# Decomposing credit spreads

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**Abstract**

This paper investigates the information contained in the yields of corporate debt securities using a structural credit risk model. As previous studies have found, credit risk is not the only factor that affects corporate yield spreads. The aim is to decompose credit spreads, using a structural model of credit risk, into credit and non-credit risk components. The contribution relative to the existing literature is the use of contemporaneous forward-looking information on equity risk premia and equity value uncertainty in a structural model. In particular, implied equity risk premia from a three-stage dividend discount model that incorporates analysts' long-term earnings forecasts are used, together with implied measures of equity value uncertainty from option prices. The paper examines the evolution of the different components of spreads across time as well as the effect of particular events. It also analyses the relationship between the derived components and other financial variables, such as swap spreads and the equity risk premium.

Key words: Default, credit spreads, risk premia.

*JEL* classification: G12, G15.

**Summary**

Corporate credit spreads are important indicators for both monetary policy and financial stability purposes. The Bank of England therefore regularly monitors movements in such spreads, both domestically and internationally. Credit spreads contribute to the cost of external debt financing for the corporate sector, which forms part of the cost of capital that affects firms' investment decisions. Spreads also reflect perceptions about the financial health of the corporate issuers, and can thus indicate potential stress in specific sectors in the economy.

This paper addresses the factors behind credit spread movements. We know that compensation for expected default is only one component of credit spreads. Another component can also be related to credit risk, that is, compensation for the uncertainty about the probability of default. The final component is due to non-credit risk factors, which are driven by differences between government and corporate bonds and the markets in which they are traded, such as liquidity, regulation and tax. The implications for policy of an increase in spreads driven by higher expected default are different from those due to an increase driven by changes in liquidity.

The technical issue raised in this paper is the quantification of the above components. In particular, we perform two exercises. The first is to calibrate a structural model of credit risk to firms' historical default frequencies, both investment-grade and high-yield. We choose the Leland and Toft model developed in 1996 because of its simplicity and intuition, and use US data, as default frequency data for UK companies are insufficient for this purpose. UK data are available for a much shorter period and a smaller sample of companies. The purpose of this exercise is to assess the ability of the model to match firms' historical default behaviour by not only examining the fit of the model to historical default frequencies, but also the plausibility of the derived estimates of asset volatility and risk premium. In addition, this exercise allows us to calculate an average historical compensation for credit risk and compare it with the average observed spread.

The second exercise involves the use of contemporaneous forward-looking information for the equity risk premium and equity volatility, along with the Leland and Toft model, to generate time-series decompositions for the observed credit spreads of UK and US investment-grade companies, as well as US high-yield companies.

The results from these two exercises are as follows:

- The historical estimates generated for US investment-grade companies are around 20.6% for asset volatility and an asset risk premium just above 4%. This is equivalent to an equity volatility of 35% and an equity risk premium of 6.3%. The estimates for high-yield firms are 25.4% for asset volatility and 7.5% for the asset risk premium. This is equivalent to an equity volatility of 78% and an equity risk premium of 11.5%.

- These parameters imply that the average compensation for credit risk factors is 72 basis points for investment-grade firms, 55% of the average observed spread of 136 basis points. We conclude that a large part of the investment-grade credit spread is due to non-credit risk factors. The corresponding numbers for high-yield firms are 430 and 523 basis points. We therefore find that a higher proportion, 82%, of the spread is explained by credit risk for riskier (high-yield) debt.

- The contemporaneous decomposition shows that, on average, a significant proportion of the observed credit spread is due to non-credit risk factors. This is consistent with the historical decomposition. The actual spreads and the compensation for credit risk we calculate are highly correlated. The component that compensates investors for expected default, which is the only credit risk compensation risk-neutral investors would require, is significantly more stable than the spreads we observe.

- The non-credit risk component is closely related quantitatively to swap spreads for our investment-grade decomposition. Previous studies have found that a small proportion of variation in swap spreads is due to credit risk. This provides support for identifying the residual of our decomposition with the non-credit risk component. For high-yield companies, the non-credit risk component is significantly higher than swap spreads, although they follow similar patterns. This may reflect higher liquidity premia required in the high-yield corporate bond markets.

The above results imply that the information content of credit spreads as a macroeconomic indicator or predictor of corporate sector default rates would depend on the source of the shock. However, data availability restricts the available history of the decomposition. As more data become available we would be able to test the predictive ability with respect to both default rates and future growth. Another avenue for future research would be to quantify the possible determinants of the non-credit risk component of credit spreads by closely examining the structure of the different markets.

## 1 Introduction

This paper analyses the information contained in the yields of corporate debt securities using structural credit risk models. The yields of corporate bonds are higher than those of government bonds to compensate investors for the higher default probability. However, as previous studies have found,(1) credit risk is not the only factor that affects corporate yield spreads. Liquidity, asymmetric regulatory (eg Minimum Funding Requirement in the United Kingdom) or tax effects of corporate and government bonds and option-type features (eg call, conversion) can also affect corporate yield spreads. The purpose of this study is to decompose these spreads using structural models of credit risk into credit and non-credit risk components. This is important as the Bank of England regularly monitors movements in such spreads, both in the United Kingdom and abroad. The implications for policy of an increase in spreads driven by credit risk components are different than if the increase was driven by changes in non-credit risk components. Previous studies focused on average historical spreads. In this study we also provide a contemporaneous decomposition using forward-looking measures of the equity risk premium and equity price uncertainty in a structural model.

Merton (1974) pioneered the theory of credit risk determination by introducing a model where a company's equity is an option on the assets of the company, with strike price equal to the repayment required on the debt. The company defaults when the value of its assets reaches the book value of total liabilities, that is, when the market value of the assets is insufficient to repay the liabilities. The main determinants of default in this framework are the value of the firm's assets, the uncertainty of the assets' value and leverage (that is, the book value of liabilities relative to the market value of the assets).

Since then, several authors have extended the structural approach. Longstaff and Schwartz (1995) incorporated stochastic interest rates, an exogenous default barrier,(2) and an exogenous recovery rate as a fraction of the value of assets at the default boundary. They derived closed-form solutions for coupon-paying bonds. Strategic default models were introduced by Anderson and Sundaresan (1996), Anderson, Sundaresan and Tychon (1996) and Mella-Barral and Perraudin (1997). In these models equity holders default strategically to extract concessions from bondholders and the default boundary and recovery rate are endogenous. Black and Cox (1976), Leland (1994) and Leland and Toft (1996) introduced models with an endogenous default boundary. The default decision is made by managers who attempt to maximise the value of equity. If it is not optimal to default, firms issue equity to service their debt coupon payments, thus delaying their default as much as possible. Another class of models assumes a mean-reverting leverage ratio. In Collin-Dufresne and Goldstein

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(1) See for example Huang and Huang (2002).
(2) The exogenous default barrier (the firm's asset value that triggers default) can be smaller than the book value of total liabilities. The long-term nature of some liabilities allows firms to continue operate with negative net worth, consistent with empirical evidence. The default point lies between total liabilities and current or short-term liabilities.

(2001), firms adjust their debt levels in response to changes in firm value, which makes their leverage ratio mean-reverting. Their model captures dynamic restructuring, that is, the notion that firms tend to issue debt when their leverage falls below some target level or avoid replacing maturing debt when leverage is above target. In contrast to Longstaff and Schwartz (1995),[3] their model implies a stationary leverage ratio.

In this paper we focus on one model, the Leland and Toft (1996) model. The comparison of different structural models has been examined extensively in the literature. Huang and Huang (2002) calibrated a range of different structural models to historical default frequencies and they showed that all these models imply similar results about the theoretical value of credit spreads for different credit quality issuers. The Leland and Toft (1996) model was chosen because it provides a simple solution for coupon-paying non-callable bonds with fixed maturity, which are the focus of this study. In contrast strategic default models assume debt with indefinite maturity. In addition, Huang and Huang (2002) found that the calibrated strategic default models had endogenous defaults that occurred at firm value above the debt's face value. This is in contrast to the empirical evidence that default levels are usually lower than debt principal. Furthermore, the Leland and Toft (1996) model endogenises the default boundary, extending the model of Longstaff and Schwartz (1995). A disadvantage of the Leland and Toft (1996) model relative to Collin-Dufresne and Goldstein (2001) is that, by assuming a static debt structure, it implies an exponentially declining leverage ratio. However, as explained below, the drift of the log asset value of the firm is small, and so the potential decline in leverage (the logarithm of the book value of liabilities to asset value) for maturities less than ten years would also be small. Moreover, the Collin-Dufresne and Goldstein (2001) model is more difficult to calibrate because of the presence of the mean-reversion parameter for leverage. For these reasons and because Huang and Huang (2002) showed that yield spreads are highly stable across different models, we decided to use the Leland and Toft (1996) model.

