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Modelling the cross-border use of collateral in payment systems

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Abstract

Banks often rely on collateralised intraday liquidity from the central bank in order to be able to effect payments in a real-time gross settlement (RTGS) payment system. If a bank is holding insufficient eligible collateral in a particular country, and therefore cannot obtain credit from the local central bank, it may have to delay payments. This constitutes a liquidity risk to the system. Furthermore, a bank operating in multiple systems may face a mismatch between the location of its collateral holdings and liquidity needs. In this paper, we examine the extent to which the liquidity risk arising from such a mismatch may be mitigated by allowing cross-border use of collateral. We develop a two-country, two-bank model in which risk-neutral banks minimise expected costs with respect to their collateral choice in each country. In our baseline model, in which each bank faces a liquidity need in only one country, we find that liquidity risk is indeed reduced by cross-border use of collateral. This result holds despite the fact that banks may find it optimal to economise on their total holdings of collateral. However, when we extend the model to allow for the possibility that a bank faces liquidity needs in both countries simultaneously, the total quantum of collateral held is important. Indeed, when a bank finds it optimal to reduce its total holdings, there may be an increase in liquidity risk in at least one country when simultaneous liquidity demands are realised.

Key words: Payment systems, collateral, liquidity risk.

Summary

Over the past decade, there has been a decisive shift towards real-time gross settlement (RTGS) of high-value payments, typically across accounts at a central bank. Settlements in such a system can only be completed if the paying bank has sufficient funds in its account; ie if it has adequate liquidity. Hence, the focus has shifted away from credit risk and towards liquidity risk.

Central banks typically address this risk by making intraday liquidity available to settlement banks on favourable terms. A commonly adopted policy is to provide intraday credit on an unlimited, free, but fully collateralised basis. But given the potential for high collateral costs to encourage payment delays and risk gridlock in the system, it will be optimal for central banks to accept a wide variety of assets as collateral, thereby allowing the efficient management of commercial banks' collateral portfolios.

Restricted eligible collateral lists are a particular issue for banks operating in multiple countries and hence facing settlement obligations in a number of payment systems. If, in each country, the central bank accepts domestic securities only, a bank must hold sufficient (costly) collateral assets to meet its expected liquidity needs wherever it is active. Once liquidity demands have been realised, a bank may find itself with a shortage of collateral in one market and abundant collateral in another. This collateral will then lay idle while the bank obtains additional assets in the country in which it experiences a shortfall. Such a mismatch is inefficient. It also contributes to liquidity risk in the payment system because there could be some disruption to the bank's payments activity while it enters the market to acquire the necessary eligible assets.

One potential policy response is then to broaden the eligible collateral list to include foreign assets; ie to allow cross-border use of collateral. But any change in policy on collateral eligibility may result in a change in market participants' incentives, so the implications of cross-border use of collateral for liquidity risk in payment systems are best analysed in the context of a model that captures full optimising behaviour on the part of commercial banks. In this regard, we develop a stylised two-country, two-bank model in which risk-neutral banks minimise expected costs with respect to their collateral choice in each country. Banks are active in both countries' payment systems and make collateral choices under uncertainty as to both the size and the location of their liquidity needs. In our baseline model, we assume that each bank only realises a liquidity demand in one country at any one time; in other words, banks' liquidity demands are negatively correlated across countries. Using our baseline model, we compare outcomes for liquidity risk for cases in which: (i) there is no cross-border use of collateral; and (ii) both central banks permit cross-border use of collateral. A number of key results emerge.

First we show that, when both countries permit cross-border use of collateral (which we refer to as the symmetric cross-border use of collateral), banks will concentrate their holdings in the country with the lowest collateral costs and may, with sufficiently high costs of experiencing a shortfall (relative to start-of-day collateral costs), reduce collateral holdings in each country. Importantly, even with a decline in total collateral holdings, we find that liquidity risk, as measured by expected

collateral shortfalls, will fall in both countries. This reflects the fact that it will always be optimal for a bank to hold a larger amount of collateral across two connected countries than in a single unconnected location. Hence, there will always be a larger pool from which to draw to meet a liquidity need in a single country.

