The stock market and capital accumulation: an application to UK data

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Abstract

Because of the difficulty in measuring investment in intangible assets and frequent data revisions, estimates based on National Accounts investment data provide an imperfect measure of the capital stock. Following the influential work by Robert Hall for the United States, this paper provides an alternative measure of the UK capital stock based on asset prices. This market-based measure reflects the premise that in fair-valued financial markets the value of firms' securities reflects the value of their productive assets. In line with Hall's results for the United States, the paper suggests that for a range of adjustment costs, depreciation rates and starting values, market-based estimates of the UK capital stock have differed substantially from those based on National Accounts investment data. Despite some advantages over National Accounts based measures, market-based measures are likely to be more volatile, because financial markets' assessment of the value of intangible assets can potentially change rapidly. Nevertheless, they can be a useful cross-check of the National Accounts based measures of the UK capital stock.

Key words: Capital stock, adjustment costs, investment, *Q*.

JEL classification: E44, G12.

Summary

Estimates of the capital stock derived from National Accounts investment data suffer from a number of potential shortcomings. These are related to the difficulty in measuring investment in intangible assets and frequent data revisions. Provided that they are fairly valued, financial markets measure the value of firms' productive assets, ie their capital stock. Being less prone to revisions and arguably better suited to measure intangible assets, such market-based estimates address some of the shortcomings of National Accounts estimates. In his influential work, Robert Hall provides such market-based estimates for the US capital stock and shows that they differ substantially from National Accounts based estimates. His model is based on the well-known result that, under the assumptions of constant returns to scale in technology and in the adjustment cost function and the firm being a price taker, *marginal q*, as derived from the first-order condition of the market value maximising firm, equals *average q*. In this framework, and under certain assumptions about adjustment costs, the volume of the capital stock can be derived by equating *marginal q* to *average q*. This paper applies Hall's model to the United Kingdom to provide a market-based estimate of the UK business sector capital stock. Qualitatively, the results for the United Kingdom mirror those of Hall for the United States, with substantial discrepancies between the market-based and National Accounts based estimates. In particular, market-based estimates of the UK business capital stock were higher in the late 1990s than National Accounts based estimates. These results are robust across a range of different depreciation rates and starting values, and for all but the largest adjustment costs. These differences could reflect financial markets better capturing intangible assets than the National Accounts. However, they could also reflect an asset price bubble or economic rents that the model would mistakenly interpret as intangible assets. The results differ from Hall, in that they show a prolonged period of 'negative intangibles' for the United Kingdom. The sensitivity analysis suggests that this result is qualitatively robust throughout a wide range of adjustment costs, depreciation rates and starting values. In spite of the possible explanations for periods of 'negative intangibles', the length and magnitude of 'negative intangibles' in the United Kingdom are puzzling.

1 Introduction

Statistical agencies commonly estimate the stock of productive capital in the economy from the associated investment flows, the so-called perpetual inventory method (PIM). For example, the Office for National Statistics (ONS) derives annual UK capital stock estimates by sector and asset class from National Accounts estimates of gross fixed capital formation. In these estimates, the aggregate capital stock for a sector, or the economy as a whole, is a sum of the stock of the different assets. In other words, each asset is weighted by the asset price, with the aggregate capital stock thus representing a measure of wealth. The ONS estimates of the UK capital stock are described in detail in Vaze *et al* (2003). In addition Oulton and Srinivasan (2003) provide quarterly estimates. Oulton (2001) argues that the wealth concept of capital is not appropriate for a production function or for a measure of capacity utilisation but that a measure of capital services is needed. Oulton and Srinivasan (2003) provide such a measure — a so-called volume index of capital services (VICS) — for the United Kingdom. Although the VICS and the wealth-based measures of the capital stock differ in the way in which different types and ages of assets are aggregated together,[(1)] and serve different purposes, they share common features: both measures apply the perpetual inventory method to National Accounts investment data. But such National Accounts PIM-based capital stock estimates suffer from a number of potential shortcomings.

First, National Accounts investment data tend to get revised periodically. Because the flow of investment is small relative to the level of the capital stock, such revisions generally do not affect the level of the capital stock by much. But at times these revisions to the investment data can affect capital stock estimates by enough to lead to revisions of the estimate of the supply potential of the economy that are significant for monetary policy purposes.

Second, Nakamura (1999 and 2001) and Lev (2003) argue that National Accounts data ignore much of what could be classified as intangible investment. For example, Nakamura (2001) argues that although the US Bureau of Economic Analysis (BEA) now records software as intangible investment in the National Accounts it omits many others. He lists R&D, movie and book production, designs and blueprints, and the advertising associated with new products as important sources of wealth creation that should be recorded as intangible investment rather than being measured as intermediate goods or services as is currently the case. Nakamura (2001) estimates that, in recent years, the value of investment in intangibles in the United States was in a range of $700 billion to $1.5 trillion per year. Compared to non-residential business investment in tangible assets of $1.1 trillion in 2000, this is large and would — over time — have made a substantial contribution to the US capital stock. We are not aware of similar estimates for the United Kingdom, but qualitatively, the same arguments apply. For example, in the United Kingdom the National Accounts estimate of intangible fixed assets includes mineral exploration, computer software and entertainment and literary or artistic originals. Expenditure on them is part of gross fixed capital formation. But gross fixed capital formation excludes non-produced intangible assets such as patented entities, leases, transferable contracts and purchased goodwill, expenditure on which is recorded as intermediate consumption (ONS (1998, pages 317 and 625)). Some of these intangibles, such as brand capital, create rents for firms but do not represent capital

(1) For details see Oulton (2001).

for the economy as a whole. But other intangibles do form part of the capital stock and — to the extent to which they have become increasingly important to a modern economy — National Accounts based estimates may be severely underestimating total investment and by extension the capital stock. To the extent to which this affects measures of the output gap, it could have repercussions for monetary policy. Nelson and Nikolov (2001) conclude that output gap measurement errors caused monetary policy errors and made a significant contribution to average UK inflation in the 1970s and 1980s. But underestimating intangible investment would have wider macroeconomic implications. Hall (2000) notes that higher investment in intangibles implies higher corporate savings and would require a whole new set of national income and product accounts. As Pickford, Smithers and Wright (2001) point out, these would not only reveal higher US real GDP growth, but also a higher share of investment and profits in GDP.(2)

These problems associated with National Accounts based capital stock estimates suggest that it may be useful to draw on additional information when forming a view about the likely level of the capital stock and/or its development over time. Financial markets can provide this information as firms' valuations will be related to the value of their productive assets, ie their capital stock.

Hall (2001a) formalises this idea in a model that allows inference of the volume of capital from the observed values of securities (ie a market-based capital stock estimate). He uses Hayashi's (1982) result that under the assumptions of constant returns to scale in technology and in the adjustment cost function and the firm being a price taker, *marginal q* equals *average q*. This allows equating Tobin's (1969) *q* (*average q* in equation **(1)**) to the first-order condition of the market value maximising firm (*marginal q* in equation **(2)**) and solving for the volume of the capital stock:

$$q_t = V_t \Big/ P_t^{i,gdp} k_t \qquad \textbf{(1)}$$

$$q_t = 1 + C_i(i_t, k_{t-1}) \qquad \textbf{(2)}$$

Here k_{t-1} is the end-of-period capital stock in volume terms (ie the capital stock available for production during period *t*), $P_t^{i,gdp}$ is the price of capital goods, V_t is the market value of the firm and $C_i(i_t, k_{t-1})$ is the derivative of the constant return to scale adjustment cost function $C(i_t, k_{t-1})$ with respect to investment.(3)(4)

The market-based estimate corresponds to the wealth concept of the capital stock rather than the capital services concept. The natural benchmark for comparison is thus a National Accounts PMI-based measure that also corresponds to the wealth concept of the capital stock. Hall (2001a) finds that the market-based estimate of the US non-farm, non-financial business sector capital stock differs substantially from US National Accounts capital stock data. In particular, during the 1990s, the market-based capital stock estimate by far exceeds the National Accounts based estimate.

(2) They calculate that, based on Hall's estimates, US GDP growth was understated by at least 1.3% per annum during the 1990s.

(3) Hall (2001a) uses the linear homogeneity of the adjustment cost function to write adjustment costs as $C = c(i_t / k_{t-1}) k_{t-1}$.

(4) In Hall (2001a), the real price of capital is one, ie the investment deflator equals the GDP deflator.

Hall argues that this discrepancy reflects US corporations owning substantial amounts of intangible capital not recorded in corporate accounts or government statistics. Hall expands on this concept of intangibles in Hall (2000), where — focusing on the 1990s — he points to the large discrepancy between many companies' book and market value, which he interprets as reflecting a body of technological and organisational know-how — so-called 'e-capital' — that is not measured elsewhere.[(5)] [(6)]

It is important to note though that Hall's results critically rely on the above assumptions of perfect competition, constant returns to scale and market efficiency. As a result, the view that financial market data may be better suited to value intangible assets than the National Accounts is not universally shared.

