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Fiscal rules for debt sustainability in emerging markets: the impact of volatility and default risk

Adrian Penalver and Gregory Thwaites

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Abstract

The determinants of public debt dynamics – real interest rates, the real exchange rate, output growth and the primary fiscal balance – are typically more volatile in emerging market economies than in industrialised countries. Capital markets also typically demand higher interest rates from emerging markets when their debt dynamics deteriorate. This paper considers how these characteristics affect the choice of fiscal policy rules in emerging markets.

We estimate an econometric model of the determinants of public debt dynamics on Brazilian data and use this model to simulate the effect of different fiscal policy rules for future paths of debt. We then derive the set of fiscal policy rules which stabilise public debt dynamics. We find that macroeconomic forecast uncertainty and feedback among the endogenous variables (principally from the debt-GDP ratio to interest rates) force the policy rule to be significantly more responsive to changes in public debt. Rules that would stabilise debt in a fully known world may not do so when the policymaker is faced with a realistic pattern of shocks. The method we employ may be a useful addition to the toolkit of domestic and international policymakers when assessing fiscal rules and debt sustainability.

Summary

The prospects of receiving full payment of emerging market sovereign debt cannot be established with certainty. In emerging market economies (EMEs), key macroeconomic variables – the primary budget balance, economic growth, inflation, domestic and foreign interest rates and the exchange rate – are typically volatile, making it difficult to predict the future with confidence. These macroeconomic variables are also usually correlated. For example, an adverse terms-of-trade shock can slow output growth, result in an exchange rate depreciation and raise the risk premium on interest rates, all of which will worsen a sovereign's debt position. The volatility, correlation and persistence of shocks in emerging markets mean that assessing debt sustainability on a single future path of these variables is too simplistic. Forecasts based solely on historical averages of these variables may therefore erroneously neglect a chance that sovereign debt and fiscal policy are unsustainable.

Using a simple econometric model estimated on a representative EME, this paper measures how uncertainty about the future and the effect of the risk of sovereign default on interest rates alters the probability of future debt to GDP outcomes. Simulations of this model under alternative fiscal policy regimes show that any stabilising fiscal policy must react strongly to innovations in the debt-GDP ratio. Forecast uncertainty and feedback from the debt level to real interest rates impose material constraints on the set of fiscal policy rules which stabilise debt.

These techniques and analysis have practical policy implications. A quantitative analysis of the uncertainty that surrounds debt projections could help the IMF when assessing members' debt sustainability before agreeing to financial assistance programmes. It could also support IMF surveillance of fiscal policy and thereby contribute to crisis prevention. The technique may also be useful to policymakers in EMEs when determining their medium-term fiscal policy strategy. It would be particularly useful if a country is considering the introduction of fiscal policy rules.

1 Introduction

Assessing the sustainability of emerging market sovereign debt is not a precise exercise. As explained in Ferrucci and Penalver (2003), the key macroeconomic variables – the primary budget balance, economic growth, inflation, domestic and foreign interest rates and the exchange rate – are often volatile making it difficult to predict the future with confidence. This macroeconomic volatility is partly attributable to poor policymaking and the economic and financial crises that can occur as a result. But emerging markets are also generally more vulnerable to terms of trade changes, which are more volatile than in industrialised countries (see Hausmann (2004)). Shocks to a country's terms of trade can directly affect sovereign debt sustainability when royalties from commodity exports are a major source of revenue. They can also affect debt sustainability indirectly via the exchange rate if a significant proportion of the sovereign's debts are denominated in foreign currency.

As well as being volatile, movements in these macroeconomic variables are also usually correlated. For example, an adverse terms-of-trade shock can slow output growth, depreciate the exchange rate and raise the risk premium on interest rates, all worsening the sustainability of the sovereign's debt position. These feedback mechanisms can magnify the uncertainty associated with a given pattern of shocks.

The volatility, correlation and persistence of shocks in emerging markets mean that assessing debt sustainability on a single future path of these variables is too simplistic. The use of historical averages of these variables or central forecast projections – the primary method used by the IMF (IMF (2003b)) – is likely to lead to overly optimistic judgements about sovereign debt sustainability. These misjudgements can have practical consequences if a sovereign subsequently defaults. The IMF's assessment of fiscal underperformance in programme countries revealed that adverse growth and exchange rate shocks were important factors in explaining a large gap between actual and projected public sector debt ratios (IMF (2004)). As a result, policymakers in these countries had a mistaken impression about the sustainability of their policies.[1]

To avoid the prospect of default, therefore, those setting fiscal policy should be cognisant of the potential for correlated and persistent shocks to undermine debt sustainability. In the aftermath of

(1) The IMF has recognised the weakness in their approach and are making some steps to revise their method of assessing debt sustainability (see Celasun, Debrun and Ostry (2005)).

the Asian crisis, Dornbusch (1998) argued that every member of the IMF should be required to perform a probabilistic debt sustainability analysis. Intuitively, the more volatile the underlying shocks, the more conservative fiscal policy needs to be to build up a buffer against a run of adverse outcomes and/or the more reactive policymakers need to be to the first signs of trouble. In recent years a number of countries have formalised this intuition by adopting explicit fiscal policy rules (see Kopits (2004)).[2] By limiting the circumstances in which sovereigns can run fiscal deficits, these fiscal policy rules are intended to add credibility to their commitment to debt sustainability.

This paper has two purposes.

- First, it extends the technique developed by Ferrucci and Penalver (2003) and Garcia and Rigobon (2005) to quantify the risks to sovereign debt sustainability from the volatility, correlation and persistence of shocks. Both these papers used simple dynamic macroeconometric models to produce probability distributions of the future path of sovereign debt. The current model innovates with respect to this previous work by incorporating an effect from the degree of indebtedness on the interest rates the government must pay. To illustrate the technique we estimate a simple model of the Brazil economy to produce probabilistic forecasts of the debt-GDP ratio.
- Second, it demonstrates that the volatility, correlation and persistence of macroeconomic shocks limit the feasible set of debt-stabilising fiscal policy rules. The results from the model of Brazil show that forecast uncertainty and feedback from the debt level materially restrict the set of fiscal policy rules which stabilise debt.