We make a number of extensions to the baseline model to relax some of its simplifying assumptions. First, in the absence of co-ordinated policy, it may be that only one central bank permits cross-border use of collateral. In this case of asymmetric cross-border use of collateral, we show that banks' collateral choices will be driven by two potentially offsetting factors. On the one hand, banks will shift collateral holdings towards the collateral that is eligible in both countries. On the other, banks will still be inclined to accumulate larger holdings of the cheaper collateral. When the cheaper collateral can be used across borders, these two factors are mutually reinforcing and the outcome will be the same as if there were symmetric cross-border use of collateral. When the collateral eligible in both countries is only slightly more expensive, banks will still hold only this collateral, but slightly less will be held overall than in the symmetric case. Again, liquidity risk will decline in both countries. Finally, when the collateral eligible in both countries is significantly more expensive, collateral will be held in both countries and the expected shortfall in the country accepting foreign collateral will be the same as in the case with no cross-border use.

Another extension allows some probability that a bank experiences a liquidity need in both countries simultaneously. In our model, banks adjust collateral holdings to take account of this possibility. But as there remains a chance that banks could experience a liquidity need in just one country, it may, under certain conditions, still be optimal to reduce total collateral holdings relative to the case with no cross-border use of collateral. Such a reduction would imply higher expected shortfalls in at least one country when a bank faces simultaneous liquidity needs, compared to the case with no cross-border use of collateral. The size of the respective shortfalls experienced in each country will depend on how the available collateral is ultimately allocated between countries.

Finally, we also consider an extension in which central banks have the option of accepting collateral in stressed situations only. Under such a regime, and with a sufficiently low probability that the emergency facility will be triggered, banks' reductions in collateral holdings may be more muted than if cross-border use of collateral were allowed routinely. As a result, should a stressed situation arise in one country, banks may have a larger pool of collateral to draw upon than they would have in the case of routine cross-border use of collateral. Expected shortfalls would, in such a case, be lower. If central banks place a higher weight on liquidity risk mitigation in times of stress, and recognise that it may be more difficult to access additional collateral during a crisis, such a policy may be attractive.

1 Introduction

Over the past decade, there has been a decided shift towards real-time gross settlement (RTGS) of high-value payments in central bank money (Bank for International Settlements (2005)). Given the asynchronous arrival of payments, settlements in such a system can only be effected if the paying bank has sufficient funds in its account with the central bank; ie if it has adequate liquidity. Hence, the focus has shifted away from credit risk and towards liquidity risk.(1)

The payments literature has devoted considerable attention to this issue in recent years. For instance, Angelini (1998), Chakravorti (2000), Kahn and Roberds (2001), and Bech and Garratt (2003) all emphasise the risk that high (and hence costly) liquidity requirements will encourage banks to delay their payments, preventing the efficient recycling of liquidity in the system and imposing welfare costs on banks' customers. Furfine and Stehm (1998) note that, in the extreme, payment delays can lead to gridlock in the system; ie the situation in which all participants are waiting for incoming payments and making no outgoing payments. The cost of intraday credit may also be a significant factor in a commercial bank's decision whether or not to participate directly in an RTGS system at all. To the extent that a bank chooses to by-pass an RTGS system by entering into bilateral agreements with other banks or by shifting flows to a competing deferred net settlement (DNS) system, systemic risk may be increased (Rochet (2005)).

Furfine and Stehm (1998) go on to assess central bank policies to mitigate such liquidity risk, focusing in particular on the extent to which liquidity is injected into an RTGS system via intraday central bank credit to settlement members. In particular, the authors consider the terms on which such liquidity may be provided, examining the conditions under which each of the various pricing or collateralisation options may be chosen. A commonly adopted policy is to provide intraday credit on an unlimited, free, but fully collateralised basis. Given the potential adverse incentives generated by high collateral costs, the authors argue that a sufficient condition for a central bank to choose such a policy is that the opportunity of posting collateral faced by commercial banks tends to zero.

