Working Paper no. 279

Monetary policy and private sector misperceptions about the natural level of output

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October 2005

Bank of England

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The views expressed in this paper are those of the authors, and not necessarily those of the Bank of England. We are grateful for comments from Peter Andrews, Charles Bean, Matt Canzoneri, Martin Ellison, Charlotta Groth, Tony Yates, two anonymous referees and seminar participants at the Bank of England. This paper was finalised on 20 July 2005.

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Publications Group, Bank of England, Threadneedle Street, London, EC2R 8AH; telephone +44 (0)20 7601 4030, fax +44 (0)20 7601 3298, email mapublications@bankofengland.co.uk.

ISSN 1368-5562

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Abstract

In this paper we illustrate, using a simple model of monetary policy, the welfare costs of the private sector and/or the central bank being uncertain about the natural level of output. It turns out that monetary policy strategies that put less weight on output stabilisation can offset some of these welfare costs.

Summary

There is ongoing debate about by how much the real world differs from the world described by models of rational expectations. This paper describes a simple model that offers some insight into the consequences for monetary policy design of problems the private sector and the central bank might have in estimating the natural level of output.

The paper uses a simple model with two agents, a private sector and a policymaker. The private sector bases its behaviour on its perception of the sustainable level of output and on its perception of the objectives and actions of the policymaker. The policymaker sets policy to keep inflation stable around an inflation target, and to keep output stable around its sustainable level. The paper assumes that the private sector and the policymaker have asymmetric information sets. These asymmetries cause the private sector and/or the central bank to have mistaken expectations – misperceptions – about the natural level of output. Furthermore, these misperceptions are not a function of the fundamentals contained in the model, but rather are some non-modelled factor. Three variants of the misperceptions problem are considered. In the first two cases, only the private sector has misperceived natural output, while in the third case, both the private sector and the central bank have misperceived natural output. In the first case, the private sector misperception is known by the policymaker, while in the second case, the misperception is unobserved by the central bank. In the third case, both agents' misperceptions are stochastic and unobservable (to the other agent). In all three variants it is found that, in the face of a private sector misperception, appointing a monetary policy maker who will be tougher on deviations of inflation from target than society can partially offset the negative effects of the private sector misperception.

1 Introduction

In many economic models, expectations play a key role in determining behaviour, and it is normal to assume that these expectations are formed rationally by agents who all have the same information set. However, although expectations almost certainly play a key role in determining real world behaviour, it is less certain whether these expectations are best described by rational expectations. For example, information asymmetries may cause agents' expectations to differ from model consistent rational expectations. In this paper, we consider the implications for monetary policy when there are information asymmetries between the private sector and the central bank. These asymmetries cause the private sector and/or the central bank to have mistaken expectations – misperceptions – about the natural level of output.

To study the interaction of misperceptions and monetary policy we use a simple model with two sets of agents, a private sector and a central bank. We allow each set of agents to have their own information set, where this may differ from the true information set and from the other agent's information set. We do not analyse the source of the information asymmetries, but we assume they are *not* a function of the model, and so describe them as being non-fundamentals based. We consider three variants of the misperceptions problem. In the first two cases, only the private sector has misperceived natural output, while in the third case, both the private sector and the central bank have misperceived natural output. In the first case, we assume that the private sector misperception is known by the policymaker. In the second case, we assume that the misperception is unobserved by the central bank. In the third case, both agents' misperceptions are stochastic and unobservable (to the other agent).

In all three variants we find, unsurprisingly, that when the private sector misperceives the true level of natural output welfare is unambiguously lower. But we find that the policymaker can reduce the expected social losses from misperceptions in all three cases by pre-announcing that it will place less weight on output gap fluctuations than it otherwise would – ie by appointing a 'conservative' central banker. Our result provides a new rationale for appointing a conservative central banker. The standard motivation for appointing a conservative central banker (ie Rogoff (1985)) is that the policymaker is trying to achieve some 'socially optimal' level of output which leads to an 'inflation bias' problem (as described, for instance, by Barro and Gordon (1983 a,b)). And, although Canzoneri (1985) discusses conservative central bank appointment as a solution to a private information problem, the result is partly generated by the presence of this 'inflation bias'. However, our rationale is generated purely by the presence of misperceptions. In our model, if there were no private sector misperceptions, there would be no incentive to appoint a conservative central bank because there is no 'inflation bias' problem.

