result, information is incorporated into prices very quickly. Foster and Viswanathan (1996) introduce heterogeneous information in a Kyle (1985) context showing that trading outcomes depend critically on the initial correlation of private information that traders possess. They show the lower the degree of initial correlation of traders' information – namely the more heterogeneous information becomes – the higher the degree of their monopoly power, with respect to their information advantage, which then gives rise to an *attrition trickle* and an incentive to trade less aggressively. Given that a public signal about fundamentals, such as a public rating, could increase the initial correlation (ie reduce heterogeneity) of traders' information, ratings could possibly be viewed as inducing strategic traders to trade more aggressively and prices to incorporate information more quickly.

This paper is in the line of literature initiated by Grossman and Stiglitz (1980) and Hellwig (1980). In the context of a discrete-time asset pricing model of infinite horizon, we consider a competitive asset market where market participants are asymmetrically informed and able to place their orders with a Walrasian auctioneer conditionally on prices.[6] However, in addition to private information that market participant may possess, we introduce a public signal (rating) in every trading round that is produced by a non-trading and non-strategic party (rating agency). Such a public signal is assumed to be produced on the basis of a stylised, time-invariant process (the rating process), which is consistent with investors' beliefs.[7] Thus, in this paper we are able to discuss possible asset pricing implications both from the use of ratings for their information content and, in addition, the impact that arises from benchmarking investment decisions and capital requirements on ratings.

Our solution approach involves the calculation of a rational expectations equilibrium (REE) of a

(5) For example, Michener and Tighe (1991), Holden and Subrahmanyam (1992) and Foster and Viswanathan (1993).

(6) See also Hellwig (1980), Diamond and Verrecchia (1981) and Allen, Morris and Shin (2003).

(7) In this paper, we adopt a reduced-form approach to ratings process, by abstracting from the information economics of ratings (eg Diamond (1985) and Veldkamp (2003)), from the financial intermediation underpinnings of ratings agencies (eg Millon and Thakor (1985)) and the possibility of strategic information revelation (*cheap talk*) by rating agencies.

securities market with ratings, assuming that the true state variables of the market are never perfectly revealed neither to investors, nor to the rating agency, but they are observed with some error. Thus, agents need to filter information from the variables that they observe. In particular, the rating agency is assumed to apply a Kalman filter approach to update its ratings on the basis of private information that it observes, while investors are assumed to fit linear econometric models on observable variables. In addition, our modelling approach allows for higher order beliefs to have a material impact on asset prices.(8) That is in line with Bacchetta and van Wincoop (2003) and Allen, Morris and Shin (2003) who argue that under beliefs of higher order asset prices may become biased towards the public information, regardless how sound that information might be. Thus it would be of interest to examine whether a similar result also follows in our set-up and whether a world without ratings would be preferable to a world with, but imprecise, ratings.

However, in discrete-time models with asymmetric information, agents' rationality requires one to address the inferences that agents make from observable variables, knowing that others act in a similar fashion. Thus, higher order beliefs become hidden state variables and the dimension of the state vector, associated with agents' signal extraction problems, becomes unbounded. In order to deal with the problem of infinite regress in expectations, we apply the techniques of Sargent (1991), as applied by Hussman (1992). More specifically, we extend Hussman (1992) by allowing rating announcements to augment investors' information sets and introducing ratings-based frictions, such as ratings-based capital requirements and benchmarking of investment decisions to ratings.

In equilibrium, investors' subjective beliefs have to be consistent with the actual law of motion that those beliefs generate. Thus, equilibrium in our model is calculated as a *fixed point* in the mapping from investors perceived laws of motion to the actual law of motion that investors' perceptions generate. This is by taking as given the econometric techniques that investors apply and assuming that those techniques belong to the same class of linear models. In particular, we focus on the situation where investors fit first-order vector autoregressive moving average (ARMA) models. As of Sargent (1991) and Hussman (1992), the equilibrium in first-order ARMA models is consistent with higher order beliefs and is such that investors have no incentive to increase the order of either

(8) Higher order beliefs is a basic feature of asset pricing under asymmetric information and it refers to the situation where opinions of other investors' opinions, and higher order than that, may have a material impact on asset prices. That is in line with Keynes' (1936) famous metaphor that the market is similar to a beauty contest, where an agent's subjective payoff from choosing the prettiest face from a list of contestants depends on how close her prediction were to the average opinion of other agents.

the AR or the MA component of their forecasting rules in order to improve their forecasts.

However, for purposes of comparison with the ARMA case, we also describe equilibria (with and without ratings) when investors' forecasting rules are restricted to be first-order vector autoregressive (AR) processes. As we know from Townsend (1983), those first-order autoregressions are always *too short* to give optimal forecasts because of the infinite regress problem. That is, in equilibrium, the prediction errors from first-order vector autoregressions will never be orthogonal to information that lagged two periods or more.[(9)] We consider the case where investors use vector AR forecasting rules as a proxy for *low market sophistication*, in contrast to *high market sophistication* when investors run ARMA models.

The analysis shows that, when ratings are used for price discovery alone they may increase price volatility, but this is consistent in the model with an increase in price efficiency (ie prices become more correlated with fundamentals). Also, the type of forecasting techniques that market participants use to form their beliefs matters for trading outcomes. Moreover, for reasonable levels of rating-based capital requirements, the volatility of prices drops, although at a cost of lower price efficiency.

Yet benchmarking of asset holdings on ratings may cause both a reduction in price efficiency and an increase in volatility. This is despite an optimistic presumption in the model that agents have common knowledge of how the economy works, there are no structural breaks in the economy and investors trust the rating agency in its objective to produce timely, accurate and objective information. In fact, regulatory and other constraints that force a residual class of market players to link their investment decisions to ratings, may generate a sequence of perceived mispricings in the market and drive other investors to overreact to news about fundamentals. That way, benchmarking magnifies the effect of news on prices in such a way that prices may respond to changes in fundamentals even in excess of the full-information case.[(10)]

The remainder of the paper is organised as follows: Section 2 describes the model and the solution method. Section 3 presents the results under no rating-based frictions and discusses persistence implications and comparative statics. Section 4 introduces rating-based frictions, such as

(9) This is, the prediction errors from first-order vector autoregressions will never be orthogonal to the Hilbert space that is generated by all past history of investors' information.

(10) This is, the hypothetical situation where investors observe perfectly any innovation in fundamentals and they do not need to solve filtering problems.

rating-based capital requirements and benchmarking of asset holdings on ratings, and discusses equilibrium implications. Section 5 concludes. Technical details and charts are included in the appendix.

2 The model

We consider a competitive market for a risky asset that pays a risky pay-off D_t that varies over time t. Pay-off D_t consists of two independent factors θ_{1t} and θ_{2t} – hereinafter called *fundamental factors* – that have some persistence over time, as well as of a transitory component u_t

$$D_t = \theta_{1t} + \theta_{2t} + u_t \tag{1}$$

We assume that factors where θ_{jt} $(j = 1, 2)$ evolve according to the following first-order autoregressive processes

$$\theta_{jt} = \rho\theta_{jt-1} + \upsilon_{jt} \quad j = 1, 2 \tag{2}$$

with $\{u_t\}$, $\left\{\upsilon_{jt}\right\}$ be *i.i.d.* white noise innovations with mean zero and variances σ_u^2 and σ_υ^2. For simplicity and without loss of generality of our analysis, we assume that the persistence ρ is the same for the two fundamental factors, but equally one could consider different degrees of persistence, where one of the factors could be thought of as long term and the other as short term.[(11)] In addition, we assume that the fundamental factors are stationary, ie they do not grow explosively for ever, by assuming that $|\rho| < 1$.

The market is populated by N privately informed investors that belong to classes indexed by $j = 1, 2$ depending on the type of private information that they observe. Proportion α belong to class 1 and observe private signals about factor θ_1, while proportion $1 - \alpha$ to class 2, observing signals about θ_2. We consider an overlapping generation of those investors who live for two periods and their preferences over future wealth demonstrate constant absolute risk aversion (CARA) with coefficient $\frac{1}{\phi_j}$. Informed investors are able to trade conditionally on prices – ie place limit orders – in the first period and invest their wealth in the risky asset or, alternatively, in a safe asset yielding a return R.

There is also a residual set of traders, called noise traders, who trade both for non-fundamental (liquidity) purposes and for benchmarking reasons, whereby they link their supply of the risky

(11) However, that would increase the computational intensity of our calculations when we would have to derive an equilibrium of our asset market.

asset to some public information.[12] Non-fundamental trade implies a random supply $\{\varsigma_t\}$ of the asset, which is *i.i.d.* normal with mean zero and variance σ^2_ς, while noise trading for benchmarking reasons is introduced in Section 4.2.

Informed investors of class $j = 1, 2$ are assumed to observe private signals s^j_t about the actual realisation of fundamental factor θ_{jt}, which are subject to an element of idiosyncratic noise η_{jt}

$$s^j_t = \theta_{jt} + \eta_{jt} \qquad j = 1, 2 \tag{3}$$

where $\{\eta_{jt}\}$ are $i.i.d.$ white noise innovations, orthogonal to $\{e_{jt}\}$ and $\{\upsilon_{jt}\}$, with zero mean and variance σ^2_η. Thus we consider informed investors as having special price discovery skills (eg *macro* versus *sector* funds), while such an information structure as given exogenously without modelling explicitly the actual decision of investors to acquire information, as in Grossman and Stiglitz (1980) for example. Instead, we focus exclusively on informed investors' problem of filtering information about fundamentals from observable variables, including their private signals $\left\{s^j_t\right\}$.

2.1 Ratings

In addition to private signals that informed investors observe and to publicly observed prices and asset pay-offs, we assume that in every trading period a non-trading, independent and non-strategic party (henceforth called the *rating agency*) produces a public signal r_t (henceforth called the *rating*) about the factors that affect the pay-offs of the risky asset.[13] Consistently with real-world features of ratings, we assume that ratings are public signals in the form of *summary statistics*, ie they summarise all the information that the rating agency has received over time about the fundamental factors that affect asset pay-offs.[14] In addition, as we discuss below, ratings in this model are updated on the basis of a recursive process, which is in analogy to the rating process outlined in *rating policy guidelines* of rating agencies. Finally, we assume that the rating agency uses only its private signals in order to produce its ratings, ignoring any element of public information such as prices,[15] and does not publicly announce the individual elements of its private information.

(12) That is discussed in more detail in Section 4.2.
(13) In this paper, we abstract from the information economics that underpin the existence and functioning of rating agencies, as well as from possible principal-agent problems in the disclosure of information to the ratings agency.
(14) In the real world, this may allow the agency to obfuscate the reason behind the rating change when this is based on confidential information.
(15) This is consistent with the Standard & Poor's approach to ratings, as outlined in their *rating policies guidelines*.

Thus, the rating r_t is assumed to be an unbiased estimator of the sum of the two fundamental factors conditional on all private signals that the rating agency has observed up to that period. In particular, the rating agency is assumed to be receive noisy private signals s^r_{1t} and s^r_{2t} of the form

$$\begin{aligned} s^r_{1t} &= \theta_{1t} + e_{1t} \\ s^r_{2t} &= \theta_{2t} + e_{2t} \end{aligned} \tag{4}$$

where $\{e_{jt}\}$ are *i.i.d.*white noise innovations, orthogonal to $\{u_t\}$, $\{\upsilon_{jt}\}$ and $\{\eta_{jt}\}$, with mean zero and variance σ_e^2. By assuming that the rating agency possesses information about both fundamental factors, while individual investors are separated in two groups with each one receiving a different signal, we aim to address possible information advantages of rating agencies relative to individual market participants. That is supported by the adoption of Regulation Fair Disclosure (Regulation FD) by the US Securities and Exchange Commission in October 2000, which prohibits selective disclosure of non-public information by firms, but provides an exception for rating agencies. Having said that, the rating process in this model is given by

$$r_t = E\left[\theta_{1t} + \theta_{2t} \mid s^r_{1s}, s^r_{2s}, s < t\right] \tag{5}$$

Given that the rating r_t in **(5)** depends on all past history of signals s^r_j, we may express it in a recursive form as a function of the previous rating r_{t-1} and the signals observed in period t. That can be achieved by using the following Kalman filter representation.

Lemma 1 The rating process $\{r_t\}$, as defined by **(5)**, exhibits positive autocorrelation and is generated by the following on-line algorithm.

$$r_t = \rho\left[1 - \Sigma\left(\Sigma + 1\right)^{-1}\right] r_{t-1} + \rho\Sigma\left(\Sigma + 1\right)^{-1}\left[s^r_{1t-1} + s^r_{2t-1}\right] \tag{6}$$

where, parameter Σ is given by

$$\Sigma = \frac{1}{2}\left[\frac{\sigma_\upsilon^2}{\sigma_e^2} - \left(1-\rho^2\right) + \sqrt{\left[\frac{\sigma_\upsilon^2}{\sigma_e^2} - \left(1-\rho^2\right)\right]^2 + 4\frac{\sigma_\upsilon^2}{\sigma_e^2}}\right] \tag{7}$$

Proof. See appendix.

From **(6)** and **(7)**, the degree of serial correlation in the rating process depends on the relative precision of the rating agency's signal errors, relative to that of fundamental innovations, rather than on the actual levels. Although this is a standard Kalman filter result, in the context of our ratings representation it suggests that the better the access of a rating agency to information the more confident the agency will be to rate more aggressively and to give a rating that may contradict a previous one. Also, the rating process in **(6)** was evaluated on the basis of a steady-state assumption,(16) assuming that the market runs for a long time. That assumption may not fit well in a situation of a regime change (eg industry liberalisation), or an economy in transition, but in those cases, rational expectations and common knowledge of model parameters would not fit well either.

2.2 Timing of events and information

Within every trading round t we consider the following sequence of events:

1. Rating r_t is publicly announced, based on information up to $t-1$.
2. Fundamentals are updated, investors observe private signals about fundamentals, as well as public information, including prices p_t, ratings r_t and pay-offs D_t. This information affects investors' optimal demands and the market clearing price p_t in equilibrium.
3. The rating agency receives information about the current level of fundamentals and, once again, makes a rating in period $t+1$.

The above sequence of events aims to capture the conventional wisdom that *ratings lag the market*, in the sense that rating agencies respond later than the market to changes in fundamentals. That is, new information about fundamentals is reflected first into prices and then into ratings because market participants can trade on new information instantly, while the rating agency follows by a natural time lag because it does not trade, but rather sets ratings at discrete-time intervals.

2.3 Definition of rational expectations equilibrium

Informed investors of class $j=1,2$ are characterised by the information set $I_{jt}=\left\{p_s,\ D_s,\ r_s,\ s_s^j;\ s\leq t\right\}$, which is a record of data z_{jt} of the form

$$z'_{jt}=\left[\begin{array}{cccc} p_t, & D_t, & r_t, & s_t^j \end{array}\right] \qquad \textbf{(8)}$$

Let also ζ_j $(j=1,2)$ be conditional forecast errors, conditional on investors' information sets and on the type of forecasting techniques that investors use to form their beliefs. Let also ς_t be the net supply of the risky asset in period t, where $\left\{\varsigma_t\right\}$ are assumed to be $i.i.d.$ white noise innovations. Then, the state vector z_t that describes the market for the risky asset in period t is

$$z'_t=\left[\begin{array}{cccccccccc} p_t & D_t & r_t & s_t^1 & s_t^2 & \theta_{1t} & \theta_{2t} & \varsigma_t & \zeta_{1t} & \zeta_{2t} \end{array}\right] \qquad \textbf{(9)}$$

(16) That is, we use the unconditional variance of the rating forecast error. More details are discussed in the appendix.

State vector z_t includes all variables that are directly and collectively observed by investors, as well as the two latent factors θ_1 and θ_2, the random supply ς of the risky asset and investors' forecast errors ζ_j , $j = 1, 2$. Also, the noise of the model at t is specified by a vector ε_t which includes all the white noise innovations

$$\varepsilon'_t = \begin{bmatrix} u_t & \eta_{1t} & \eta_{2t} & \upsilon_{1t} & \upsilon_{2t} & e_{1t-1} & e_{2t-1} & \varsigma_t \end{bmatrix} \qquad \textbf{(10)}$$

where, the white noise innovations $\{u\}$, $\{\upsilon_j\}$, $\{\eta_j\}$ and $\{e_j\}$ are defined by **(1)**, **(2)**, **(3)**, **(4)** and $\{\varsigma\}$ are shocks to the aggregate supply of the risky asset.

