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Modelling manufacturing inventories

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Abstract

This paper presents and applies a stage-of-fabrication inventory model to the UK manufacturing sector. The model emphasises the interaction between input (raw materials and work-in-process) and output (finished goods) inventories. This interaction is an important empirical regularity and proves critical for the ability of the model to fit the data. Decisions about input and output inventory investment cannot be considered in isolation from each other, but must be analysed jointly. Overall, the stage-of-fabrication model receives considerable support. Maximum likelihood estimation of the model's decision rules yields correctly signed and significant parameter estimates. In terms of producer behaviour, the results imply rising marginal costs of production and significant costs of adjusting production.

Key words: Inventories, maximum likelihood, manufacturing.

JEL classification: E22.

Summary

Changes in the stock of firms' inventories are an important component of the business cycle. In fact, discussion about the timing of a recovery following economic recessions often focuses on inventories. But there is no consistent explanation for their behaviour. Most modelling work focuses on the so-called production smoothing model–where the firm maintains a smooth production plan and uses inventories to satisfy unforeseen changes in demand. Moreover, this model has generally only been applied to manufacturers' finished goods inventories.

This paper offers an extended stage-of-fabrication inventory model that considers not only finished goods inventories, but also input inventories–the sum of raw materials and work-in-process inventories. Stylised facts for UK manufacturing reveal that input inventories are empirically more important than finished goods inventories. This is true not only in terms of their size but also in terms of their volatility.

One of the key facts of the UK manufacturing sector is the significant interaction between finished goods and input inventories. The covariance between input and finished goods (or output) inventories can explain over one quarter of the variance in manufacturing inventory investment. This is an important finding because it points to linkages between different aspects of production. More importantly, it implies that finished goods inventories cannot be considered in isolation from input inventories. Intuitively, an optimising firm that decides to draw down finished goods inventories (as often happens following an unexpected demand shock) will typically increase production in the future to correct this imbalance. This correction will affect input inventories as well because the firm has to draw down input inventories in order to increase production.

The paper demonstrates that ignoring input inventories yields misleading results. In particular, the precision and plausibility (relative magnitudes) of the estimated parameters in the joint model differs from those when input inventories are ignored. To estimate the model, a maximum likelihood approach is used that is shown to be superior to the often-used generalised method of moments estimators (GMM). The sizable interaction between input and finished goods inventories yields very precise estimates. One of the key findings of the model is the familiar production-smoothing result. The estimation results suggest that firms satisfy unexpectedly strong demand from finished goods inventories, resulting in the latter falling below companies' desired levels. Given that estimated costs of changing production are large relative to stockout costs (of deviations of inventories

from their desired level), this imbalance corrects rather slowly, implying that inventories deviate from target for long periods. On the cost side of the model, when materials become more expensive companies prefer to cut production temporarily: cutting production implies that companies save on the expensive materials. With sales unchanged, this shortfall is satisfied out of finished goods inventories causing them to fall from their desired level. Moreover, despite the presence of input inventories–where fixed costs of ordering may be substantial–the estimated aggregate marginal cost function is a rising function of output, thus implying decreasing returns to scale in manufacturing.

1 Introduction

Inventories—ie, the stocks of finished goods, raw materials, and work-in-process—have been the subject of extensive economic analysis on many fronts.[(1)] The importance of and subsequent interest in inventory behaviour was stimulated by Metzler (1941) who demonstrated that an inventory accelerator mechanism can produce cycles in simple Keynesian models. The introduction of the production-smoothing inventory model of Holt, Modigliani, Muth and Simon (1960), has spurred a considerable theoretical and empirical literature that has used different versions to explain and test the microeconomic inventory behaviour of the firm. The literature, as summarised by Blinder and Maccini (1991) has almost entirely focused on the analysis of manufacturers' finished goods (output) inventories. As Blinder and Maccini (1991) put it 'Much of the research is thus applying a model that fits the data only with difficulty to a component of inventories that is relatively unimportant.' The attention to output inventories has 'crowded out' consideration of the empirically dominant component of input inventories.[(2)] Recently however, input inventories—defined as the sum of raw materials and work-in-process—have generated some interest. Ramey (1989) developed an optimising model of inventories at different stages of the production process, but treated the stocks of materials, work-in-process and finished goods as factors of production in order to derive demand functions for different types of inventories. Conceptually this approach is not consistent with the micro-theory of the firm unless one is willing to accept that the flow out of inventories into production is proportional to the stock of inventories. In other words the stock-flow aspect of inventory holding behaviour must be carefully distinguished in an optimising inventory model. At the firm level the decision to order and use materials in production is inherently different from the benefits and costs of holding stocks of inventories. In a recent paper that addresses this important issue, Humphreys, Maccini, and Schuh (2001) analyse and estimate an inventory model for the US manufacturing sector incorporating the empirically important stock of input inventories. Their results suggest that a model of joint determination of input and output inventories is critical for fitting the data.

There are theoretical and empirical arguments against ignoring input inventories. Humphreys, Maccini and Schuh (2001) describe a theoretical consideration. 'Input inventories are the linchpin of the stage-of-fabrication production process. Input inventories arise whenever the delivery and usage of materials differ; in other words, firms do not instantaneously obtain and use materials in production. Furthermore, since the usage of input materials is a factor of production, decisions about production smoothing

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(1) The terms inventories and stocks will be used interchangeably.
(2) In manufacturing, input inventories are larger and more volatile than output inventories.

and output inventory investment are inherently related to decisions about drawing down input inventories. In turn, the ability of the firm to draw down input inventories depends on supplier relationships, materials prices, and factor substitutability–ie the spectrum of production decision making.' Also, input inventories are empirically more important than output inventories. Stylised facts for the United Kingdom indicate that input inventories are larger and more volatile than output inventories in manufacturing.[3]

This paper attempts to shed light on the inventory behaviour of the UK manufacturing sector. The focus is on manufacturing because the data indicate that this sector is a large contributor to inventory movements over the business cycle. In addition, input inventories arise exclusively in this sector. For the reasons outlined above the paper utilises an approach that emphasises the joint treatment of input and output inventories. The idea is not novel. The methodology for this paper is similar to Humphreys *et al* (2001), henceforth referred to as HMS (2001). Their study recognises the serious limitation of the applied inventory literature and develops a stage-of-fabrication (SOF) model that includes separate decisions to order and use input inventories. The stage-of-fabrication (SOF) model extends the linear quadratic model of output inventories by adding the joint determination of input inventories.[4]

By and large, the most striking feature of the UK manufacturing inventory data is the strong co-movement between input and output stocks. As much as 26.7% of the total inventory volatility can be accounted for by the covariance between input and output inventories. This finding points to considerable interaction between the two types of inventory stocks and proves to be critical to the success of the model fitting the data. By comparison to the HMS (2001) study, the extent of the inventory interaction found in UK data is considerably larger than that found in US data. The strength of the co-movement is reflected in very precise estimates of the SOF model.

The econometric results provide substantial support for the SOF model. First, the data clearly reveal evidence of stage-of-fabrication interactions between inventory stocks, and among inventory stocks and other facets of production. Second, the data indicate that the aggregate cost function in manufacturing is convex; ie marginal cost slopes upward. Third, the data show evidence of inventory saving technology.

The rest of the paper proceeds as follows. Section 2 presents UK inventory stylised facts.

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(3) A finding that is consistent with US studies.

(4) The linear quadratic output inventory model was developed by Holt *et al* (1960). It has since served as the workhorse for empirical inventory work. Stylised versions can be found in West (1993) and Ramey and West (1999).

Section 3 describes the model. Section 4 reports and discusses the econometric estimates. Section 5 examines dynamic properties and Section 6 concludes.