In particular, we perform two exercises. The first exercise is to calibrate the Leland and Toft (1996) model to US firms' historical default frequencies, both investment-grade and high-yield. Default frequency data for UK companies are insufficient for this purpose. They are available for a much shorter period and a smaller sample of companies. Since these default frequencies depend on the actual growth rate of underlying assets value, this calibration provides some estimates for the firms' asset volatility and the asset risk premium. The purpose of this exercise is to assess the ability of the model to match firms' historical default behaviour by not only examining the fit of the model to historical default frequencies but also the plausibility of the derived estimates of asset volatility and risk premium. In addition, this exercise allows us to calculate an average historical compensation for credit risk and compare it with the average observed spread. The compensation for credit risk is

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[3] Since the firm value increases exponentially over time in Longstaff and Schwartz (1995), and the default boundary is a monotonic function of the book value of liabilities, a constant over time default boundary implies exponentially declining leverage ratios.

the sum of the compensation due to expected default (also called risk-neutral spread or 'actuarial' spread as it ignores the compensation for uncertainty about default) and the credit risk premium (compensation for uncertainty about default). That is, compensation for credit risk = risk-neutral spread + credit risk premium. The compensation for credit risk can be different from the observed credit spreads due to the presence of liquidity or other non-credit risk related factors in the observed credit spreads.

The second exercise involves the use of contemporaneous forward-looking information for the equity risk premium and equity volatility to generate time-series decompositions for the observed credit spreads of UK and US investment-grade companies, as well as US high-yield companies. We do not generate decompositions for the observed credit spreads of UK high-yield companies, as the market is not sufficiently large. That is, at each point of time, we use the Leland and Toft (1996) model and forward-looking information on equity risk premium and equity volatility to generate the part of the spread which reflects compensation due to expected default, the part due to the uncertainty about default, and the 'non-credit risk component' due to the presence of liquidity or other non-credit risk related factors in the observed credit spreads.

The paper is structured as follows. Section 2 describes the Leland and Toft (1996) model. Section 3 calibrates the model to US historical default frequencies and derives estimates of the average historical equity risk premium and volatility (exercise 1). Section 4 uses forward-looking information of the equity risk premium, uncertainty, the interest rate, and the actual spread being paid at the time to derive time-varying estimates of the compensation for credit risk and its components (exercise 2). It describes the empirical results for US and UK investment-grade firms as well as US high-yield firms. Section 5 concludes.

## 2 The Leland and Toft (1996) model

As with Merton (1974) and most structural models, the Leland and Toft (1996) model assumes a Geometric Brownian Motion for the value $V(t)$ of the firm's productive assets. That is, the firm's asset value grows on average at a rate equal to the cost of capital reduced by the cash flow paid out as coupons or dividends. The uncertainty about the proportional changes of $V(t)$, that is, is the volatility of the firm's assets $\sigma$ is constant. This is shown in equation **(1)**:

$$\frac{dV(t)}{V(t)} = (\mu - \delta)\cdot dt + \sigma \cdot dW(t), \text{ with } V_0 = 100 \qquad \textbf{(1)}$$

where $\mu = r + \lambda$ is cost of capital, the sum of the risk-free rate $r$ and the asset risk premium $\lambda$. The asset risk premium is a leverage weighted average of the debt yield spread and the equity risk premium. $\delta$ is the constant payout ratio (cash flow paid out) and is equal to the sum of the debt coupon $C$ and dividend payments $Div$ in a year, as a proportion of firm value. That is, $\delta = \frac{C + Div}{V_o}$. $\{W\}$ is a standard Wiener process. The firm asset value and the payout ratio are assumed to be unaffected by changes in the leverage; that is, investment does not change with capital structure. This is consistent with the Modigliani-Miller framework.

In equation **(1)** the process of firm's asset value, $V(t)$, relates to the 'subjective' probability measure. Therefore the drift term $\mu = r + \lambda$ includes a positive asset risk premium term $\lambda$. Under the risk-neutral probability measure, the asset risk premium vanishes and as a result the drift, equal to the risk-free rate $r$, is smaller than under the 'subjective' probability measure. Thus, under the 'subjective' probability measure the asset value is more likely to be above the default barrier and therefore the probability of default is lower to that under the risk-neutral probability measure.

The firm process continues without limit until the asset value reaches the default trigger level $V_B$. As explained later, the default trigger level is determined endogenously rather than imposed by a covenant such as net worth requirement. Obviously $V_B$ can be time varying depending on the capital structure. If a company has a single debt issue, debt service payments are much greater at maturity and the company needs more asset value at maturity to avoid bankruptcy. However, a time-varying $V_B$ level complicates debt valuation. To overcome this problem, Leland and Toft (1996) assumes a debt structure with constant debt service payments. This leads to a constant default trigger level $V_B$.

In particular, new bond principal of maturity $T$ is issued at a rate $\frac{P}{T}$ per year, so that, the total debt outstanding principal $P$ is constant with maturity uniformly distributed over the next $T$ years. The same amount of principal is redeemed per year as bonds mature. This means that the average bond maturity of the debt is $\frac{T}{2}$. Bonds pay coupon at a constant rate $\frac{C}{T}$, so that the total coupon paid in any particular year is $C$. To service its debt, the firm pays a total amount of $C + \frac{P}{T}$ per year. Because of the static debt structure, the value of the debt $D$ has no explicit time dependence, that is, $D = D(V, V_B, T)$. Because of this, the default barrier level is also time independent, and so simple barrier option formulas can be used to derive closed-form solutions for the value of debt.

In the case of bankruptcy bondholders get a fraction $(1-\alpha)$ of the asset level at default $V_B$, that is, $(1-\alpha) \cdot V_B$. Equity holders receive nothing under the assumption of strict seniority. The parameter $\alpha$ determines bankruptcy costs. The recovery rate, which is defined as the ratio of the value available to bondholders at default divided by the principal value $P$ of total debt outstanding, is equal to $R = \frac{(1-\alpha)\cdot V_B}{P}$.

The total firm value is equal to the sum of the asset value $V$ and the value of the tax deduction of coupon payments less the value of bankruptcy costs. The value of bankruptcy costs is positively related to $V_B$, as it is equal to the present value of $\alpha \cdot V_B$ contingent on future bankruptcy. The value of the tax deduction of coupon payments is negatively related to $V_B$, as a higher default trigger asset level (for given $V$) makes bankruptcy (with the resulting loss of tax benefits) more likely. Therefore, the value of the equity is the total firm value reduced by the value of the debt.

The equity holders face a question at every point: is it worth meeting the debt service payments? If it is, then they will do so, even if it requires additional equity contributions to achieve this. The value of $V_B$ is chosen so that the expected change in equity value at bankruptcy level equals the additional cash flow required by equity holders for debt service payments (coupon and principal expenses less the cash flow available for payout generated by firm's activities). When $V > V_B$, expected equity appreciation exceeds the required cash flow to keep the firm solvent. Current shareholders will be willing to sell new equity to avoid bankruptcy since the alternative of bankruptcy implies zero equity value. The formula of the optimal value of $V_B$, which is time independent because of the static debt structure, is shown in Appendix 1.

$V_B$ depends on maturity $T$, on asset volatility $\sigma$, debt principal $P$ and coupon $C$ and the effective tax advantage of debt $\tau$.[4] The dependence on maturity results in a bankruptcy asset level that is less than the principal value of debt, for long-term debt, that is, at default, the firm value $V_B$ is less than the debt principal $P$ (negative net worth). As explained in Huang and Huang (2002) and Leland (2002), this is consistent with empirical evidence of recovery rates of around $R = 50\%$, while bankruptcy costs are only around $\alpha = 30\%$. When debt is short term it is possible that $V_B > P$. This is because the expected equity appreciation cannot compensate for the required debt service payments.