In studying the effects of non-fundamentals based expectations on behaviour and policy, this paper follows on from the work of Bernanke and Gertler (2001), Cecchetti *et al* (2000), Bordo and Jeanne (2002) and Bean (2004). However, while these papers focus on the implications of non-fundamentals based expectations about firms' valuations and hence discuss asset price bubbles, we examine the implications when private sector expectations about the natural level of output are influenced by some non-fundamental factor.

Our finding, that policymakers should react less to output gap fluctuations when the private sector misperceives the true model of the economy, is very similar to the results of Orphanides and Williams (2003) and Jääskelä and McKeown (2005). In the Orphanides and Williams paper, the private sector does not know - misperceives - the structural parameters of the true model and has to learn them using recursive least squares. Jääskelä and McKeown (2005) model misperceptions as persistent, additive demand shocks in a standard New Keynesian model and study optimised Taylor rules. Both papers find that welfare can be improved by placing relatively less weight on output fluctuations.

The paper is structured as follows. In Section 2, we set out the model. In Section 3, we introduce misperceptions and consider the three variants of the misperceptions problem described above. In Section 4, we analyse whether monetary policy can do anything to offset the effects of misperceptions. Section 5 offers some conclusions.

2 A simple model of policy and misperceptions

Our model consists of: an objective function for the policymaker; an expectations augmented Phillips curve; and statements about the information sets of private agents and policymakers. The information sets are allowed to differ across agents and from the true information set.

$$y_t = y_t^{n,t} + \alpha(\pi_t - \pi_t^e) + \varepsilon_t \quad \textbf{(1)}$$

The Phillips curve is given by **(1)**, where y_t is (the log of) actual output, $y_t^{n,t}$ is (the log of) 'true' natural output, π_t is the inflation rate (set by the policymaker), and π_t^e is the expected inflation rate.

In this model, the private sector and the policymaker know the model structure and all the parameters, and form expectations rationally with respect to their information sets. However, both sets of agents have different information sets, denoted by I_{t-1}^{pm} for the policymaker and I_{t-1}^{ps} for the private sector. The differences in information sets can cause both the private sector and the policymaker to misperceive the true level of natural output.

The private sector's (rational) expectation of inflation is determined as,

$$E(\pi_t | I_{t-1}^{ps}) \equiv \pi_t^e = \lambda\alpha[E(y_t^* | I_{t-1}^{ps}) - E(y_t^n | I_{t-1}^{ps})] \quad \textbf{(2)}$$

where $E(y_t^n | I_{t-1}^{ps})$ is the private sector's perception of the level of natural output and $E(y_t^* | I_{t-1}^{ps})$ is the private sector's perception of the level of natural output being used to set policy.

The policymaker sets inflation to minimise the following loss function,

$$E(L_t | I_{t-1}^{pm}) = \frac{1}{2}E([\pi_t^2 + \lambda(y_t - y_t^*)^2] | I_{t-1}^{pm}) \quad \textbf{(3)}$$

$$E(L_t | I_{t-1}^{pm}) = \frac{1}{2}E([\pi_t^2 + \lambda(\alpha(\pi_t - \pi_t^e) + y_t^{n,t} - y_t^*)^2] | I_{t-1}^{pm})$$

where $y_t^* \equiv E(y_t^n | I_{t-1}^{pm})$ is policymaker's estimate of the natural level output and the inflation target is set to zero for simplicity. The first-order condition (FOC) of the policymaker's problem is given by **(4)**.

$$\pi_t = \lambda\alpha(y_t^* - y_t) \quad \textbf{(4)}$$

The policymaker sets inflation conditional on: its perception of expected inflation; its perception of natural output; and the realisation of the shock ε_t. The key to this model is that by allowing both agents to have different information sets, we allow them to have different perceptions of the level of natural output. These differences in information sets are not a function of variables or parameters of the model, and so cannot be learned by the other agent. Misperceptions are defined as expectations of natural output which differ from true natural output. We think of private sector misperceptions as being due to over-optimism or 'animal spirits'. policymaker misperceptions, on the other hand, can be thought of as being analagous to mismeasurement. An important feature of our model set-up is that y_t^* will only be different to y_t^n due to misperceptions - the policymaker is not trying to target some 'socially optimal' level of output that is different from y^n, as in Barro and Gordon (1983 a,b) and Canzoneri (1985). In short, there is no 'inflation bias'.