The fundamental requirement that a REE must satisfy is that equilibrium prices have to be consistent with the presumption that investors know the actual law of motion of the securities market and choose their demands schedules accordingly. Within a given class of linear forecasting rules (eg ARMA), a competitive REE for our securities market is defined as follows:

Definition 1

1. *Investors make conjectures about the law of motion of the variables they observe.*[(17)] *Given their information sets, investors use statistically optimal predictors to derive the perceived law of motion for their observable variables.*
2. *Investors select their demand schedules* q_t^j *so as to maximise their expected utilities. In order to calculate expected utilities, investors use their perceived laws of motion of the variables they observe.*[(18)]
3. *Given investors' demand schedules, the price* p_t *of the risky asset clears the market.*
4. *Investors' perceived laws of motion are correct. That is, there is a fixed point in the correspondence that maps investors' perceived laws of motion to the actual law of motion that those perceptions generate.*

In general, the properties of a REE of our securities market will depend on the type of linear forecasting models that investors are assumed to run. Following Sargent (1991), if an equilibrium is such that investors find it optimal to form their beliefs by fitting more complicated (linear) models on their observable variables, that equilibrium would be defined as a *reduced-order* equilibrium. In contrast, a *full-order equilibrium* would be one where investors have no incentive

(17) As we discuss below, conjecturing a law of motion about observable variables is equivalent to assume that investors conjecture an actual law of motion for the state vector z_t.
(18) In other words, conditional on their perceived laws of motion, investors form subjective beliefs about the variables they observe and the variance of their forecast errors.

to increase the order of either the AR or the MA part of their forecasting rules. Given the structure of information that is stipulated in this model, Sargent (1991) and Hussman (1992) have shown that an equilibrium that is calculated on the assumption of investors fitting ARMA(1,1) models on observable variables is of full-order and we focus on that type of equilibrium. Thus, conditioning investors' forecasts on an infinite history of data is equivalent to conditioning those forecasts only on first-order lags and the information sets I_{jt} $(j = 1, 2)$ can be restated as:
$I_{jt} = \left\{p_t,\ D_t,\ r_t,\ s_t^j\right\}$.

2.4 Beliefs

Following Sargent (1991) and Hussman (1992), informed investors' perceptions about the law of motion of their observable variables are assumed to be of the general ARMA(1,1) form

$$z_{jt+1} = \mathbf{A}_j z_{jt} + \zeta_{jt+1} + \mathbf{C}_j \zeta_{jt} \qquad j = 1, 2 \tag{11}$$

where $z'_{jt} \equiv \begin{bmatrix} p_t, & D_t, & r_t, & s_t^j \end{bmatrix}$, ζ_{jt+1} is the vector of conditional forecast errors and $\mathbf{A}_j$, $\mathbf{C}_j$ are matrices of ARMA coefficients that can be recasted such that **(11)** becomes

$$x_{jt+1} = \mathbf{B}_j x_{jt} + \mathbf{v}_{jt+1} \qquad j = 1, 2 \tag{12}$$

with $x_{jt} \equiv \begin{bmatrix} z_{jt} \\ \zeta_{jt} \end{bmatrix}$ be the vector of variables that privately informed investors observe in every period, including their realised forecast errors ζ_{jt}, $\mathbf{v}_{jt+1} = \begin{bmatrix} \zeta_{jt+1} \\ \zeta_{jt+1} \end{bmatrix}$, $\mathbf{B}_j \equiv \begin{bmatrix} \mathbf{A}_j & \mathbf{C}_j \\ \mathbf{0}_4 & \mathbf{0}_4 \end{bmatrix}$ and $\mathbf{0}_4$ be 4×4 matrices of zeros. Given **(12)**, informed investors can forecast x_{jt+1} on the basis of observable x_{jt}

$$E\left[x_{jt+1} \mid x_{jt}\right] = \mathbf{B}_j x_{jt} \tag{13}$$

Beliefs **(13)** affect investors' optimal demands for the risky asset and, as a result, prices in equilibrium. In Sections 2.5 and 2.6 we discuss the solution to investors' optimal portfolio choice problem and we solve for equilibrium prices.

2.5 Investor optimisation

We assume that investors of class $j = 1, 2$ demonstrate CARA preferences over future wealth w^j with coefficient of constant absolute risk aversion $\frac{1}{\phi_j}$. We also assume that ratings may influence investment decisions not only through the information they convey to market participants, but also through ratings-based capital requirements. Such capital requirements are assumed to imply an opportunity cost of funds that investors need to set aside as capital, which is proportional to the risky-asset holdings of each individual investor.

Thus, in this model, we examine the possibility that ratings-based capital requirements may have an impact on investment decisions by focusing on the opportunity cost of funds that such requirements would imply for market participants. In particular, we adopt a reduced-form approach to ratings-based capital charges whereby investors face an opportunity cost (gain) due to capital requirements at a given period, which is proportional to the extent of deterioration (improvement) in the rating quality of the risky asset over that period. For example, if the rating of the risky asset decreases, then an investor with positive asset holdings would face an opportunity cost of funds due to capital charges, proportional to the quantity of his risky-asset holdings.(19) In particular, we assume that investors of class $j = 1, 2$ choose their optimal demands q_t^j for the risky asset in order to maximise their expected utility over next period's wealth w^j

$$q_t^j = Arg \max_{q_t^*} E\left[-\exp\left(-w_{t+1}^j/\phi_j\right) \mid I_{jt}\right] \qquad j = 1, 2 \qquad \textbf{(14a)}$$

subject to

$$w_{t+1}^j = R\left(w_t^j - q_t^* p_t\right) + q_t^*\left(p_{t+1} + D_{t+1}\right) + kq_t^*\left(r_{t+1} - r_t\right) \qquad \textbf{(14b)}$$

where R is the constant gross interest rate on an alternative risk-free investment and, as said before, parameter k is aimed to capture the opportunity cost of funds that investors have to set aside as capital.(20) The above maximisation problem gives the following optimal demands

$$q_t^j = \phi_j \frac{E\left[p_{t+1} + D_{t+1} + kr_{t+1} \mid I_{jt}\right] - Rp_t - kr_t}{Var\left[\zeta_{t+1}^p + \zeta_{t+1}^D + k\zeta_{t+1}^r \mid I_{jt}\right]} \qquad j = 1, 2 \qquad \textbf{(15)}$$

2.6 Market clearing

We assume that investors' optimal demands are aggregated by a central auctioneer who finds, if possible, a market-clearing price.(21) At a rational expectations equilibrium the price p_t must clear

(19) In order to preserve the linearity of our model, we are going to assume that a rating increase would imply the release of some capital and, as a result, the investor would face a negative opportunity cost (ie a gain).
(20) Given that the alternative investment that we consider is the risk-free asset, the opportunity cost k must be inversely related to the level of risk-free interest rates R. Moreover, the opportunity cost parameter k has to take into account the slope in the risk-weights scale that is specified by regulators. For example, the Standardised Approach, under the proposed New Basel Capital Accord, stipulates the following (discrete) scale of risk weights: 0% for assets that are rated between AAA and AA-, 20% for A+ to A-, 50% for BBB+ to BBB-, 100% for BB+ to B- and 150% for assets with a rating below B-.
(21) That formulation differs from Kyle (1985) and its extensions, where prices are set by a market-maker on the basis of a *semistrong market efficiency* rule.

the market

$$\alpha N q_t^1 + (1-\alpha) N q_t^2 = \varsigma_t \tag{16}$$

where q_t^1 and q_t^2 are agents' optimal demands for the risky asset, as given by **(15)**, and $\{\varsigma_t\}$ are $i.i.d.$ white noises with mean zero, variance σ_ς^2 and mutually orthogonal in all lags to any other noise term in the model. From **(15)** and **(16)** the price process p_t becomes

$$p_t = \Lambda^{-1}\left[\alpha\sigma_2^2 N\phi_1 E_1[\cdot] + (1-\alpha)\sigma_1^2 N\phi_2 E_2[\cdot] - Mr_t - \sigma_1^2\sigma_2^2\varsigma_t\right] \tag{17}$$

where $E_j[\cdot] \equiv E\left[p_{t+1} + D_{t+1} + kr_{t+1} \mid I_{jt}\right]$, $\sigma_j^2 \equiv Var\left[\zeta_{t+1}^p + \zeta_{t+1}^D + k\zeta_{t+1}^r \mid I_{jt}\right]$, for $j = 1, 2$, and parameters Λ, M are given by

$$\Lambda \equiv RN\left[\sigma_2^2\alpha\phi_1 + \sigma_1^2(1-\alpha)\phi_2\right]$$
$$M \equiv kN\left[\sigma_2^2\alpha\phi_1 + \sigma_1^2(1-\alpha)\phi_2\right]$$

Both, subjective beliefs $E_j[\cdot]$ and subjective measures of riskiness σ_j^2 are determined in equilibrium on the basis of investors' perceived laws of motion, as discussed in Section 2.4.

2.7 Solving for a REE

We assume that investors conjecture that the state vector z_t evolves according to the following law of motion

$$z_t = T(\mathbf{B})\, z_{t-1} + V(\mathbf{B})\, \varepsilon_t \tag{18}$$

where $\mathbf{B} \equiv [\mathbf{B}_1\ \mathbf{B}_2]$ and $T(\mathbf{B})$, $V(\mathbf{B})$ are matrices of actual coefficients. If all eigenvalues of $T(\mathbf{B})$ lie inside the unit circle,(22) then equation **(18)** determines a covariance-stationary distribution for the state vector z_t, whose moment matrix $\mathbf{M}_z$ solves

$$\mathbf{M}_z = T(\mathbf{B})\,\mathbf{M}_z T(\mathbf{B})' + V(\mathbf{B})\,\Omega V(\mathbf{B})' \tag{19}$$

where $\mathbf{\Omega}$ is the moment matrix of the vector ε_t of white noise innovations and $\mathbf{B} \equiv [\mathbf{B}_1\ \mathbf{B}_2]$. Given that matrix $V(\mathbf{B})\,\Omega V(\mathbf{B})'$ is symmetric, equation **(19)** defines a discrete-time Lyapunov equation. Then, with all eigenvalues of $T(\mathbf{B})$ less than unity in modulus, there is a unique(23) symmetric matrix M_z that solves equation **(19)**. With $\mathbf{M}_z$ in hand we can derive the variance covariance matrices $\mathbf{M}_{x_j}$ of investors' observable variables x_{jt} and the covariance matrix $\mathbf{M}_{zx_j}$ of the state vector z_t with the vector of observable variables x_{jt}, $j = 1, 2$. Using an appropriate selector

(22) It can be easily verified that in our model all eigenvalues of matrix $T(\mathbf{B})$ lie inside the unit circle. This is because of the assumption that the autoregressive parameters are such that $|\rho_j| < 1$ $(j = 1, 2)$.

(23) From standard theory, there is a unique symmetric matrix $\mathbf{M}_z(\mathbf{B})$ that solves **(19)** *i.f.f.* no eigenvalue of $T(\mathbf{B})$ is the reciprocal of any other eigenvalue of $T(\mathbf{B})$. This is, *i.f.f.* $\mathbf{eig}[T(\mathbf{B})]\,\mathbf{eig}[T(\mathbf{B})]' - \mathbf{1} \neq \mathbf{0}$. Given that all eigenvalues of $T(\mathbf{B})$ lie inside the unit circle, none of them can be the reciprocal of another eigenvalue of $T(\mathbf{B})$.

matrix $\mathbf{u}_j$, matrices $\mathbf{M}_{x_j}$ and $\mathbf{M}_{zx_j}$ are given by

$$\begin{aligned} \mathbf{M}_{x_j} &= \mathbf{u}_j \mathbf{M}_z \mathbf{u}'_j \\ \mathbf{M}_{zx_j} &= \mathbf{M}_z \mathbf{u}'_j \end{aligned} \quad \textbf{(20)}$$

Let us now consider the linear projection of vector x_{jt+1}, of investor's j observable variables, on its previous realisation x_{jt}

$$E\left[x_{jt+1} \mid x_{jt}\right] = \mathbf{S}_j\left(\mathbf{B}\right) x_{jt} \qquad j = 1, 2 \quad \textbf{(21)}$$

Using matrices $\mathbf{M}_{x_j}$ and $\mathbf{M}_{zx_j}$, we are able to evaluate the matrix $\mathbf{S}_j\left(\mathbf{B}\right)$ of statistically optimal estimators as follows

$$\mathbf{S}_j\left(\mathbf{B}\right) = \mathbf{u}_j T\left(\mathbf{B}\right) \mathbf{M}_{zx_j} \mathbf{M}_{x_j}^{-1} \quad \textbf{(22)}$$

where $\mathbf{M}_{x_j}$ and $\mathbf{M}_{zx_j}$ are given by **(20)** and $\mathbf{u}_j$ is a matrix that selects the subvector of observable variables x_{jt} from the state-space vector z_t.

Let $\mathbf{S}\left(\mathbf{B}\right) \equiv \left[\mathbf{S}_1\left(\mathbf{B}\right)\ \mathbf{S}_2\left(\mathbf{B}\right)\right]$, then, a rational expectations equilibrium is a fixed point in the correspondence that maps investors' perceptions – as defined by the VAR coefficients **B** in **(13)** – into statistically optimal projections $\mathbf{S}\left(\mathbf{B}\right)$, given the actual law of motion **(18)** that investors' perceptions generate. It is worth emphasising that, in this model, conjectures about the coefficient matrix **B** are equivalent to conjectures about the actual law of motion **(18)** of the state vector z_t. Such an equivalence stems from the fact that, for a given coefficient matrix **B**, equation **(19)** defines a *unique* moment matrix $\mathbf{M}_z$ for the state vector z_t, which in turn, defines matrices $T\left(\mathbf{B}\right)$ and $V\left(\mathbf{B}\right)$ of the actual coefficients. In other words, there is a one-to-one relationship between conjectures about coefficient matrix **B** and matrices $T\left(\mathbf{B}\right)$, $V\left(\mathbf{B}\right)$. That becomes evident in Section 6.2 where we outline the fixed-point solution algorithm and how matrices $T\left(\mathbf{B}\right)$ and $V\left(\mathbf{B}\right)$ are evaluated.

3 Ratings and price discovery

In this section we examine the effect of ratings on price volatility and efficiency under the presumption that they are used solely for price discovery and not for any other purpose, such as to benchmark investment decisions or to set capital requirements. The equilibrium coefficients of investors' forecasting models were calculated using Matlab programs under the following basic parameterisation

Risk tolerance	ϕ_1=ϕ_2=1
Investor proportions	a=0.5
Gross interest rate	R=1.02
Constants	N=1, σ_u^2=1
Persistence of fundamentals	ρ=0.8
Variance of fundamental innovations	σ_υ^2=0.1
Variance of errors in investors' private signals	σ_η^2=1
Variance of errors in rating agency's signal	σ_e^2=0.1
Variance of noise in the supply of the risky asset	σ_ς^2=0.01

The above parameterisation was chosen mainly to illustrate the potential impact of rating announcements on asset prices, but has not been calibrated to match any actual data. Moreover, it allows us to search for a symmetric equilibrium, whereby the coefficients in the forecasting models of each class of investors are equal. In Section 3.3, we present a comparative statics analysis where we examine the sensitivity of our results to different levels of risk aversion and precision of rating information.

In order to gauge the impact of ratings on asset prices, we consider two benchmark cases, namely, the case with asymmetric information, but without ratings, and the case of full information.(24) Given the linearity of the model and the assumption that all innovations in the model are normally distributed, correlations are considered in terms of the coefficient of linear correlation. Market efficiency is then considered with respect to the informativeness of prices and the extent to which prices correlate with fundamentals.