Again simple barrier option formulas can be used to derive the first-passage time cumulative probability function, that is, the expected default frequency up to time $t$ is:

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[4] As explained in Leland (2002), the effective tax advantage of debt represents the corporate tax rate offset by the personal tax advantage of equity returns.

$$EDF_t = N[\frac{-b-(\mu-\delta-0.5\sigma^2)t}{\sigma\sqrt{t}}] + e^{-\frac{2b(\mu-\delta-0.5\sigma^2)}{\sigma^2}} N[\frac{-b+(\mu-\delta-0.5\sigma^2)t}{\sigma\sqrt{t}}] \quad \textbf{(2)}$$

where $b = \ln\left(\frac{V_0}{V_B}\right)$. This formula can be used to generate default probabilities under both the 'subjective', when $\mu = r + \lambda$, and risk-neutral probability measure, when $\mu = r$. The first part of equation **(2)** is the Merton (1974) formula that expresses the probability that the value of the firm's assets would be below the default barrier at time *t*. The second term accounts for the probability of paths for the firm's asset value that would be above the barrier at time *t* but cross the default barrier prior to this. As already mentioned, the default probability is lower under the 'subjective' probability measure because the value of the firm's assets drifts at a higher rate under the 'subjective' probability measure and it is therefore more likely to finish above the default boundary.

## 3 Fitting the model to US firms' historical default frequencies

In this section, we describe the process and the results of the first exercise, that is, to calibrate the Leland and Toft (1996) model to historical default frequencies and provide some estimates for the representative firm's asset volatility and the asset risk premium. The purpose of this exercise is to assess the model's ability to match firms' historical default behaviour by examining the fit to default frequencies, as well as the plausibility of the derived estimates of asset volatility and asset risk premium.

Before explaining the details of our calibration, we should emphasise that our focus is on explaining aggregate spreads and aggregate default behaviour, and so our results correspond to a *representative* firm of either investment-grade or high-yield rating. Therefore the data that we use in both this section and the next section as inputs to the model are average data rather than that of an individual issuer.

We first parameterise the model to fit the actual default experience of US companies since 1970.(5) Default frequency data for UK companies are insufficient for this purpose. They are available for a much shorter period and a smaller sample of companies. Average default frequencies are defined as the percentage of companies that defaulted over a specific time period starting at any point in the past. For example, 2.7% of US investment-grade bonds defaulted on average within ten years.

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(5) Using data from Moody's Investors Service.

The data from Moody's Investors Service contains the credit histories of nearly 10,000 corporate and sovereign entities and over 80,000 individual debt securities since 1970. We use Moody's 'Credit Risk Calculator' to generate weighted average cumulative default rates from January 1970 to January 2001 for investment-grade and high-yield bonds. The averaging occurs across cohorts of bonds that existed at the start of each year. The weighting is done according to the size of the cohorts.

Based on historical evidence in the United States since 1970, we set the average government long (ten-year) interest rate $r$ to 8%, and the dividend yield to 4% for investment-grade and 2% for high-yield firms. The average spread is 136 basis points for investment-grade[(6)] and 523 basis points for high-yield firms.[(7)] We also follow Leland (2002) in setting bankruptcy costs $\alpha = 30\%$. We use data from Standard and Poor's (2001) and the market values of the Merrill Lynch bond index by rating to generate an average leverage for investment-grade firms of $P = 41.6\%$.[(8)] The equivalent estimate of leverage for high-yield firms is 68.4%. Based on the above parameters, the payout ratio $\delta = \frac{P(r + spread)}{V_0} + \frac{dividend\ \ yield(100 - P)}{V_0}$ is 6% for investment-grade and 10% for high-yield. The effective tax advantage of debt is set to $\tau = 13.3\%$ for the United States, as in Leland (2002). The debt maturity is set to $T = 10$ years, so that the average maturity of the model, which is equal to $\frac{T}{2}$, matches the historically observed average debt maturity of about 5 years.

We use the above parameters and equation **(2)**, under the 'subjective' probability measure, to find [(9)] the values of the remaining parameters $\sigma$ and $\lambda$ of the model that match the historical default frequencies taken from Moody's Investors Service. We effectively derive historical estimates of the average asset risk premium $\lambda$ and asset volatility $\sigma$ of US investment-grade and high-yield firms since 1970.

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(6) The average investment-grade spread of Merrill Lynch index since 1997 is 134 basis points. The corresponding average spread using the Lehman bond index data from 1973-93 in Huang and Huang (2002), and the Merrill Lynch index weights of rating baskets is 137 basis points. The average between the two estimates is 136 basis points.
(7) The average high-yield spread of Merrill Lynch index since 1997 is 579 basis points. The corresponding average spread using data in Huang and Huang (2002) is difficult to compute since they do not report an average for C-rated debt. Taking the average C-rated spread from Merrill Lynch since 1997 and using the Merrill Lynch weights of rating baskets in the BB and B average spreads from Huang and Huang (2002), gives an estimated average spread of 466 basis points. The average between the two estimates is 523 basis points.
(8) This is larger than the estimates in Huang and Huang (2002), due to their use of Standard and Poor's (1999) data. Although both take a snapshot and are therefore not ideal, we would argue that the stock market might have been less overvalued in 2001.
(9) We use the Newton-Raphson optimisation technique to search for the optimal parameters.

**Results**

The estimates generated for investment-grade companies are around 20.6% for asset volatility[(10)] and just above 4% for asset risk premium. This is equivalent[(11)] to an equity volatility of 35% and an equity risk premium of 6.3% for a representative investment-grade firm when leverage is 41.6%. The equity risk premium of 6.3% is in line with that usually reported for equity indices of about 3%-6% (see for example Dimson, Marsh and Staunton (2002)) and the equity volatility of 35% is higher than the approximately 20% reported for equity indices. This is due to the fact that volatility that we derive corresponds to that of a representative investment-grade firm and not to that of an equity index. The volatility of an equity index is significantly lower than that of an average individual firm as the equity returns of the constituent companies are less than perfectly correlated.

The estimates for high-yield firms are 25.4% for asset volatility and 7.5% for the asset risk premium. This is equivalent to an equity volatility of 78% and an equity risk premium of 11.5% when leverage is 68.4%. Again this is in line with a historically significantly greater volatility for high-yield firms. The higher excess return for the representative high-yield firm relative to the investment-grade is likely to reflect a significantly priced size factor as documented in many cross-sectional equity return studies (see for example Fama and French (1993)). High-yield firms have on average lower capitalisation than investment-grade firms and thus the size factor could account for the difference in the derived equity risk premium.

To assess the fit of the model, we plot the fitted and actual default frequencies in Charts 1 and 2. It is possible to fit the default frequencies well, albeit with a systematic underestimation of default probabilities at the shortest horizons.

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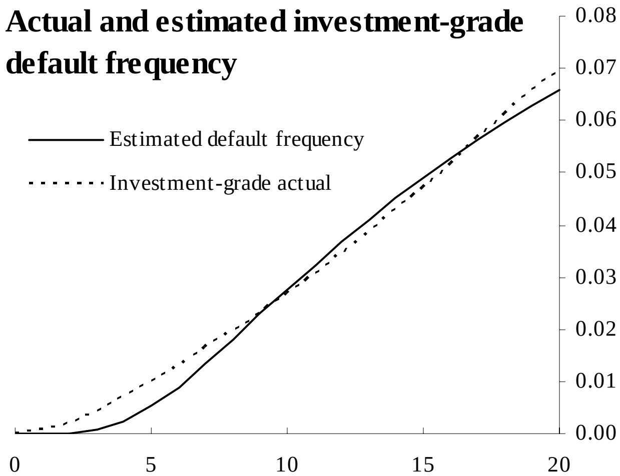
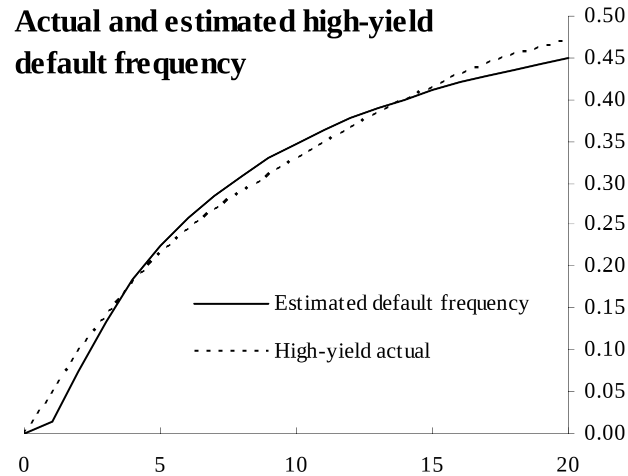
(10) Asset volatility is equal to the volatility of the sum of the firms' debt and equity values, expressed as a percentage.
(11) To derive the equity risk premium from asset risk premium we simply use the definition that the asset risk premium is a leverage-weighted average of the equity risk premium and the observed spread. See Appendix 3 for more details on how we derive equity volatility from asset volatility.