3 Introducing misperceptions

To study the implications of misperceptions, we consider three scenarios. In the first scenario, we assume that the private sector has misperceived the natural level of output by an amount which the policymaker can observe. In this case, we assume that the policymaker has no misperceptions. We call this first case premonition, because the policymaker can observe the misperception. In the second scenario, the private sector misperception is stochastic and unobservable to the policymaker, who makes no misperceptions. Finally, we consider unobservable stochastic misperceptions from both the private sector and the policymaker. We analyse each of these scenarios in turn before turning to consider how policy might be designed to offset the effects of misperceptions in the next section.

3.1 Private sector misperceptions: premonition

Let us assume there is a private sector misperception about the natural level of output and that the size of this misperception is given by k_t, where k_t could be either positive or negative, such that $E(y_t^n | I_{t-1}^{ps}) = y_t^n + k_t$. Turning to equation **(2)**, we see that inflation expectations are conditional on the private sector's perception of the level of natural output used by the policymaker – $E(y_t^* | I_{t-1}^{ps})$ – as well as their own perception of natural output. In this scenario, the asymmetric information means that the private sector thinks it has extra information about the natural level of output which the policymaker does not have. So the difference between the private sector's perception of natural output and their perception of the policymaker's perception of natural output will be equal to k_t ($ie\ E(y_t^*|I_{t-1}^{ps}) - E(y_t^n|I_{t-1}^{ps}) = k_t$). Substituting this into **(2)** we get the private sector expectation for inflation in the face of a misperception.

$$\pi_t^e = \lambda\alpha k_t \qquad \textbf{(5)}$$

To work out the policymaker's response we can use the policymaker's FOC, given by **(4)**. To evaluate **(4)** we first calculate y_t by substituting π_t^e from **(5)** into the equation for output, given by **(1)**. We also need to know what the policymaker's estimate of natural output is. In this scenario, we assume that, although the private sector has misperceived the true level of natural output, the policymaker has not – so $y_t^* = y^n$. Substituting this into **(4)** yields inflation when there is a private sector misperception.

$$\pi_t = -\frac{\lambda\alpha}{1+\lambda\alpha^2}\{\varepsilon_t - \alpha^2\lambda k_t\} \qquad \textbf{(6)}$$

When the private sector misperceives too high (low) a level of natural output, k is negative and so both actual and expected inflation will be lower (higher) than they would be if there was no misperceptions – with no misperception $k_t = 0$. The size of this distortion will increase as the size of the misperception increases. What will the effect of the misperception be on output? In this set-up, actual output is determined by the true level of natural output and not by the perceived level of natural output - see equation **(1)**.(1) Substituting inflation expectations and inflation as derived above into **(1)**, we find that with a private sector misperception output is given by,

$$y_t = y_t^* - \frac{\lambda\alpha^2}{1+\lambda\alpha^2}k_t + \frac{1}{1+\lambda\alpha^2}\varepsilon_t \qquad \textbf{(7)}$$

If the private sector misperceives too high (low) a level of natural output, the private sector misperception has the effect of *raising* (*lowering)* output because inflation falls (rises) by less than

(1) An obvious implication of this is that in this static model there is no way for agents to learn that their expectations are wrong - once agents get their expectation for natural output wrong they are stuck with this expectation. In the real world we would expect agents to notice a permanent disparity between actual output and their perceived natural rate and hence revise their erroneous expectation for natural output.

expected inflation.[2] To consider the overall effect of the misperception on the economy, we can plug **(7)** and **(6)** into the expected loss function evaluated using the 'true' information set – which combines the information sets of the private sector and the policymaker – because this is not distorted by misperceptions.

$$E(L_t|I_{t-1}^T) = \frac{\lambda}{2(1+\lambda\alpha^2)}\left[\sigma_\varepsilon^2 + \lambda^2\alpha^4 k_t^2\right] \quad \textbf{(8)}$$

It is clear that the impact of a private sector misperception is to unambiguously increase expected losses – with no misperception $k_t = 0$ and the last term on the right-hand side of **(8)** disappears.