However, as Green (2005) argues, eligible collateral assets typically have lower rates of return than other assets, and hence do entail a non-negligible opportunity cost. Indeed, many central banks' eligible lists are restricted to high-quality domestic marketable debt securities. Liquidity risk in payment systems might, therefore, be mitigated if central banks accepted a wide variety of assets as collateral and facilitated the efficient management of commercial banks' collateral portfolios.(2)

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(1) The Basle Committee on Payment and Settlement Systems (2003) defines liquidity risk as 'the risk that a counterparty (or participant in a settlement system) will not settle an obligation for full value when due. Liquidity risk does not imply that a counterparty or participant is insolvent since it may be able to settle the required debit obligations at some unspecified time thereafter'.

(2) One possibility, suggested by Green (2005), but not explored further here, might be for central banks to accept less liquid, and hence less expensive, collateral. The rationale here is that central banks are in the unique position of being able to generate liquidity at will, and hence they could optimally accept less liquid collateral than could other secured lenders.

Restricted eligible collateral lists are a particular issue for banks operating in multiple countries and hence facing payment obligations in a number of payment systems. If, in each country, the central bank accepts domestic securities only, a bank must hold sufficient (costly) collateral assets to meet its expected liquidity needs in each payment system in which it is active. Once liquidity demands have been realised, a bank may, given imperfectly correlated liquidity demands, find itself with a shortage of collateral in one market and abundant collateral in another. This collateral will then lay idle while the bank obtains additional assets in the country in which it experiences a shortfall. Such a mismatch is not only inefficient, but also contributes to liquidity risk in the payment system because there could be some disruption to the bank's payments activity while it enters the market to acquire the necessary eligible assets. One potential policy response is then to broaden the eligible collateral list to include foreign assets; ie to allow the cross-border use of collateral.

Such issues have become increasingly prominent in policy circles. The Basle Committee on Payment and Settlement Systems (CPSS) recently initiated a workstream on this topic, in part in response to a call by the Payments Risk Committee, a New York-based consortium of internationally active banks, for increased cross-border use of collateral within G10 countries. A number of central banks already accept foreign collateral on a routine basis, employing a variety of institutional and infrastructural arrangements to mobilise assets (Bank for International Settlements (2006)). For example, within the European System of Central Banks (ESCB), the Correspondent Central Banking Model (CCBM) was introduced in 1999 to facilitate the cross-border use of collateral to support Eurosystem credit operations, or to obtain liquidity in TARGET (European Central Bank (2003)). Under this arrangement, central banks act as custodians for one another, allowing the use throughout the system of all eligible securities issued and located in any Eurosystem country. Around 35% of all collateral posted within the Eurosystem is delivered via this arrangement. The European Central Bank has also assessed a number of links between national securities settlement systems (SSSs) that serve as an alternative, but less regularly used, vehicle for mobilising eligible securities across borders. Some other EU central banks, such as the Bank of England and Swedish Riksbank, also allow cross-border use of collateral. The Bank of England, for example, accesses the CCBM to receive euro-denominated securities. Indeed, some 75% of the collateral used to support banks' intraday liquidity needs is received in this way.[(3)] The Bank of England also stands ready to extend its collateral list to encompass US Treasuries in exceptional circumstances.[(4)] Outside of the ESCB, the Swiss National Bank employs collateral management services offered by the local central securities depository (CSD) to receive euro-denominated securities from several EU countries in support of its repo operations. Again, this is a widely used arrangement, used to deliver some 60% of total collateral posted. Finally, the US Federal Reserve receives a host of foreign collateral assets into its accounts at Euroclear and Clearstream, primarily to support discount window lending.