With regard to the assumption of market efficiency Bond and Cummins (2000) argue that the logic underlying the advantage of market-based measures relative to National Accounts based measures is circular: '*...accounting principles for intangible assets are unsatisfactory, making it difficult for market participants to value companies; but strong market efficiency is assumed in order to assign a value to intangibles*'. This criticism highlights an important issue: the results of Hall's model are based on the premise that there are no bubbles in security markets.[(7)] If an increase in the stock market were to reflect fundamentals,[(8)] this would warrant an increase in the capital stock. However, if an increase in the stock market were to reflect a bubble rather than changes in fundamentals an increase in the capital stock would not be warranted. Despite this, the model would record a bubble-driven rise in the stock market as an increase in q and subsequently — because of the delay caused by the adjustment cost function — as an increase in quantity of capital (k). So in the presence of bubbles, the model would measure the capital stock incorrectly. Hall (2001a and 2001b) discusses the issue of valuation in some detail and concludes that there is '*...nothing in the data that demonstrates affirmatively a systematic failure of the standard valuation principle—that the value of the stock market is the present value of future cash payouts to shareholders*'. But there is no consensus in the academic literature, with others (eg Shiller (2000)) suggesting that stock market bubbles have occurred frequently.

With regard to the assumption of perfect competition Lafourcade (2003) points out that in addition to firms' capital stock, market valuations could also reflect capitalised future monopoly rents. Although these rents may be derived from intangibles such as brand capital or patents, they do not represent productive capital for the economy as a whole. The presence of such economic rents presents another possible distortion to the estimate of the stock of intangibles. Cummins (2000) argues that the assumption of perfect competition with constant returns to scale may not be a good representation of the real world, in particular for many of the companies in the

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(5) These results are not contradicted by Hall's analysis of the flow of earnings in Hall (2002). The reason is that the flow of expected earnings also includes the flow of value that firms receive from their stock of intangible capital.

(6) This definition sounds somewhat similar to technical progress and in the discussion of Hall's (2000) paper, Daniel Sichel wondered whether Hall's measure of capital might in part pick up technological improvements. In theory Hall's measure should be free of such 'contamination'. If an increase in total factor productivity (TFP) were to lead to a rise in the value of the firm, the increase in q should lead firms to add to the capital stock. So the k measured should reflect only tangible and intangible capital, rather than elements of TFP.

(7) Bubbles may mean different things to different people. In the context of this paper they are defined as the value of an asset not reflecting the present discounted value of expected future payoffs from the asset.

(8) Fundamentals could reflect warranted changes in future earnings or time-varying risk premia. Lettau and Ludvigson (2002) discuss the implications of time-varying risk premia on q-theory in some detail.

IT sector cited in Hall (2000). If imperfect competition was a feature of the economy as a whole rather than of some individual firms, the difference between the National Accounts and the market-based capital stock measure would — at least in part — reflect the value of economic rents. In this case, the estimate of the capital stock of intangible assets would be biased upwards.

Relaxing either the assumption of market efficiency or perfect competition and constant returns to scale would result in Hall's approach measuring the capital stock incorrectly. Trying to quantify these possible effects by relaxing some of the assumptions in Hall's model is left for future work. This paper instead focuses on applying Hall's original approach to UK data, to generate a market-based estimate of the business sector capital stock in the United Kingdom.

The remainder of this paper is organised as follows: Section 2 derives Hall's model and calibrates it for the United Kingdom. The data are described in Section 3 and the results are presented in Section 4. Section 5 provides a sensitivity analysis and Section 6 concludes.

2 The model

2.1 The q model

Tobin's (1969) *q*-model can be derived from the value maximisation problem of the firm.[(9)] Assuming perfect foresight, the firm's ex-dividend market value on date *t,* V_t is given by:[(10)]

$$V_t = \frac{D_{t+1}}{1+r} + \frac{V_{t+1}}{1+r} \qquad \textbf{(3)}$$

where *D* are payouts to bond and equity holders,[(11)] and *r* is the real discount rate (assumed to be constant for simplicity).[(12)] Repeated substitution and ruling out self-fulfilling speculative asset price bubbles (ie $\lim_{T\to\infty}\left(\frac{1}{1+r}\right)^T V_{t+T} = 0$), yields equation **(4)**. Thus the firms' market value on date *t* is the present discounted value of all expected future payouts.

$$V_t = \sum_{s=t+1}^{\infty}\left(\frac{1}{1+r}\right)^{s-t} D_s \qquad \textbf{(4)}$$

Payouts are defined as the residual of output after labour, investment and capital adjustment costs.[(13)]

$$d_s = F(k_{s-1}, l_s) - w_s l_s - C(i_s, k_{s-1}) - p_s i_s \qquad \textbf{(5)}$$

where *d* represents real (ie payouts deflated by the GDP deflator), $F(\bullet)$ is the production function, *k* is the end of period capital stock, *l* is labour, *i* is investment, *w* is the real wage, $C(\bullet)$

(9) See Hasset and Hubbard (1996) or the textbook treatment in Obstfeld and Rogoff (1998). Note that in an uncertain world, *r* would be a risky rate.
(10) Ex-dividend because the firm has already paid the dividend at *t,* d_t.
(11) It is important to note that since *V* is the value of the firm—and not just the value of equity—*D* represents not just dividends but all payouts to all owners. This includes cash flows through the repurchase of securities. For example, Robertson and Wright (2003) show that net share repurchases and cash-financed acquisitions have been an important source of cash flow to equity holders in recent years.
(12) We deflate nominal values by the GDP deflator.
(13) Following Hall (2001a), we ignore taxes and investment allowances.

is the adjustment cost function and p is the *real* price of investment goods, ie the ratio of the investment deflator to the GDP deflator. This formulation is more general than that in Hall (2001a), who assumes that the real price of investment goods is 1. As will be shown later, this is a restrictive assumption in the light of actual trends in the data.(14)

In real terms, equation **(4)** becomes **(4')**:

$$v_t = \sum_{s=t+1}^{\infty} \left(\frac{1}{1+r} \right)^{s-t} \{ F(k_{s-1}, l_s) - w_s l_s - C(i_s, k_{s-1}) - p_s i_s \} \tag{4'}$$

The firm maximises the present value of current and future payouts, where $\tilde{v}_t = v_t + d_t$ is the value of the firm before payouts to all owners (ie cum dividend):

$$\tilde{v}_t = v_t + d_t = \sum_{s=t}^{\infty} \left(\frac{1}{1+r} \right)^{s-t} \{ F(k_{s-1}, l_s) - w_s l_s - C(i_s, k_{s-1}) - p_s i_s \} \tag{6}$$

subject to the constraint that gross investment is given by

$$i_t = k_t - (1-\delta) k_{t-1} \tag{7}$$

where δ is the depreciation rate.

Denoting the Lagrange multiplier (or shadow price of an additional unit of investment) as q, the Lagrangian is:

$$L = \sum_{s=t}^{\infty} \left(\frac{1}{1+r} \right)^{s-t} \{ F(k_{s-1}, l_s) - w_s l_s - C(i_s, k_{s-1}) - p_s i_s - q_s [k_s - (1-\delta) k_{s-1} - i_s] \} \tag{8}$$

Under perfect foresight, the first-order conditions are:

$$\frac{\partial L}{\partial i_s} \longrightarrow q_s = p_s + C_i(i_s, k_{s-1}) \tag{9}$$

$$\frac{\partial L}{\partial l_s} \longrightarrow w_s = F_l(k_{s-1}, l_s) \tag{10}$$

$$\frac{\partial L}{\partial k_s} \longrightarrow \frac{F_k(k_s, l_{s+1}) - C_k(i_{s+1}, k_s) + (1-\delta) q_{s+1}}{(1+r)} = q_s \tag{11}$$

Equation **(9)** states that the *marginal* value of an additional unit of investment equals its marginal cost. This result differs slightly from equation **(1)** as Hall (2001a) assumes that the real price of investment goods, p, is one. For s=t, we therefore define *marginal q* as $\breve{q}_t = \frac{q_t}{p_t}$ and equation **(9)** becomes

$$\breve{q}_t = 1 + \frac{C_i(i_t, k_{t-1})}{p_t} \tag{9'}$$

(14) We only have a one-good economy in which a relative price is not clearly defined. To allow for the downward trend in the real price of investment observed in the data, we assume a non-specified segmentation that allows for a relative price.