3.2 Private sector misperceptions: unobserved misperceptions

The previous section considered the case of premonition, in which there was a private sector misperception which the policymaker could observe in advance. However, a more realistic case would be one in which the policymaker cannot use premonition. In this case, we treat k_t as a random variable and assume that although the policymaker does not know what k_t is at any point in time, it knows the distribution of k_t.[3] In particular, we assume that k_t is normally distributed around a zero mean, with a variance of σ_k^2. All of the other assumptions remain unchanged. We can now re-do the above analysis treating k_t as unknown to the policymaker. Because all that has changed is the assumption about the observability of k_t, inflation expectations are unchanged and still given by **(2)**. However, the unobservability of the misperception means that the policymaker has to set policy based on its expectation of the misperception, given its information set. Given the distribution of k_t, the policymaker's best guess of the misperception is $k = 0$, the mean of the distribution. So, with an unobservable private sector misperception, inflation is given by,

$$\pi_t = -\frac{\lambda\alpha}{1+\lambda\alpha^2}\{\varepsilon_t\} \quad \textbf{(9)}$$

(2) An interesting case to consider is that in which the private sector may have misperceived the true level of natural output but assumes that the policymaker is targeting the same level of output. In this case, the misperception has no effect on inflation expectations and hence no effect on inflation or output. This case is interesting because it suggests that if the private sector trusts the policymaker to do its job perfectly, then misperceptions would have no effect on the economy. However, this is not the case we concentrate on in the remainder of the paper.

(3) Or in a real world setting, the policymaker can form a reasonable judgement about what the variance of the misperception is likely to be.

Output will now be given by,

$$y_t = y_t^* - \lambda\alpha^2 k_t + \frac{1}{1+\lambda\alpha^2}\varepsilon_t \tag{10}$$

As before, we can now obtain the expected loss,

$$E(L_t|I_{t-1}^T) = \frac{1}{2}\left[\frac{\lambda}{(1+\lambda\alpha^2)}\sigma_\varepsilon^2 + \lambda^3\alpha^4\sigma_{k_t}^2\right] \tag{11}$$

In this case the loss is greater, because the coefficient on the variance of inflation is no longer divided by $1+\lambda\alpha^2$. In the face of an unknown private sector misperception losses are higher than in the face of a known misperception. Intuitively, this is because the policymaker knows less in this scenario, and so simply assumes $k_t = 0$.

3.3 Introducing policymaker misperceptions

The final case to consider is that in which both the private sector and the policymaker are subject to misperceptions. In this case we assume that the private sector misperceptions are the same as those described in the *unobserved misperceptions* section. In addition, let us now assume that the policymaker also has a misperception about the natural rate of output, such that $E(y_t^n|I_{t-1}^T) = y_t^n + \upsilon_t$. This misperception may be more intuitively thought of as arising from mismeasurement, rather than from the 'non fundamentals' which drive private sector misperceptions. We assume that the distribution of the policymaker's misperception is the same as for the private sector misperception: $\upsilon_t \sim N(0, \sigma_\upsilon^2)$. We assume that the private sector can observe their own misperceptions but not that of the policymaker. However, because we are treating the policymaker misperception as a measurement error, we do not allow the policymaker to observe υ_t – $E(y_t^n|I_{t-1}^{pm}) = y_t^n$. The policymaker must base policy on the distribution of its own misperceptions.

What will the inflation expectations of the private sector be? As the information set of the private sector is the same as in the previous section, their inflation expectation is still given by **(5)**. Inflation and output will also be the same as it was in the previous section, and given by **(9)** and

(10) respectively. This is because although the policymaker knows that it is making a mistake, its best guess of its own misperception at a point in time is zero. Similarly, although it knows the private sector is making a mistake, the policymaker's best guess of that misperception is also zero.