**Chart 1**




Tenor of one year

**Chart 2**




Tenor of one year

Based on the above parameters we can calculate the model-based average historical compensation for credit risk for both investment-grade and high-yield firms. To do this we use the calibrated $\sigma$ along with all other parameter values used in the calibration, to generate an annualised par coupon rate. From this par coupon rate we subtract the assumed risk-free rate $r$, to estimate the spread. The exact formulas used in the calculation of the compensation for credit risk are shown in Appendix 2. Because a risk-neutral valuation method can be used, the asset risk premium $\lambda$ is not needed in the calculation of the compensation for credit risk, that is, the compensation for credit risk depends on asset volatility $\sigma$ and the other structural parameters of the model.

The calculated average historical compensation for credit risk is a spread of 72 basis points for the representative investment-grade firm compared to the average observed spread of 136 basis points. This implies that around 55% of the investment-grade credit spread is due to credit risk. This estimate is greater than that of Huang and Huang (2002) for two reasons: our leverage estimate is higher, and the two added years of default data we gain had historically high default rates. However, the conclusion that a large part of the investment-grade credit spread is due to non-credit risk factors remains. The calculated average historical compensation for credit risk in the case of high-yield firms is a spread of 430 basis points, compared to the average observed spread of 523 basis points. We therefore find that a higher proportion (82%) of the spread is explained by credit risk for riskier (high-yield) debt.

## 4 Using contemporaneous forward-looking information

We now turn to the second exercise, that is, to examine whether movements in the Merrill Lynch US investment-grade and high-yield firm spreads and UK investment-grade firm spreads since 1997 could be explained using contemporaneous forward-looking variables available at the time. To do this we collect end of month data on firm implied volatility, the interest rate, the actual spread being paid at the time, and the cost of equity.

For information on corporate spreads we use indices from the Merrill Lynch Global Index System. This system contains all corporate bond issues with a face value over $150 million, with a time to maturity of one year or above. The indices are denominated by currency of issue, rather than domicile of issuer, so there is not an exact match between the countries we state above, and the domiciles of the firms in the index. The credit spreads we examine are for the sterling investment-grade, dollar investment-grade and dollar high-yield indices.(12) They are option-adjusted spreads, so take account of any embedded options. These spreads are taken over government bonds. An alternative would be to use spreads over swap rates (asset swap spreads). This issue is discussed later in this section.

We collect end of month data on firm volatility, the risk-free interest rate, the actual spread being paid at the time, and the cost of equity. As explained in the previous section, the analysis is conducted with respect to a representative firm, either investment-grade or high-yield. Therefore, average data are used as inputs to the model. To generate an average for the equity volatility of the representative firm, we need to take an average of firm-specific equity volatilities, weighted by market capitalisation. This is because the equity index volatility cannot proxy for the representative firm implied volatility as less than perfect correlation between individual index constituent company returns significantly lower the volatility of the index.

Ideally, we would use an average over different firms of option implied (forward-looking) equity volatility with maturity $T$. However, the data we get from Bloomberg on individual firm option implied volatility has a maturity of one year, so this number may well be more variable than the actual expected volatility over the maturity of the debt. We assume that investors believe that equity volatility will converge linearly over the maturity $T$ from that implied from option data over the next year to a historical volatility averaged over the same sample of firms. To estimate the average historical volatility we use weekly data and a ten-year moving window. This means that the large day-to-day movements seen in implied volatility will be damped, while investors' views about the 'long-run' level of volatility can change. The assumption of linear convergence of implied volatility to the average historical volatility is *ad hoc*, but it is not possible to empirically assess the time-series properties of one-year equity implied volatilities for individual firms over any significant time period. The result of our linear convergence assumption is that the average equity volatility used as

(12) Codes UC00, C0A0 and H0A0 on the Merrill Lynch website or Bloomberg page IND.

an input to the model is an half way in between the average option implied volatility and the average[13] historical volatility.

Unfortunately, equity implied volatility data are not available for all the firms included in the aggregate Merrill Lynch indices. We collect the firm-specific information for all firms in the UK investment-grade index, the 300 largest issuers in the US investment-grade index and the 100 largest issuers in the US high-yield index, for which data is available.[14]

In particular, as of November 2003, the sample of the firms for which equity volatility data are available covers (in terms of capitalisation) 85% of the US investment-grade index, 62% of the UK investment-grade index, and an even smaller proportion for the US high-yield index. An important issue for our exercise is whether less than 100% coverage introduces biases in the calculation of the average equity volatility used as input in the Leland and Toft model. The first indication is by looking at the size of the average equity volatility (based on the firm-specific data in Exercise 2) relative to that derived by the model calibration in Exercise 1. There appears to be a bias in the case of US high-yield for which we provide more details later in this section. For investment-grade in both the United Kingdom and United States, the average equity volatility used as an input to the model, has been on average since 1997 close to 35%, the value found in Exercise 1. The second indication comes from examining two different samples of up to 100 and up to 300 firms in the case of US investment-grade companies, depending on availability at each point in time. The time series of the average equity volatility for these two different samples are almost identical, indicating that there is no evidence of any significant bias in the mean equity volatility due to sample selection.

To transform equity volatility to asset volatility we use the methodology described in Hull (2000), which is based on the Merton (1974) model. It provides a pair of simultaneous equations that we solve numerically for $\sigma$. The Leland and Toft (1996) model is not equivalent to the Merton model so this transformation is an approximation. These formulae are shown in Appendix 2.

The decision on whether to include time-varying leverage or a constant historical average is a difficult one. Theoretically, the appropriate figure in the model is the expected leverage over the next $T$ years. If firms have a target leverage or stationary capital structure, a historical average of leverage would be a good proxy for expected long-run leverage in the future. Although a firm with higher leverage must be, *ceteris paribus*, closer to default, firms can adjust their payout ratio over

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(13) The average over different firms is calculated using weights based on each firm's debt capitalisation in Merrill Lynch indices.

(14) Merrill Lynch indices are rebalanced monthly, meaning that the bonds included in the index can change if they no longer fulfil the criteria for entry, or new bonds are issued. Since the index rebalancing takes place on the last day of the month and we use end-month data, this is not an issue for our analysis. We rebalance each year the sample of the firms for which we collect equity implied volatility data. In practice the composition of the indices tends to change slowly over time, therefore any discrepancy should not be large.

time to return their leverage close to their target. The ratings agencies claim to look through the cycle, and therefore any significant changes in leverage that are not coupled with a ratings action could be assumed to be temporary. Further, a practical problem with using time-varying leverage is that most companies report their accounts quarterly in the United States (and less frequently in the United Kingdom). The data are therefore not available fully contemporaneously. We therefore choose to assume constant leverage over time as a base case, but historical data allow us to see the effects of including time-varying leverage figures and we show that below.

The cost of equity is the sum of the risk-free rate and the equity risk premium. The latter is measured by the residual of a three-stage dividend discount model (DDM), which uses long-term earnings per share growth expectations from Institutional Brokers' Estimate System (IBES) analysts' forecasts for S&P 500 and FTSE 100.[(15)] That is, the equity risk premium is derived by matching the observed value of the index with that implied by the DDM, given the current IBES long-term earnings projections of the index, its dividend yield and the risk-free rate (proxied by the ten-year zero coupon yield). The advantage of the three-stage relative to the one-stage DDM is that the three-stage DDM allows us to derive a forward-looking measure for the equity risk premium by quantifying the effect of analysts' earnings projections. In particular, earnings growth is expected to grow according to analysts' long-term forecasts (ie over a business cycle period specified to be between three to five years by IBES) for the first four years and then revert towards an equilibrium value that is specified by the condition that return on equity is equal to the cost of equity. This equilibrium condition excludes abnormal earnings in the long run, that is, it assumes that in the long run the firm is unable to generate profits at a rate that is higher than that corresponding to the 'perceived' (by the market) riskiness of its investment projects.