Marginal q is unobservable, but Hayashi (1982) has shown that under the assumption of constant returns to scale in technology and in the adjustment cost function, and the firm being a price taker, the unobservable *marginal q* equals *average q*. Put differently, the shadow value of an additional unit of capital equals the shadow value of existing capital. It can be shown that *average q* equals:[(15)]

$$q_t = \frac{v_t}{k_t} \qquad \textbf{(12)}$$

and $\breve{q}_t = \frac{q_t}{p_t} = \frac{v_t}{p_t k_t} = \frac{P_t^{GDP} v_t}{P_t^i k_t} = \frac{V_t}{K_t}$ [(16)] **(13)**

where P_t^i is the investment deflator. Thus *average q* is the value of the firm's *existing* capital (V) relative to its replacement cost (K). Combining equations **(9')** and (**13)** the model becomes:

$$\frac{v_t}{p_t k_t} = 1 + \frac{C_i(i_t, k_{t-1})}{p_t} \qquad \textbf{(14)}$$

To make the model operational, we need to specify the adjustment cost function $c(\bullet)$, and calibrate a number of parameters.

2.2 Adjustment costs

Adjustment costs represent a deadweight loss the firm has to pay over and above the actual costs of purchasing new capital goods. Hall (2001a) specifies the adjustment cost function as piecewise quadratic. It can be written as:

$$C(i_t, k_{t-1}) = \frac{\alpha^+}{2} P\left(\frac{i_t}{k_{t-1}}\right)^2 k_{t-1} + \frac{\alpha^-}{2} N\left(\frac{i_t}{k_{t-1}}\right)^2 k_{t-1} \qquad \textbf{(15)}$$

where P and N are the positive and negative parts. This quadratic formulation implies an increasing marginal cost of investment, with respect to the level of investment ($C_i(\bullet)>0$ and $C_{ii}(\bullet)>0$). It also implies that a faster pace of change in investment requires a greater than proportional rise in adjustment costs.[(17)] To capture irreversibility of investment Hall (2001a) assumes that the downward adjustment-costs parameter α^- is positive and substantially larger than the upward parameter α^+.[(18)]

Differentiating equation **(15)** with respect to investment:

$$C_i = \alpha\left(\frac{i_t}{k_{t-1}}\right) \qquad \textbf{(16)}$$

[(15)] See appendix.
[(16)] Hayashi (1982) suggests that the measure of q should be modified to take account of taxes and depreciation allowances. He calls this measure 'modified q' ($\widetilde{q}$).
[(17)] See Dixit and Pindyck (1994, page 385).
[(18)] Dixit and Pindyck (1994, page 385) provide examples of different adjustment cost functions. For example α^-=-α^+ would imply that the full costs of purchase could be recovered and investment is fully reversible. $\alpha^->0$ as in Hall's (2001a) paper implies that the firm must pay costs such as restoration costs for sites or redundancy costs for workers, when sites or machines are shut down.

where α is either α^- or α^+. α^+ represents the doubling time for the capital stock in the face of a doubling of $\breve{q}$, whereas α^- is the time required for the capital stock to halve, when $\breve{q}$ has halved.[19]

Equations **(7)**, **(14)** and **(16)** can now be combined to solve for *k*.[20] [21]

$$k_t = -\frac{(p_t - \alpha(1-\delta))k_{t-1}}{2\alpha} + \frac{\sqrt{([(p_t - \alpha(1-\delta))k_{t-1}]^2 - 4\alpha v_t k_{t-1})}}{2\alpha} \qquad \textbf{(17)}$$

It is worthwhile pointing out at this stage that the model assumes that the value of securities represents the present discounted value of expected future cash flows, ie it rules out asset price bubbles. This issue will be discussed in more detail later.

2.3 Calibration

Hall (2001a) uses a fixed annual depreciation rate of 10%, ie a value for δ = 0.026 in the quarterly model. The adjustment costs parameter α^+ is set at eight quarters and α^- at ten times α^+, ie 80 quarters. In other words 50% of the adjustment takes place within one year in the case of α^+ and 5% in the case of α^-. The value for α^+ is based on his interpretation of Shapiro's (1986) findings, but Hall (2001a) does not expand on the calibration of α^- or δ.[22]

We initially follow this calibration to allow for a better comparison of our results for the United Kingdom to Hall's results for the United States. We subsequently allow the calibration of the UK model to differ from Hall (2001a) in a number of places.

First, we do not use Hall's (2001a) depreciation rate of 10%. Instead — to facilitate comparison of our market-based estimate with the National Accounts PMI-based estimates by Oulton and Srinivasan (2003) and those of the ONS — we provide estimates based on the variable depreciation rate implied by these National Accounts based measures.

Second, as pointed out in the previous section, we do not restrict the real price of capital goods (*p*) to equal one. Given the rapid fall in the price of investment goods relative to the GDP deflator in both the United Kingdom and the United States (Chart 1) we think that this more general specification may be preferable. However, it turns out that re-estimating Hall's (2001a) model with

(19) This becomes more obvious by substituting equation **(16)** into **(9')** and rearranging into an investment equation: $\frac{i_t}{k_{t-1}} = \frac{1}{\alpha} p_t(\breve{q}_t - 1)$. A doubling of $\breve{q}$, say from one to two will initially raise the investment-capital ratio by $\frac{1}{\alpha} p_t$. To cumulate to a unit increase (assuming p_t=1 for simplicity), the flow must continue at this level for α periods.

(20) Multiplying out $\frac{1}{p_t}\alpha\left(\frac{k_t - (1-\delta)k_{t-1}}{k_{t-1}}\right) + 1 = \frac{v_t}{p_t k_t}$ yields $\alpha k_t^2 + (p_t - \alpha(1-\delta))k_{t-1}k_t - v_t k_{t-1} = 0$. Remembering that the solution for a quadratic formula of the form $ax^2 + bx + c = 0$ is given by $\bar{x}_1, \bar{x}_2 = -\frac{b}{2a} \pm \frac{\sqrt{(b^2 - 4ac)}}{2a}$, *k* can be solved for as $k = -\frac{(p_t - \alpha(1-\delta))k_{t-1}}{2\alpha} \pm \frac{\sqrt{([(p_t - \alpha(1-\delta))k_{t-1}]^2 + 4\alpha v_t k_{t-1})}}{2\alpha}$.

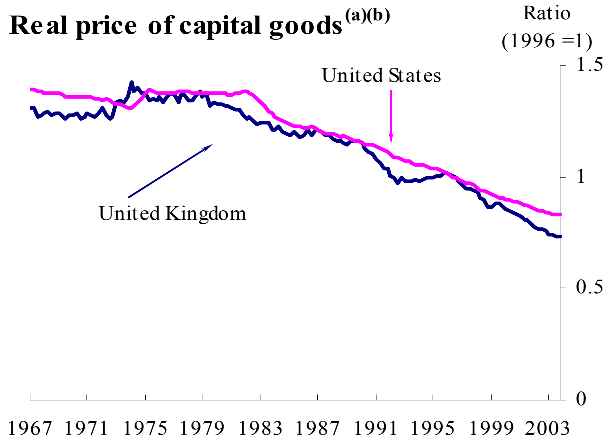
(21) This formula differs from Hall (2001a) as we allow $p_t \neq 1$ and as a consequence deflate *v* by the GDP deflator, rather than by the investment deflator.

(22) Appendix C in Hall (2001a) provides details on the interpretation of Shapiro (1986). Hall (2001a) also provides a sensitivity analysis for adjustment cost parameters α^+= 32 quarters and α^-= 320 quarters.

$p \neq 1$ does not have a noticeable impact on the capital stock estimates (Chart 2). In particular, the difference in the level of the capital stock appears small towards the end of the sample.[23]

Chart 1

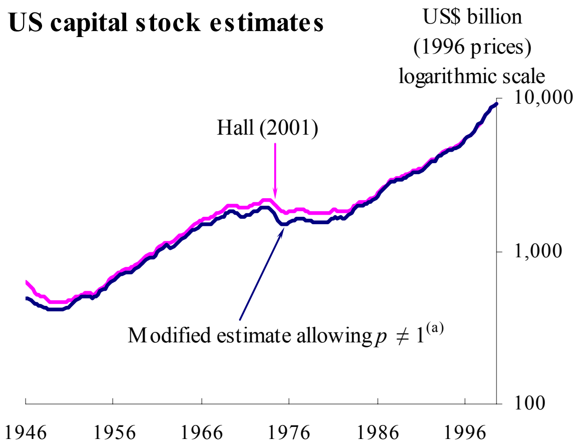
Real price of capital goods[(a)(b)]



(a) United Kingdom: Business investment deflator/GDP deflator.
(b) United States: Non-residential investment deflator/GDP deflator.

Chart 2

US capital stock estimates



(a) Implicit price deflator of non-residential investment divided by the GDP deflator.