We can again obtain the expected loss as before. In this case, although inflation and inflation expectations are not affected by the policymaker's misperception, the loss is.

$$E(L_t|I^T_{t-1}) = \frac{1}{2}\left[\frac{\lambda}{(1+\lambda\alpha^2)}\sigma^2_\varepsilon + \lambda^3\alpha^4\sigma^2_{k_t} + \lambda\sigma^2_\upsilon\right] \qquad \textbf{(12)}$$

(12) shows that the loss is greatest in this case. Once again, this result is driven by the fact that with less information, losses are higher. In fact, the coefficient on the policymaker's misperception is larger than the coefficients on the shock, ε_t. The impact of the policymaker misperception is larger because υ_t realises after policy is set, whereas we have allowed policy to respond to ε_t.

4 Can policy be designed to offset the effects of misperceptions?

In this section, we consider whether anything can be done to improve the outcomes outlined above: can policy act to offset the effects of misperceptions? We consider whether welfare can be improved by the appointment of a policymaker with different preferences to the rest of society. In particular, we are interested in analysing how expected losses vary with the relative weight placed on output stabilisation – λ. Put differently, we are considering whether the appointment of a conservative policymaker can reduce losses, in a similar way to Rogoff (1985). In this section, we concentrate on the case where there are both private sector and policymaker misperceptions – the misperceptions problem is that set out in *introducing policymaker misperceptions*. We go on to show that the analysis presented here also applies to the cases where there are only private sector misperceptions (both known and unknown).

In this section, we introduce a third player into the economy: a government.[4] We assume the government is trying to minimise the social loss function evaluated using the true information set. We assume that the government can force the monetary policymaker to set policy using some λ^*–

(4) In doing so we follow a standard assumption in the delegation literature. Our assumption is comparable, for instance, to Svensson (1997).

different to λ – which minimises social loss. Assuming that some optimal λ^* exists: the government acts first and sets the optimal λ^*; the private sector sets expectations as given above conditional on λ^*; and the policymaker sets inflation according to **(4)** conditional on λ^* and its perception of the private sector expectation of inflation.

In order to find if an optimal λ^* exists, and what it is, we re-write **(12)** as;

$$E(L_t|I_{t-1}^T) = \frac{1}{2}\left[\frac{\left(\lambda + \lambda^{*^2}\alpha^2\right)}{(1+\lambda^*\alpha^2)^2}\sigma_\varepsilon^2 + \lambda\lambda^{*^2}\alpha^4\sigma_k^2 + \lambda\sigma_\upsilon^2\right] \qquad \textbf{(13)}$$

In **(13)** we notice that there are terms in both λ^* and λ. This is because although policy will be set using λ^*, the true social loss will be evaluated using λ. Minimising **(13)** with respect to λ^*, we obtain the FOC that implicitly defines the optimally appointed policymaker;

$$\frac{\partial EL(.)}{\partial\lambda^*} = \frac{\alpha^2(\lambda^* - \lambda)}{(1+\lambda^*\alpha^2)^3}\sigma_\varepsilon^2 + \lambda\lambda^*\alpha^4\sigma_k^2 \equiv G(\lambda^*;\lambda,\alpha) \qquad \textbf{(14)}$$

Can we say anything about whether the appointed policymaker places more or less weight on output deviations than society? Setting $\lambda^* = 0$ we have,

$$G(\lambda^* = 0;\lambda,\alpha) = -\alpha^2\lambda\sigma_\varepsilon^2 < 0$$

And setting $\lambda^* = \lambda$, we have,

$$G(\lambda^* = \lambda;\lambda,\alpha) = \lambda^2\alpha^4\sigma_k^2 > 0$$

Because $G(\lambda^* = 0;\lambda,\alpha) < 0$ and $G(\lambda^* = \lambda;\lambda,\alpha) > 0$, we know that at least one solution must exist for λ^*in the interval $[0,\lambda]$. In fact, there is a unique optimum, $\lambda^* < \lambda$, in this interval such that the optimally appointed policymaker will place less weight on the output gap than society would.[(5)] Misperceptions reduce welfare because a misperception causes a wedge to be driven