2 Stylised facts and key interactions

This section lays out basic UK inventory facts. There are three key empirical regularities one can identify in UK inventory data.

1. UK inventories are procyclical.
2. Manufacturing inventories account for a large share of total inventory volatility.
3. Within manufacturing, input inventories are more volatile and interact considerably with output inventories.

One important reason to study inventories is their crucial role in the business cycle. Table A below illustrates the importance of inventories in UK recessions. Table A columns 2 and 3 report contributions of inventory investment (for whole-economy and manufacturing stocks respectively) as a percentage of the decline in GDP during post-war UK recessions.

Table A: Arithmetical importance of inventory changes in UK recessions

Peak quarter Trough quarter*	Total	Manufacturing
1973:2–1974:1	46†	37
1975:1–1975:3	-29	-120
1979:4–1981:1	39	7
1990:2–1991:3	67	13

* Recessions are defined as periods accounting for at least two consecutive quarters of real GDP decline.

†The numerator of the ratio is defined as the inventory investment contribution while the denominator is defined as GDP in trough quarter minus GDP in peak quarter and is by definition negative. A negative entry means that the change in the inventory change is positive.

Table A establishes that inventory liquidation is an important feature of the UK business cycle. Excluding the 1975 recession (where inventories moved countercyclically), all other episodes are associated with stock liquidation. For example, in the recession of the early 1990s (1990:2–1991:3) inventory investment accounts for a tiny 0.7 of a per cent of GDP,[5] but its contribution to the decline in GDP is orders of magnitude higher (67%) and

[5] It is worth emphasising that whereas inventory investment enters the GDP flow accounting identity, the size of inventories as a stock relative to GDP is much higher, around 60% of GDP.

even higher than the contribution of any other expenditure component.[6] Moreover, in other G-7 countries the corresponding contribution of inventories is of similar magnitude.[7] Hence inventories are an important feature of cyclical fluctuations and a potential useful resource in business cycle analysis.

In what follows I present evidence to support the three facts outlined above based on quarterly UK data from 1959 to 2003. Procyclicality of inventory movements can be documented in several ways. A simple indication that inventories move procyclically is a positive correlation between inventory investment and final sales. The correlation between inventory investment and sales is related to the relative variances of production and sales.[8]

Table B: Variance of inventory investment across sectors

Time period	Wholesale (a)	Retail (b)	Manufacturing (c)	Total (d)	$\frac{var(c)}{var(d)}$ $(e)^*$
1959:2–2003:4	0.09	0.06	1.04	1.43	0.72
1975:1–2003:4	0.13	0.07	0.69	1.11	0.62
1994:1–2003:4	0.19	0.04	0.35	0.47	0.72

In column (d), total is the sum of distribution (wholesale and retail) and manufacturing.

Entries in Table B are share weighted.

Table B presents a variance decomposition exercise for the sectors of the economy that hold the majority of inventories. Arithmetically, manufacturing stocks account for 36$\%$, distribution (retail and wholesale trade) for 34$\%$ and ‘other’ industries for 27$\%$ of total stocks in the United Kingdom.[9] As can be seen from Table B, manufacturing accounts for a large portion of the total inventory volatility, ranging from 62$\%$ to 72$\%$ depending on the time period, and this ratio does not seem to be falling despite the reduction in total inventory volatility observed in recent time periods (last row of Table B).

(6) Repeating the same calculation we find that fixed investment contributes positively to the GDP decline by 62$\%$, consumption by 52$\%$, while net exports contribute negatively (ie, countercyclically) by 59$\%$, and government consumption by 33$\%$.

(7) For example, using annual data, we can calculate a contribution of 12$\%$ for the United States, 50$\%$ for Canada, 71$\%$ for France, 19$\%$ for Germany, and 30$\%$ for Italy, in the recession of the early 1990s.

(8) The correlation between inventory investment and final sales in UK data is 0.05. Letting Y denote production, X sales and N inventories and using the identity $var(Y) = var(X) + var(\Delta N)$ $+2cov(X, \Delta N)$, the positive correlation implies $var(Y) > var(X)$. Thus inventories seem to amplify rather than mute movements in production. This finding is consistent with evidence from a cross-section of countries. See Ramey and West (1999) for an excellent survey.

(9) As of end-2003. ‘Other’ include industries such as energy, construction, motor trades.

Finally, Table C presents a stage-of-fabrication variance decomposition within UK and US manufacturing that documents the importance of input inventories. This exercise highlights the extent of inventory interaction and motivates the joint treatment of input and output inventories. In particular, Table C shows that almost 27% of the variance in UK manufacturing inventory investment can be accounted for by the covariance between input and output inventories. When the inventory stocks are disaggregated by stage of processing, ie, finished goods, work-in-process, and materials and supplies, the covariance terms increase significantly to 44.7% of the total variance. In other words the data indicates a strong interaction between input and output stocks. By contrast the corresponding US numbers are significantly lower (21% and 35.7% respectively). As is explained in detail in Section 4 this strong interaction is reflected in the ability of the SOF model to fit the data. As it turns out when applied to UK data, the SOF model produces more precise parameter estimates relative to the estimates that HMS (2001) obtain using US manufacturing data. This difference can easily be explained in terms of the likelihood function of the model given the data. Because the SOF model relies on stock interaction, the higher covariance terms in UK manufacturing sharpens the likelihood surface, thereby producing more precise estimates.

Table C: Variance decomposition of manufacturing inventory investment

	UK		US	
	Mfg.	(%)	Mfg.	(%)
Total	2	100	0.889	100
Finished goods	0.42	21.3	0.148	16.7
Input	1.04	52	0.554	62.3
Covariance	0.53	26.7	0.185	21
Finished goods	0.42	21.3	0.148	16.7
Work-in-process	0.283	14	0.213	24
Materials and supplies	0.40	20	0.206	23.3
Covariance terms	0.884	44.7	0.318	35.7

*Quarterly 2000 data over the period 1967:2 to 1997:4.

The stylised facts presented in this section suggest that complete analysis of manufacturing inventory behaviour requires the modelling of input inventories.

3 The stage-of-fabrication model

The SOF model developed by HMS (2001), is an extension of the linear-quadratic model of output inventories. The key idea behind this model is to introduce stage-of-fabrication linkages in production; that is, firms produce goods in stages. A typical firm will acquire materials from an outside supplier, and then combine them with capital and labour to produce finished output. Often during the production process the firm generates its own intermediate output as well. In addition, many firms sell their finished output to other firms, which view the output as input material. These stage-of-fabrication linkages—within and between firms—imply that optimising firms will be characterised by joint interaction among all aspects of production, including inventories. The SOF model is fully structural with intertemporal cost minimisation under rational expectations, and the total cost function employs several quadratic approximations similar to those employed in conventional output inventory models.[10]

The rest of this section presents the SOF model in depth.

3.1 Economic environment

The basic set up of the model features a representative firm that minimises costs intertemporally under rational expectations. More specifically, each period, the firm combines labour (L), materials used in production (U), and capital (K) to produce finished goods. Materials used in production are obtained from the on-hand stock of input inventories (M), which is continually replenished by deliveries (D) of materials. Production (Y) of final goods is added to the stock of output inventories (N), which are used to meet final demand (X). The firm takes final demand, the wage rate (W), and the price of material deliveries (V) as exogenous. There are two simplifying assumptions that need to be mentioned at the outset. First, because inventory investment is a short-run decision, it is assumed that the capital stock is fixed.[11] This assumption allows us to suppress capital in the production function. Second, the firm purchases intermediate inputs and materials from outside suppliers.[12] Thus intermediate goods are analogous to raw materials so work-in-process can be lumped together with materials inventories. It has to be emphasised that the latter assumption is perfectly sensible from the point of view of the representative firm and the model's stage of fabrication implications. Whereas in certain 'production to order' industries work-in-process may help smooth production relative to sales fluctuations, 'production to stock' industries mostly rely on finished goods. However, the model is estimated with aggregate manufacturing data which do not allow for a

(10) The main reason for employing quadratic approximations is to obtain linear decision rules that can be estimated by maximum likelihood.