3 The data

3.1 Measures of value: balance sheet data versus stock market capitalisation

Balance sheets usually show all the firm's assets on one side of the balance sheet and all liabilities (including the residual claim of shareholders) on the other side. The accounting framework presented in Hall (2001a) differs from this convention. To arrive at an estimate for the value of the firm's *productive* assets, he subtracts the financial assets of the firm (eg cash, accounts receivables etc) from the firm's financial liabilities. This is illustrated in Table 1 which shows non-financial assets (eg plant, equipment, land and intangibles) on the left-hand side and *net* financial liabilities (which include shareholders claims) on the right-hand side of the modified 'balance sheet'.[24] These *net* financial liabilities serve as an estimate for the value of the firm's productive assets (V).[25]

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[23] The degree to which this more general specification will quantitatively alter the results depends on whether the 'true' investment deflator (ie the deflator which accounts for all intangible assets) has moved in line with the National Accounts investment deflator which only includes a subset of intangible assets. Using the deflator of those intangible assets recorded in the UK National Accounts (mineral exploration, computer software and entertainment, literary or artistic originals) as an indication for overall price trends of intangibles suggests that this has not been the case. Indeed, rather than showing a similar fall as in Chart 1, the ratio of the UK National Accounts intangible investment price deflator to the GDP deflator has changed little over the past ten years.
[24] See also Hall (2001b).
[25] The US Flow of Funds Accounts do not report the market value of long-term bonds and Hall makes an adjustment himself (see Appendix B in Hall (2001a) for details). We do not need to make a similar adjustment to our data, since UK financial balance sheet data value securities other than shares, such as money market instruments and bonds, already at current market prices including the value of accrued interest (see ONS (1998, page 400)).

Table 1

MODIFIED 'BALANCE SHEET'

Non-financial assets	**Net financial liabilities**
Value of tangible assets	Value of financial liabilities
Land (+)	Shareholder's equity (+)
Plant (+)	Debt (+)
Equipment (+)	Accounts payable etc (+)
Inventories (+)	
Value of intangible assets	Value of financial assets
Organisational capital (+)	Cash (-)
Intellectual property (+)[a]	Accounts receivable (-)
Brand capital (+)[a]	Other financial claims on others (-)

[a] These create rents for the firm, but do not represent capital for the economy as a whole. Hall expands on the definition of intangibles in Hall (2000) by introducing the concept of 'e-capital'. He defines this e-capital as a body of technical and organisational know-how.

Hall (2001a) uses Financial Accounts balance sheet data as the basis for his market-based estimates of the capital stock. The disadvantage of this is that Financial Accounts are subject to revisions and are not as timely as the market capitalisation of a broad equity index.[26] But the alternative, a market-based measure based on stock market capitalisation, also suffers from a number of shortcomings. On the one hand, stock market capitalisation understates the value of productive assets in the United Kingdom by excluding (a) non-quoted firms in the United Kingdom, (b) firms listed abroad that have operations in the United Kingdom and (c) other liabilities such as bonds. On the other hand, it overstates the value of productive assets in the United Kingdom by (a) valuing all assets (including financial assets) and (b) including multinationals listed in the United Kingdom. The latter problem is also present in Financial Accounts data, but overall, the arguments favour the use of Financial Accounts balance sheet data to estimate firm value. We therefore follow Hall (2001a) and use data from the UK Financial Accounts as the basis for the market-based capital stock estimates.

3.2 Financial Accounts data

In the United Kingdom, the business sector is comprised of private, non-financial corporations (PNFCs), financial corporations and public corporations. We follow this definition when using the Financial Accounts data to construct the market-based estimate of the value of UK productive assets. In particular, the inclusion of public corporation in the overall measure avoids distortions resulting from large-scale privatisations in the 1980s, when formerly public corporations were reclassified as PNFCs.[27]

[26] For example, business sector net financial liabilities for 2002 Q1 were estimated at £1,759 billion in August 2002. They have since been revised up by almost 9%. But because of the presence of adjustment costs, revisions to the most recent data will primarily affect the estimate of the price of existing capital (*q*) and have less of an effect on the estimate of the quantity of capital (*k*).

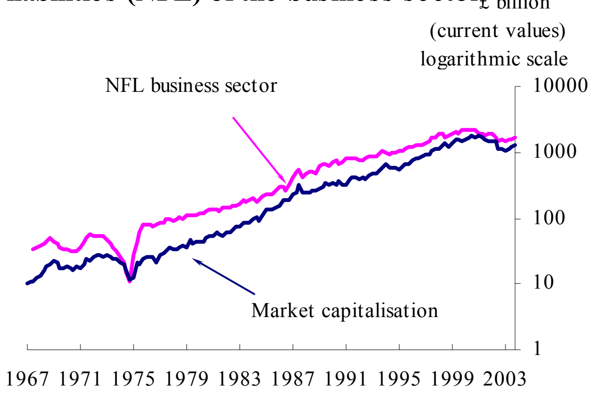
[27] Vaze *et al* (2003, page 12) provides a summary of the largest privatisations.

Quarterly data consistent with the European System of National Accounts (*ESA 95*) for all data series used are available back to 1987. For the period 1975 Q2 to 1986 Q4 we used the equivalent quarterly data based on ESA 79. The level of both series is very close for the business sector, in particular in 1987, the date when we splice the two series together. Data for 1966 to 1975 are only available annually (end-year levels). To obtain a quarterly series for that period we use a linear interpolation of the annual data.[28] This will mean that the results prior to 1975 are subject to greater uncertainty than the post-1975 results.

Chart 3 shows these data and compares them to a measure of stock market capitalisation (measured by the Datastream Total Market Index). It shows that despite a noticeable difference in the level, the predominance of equity financing in the United Kingdom means that both series move closely together over the whole sample period. The data highlight the large volatility in equity prices during the late 1960s and early 1970s. Because of the presence of adjustment costs in the model, this variability will primarily affect the values of q and will have less of an effect on the market-based capital stock estimates.

Chart 3

Stock market capitalisation and net financial liabilities (NFL) of the business sector



3.3 Benchmarks for comparison

We construct three National Accounts PMI-based measures of the UK business sector capital stock as benchmarks for comparison to the market-based measure of the business sector capital stock.

The first measure (henceforth Benchmark 1) uses Hall's assumption of a fixed depreciation rate of 10% per year and constructs UK business sector capital stock estimates from the perpetual inventory method. We take a starting value of £498.2 billion for the end of 1966. This estimate is the sum of the ONS net capital stock estimates for PNFCs, public corporations and non-financial corporations at 2000 prices.[29] The second measure (Benchmark 2) is based on the ONS's annual data for net capital stock estimates for PNFCs, public corporations and non-financial corporations at 2000 prices. These data are then interpolated to a quarterly

[28] Annual data are not available for the total financial assets and liabilities of life assurance, pension funds and other financial institutions between 1966 and 1981, so we use the growth rates for banks (the biggest constituent of the financial institutions group) to extrapolate these.
[29] ONS four-letter codes are GUAP, GUBQ and GSVY.

frequency that takes account of the profile of business investment during the year. The third measure (Benchmark 3) is based on an updated estimate of the ONS1 capital stock estimate by Oulton and Srinivasan (2003). Because their measure relates to the whole-economy capital stock excluding dwellings, we make some adjustments to calculate a comparable estimate for the business sector capital stock.(30) These benchmarks — which all correspond to the wealth concept of the capital stock — are shown in Chart 4. The difference in level and the time profile primarily reflects the assumptions about depreciation rates, which are shown in Chart 5.(31)

Chart 4

Benchmark UK National Account based capital stock estimates

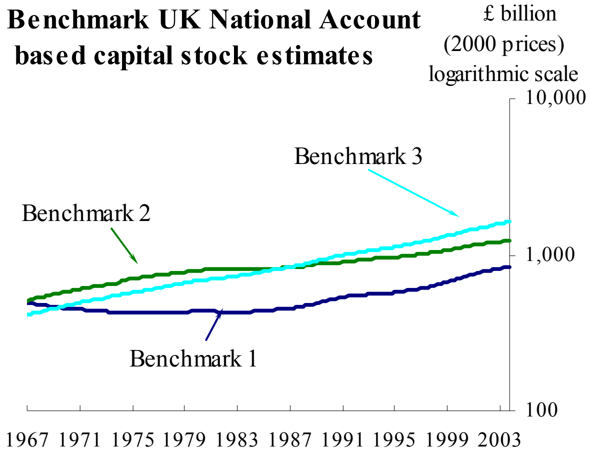
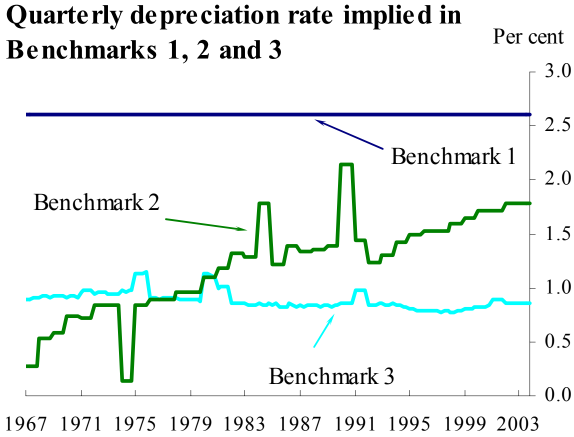


Chart 5

Quarterly depreciation rate implied in Benchmarks 1, 2 and 3



4 Results

4.1 Comparing UK and US estimates

Chart 6 shows a variant of Hall's (2001a) market-based estimates for the US non-farm, non-financial capital stock and an estimate derived from the perpetual inventory method using US National Accounts data.(32) It shows that a large discrepancy between the two estimates has developed since the 1990s. Chart 8 shows the comparable estimate of q.