(24) Under the full information benchmark, investors are assumed to observe perfectly the realisation of both fundamental factors, but they still remain uncertain about future realisations of these factors. As in Hussman (1992), one can show that for fundamental shocks υ_{jt}, $j = 1, 2$, the price process (p_t) under full information is given by:

$$p_t = \frac{\rho}{(R-\rho)(1-\rho L)}(\upsilon_{1t} + \upsilon_{2t}) - Constant$$

which implies the following expression for the unconditional variance of prices:

$$Var(p_t) = 2\left(\frac{\rho}{R-\rho}\right)^2 \left(\frac{1}{1-\rho^2}\right)\sigma_\upsilon^2$$

and the covariance of prices with fundamentals θ_{jt}, $j = 1, 2$:

$$Cov\left(p_t, \theta_{jt}\right) = \frac{\rho}{(R-\rho)\left(1-\rho^2\right)}\sigma_\upsilon^2$$

Using the above expressions and the fact that the unconditional variance of fundamentals is $Var\left(\theta_{jt}\right) = \frac{\sigma_\upsilon^2}{1-\rho^2}$, we can derive the coefficient of linear correlation of prices with fundamentals under full information.

Section 6.3 in the appendix presents the equilibrium coefficients of investors' forecasting techniques and second moments of prices when investors are assumed to run vector ARMA(1,1) models. The ARMA(1,1) case is called *the high-sophistication case*, in the sense that investors cannot improve further their predictions by incorporating more lags in their forecasting models.[25] Also in the appendix we present the equilibrium when investors' forecasting techniques are restricted to a first-order vector AR(1) process. This allows for an examination of the extent to which our results might be sensitive to the assumption of the type of forecasting techniques that investors are using at the REE. The case where investors run simple AR(1) models is called *the low-sophistication case*, in a sense that investors could further improve their forecasts by adding more lags in their time-series models.

Based on the results that we derive under both the high and low-sophistication case, we discuss how the use of ratings for price discovery may impact on market efficiency and price volatility.

3.1 Results

Tables I and II below, compare the equilibrium results when there is incomplete information under both the highly sophisticated (ARMA) and less sophisticated (AR) forecasting rules, both with and without ratings. Table I reports the equilibrium variance of asset prices in the different cases, while Table II shows the impact on price efficiency (ie how much prices correlate with fundamentals). The benchmark case of full information is also shown in the following tables.

Table I: Price volatility

Full-information benchmark	
7.3462	
Incomplete information without ratings	
High sophistication	Low sophistication
4.3608	0.3988
Incomplete information with ratings	
High sophistication	Low sophistication
5.0634	4.5017

Table II: Price efficiency

Full-information benchmark	
0.7071	
Incomplete information without ratings	
High sophistication	Low sophistication
0.4914	0.3927
Incomplete information with ratings	
High sophistication	Low sophistication
0.5429	0.5400

We observe that, in the incomplete information equilibrium, and regardless of the forecasting

(25) See, for example, Sargent (1991) and Hussman (1992).

techniques used, the introduction of ratings increases the volatility of prices but they also enhance price informativeness. In particular, Table II shows that, under both the ARMA and the AR case, the introduction of ratings increases the correlation of prices with fundamentals θ_j $(j = 1, 2)$ albeit at a cost of higher price volatility. The increase in price volatility is much stronger under the low-sophistication case where the introduction of ratings results in an increase in volatility from, approximately, 0.4 to 4.5. However, under the high-sophistication case, the increase is less striking from approximately 4.4 to 5.1.

In the following section we examine how non-fundamental shocks (ς_t) impact on prices under both the ratings and no-ratings case. We also consider the impact of a one-off shock in fundamentals ν_t – ie the impact of a single shock in fundamentals that is isolated from the impact of any other shock in the model – as well as the impulse response of prices to pay-off innovations u_t and private signal errors η_t.

3.2 Persistence

Under the full-information benchmark, non-fundamental shocks have no persistence on prices because non-fundamental shocks themselves have no persistence. However, when the full information assumption is relaxed, non-fundamental shocks may have a persistent effect on prices. That is because fundamentals are latent variables and investors rely on past values of observable variables to filter information about fundamentals and form their beliefs. Given that prices are affected, through market clearing, by one-off non-fundamental shocks, those shocks may continue to affect prices in future periods through investors' filtering problems. In other words, persistence of non-fundamental shocks on prices is driven by, what Bacchetta and van Wincoop call, persistence of investors' *rational confusion* that eventually dissipates as investors gradually learn about the realisation of fundamentals in previous periods.

Similarly, rational confusion may inhibit investors from responding effectively to fundamental shocks ν_t and it may also drive them to misinterpret non-fundamental noise u_t in asset pay-offs as being fundamental information. The extent to which ratings ameliorate investors' rational confusion and facilitate the incorporation of fundamental information into prices will determine to what extent rating agencies provide a useful service to the market. Finally, errors in investors' private signals may have a different impact on prices under the ratings and the no-ratings case.

Private signal errors are expected to affect prices through channels of both subjective beliefs and subjective measures of riskiness. As with all other types of shocks that we consider, the impulse response of prices to private signal errors will be determined by the sign and relative importance of elements in the VAR matrix T (**B**), as defined by equation **(18)**.

The impulse response of prices to various shocks in the model will be determined by the sign and relative importance of elements in the VAR matrix $T\ (B)$, as defined by equation **(18)**. From **(18)**, the impulse response of prices to a one standard deviation shock in the i^{th} element of innovations vector ε_t, as defined in **(10)**, is given by the following function:

$$f\ (t) = \left[T\ (\mathbf{B})^{t-1}\ V\ (\mathbf{B})\right]^{(1,i)} \sigma_i \qquad \textbf{(23)}$$

where T (**B**) and V (**B**) are defined by **(18)**, σ_i is the standard deviation of the i^{th} element of vector ε_t, superscript $(1, i)$ refers to the i^{th} element in the first row of the matrix in brackets and $t = 1, 2, ...\infty$.

Chart 1 illustrates the impulse response of prices to a one standard deviation shock in non-fundamental trade. Under the no-ratings case, the rational confusion that follows the shock induces a price overreaction almost three times larger than the case with ratings. It then takes around 13 trading rounds for most of the rational confusion to unwind, compared to eight trading periods under the ratings case. Similarly, Chart 2 shows how prices respond to an idiosyncratic shock u in asset pay-offs. We observe that ratings mitigate any undue price impact of a one-off shock in asset pay-offs that is not related to fundamentals.

Chart 3 shows the price response to a shock of one standard deviation in fundamentals. Although, initially, the price responds to the shock in the same fashion under both the ratings and the no-ratings case, over the next couple of trading rounds prices tend to move closer towards the full-information benchmark under the ratings case, compared with the no-ratings case. This confirms our earlier finding – by using the equilibrium variance/covariance matrix of our state variables – that ratings improve the informativeness of prices. Finally, in Chart 4, we report the impact on prices of a one standard deviation shock in private signal errors. The non-monotonicity in the impulse response is due to a particular combination of positive and negative elements in the VAR coefficient matrix T (**B**), the endogenous nature of prices and forecast errors and the fact that private signals may play a more pronounced role in affecting investors' forecast errors than any other state variable.

3.3 Comparative statics

In this section we present a comparative statics of different degrees in risk aversion and of the precision of rating information relative to that of privately informed investors.

Risk aversion:

According to financial economics, agents trade securities for two different motives: (i) to share risk when they are endowed with different quantities of the risky asset and (ii), to exploit information when they have access to different information sources and possess different assessments of risky-asset pay-offs. The two motives for trading may combine together and affect prices in various ways depending on the model parameters. In particular, as with Hellwig's (1980) static model, as risk aversion increases in the market the risk-sharing motive dominates that of exploiting information. As a result, risk aversion results in less informative prices, which is consistent with the results that our dynamic model produces and are shown in Chart 5.

As far as price volatility is concerned, it depends both on the degree of price informativeness and serial correlation. On the one hand, we have seen already that price informativeness increases with ratings towards that of the full-information benchmark and this is mainly because prices, by becoming more informative, respond better to fundamental innovations. On the other hand, the higher the serial correlation (in absolute terms) of prices the higher the unconditional variance of the price process. Prices, however, may become serially correlated as a result of serially correlated fundamentals, strong risk-sharing motives, filtering problems, or other externalities that may induce investors to trade with less confidence on private information and place more weight on publicly observed signals, such as prices.

Chart 6 reports the impact of risk aversion on price volatility and Chart 7 the relationship between risk aversion and investors' modelled risk-perceptions. Charts 6 and 7 illustrate that, *ceteris paribus*, in a market with high risk aversion rational investors are aware that risk-sharing motives dominate those of information exploitation and prices become less informative.[26] As a result, the accuracy of investors' optimal forecasts, which depend among other things on how informative prices are, diminishes. Given that the long-run (unconditional) mean of prices is common

(26) This is consistent with Hellwig (1980).

knowledge among investors and prices are competitive, less accurate forecasts induce investments to weigh more on the fact that any price deviations from its unconditional mean will be reversed afterwards.[27] Consequently, prices are characterised by strong mean reversion and, as a result, high serial correlation and price volatility.

Rating precision:

As far as the effect of the precision of ratings on prices is concerned, Chart 8 shows that the lower the precision of rating information the less precise investors' optimal forecasts become, but they still remain more precise than in the no-ratings case. Consequently, as the precision of rating information diminishes relative to that of investors' private signals, investors trade less aggressively for information reasons and the informativeness of prices drops towards the no-ratings case benchmark. The relationship between price informativeness and the precision of rating information is illustrated in Chart 9.

Moreover, as the precision of ratings decreases, relative to that of investors' private information, the market turns out to ignore ratings and the volatility of prices drops towards the level under the no-ratings case. This effect is quite distinct from the impact of risk aversion on prices; while risk aversion induces higher serial correlation in prices and, as a result, higher unconditional volatility of prices, the lower the precision of ratings, relative to that of investors' private signals, the more rational investors tend to ignore ratings and focus more on their private information. This point is illustrated in Chart 10.

4 Ratings and benchmarking

We now turn to examine how prices may be affected by frictions that relate to the use of ratings not only for pure information discovery purposes, but also for rating-based capital requirements and benchmarking of investment decisions on ratings.

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(27) We could argue that the less accurate investors' forecasts become, the price tends to become a *focal point* around which investors co-ordinate their beliefs. As a result, long-run (unconditional) mean reversion of prices becomes self-fulfilled at earlier trading rounds and it becomes more likely to affect the decisions of currently lived investors. This is consistent with the results of Allen, Morris and Shin (2003) who solve a similar type of equilibrium but with three trading rounds, totally uninformative prices and a public signal about fundamentals that acts as a focal point and *skews* agents beliefs towards it. In our case, however, the focal point is still the price signal itself.

4.1 Rating-based capital requirements

Regulatory rules often allow regulated entities, such as banks and securities houses, to use credit ratings for capital adequacy purposes. The usual requirement that those entities have to meet is to deduct from capital a certain percentage of the value of their security holdings, depending on the rating that those securities receive from recognised rating agencies. In addition, regulated entities are required by law to maintain a minimum level of capital to withstand potential future losses and, should their capital fall towards that level, they have either to reduce their exposures to risky investments, or to recapitalise.

But setting capital aside for prudential regulation purposes entails an opportunity cost of foregone interest from investing in more profitable risky assets rather than in risk-free securities. This is especially the case when an investor's internal assessment of the fundamental value of traded securities conflicts with that of a rating agency. Consequently, via rating-based capital requirements, ratings could impose a constraint on investment decisions, forcing investors to respond to rating changes in a way that is possibly contrary to their private assessments. That, in turn, could have a material impact on both price efficiency and volatility. What such an impact could be is an open question that we attempt to address through our stylised model in this section.

From Section 2.5, parameter k captures the opportunity cost of funds due to rating-based capital charges. So far, k has been set equal to zero, but now turn to examine the case with rating-based capital requirements, that is when $k > 0$, and their impact on the informativeness and volatility of prices. Chart 11 shows that the informativeness of prices decreases the higher the parameter k, namely, the higher the incentives that capital adequacy rules offer to investors to forecast next period's rating. At the same time, the volatility of prices drops for an initial range of parameter k and then increases as investors' incentives to forecast the rating process increase further. This is illustrated in Chart 12.

However, high levels of parameter k would be far from relevant to existing rating-based capital adequacy rules. In particular, the risk-weighting scale of asset holdings under the proposed New Basel Accord, along with the 8% Basel ratio, and low levels of world interest rates would imply a relatively modest level of incentives to forecast ratings for capital adequacy purposes. Thus, any realistic set of rating-based capital rules would be expected to imply a low k, under which both price efficiency and volatility would possibly drop. Moreover, in the real world, the dispersion of information across investors would possibly be higher than in our model, where only two classes of informed investors have been assumed. Higher dispersion of beliefs across investors would lead to greater heterogeneity in asset holdings across portfolios and, as a result, the impact of rating-based capital requirements on the informativeness of prices at an aggregate level would be less pronounced than what our model implies.

4.2 Benchmarking noise trades to ratings

An increasing number of policymakers and market participants, including the rating agencies themselves, have pointed to the fact that the use of ratings for reasons other than their information content may impose a negative externality on the efficient functioning of securities markets. In particular, linking investment decisions to ratings, with the most notable example the dichotomy between *investment* and *subinvestment* grade credits, may distort financial markets from pooling information and allocating financial resources in an efficient way.

Such a distortion could arise as a result of both regulatory rules and market practices. In particular, many institutional investors are forced by law, or their own charter, to sell bonds whose credit rating has crossed some critical threshold level. In the United States, for example, regulators place restrictions on the quality of assets pension funds and insurance companies can invest in and those restrictions are explicitly linked to the credit ratings produced by the Nationally Recognised Statistical Rating Organisations (NRSROs). Although these rating-linked constraints may not be necessarily *hard* – in a sense of prescribing *immediate* liquidation of affected assets – they may adversely interfere with investment decisions and drive investors' interest away from assets whose economic value would, otherwise, warrant a better treatment by the market.

In this section we attempt to touch upon the issue of linking investment decisions to ratings and to examine the efficiency implications of such practices. However, the idea of having to liquidate a position in an asset, whose rating has fallen below a certain threshold, implies an optimal investment strategy that allows for the possibility of *downgrade-and-sell* scenarios.[28] In a multi-period context that would require us to track individual investors' asset holdings over time and to incorporate them into the state-space representation of our securities market.

(28) In that case, evaluating optimal holdings in the risky asset would require techniques similar to those for pricing barrier contracts.

To avoid such a complication, we consider the situation where both fundamental and non-fundamental trade takes place on the basis of a one-period horizon. Then, we relate the non-fundamental trade to ratings by assuming that there is a set of residual market participants who supply the risky asset proportionately to the probability the rating next period will fall below a certain threshold $\overline{r}$. Threshold $\overline{r}$ is assumed common knowledge among investors. For simplicity, we also assume that the residual investors do not learn from prices or asset pay-offs, but they only consider ratings.[29] Thus, the empirical ratings distribution of those investors is conditional only on past rating information. Given the Kalman filter representation of the rating process in lemma 1, the above conditionality can be stated simply in terms of the currently observed rating r_t and not on the basis of the whole history of ratings up to period t.