The structure of the paper is as follows. Section 2 explains our method of estimating a model to quantify this framework. Section 3 discusses a class of simple fiscal rules which can be used to promote debt sustainability, and uses the empirical model to conduct comparative-dynamic simulations to show the degree to which uncertainty and default risk affect the set of stabilising fiscal policies. Section 4 discusses the policy implications of these results, and Section 5 concludes.

(2) Kopits and Symansky (1998) define a fiscal policy rule as a permanent constraint on fiscal policy expressed in terms of a summary indicator of fiscal performance, such as the budget deficit, borrowing, debt or a major component thereof.

2 Empirical method

The starting point for debt sustainability analysis is the familiar equation of motion for real debt D[3] as a function of the real primary budget surplus PB and the real interest rate r:

$$D_t = (1 + r_t)D_{t-1} - PB_t \quad \textbf{(1)}$$

where debt is measured at the end of a period. This simple equation can be extended by splitting sovereign debt into domestic and foreign currency denominated debt. Dividing through by real GDP (which grows at rate g_t), linearising, and denoting these ratios by lower-case letters, the equation of motion can be rewritten as

$$d_t = \left(1 + (1 - \phi_t)\, r_t^d + \phi_t \left(r_t^f + \Delta s_t\right) - g_t\right) d_{t-1} - pb_t \quad \textbf{(2)}$$

where r^f is the foreign real interest rate, Δs_t is the change in the real exchange rate and ϕ_t is the share of the public debt denominated in foreign currency.[4] To use equation **(2)** to make projections about the evolution of the debt-GDP ratio therefore requires estimating possible paths for r_t^d, r_t^f, Δs_t, g_t and pb_t. And in order to perform a full stochastic simulation, we require a method which can capture the volatility, correlation and persistence of shocks to these variables. There are several options to choose from. One could employ an estimated structural macroeconomic model such as Caporale *et al* (1998) but these models rely on specific and contentious assumptions about how the variables in the model interact. A further alternative would be to apply the value-at-risk technique from corporate finance to a country's balance sheet (see Barnhill and Kopits (2003)). Our method is to capture and approximate these relationships with a vector autoregression (VAR). Similar methods have been used in Ferrucci and Penalver (2003) and Garcia and Rigobon (2005). The key change with respect to these papers is to condition the variables on the debt-GDP ratio in the specification to allow for feedback effects from the impact of increasing indebtedness on default risk, and therefore the real interest rate.

There is a range of techniques available to estimate the parameters of the VAR. In particular, one could estimate a panel VAR across a sample of EMEs. However, we have little reason to believe that the parameters are stable across countries and, given the likely small sample size for which data would be available, a random coefficients model would quickly use up the few available degrees of freedom. Instead, we illustrate the technique with data from one country, estimating a VAR on quarterly Brazilian data from 1999 Q2 - 2005 Q1.

(3) The analysis here uses real local currency variables, but equally could be done in nominal terms.
(4) Assumed for the purpose of the exercise to be fixed.

Brazil is an interesting example because over the sample period it paid a large and volatile risk premium in external credit markets – the JPMorgan EMBI spread ranged between 372 and 2451 basis points, and averaged 858. This suggests that, over the sample period, the markets viewed Brazil's default risk as time-varying, and material on average. The start date was chosen to begin after the flotation of the real in early 1999. To test for stationarity, we run ADF tests on each of the series, choosing the lag length of the test with the Schwarz information criterion. We can reject the null hypothesis of a unit root at the 10% level or below for all of the variables except the external real interest rate.(5) Test statistics and lag lengths are shown in Table B in the appendix.

The VAR is specified as

$$x_t = A_0 + A_1 x_{t-1} + A_2 d_{t-1} + \varepsilon_t$$

$$x_t = [g_t, r_t^d, r_t^f, pb_t, s_t]$$

In the vector x, g is the change in log GDP, r^d and r^f are *log* domestic and foreign real interest rates, s is the *log* real US$ exchange rate (defined such that a depreciation is a rise in s), d is the public debt-GDP ratio.(6) The innovation with respect to Ferrucci and Penalver (2003) and Garcia and Rigobon (2005) is the inclusion of the d_{t-1} term designed to capture the effect of changes in government indebtedness on the probability of default and the risk premium demanded by creditors. Note that specifying the model in terms of the logarithm of real interest rates has two key effects. First, the model will never predict negative real interest rates. Second, the convex (exponential) relationship between the real interest rate and the debt-GDP ratio means that a given increase in the debt-GDP ratio has a successively larger effect on interest rates as debt increases. This captures the idea that the expected probability of default rises with the debt-GDP ratio at an increasing rate. However, this specification has no particular economic justification relative to other functional forms with these two properties.

The technique, rather than the specific parameter estimates, are the emphasis of this paper. However, the estimated VAR system merits some discussion and is shown in Table A. The parameter estimates imply that a 1 percentage point increase in the debt-GDP ratio raises the external real interest rate by around 6% and weakens the real exchange rate by 2%; these parameters are significantly positive at standard levels. There is some evidence that the primary balance increases in response to an increase in the debt-GDP ratio (the central estimate of the coefficient is 0.2, but the corresponding t-statistic is only 1.3), and a counterintuitive but also weak

(5) This last finding may be related to the steady decline in world interest rates observed over the sample period.
(6) Longer lags can be used where appropriate and if the data permit.

finding that a higher debt-GDP ratio is associated with faster GDP growth. The size and significance of these parameter estimates supports the innovation of including lagged debt in the VAR.