(30) First we assume that the ratio of the whole-economy capital stock less dwelling to the business capital stock in 1967 Q1 is the same as the ratio of whole-economy less dwellings gross fixed capital formation to business sector capital formation. This provides us with an estimate of the business sector capital stock in 1967 Q1. We then calculate the depreciation rate implied in their original capital stock estimate and assume that the same depreciation rate applies to the business sector, too. Based on the starting value, the depreciation rate and gross fixed capital formation in the business sector, a time series for the business sector capital stock can be calculated using the perpetual inventory method.

(31) It should be noted that in contrast to the market-based capital stock estimates, the National Accounts PIM-based estimates (Benchmark 1, 2 and 3) do not make *explicit* assumptions about adjustment costs. But they will *implicitly* capture adjustment costs: the investment recorded in the National Accounts is the result of firms' investment decisions which in turn will be influenced by adjustment costs.

(32) We use Hall's (2001a) estimates derived form the perpetual inventory method and his estimate for the non-farm, non-financial corporate sector based on the q-model. The latter only differs from that presented in Chart 10 of Hall (2001a) in that we set q=1 in 1967 Q1 (as opposed to 1946 Q1).

Chart 6

US capital stock estimates

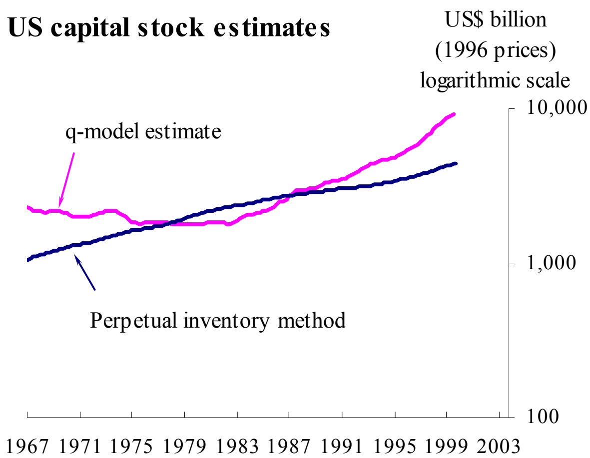


Chart 7

UK capital stock estimates
(Using p=1 and δ fixed)

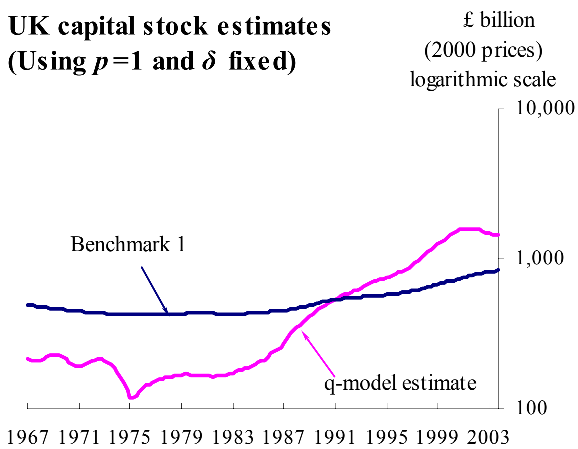


Chart 8

US q estimate

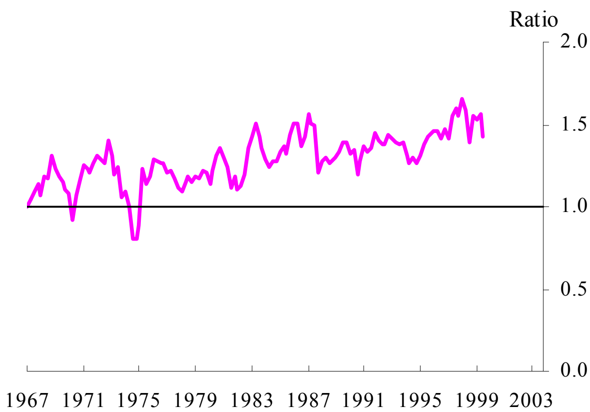
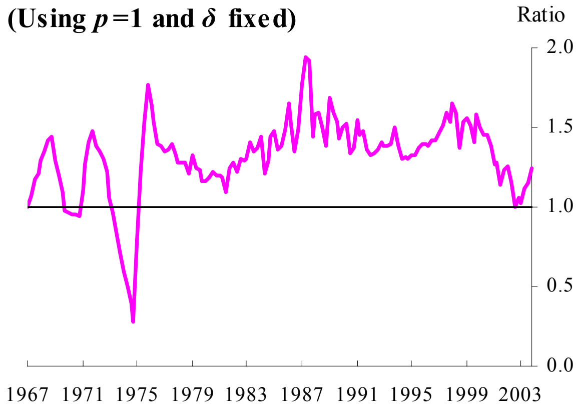


Chart 9

UK q estimate
(Using p=1 and δ fixed)



Charts 7 and 9 above replicate Hall's (2001a) results for the United Kingdom. The market-based estimates are based on Hall's (2001a) calibrations for the US data (ie a fixed depreciation rate of 10% and adjustment cost parameters α^+= 8 quarters and α^-= 80 quarters and p = 1), with a starting value of q = 1 in 1967 Q1. The National Accounts estimate used for comparison (Benchmark 1) has been described above. The estimate of q at the start of the sample appears more volatile in the UK data (Chart 8) than in the US data (Chart 9). To a large degree this reflects the volatility in UK equity prices during this period (see Chart 3 above). But as discussed in Section 3.2, it may in part also reflect less reliable Financial Accounts data prior to 1975.

Focusing on the period from the mid-1970s, the results are very similar to Hall's (2001a) findings for the United States. In particular, the capital stock estimates derived from the q-model are more than twice as large as those derived from the perpetual inventory method towards the end of the 1990s. Hall (2001a) argues that the large discrepancy between the National Accounts and market-based estimates of the capital stock reflects US corporations owning substantial amounts of intangible capital not recorded in corporate accounts or government statistics. The arguments in favour of and against this proposition were discussed in the introduction.

But Chart 7 also shows a prolonged period from the start of the sample to the late 1980s during which the market-based estimate is well below Benchmark 1. Hall (2001a) finds similar results for the United States. However, the magnitude and duration differs substantially, with the market-based estimate for the United States (Chart 6) falling only slightly below the National Accounts PIM-based capital stock estimate. Hall acknowledges that such 'negative intangibles' are somewhat puzzling, but provides a number of possible explanations for this phenomenon.

First, Hall (2001a) refers to work by Greenwood and Jovanovic (1999). They note that although the overall effect of IT on productivity should be positive, the stock market falls in the early 1970s coincided with the implications of the information technology (IT) revolution becoming apparent. To rationalise this apparent paradox, Greenwood and Jovanovic (1999) argue that existing firms with human and physical capital tied to existing practices were not able or willing to exploit the benefits of IT and lost in value. At the same time, new firms destined to exploit the IT revolution had not yet been founded. The result was a fall in stock market valuations. Greenwood and Jovanovic (1999) support this view with the observation that in aggregate, the stock market value of firms present in 1968 fell sharply over the next three years and never recovered. Moreover, the rise in the overall stock market capitalisation was driven by firms which entered after 1968. A second related argument could be made with reference to the oil price shocks that hit the global economy in the 1970s (Bailey (1981)). These may have made much of the existing capital stock obsolete. Third, Hall (2001b) points out that shareholders have the last claim on corporate revenue and may during the early 1970s have lost to other stakeholders such as suppliers, workers, managers or governments. In addition, Smithers and Wright (2000) argue that the low market valuations relative to the capital stock recorded in National Accounts may reflect a systematic under-depreciation in the national and corporate accounts. All these factors could conceivably have been more important in the United Kingdom than in the United States, thereby contributing to a longer period of 'negative intangibles' in the United Kingdom. Even so, the length and magnitude of 'negative intangibles' suggested in Chart 7 is puzzling.