(11) This assumption is clearly invalid in the long run, but for short-run analysis it is a reasonable conditioning assumption.

(12) One can imagine firms producing intermediate output—and thus work-in-process inventories—but that requires us to model production of both intermediate and finished goods, a serious extension of the SOF model.

different treatment of work-in-process inventories. More importantly, the SOF model does not limit production smoothing to finished goods inventories. Due to the existence of strong stage of fabrication linkages input, and hence work-in-process inventories, can act in much the same way; ie, smooth production relative to sales.[13]

The firm optimises in a dynamic stochastic environment. In the short run, with capital fixed, the firm chooses *U*, *M*, and *N* to minimise the present value of total costs, given *V*, *W* and *X*. Six random shocks (ε_i)—one for each equation in the model—buffet the firm's production environment. One shock is a demand shock (ε_x). The other shocks comprise a disaggregation of the traditional 'supply' shock: a technology shock (ε_y) affects the production function; inventory holding cost shocks (ε_{hm}, ε_{hn}) affect the costs of carrying inventory stocks; a real wage shock (ε_w) affects labour costs; and a real raw materials price shock (ε_v) affects material costs.

The model generalises the traditional linear-quadratic model of output inventories. The central extension in the stage-of-fabrication model is the explicit introduction of input inventories, which must be chosen simultaneously with output inventories. Input inventory investment is controlled by varying the usage of materials in production and the deliveries of materials. Total costs which consist of labour costs, inventory holding costs, and delivery costs are approximated with a generalised quadratic form.

3.2 Model equations

3.2.1 Production function

The short-run production function is

$$Y_t = F(L_t, U_t, \varepsilon_{yt}) \qquad \textbf{(1)}$$

where U_t is the flow of materials used in the production process, not the stock of materials inventories. Because Y_t is gross output, equation **(1)** is referred to as the gross production function. Furthermore, we assume that capital is a fixed factor of production with no short-run variation in utilisation.[14] Given this assumption, materials usage and labour

[13] In Appendix C the sensitivity of the model's estimates to this assumption is tested. The results in Table H clearly demonstrate the validity of this assumption.
[14] Larsen, Neiss and Shortall (2002) have examined relaxing this assumption in a productivity measurement study.

possess positive and non-increasing marginal products, and the short-run production function exhibits decreasing returns to scale. An important specification issue for the production function is whether U_t is additively separable from the other factors of production. If U_t is separable, then the production function can be written as

$$Y_t - U_t = G(L_t, \varepsilon_{yt}) \qquad \textbf{(2)}$$

where $Y_t - U_t$ is value added. Note that equation **(2)** is a special case of **(1)** with the restrictions $F_U = 1$ and $F_{LU} = F_{\varepsilon_y U} = 0$. This form is referred to as the value-added production function.

3.2.2 Cost structure

The firm's total cost consists of three major components: labour costs, inventory holding costs, and materials costs.

Labour costs are

$$LC_t = W_t L_t + A(\Delta L_t) \qquad \textbf{(3)}$$

where $\Delta L_t = L_t - L_{t-1}$. The first component is the standard wage bill. The second component, $A(\Delta L_t)$, is a standard adjustment cost function intended to capture the hiring and firing costs associated with changes in labour inputs. To focus on inventory decisions, we eliminate labour input. Inverting the production function yields the labour requirements function

$$L_t = L(Y_t, U_t, \varepsilon_{yt}) \qquad \textbf{(4)}$$

Substituting equation **(4)** into **(3)** yields

$$LC_t = W_t L_t + A(L(Y_t, U_t, \varepsilon_{y,t}) - L(Y_{t-1}, U_{t-1}, \varepsilon_{y,t-1})) \qquad \textbf{(5)}$$

Finally, using a generalised quadratic function approximation yields the following equation for the labour cost function[15]

$$LC_t = (\frac{\gamma_1}{2})Y_t^2 + (\frac{\gamma_2}{2})U_t^2 + \gamma_3 Y_t U_t + W_t[\gamma_4 Y_t + \gamma_5 U_t] + \\ (\frac{\varphi}{2})[\gamma_6 \Delta Y_t + \gamma_7 \Delta U_t]^2 + \varepsilon_{yt}(\gamma_8 Y_t + \gamma_9 U_t) \qquad \textbf{(6)}$$

The properties of the production function and the labour adjustment function imply parametric restrictions on γ_s .[16] Nevertheless, the cost function is extensively over parameterised, and it would be useful to impose some restrictions in order to be able to estimate parameters precisely. The value-added production function specification provides one such set of restrictions.

Value added: To obtain the value-added specification, let

$$\gamma_1 = \gamma_2 = -\gamma_3 = \gamma > 0, \quad \gamma_4 = -\gamma_5 > 0$$

$$\gamma_6 = -\gamma_7 = -\gamma_8 = \gamma_9 = 1$$

These restrictions make value added, $Y_t - U_t$, a factor in the inverted production function, rather than Y_t and U_t separately. Then the value-added (v) cost function is

$$LC_t^v = (\frac{\gamma}{2})(Y_t - U_t)^2 + \gamma_4 W_t(Y_t - U_t) + (\frac{\varphi}{2})(\Delta Y_t - \Delta U_t)^2 - \varepsilon_{yt}(Y_t - U_t) \qquad \textbf{(7)}$$

(15) For example, a quadratic approximation similar to the one used in this paper can be found in Ramey and West (1999).
(16) The restrictions follow from the strict concavity of the production function, and the strict concavity of the labour adjustment function.

Note that the value-added specification imposes adjustment costs on the change in value added, which implies that adjustment costs depend on the change in materials usage (ΔU_t) as well as on the change in gross output (ΔY_t).

The rationale for imposing those restrictions is to achieve a more parsimonious econometric model, while being able to encompass the standard output inventory model as a special case.

3.2.3 Inventory holding costs

The holding costs for output inventories are a quadratic approximation of the form

$$HC_t^N = (\delta_0 + \varepsilon_{hnt})N_t + (\frac{\delta}{2})(N_t - N_t^*)^2 \tag{8}$$

where ε_{hnt} is the white noise innovation to output inventory holding costs, N_t^* is the target level of output inventories that minimises output inventory holding costs, and $\delta > 0$. An analogous formulation is adopted for input inventories

$$HC_t^M = (\tau_0 + \varepsilon_{hmt})M_t + (\frac{\tau}{2})(M_t - M_t^*)^2 \tag{9}$$

where ε_{hmt} is the white noise innovation to input inventory holding costs, M_t^* is the target level of input inventories that minimises input inventory holding costs, and $\tau > 0$. The quadratic inventory holding cost structure balances two forces. Holding costs rise with the level of inventories, M_t and N_t, due to increased storage costs, insurance costs, etc. But holding costs fall with M_t and N_t because—given expected M_t^* and N_t^*—higher M_t and N_t reduce the likelihood that the firm will 'stock out' of inventories.