Table A: VAR parameter estimates

	t statistics in parentheses g	r^f	r^d	s	pb
g_{t-1}	0.018	-7.245	-6.195	-0.451	0.234
	[0.074]	[-1.474]	[-1.304]	[-0.224]	[0.587]
r^f_{t-1}	-0.002	1.0274	-0.073	0.095	-0.017
	[-0.222]	[7.178]	[-0.530]	[1.612]	[-1.495]
r^d_{t-1}	0.003	-0.101	0.738	-0.057	0.027
	[0.370]	[-0.705]	[5.338]	[-0.972]	[2.357]
s_{t-1}	-0.047	-1.701	-0.143	0.291	-0.012
	[-1.761]	[-3.069]	[-0.267]	[1.282]	[-0.273]
pb_{t-1}	0.063	-0.542	-6.198	1.062	-0.150
	[0.494]	[-0.204]	[-2.414]	[0.975]	[-0.697]
constant	0.141	5.142	-0.169	2.404	-0.003
	[1.455]	[2.565]	[-0.087]	[2.924]	[-0.019]
d_{t-1}	0.183	5.786	0.617	2.138	0.228
	[1.806]	[2.755]	[0.304]	[2.482]	[1.338]
Adjusted R-squared	0.122	0.757	0.713	0.797	0.173
Standard error	0.008	0.169	0.164	0.069	0.014
Mean dependent	0.006	-2.268	-2.221	4.900	0.033
Standard deviation dependent	0.009	0.344	0.306	0.154	0.015

3 Comparative dynamics

3.1 Baseline model

Combining the results from the estimated VAR for the Brazilian economy with the linearised transition equation for the debt-GDP ratio (equation **(2)**) we can produce a joint distribution of future paths for GDP growth, foreign interest rates, domestic interest rates, changes in the exchange rate (all in real terms) and the primary balance-GDP ratio. We employ one and five-year historical averages of the domestic and foreign marginal interest rates respectively (denoted $\overline{r}_t^d$ and $\overline{r}_t^f$), to reflect approximately the typical average maturities of these debts in Brazil. The existence of multi-period debt smooths the pass-through of marginal interest rate shocks to debt dynamics.[7]

3.2 Changes in fiscal policy

Our estimated model contains an equation for fiscal policy because the primary fiscal balance is estimated as a function of the other variables in the model. In this section we want to analyse how the forecast distribution for the debt-GDP ratio changes if we alter the fiscal policy function. This amounts to changing the parameters in the reduced-form primary balance equation. But changing one equation in an interdependent system may give misleading results, unless certain restrictive statistical and economic conditions are satisfied.

From a *statistical* point of view, the coefficients in each of our reduced-form VAR equations are functions of the unknown structural parameters. There are fewer estimated reduced-form coefficients than underlying structural parameters, so we need some non-sample information to identify them. Without identifying the structural parameters, we cannot be sure that changing the coefficients in one equation of the reduced-form system has no implications for the others.

A sufficient (but not necessary) condition for changes in the primary balance equation to have no econometric implication for the other equations is that there are no contemporaneous structural dependencies between the primary balance and the other variables. This condition is also sufficient (but not necessary) to produce zero covariance between the reduced-form residuals in the primary balance equation and the other equations. Economics does not provide strong priors as to the true sign of these covariances. On the one hand, automatic stabilisers would induce a positive contemporaneous correlation between the primary balance and GDP growth, on the other hand, shocks to GDP growth affect the debt-GDP ratio and the primary balance may need to be raised to reduce the risk of default. Furthermore, the contemporaneous impact of the primary balance on capital market variables will be affected by the fact that the former is measured with a lag.

Based on a test of the relevant terms of the covariance matrix, we cannot reject the joint

(7) A more detailed model would reflect the fact that the maturity and currency structure of debt are endogenous choice variables, and that a country with some flexibility over the timing of debt issuance can concentrate it in periods when interest rates are low. But we have not attempted to model these features here.

hypothesis that the covariances of the primary balance shocks with those of the other equations are all equal to zero (p-value=0.16). If the true covariances are in fact zero, we would argue that the absence of the contemporaneous structural dependencies – the condition we are looking for – is economically the most likely explanation. However, we are vulnerable to a type II error here. We are unable to reject the hypothesis that the relevant covariances are zero but this does not imply that the hypothesis is in fact true.[(8)]

From an *economic* point of view, the practice of evaluating a change in policy on the basis of statistical relationships is vulnerable to the Lucas Critique. Put another way, our quantitative model describes relationships that depend on the policy regime, so changes in this regime will, in general, change the other relationships in the model – both the 'structural' and 'reduced' forms of the VAR representation. In particular, if the government were to adopt a credible debt-stabilising fiscal rule, the relationship between indebtedness and the interest rates the capital markets require would change. For example, the market should not demand a default risk premium when a country is running a debt-stabilising fiscal policy. Our model does not capture this effect. One might therefore interpret this technique as embedding the implicit assumption that the market does not find credible the change in fiscal policy introduced in the simulation.

In summary, we have some statistical and economic evidence to suggest that sufficient conditions may hold for us to interpret changes in the primary balance equation as changes in the fiscal regime. It is fair to say that we have an 'amber light' for treating the primary balance equation independently and using the model to predict what happens if we alter it. With these caveats in mind, we search for fiscal policy rules within a general class of possible rules. In this paper we consider rules in the class of functions piecewise quadratic in the debt-GDP ratio, ie over parameters $\{\pi_0, \pi_1, \pi_2\}$ in the function:

$$pb_t = \pi_0 + \pi_1(d_{t-1} - d^*) + \pi_2 \left(\frac{d_{t-1} - d^*}{|d_{t-1} - d^*|} \right) (d_{t-1} - d^*)^2 \qquad \textbf{(3)}$$

where d^* is some reference value. In this case we have set $d^* = 0.45$ which is the mid-point of the target range.[(9)] We chose this class because the coefficients $\{\pi_0, \pi_1, \pi_2\}$ are easy to interpret, and because it nests a linear rule – the simplest possible form of reactive rule. It clearly would be

(8) This logic is implicit in the testing down of most regression specifications.
(9) The term immediately after π_2 clearly gives $+1$ if d is above d^* and -1 if it's below, and 0 if $d = d^*$. Its purpose is to make $\frac{\partial^2 pb}{\partial d^2}$ the same sign as π_2. Note that d^* is *not*, in general, the value of the debt-GDP ratio at which debt will stabilise, but rather locates the point of inflexion at which the second derivative of the reaction function changes sign.