In this paper, we seek to establish the implications of cross-border use of collateral for liquidity risk in payment systems. Given that a change in policy on collateral eligibility may elicit a change in

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[(3)] The denominator here is total collateral repo'd to the Bank of England intraday by settlement banks, excluding self-collateralising repos generated automatically in CREST, the securities settlement system.
[(4)] Announced in the Bank of England's Operational Notice (November 2004).

incentives, we adopt an approach which allows for full optimising behaviour on the part of commercial banks. More specifically, we develop a stylised two-country, two-bank model in which risk-neutral banks minimise expected costs with respect to their collateral choice in each country. Banks are active in both countries' payment systems and face uncertainty as to the location and size of their liquidity demands.

The fundamental structure of the model we present can be traced back to work on the precautionary demand for reserves: for example, Olivera (1971) and Baltensperger (1974). In these models, the agent chooses a level of reserves so as to minimise a total expected cost function, which has two key components: (i) the cost of accumulating a reserve inventory at the start of the period; and (ii) the cost of adjusting the reserve inventory in the face of a deficiency. The basic form of the problem is given by equation **(1)**, in which a is the marginal cost of the choice variable, X (in these models, the choice variable is reserves; in ours, it is collateral). The term under the integral represents the expected shortfall in the choice variable (Z being the draw from a distribution of required amounts), conditional on the size of the holding at the start of the period. And p represents the cost associated with facing such a shortfall:

$$E[C] = aX + p\int_{X}^{\infty}(Z - X)\, f(Z)dZ \qquad \textbf{(1)}$$

Similar approaches have been adopted in more recent related work, such as Heller and Lengwiler (2003), and, firmly in a payment systems context, by Kobayakawa (1997) and Angelini (1998).

In the context of our problem, each bank faces a similar cost function, comprising the cost of collateral accumulated at the start of the day, and the cost of experiencing a collateral shortfall. The cost of experiencing a shortfall reflects the cost of acquiring additional collateral and any costs, both pecuniary and non-pecuniary, associated with delaying payments while additional collateral is sought in the market. The size of the expected shortfall is the metric we adopt in this paper for liquidity risk in the system.

Minimising **(1)**, subject to the constraint that $X = 0$, the choice of X must satisfy the first-order condition given in **(2)**. The intuition of this condition is that the agent will continue to accumulate X until the expected cost of experiencing a shortfall, conditional on the choice of X, is equal to the marginal cost of acquiring X in advance.

$$a \geq p\int_{X}^{\infty} f(Z)dZ \qquad \textbf{(2)}$$

Our model extends the structure in equation **(1)** to incorporate expected collateral and shortfall costs in two countries, and we solve for collateral choices in each country. In our baseline model, we assume that each bank only realises a liquidity demand in one country at any one time. Using our baseline model, we compare outcomes for liquidity risk for cases in which: (i) there is no cross-border use of collateral; and (ii) both central banks permit cross-border use of collateral (which we refer to as the symmetric cross-border use of collateral case). A number of key results emerge.

First we show that, when both countries permit cross-border use of collateral, banks will concentrate their holdings in the country with the lowest collateral costs. Banks will decrease (increase) total holdings of collateral if the costs of experiencing a shortfall are sufficiently high (low) relative to start-of-day collateral costs. Importantly, even if there is a decline in total collateral holdings, we find that liquidity risk, as measured by expected collateral shortfalls, will fall in both countries. This reflects the fact that it will always be optimal for a bank to hold a larger quantum of collateral in total across two connected countries than it would hold in a single country in the absence of cross-border use of collateral. Hence, there will always be a larger pool from which to draw to meet a given liquidity need in a single country when there is symmetric cross-border use of collateral.