Finally, it remains to specify the inventory target stocks. Following Ramey and West (1999) the output inventory target stock is

$$N_t^* = \alpha X_t + \theta_n t \tag{10}$$

where $\alpha > 0$ and t is a linear time trend. The output inventory target depends on sales because the firm incurs costs due to lost sales when it stocks out of output inventories. In addition the parameter θ_n serves as a proxy for the introduction of technologies that affect the cost minimising level of inventories, ie, inventory saving technologies such as the just-in-time production technique.[17] Similarly, the input inventory target stock is

$$M_t^* = \theta Y_t + \theta_m t \tag{11}$$

where $\theta > 0$. The input inventory target depends on production ($Y_t = X_t + \Delta N_t$) because stocking out of input inventories entails costs associated with production disruptions. Similarly, this optimal stock is also affected by a time trend through the parameter θ_m that intends to proxy inventory saving technology.

3.2.4 Input materials costs

Input materials costs consist of purchase and adjustment costs. Specifically input materials costs are

$$MC_t = V_t D_t + (\frac{\phi}{2}) D_t^2 \tag{12}$$

The first term on the right-hand side of equation **(12)** is the cost of ordering and purchasing input materials at the 'base' price each period. The second term is a quadratic approximation for adjustment costs on purchases of materials and supplies.[18]

3.2.5 The firm's problem

Given demand (X_t) and factor prices (V_t, W_t) the firm chooses $\{U_t, M_t, N_t\}$ to minimise the present discounted value of total costs

$$E_0 \sum_{t=0}^{\infty} \beta^t (LC_t + HC_t^N + HC_t^M + MC_t)$$

[17] The data show a trend decline until 1994 for both types of stocks.

[18] For example, for $\phi > 0$ the firm faces increasing marginal costs of acquiring materials. This for example can arise if the firm must pay a premium for faster delivery of materials.

The two laws of motion governing inventory stocks,

$$\Delta N_t = Y_t - X_t$$

$$\Delta M_t = D_t - U_t$$

will be used to substitute for production (Y_t) and deliveries (D_t).

3.2.6 Euler equations

The model yields Euler equations for U_t, M_t, and N_t. However, we are going to utilise only the Euler equations for input (M_t) and output (N_t) inventory stocks,[19] since usage (U_t) is unobservable at high frequencies.

To present the Euler equations concisely, define the lag operator as *P*, which works as a lead operator when inverted ($P^{-1}Y_t = Y_{t+1}$), and a variable Z_{ij} that denotes quasi-differences ($1 - \beta P^{-1}$) of model variables. Subscript i indicates the variable being quasi-differenced, subscript j indicates the number of quasi-differences, $(1 - \beta P^{-1})^j$, and $\Delta = (1 - P)$ is the standard difference operator. For example, $Z_{Y1} = (1 - \beta P^{-1})Y_t$ is the quasi-first difference of Y_t, and $Z_{Y2} = (1 - \beta P^{-1})^2 Y_t$ is the quasi-second difference of Y_t. $\Delta Z_{Y2} = (1 - P)(1 - \beta P^{-1})^2 Y_t = (1 - \beta P^{-1})^2 \Delta Y_t$ is the change in the quasi-second difference.

The Euler equations for the value-added specification follow.

Input inventories

$$\begin{gathered} E_t\{\gamma\phi Z_{\Delta M1} + \phi\varphi Z_{\Delta M2} + \gamma Z_{V1} + \phi\gamma_4 Z_{W1} + \varphi\Delta Z_{V2} \\ +\gamma\phi Z_{Y1} + \phi\varphi\Delta Z_{Y2} + \tau(\gamma+\phi)[M_t - \theta Y_t - \theta_m t] \\ +\tau\varphi[(\Delta M_t - \theta\Delta Y_t - \theta_m\Delta t) - \beta(\Delta M_{t+1} - \theta\Delta Y_{t+1} - \theta_m\Delta(t+1))] \\ +\tau_0 - \phi Z_{\varepsilon_{y1}} + (\gamma+\phi)\varepsilon_{hmt} + \varphi\Delta Z_{\varepsilon_{hm1}}\} = 0 \end{gathered} \quad \textbf{(13)}$$

[19] Usage will be eliminated by solving its Euler equation in terms of the rest of the variables.

Output inventories

$$
\begin{aligned}
&E_t\{\delta(\gamma+\phi)[N_t-\alpha X_t-\theta_n t]+\delta\varphi[\Delta N_t-\alpha\Delta X_t-\theta_n\Delta t] \\
&-\tau(\gamma+\phi)[(1+\theta)(M_t-\theta Y_t-\theta_m t)-\theta\beta(M_{t+1}-\theta Y_{t+1}-\theta_m(t+1))] \\
&-\tau\varphi[(1+\theta)(\Delta M_t-\theta\Delta Y_t-\theta_m\Delta t)-\theta\beta(\Delta M_{t+1}-\theta\Delta Y_{t+1}-\theta_m\Delta(t+1)] \\
&+\delta_0+(\gamma+\phi)\varepsilon_{hnt}+\varphi\Delta Z_{\varepsilon_{hn1}}-(\gamma+\phi)\varepsilon_{hmt}-\varphi\Delta Z_{\varepsilon_{hm1}}\}=0 \qquad \textbf{(14)}
\end{aligned}
$$

To gain some insight consider the input (M_t) Euler equation. This optimality condition states that the firm balances the marginal cost of ordering and holding input inventories this period against the cost of ordering input inventories next period. The firm attempts to set the input inventory stock equal to its target subject to several dynamic frictions. First, the adjustment costs associated with purchases and deliveries of materials, as captured by ϕ, prevent the firm from instantaneously eliminating input inventory gaps, $M_t - M_t^*$. Second, time variation in expected materials prices gives the firm an incentive to intertemporally substitute deliveries of input materials. However, bargains on input materials must be large enough to offset adjustment and stockout costs. Finally, higher output inventory stocks induce the firm to raise gross production (Y_t), which in turn requires higher materials usage and thus leads to drawing down materials inventory stocks.

The output (N_t) inventory Euler equation states that the firm balances the marginal cost of producing a good and storing it as output inventory this period against the cost of producing the good in the future. The equation incorporates input inventories (M_t) that affect the output inventory investment decision through the gap terms, $M_{t+i} - \theta Y_{t+i} - \theta_m(t+i)$, $\Delta M_{t+i} - \theta\Delta Y_{t+i} - \theta_m\Delta(t+i)$. The firm is attempting to eliminate deviations from the cost minimising level of stocks, M_t^*, N_t^*. The Euler equation makes clear that this goal is being achieved by balancing input and output inventory gaps. The parameters $\delta, \varphi, \alpha, \theta, \phi$ indicate the relative frictions that these actions entail. For instance, all else being equal, an increase in the input inventory gap ($M_t - M_t^*$) will affect the output inventory gap ($N_t - N_t^*$), and hence the output inventory decision, because being away from the cost minimising level, N_t^* entails a stockout cost. An increase in the input inventory gap entails a stockout cost as well. Because the stockout costs for both types of inventories are quadratic it is cost minimising to spread the stockout costs between inventory stocks. It is also evident from the Euler equation that stock adjustment will not be instantaneous. Depending on the relative magnitude of the structural parameters that multiply the gap terms, stocks will exhibit some degree of persistence, ie, the speed by which deviations from target stocks, M^* or N^* are corrected.

In sum, input and output inventory stocks interact directly and indirectly in the stage-of-fabrication model. As can be seen from equation **(14)**, input inventories *directly* affect output inventories through the input inventory gap terms in the output Euler equation. All else being equal, an increase in the input inventory gap raises current and, due to adjustment costs, future output inventories. On the other hand, output inventories *indirectly* affect input inventories through the input inventory target stock, M_t^* in equation **(13)**. All else being equal, an increase in output inventories raises production, Y_t, and thus M_t^* by a factor of θ, thereby affecting the input inventory gap, $(M_t - M_t^*)$. The firm's optimal response to this change is to increase input inventories, though less than completely due to adjustment cost frictions.