4.2 Comparing UK estimates

The calibration above serves to highlight the main difference between market-based and National Accounts PIM-based measures of the capital stock. But the depreciation rate used is well above that used in the widely used National Accounts PMI-based measures of the UK business sector capital stock represented by Benchmarks 2 and 3. To construct market-based measures that are comparable with Benchmarks 2 and 3 we need to make a number of changes to Hall's calibration.

First, we allow the real price of capital goods to differ from unity. Charts 10 and 11 show that, as for the United States (Chart 2), allowing the real price of investment goods to differ from unity does not affect the market-based estimates of the capital stock by much. Despite the small quantitative difference, we prefer to use this more general specification for the remainder of this paper.

Chart 10

UK capital stock estimates (δ fixed at 10% p.a.)

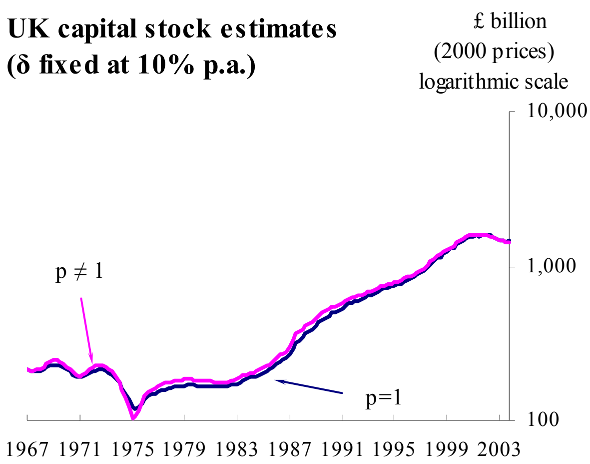
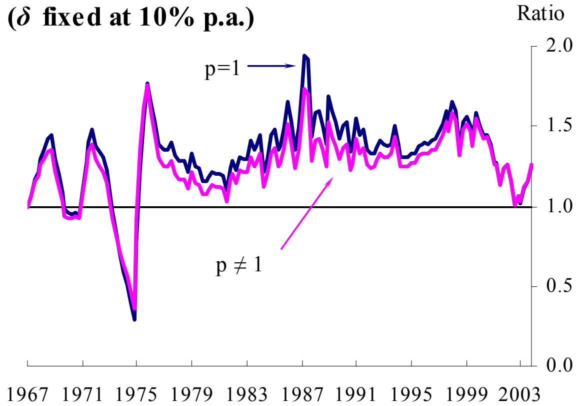


Chart 11

UK *q* estimate (δ fixed at 10% p.a.)



Second, while we maintain Hall's assumptions about the adjustment cost parameters, we use the variable depreciation rates for Benchmarks 2 and 3 shown in Chart 5 above. Since the depreciation rates implicit in Benchmarks 2 and 3 change over time according to shifts in the asset composition of the net capital stock, the capital goods deflator used as the numerator in the real price of investment should allow for the effect of such compositional changes, too. The business investment deflator allows for the effects of such compositional changes.

These estimates are shown in Charts 12 and 13, with the related values for $\tilde{q}$ shown in Charts 14 and 15. A number of features stand out. First, despite some visible differences, the market-based estimates in Charts 12 and 13 and the values for $\tilde{q}$ in Charts 14 and 15 are very similar despite different depreciation rates used (see Chart 5). In part this reflects the choice of a log scale, which makes the levels differences less visible. But at an average of 1.2% and 0.9% respectively, the quarterly depreciation rates in Benchmarks 2 and 3 are not very different in any case, resulting in only small level differences between the two estimates. Second, compared to Chart 7, Charts 12 and 13 show an even larger difference between the market-based estimate and the National Accounts PMI-based estimates. Indeed, the market-based estimates only exceed the National Accounts based estimates at the end of the equity market boom in the late 1990s. This positive gap has narrowed in recent years as the effect of the sharp falls in equity prices between 2001 and 2003 has reduced the market valuations of firms' productive assets. Because of the presence of large negative adjustment costs, this is primarily reflected in a sharp fall in $\tilde{q}$ (Charts 14 and 15), with little effect so far on the market-based estimate of the UK capital stock.

Chart 12

UK capital stock estimates
(Using $p \neq 1$ and δ from BM2)

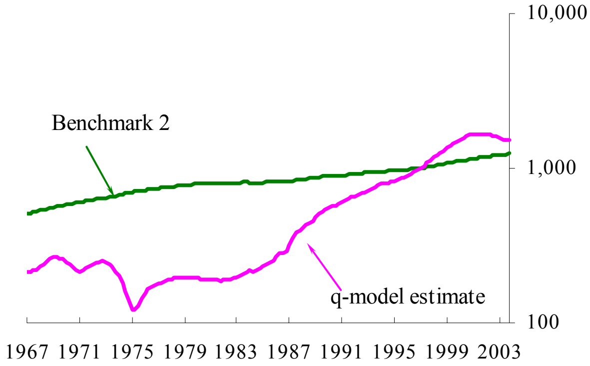


Chart 13

UK capital stock estimates
(Using $p \neq 1$ and δ from BM3)

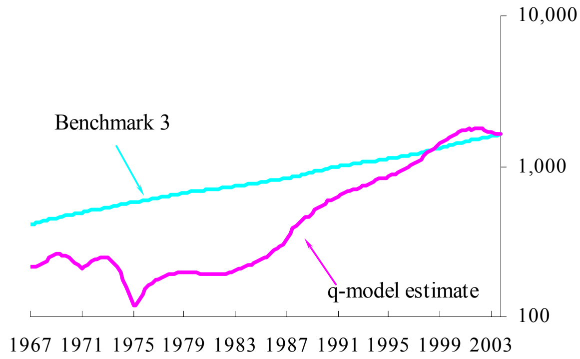


Chart 14

UK $\tilde{q}$ estimate
(Using $p \neq 1$ and δ from BM2)

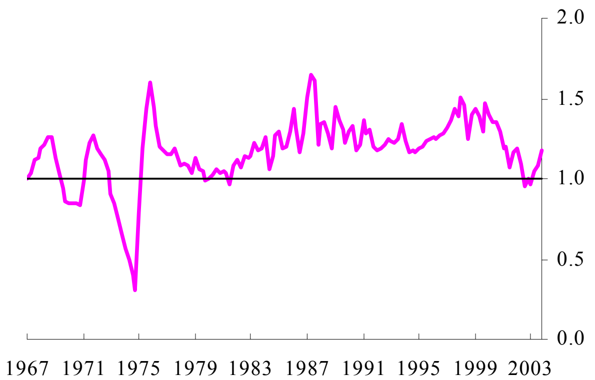
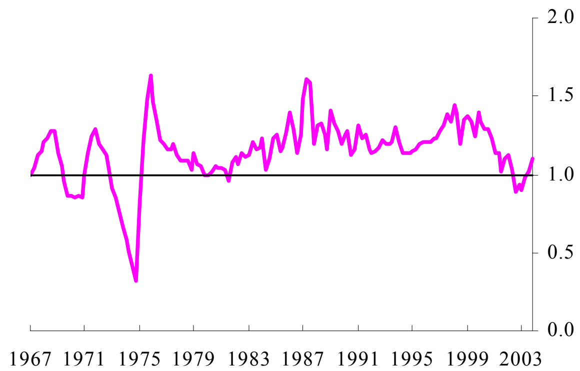


Chart 15

UK $\tilde{q}$ estimate
(Using $p \neq 1$ and δ from BM3)



5 Robustness

5.1 Some caveats

Stock market efficiency and the requirement of the absence of bubbles have been discussed already. But there are a number of further potential caveats.

First, the market-based capital stock estimates will be affected by the choice of adjustment cost parameters. The literature says little about likely parameter values for α^+ and α^- or whether they should be constant over time and Hall (2001a) does not address this issue in much detail. To complicate matters further, it is likely that the adjustment cost parameters α^+ and α^- depend on the type of capital installed. So if the share of investment in intangible capital in total investment has increased rapidly as claimed by Hall (2001a) and Nakamura (2001), the aggregate adjustment cost parameters may not be constant over time. Focusing on the 1990s, Hall (2000) distinguishes between different types of capital — physical capital and intangible capital that he labels ‘e-capital’ — but assigns the same adjustment cost parameters to both types. This may not be appropriate, because scrapping intangibles could be less costly than scrapping tangibles. For example, while replacing a machine or other tangible capital is costly and a large positive parameter for α^- may be justified, deleting a software programme entails little costs. By the same token, the upward adjustment cost parameter α^+ could differ for intangibles. Focusing on computer adjustment costs, Mun (2002) finds that technology adoption accounts for only 28% of the marginal adjustment cost of computer investment, while quantity expansion explains the remaining 72%. But it is conceivable that creating the knowledge base to embark on a new R&D project could be more costly than say building a new laboratory. It is possible that aggregation may reduce such differences in the adjustment cost parameters for intangible assets and tangible assets as a whole. But changes in the overall adjustment cost parameter for tangible and intangible investments over time cannot be ruled out.