The implication that with cross-border use of collateral banks will shift collateral holdings to the cheapest location is consistent with evidence from the Eurosystem. Data as at December 2003 reveal that some 63% of the collateral used across borders was issued by non-AAA, and hence 'cheaper', issuers.[5]

A further implication of symmetric cross-border use of collateral is that banks' total expected costs also decline. Hence, payment system efficiency will also be improved. Although this result is not central to our analysis in this paper, it may provide further support for a policy of cross-border use of collateral.

We make a number of extensions to the model, so as to relax some of the simplifying assumptions in the baseline model. First, in the absence of co-ordinated policy, it may be that only one central bank permits cross-border use of collateral. Hence, this case warrants some attention. In this case of asymmetric cross-border use of collateral, we show that banks' collateral choices will be driven by two potentially offsetting factors. On the one hand, banks will shift collateral holdings towards the collateral that is eligible in both countries. On the other, banks will still be inclined to accumulate larger holdings of the cheaper collateral. When it is the cheaper collateral that can be used across borders, these two factors are mutually reinforcing and the outcome will be the same as in the case with symmetric cross-border use of collateral. When the collateral eligible in both countries is only slightly more expensive, banks will still only hold this collateral, but slightly less will be held overall than in the symmetric case. Again, liquidity risk will decline in both countries. Finally, when this collateral is significantly more expensive, collateral will be held in both countries and the expected shortfall in the country accepting foreign collateral will be the same as in the case with no cross-border use.

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[5] This 63% is comprised, as follows: Italy, 18.4%; Belgium, 15.6%; Greece, 4.4%; Portugal, 2.4%; and international non-sovereign, 22.2%. These data are sourced from the European Central Bank, and may be found at: www.ecb.int/stats/payments/securities/html/coll2.en.html.

In the baseline model, there is no cost associated with cross-border use of collateral; for example, central banks charge no 'haircuts'[(6)] to reflect foreign exchange risk or differential credit quality. Therefore, banks take full advantage of any collateral cost differential and concentrate holdings in the cheapest country. In practice, there may be some cost to cross-border use of collateral, one of which is the potential for an operational disruption in infrastructures to prevent the mobilisation of collateral assets. Extending the model to incorporate the possibility of operational problems, we find that a bank's incentive to shift collateral entirely towards the cheapest country under symmetric cross-border usage will be tempered if the probability of a problem is sufficiently high.

A third extension relaxes a crucial assumption of the baseline model, allowing for some probability that a bank experiences a liquidity need in both countries simultaneously. A bank will adjust its collateral holdings to take account of this possibility, but as there remains a probability that banks experience asynchronous liquidity needs, it may, under certain relative shortfall and start-of-day collateral costs, still reduce total collateral holdings relative to the case with no cross-border use of collateral. While in the baseline model, liquidity risk falls even if banks economise on total collateral holdings, this may no longer be the case in this extended model. Rather, compared to the case with no cross-border use of collateral, a reduction in collateral holdings would imply higher expected shortfalls in at least one country in the state of the world in which a bank realised simultaneous liquidity needs. The ultimate size of the shortfalls experienced in each country will depend on how the available collateral is ultimately allocated between countries.

Finally, recognising that central banks may perceive certain costs[(7)] to routine acceptance of foreign collateral, we also consider an extension in which central banks have the option of only accepting collateral in stressed situations where liquidity demands are very high. With a sufficiently low probability that the emergency facility will be triggered, banks' reductions in collateral holdings may be more muted than if cross-border use of collateral were allowed routinely. As a result, should a stressed situation arise in one country, banks may have a larger pool of collateral to draw upon than they would have in the case of routine cross-border use of collateral. Hence, expected shortfalls in such an event will be lower. If central banks place a higher weight on liquidity risk mitigation in times of stress (perhaps because it may be more difficult to access additional collateral during a crisis) such a policy may be attractive.