Most of the firms in the S&P 500 and FTSE 100 indices are investment-grade firms. Therefore, the cost of equity implied by the three-stage DDM is a good proxy for the cost of equity of the average firm in the Merrill Lynch investment-grade indices. As explained in the previous section, the equity risk premium does not affect the compensation for credit risk as this is priced under the risk-neutral probability measure. The effect of the equity risk premium is to determine the amount of the compensation for credit risk that is due to expected default loss and that due to credit risk premium (compensation for the uncertainty about the default probability and/or the recovery rate). That is, at each point in time, we use the Leland and Toft (1996) model and forward-looking information on the equity risk premium and equity volatility to generate the two components of the compensation for credit risk: the compensation due to expected default and the credit risk premium.

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[(15)] See Panigirtzoglou and Scammell (2002). Their estimate of an equity risk premium of about 4%-6% for the S&P 500 in the 1990s is consistent with the historical estimate of 6.3% obtained by fitting historical default frequencies of investment-grade US companies since 1970.

In particular, to quantify the expected default component or 'actuarial spread', we set the asset risk premium $\lambda$ to its value under the 'subjective' probability measure (that is, the value of the firm's assets drifts at a rate $r + \lambda$ that is higher than the risk-free rate *r*). At the same time the discount rate is equal to the risk-free rate; that is, the discount rate corresponds to that of a risk-neutral investor that doesn't require compensation for risk. In this way we can isolate the part of the compensation for credit risk that is due to expected default under the 'real-world' or 'subjective' probability measure, but exclude the compensation for uncertainty that 'real-world' or risk-averse investors require. This decomposition is explained in more detail in Appendix 2.

The credit risk component can be different to the observed credit spreads due to the presence of liquidity or other non-credit risk related factors in the observed credit spreads. We call this difference the 'non-credit risk component'.

**Results**

Charts 3 and 4 show the base case for US and UK investment-grade by keeping leverage constant.[16] The risk-neutral spread is that which investors would require should they only desire compensation for expected loss. It is clear that the movements in both actual spreads and the compensation for credit risk we calculate are correlated. The correlation coefficients are 0.87 for the United States and 0.75 for the United Kingdom. There is a marked increase in both the observed and model-based spreads after the LTCM crisis in 1998. This is because the equity implied volatilities increased substantially in the LTCM crisis. As explained in Appendix 2, the main determinant of the model-based spread is equity volatility; the asset or equity risk premium is not needed as a risk-neutral valuation method is used in its calculation. Therefore, the compensation for credit risk derived by the model and the equity volatility used are very highly correlated. Charts 3 and 4 also show that there are episodes where the credit risk component and the non-credit risk component move in opposite directions. These episodes correspond to cases where the average equity volatility (the main determinant of the credit risk component) moved in the opposite direction to observed spreads.

The correlation between the US and UK investment-grade spread components that are driven by fundamental credit risk factors is higher than the correlation between the components driven by non-credit risk factors. That is liquidity, tax or regulatory effects are more country specific.

We use the same leverage of 41.6% for both US and UK investment-grade firms. It is possible for the United Kingdom that the long-run average leverage level is lower than that of the United States,

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(16) The United Kingdom is parameterised with a higher debt maturity $T = 20$, reflecting the higher average maturity historically, and the effective tax advantage of debt is set to $\tau$ =30% reflecting the non-differentiation between tax on interest and dividend income in the United Kingdom.

which would have the affect of reducing the two solid lines in Chart 4. For instance, the ONS capital gearing based on market values averaged 19% from 1970-2002. However, using firm-specific data, the average gearing (defined slightly differently to the ONS measure) in 2001 of those firms who were in the bond index was 18% higher than a sample of all listed UK firms. This back-of-the-envelope calculation would therefore suggest an average leverage of around 37%.

**Chart 3**

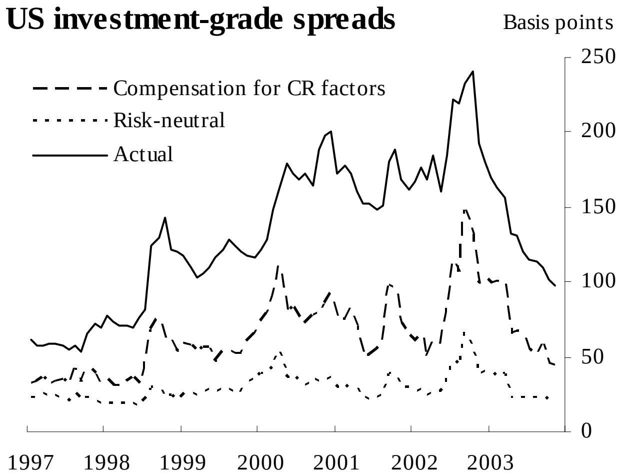
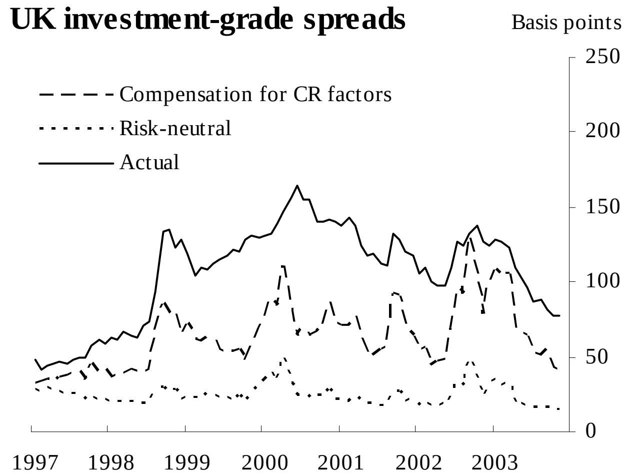



**Chart 4**




It is interesting to examine the proportions of the spread that are attributed to different factors using this model (Charts 5 and 6). There are two possible hypotheses here. One is that the spread attributable to non-credit risk factors (Charts 5 and 6) is constant through time. The other is that it is a constant proportion of the total spread. The above charts show that the second hypothesis is more likely. The additional premium attributable to non-credit risk factors, eg liquidity, is roughly proportional to the credit risk component. This is consistent with the stylised fact of bid-ask spreads increasing for riskier debt issuers.(17) In mid-2002 there was a marked increase in equity volatility that accompanied the equity price falls. But actual spreads did not rise as much. That is, the model attributes this to a significant fall of the proportion of the non-credit risk component.

(17) For a more detailed description of this fact in the credit default swap market see Blanco, Brennan and Marsh (2004).

**Chart 5**

**Proportions of US investment-grade credit spreads attributed to different factors**

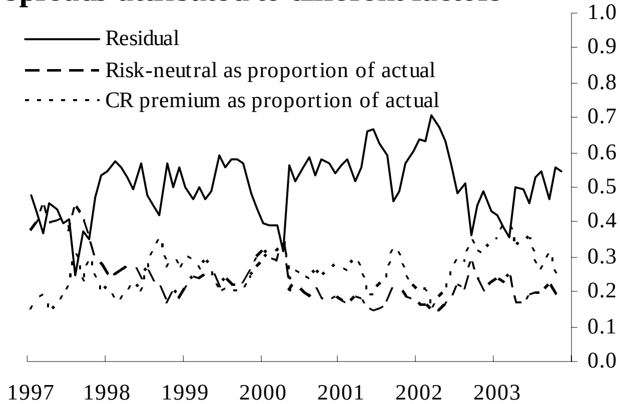
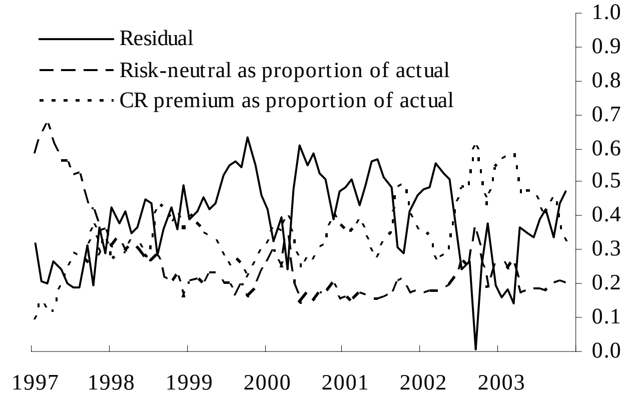



**Chart 6**

**Proportions of UK investment-grade credit spreads attributed to different factors**




Examining the residuals from the model, we can see how they compare to some other financial variables that we would expect to be related. For instance, Charts 7 and 8 show there appears to be some relationship between the residual and the maturity matched swap spread in both the United Kingdom and United States. Most studies have found that a small proportion of variation in the swap spread is due to credit risk.(18) This supports the argument that the residual component in our decomposition is related to non-credit risk factors. These factors can include not only differences in liquidity between the corporate bond and government bond market but also regulatory or tax differences. For example the Minimum Funding Requirement in the United Kingdom put substantial downward pressure on government bond yields between 1998-2001, increasing at the same time UK swap and corporate spreads.