3.2.7 The complete model

The stage-of-fabrication (SOF) model consists of five equations: two Euler equations for the inventory stocks, M_t and N_t, and three auxiliary equations for the exogenous variables, V_t, W_t and X_t. Each exogenous variable is assumed to follow a general autoregressive ($AR(p)$) process.

The SOF model can be written in matrix difference equation form as

$$E_t\{\sum_{i=-lags}^{leads} H_i S_{t+i} - G\varepsilon_t|\Omega_t\} \quad \textbf{(15)}$$

where

$$S_t = [M_t, N_t, V_t, W_t, X_t] \quad \textbf{(16)}$$

is the vector of system variables;[20] the H_i are conformable square matrices containing model parameters;

$$\varepsilon_t = [\varepsilon_{mt}, \varepsilon_{nt}, \varepsilon_{xt}, \varepsilon_{vt}, \varepsilon_{wt}]'$$

is the vector of structural disturbances; *G* is a conformable square matrix that may contain model parameters and/or the lag operator; and Ω_t is the information set available at time *t*. The fundamental random shock ε_t is distributed iid N(0,Σ). In order to estimate equation

(20) A constant and a trend are included in matrix equation **(15)**.

(15) we need to replace the terms involving expectations of future variables. The numerical procedure that achieves this has been developed by Anderson and Moore (1985).[21] This procedure solves the model by replacing unobserved expectations with model consistent expectations—which are functions of observables— and derives a set of decision rules containing only current and lagged variables. Applying this procedure to equation **(15)** yields the *observable* structure of the SOF model

$$\sum_{i=-lags}^{0} A_i S_{t+i} = \varepsilon_t \qquad \textbf{(17)}$$

Equation **(16)** is a structural representation of the model because it is driven by the structural disturbance, ε_t; the coefficient matrix A_0 contains the contemporaneous relationships among the elements of S_{t+i}. Notice that equation **(16)** only contains current and lagged model variables. Equation **(16)** is then used to form the concentrated log likelihood function

$$\mathcal{L} = T(\log|\jmath| - 0.5\log|\hat{\Sigma}|) \qquad \textbf{(18)}$$

where $\hat{\Sigma}$ is the estimated covariance matrix of ε_t, and $\jmath$ is the Jacobian linking $\hat{\varepsilon}_t$ to the portion of the S_{t+i} data pertaining to stochastic equations. The likelihood function is maximised with a sequential quadratic programming algorithm using numerical derivatives, and the covariance matrix, $\hat{\Sigma}$ is obtained by numerically evaluating the Hessian at the final parameter estimates. Standard errors are the roots of the diagonal elements of the inverted Hessian.

The estimation method employed is a two-step approximation to full information maximum likelihood following Fuhrer, Moore and Schuh (1995). In the first step, parameters of the $AR(p)$ auxiliary equations—that is X_t, V_t, W_t—are estimated with OLS. In the second step, the structural parameters are estimated with maximum likelihood, conditional on the OLS estimates of the $AR(p)$ processes. The two-step estimator is asymptotically equivalent to full-information estimation but less efficient.

More importantly, the choice of maximum likelihood estimation for the SOF model is motivated by several studies that demonstrate the superiority of maximum likelihood (ML)

[21] The Anderson and Moore (1985) technique is a generalisation of Blanchard and Kahn (1980).

compared to the generalised method of moments (GMM) estimator. The most compelling evidence is presented in Fuhrer *et al* (1995). As Fuhrer *et al* (1995) demonstrate, GMM applied to a linear quadratic inventory model (similar to the one studied in this paper) is dominated along every dimension by ML.[22]

4 Econometric results

This section presents and discusses estimation results for the SOF model. Section 4.1 presents estimates of the $AR(p)$ processes for the exogenous variables that are imposed on the ML estimation of the SOF model. Section 4.2 then presents the econometric estimates for the full SOF model and two additional simplified specifications that are nested in the full model. These simplified models are an output model derived from the full model after all parameters and data associated with the input stock are equal to zero, and an input model derived from the full model after all parameters and data associated with the output stock are equal to zero.

All three models are estimated from 1975:1 to 2003:4 using aggregate manufacturing data.[23]

4.1 Auxiliary equation estimates

Table D contains regression results for the first-step estimates of the auxiliary equations for the exogenous variables.[24] Each exogenous variable has been estimated as a general $AR(p)$ process and a lag length p has been chosen, such that residual serial correlation is eliminated. This can be inspected by the Q(12) statistics at the bottom of Table D. This procedure suggests estimating sales as an $AR(4)$ process,[25] real materials price as an $AR(6)$, and the real wage as an $AR(4)$ process. The estimates in Table D are imposed on the maximum likelihood estimation of the SOF models.[26]

[22] 'In small samples, ML estimates generally are unbiased, correctly signed, and statistically precise. However, GMM estimates are imprecise and biased—often sufficiently biased that the median parameter estimates have the incorrect sign. Even when the model is misspecified in certain respects, ML dominates GMM. Asymptotically, ML is orders of magnitude more efficient than GMM.'(page 150).

[23] Appendix A describes the data used in this paper.

[24] Appendix B presents stationarity tests for the exogenous variables.

[25] Note that this specification allows for the presence of a unit root in sales as the ADF test in Appendix B indicates.

[26] The parameter estimates of the SOF model presented in Section 4.2 are to some degree insensitive to the lag length of the AR models for the exogenous variables. In an earlier version of this paper $AR(1)$ models for the exogenous variables are estimated; even though the fit of the model is worse, the estimates do not differ significantly in order to alter the economic interpretation.

Table D: Equation estimates for exogenous variables

	Sales(X)	Wage(W)	Materials price(V)
X_{t-1}	0.73^*		
	(0.09)		
X_{t-2}	0.3^*		
	(0.11)		
X_{t-3}	$0.18^\dagger$		
	(0.11)		
X_{t-4}	-0.28^*		
	(0.08)		
W_{t-1}		0.92^*	
		(0.092)	
W_{t-2}		0.15	
		(0.12)	
W_{t-3}		-0.21^*	
		(0.088)	
V_{t-1}			1.26^*
			(0.089)
V_{t-2}			-0.5^*
			(0.14)
V_{t-3}			$0.29^\dagger$
			(0.15)
V_{t-4}			-0.12^*
			(0.143)
V_{t-5}			-0.33^*
			(0.12)
V_{t-6}			0.26^*
			(0.07)
trend		0.001^*	-0.0007^*
		(0.00028)	(0.0002)
R^2	0.96	0.99	0.98
Q(12)	12.5	15.55	16.54
	(0.4)	(0.212)	(0.167)

Asymptotic std. errors and p-values in parenthesis. * Significant at the 5% level.
† Significant at the 10% level.
$Q(p)$ is the Ljung-Box statistic for serial correlation with p lags.

4.2 Model estimates

This section presents econometric estimates for the three different models; that is, the full-blown SOF model (ie, joint input-output determination); the input model; and the output model.

For convenience Table E below lists the structural parameters to be estimated.[27]

[27] The discount factor β is preset at 0.995 rather than estimated.