Noise traders are assumed to benchmark their supply of the risky asset to some measure of the probability the rating next period will fall below a given threshold $\overline{r}$. Benchmarking, in this way, can be rationalised as the result of forced sales by a class of regulated investors that are restricted to hold the asset only if its rating is above $\overline{r}$ and unload their holdings to the market proportionally to the probability such *downgrading* will take place. For computational convenience and without loss of generality we assume that noise traders consider only ratings for computing such a probability and do not filter information from prices and asset pay-offs. Thus, the total net supply S_t of the risky asset in period t is assumed to be of the form:

$$S_t \simeq A \Pr\left(r_{t+1} \leq \overline{r} | r_t\right) + \varsigma_t \tag{24}$$

where A is a constant that captures the extent of benchmarking of noise trades to ratings. Normality is preserved by conditional expectations, thus, by taking the first-order Taylor expansion of the probability term in **(24)** we may express S_t as

$$S_t = A\left(\frac{1}{2} + \frac{1}{\sqrt{2\pi Var\left(r_{t+1} | r_t\right)}}\left[\overline{r} - E\left(r_{t+1} | r_t\right)\right]\right) + \varsigma_t \tag{25}$$

where by application of the *Projection Theorem* and from **(6)**

$$E\left(r_{t+1} | r_t\right) = \left\{\rho\left[1 - \Sigma\left(\Sigma + 1\right)^{-1}\right] + 2\rho\Sigma\left(\Sigma + 1\right)^{-1}\frac{Cov\left(r, \theta_j\right)}{Var\left(r\right)}\right\} r_t \tag{26}$$

and

$$Var\left(r_{t+1} | r_t\right) = 2\left[\rho\Sigma\left(\Sigma + 1\right)^{-1}\right]^2\left[Var\left(\theta_j\right) - \frac{Cov\left(r, \theta_j\right)^2}{Var\left(r\right)}\right] \tag{27}$$

(29) This assumption is without loss of generality and is imposed in order to avoid the complication of having to consider non-fundamental investors running econometric models. In a real-world context, one could think of a competitive intra-dealer market and institutional investors with limited price discovery capabilities and restricted access to competitive prices. This would be possibly not far from the realities of corporate bond markets.

where parameter Σ is given by **(7)** and all unconditional second moments in **(26)** and **(27)** can be derived from the equilibrium moment matrix M_z of state vector z_t. By substituting **(25)** into the price equation **(17)** we can derive an expression for the state-space representation of the new price process. We can then compute the price dynamics in the new REE and consider the efficiency implications of benchmarking investment decisions to ratings.

It is worth reiterating that both the benchmarking parameter A and the rating threshold $\overline{r}$ are assumed common knowledge among investors. Thus, the supply of the risky asset that is due to benchmarking of asset holdings on ratings is also common knowledge in every period. That allows us to avoid any further complication of having to consider higher order beliefs about the extent of ratings benchmarking in the market and investors' individual threshold levels. Moreover, regarding the supply of the risky asset, no more noise was added in the model and, as a result, the extent of noise trading ς_t in our securities market remains unaltered. Despite that, however, we will see next that the effect of benchmarking on the second moments of asset prices, and consequently on efficiency, is non trivial.

Assuming a relative precision of 0.9 between private and rating information (30) and by varying the level of benchmarking parameter A, we show that price efficiency drops with the extent of benchmarking (A) in the market while volatility increases, as illustrated in Charts 13 and 14. That occurs despite informed investors being fully rational and no extra source of noise was added in the model. In fact, given the timing of events that we discussed in Section 2.2, investors observe the realisation of the rating at the beginning of each period and, by the time investment decisions are made, everyone knows exactly the amount of concurrent residual supply that is due to benchmarking. (31) But, instead of that having a trivial levels-impact on prices, benchmarking on ratings has a material impact on the second moments of prices.

Such an impact of benchmarking on asset prices can be justified on the grounds that perceived changes in fundamentals feed into prices not only through changes in perceptions about future income from holding the asset, but also through beliefs about capital gains that depend on the net supply of the asset. Given that benchmarking renders the net supply of the risky asset partly forecastable, informed investors are inclined to trade more aggressively on any item of

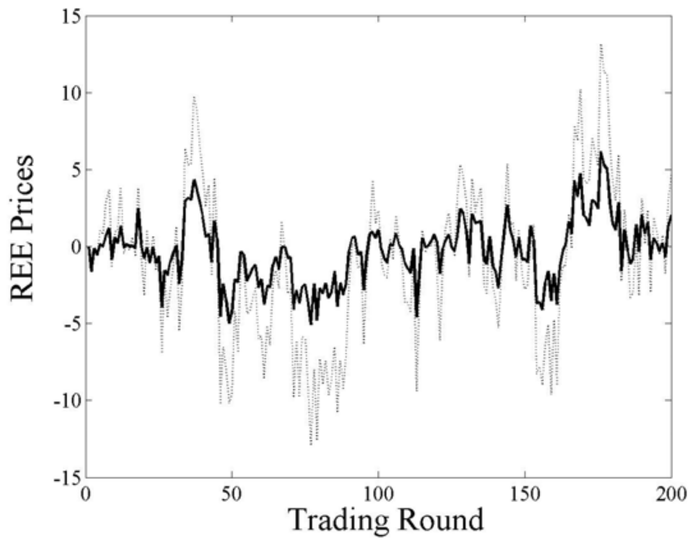
(30) We have repeated the analysis using different levels of relative information precision and the results look qualitatively the same.
(31) In reality, uncertainty about the extent of benchmarking on ratings may further amplify the loss of efficiency.

information in order to exploit perceived mispricings and become more prone to misinterpret any item of news as information about fundamentals.

More formally, Charts 15 to 18 report the impulse response of prices to a one standard deviation shock in various noise terms in the model, demonstrating how rational confusion due to asymmetric information could impact on prices. Charts 15 to 17 show that benchmarking magnifies any undue price response to non-fundamental shocks in pay-offs and errors in private signals and ratings.[32] Chart 18 shows that, in the presence of benchmarking of noise trades to ratings, prices overreact to innovations in fundamentals. That is by overshooting even the full information case, which captures the basic accounting identity between prices and asset pay-offs and is represented by a dotted, downward-sloping line in Chart 18.

The chart below presents a simulation of REE prices with and without benchmarking (solid line), illustrating the magnifying impact of benchmarking on price variations.

REE prices with benchmarking of noise trades to ratings



(32) Notice that it is only informed traders who observe pay-offs and private signals. As a result, any price overreaction at least to non-fundamental pay-off shocks and private signal errors is due to trading by informed traders rather than stemming directly from noise trading.

Consequently, benchmarking of noise trades to ratings could induce informed traders to trade aggressively on any item of news in order to exploit perceived mispricings in the traded asset. In that case, even relatively unimportant news – ie news that is unrelated to fundamentals – could lead to large price swings, resulting in excess asset price volatility and low price efficiency.

5 Conclusions

The role and importance of rating agencies in capital markets has been criticised in recent years because agencies have failed to foresee a number of high-profile credit events, such as the Asian crisis in 1997, the Russian default in 1998 and the Enron bankruptcy in 2001. Agencies have also been criticised for increasing volatility in financial markets, while there have also been voices arguing that ratings are of marginal value to financial markets because the information they provide is *stale* and has already been reflected into share prices.[(33)]

The model presented in this paper demonstrated that, even if ratings lag the market, they may enhance price efficiency when they are used solely for price discovery by market participants and not for other purposes, such as benchmarking of asset holdings on ratings or rating-based capital requirements. On the other hand, the introduction of ratings could add to asset price volatility, but this was found to be consistent with improved market efficiency. This is under the presumption that investors believe that what the rating agency announces is its *best guess* about fundamentals, and investors, despite having different information, have common knowledge of how the economy works.

We also showed that the quantitative impact resulting from the use of ratings for price discovery purposes may depend on the way that rating information is rationally processed by investors. The lower the sophistication of the forecasting techniques used, the more pronounced the impact of ratings on market outcomes. Qualitatively, however, our results remain robust to the type of forecasting techniques that are used by investors.

Regarding the use of ratings for reasons other than price discovery, we distinguished between two types of ratings-related frictions: (i) rating-based capital requirements that apply to investors on the basis of their individual holdings of a rated asset, (ii) benchmarking of asset holdings to ratings

(33) See, for example, J. DuPratt White Professor of Law at Cornell Law School, testimony in front of the US Senate's Committee on Governmental Affairs, 21 March 2002.

from a residual set of investors (eg pension funds, insurance companies) whose sole concern is to sell assets whose rating is likely to fall below a certain threshold.

As far as rating-based capital requirements are concerned, our analysis indicated that if investors' incentives to track ratings for capital adequacy purposes are relatively modest then, rating-based capital requirements may reduce price volatility, yet at the cost of lower price efficiency. However, if incentives to track ratings are sufficiently strong then, rating-based capital requirements could, under certain conditions, add to asset price volatility.

In order to analyse the impact of benchmarking asset holdings to ratings, we considered a residual class of (noise) traders that link their net supply of the risky asset to some measure of the probability that the rating next period will fall below a certain threshold. Benchmarking, in this way, was rationalised as the result of forced sales by a class of regulated market participants who face restrictions on the rating quality of assets they hold.

Our results showed that benchmarking of asset holdings to ratings by certain classes of market participants could induce informed investors to overreact to any item of news about fundamentals, leading to lower price efficiency and higher asset price volatility. We argued that this is because perceived changes in fundamentals feed into prices not only through changes in perceptions about future income from holding the asset, but also through beliefs about capital gains that depend on the net supply of the asset. Given that benchmarking renders the net supply of the risky asset partly forecastable, informed traders are inclined to trade more aggressively on any item of news that could imply a change in fundamentals, even if they face no restrictions on the rating quality of assets they hold. As a result, informed investors become more prone to misinterpret any item of news as information about fundamentals leading to less informative and more volatile prices.

At this point, it is worth drawing a parallel between our results, in case of benchmarking asset holdings to ratings, and the UK market experience in the second half of 2002. Market commentators at the time attributed the rapid swings in market sentiment partly to a regulatory *resilience test* that applies to life insurance companies. According to that test, firms have to demonstrate solvency in the face of a further 25% decline in their asset holdings. In view of a rapid decline in stock prices that period, the resilience test was suspended for several weeks in order to mitigate forced sales of stocks by major market players.[34]

In a sense, the resilience test that applies to life insurers is a form of benchmarking similar in nature to the rating-based benchmarking that we discussed in this paper. That is, in both cases, a class of market participants benchmarks its investment decisions on a public signal which also conveys information about fundamentals. In our model that public signal was the rating; regarding the resilience-test case, that signal was the price. Similar parallels one could draw with respect to the1987 stock market crash and the role of portfolio insurance, as another form of benchmarking on prices, in exacerbating market turbulence.

Looking forward, the model could be extended to incorporate an explicit objective, by the rating agency, to smooth the rating process (eg to avoid rating reversals) and to examine how that might impact on market outcomes. That, of course, would require us to introduce an adjustment cost in the rating process and the rating agency, in the model, to solve a dynamic programming problem rather than running a simple Kalman filter to assign its ratings. Moreover, a different, though still time invariant, rating process could be adopted that would share more similarities with the actual way that ratings are announced in the marketplace, namely, not in every trading round. A good candidate could be a Markov arrival of rating information under which a rating would be announced in randomly selected periods according to a Markov process. From a modelling perspective, an appealing feature of a Markov formulation would be that, as with the Kalman filter, it has a state-space representation and can be easily incorporated into our framework.

Finally, it would be worth exploring how the results would be affected by an increase in the information dispersion among investors about fundamentals and consider more than two classes of privately informed investors. That would possibly allow us to compare our results with earlier findings on the impact of public information on asset prices, such as in Allen, Morris and Shin (2003).

(34) See, FSA Guidance Note 4 (2002), ‘Resilience test for insurers’.

6 Appendix

6.1 Proof of Lemma 1

Let $\mathbf{s}^r_{jt-1}$ be the vector of signals that the rating agency receives up to $t-1$ about factor θ_j, $j = 1, 2$. Given normality of θ_{jt} and signal vector $\mathbf{s}^r_{jt-1}$, the conditional distribution of θ_{jt}, conditional on signal vector $\mathbf{s}^r_{jt-1}$, is also normal with conditional mean and variance

$$\overline{\theta}_{jt|t-1} \equiv E\left(\theta_{jt} \mid \mathbf{s}^r_{jt-1}\right) \tag{28}$$

$$\Sigma_{t|t-1} \equiv Var\left(\theta_{jt} \mid \mathbf{s}^r_{jt-1}\right) \tag{29}$$

Let us suppose that the conditional mean $\overline{\theta}_{jt|t-1}$ and variance $\Sigma_{t|t-1}$ have been calculated and with those in hand we are able to evaluate $\overline{\theta}_{jt+1|t}$ and $\Sigma_{t+1|t}$. From **(4)** we easily derive the conditional expectation of the signals that the rating agency receives in period t, conditional on the agency's signal information up to period $t-1$

$$E\left(s^r_{jt} \mid \mathbf{s}^r_{jt-1}\right) = E\left(\theta_{jt} \mid \mathbf{s}^r_{jt-1}\right) = \overline{\theta}_{jt|t-1} \tag{30}$$

Moreover, the forecast error $s^r_{jt} - E\left(s^r_{jt} \mid \mathbf{s}^r_{jt-1}\right)$ is

$$s^r_{jt} - E\left(s^r_{jt} \mid \mathbf{s}^r_{jt-1}\right) = \left(\theta_{jt} - \overline{\theta}_{jt|t-1}\right) + e_{jt} \tag{31}$$

Since e_{jt} are independent over time and orthogonal to θ_{jt}, they are also independent of $\overline{\theta}_{jt|t-1}$. This implies that the conditional variance of the forecast error **(31)** is

$$Var\left[\left(s^r_{jt} - E\left(s^r_{jt} \mid \mathbf{s}^r_{jt-1}\right)\right)\right] = \Sigma_{t|t-1} + \sigma^2_e \tag{32}$$

where $\sigma^2_e \equiv Var\left[e_{jt}\right]$. Similarly, the conditional covariance between the forecast errors $s^r_{jt} - E\left(s^r_{jt} \mid \mathbf{s}^r_{jt-1}\right)$ and $\theta_{jt} - E\left(\theta_{jt} \mid \mathbf{s}^r_{jt-1}\right)$ is

$$\begin{aligned}
& Cov\left[s^r_{jt}, \theta_{jt} \mid \mathbf{s}^r_{jt-1}\right] \\
= \; & E\left[\left(\theta_{jt} - \overline{\theta}_{jt|t-1} + e_{jt}\right)\left(\theta_{jt} - \overline{\theta}_{jt|t-1}\right)\right] \\
= \; & \Sigma_{t|t-1}
\end{aligned} \tag{33}$$

From **(28)**, **(29)**, **(30)**, **(32)** and **(33)** we get the conditional joint distribution of signal s^r_{jt} and fundamental factor θ_{jt}, conditional on signal information $\mathbf{s}^r_{jt-1}$ up to period $t-1$

$$\begin{bmatrix} s^r_{jt} \mid \mathbf{s}^r_{jt-1} \\ \theta_{jt} \mid \mathbf{s}^r_{jt-1} \end{bmatrix} \sim N\left(\begin{bmatrix} \overline{\theta}_{jt|t-1} \\ \overline{\theta}_{jt|t-1} \end{bmatrix}, \begin{bmatrix} \Sigma_{t|t-1} + \sigma^2_e & \Sigma_{t|t-1} \\ \Sigma_{t|t-1} & \Sigma_{t|t-1} \end{bmatrix}\right) \tag{34}$$

Let us now define $\overline{\theta}_{jt|t}$ as the conditional expectation of factor θ_{jt} conditional on signal vector $\mathbf{s}^r_{jt}$, namely, all signals s^r_j up to period t

$$\begin{aligned}\overline{\theta}_{jt|t} &\equiv E\left(\theta_{jt} \mid \mathbf{s}^r_{jt}\right) \\ &= E\left(\theta_{jt} \mid s^r_{jt} \middle| \mathbf{s}^r_{jt-1}\right)\end{aligned} \quad \textbf{(35)}$$

The conditional expectation $\overline{\theta}_{jt|t}$ and the conditional variance $\Sigma_{t|t}$ of the forecast error can be evaluated by applying the *Projection Theorem,* using the join distribution in **(34)**

$$\overline{\theta}_{jt|t} = \overline{\theta}_{jt|t-1} + \Sigma_{t|t-1}\left(\Sigma_{t|t-1} + \sigma^2_e\right)^{-1}\left(s^r_{jt} - \overline{\theta}_{jt|t-1}\right) \quad \textbf{(36)}$$

$$\Sigma_{t|t} = \Sigma_{t|t-1} - \Sigma^2_{t|t-1}\left(\Sigma_{t|t-1} + \sigma^2_e\right)^{-1} \quad \textbf{(37)}$$