A corporate issuer can issue debt in different currencies and can use currency and interest rate swaps to achieve the desired payment structure; eg floating rate payments in the domestic currency. The cheapest funding corresponds to the currency, which offers the smallest spread of the coupon the company would have to pay on a par bond in this currency, over the respective swap rate.(19) The spread over government yields does not affect the company's decision on which currency to issue. Interest rate swaps are therefore considered to be more appropriate benchmarks for corporate debt than government yields. Charts 7 and 8 further support this. They imply that measuring investment-grade corporate spreads with respect to the swap rate curve as opposed to the government

(18) For example, Longstaff, Liu and Mandell (2002) find that most of the variation in swap spreads is due to changes in the liquidity of Treasury bonds rather than to changes in the default risk associated with the swap curve. Huang (2004) states that the prevailing view among swap traders is that the swap spread serves mainly as an indicator of market liquidity.

(19) For a more detailed description of funding options see Scholtes (2000).

bond yield curve, will result in a residual component (non-credit risk component) that is much smaller.

**Chart 7**

**Comparison of residual with US 5-year swap spread**

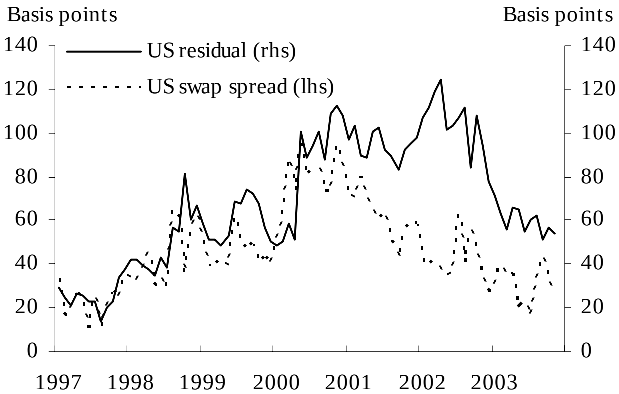
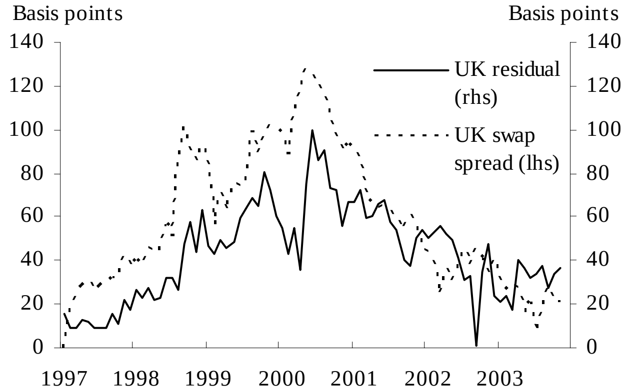



**Chart 8**

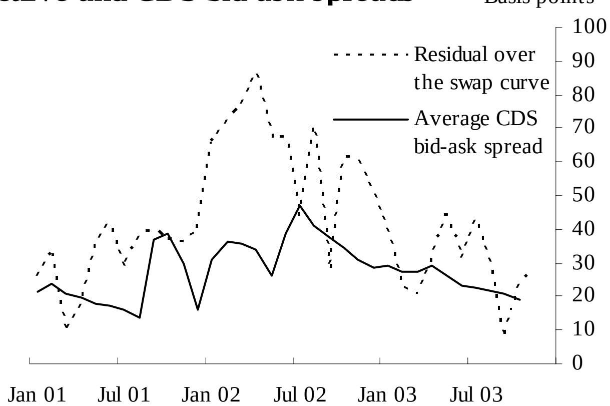
**Comparison of residual with UK 10-year swap spread**




The residual that remains once credit spreads are measured with respect to the swap curve is shown in Chart 9. The size of this remaining residual is comparable to the bid-ask spread of credit default swaps (CDS) of a sample of US investment-grade companies since 2001.[20] This is consistent with liquidity being the driver of this component.

**Chart 9**

**Comparison of residual over the swap curve and CDS bid-ask spreads**

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[20] This sample includes around 50 benchmark quotes for US investment-grade CDS traded with CreditTrade (inter dealer broker).

As expected, there is a strong relationship since the Russian default and LTCM in 1998, between credit risk premia from this model and equity risk premia from the three-stage DDM (Charts 10 and 11). This strong relationship is the result of the fact the equity risk premium is an important driver of the asset risk premium. These charts show that perceptions of risk in the bond market changed significantly following these events. The credit risk premium implied by the model as a proportion of the assumed equity risk premium, increased by a factor of two from about 1/20 (from 1997 to Aug. 1998) to about 1/10 (from Sep. 1998 to 2003), for both UK and US investment-grade bonds.

To justify this, consider a long position in a corporate bond and a short position in a corresponding government bond. In a standard CAPM framework the credit risk premium as a proportion of the market risk premium would depend on the ratio of the volatility of this position (which is closely related to spread volatility) to the market volatility, and the correlation between this portfolio's returns and market returns. The ratio of the volatilities did not change significantly in the period considered although volatility increased in both markets. However, the return correlation increased by a factor of about three for US investment-grade (using the S&P 500 as a rough market proxy). This number is reasonably similar to the factor of two mentioned above.

The correlation between the credit risk premia that are reported in Charts 10 and 11 is very high (0.96). It is higher than the correlation between the assumed equity risk premia (although this correlation is also high; 0.80). This suggests that perceptions of risk are driven more by a common factor in the case of corporate debt, than equities.

**Chart 10**

**Comparison of US credit risk premium with S&P 500 risk premium from 3-stage DDM**

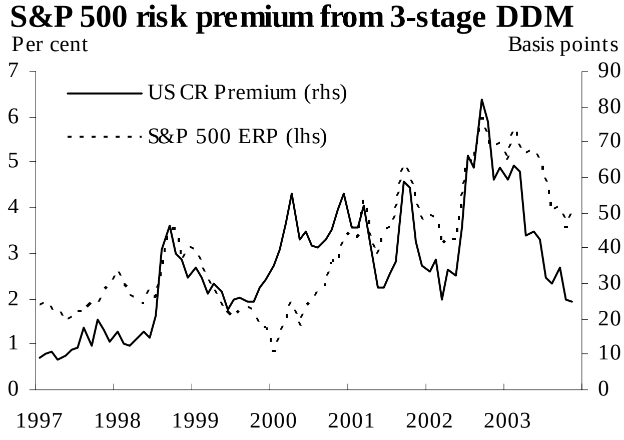
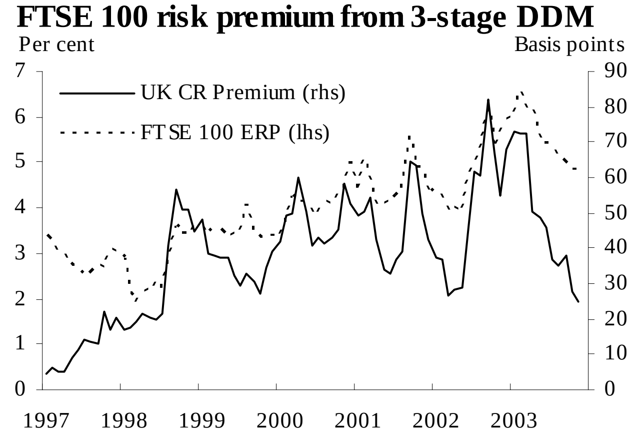



**Chart 11**

**Comparison of UK credit risk premium with FTSE 100 risk premium from 3-stage DDM**




We can now examine the sensitivity of the model to including time-varying leverage. We have data for leverage up to 2002 Q4 for the United States, on a quarterly basis. We interpolate between figures to generate the monthly series. Chart 12 shows the results from the model if investors assume that the current leverage is the long-run leverage rate, in Chart 13 investors assume that the long-run leverage rate is half way between the current leverage and the historical average of 41.6. The main difference between these and the base case is to generate less default risk while the stock markets were high. It can be seen that the leverage does not significantly alter the shape seen in the charts, as leverage is a less volatile series than the equity volatility.