The paper is structured, as follows. In Section 2, we introduce the baseline model, in which banks face liquidity demands in only one of the countries. In Section 3 we solve the model for the cases with no cross-border use of collateral and symmetric usage. For these cases, we discuss the implications for total collateral holdings and compare outcomes for expected collateral shortfalls (our

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[(6)] The term 'haircut' refers to the additional collateral a central bank typically requires over and above the value of credit (liquidity) extended to protect itself against potential adverse changes in the market value of the collateral posted.
[(7)] Bank for International Settlements (2006) considers a number of potential policy costs, including effects on competition in the banking sector; concentration in the central bank collateral portfolio, as banks gravitate towards the 'cheapest to deliver' collateral; and the potential for greater interlinkages between systems to increase the likelihood of contagion.

metric for liquidity risk). Section 4 introduces the extensions to the baseline model. Section 5 concludes.

2 A model of the cross-border use of collateral in payment systems

In this section, we present our baseline model, sketching the environment and introducing the timeline for banks' actions. Our model is necessarily stylised, but is designed to capture some of the essential features of a commercial bank's collateral decision: (i) decision-making under uncertainty; (ii) cost minimisation; and (iii) imperfectly correlated liquidity needs in multiple countries.

2.1 The set-up

There are two countries, indexed by $i \in \{1,2\}$. Each is home to a central bank, an RTGS payment system, a central securities depository (CSD), and a deep and liquid securities market. We assume that the two countries lie in the same time zone, so that the payment systems, CSDs and securities markets operate concurrently. The assumption of deep and liquid securities markets allows us to treat prices as given.

There are also two *ex-ante* identical banks.[(8)] Each bank is a settlement member of both payment systems and a participant in both CSDs. Banks are risk-neutral.[(9)] Events take place over the course of a single day, with each bank making a single payment to the other bank in each country (there are no cross-border payments). Payments between banks in country *i* are of equivalent value and are settled across accounts at the local central bank on a real-time gross basis. Thus, a bank can only settle a payment obligation in full when its account balance at the central bank is at least the value of the payment. We assume that each bank begins the day with a zero balance at each central bank.

In each country, payments are made sequentially; thus the first payer in each country must obtain credit from the local central bank to ensure that it has sufficient funds to make its payment. Credit will only be granted on the basis of adequate and eligible collateral and hence each bank must make a choice as to how much collateral to hold.[(10)] This choice is made at the start of the day, under uncertainty as to both the country in which it will be first payer, *and* the value of the payment.

Should the first payer's collateral choice in country *i* prove insufficient to meet its liquidity need, it must obtain additional collateral. If cross-border use of collateral is permitted, any idle collateral in the other country may be brought across to mitigate any shortfall. If cross-border use of collateral is not permitted, or if the bank has insufficient idle collateral in the other country, the first payer makes a payment up to the value of collateral available but must then enter the market for collateral assets. It is assumed that this will delay the full execution of its payment obligation. Because each bank's

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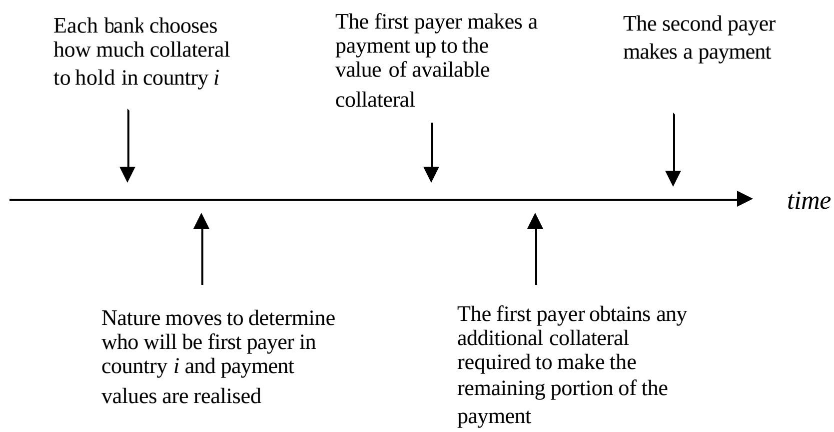
(8) We could interpret the two banks as two sets of banks, each set containing *n* identical banks of the same type.
(9) If the banks were risk-averse we would expect that they would choose to hold more collateral. We leave for future research exactly how other results might differ if banks were risk-averse rather than risk-neutral.
(10) For simplicity, we refer to banks making a choice over how much 'collateral' to hold, rather than making a choice over their holdings of assets which are eligible for use as collateral.

payment obligation in country i has the same value, the second payer is able to recycle the liquidity generated by the incoming payment. The first payer is then able to repay the local central bank.