Moreover, from **(2)** and also the fact that υ_{jt} are orthogonal to every element of the signal vector $\mathbf{s}^r_{jt}$, we get

$$\begin{aligned}\overline{\theta}_{jt+1|t} &\equiv E\left(\theta_{jt+1} \mid \mathbf{s}^r_{jt}\right) \\ &= E\left(\rho\theta_{jt} + \upsilon_{jt} \mid \mathbf{s}^r_{jt}\right) \\ &= \rho\overline{\theta}_{jt|t}\end{aligned} \quad \textbf{(38)}$$

$$\begin{aligned}\Sigma_{t+1|t} &\equiv Var\left(\theta_{jt+1} \mid \mathbf{s}^r_{jt}\right) \\ &= Var\left(\rho\theta_{jt} + \upsilon_{jt} \mid \mathbf{s}^r_{jt}\right) \\ &= \rho^2\Sigma_{t|t} + \sigma^2_\upsilon\end{aligned} \quad \textbf{(39)}$$

Combining **(36)** with **(38)**, and **(37)** with **(39)** we derive the following Kalman filter representation that gives the one-period forecast $\overline{\theta}_{jt+1|t}$ as a function of $\overline{\theta}_{jt|t-1}$

$$\overline{\theta}_{jt+1|t} = \rho\overline{\theta}_{jt|t-1} + \rho\Sigma_{t|t-1}\left(\Sigma_{t|t-1} + \sigma^2_e\right)^{-1}\left(s^r_{jt} - \overline{\theta}_{jt|t-1}\right)$$

or

$$\overline{\theta}_{jt+1|t} = \rho \left[1 - \Sigma_{t|t-1}\left(\Sigma_{t|t-1} + \sigma_e^2\right)^{-1}\right]\overline{\theta}_{jt|t-1} + \rho\,\Sigma_{t|t-1}\left(\Sigma_{t|t-1} + \sigma_e^2\right)^{-1} s_{jt}^r \qquad \textbf{(40)}$$

where $\Sigma_{t+1|t}$ solves

$$\Sigma_{t+1|t} = \rho^2 \Sigma_{t|t-1} - \rho^2 \Sigma_{t|t-1}^2 \left(\Sigma_{t|t-1} + \sigma_e^2\right)^{-1} + \sigma_v^2 \qquad \textbf{(41)}$$

Given that $|\rho| < 1$, $\sigma_e^2 > 0$ and $\sigma_v^2 > 0$, the conditional variance $\Sigma_{t|t-1}$ converges to a unique (positive) steady-state constant Σ^* that solves(35)

$$\Sigma^* = \rho^2 \Sigma^* \left[1 - \Sigma^* \left(\Sigma^* + \sigma_e^2\right)^{-1}\right] + \sigma_v^2 \qquad \textbf{(42)}$$

It is easy to show that the solution to **(42)** is

$$\Sigma^* = \frac{1}{2}\sigma_e^2 \left[\frac{\sigma_v^2}{\sigma_e^2} - \left(1-\rho^2\right) + \sqrt{\left[\frac{\sigma_v^2}{\sigma_e^2} - \left(1-\rho^2\right)\right]^2 + 4\frac{\sigma_v^2}{\sigma_e^2}}\right] \qquad \textbf{(43)}$$

Independence between θ_1 and θ_2, s_1^r and s_2^r implies that the rating process r_t is given by

$$\begin{aligned} r_t &= E\left[\theta_{1t} + \theta_{2t} \mid s_{1s}^r, s_{2s}^r, s < t\right] \\ &= \overline{\theta}_{1t|t-1} + \overline{\theta}_{2t|t-1} \end{aligned}$$

or, from **(40)**

$$r_t = \rho\left[1 - \Sigma\left(\Sigma + 1\right)^{-1}\right] r_{t-1} + \rho\Sigma\left(\Sigma + 1\right)^{-1}\left[s_{1t-1}^r + s_{2t-1}^r\right] \qquad \textbf{(44)}$$

where $\Sigma = \frac{1}{2}\left[\frac{\sigma_v^2}{\sigma_e^2} - \left(1-\rho^2\right) + \sqrt{\left[\frac{\sigma_v^2}{\sigma_e^2} - \left(1-\rho^2\right)\right]^2 + 4\frac{\sigma_v^2}{\sigma_e^2}}\right]$.

Q.E.D.

6.2 Fixed-point solution algorithm

Following Hussman (1992), we outline here the main steps we need to follow in order to calculate a linear REE equilibrium of our securities market. To derive such an equilibrium we need to evaluate matrices $T(\mathbf{B})$ and $V(\mathbf{B})$ of the actual law of motion **(18)**. We start by choosing arbitrary values for their first row, which corresponds to the price process, and for the conditional variances σ_j^2 and coefficient matrices $\mathbf{B}_j$, $j = 1, 2$. We also define selector matrices $\mathbf{e}_1$, $\mathbf{e}_2$, $\mathbf{u}_1$, $\mathbf{u}_2$ that satisfy

(35) See, for example, Hamilton (1994), Proposition 13.1, page 390.

the following set of equations

$$
\begin{aligned}
z_{1t} &= \mathbf{e}_1 z_t \qquad x_{1t} = \mathbf{u}_1 z_t \\
z_{2t} &= \mathbf{e}_2 z_t \qquad x_{2t} = \mathbf{u}_2 z_t \\
r_t &= \mathbf{e}_r z_t \qquad \varsigma_t = \mathbf{e}_s \varepsilon_t
\end{aligned} \tag{45}
$$

Let also matrix $\mathbf{c}$ be such that $p_{t+1} + D_{t+1} + kr_{t+1} = \mathbf{c}x_{jt+1}$. Given **(45)**, we can easily see that $E\left[p_{t+1} + D_{t+1} + kr_{t+1} \mid I_{jt}\right] = \mathbf{cB}_j\mathbf{u}_j z_t$ $(j = 1, 2)$ and the equilibrium price **(17)** can be restated as

$$p_t = \Lambda^{-1}\left[\alpha\sigma_2^2 N\phi_1 \mathbf{cB}_1\mathbf{u}_1 + (1-\alpha)\,\sigma_1^2 N\phi_2 \mathbf{cB}_2\mathbf{u}_2 - M\mathbf{e}_r\right] z_t - \Lambda^{-1}\sigma_1^2\sigma_2^2 \mathbf{e}_s \varepsilon_t \tag{46}$$

Substituting z_t from **(18)** into the price equation **(46)** we derive the following expression for the price process

$$p_t = \mathbf{d}_p z_{t-1} + \boldsymbol{\epsilon}_p \varepsilon_t$$

where row matrices $\mathbf{d}_p$ and $\boldsymbol{\epsilon}_p$ define the first row of $T\,(\mathbf{B})$ and $V\,(\mathbf{B})$, respectively, and they are given by

$$
\begin{aligned}
\mathbf{d}_p &\equiv \Lambda^{-1}\left[\alpha\sigma_2^2 N\phi_1 \mathbf{cB}_1\mathbf{u}_1 + (1-\alpha)\,\sigma_1^2 N\phi_2 \mathbf{cB}_2\mathbf{u}_2 - M\mathbf{e}_r\right] T\,(\mathbf{B}) \\
\boldsymbol{\epsilon}_p &\equiv \Lambda^{-1}\left[\alpha\sigma_2^2 N\phi_1 \mathbf{cB}_1\mathbf{u}_1 + (1-\alpha)\,\sigma_1^2 N\phi_2 \mathbf{cB}_2\mathbf{u}_2 - M\mathbf{e}_r\right] V\,(\mathbf{B}) - \Lambda^{-1}\sigma_1^2\sigma_2^2 \mathbf{e}_s
\end{aligned}
$$

The second row of $T\,(\mathbf{B})$ and $V\,(\mathbf{B})$, which corresponds to the pay-off process D_t, is implied by **(1)**, while the third row, which corresponds to the rating process, is implied by lemma 1. The fourth and fifth row of $T\,(\mathbf{B})$ and $V\,(\mathbf{B})$, which correspond to investors' private signals s_t^j $(j = 1, 2)$ are implied by **(3)**, and the sixth and seventh row by **(2)**. Row eight of $V\,(\mathbf{B})$ corresponds to supply of the risky asset and is set equal to

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

With respect to investors' forecast errors ζ_{jt} $(j = 1, 2)$ we define selector matrices $\mathbf{e}_{zj}$ such that

$$\zeta_{jt} = \mathbf{e}_{zj} z_t$$

From the actual law of motion **(18)**, from investors' perceptions **(11)** and from selector matrices $\mathbf{e}_{zj}$ and $\mathbf{e}_j$, the forecast errors ζ_{jt} can be written as

$$
\begin{aligned}
\zeta_{1t} &= \left[\mathbf{e}_1 T\,(\mathbf{B}) - \mathbf{A}_1\mathbf{e}_1 - \mathbf{C}_1\mathbf{e}_{z1}\right] z_{t-1} + \mathbf{e}_1 V\,(\mathbf{B})\,\varepsilon_t \\
\zeta_{2t} &= \left[\mathbf{e}_2 T\,(\mathbf{B}) - \mathbf{A}_2\mathbf{e}_2 - \mathbf{C}_2\mathbf{e}_{z2}\right] z_{t-1} + \mathbf{e}_2 V\,(\mathbf{B})\,\varepsilon_t
\end{aligned} \tag{47}
$$

Equations in **(47)** define the following matrices $\mathbf{d}_\zeta$ and $\mathbf{e}_\zeta$

$$\mathbf{d}_\zeta \equiv \begin{bmatrix} \mathbf{e}_1 T\,(\mathbf{B}) - \mathbf{A}_1\mathbf{e}_1 - \mathbf{C}_1\mathbf{e}_{z1} \\ \mathbf{e}_2 T\,(\mathbf{B}) - \mathbf{A}_2\mathbf{e}_2 - \mathbf{C}_b\mathbf{e}_{z2} \end{bmatrix}$$

$$\mathbf{e}_\zeta \equiv \begin{bmatrix} \mathbf{e}_1 V\,(\mathbf{B}) \\ \mathbf{e}_2 V\,(\mathbf{B}) \end{bmatrix}$$

Matrix $\mathbf{d}_\zeta$ defines rows 9 to 16 of $T(\mathbf{B})$, while matrix $\mathbf{e}_\zeta$ defines rows 9 to 16 of $V(\mathbf{B})$. It is worth noting that in equations **(47)** selector matrices $\mathbf{e}_1$ and $\mathbf{e}_2$ select elements only from the first five rows of matrices $T(\mathbf{B})$ and $V(\mathbf{B})$. However, the rows of matrices $T(\mathbf{B})$ and $V(\mathbf{B})$ that are relevant to ζ_{1t} are rows 9 to 12, while for ζ_{2t} rows 13 to 16. Consequently, $\mathbf{e}_1$ and $\mathbf{e}_2$ do not select any of the coefficients of matrices $T(\mathbf{B})$ and $V(\mathbf{B})$ that are relevant to the evaluation of forecast errors ζ_{1t} and ζ_{2t}. Thus, there is no need to evaluate a fixed point for the rows of $T(\mathbf{B})$ and $V(\mathbf{B})$ that correspond to investors' forecast errors.

6.3 Equilibrium in the high-sophistication case

Under the benchmark case without ratings, the equilibrium ARMA(1,1) coefficients of the observable variables $\begin{bmatrix} p_t & D_t & s_t^j & \zeta_t^j \end{bmatrix}$ are calculated to be

$$\mathbf{B}_j = \begin{bmatrix} 0.5527 & 0.9359 & -0.0734 & -0.3720 & -0.3491 & 0.2081 \\ -0.0000 & 0.8000 & -0.0000 & 0.0356 & -0.6326 & 0.0793 \\ 0.0000 & -0.0000 & 0.8000 & -0.0120 & 0.0952 & -0.6797 \\ 0.0000 & -0.0000 & 0.0000 & -0.0000 & 0.0000 & -0.0000 \\ -0.0000 & 0.0000 & -0.0000 & 0.0000 & -0.0000 & 0.0000 \\ -0.0000 & 0.0000 & -0.0000 & 0.0000 & -0.0000 & 0.0000 \end{bmatrix}$$

The last three rows of $\mathbf{B}_j$ give the coefficients in the projection of forecast errors ζ_t^j on p_{t-1}, D_{t-1}, s_{t-1}^j and ζ_{t-1}^j. That these coefficients are zero is a necessary condition for ζ_t^j to be conditional vector white noise, conditional on observable information of investors of type $j = 1, 2$. At the REE, from the moment matrix $\mathbf{M}_z$, we derive the following variance-covariance matrix $\mathbf{M}$ for the variables $\begin{bmatrix} p_t & D_t & s_t^1 & s_t^2 & \theta_{1t} & \theta_{2t} & \varsigma_t \end{bmatrix}$

$$\mathbf{M} = \begin{bmatrix} 4.3608 & 2.0202 & 0.6740 & 0.6740 & 0.5409 & 0.5409 & -0.0736 \\ 2.0202 & 1.5556 & 0.2778 & 0.2778 & 0.2778 & 0.2778 & -0.0000 \\ 0.6740 & 0.2778 & 1.2778 & 0.0000 & 0.2778 & 0.0000 & 0.0000 \\ 0.6740 & 0.2778 & -0.0000 & 1.2778 & 0.0000 & 0.2778 & -0.0000 \\ 0.5409 & 0.2778 & 0.2778 & 0.0000 & 0.2778 & 0.0000 & 0.0000 \\ 0.5409 & 0.2778 & -0.0000 & 0.2778 & 0.0000 & 0.2778 & -0.0000 \\ -0.0736 & -0.0000 & 0.0000 & -0.0000 & 0.0000 & -0.0000 & 0.0100 \end{bmatrix}$$

By introducing ratings in the information sets of investors, the equilibrium ARMA(1,1) coefficients of the observable variables $\begin{bmatrix} p_t & D_t & r_t & s_t^j & \zeta_t^j \end{bmatrix}$ are calculated to be

$$\mathbf{B}_j = \begin{bmatrix} 0.3364 & 1.7162 & -0.0002 & -0.0601 & -0.2136 & -1.2186 & 1.6445 & 0.2319 \\ 0.0000 & 0.8000 & -0.0000 & -0.0000 & 0.0277 & -0.6607 & 0.4579 & 0.0659 \\ 0.0000 & 0.4624 & 0.3376 & -0.0000 & 0.0160 & -0.3819 & 0.2647 & 0.0381 \\ 0.0000 & -0.0000 & 0.0000 & 0.8000 & -0.0138 & 0.0798 & 0.2331 & -0.6844 \\ 0.0000 & -0.0000 & -0.0000 & 0.0000 & -0.0000 & 0.0000 & 0.0000 & -0.0000 \\ -0.0000 & 0.0000 & 0.0000 & -0.0000 & 0.0000 & -0.0000 & -0.0000 & 0.0000 \\ -0.0000 & 0.0000 & 0.0000 & -0.0000 & 0.0000 & -0.0000 & -0.0000 & 0.0000 \\ 0.0000 & -0.0000 & 0.0000 & 0.0000 & -0.0000 & 0.0000 & -0.0000 & -0.0000 \end{bmatrix}$$

From the moment matrix $\mathbf{M}_z$ we derive the following variance-covariance matrix $\mathbf{M}$ for the variables $\begin{bmatrix} p_t & D_t & r_t & s_t^1 & s_t^2 & \theta_{1t} & \theta_{2t} & \varsigma_t \end{bmatrix}$

$$\mathbf{M} = \begin{bmatrix} 5.0634 & 2.0202 & 1.0239 & 0.7806 & 0.7806 & 0.6439 & 0.6439 & -0.0668 \\ 2.0202 & 1.5556 & 0.2816 & 0.2778 & 0.2778 & 0.2778 & 0.2778 & 0.0000 \\ 1.0239 & 0.2816 & 0.2816 & 0.1408 & 0.1408 & 0.1408 & 0.1408 & -0.0000 \\ 0.7806 & 0.2778 & 0.1408 & 1.2778 & 0.0000 & 0.2778 & 0.0000 & -0.0000 \\ 0.7806 & 0.2778 & 0.1408 & 1.2778 & 1.2778 & 0.0000 & 0.2778 & 0.0000 \\ 0.6439 & 0.2778 & 0.1408 & 0.2778 & 0.0000 & 0.2778 & 0.0000 & 0.0000 \\ 0.6439 & 0.2778 & 0.1408 & 0.0000 & 0.2778 & -0.0000 & 0.2778 & 0.0000 \\ -0.0668 & -0.0000 & -0.0000 & 0.0000 & 0.0000 & -0.0000 & -0.0000 & 0.0100 \end{bmatrix}$$

We now briefly present the equilibrium forecasting techniques under the *low-sophistication* case where investors are restricted to run vector AR(1) models.(36)

6.4 Equilibrium in the low-sophistication case

Under the benchmark case of asymmetric information without ratings, the equilibrium AR(1) coefficients of the observable variables $\begin{bmatrix} p_t & D_t & s_t^j \end{bmatrix}$ are calculated to be

$$\mathbf{B}_j = \begin{bmatrix} 0.0510 & 0.1044 & 0.0503 \\ 0.1081 & 0.2221 & 0.1068 \\ -0.0235 & 0.1258 & 0.1508 \end{bmatrix}$$

(36) As with the ARMA case, the NREE when the market is using vector AR techniques is calculated using Matlab programs.