possible to search over other classes of rule, altering the functional form and/or the variables to which fiscal policy can react. For example, a fiscal policy rule might respond to innovations in interest rates and growth that occur independently of changes to the debt-GDP ratio. Our first experiment is intended to demonstrate that fiscal policy must react to shocks in order to stabilise debt dynamics. Chart 1 shows the effect of 'switching off' the estimated fiscal reaction function and replacing it with a rule that holds the primary balance constant at the sample average. In this case we set:

$$
\begin{aligned}
pb_t &= \pi_0 = 0.03 \\
\pi_1 &= \pi_2 = 0
\end{aligned}
$$

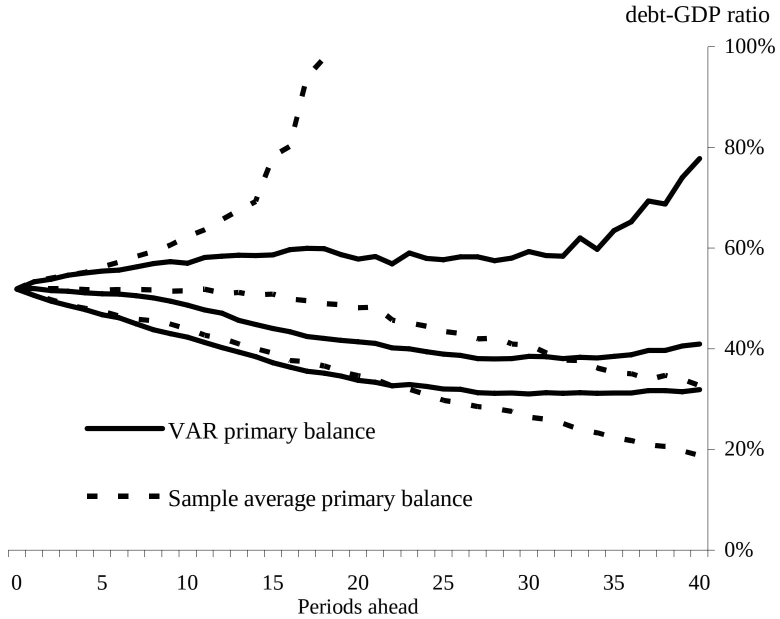
The solid lines show the median and 25th and 75th percentiles of the forecast debt-GDP ratio when the estimated fiscal reaction function is allowed to operate, while the dashed lines show the same statistics in the case where the primary balance is held constant. The chart clearly shows that the implicit fiscal reaction function in the VAR model responds strongly over the past to innovations in the debt-GDP ratio[(10)] and approximately stabilises it. In contrast, and as expected, a fixed surplus causes the forecast variance to expand without limit as the horizon increases. The feedback between debt levels and interest rates mean that adverse shocks have a bigger effect on the debt level than benign shocks. So even though shocks are symmetric, the impact on debt levels is skewed upwards over time.

3.3 Debt-stabilising fiscal policy rules

Our objective here is to determine which fiscal policies stabilise the forecast distribution of debt. Once again, our approach is to 'turn off' the estimated fiscal reaction function, replace it with a hypothetical alternative, and continue to use the other parameters of the VAR.

(10) The estimated coefficient (0.23) is very close to that found by Wyplosz (2004), who employed monthly data over a subset of our sample period.

Chart 1: Forecasts of debt-GDP ratio: interquartile range



Our working definition of debt sustainability is to require that the 10th and 90th percentiles of the debt-GDP ratio both lie within the interval [0.2,0.7] for 50 simulated years.[(11)] Clearly, alternative criteria are available – different admissible ranges, lengths of time, and sample statistics.[(12)] Since the main purpose of this paper is to present the technique, we propose this criterion as one of many plausible alternatives. We also need to impose some structure on the set of possible fiscal policy rules. We consider two classes of rule: rules that determine the primary balance only as a function of debt, and rules that allow fiscal policy also to depend on the economic cycle.

3.3.1 Primary balance as a function of debt-GDP ratio

For this exercise, we search for fiscal policy rules using the full flexibility of equation **(3)**. We simulate 200 realisations of the augmented VAR system for different parameterisations of the fiscal policy rule, calculating debt-GDP paths for each set of realisations at each period in time and halting the recursion when the system violates the stability conditions we defined above. If

(11) There is a non-trivial implicit assumption in this definition. If public sector borrowing could yield a greater return (through higher g) than the cost of borrowing, then any positive stock of debt would be sustainable. Imposing an upper bound on the debt to GDP ratio which is stable implies that public expenditure does not yield a large enough realisable rate of return.

(12) The IMF *WEO* September 2003 noted, for example, that the probability of default increased significantly for debt levels about 50% of GDP (IMF (2003c, page 128)).

the debt paths stayed within the required range for the duration of the whole simulation, the corresponding reaction function is classed as stabilising.

Our results consist of regions in $\{\pi_0, \pi_1, \pi_2\}$ space that represent stabilising reaction functions. To present them in two dimensions, we display diagrams in $\{\pi_0, \pi_1\}$ space for fixed values of π_2. For example, Chart 2 shows the contours that bound the set of stabilising combinations of $\{\pi_0, \pi_1\}$ for $\pi_2 \in \{0, 0.5\}$.[(13)] Fiscal policy becomes unconditionally tighter as we move north, and more reactive as we move east. Sensitivity analysis indicates that the starting value of the debt-GDP ratio can affect the set of stabilising policies we calculate: with a higher initial value, the debt-GDP ratio can be pushed outside of the specified range with less extreme, and therefore less unlikely, adverse shocks.[(14)]

Chart 2 illustrates two important points:

- As expected, no constant primary surplus can stabilise the debt-GDP ratio (ie $\min \pi_1 > 0$). Fiscal policy must be reactive to be stabilising, even under our arguably loose definition of stability.
- Unsurprisingly, setting $\pi_2 = 0.5$ permits a lower value of π_1. Differentiating the two reaction functions with respect to d shows that the derivative of the quadratic function is larger when $d > 0.68$, but smaller below this point. So setting the quadratic term at 0.5 makes the reaction function increasingly sensitive to d as extreme values in the range, but less sensitive when d is close to d^*.