Second, the estimates will be affected by the choice of starting value. But this is also the case with any measure based on the perpetual inventory method. It therefore does not pose a problem for the comparison between the two measures.

Third, we need to make assumptions about the depreciation rate δ. If information technology and knowledge-based capital had become increasingly important over time, the faster depreciation of this type of capital would suggest that a depreciation rate based on National Accounts data — which may omit many intangible investments — would be too low.

An explicit model of different types of capital is beyond the scope of this paper. But the sensitivity analysis below may provide some gauge about the quantitative importance of the assumptions about adjustment costs, the starting value and the depreciation rate.

5.2 Robustness to different adjustment costs

This section evaluates the robustness of the results for different adjustment costs. We conduct two experiments around the capital stock estimates and the corresponding values for $\tilde{q}$ that were shown in Charts 12 and 14 respectively. In all instances, the starting capital stock for 1967 Q1 is set such that $\tilde{q} = 1$, the depreciation rate is the variable rate implied in Benchmark 2 and the real price of capital is allowed to differ from unity.

In the first experiment we maintain the assumption that $\alpha^{-} = 10\alpha^{+}$ and compare the base case of $\alpha^{+} = 8$ as shown in Charts 12 and 14 to scenarios in which $\alpha^{+} = 2$ and $\alpha^{+} = 32$ (the latter is the ‘extreme case’ suggested by Hall (2001a)). The results are shown in Charts 16 and 18 below. Not surprisingly, the level of the capital stock and the value of $\tilde{q}$ are very sensitive to the choice of the adjustment cost parameters. In the case of low adjustment costs (ie $\alpha^{+} = 2$), the rapid rise of the UK stock market in the late 1990s and the subsequent fall from 2001 feed very quickly into the market-based estimates of the capital stock, with the value of $\tilde{q}$ close to unity.

Chart 16

UK capital stock estimates for different adjustment costs (Using $p \neq 1$ and δ from BM2)

£ billion (2000 prices) logarithmic scale

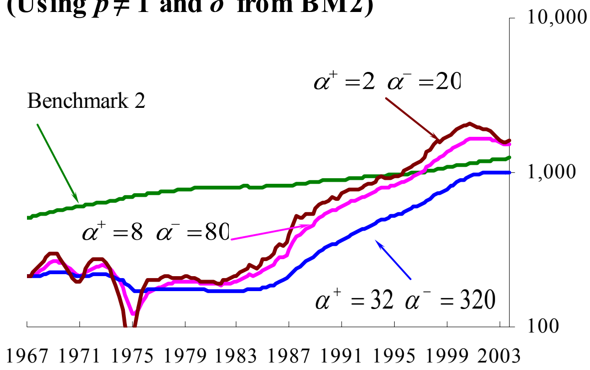


Chart 17

UK capital stock estimates for different adjustment costs (Using $p \neq 1$ and δ from BM2)

£ billion (2000 prices) logarithmic scale

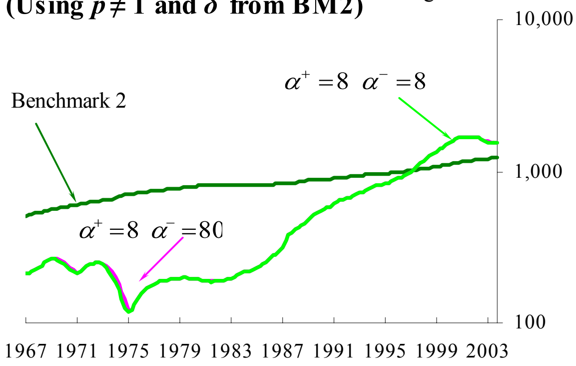


Chart 18

UK $\tilde{q}$ estimate for different adjustment costs (Using $p \neq 1$ and δ from BM2)

Ratio

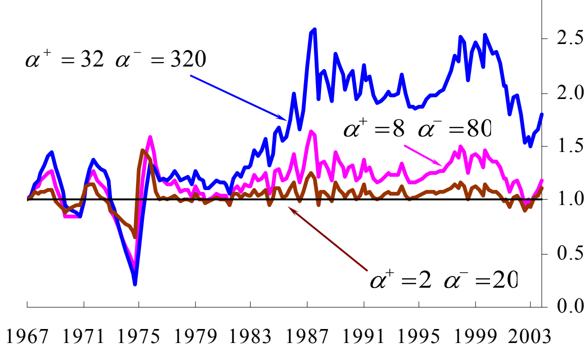
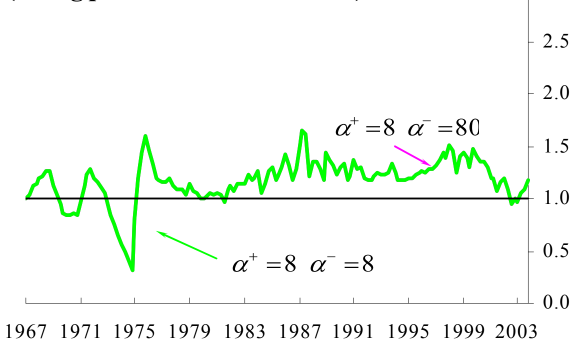


Chart 19

UK $\tilde{q}$ estimate for different adjustment costs (Using $p \neq 1$ and δ from BM2)

Ratio



In the second experiment, we compare the capital stock estimates and corresponding values for $\tilde{q}$ in the base case $\alpha^+ = 8$ and $\alpha^- = 80$ to the case in which $\alpha = \alpha^+ = \alpha^- = 8$. The results are shown in Charts 17 and 19 above. It turns out that there is hardly any difference between the estimates. This result is robust to other specifications of the adjustment cost function.[(33)] It reflects the fact that negative adjustment costs take effect very rarely. In other words, the falls in value of firms' productive capital have very rarely been large enough to suggest any scrapping of capital. Indeed, the model suggests that most falls in the value of firms reflect gross investment levels below the levels required for replacing depreciating capital. While true for the aggregate level, it has to be acknowledged that an analysis at the industry or firm level would likely alter this latter result, as certain sectors of the stock market (eg IT) have experienced stronger falls. Thus, estimating the model on industry data and constructing a 'bottom-up' estimate of the economy-wide capital stock from the sum of such industry estimates would likely differ somewhat from the 'top-down' estimate presented in this paper. Because of data constraints, such a 'bottom-up' estimate is not straightforward and has not been pursued further in this paper.[(34)]

---

(33) An alternative standard formulation is $C = \frac{\alpha}{2}\left[\frac{i_t}{k_{t-1}} - (g+\delta)\right]k_{t-1}$, where g is the steady-state growth rate and δ the steady-state depreciation rate, implying no adjustment costs in the steady state.

(34) With daily or monthly volatility averaging out over longer periods, a higher frequency estimate may also trigger the negative adjustment costs more often. Because of data constraints a high-frequency estimate is not feasible.

5.3 Robustness to different starting values

This section evaluates the robustness of the results to different starting values. As in the previous section, we conduct the experiment around the capital stock estimates and the corresponding values for $\tilde{q}$ that were shown in Charts 12 and 14. In all instances, the depreciation rate is the variable rate implied in Benchmark 2, α^{+}= 8 and α^{-}= 80 and the real price of capital can differ from unity.

In the experiment we compare the results of setting the starting capital stock for 1967 Q1 such that $\tilde{q}=1$ (as shown in Charts 12 and 14), with setting the starting value to equal the National Accounts PIM-based estimate in Benchmark 2. The results are shown in Charts 20 and 21. They show that despite the substantial difference in the starting value in 1967 Q1 (a difference of over 200%) the effects die out quickly for both the capital stock estimates and the estimates of $\tilde{q}$. This result is familiar from similar experiments using National Accounts PIM-based estimates that also tend to be little affected by the choice of starting values. The length for which the difference persists will be affected by the choice of adjustment cost parameter and the depreciation rates. But overall, the results are robust across different adjustment costs and depreciation rates.