(5) See Appendix 1 for the proof.

between actual and expected inflation and this causes output to deviate from its natural level. In order to reduce the effect of this distortion on welfare, the policymaker will want to place relatively less weight on output fluctuations than it would want to if output was not distorted by misperceptions. In the absence of misperceptions the optimal $\lambda^* = \lambda$.[6] Through the appointment of a policymaker who is going to be tough on inflation and respond less to output, the expectations of the private sector are altered so as to partially offset the effect of any misperception.

Can we say anything about the cases in which we only have private sector misperceptions? In the case of stochastic misperceptions $G(\lambda^*; \lambda, \alpha)$ is exactly the same as in **(14)**, so the analysis above carries through exactly such that the optimal λ^* is exactly the same. This means that in the presence of unknown private sector misperceptions the optimally appointed policymaker would be the same, whether there are policymaker misperceptions or not. The intuition for this result is that because policymaker misperceptions do not influence inflation, expected inflation or output, monetary policy design cannot help offset their effects.

In the case of premonition, the result is qualitatively the same – appointing a policymaker that will place less weight on output can partially offset the effects of misperceptions. The equations are slightly different however, and the details are provided in Appendix 2.

Our finding, that policymakers should react less to output gap fluctuations when the private sector misperceives the true model of the economy, is very similar to the results of Orphanides and Williams (2003) and Jääskelä and McKeown (2005). And despite these papers using more complex models, the logic driving the results – that inducing extra conservatism in policy keeps inflation expectations closer to actual inflation by making them less responsive to misperceptions – is similar in all three papers.

5 Conclusions

In this paper, we have analysed some possible implications of private sector and policymaker misperceptions about the natural level of output for monetary policy. These misperceptions arise due to asymmetric information. We find that when the private sector misperceives the true level of

(6) Consider equation **(10)** when $k = 0$. In this case it can be easily seen that loss is minimised when $\lambda^* = \lambda$.

natural output the expected loss is unambiguously higher. This result holds whether the policymaker can observe the misperceptions or not, although if it cannot losses are even higher. When we allow interaction between a private sector misperception and a policymaker who may be mismeasuring the true level of natural output, we find that the expected social losses increase further. In all the cases we consider, the policymaker can partially offset the effects of misperceptions by pre-announcing it will place less weight on output gap fluctuations than it otherwise would – by appointing a conservative central banker.

The paper also suggests some interesting avenues for further research. For example, in order to properly assess the ability of monetary policy to react to misperceptions we need to consider where misperceptions come from. Understanding this may help us to describe how policy might act to correct misperceptions, rather than just trying to counteract their uncertain effects.

Appendix 1

In order to prove that a unique solution exists for λ^* in the interval $[0, \lambda]$ we need to rule out multiple solutions. To do this we need to prove that $G(\lambda^*; \lambda, \alpha)$ is monotonic in λ^* for values in the interval $[0, \lambda]$.

$$G(\lambda^*; \lambda, \alpha) \equiv \frac{\alpha^2 (\lambda^* - \lambda)}{(1 + \lambda^* \alpha^2)^3} \sigma_\varepsilon^2 + \lambda \lambda^* \alpha^4 \sigma_{k_t}^2$$

The slope of $G(\lambda^*; \lambda, \alpha)$ is given by;

$$\frac{dG(\lambda^*; \lambda, \alpha)}{d\lambda^*} = \frac{\left[\alpha^2 \left(1 + \lambda^* \alpha^2\right)^3 - 3\alpha^4 \left(1 + \lambda^* \alpha^2\right)^2 (\lambda^* - \lambda)\right]}{\left(1 + \lambda^* \alpha^2\right)^6} \sigma_\varepsilon^2 + \lambda \alpha^4 \sigma_{k_t}^2$$

$$= \frac{1 + \lambda^* \alpha^2 - 3\alpha^2 (\lambda^* - \lambda)}{\left(1 + \lambda^* \alpha^2\right)^4} \sigma_\varepsilon^2 \alpha^2 + \lambda \alpha^4 \sigma_{k_t}^2$$