At the REE, from the moment matrix $\mathbf{M}_z$ we derive the following variance-covariance matrix $\mathbf{M}$ for the variables $\left[\begin{array}{ccccccc} p_t & D_t & s_t^1 & s_t^2 & \theta_{1t} & \theta_{2t} & \varsigma_t \end{array}\right]$

$$\mathbf{M} = \left[\begin{array}{ccccccc} 0.3988 & 0.6408 & 0.2220 & 0.2220 & 0.1307 & 0.1307 & -0.0339 \\ 0.6408 & 1.5556 & 0.2778 & 0.2778 & 0.2778 & 0.2778 & -0.0000 \\ 0.2220 & 0.2778 & 1.2778 & 0.0000 & 0.2778 & -0.0000 & -0.0000 \\ 0.2220 & 0.2778 & -0.0000 & 1.2778 & -0.0000 & 0.2778 & 0.0000 \\ 0.1307 & 0.2778 & 0.2778 & -0.0000 & 0.2778 & -0.0000 & -0.0000 \\ 0.1307 & 0.2778 & -0.0000 & 0.2778 & -0.0000 & 0.2778 & -0.0000 \\ -0.0339 & 0.0000 & -0.0000 & 0.0000 & 0.0000 & -0.0000 & 0.0100 \end{array}\right]$$

We consider now the case with ratings that are used by investors for information discovery purposes only. In this case, the equilibrium AR(1) coefficients of the observable variables $\left[\begin{array}{cccc} p_t & D_t & r_t & s_t^j \end{array}\right]$ are calculated to be

$$\mathbf{B}_j = \left[\begin{array}{cccc} 0.0808 & 0.3467 & 2.1872 & 0.1625 \\ 0.0339 & 0.1454 & 0.4974 & 0.0681 \\ 0.0196 & 0.0840 & 0.6251 & 0.0394 \\ -0.0210 & 0.0838 & 0.3266 & 0.1317 \end{array}\right]$$

From the moment matrix $\mathbf{M}_z$ we derive the following variance-covariance matrix $\mathbf{M}$ of the variables $\left[\begin{array}{cccccccc} p_t & D_t & r_t & s_t^1 & s_t^2 & \theta_{1t} & \theta_{2t} & \varsigma_t \end{array}\right]$

$$\mathbf{M} = \left[\begin{array}{cccccccc} 4.5017 & 1.7513 & 1.0239 & 0.7312 & 0.7312 & 0.6039 & 0.6039 & -0.0572 \\ 1.7513 & 1.5556 & 0.2816 & 0.2778 & 0.2778 & 0.2778 & 0.2778 & 0.0000 \\ 1.0239 & 0.2816 & 0.2816 & 0.1408 & 0.1408 & 0.1408 & 0.1408 & 0.0000 \\ 0.7312 & 0.2778 & 0.1408 & 1.2778 & -0.0000 & 0.2778 & -0.0000 & -0.0000 \\ 0.7312 & 0.2778 & 0.1408 & -0.0000 & 1.2778 & -0.0000 & 0.2778 & -0.0000 \\ 0.6039 & 0.2778 & 0.1408 & 0.2778 & -0.0000 & 0.2778 & -0.0000 & 0.0000 \\ 0.6039 & 0.2778 & 0.1408 & -0.0000 & 0.2778 & -0.0000 & 0.2778 & 0.0000 \\ -0.0572 & 0.0000 & 0.0000 & -0.0000 & -0.0000 & 0.0000 & -0.0000 & 0.0100 \end{array}\right]$$

As expected, the moment matrices of vector $\left[\begin{array}{cccccccc} p_t & D_t & r_t & s_t^1 & s_t^2 & \theta_{1t} & \theta_{2t} & \varsigma_t \end{array}\right]$ under the ARMA and AR equilibrium differ only with respect to their first row and column that correspond to the second moments of prices. This is because the price process is the only endogenously determined process in the vector, while all other variables are assumed to be exogenous and remain unaffected by the equilibrium allocations of asset holdings among investors.

Chart 1: Non-fundamental supply shock (ς)

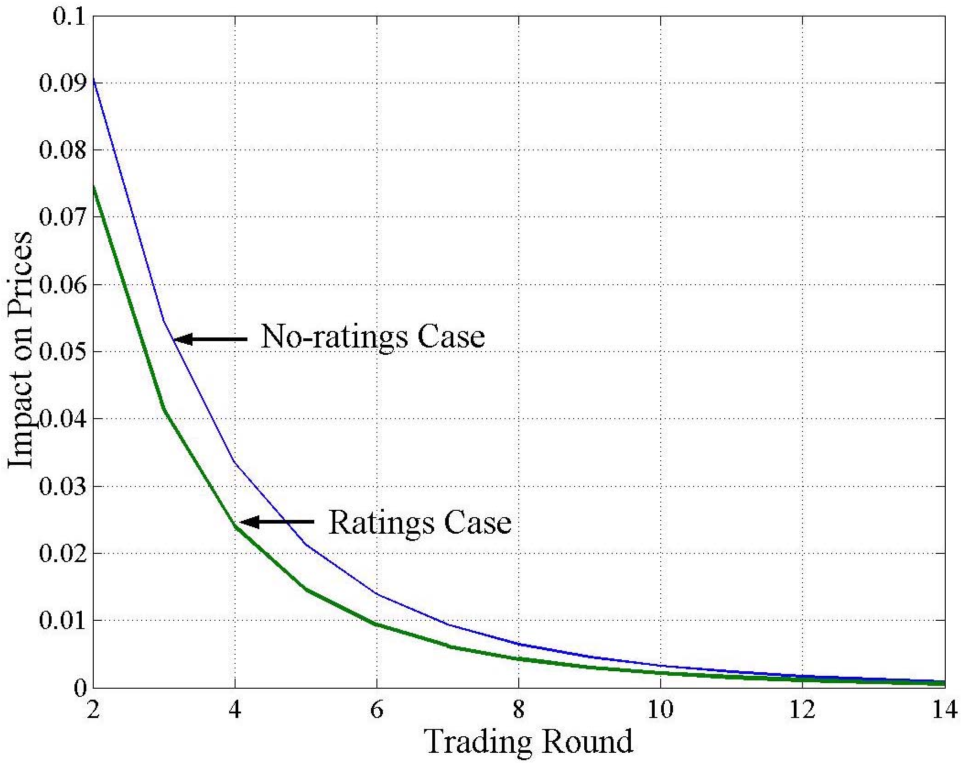


Chart 2: Non-fundamental pay-off shock (u)

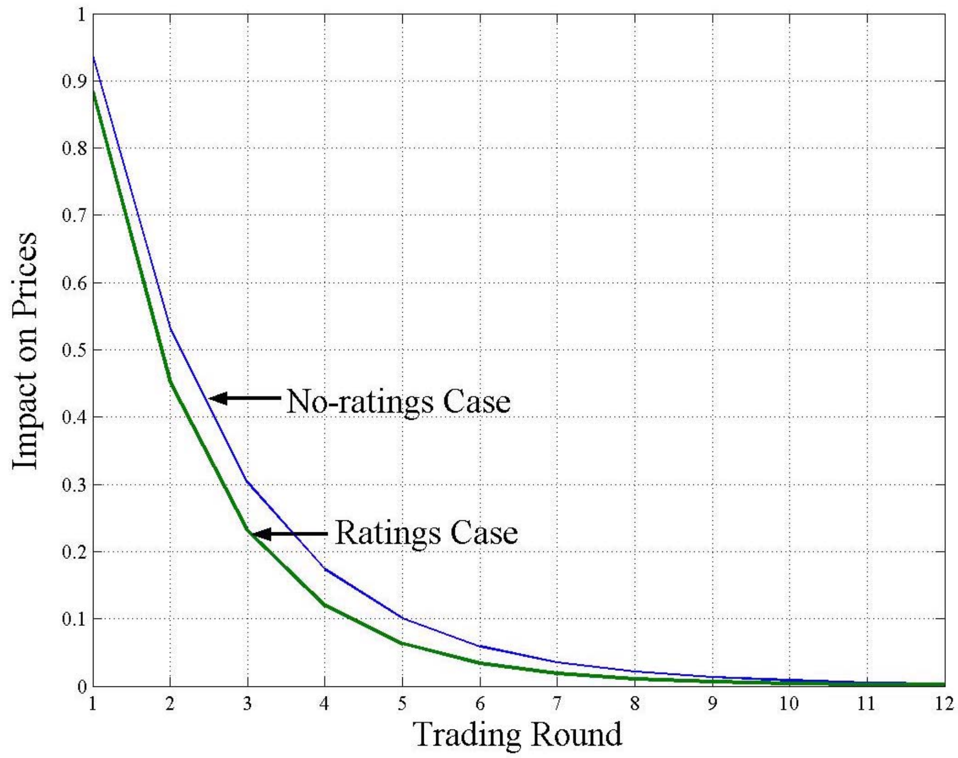


Chart 3: Fundamental shock (υ_j)

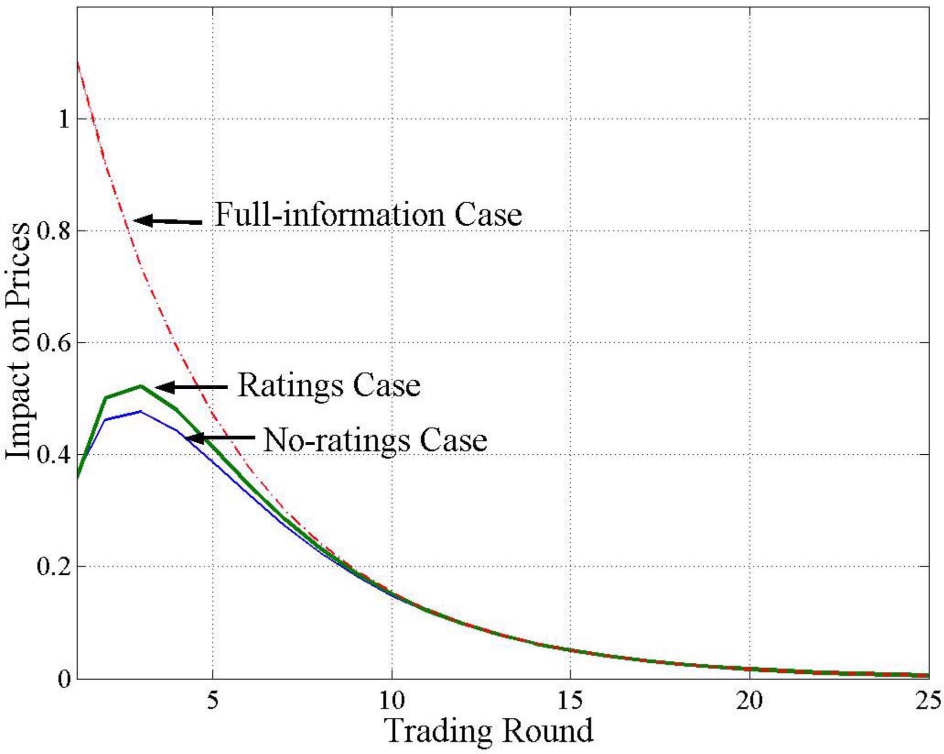


Chart 4: Private signal error (η_j)