Table E: Structural parameters

Labour costs	γ, γ_4, φ
Holding costs	δ, τ
Target stock	θ, α, θ_m, θ_n
Delivery cost	ϕ

The labour cost parameters, γ, γ_4, and φ capture the slope of the marginal production cost (holding adjustment costs constant), the sensitivity of the marginal cost with respect to real wage, and marginal adjustment cost associated with changes in labour respectively (see equation **(7)**). The holding costs parameters, δ, τ capture the marginal stockout costs for output and input inventories respectively. The target stock parameters, $\theta, \theta_m, \alpha, \theta_n$ relate optimal inventory stocks with sales (for output inventories, α), and production (for input inventories, θ). Moreover two additional parameters are specified, θ_m, θ_n that aim to capture inventory saving technology in input and output stocks respectively.[28] Finally, ϕ governs the slope of the marginal delivery cost associated with purchases and deliveries of materials. Table F below presents the estimates obtained from the three models. As a first pass of the SOF model note that all estimates agree with the signs predicted by theory. Every parameter for the full (joint) model is estimated significantly at the 1% level. By contrast, the estimates for the input or output single equation models (Columns 3 and 4) in general differ in magnitude from the joint model. Two points are worth noting. First, single equation estimates may suffer from misspecification bias because they fail to account for stock interaction. This can be most easily seen if we recall the output Euler equation in Section 3.2.6. The output Euler equation is essentially an equation that balances input and output inventory gaps. Without this source of interaction we are likely to obtain biased estimates for the rest of the parameters. Second, the estimates obtained under the joint model are estimated considerably more precisely. Therefore, the joint estimates provide strong econometric support for the importance of interaction between input and output inventories.

The estimates from the joint model appear reasonable and comparable to similar studies.[29] The target stock parameters imply that firms aim to hold around four months' worth of sales in finished goods inventories (α), and three months' worth of production

(28) In the estimation a broken trend assumption has been imposed that ends in 1994 Q4 as the data indicate that any time dependent saving factors are not important for the remainder of the sample.
(29) The two studies that present comparable estimates are Fuhrer *et al* (1995) and HMS (2001). Both estimate inventory models for the US manufacturing sector.

Table F: SOF model estimates

Parameters	Predicted Sign	Joint	Input	Output
α	+	$+1.37^{**}$	n/a	$+1.02^{**}$
		(0.04)		(0.049)
θ	+	$+1.04^{**}$	$+0.20^{**}$	n/a
		(0.012)	(0.087)	
θ_m	-	-0.0028^{**}	-0.00059^{**}	n/a
		(0.0002)	(0.0002)	
θ_n	-	-0.02^{**}	n/a	-0.002^{**}
		(0.004)		(0.0004)
γ	+	$+0.47^{**}$	$+0.42^{**}$	$+0.05^{**}$
		(0.018)	(0.06)	(0.005)
γ_4	+	$+0.74^{**}$	+0.41	n/a
		(0.22)	(0.53)	
δ	+	$+0.08^{**}$	n/a	$+1.91^{**}$
		(0.01)		(0.38)
τ	+	$+0.072^{**}$	$+0.20^{**}$	n/a
		(0.012)	(0.02)	
φ	+	$+2.02^{**}$	$+2.7^{**}$	$+0.35^{**}$
		(0.14)	(0.49)	(0.08)
ϕ	+	$+2.00^{**}$	$+1.81^{**}$	n/a
		(0.13)	(0.34)	
$\partial^2 TC/\partial Y^2$		4.50^{**}		
		(0.0839)		
$Q(12)$	ϵ_m	18.64	15.73	n/a
		(0.11)	(0.2)	n/a
$Q(12)$	ϵ_n	9.94	n/a	14.5
		(0.62)	n/a	(0.26)
$\mathcal{L}$		2303	1777	845
$2(\mathcal{L}-\mathcal{L}^{\mathcal{R}})$		397.9		
		(0.00)		

Asymptotic std. errors in parenthesis. ** and * indicate significance at the 1% and 5% level respectively.
$Q(p)$ is the Ljung-box statistic for serial correlation with p lags. p-values for $Q(p)$ and $2(\mathcal{L}-\mathcal{L}^{\mathcal{R}})$.

in input inventories (θ).[30] Moreover the θ_m, θ_n estimates suggest that inventory saving technology is present for both types of inventories and it is affecting more input inventories, even though the rate at which this is taking place seems to be slow (the value of θ_m implies that the target input stock drops by nearly 0.03 of a per cent each quarter). Turning to other aspects of producers' behaviour there is a key qualitative similarity between the results of this study and the results obtained by Fuhrer *et al* (1995) and HMS (2001). Specifically, as in Fuhrer *et al* (1995) and HMS (2001) the results of Table F imply that stockout costs (δ, τ) are smaller and adjustment costs (φ,ϕ) are greater than production

[30] Note that these parameter estimates may suffer from an identification problem. The reason is that production and sales do not differ enough at the quarterly frequency. That is, the model attempts to identify two parameters using essentially a single source of variation. The mean inventories to sales ratios in the data are 0.95 for output and 1.01 for input inventories. Only the estimated input target parameter is reasonably close to the value in the data.

costs (γ). That is, in the UK manufacturing sector marginal adjustment costs as captured by φ are around four times higher than marginal production costs (γ), which is similar to the relative values that Fuhrer *et al* (1995) obtain, although significantly higher than those HMS (2001) obtain.[(31)] On the other hand marginal stockout costs for both types of inventories are estimated to be around 15% of marginal production costs, whereas HMS (2001) find them in the 25%-35% range.[(32)] In addition, Table F reports an estimate of the slope of the aggregate marginal cost function, $\partial^2 TC/\partial Y^2$. This value is positive and significant indicating that marginal costs are convex.[(33)] Moreover, the Q(12) Ljung-Box statistic cannot reject the null of no serial correlation, providing some indication that the model is well specified (see p-values in Table F). A finding weighting against the model, however, is the fact that the likelihood ratio test rejects the overidentifying restrictions of the SOF model, suggesting that future work should relax some of the simplifying assumptions.[(34)]

5 Impulse response functions

This section explores dynamic properties of the SOF model. Chart 1 plots the impulse responses of input and output inventories to a 1% unit sales and materials price shocks. As can be seen from the top panel of Chart 1 both input and output inventories ultimately rise in response to a positive sales shock. With higher expected sales the target stocks rise temporarily.[(35)] Hence the firm would attempt to bring actual stocks as close as possible to their new targets. Initially however, both stocks decline. This implies that the firm does not increase production as much as sales, and instead satisfies some of the extra demand out of output inventories. This is the familiar production smoothing result: inventories are used to smooth production relative to sales. This is a result that models without a joint treatment of input and output inventories have trouble capturing (see Ramey and West (1999)). A negative output inventory gap arises because the adjustment costs on output—captured by φ—are high relative to stockout costs (governed by δ). Input inventories also fall, because

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(31) HMS (2001) estimates are in the 1-2 times higher interval.

(32) A point to note is that the HMS (2001) estimate durable and non-durable industries separately, whereas the results here apply to aggregate manufacturing, so strictly speaking the estimates are not directly comparable.

(33) This finding is important especially in the presence of input inventories, where non-convexities are more likely to arise and spill over to the production process through the stage of fabrication linkages. Non-convexities can arise if for example ordering materials follows *(S,s)* type rules.