We now prove that $\frac{dG(\lambda^*; \lambda, \alpha)}{d\lambda^*} > 0$ by contradiction. First, we assume that $\frac{dG(\lambda^*; \lambda, \alpha)}{d\lambda^*} < 0$ for some $\lambda^* = \hat{\lambda}^* \in [0, \lambda]$. Then it must be the case that;

$$\left[1 + \lambda^* \alpha^2 - 3\alpha^2 \left(\widehat{\lambda}^* - \lambda\right)\right] \sigma_\varepsilon^2 \alpha^2 < -\lambda \alpha^4 \sigma_{k_t}^2 \left(1 + \lambda^* \alpha^2\right)^4$$

And as $\left(\hat{\lambda}^* - \lambda\right) < 0$ it must be the case that $LHS > 0$. So,

$$0 < -\lambda \alpha^4 \sigma_{k_t}^2 \left(1 + \lambda^* \alpha^2\right)^4$$

which is a contradiction!

So $\frac{dG(\lambda^*; \lambda, \alpha)}{d\lambda^*} \nless 0$ for some $\lambda^* = \hat{\lambda}^* \in [0, \lambda]$ (and similarly **(17)** shows that $\frac{dG(\lambda^*; \lambda, \alpha)}{d\lambda^*} \neq 0$). Hence $G(\lambda^*; \lambda, \alpha)$ is monotonically increasing in λ^* for values in the interval $[0, \lambda]$.

Appendix 2

We can re-write **(8)** as,

$$E(L_t|I_{t-1}^T) = \frac{1}{2(1+\lambda^*\alpha^2)^2}\left[\left(\lambda + {\lambda^*}^2\alpha^2\right)\sigma_\varepsilon^2 + \left({\lambda^*}^4\alpha^6 + \lambda{\lambda^*}^2\alpha^4\right)k_t^2\right]$$

And differentiating by λ^* gives us the equation for the optimally appointed policymaker,

$$\frac{\partial EL(.)}{\partial\lambda^*} = \frac{\alpha^2}{(1+\lambda^*\alpha^2)^3}\left[(\lambda^*-\lambda)\,\sigma_\varepsilon^2 + \left(2{\lambda^*}^2\alpha^2 + \lambda + {\lambda^*}^3\alpha^4\right)\lambda^*\alpha^2k_t^2\right] \equiv G(\lambda^*;\lambda,\alpha)$$

Setting $\lambda^* = 0$ we have;

$$G(\lambda^*=0;\lambda,\alpha) = -\alpha^2\left(\left(\lambda+\alpha^2\right)\sigma_\varepsilon^2\right) < 0$$

And setting $\lambda^* = \lambda$, we have;

$$G(\lambda^*=\lambda;\lambda,\alpha) = \frac{\lambda^2\alpha^4k_t^2}{\left(1+\lambda\alpha^2\right)} > 0$$

Because $G(\lambda^*=0;\lambda,\alpha) < 0$ and $G(\lambda^*=\lambda;\lambda,\alpha) > 0$, we know that at least one solution must exist for λ^*in the interval $[0,\lambda]$. The function $G(\lambda^*;\lambda,\alpha)$ is quartic in λ^*. In order to prove that a unique solution exists for λ^* in the interval $[0,\lambda]$ we need to rule out multiple solutions. To do this we need to prove that $G(\lambda^*;\lambda,\alpha)$ is monotonic in λ^* for values in the interval $[0,\lambda]$.

$$G(\lambda^*;\lambda,\alpha) \equiv \frac{\alpha^2}{(1+\lambda^*\alpha^2)^3}\left[(\lambda^*-\lambda)\,\sigma_\varepsilon^2 + \left(2{\lambda^*}^2\alpha^2 + \lambda + {\lambda^*}^3\alpha^4\right)\lambda^*\alpha^2k_t^2\right]$$