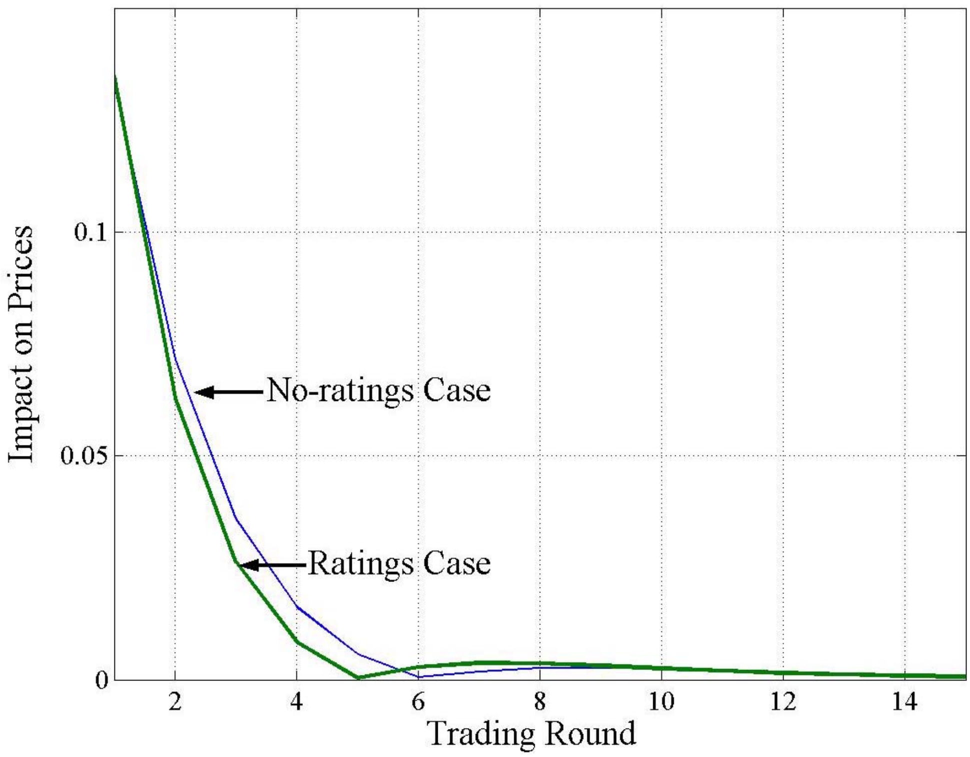


Chart 5: Risk aversion and price efficiency

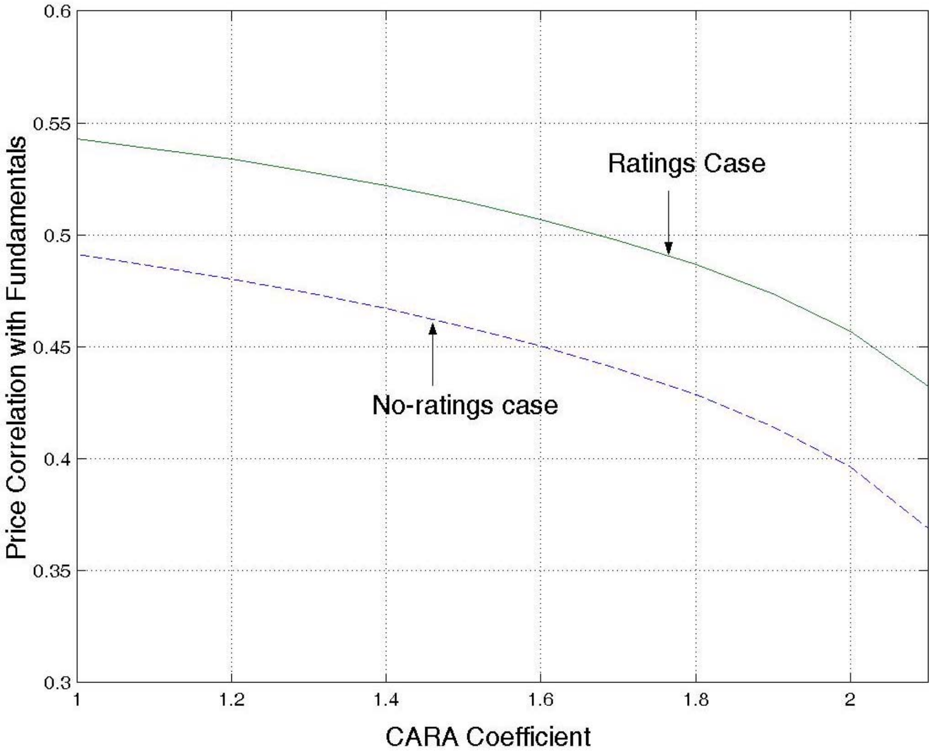


Chart 6: Risk aversion and price volatility

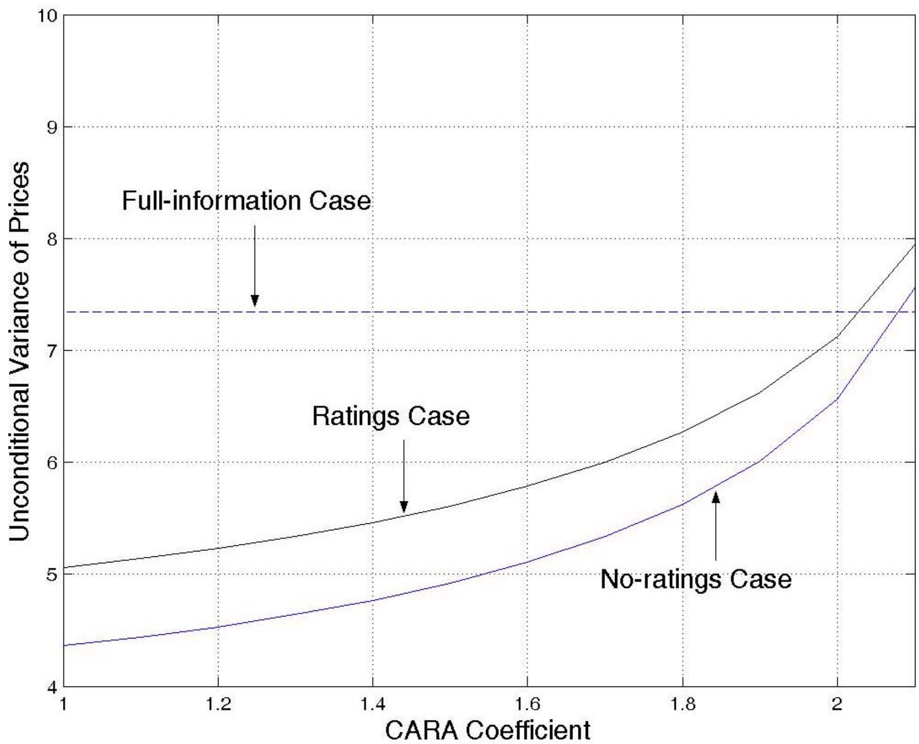


Chart 7: Risk aversion and modelled risk perceptions

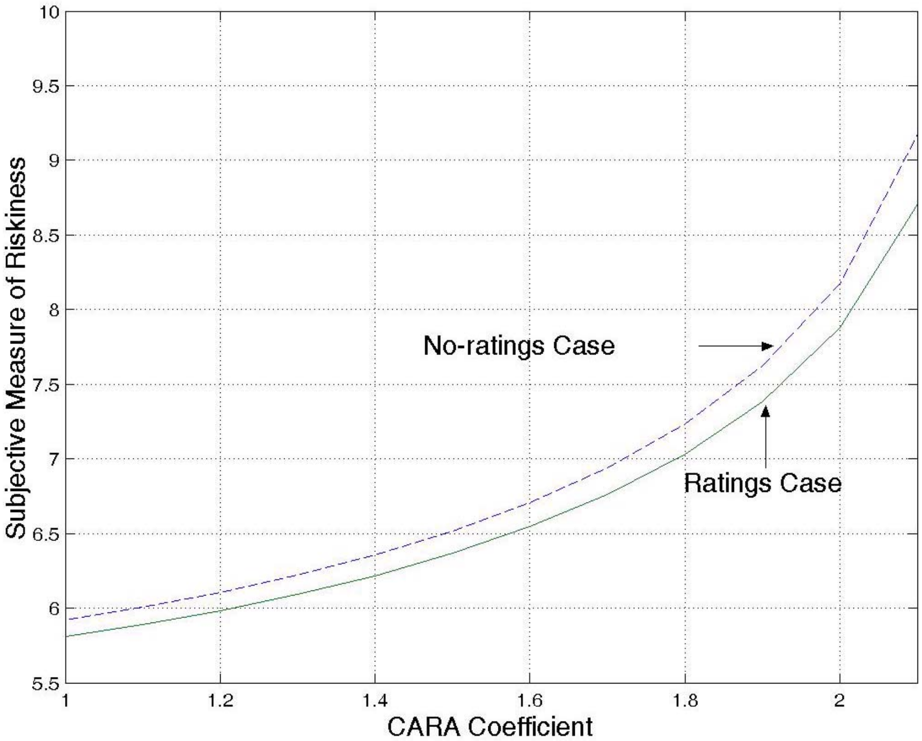


Chart 8: Rating precision and modelled risk perceptions

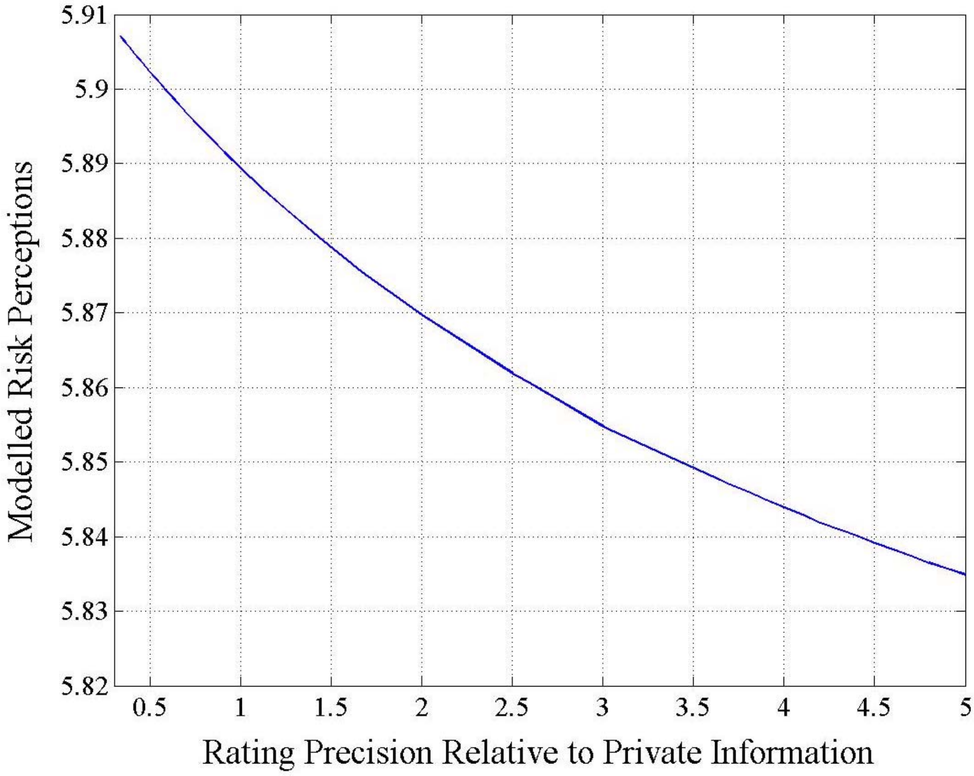


Chart 9: Rating precision and price efficiency

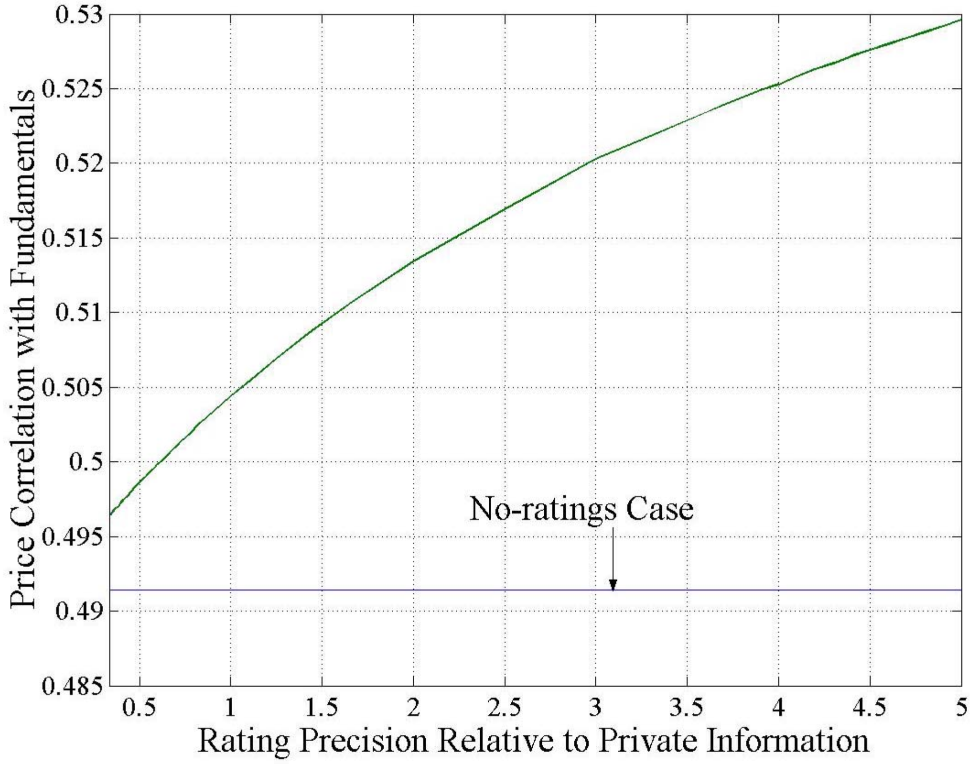


Chart 10: Rating precision and price volatility

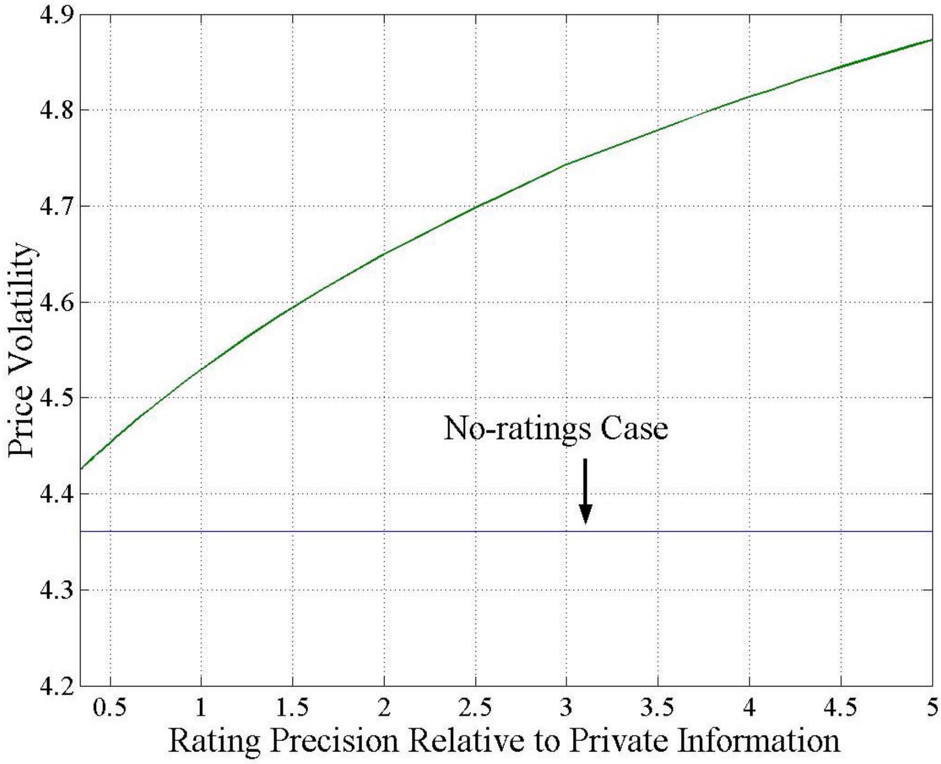


Chart 11: Capital requirements and price efficiency

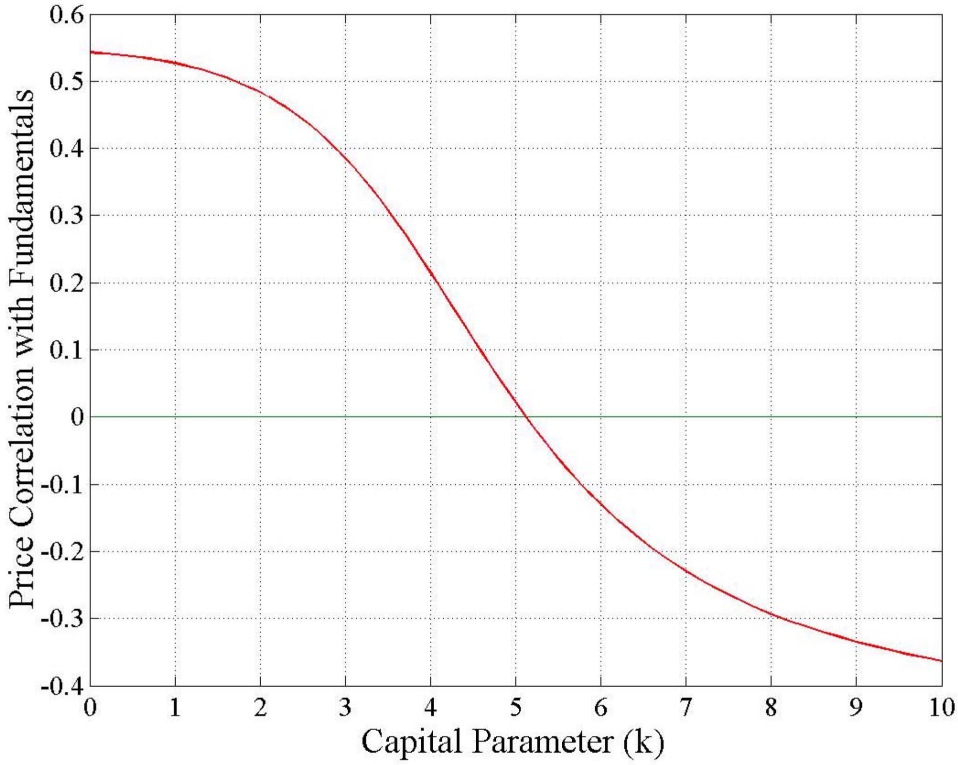