**Chart 12**

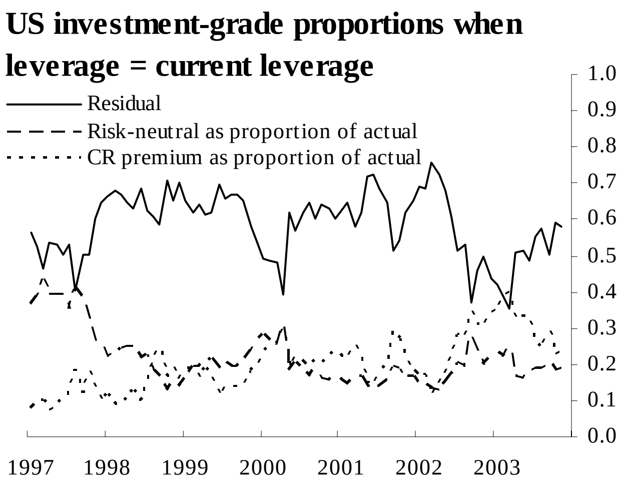
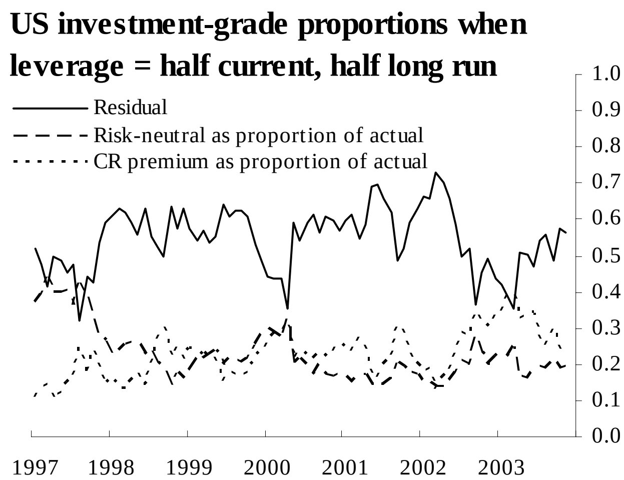



**Chart 13**




Generating a high-yield spread series since 1997 using our model is a more formidable task, mainly because of difficulties obtaining all the firm-specific data. We only try to fit these spreads for the dollar index, as the sterling high-yield market is very small. The data available to us are more limited; many firms were previously investment-grade, or are recent start-ups. Unlike the investment-grade series, we therefore have to check the rating history of all the firms, and only use data from when they were high-yield.[21] This reduces the sample size significantly, and there is possibly a bias towards larger companies, at the higher rated end of the high-yield spectrum. The equity volatilities in our series are not as high as the volatility suggested by fitting the model to historical default data. Again, this is possibly due to a bias towards larger higher rated companies for which option contracts are more likely to exist and to be traded. Therefore we scale up the equity volatilities so that the average in our model is equal to 78%, that implied by fitting the model to historical default frequencies. We also scale the cost of equity upward to account for the implied difference in equity risk premia between investment-grade and high-yield companies found when fitting the historical default frequencies. Recall that the difference between investment-grade and high-yield average historical risk premium was around 5%. We therefore scale up the cost of equity for high-yield firms by 5%. The results are shown in Chart 14.

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[21] It would be more accurate to do this for the investment-grade index too, but the numbers of firms it would affect is very small relative to the whole sample available.

Again the shape is fitted reasonably well, and, interestingly, the model implies that actual spreads may have been almost equal to the credit risk component before LTCM. After the LTCM liquidity crisis, investors were prepared to pay a higher price for liquidity, which is reflected in the increase of the non-credit risk component. As with US investment-grade bonds, the pattern of the residual is related to that of swap spreads. However, as Chart 15 suggests, the size of the residual is on average about three times larger than the swap spread. This suggests that liquidity factors play a more important role in high-yield spreads, in line with the evidence of a positive relation between credit risk and bid ask spreads in the secondary bond and CDS markets. The average bid-ask spread of CDS for a sample of US high-yield firms since 2001 is significantly higher than that for the US investment-grade firms shown in Chart 9. The average was 104 for the high-yield CDS quotes, compared to 26 for investment-grade. This is consistent with a much larger non-credit risk component for high-yield spreads over the swap curve.(22)

**Chart 14**

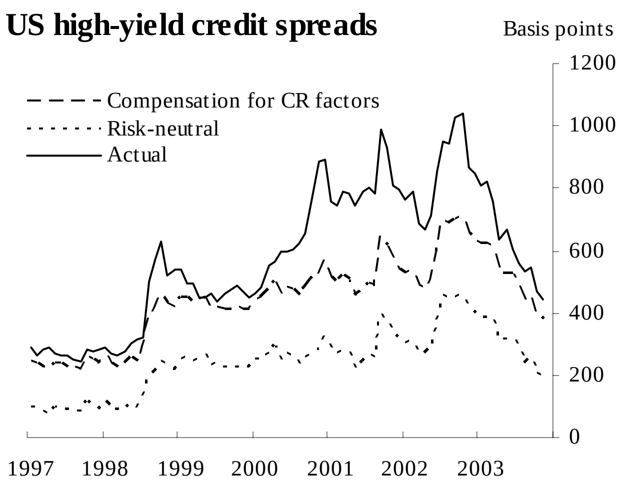
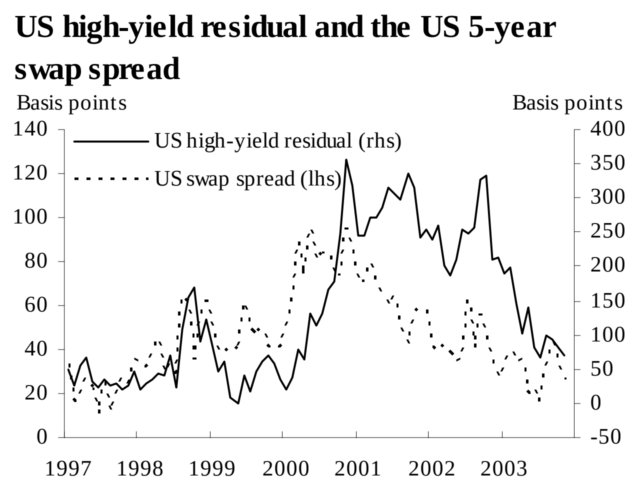



**Chart 15**




As with the investment-grade case, there is a positive relationship between the credit risk premium and the assumed equity risk premium (Chart 16). This chart shows that the credit risk premium implied by the model as a proportion of the assumed equity risk premium, did not change significantly after the LTCM crisis.

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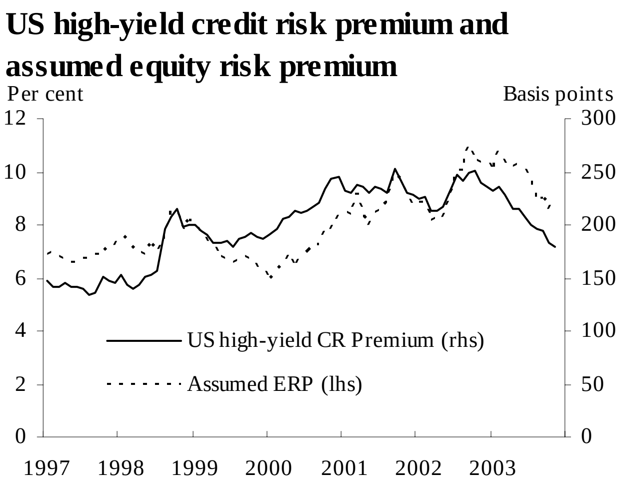
(22) The average bid-ask spread for high-yield firms is based on a very small sample of quotes for each day, so we do not show a time series.

**Chart 16**




## 5 Conclusion

The paper examines the implications of the Leland and Toft (1996) credit risk model for US and UK credit spreads. As in Leland (2002), we examine the fit of the model to the historical default experience of US firms. The model provides a good fit at all except the shortest horizons. The equity risk premium and equity volatility numbers resulting from the calibration process, are reasonable and consistent with other independent estimates.