The slope of $G(\lambda^*;\lambda,\alpha)$ is given by;

$$\frac{dG(\lambda^*;\lambda,\alpha)}{d\lambda^*} = -\left(\left(\lambda^* - \lambda\right)\sigma_\varepsilon^2 + \left(2\lambda^{*^2}\alpha^2 + \lambda + \lambda^{*^3}\alpha^4\right)\lambda^*\alpha^2 k_t^2\right)\frac{3\alpha^4}{\left(1+\lambda^*\alpha^2\right)^4}$$

$$+\left(\sigma_\varepsilon^2 + \left(6\lambda^{*^2}\alpha^2 + \lambda + 4\lambda^{*^3}\alpha^4 k_t^2\right)\right)\frac{\alpha^2}{\left(1+\lambda^*\alpha^2\right)^3}$$

We now prove that $\frac{dG(\lambda^*;\lambda,\alpha)}{d\lambda^*} > 0$ by contradiction. First, we assume that $\frac{dG(\lambda^*;\lambda,\alpha)}{d\lambda^*} < 0$ for some $\lambda^* = \hat{\lambda}^* \in [0, \lambda]$. Then it must be the case that;

$$\left(\left(\hat{\lambda}^* - \lambda\right)\sigma_\varepsilon^2 + \left(2\hat{\lambda}^{*^2}\alpha^2 + \lambda + \hat{\lambda}^{*^3}\alpha^4\right)\hat{\lambda}^*\alpha^2 k_t^2\right)3\alpha^2$$

$$> \left(\sigma_\varepsilon^2 + \left(6\hat{\lambda}^{*^2}\alpha^2 + \lambda + 4\hat{\lambda}^{*^3}\alpha^4\right)\alpha^2 k_t^2\right)\left(1 + \hat{\lambda}^*\alpha^2\right)$$

And as $\left(\hat{\lambda}^* - \lambda\right)\sigma_\varepsilon^2 < 0$ it must be the case that

$$6\hat{\lambda}^{*^3}\alpha^4 + 3\alpha^2\lambda\hat{\lambda}^* + 3\hat{\lambda}^{*^4}\alpha^6 > \left(6\hat{\lambda}^{*^2}\alpha^2 + \lambda + 4\hat{\lambda}^{*^3}\alpha^4\right)\left(1 + \hat{\lambda}^*\alpha^2\right)$$

and therefore that

$$6\hat{\lambda}^{*^3}\alpha^4 + 3\alpha^2\lambda\hat{\lambda}^* + 3\hat{\lambda}^{*^4}\alpha^6 > \left(6\hat{\lambda}^{*^2}\alpha^2 + \lambda + 4\hat{\lambda}^{*^3}\alpha^4\right) + \left(6\hat{\lambda}^{*^3}\alpha^4 + \lambda\hat{\lambda}^*\alpha^2 + 4\hat{\lambda}^{*^4}\alpha^6\right) \quad \textbf{(15)}$$

For **(15)** to be true requires that;

$$2a^2\lambda\hat{\lambda}^* > \hat{\lambda}^{*^4}\alpha^6 + 6\hat{\lambda}^{*^2}\alpha^2 + \lambda + 4\hat{\lambda}^{*^3}\alpha^4 \quad \textbf{(16)}$$

We know that $2\alpha^2\hat{\lambda}^{*^2} > 2\alpha^2\lambda\hat{\lambda}^*$. But **(16)** is only true if

$$0 > \hat{\lambda}^{*^4}\alpha^6 + 4\hat{\lambda}^{*^2}\alpha^2 + \lambda + 4\hat{\lambda}^{*^3}\alpha^4 \qquad \textbf{(17)}$$

which is a contradiction!

So $\frac{dG(\lambda^*;\lambda,\alpha)}{d\lambda^*} \nless 0$ for some $\lambda^* = \hat{\lambda}^* \in [0, \lambda]$ (and similarly **(17)** shows that $\frac{dG(\lambda^*;\lambda,\alpha)}{d\lambda^*} \neq 0$). Hence $G(\lambda^*; \lambda, \alpha)$ is monotonically increasing in λ^* for values in the interval $[0, \lambda]$.

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