Chart 12: Capital requirements and price volatility

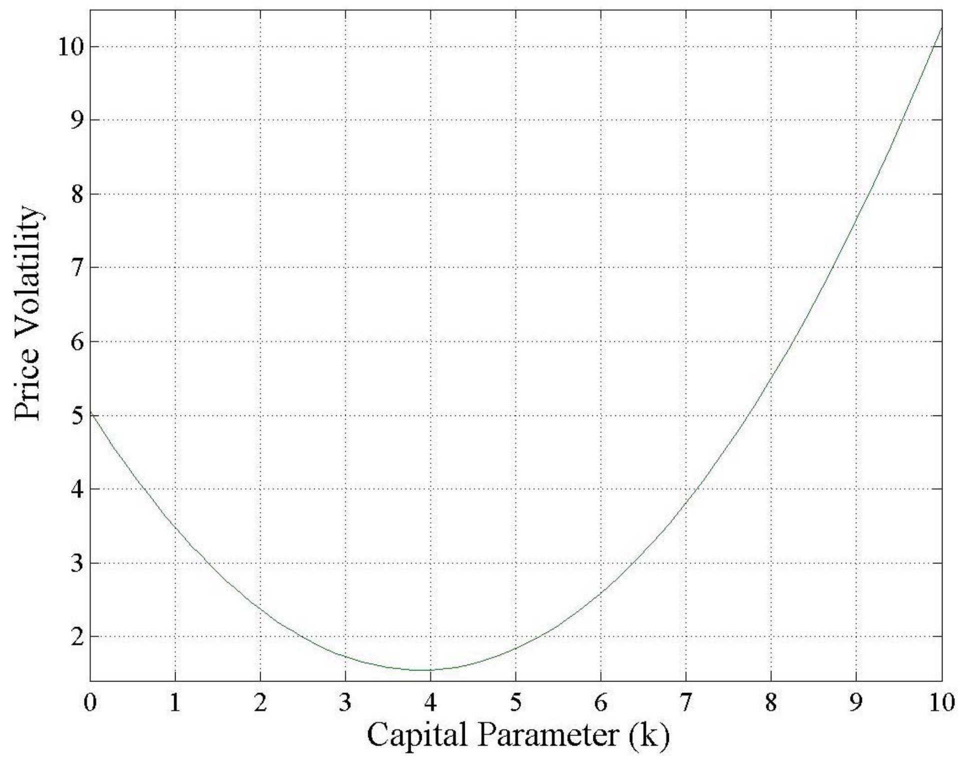


Chart 13: Investment benchmarking (*A*) and price efficiency

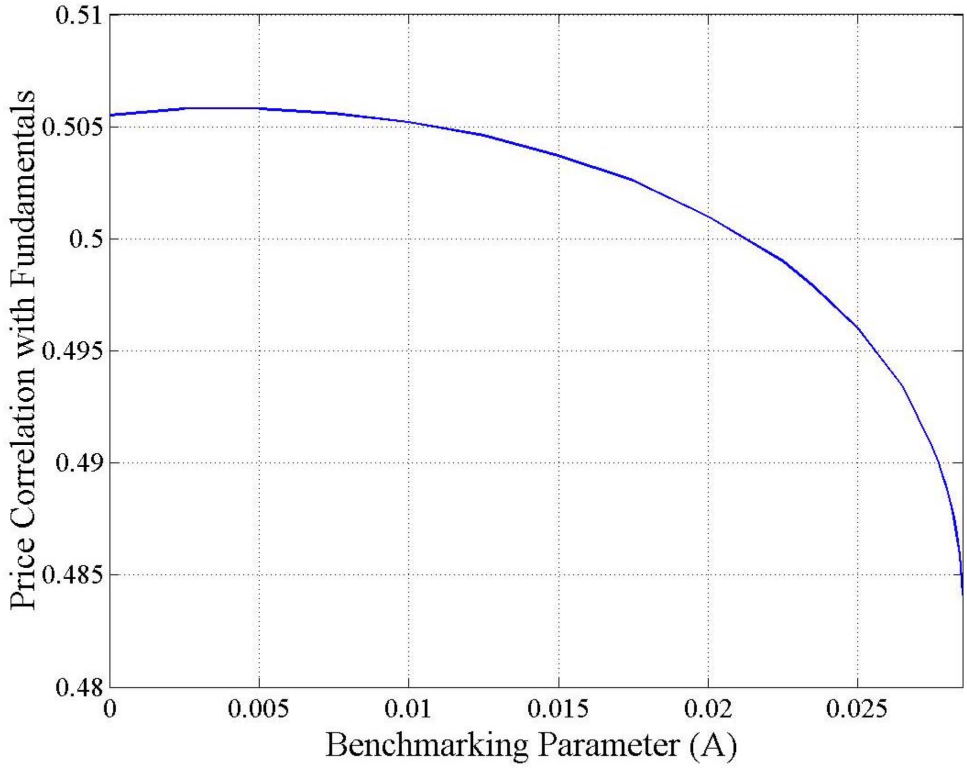


Chart 14: Investment benchmarking (*A*) and price volatility

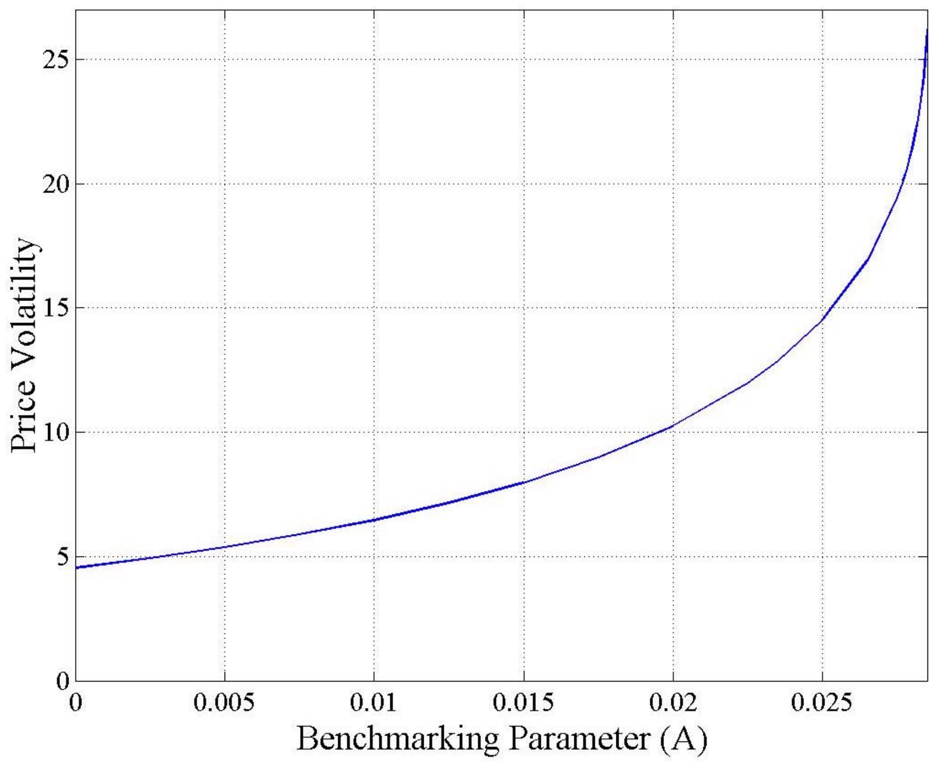


Chart 15: Non-fundamental pay-off shock (u)

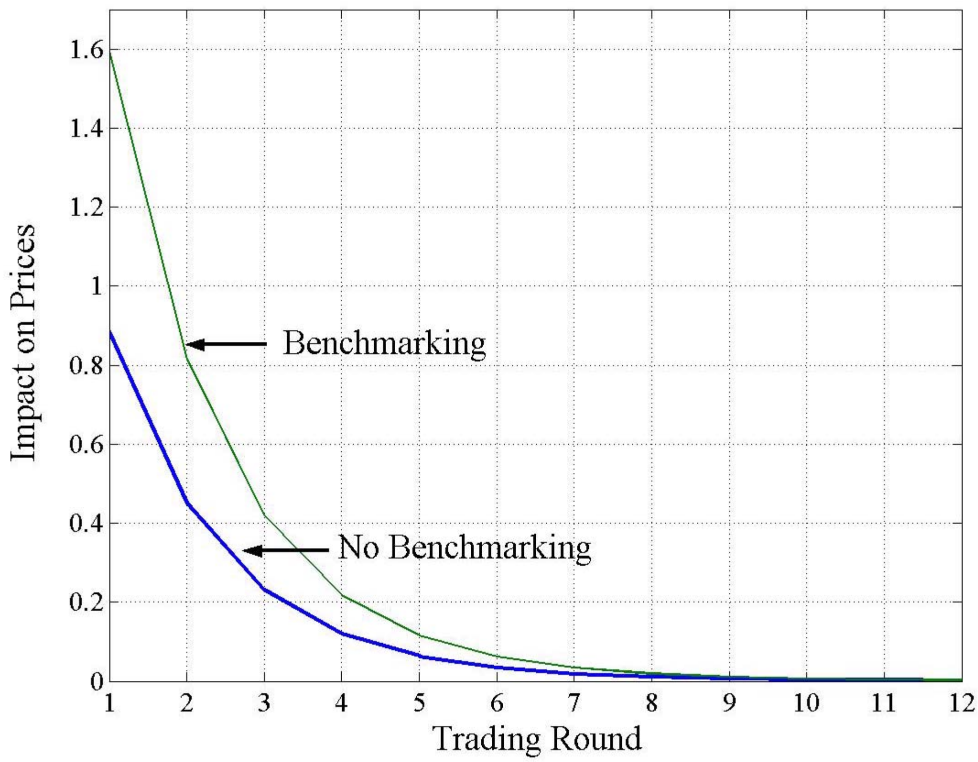


Chart 16: Private signal error (η_j)

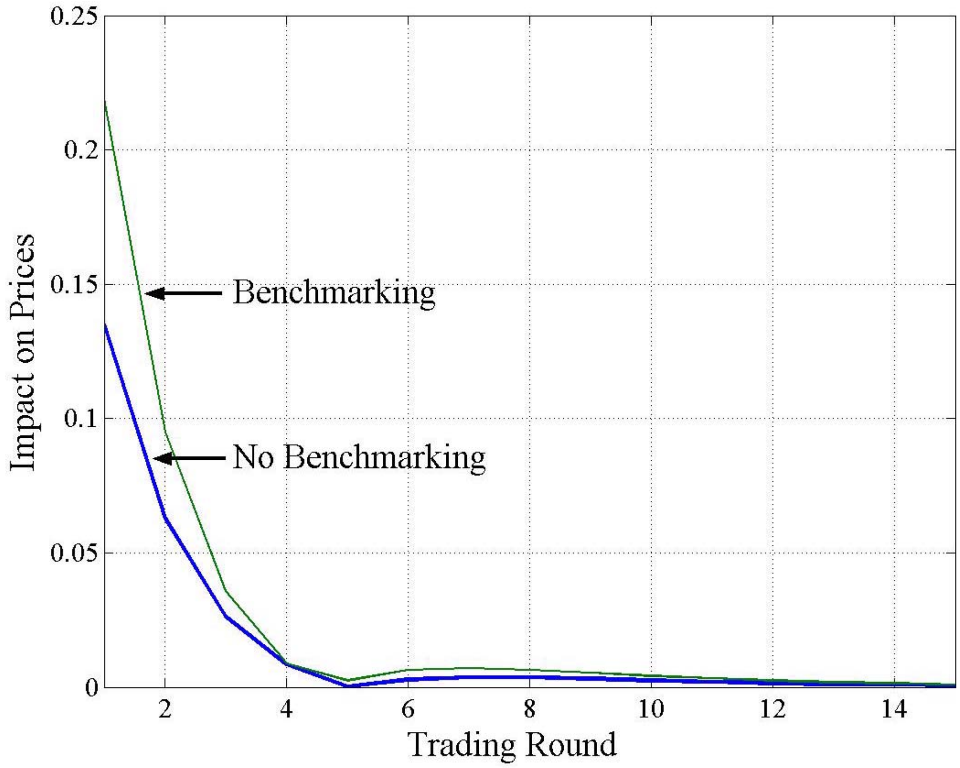


Chart 17: Rating error (e_j)

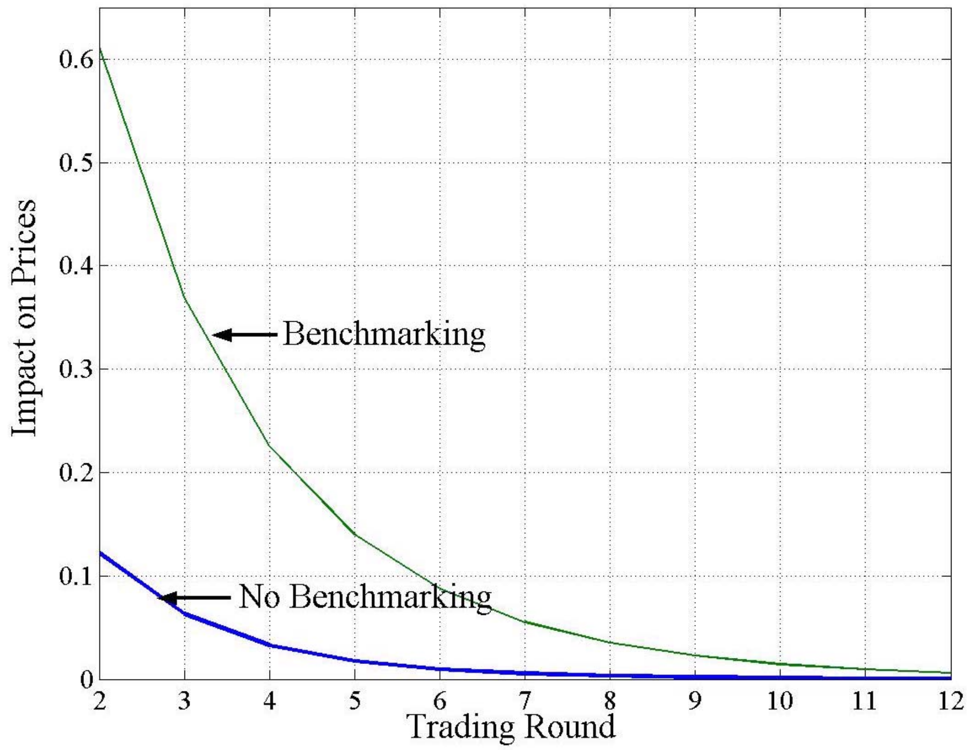
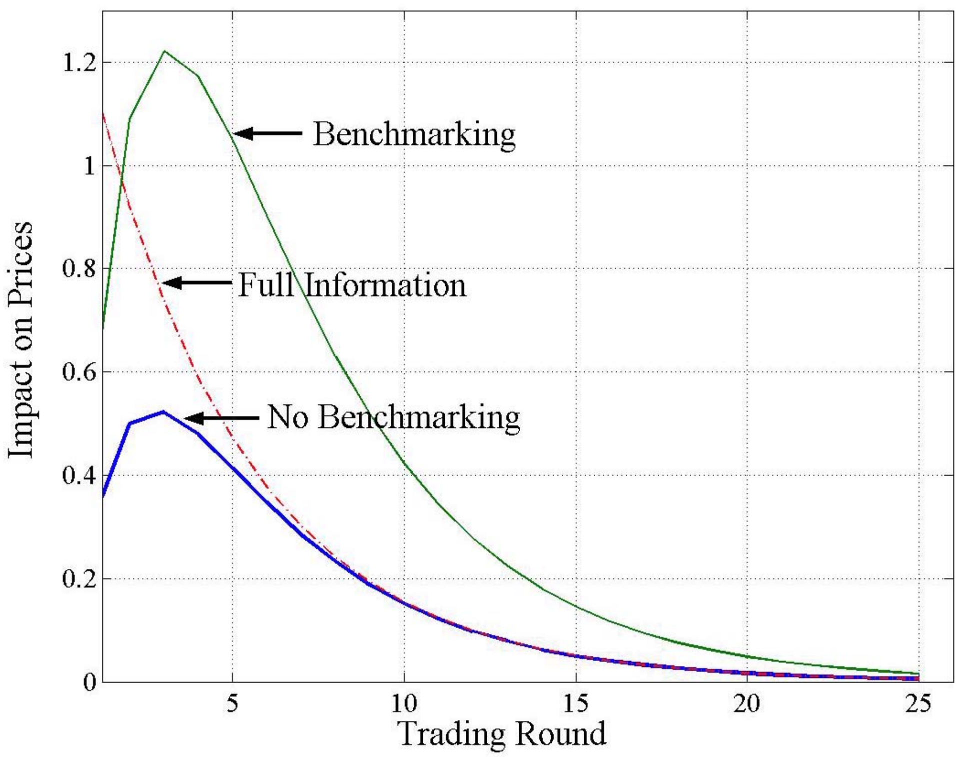


Chart 18: Fundamental shock (υ_j)



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