Chart 3 demonstrates the importance of uncertainty in constraining the set of stabilising fiscal reaction functions. The dashed line shows the feasible set when we remove all uncertainty from the determinants of debt dynamics (that is, we set to zero the covariance matrix of reduced-form shocks). The size of the gap between the two contours is a measure of the effect of uncertainty, and is clearly material, so these theoretical effects appear to matter a great deal in practice. For

(13) The estimated contour is rough because of the random component in the simulations used to estimate each point. As the number of simulations tends to infinity, the surface will become smooth.
(14) But note that, because the maturity of the debt is greater than one period, the current debt-GDP ratio is not the only state variable in the system. In these simulations, we have not allowed the counterfactual starting debt-GDP ratio to affect the interest rate at which the debt was contracted.

example, the presence of uncertainty raises the minimum necessary degree of fiscal policy responsiveness (ie the minimum value of π_1 in the stabilising set) from around 0.05 to 0.15.[15]

But how does this uncertainty translate into the size of the primary surplus in a practical situation? An answer is illustrated in Chart 4. For the purpose of comparison, two specific rules from the feasible set were chosen – in this case the least 'reactive' stabilising linear function in the no-uncertainty case and the least reactive linear and quadratic functions that stabilise debt with observed levels of uncertainty. The primary surpluses derived from these two policy rules are shown for different values of the debt-GDP ratio.[16] Chart 4 shows that the inclusion of uncertainty is material: given a linear reaction function, policymakers facing a debt-GDP ratio of 60% could run a fiscal balance of around 4.5% of GDP if there were no shocks to the macroeconomic determinants of debt dynamics. But under a realistic pattern of shocks, a linear reaction function sufficiently aggressive to stabilise debt dynamics with high probability would call for a surplus of around 6% of GDP. The chart also illustrates, as mentioned above, that if a country can tighten fiscal policy sharply at extreme debt levels, it can run a less reactive fiscal policy at intermediate levels of debt.

Charts 5, 6 and 7 illustrates how choosing alternatives rules could have produced different historical paths for the main variables. Each chart has the actual path of history and three counterfactuals – one using the estimated fiscal reaction function contained in the VAR system; one using a fixed primary balance rule; and one using the least reactive linear stabilising rule under uncertainty. The last rule is the same used as the baseline cases in Chart 4. The counterfactuals are generated by applying the different rules to the sequence of shocks actually experienced over the period starting in 1999. Unsurprisingly, the paths derived from the estimated rule from the VAR are very little different from the actual historical path. As shown above, it is clear that setting a fixed primary balance rule does not stabilise the debt ratio. The debt ratio gradually rises and with it external interest rates. Since the primary balance, by construction, does not react to the rising debt ratio, it continues on this upward path until it eventually becomes explosive. By contrast, the debt-stabilising rule leads to a lower debt path for much of the simulation. This occurs because initially the primary balance reacts to the level of the debt and

(15) Clearly the probability condition is irrelevant here; in a deterministic world the boundary condition will be satisfied with 100% or not at all.
(16) Note that our definition of sustainability has a lower as well as upper bound and therefore fiscal policy becomes considerably less restrictive as debt levels fall.