Chart 20

UK capital stock estimates for different starting values (Using $p \neq 1$ and δ from BM2)

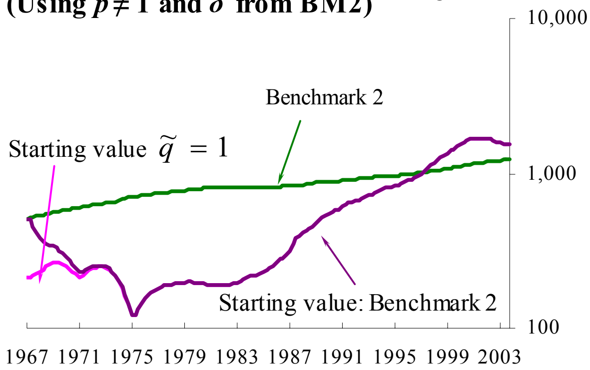
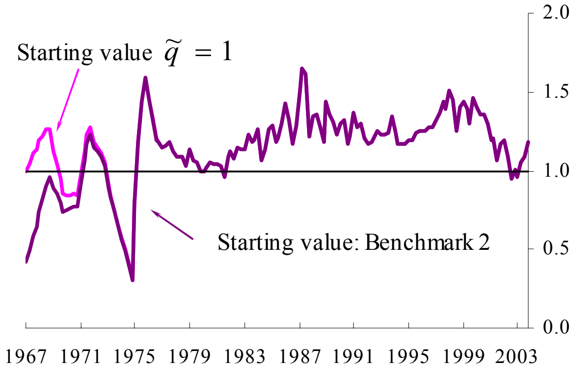


Chart 21

UK $\tilde{q}$ estimate for different starting values (Using $p \neq 1$ and δ from BM2)



5.4 Robustness to different depreciation rates

This section evaluates the robustness of the results for different depreciation rates. As in the previous sections, we conduct the experiments around the capital stock estimates and the corresponding values for $\tilde{q}$ that were shown in Charts 12 and 14. In all instances the starting capital stock for 1967 Q1 is set such that $\tilde{q}=1$, α^{+}= 8 and α^{-}= 80 and the real price of capital is allowed to differ from unity.

In the experiment we compare the results from setting the depreciation rate to the fixed depreciation rate used in Hall (2001a), the variable depreciation rate implicit in Benchmark 2 and the variable depreciation rate implicit in Benchmark 3. These depreciation rates are shown in Chart 5 above. The results in Charts 12 to 15 already indicated that the results will not change qualitatively by much. This is also reflected in Charts 22 and 23, which show that although different depreciation rates affect the *level* of the market-based estimate of the business sector capital stock and the value of $\tilde{q}$, the *time profile* changes little.

Chart 22

UK capital stock estimates for different depreciation rates (Using $p \neq 1$ and $\alpha^{+}= 8$, $\alpha^{-}= 80$)

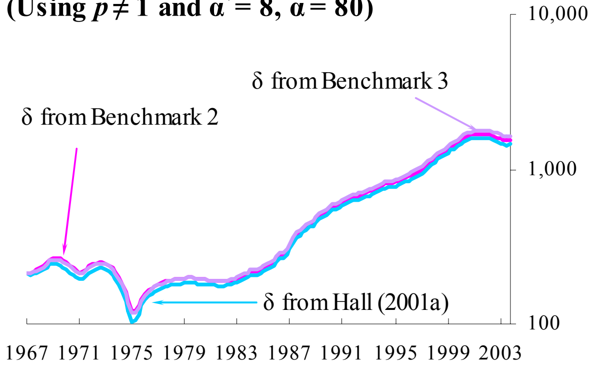
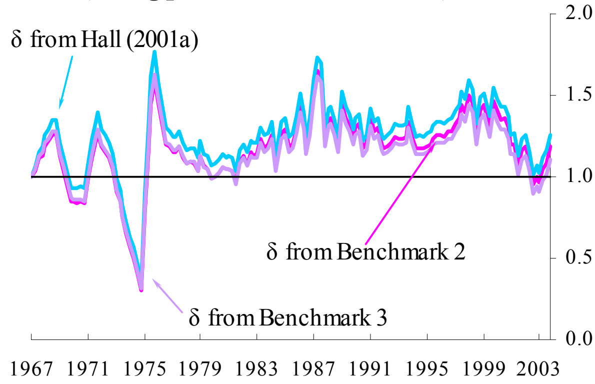


Chart 23

UK $\tilde{q}$ estimate for different depreciation rates (Using $p \neq 1$ and $\alpha^{+}= 8$, $\alpha^{-}= 80$)



6 Conclusion

Market-based capital stock estimates have a number of advantages over the traditional National Accounts PIM-based measures. In particular, it has been argued that they may be better suited to measure intangible assets. On the other hand, they are likely to be more volatile, because financial markets' assessment of the value of intangible assets can potentially change rapidly. Moreover, the market-based approach would incorrectly interpret asset price bubbles or economic rents as part of the capital stock. Nevertheless, market-based capital stock estimates can be a useful cross-check of the National Accounts based measures of the UK capital stock.

This paper applies the model used by Hall (2001a) to provide a market-based estimate of the UK business sector capital stock. Qualitatively, our results for the United Kingdom mirror those of Hall for the United States, with substantial discrepancies between the market-based and National Accounts based estimates. In particular market-based estimates of the UK business capital stock have been higher in the late 1990s than National Accounts PMI-based estimates. These results are robust across a range of different depreciation rates and starting values, and for all but the largest adjustment costs. These differences could reflect financial markets better capturing intangible assets than the National Accounts. However, they could also reflect an asset price bubble or economic rents that the model would mistakenly interpret as intangible assets.

Our results differ from Hall (2001a), in that they show a prolonged period of 'negative intangibles' for the United Kingdom. Moreover, the sensitivity analysis suggests that this result is qualitatively robust throughout a wide range of adjustment cost, depreciation rates and starting values. The impact of IT and of oil shocks on company valuations have been put forward as possible explanations for the gap between the PIM and the market-based capital stock estimates. Nevertheless, the length and magnitude of such 'negative intangibles' in the United Kingdom is puzzling.

Appendix: Deriving average *q*[35]

Multiplying equation **(11)** for *s=t* by k_t

$$\frac{F_k(k_t,l_{t+1})k_t - C_k(i_{t+1},k_t)k_t + (1-\delta)q_{t+1}k_t}{(1+r)} = q_t k_t \qquad \textbf{(A.1.1)}$$

With $F_l(k_t,l_{t+1}) = w_{t+1}$ from equation **(10)** and under constant returns to scale, ie $F(k_t,l_{t+1}) = F_k(k_t,l_{t+1})k_t + F_l(k_t,l_{t+1})l_{t+1}$ equation **(A.1.1)** can be written as:

$$\frac{F(k_t,l_{t+1}) - w_{t+1}l_{t+1} - C_k(i_{t+1},k_t)k_t + (1-\delta)q_{t+1}k_t}{(1+r)} = q_t k_t \qquad \textbf{(A.1.2)}$$

Using equation **(7)** one can rewrite **(A.1.2)** as

$$\frac{F(k_t,l_{t+1}) - w_{t+1}l_{t+1} - C_k(i_{t+1},k_t)k_t - q_{t+1}i_{t+1} + q_{t+1}k_{t+1}}{(1+r)} = q_t k_t \qquad \textbf{(A.1.3)}$$

Substituting equation **(9)**

$$\frac{F(k_t,l_{t+1}) - w_{t+1}l_{t+1} - C_k(i_{t+1},k_t)k_t - p_{t+1}i_{t+1} - C_k(i_{t+1},k_t)i_{t+1} + q_{t+1}k_{t+1}}{(1+r)} = q_t k_t \qquad \textbf{(A.1.4)}$$

Under constant returns to scale in adjustment costs, ie $C(i_{t+1},k_t) = C_k(i_{t+1},k_t)k_t + C_i(i_{t+1,}k_t)i_{t+1}$

$$\frac{F(k_t,l_{t+1}) - w_{t+1}l_{t+1} - C(i_{t+1},k_t) - p_{t+1}i_{t+1} + q_{t+1}k_{t+1}}{(1+r)} = q_t k_t \qquad \textbf{(A.1.5)}$$

Iterating for $q_{t+i}k_{t+i}$ and assuming a transversality condition on that variable

$$\sum_{s=t+1}^{\infty}\left(\frac{1}{1+r}\right)^{s-t} F(k_{s-1},l_s) - w_s l_s - C(i_s,k_{s-1}) - p_s i_s = q_t k_t \qquad \textbf{(A.1.6)}$$

The right-hand side of equation **(A.1.6)** equals equation **(4')**, so:

$$v_t = k_t q_t \qquad \textbf{(A.1.7)}$$

So $q_t = \frac{v_t}{k_t} = \frac{V_t / P_t^{gdp}}{K_t / P_t^i} = \frac{V_t}{\frac{1}{p_t}K_t}$, where P_t^i, P_t^{gdp} and p_t are the investment deflator, the GDP deflator and the real price of investment goods. So $\breve{q}_t$ is:

$$\breve{q}_t = \frac{q_t}{p_t} = \frac{v_t}{p_t k_t} = \frac{V_t}{K_t} = \frac{V_t}{P_t^i k_t} \qquad \textbf{(A.1.8)}$$

[35] This derivation follows Obstfeld and Rogoff (1998).

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