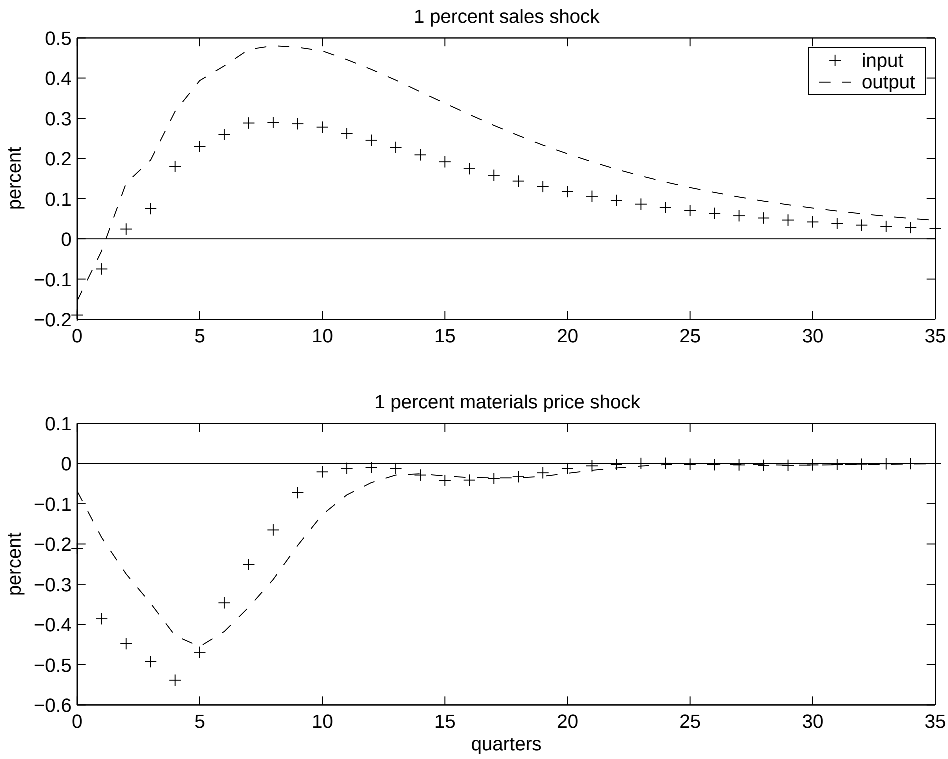
(34) Schuh (1996) finds that firm-level and detailed industry-level data do not reject the overidentifying restrictions of a benchmark output inventory model. This suggests the problem may be attributable to aggregation. Aggregation also bears in the discussion of the dynamic properties of the model in Section 5 below.

(35) This follows from equations **(10)** and **(11)** that define the target stocks N^*, M^*. Notice that even though this is a temporary shock its effects on sales are expected to persist, as is obvious from the dominant root of around 0.93 for the sales equation in Table D.

in order to increase production the firm must increase usage and this requires it to draw down input inventories. The firm does not replenish with new deliveries immediately because the adjustment costs on orders—as captured by ϕ— are high relative to stockout costs (governed by τ). Given a negative output inventory gap and convexity of the cost function it becomes optimal for the firm to spread the stockout costs across both types of stocks. This explains why the firm tolerates a negative input gap at the same time. Input inventories fall slightly more than output inventories on impact because stockouts are more costly for output relative to input inventories ($\hat{\delta} * \hat{\alpha} > \hat{\tau} * \hat{\theta}$), and stocks rise slowly over time towards their target level because adjustment costs are very high relative to stockout costs as can be seen from the parameter estimates of Table F.

High adjustment costs of changing production imply a strong smoothing motive; a demand (sales) shock as the one analysed in Chart 1 raises the target stock and so the firm adjusts production in order to bring output inventories in line with the new target. Without a strong stockout effect, the firm is reluctant to quickly eliminate the gap between actual and target stock. In sum, the slow speed of adjustment follows as an implication from the high adjustment production costs relative to stockout costs ($\hat{\varphi} >> \hat{\delta}$ and $\hat{\phi} >> \hat{\tau}$).

Chart 1: Dynamic simulations



Input and output inventories decline in response to a positive materials shock. The temporary price increase causes the firm to substitute deliveries intertemporally (postponing deliveries) and reduce input inventories. A negative input inventory gap emerges, which the firm wants to eliminate. With deliveries expensive at the time of the shock the firm cuts materials usage and hence production. With sales unchanged, output inventories decline too. This behaviour essentially follows from the specification of the stockout costs. As explained in more detail in Section 3.2.6, with convex stockout costs the firm prefers to endure two moderate gaps rather than a zero output and a larger positive input gap.

A broad picture that emerges from the impulse response plots is that both inventory stocks are very persistent or equivalently exhibit slow adjustment speeds toward their targets. For example, in response to a positive sales shock, stocks take around two and a half years to reach their peak response. This is because marginal adjustment costs are substantially higher than marginal stockout costs. While consistent with the aggregate data,[36] such behaviour requires sluggish adjustment at the firm level which is difficult to rationalise from a theoretical perspective.[37]

Finally, this analysis suggests some broad implications for monetary policy. Using the dynamic simulations of this section we can highlight just a few. First, it is important to note that the extent to which manufacturing inventories react to changes in the policy environment depends on where actual stocks are relative to their targets. The impulse response plots are conditional on actual stocks being equal to target stocks at the time of the shock. Conditional on this, and broadly interpreting the demand shock analysed in Chart 1 as a monetary policy shock we can infer that manufacturing inventories decline in response to a positive demand shock, ie, a ‘monetary easing’. In addition, the strength by which different types of stocks (input or output inventories) react to shocks depends on the relationship between input and output inventory gaps at the time of the shock. The joint interaction that the model emphasises implies that larger gaps in one stock will be met by stronger adjustment in the other stock because firms try to minimise costs by spreading imbalances more evenly across stocks. But, if for example actual stocks are far away from their targets it is not easy to draw out the implications. Firms may hold excessively low stocks at the time of the shock, and we could equally observe stocks rising instead of falling because the need to minimise imbalances will dominate the production smoothing

[36] As Ramey and West (1999) point out, one of the stylised facts of inventories is the highly persistent nature of the inventory to sales ratio, or, alternatively, a slow speed of adjustment.

[37] A noteworthy finding by Schuh (1996) suggests that firm-level inventory adjustment speeds are an order of magnitude higher than aggregate adjustment speeds. He points to aggregation bias as the most plausible source of this difference. Aggregation bias arises because the ‘true’ aggregate adjustment speed parameter is time variant, while a fixed aggregate parameter is imposed on the data.

motive.

6 Conclusions

This paper has applied a new stage-of-fabrication inventory model to the UK manufacturing sector. The model, a generalisation of the linear quadratic model of Holt *et al* (1960), was introduced in the inventory literature by HMS (2001). It makes an important step toward addressing the neglect of input inventories which, empirically, are more important than output inventories in the United Kingdom. The key finding of the paper is that stock interaction between input and output inventories is quantitatively significant, and crucial for the model to fit the UK data.

It is worth considering several extensions and/or refinements to this study. First, and most importantly, further disaggregated data will be of great help in drawing out differences between different industries. The HMS (2001) study, considers a durable, non-durable goods industry breakdown. This is important because in US data input inventories are larger and more variable in durable goods industries, and we would anticipate a similar pattern in UK data.[38] Further insights into producer behaviour, may be gained by applying this model to more disaggregated—at the industry level—data in order to gain more insights into producer behaviour. Another important extension of this model is to explicitly include work-in-process inventories rather than combine everything under input inventories. This would require the modelling of the production process to yield production of intermediate output. But if the linear quadratic approach is used, estimation of a more heavily parameterised model may be problematic. A more direct approach in specifying production and cost functions is needed. However, this could be a worthwhile extension in light of the strength of the covariance terms of input and output inventories resulting with a finer breakdown of inventories.

Another potentially fruitful area for further work would be to explore the inventory behaviour of the wholesale and retail sectors. These sectors have gained in significance over the last decade relative to manufacturing and the inventory volatility of the wholesale sector has increased relative to the past.[39] An interesting question is to examine how supplier-user relationships between manufacturers and wholesalers and/or retailers play a role in shaping inventory dynamics.