Chart 2: Debt-stabilising reaction function parameters

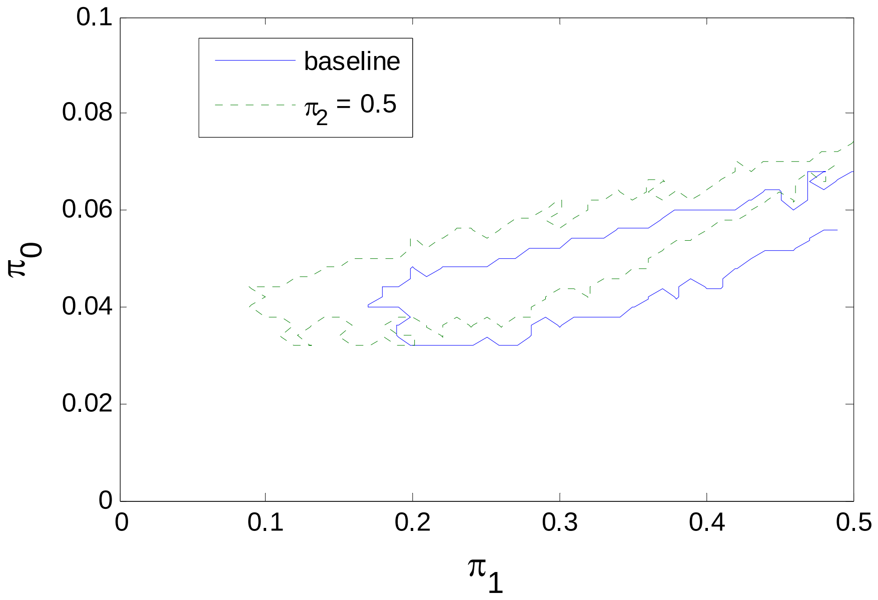


Chart 3: Debt-stabilising reaction function parameters: deterministic case

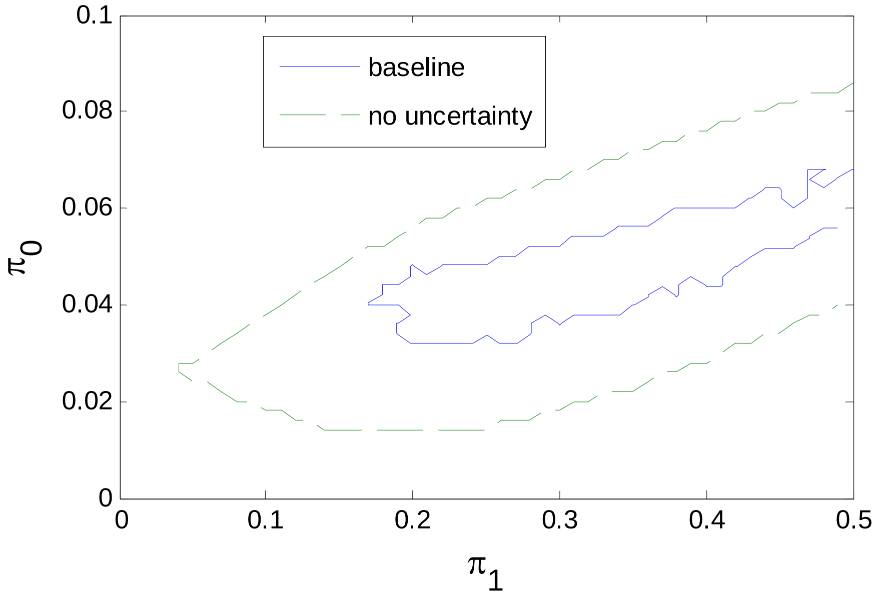
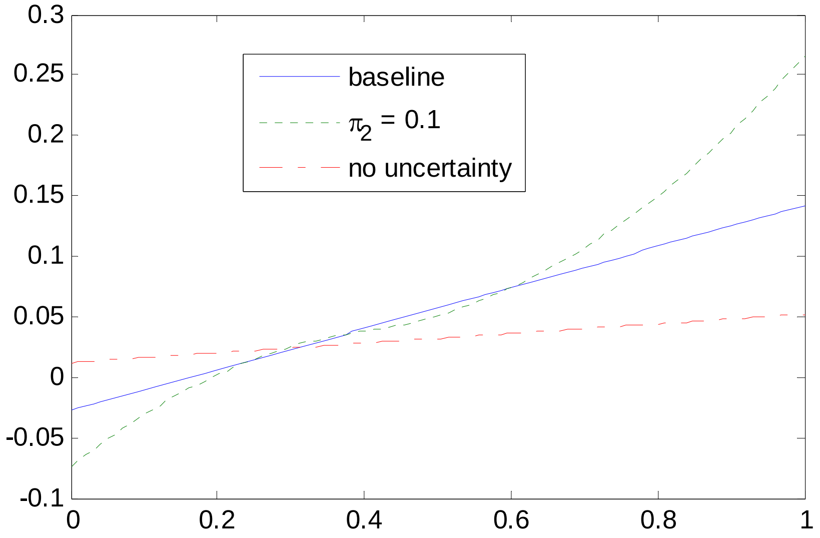


Chart 4: Stabilising reaction functions in $\{d, pb\}$ space



therefore is higher than the sample average and the historical path for a couple of years. But the external interest rate also begins to fall and is noticeably less reactive to the shocks in 2001 and 2002. Eventually as the debt ratio begins to drop, the fiscal balance reacts and falls below the sample average and that historically experienced.

4 Policy applications

The results of Section 3 showed that uncertainty and default risk have a material impact on the fiscal balance a country needs to run to achieve stable debt dynamics. Over moderate ranges of the debt-GDP ratio, taking uncertainty and default risk into account can add several percentage points to the appropriate primary balance. Put another way, policymakers ignoring the impact of these effects may react too little to innovations in the debt-GDP ratio.

Those responsible for monitoring public sector debt dynamics in EMEs are aware of these issues. The advantage of this technique is that it offers policymakers an idea of their magnitude although there are obviously statistical caveats which apply to this analysis. And any policymakers using this technique would need to choose the structure of the VAR carefully, paying due attention to the robustness of the parameters of the model for their country. However, notwithstanding these