We also use contemporaneous forward-looking information for the equity risk premium and equity uncertainty to decompose the observed credit spreads since 1997. The equity risk premium is derived from a three-stage dividend discount model that incorporates analysts' long-term earnings forecasts. The measure of equity uncertainty is derived from option prices. The part of the spread due to fundamental credit factors looks reasonable and increased substantially since the LTCM and Russian default crisis of 1998. The patterns are similar across different assumptions about leverage. As not all of the spread is due to credit risk factors, it is important not to simply take spreads at face value when analysing UK credit risk. This is particularly important for investment-grade firms, which make up the vast majority of issuers in the sterling corporate bond markets.

The non-credit risk component, attributed to liquidity, regulatory or tax effects, increases as the credit risk component increases, consistent with the empirical evidence of higher bid-ask spreads for lower quality credits. This component is closely related to swap spreads for investment-grade companies, in line with the existing literature that finds that a small proportion of swap spreads is due to credit risk. Moreover, it provides justification for the use of the swap curve as a fixed income benchmark for corporate debt. For high-yield companies, the non-credit risk component is much higher than swap spreads, suggesting a greater importance of liquidity for these lower quality corporates. In the second half of 2002, we observe small movements in the UK investment-grade credit spreads compared to the United States. Our decomposition implies that this is due to the combination of a reduction in compensation for non-credit risk factors in the United Kingdom and greater credit risk in the United States.

Another result of our decomposition is that the credit risk premium component of the credit spread as a proportion of the assumed equity risk premium, increased by a factor of two for investment-grade. We can explain this increase qualitatively, using a standard CAPM framework, by the correlation between equity market (a proxy for the market portfolio) returns and credit returns.

Data availability though restricts the available history of the decomposition. As more historical data become available we would be able to test the predictive ability with respect to both default rates and future growth.

**Appendix 1**

Default asset level in Leland and Toft (1996) model:

The default boundary is time independent in the Leland and Toft (1996) model and is given by the following equations:

$$V_B(C,T,\alpha,\delta,\sigma,\tau,P,r) = \frac{\frac{C}{r}\frac{A}{rT} - B - \frac{AP}{rT} - \frac{\tau C x}{r}}{1+\alpha x-(1-\alpha)B} \quad \textbf{(A1.1)}$$

where

$$A = 2ae^{-rT}N[a\sigma\sqrt{T}] - 2zN[z\sigma\sqrt{T}] - \frac{2}{\sigma\sqrt{T}}n[z\sigma\sqrt{T}] + \frac{2e^{-rT}}{\sigma\sqrt{T}}n[a\sigma\sqrt{T}] + z - a,$$

$$B = -(2z + \frac{2}{z\sigma^2 T})N[z\sigma\sqrt{T}] - \frac{2}{\sigma\sqrt{T}}n[z\sigma\sqrt{T}] + z - a + \frac{1}{z\sigma^2 T},$$

$$a = \frac{(r-\delta-(\sigma^2/2))}{\sigma^2},$$

$$z = \frac{((a\sigma^2)^2 + 2r\sigma^2)^{\frac{1}{2}}}{\sigma^2},$$

$$x = a + z$$

and where $r$ is the risk-free rate, $\sigma$ is the asset volatility, $T$ is the debt maturity, $\alpha$ is bankruptcy costs, $P$ is the leverage, $\tau$ is the effective tax advantage of debt, $C$ is the coupon yield, $\delta$ is the payout ratio.

**Appendix 2**

Calculating the credit risk component of the spread:

The value of a coupon bond with semi-annual coupon payments *K* is given by the following formula:

$$Value = \sum_{t=1}^{T} \frac{K}{2} e^{-rt} (1-(1-R)EDF_t(b,\mu,\delta,\sigma)) + \frac{1+K}{2} e^{-rT} (1-(1-R)EDF_T(b,\mu,\delta,\sigma)) \qquad \textbf{(A2.1)}$$

where $r$ is the risk-free rate, $\lambda$ is the asset risk premium which is set to zero (risk-neutral valuation as explained below), $\mu = r + \lambda$ is the drift of the firm's asset value, $\sigma$ is the asset volatility, $T$ is the debt maturity, $\alpha$ is bankruptcy costs, $P$ is the leverage, $\tau$ is the effective tax advantage of debt, $C = \frac{P(r + spread)}{V_0}$ is the coupon yield, $\delta = C + \frac{dividend\ yield(100-P)}{V_0}$ is the payout ratio, $V_0$ is the initial value of the firm's assets which is set to 100, $V_B(C,T,\alpha,\delta,\sigma,\tau,P,r)$ is the default trigger asset level given by equation **(A1.1)**, $R = \frac{(1-\alpha)\cdot V_B}{P}$ is the recovery rate and $EDF_t(b,\mu,\delta,\sigma)$ is the expected default frequency at horizon *t* given by equation **(2)**.

The coupon bond value in equation **(A2.1)** can be calculated under the risk-neutral probability measure, that is we set $\lambda = 0$ and at the same time all future coupon payments are discounted using the risk-free rate $r$, that is, the discount rate of a hypothetical risk-neutral investor. The par coupon spread is the value of *K* that makes the bond value equal to 1 (par value). We solve this simple optimisation using a Newton-Raphson method. The derived coupon value *K* is annualised using the formula $K_{annual} = (1+\frac{K}{2})^2 - 1$. The credit risk component (the sum of the compensation due to expected default and the credit risk premium) is then equal to $K_{annual} - r$. Because a risk-neutral valuation method can be used, the asset risk premium $\lambda$ is not needed in the calculation of the credit risk component, that is, it depends on asset volatility $\sigma$ and the other structural parameters of the model.

To decompose the compensation for credit risk into its two components, the compensation due to expected default and the credit risk premium, we need to use an estimate of the asset risk premium $\lambda$. In particular, to quantify the first component, we use equation **(A2.1)** and set $\lambda$ to its value under the 'subjective' probability measure rather than zero (that is, the value of the firm's assets drifts at a rate

$r + \lambda$ that is higher than the risk-free rate). At the same time the discount rate is equal to the risk-free rate, that is, it corresponds to that of a risk-neutral investor that does not require compensation for risk. In this way we can isolate the part of the compensation for credit risk that is due to expected default under the 'real-world' or 'subjective' probability measure but excludes the compensation for risk that 'real-world' or risk-averse investors require. The first component, the compensation due to expected default or 'actuarial' spread, is always lower than the total compensation for credit risk, as the probability of default, given by equation **(2)**, is lower under the 'subjective' probability measure than under the risk-neutral measure. This is because the value of the firm's assets drifts at a higher rate $r + \lambda$ under the 'subjective' probability measure and it is therefore more likely to finish above the default boundary. To quantify the second component, the credit risk premium, we simply subtract the compensation due to expected default from the total compensation for credit risk.

**Appendix 3**

Solving Merton's model for asset volatility:

We assume a firm with only one issue of zero-coupon debt with maturity $T$. The firm can only default at time $T$. Default is determined by the positive net worth covenant, that is the firm defaults when it's asset level at maturity $V_T$ is less than debt principal $P$. Therefore, in the Merton model the value of the equity is a call option on the firm's asset value with strike price equal to the face value of debt. The pay-off at maturity is $E_T = \max(V_T - P, 0)$. Using the Black-Scholes formula gives the value of the equity today as

$$E_0 = V_0 N(d_1) - P\exp(-rT)N(d_2) \qquad \textbf{(A3.1)}$$

where $d_1 = \left(\frac{\ln(V_0/P) + (r + \sigma_V^2/2)T}{\sigma_V\sqrt{T}}\right)$, $d_2 = d_1 - \sigma_V\sqrt{T}$, $\sigma_V$ is the asset volatility and $N$ is the cumulative distribution function of the normal distribution. From Ito's lemma we know that

$$\sigma_E E_0 = \frac{\partial E}{\partial V}\sigma_V V_0 \Leftrightarrow \sigma_E E_0 = N(d_1)\sigma_V V_0 \qquad \textbf{(A3.2)}$$

where $\sigma_E$ is the equity volatility.

It is possible to solve **(A3.1)** and **(A3.2)** simultaneously for $V_0$ and $\sigma_V$. As we already set $V_0$ = 100 in our model, we only solve for $\sigma_V$. Similarly, we can solve for $\sigma_E$ if we know $\sigma_V$ and leverage $\frac{P}{V_0}$.

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