(38) For example in US data a mere three manufacturing industries hold the majority of input inventories; Primary Metals, Fabricated Metals, and Non-electrical Machinery.
(39) See Table B.

Appendix A: Variables and data sources

The data used for this paper are published by the Office for National Statistics and cover the period 1975:1 to 2003:4. Details are as follows. Manufacturing sales data are constructed using the identity, $X = Y - \Delta N$, where Y is manufacturing production and ΔN is the change in finished goods inventories. The input inventories series M is obtained as the sum of raw materials and supplies and work-in-process inventories. The output inventories series, N, is simply finished goods inventories.[40] The raw materials price is the price index for materials and fuels purchased by all manufacturing industries. The nominal wage data are constructed as the product of unit wage cost and output per job to arrive at an index of wage per head. The resulting index is converted to a nominal figure using the Annual Survey Hours Earnings figure (2003 Q2) for average weekly earnings.

The nominal wage data and raw materials price data are deflated by the aggregate producer price output index. The index reflects manufacturing output prices excluding duty. All data are in logs, seasonally adjusted and in 2000 prices.[41] The ONS codes are:

Manufacturing output index—CKYY

Manufacturing output(finished goods) inventories—FBNH

Manufacturing work-in-process inventories—FBNG

Manufacturing materials and fuels inventories—FBNF

Manufacturing price index of materials and fuels—RNPE

Manufacturing output price index—PVNQ

Manufacturing output per job—LNNX

Manufacturing unit wage cost—LNNQ

(40) The raw inventory data form the ONS refer to changes in stocks. The levels for stocks are derived by cumulating the corresponding series.
(41) The latest data vintage (2004 Q1) following methodological revisions (reference year changed from 2000 to 2001) would differ from that used for this paper.

Appendix B: Stationarity tests for exogenous variables

Table G below presents ADF unit root and Johansen cointegration tests for the variables of the SOF model. The ADF test statistics for the real wage (W_t) and the real materials price (V_t) suggest treating these two variables as trend stationary processes (null of unit root is rejected at the 5% significance level), whereas, for sales (X_t) one cannot reject the null of unit root at conventional levels. Similarly, the Johansen test indicates one co-integrating relationship at the 5% level among inventories and production (for input inventories) or sales (for output inventories). Note however that the time series properties of the exogenous variables does not pose a problem for estimation of the SOF model since the equations to be estimated (**(13)** and **(14)**) are composed of terms that are stationary. For example, equation **(13)**, the input Euler equation includes terms that are combinations of data that are first or higher order differenced and hence stationary.

Table G: Unit root and cointegration tests

	ADF statistic	Johansen test$^\flat$	
		Trace statistic	Max eigenvalue statistic
X_t	-2.20		
V_t	-3.84^{*}		
W_t	-3.76^{*}		
ΔX_t	-4.91^{**}		
ΔV_t	-6.81^{**}		
ΔW_t	-10.47^{**}		
(X_t, N_t)		25.2^{*}	24.11^{*}
(Y_t, M_t)		20.54^{*}	20.34^{*}

For the ADF tests $^{*}, ^{**}$ indicates rejection of null of unit root at the 5% and 1% level respectively.
Linear trends and intercepts included in the ADF tests.
For the Johansen cointegration test * indicates one cointegrating relationship.
$\flat$Johansen cointegration tests are performed with linear trends.
Johansen cointegration test 5 $\%$ critical values: 15.41(trace statistic), 14.07(Max Eigen. statistic).

Appendix C: A different treatment of work-in-process inventories

This section presents estimation results that deviate from the conventional definition of output and input inventories adopted in this paper as well as in the inventory literature. The results of Table F are based on the assumption that work-in-process inventories can be lumped together with materials to define input inventories. This assumption implies a symmetrical treatment of work-in-process with raw materials. Nevertheless one can think of industries where work-in-process inventories are neither 'true' input nor output inventories. For example, it is more natural to think of 'production to order manufacturing industries' as industries that rely less on finished goods inventories to achieve production smoothing (given that they maintain a backlog of unfilled orders that can manipulate to achieve this objective). By contrast, in manufacturing industries that 'produce to stock' finished goods inventories assume the traditional role of production smoothing.[42] One can then take the extra step in arguing that work-in-process inventories would take the characteristics of output as well as input inventories in the 'production to order' industries; the implication is that it is perhaps more appropriate to exclude work-in-process from both categories.

Excluding work-in-process from the analysis also implies an indirect test of the maintained assumption of symmetrical treatment of work-in-process and raw materials. This assumption can be tested by repeating the analysis without work-in-process. Finished goods inventories continue to be defined as output inventories, but now only raw materials constitute input inventories. Comparison between the estimation results from two SOF models should highlight whether including work-in-process in input inventories is a reasonable assumption for the UK aggregate manufacturing sector. The first impression from the results in Table H below is that excluding work-in-process is not inconsistent with the data. Parameters are precisely estimated (and all but γ_4 significantly different from zero) and very similar in magnitude to the parameters of the full SOF model. For ease of comparison the results presented in Table F employ the conventional definition.

Nevertheless, the SOF model is clearly at better grips with the data, when work-in-process is part of input inventories. Table H demonstrates that the full-SOF model yields more precise estimates–as can be seen from the lower standard errors–compared to the SOF model that excludes work-in-process and attains a higher value for the likelihood function.

[42] In general, industries in the durable goods sector produce to order, while industries in the non-durable sector produce to stock. Regrettably, UK data are not available for this durable/non-durable sector classification.

Table H: SOF model estimates

Parameters	Predicted Sign	full-SOF	SOF-exl. w-in-p
α	+	$+1.37^{**}$	$+1.03^{**}$
		(0.04)	(0.12)
θ	+	$+1.04^{**}$	$+1.02^{**}$
		(0.012)	(0.13)
θ_m	-	-0.0028^{**}	-0.0014^{**}
		(0.0002)	(0.0003)
θ_n	-	-0.02^{**}	-0.007^{**}
		(0.004)	(0.002)
γ	+	$+0.47^{**}$	$+0.58^{**}$
		(0.02)	(0.07)
γ_4	+	$+0.74^{**}$	+0.77
		(0.22)	(0.43)
δ	+	$+0.08^{**}$	$+0.14^{**}$
		(0.01)	(0.03)
τ	+	$+0.072^{**}$	$+0.10^{*}$
		(0.012)	(0.04)
φ	+	$+2.02^{**}$	$+2^{**}$
		(0.14)	(0.73)
ϕ	+	$+2.00^{**}$	$+2.00^{**}$
		(0.13)	(0.79)
$\partial^2 TC/\partial Y^2$		4.50^{**}	4.57^{**}
		(0.0839)	(1.96)
$Q(12)$	ϵ_m	18.64	18.34
		(0.11)	(0.10)
$Q(12)$	ϵ_n	9.94	7.85
		(0.62)	(0.79)
$\mathcal{L}$		2303	2278
$2(\mathcal{L} - \mathcal{L}^{\mathcal{R}})$		397.9	446
		(0.00)	(0.00)

Asymptotic std. errors in parenthesis. ** and * indicate significance at the 1% and 5% level respectively.
$Q(p)$ is the Ljung-box statistic for serial correlation with p lags. p-values for $Q(p)$ and $2(\mathcal{L} - \mathcal{L}^{\mathcal{R}})$.

Including work-in-process inventories provides additional information over that contained in raw materials that helps explain the interaction between input and output inventories. Therefore, unless we know the precise role of work-in-process inventories in disaggregated manufacturing data we would not be able to disentangle their contribution in aggregate manufacturing data, and the most reasonable assumption is to define them as input inventories.

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