Chart 5: Primary balance-GDP ratio

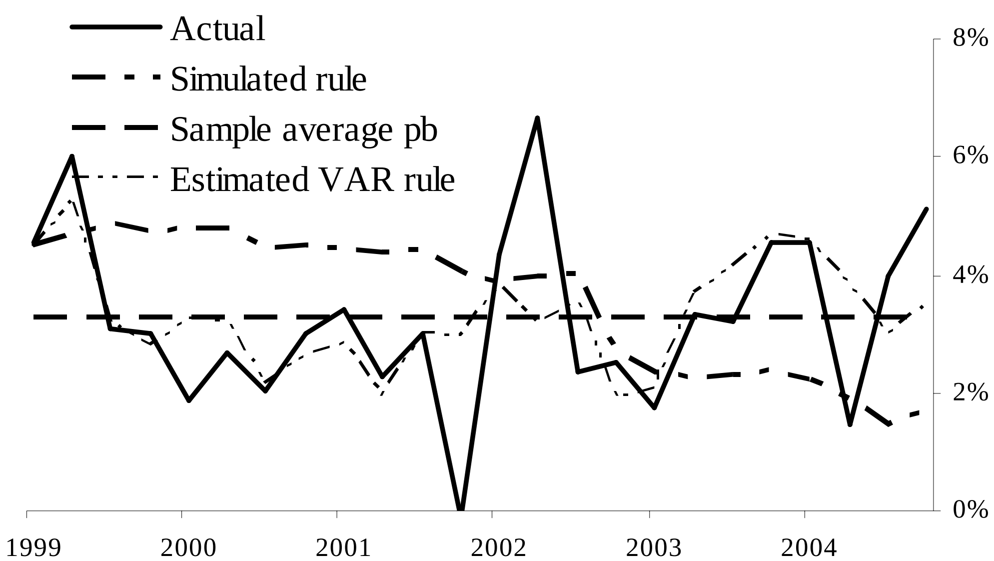


Chart 6: Debt-GDP ratio

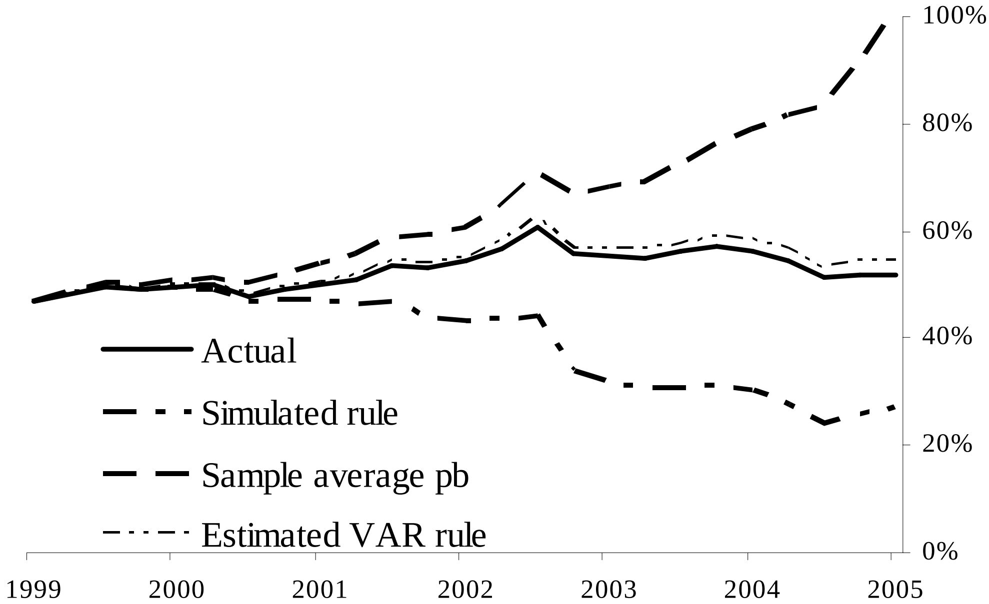
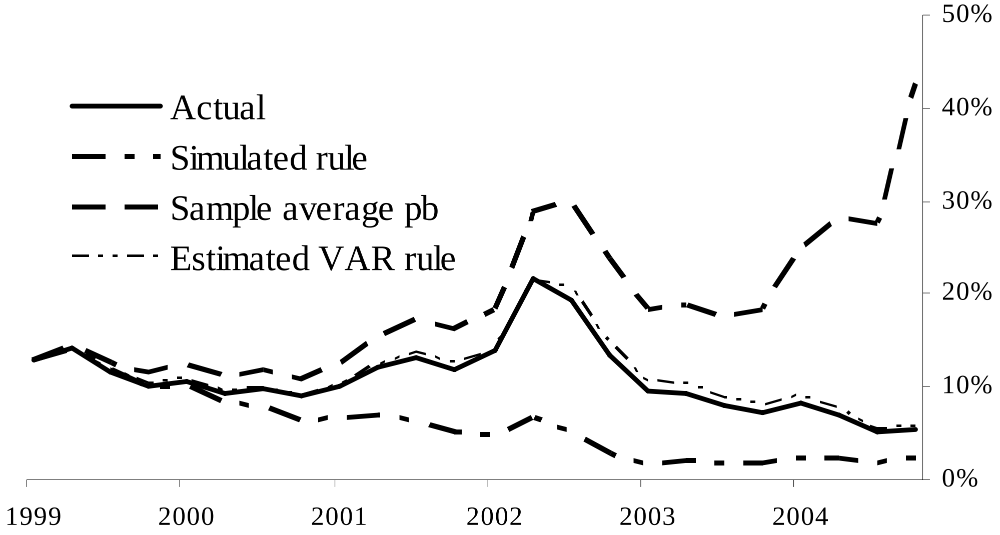


Chart 7: External interest rates



caveats, the techniques we have used in obtaining our results may have two practical applications.

- First they may be helpful for IMF assessments of debt sustainability. Under the IMF's exceptional access policy, large-scale IMF financial assistance can only be provided if 'a rigorous and systematic analysis indicates that there is a high probability that debt will remain sustainable' (IMF (2003a, page 4)). At present, the IMF's debt sustainability assessment process begins by calculating a baseline projection path for the key variables, and a baseline path for the debt to GDP ratio, assuming full implementation of the programme conditions (see IMF (2003b)). Sustainability is assessed on the basis of whether this ratio ends below its starting value or, if initially rising, is on a downward path by the end of the projection. Alternative paths are considered by setting the variables on the basis of country-specific scenarios, historical averages, market forecasts, or a number of standard deviations away from the central projection.

  This approach explicitly recognises that IMF programmes are designed to mark a break from the past. However, in contrast to the techniques presented here, the IMF approach pays insufficient attention to the statistical dependence of shocks, and does not quantify the probability of alternative scenarios. Furthermore, the IMF may find an assessment of debt sustainability not conditional upon a structural change in economic performance a useful benchmark for judging the likely success of a programme.

- Second, they may be useful for fiscal programming. Our analysis suggests that simple fiscal policy rules, such as a primary surplus which is a constant fraction of GDP, do not stabilise debt in the long run. This is particularly true when we take into account the interaction between future macroeconomic uncertainty and the feedback of debt to default risk premia. The public debt-GDP ratio may therefore make a more suitable medium-term target variable. The technique we use may help to determine the rule with which a policymaker implicitly implements the debt-GDP target. For example, Brazil currently employs the primary surplus-GDP ratio as its operational legal fiscal policy target, setting the surplus at a level consistent with current macroeconomic projections and the debt targets contained in its Fiscal Responsibility Law (Goldfajn and Guardia (2004)). This technique may help to link the former to the latter.

5 Conclusion

This paper has presented a simple framework for producing probabilistic forecasts of public debt sustainability and used this model to assess the adequacy of different fiscal policy rules. The empirical estimates we presented show that the interaction of shocks to the determinants of debt sustainability with feedback from the debt-GDP ratio can be quantitatively important for a representative EME. In the example we present, the primary balance must respond aggressively to increases in the debt-GDP ratio.

The technique presented could also be a practical tool for domestic and international policymakers. A quantitative stochastic analysis of debt projections could help the IMF when assessing the probability that a member's debts are sustainable before agreeing to financial assistance programmes. Our technique is explicitly probabilistic and may therefore serve as a useful complement to the existing method of assessment. It could support IMF surveillance of fiscal policy and thereby contribute to crisis prevention. The technique may also be useful to policymakers in emerging markets when determining medium-term fiscal policy strategy, particularly if a country is considering the introduction of fiscal policy rules.

Appendix

Data sources

- GDP growth

  Seasonally adjusted quarterly real GDP growth (source: Banco Central do Brasil)
- Domestic real interest rates

  Quarterly average of SELIC rate less expected inflation from market survey where available, lead of inflation otherwise (source: Banco Central do Brasil)
- External real interest rates

  Brazil EMBI spread plus yield on US indexed-linked bonds (source: JPMorgan, US BLS)
- Primary surplus

  Quarterly average consolidated public sector primary surplus as a percentage of GDP (source: Banco Central do Brasil)
- Change in real exchange rate

  Change in US$/R$ exchange rate, deflated by producer prices (source: Banco Central do Brasil)
- Debt-GDP ratio

  Quarterly average consolidated public sector net debt-GDP ratio (source: Banco Central do Brasil)

Econometric results

Table B: ADF unit root tests - p values for rejecting null

Variable	p value	Lag length
g	0.0021	0
r^f	0.6556	7
r^d	0.0157	6
s	0.0254	0
pb	0.0011	0
d	0.0631	0

Chart 8: Real exchange rate and debt-GDP ratio

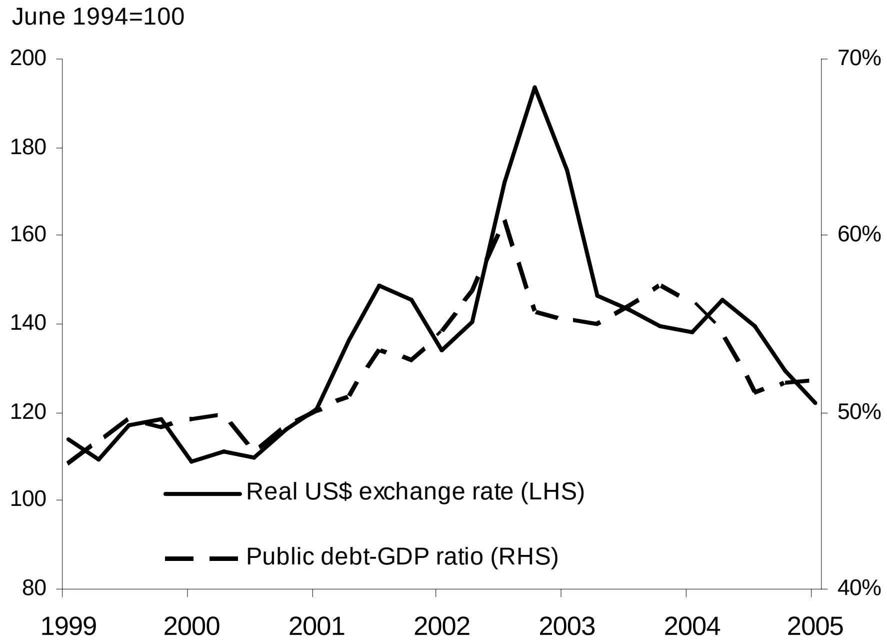


Chart 9: Quarterly GDP growth and the primary surplus-GDP ratio

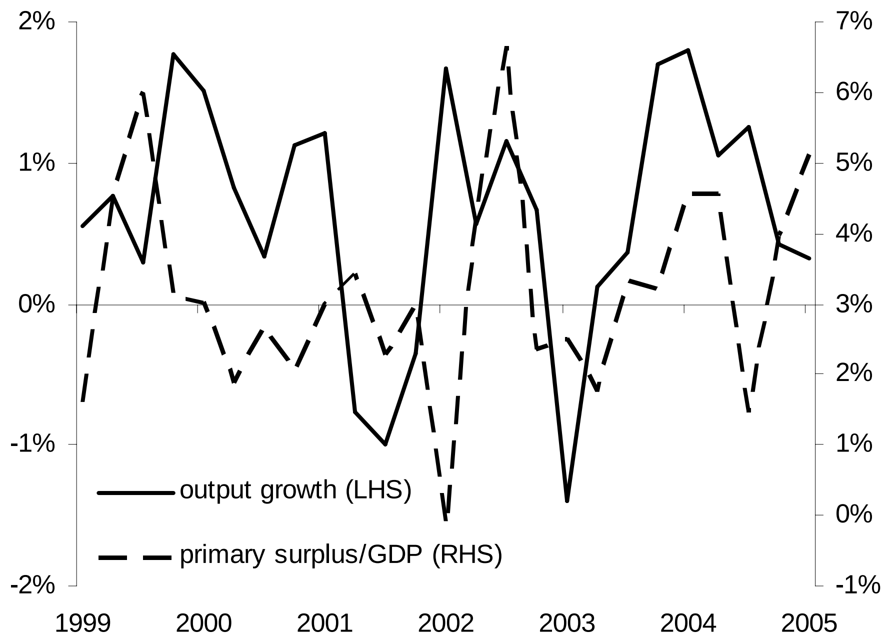


Chart 10: Real domestic and external interest rates

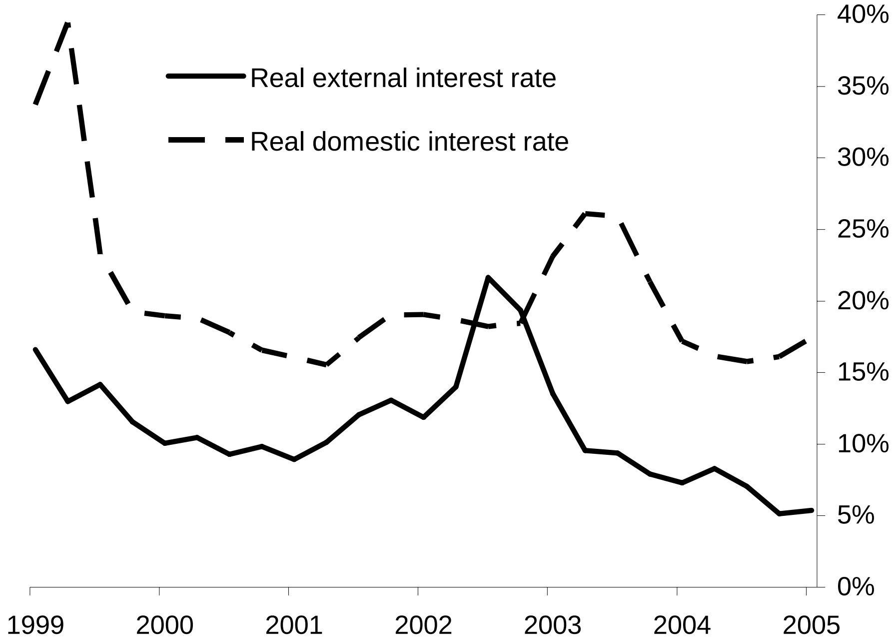


Table C: Correlation matrix of residuals

	g	**r^f**	**r^d**	**s**	**pb**
g	1	-0.56	0.13	-0.64	-0.41
r^f	-0.56	1	0.06	0.79	0.48
r^d	0.13	0.06	1	-0.10	0.06
s	-0.64	0.79	-0.10	1	0.47
pb	-0.41	0.48	0.06	0.47	1

An F-test that the covariances between the reduced-form shocks in the primary balance equation with those from the other equations are all zero returns a p-value of 0.82.

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