For simplicity, we assume that a bank will only be first payer in one country. With this assumption, we restrict attention in the baseline model to bank-specific local liquidity needs.[11] (However, we later extend the framework to allow for a bank to face simultaneous liquidity demands across the two countries.) The identity of the first payer is drawn by nature, with a bank equally likely to be first payer in each country. The sequence of events over the course of the day in each country is summarised in Chart 1.

Chart 1: A timeline for actions in country *i*



2.2 Costs

The choice variable in our model is the amount of collateral to be held in each country, c_i, which is chosen so as to minimise total expected costs, $E[C]$. The per-unit (opportunity) cost of obtaining collateral at the start of the day in country i is g_i.[12]

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[11] This assumption is consistent with certain sources of liquidity demand. For example, liquidity demands associated with pay-ins to the Continuous Linked Settlement (CLS) system for foreign exchange transactions will occur only in those currencies in which a bank is a net seller; elsewhere, the bank is a net receiver of funds. Another interpretation might be that there is a *certain* and an *uncertain* component to banks' routine liquidity needs. Hence, our framework might be thought of as modelling uncertain needs (liquidity *surprises*) only, with the certain component normalised to zero. For simplicity, we model such surprises for each bank as arising in one country only in the baseline model.

[12] The cost, or opportunity cost, of obtaining collateral may differ across markets for a number of reasons. For example, there may be differences in creditworthiness or liquidity or countries may have differing prudential liquidity regimes, with implications for the stock of eligible assets a bank actually holds. James (2003) considers the cost of posting collateral in CHAPS Sterling, the large-value payment system of the United Kingdom. He suggests that a reasonable cost measure might be the reverse-repo spread (ie the spread between secured and unsecured borrowing costs). The author also recognises that policy may influence the opportunity cost of collateral: for example, the existence of a quantitative prudential liquidity requirement in the United Kingdom significantly lowers the opportunity cost of posting collateral for domestic banks that are subject to the regime.

Should the first payer in country i find that it has insufficient collateral available (either across the two countries when cross-border use of collateral is permitted or in country i alone when it is not) it must source additional collateral assets in the market. This entails a cost, w_i, which is the sum of the per-unit cost of acquiring extra collateral late in the day and the per-unit cost of delaying payments while additional collateral is sought.

When foreign collateral is not eligible in country i, any additional collateral required must be obtained in country i itself. We assume that this entails a per-unit cost of $a g_i$, where $a \geq 1$ to capture the possibility that it is more costly for banks to obtain extra collateral at short notice.(13) When foreign collateral is eligible, the per-unit cost of obtaining additional collateral is $a \min\{g_1, g_2\}$, because a bank will source the extra collateral in whichever country it is cheapest to do so. We assume in our baseline model that there are no transactions (or other) costs to mobilising collateral across borders, although this assumption is relaxed when we allow for operational risk in one of our extensions.

The 'delay cost', *d*, is assumed to be equivalent in the two countries. Kobayakawa (1997) describes delay costs in terms of the costs of customer dissatisfaction should payments not be made on time. We would like to think of these costs as encompassing a broad range of pecuniary and non-pecuniary costs, such as: financial penalties for missing time-critical payments;(14) sanctions associated with failure to meet 'throughput requirements'; and reputational costs.

2.3 *The value of payments*

The value of banks' payments in country i, v_i, is distributed with a probability density function $f(v_i)$ and a cumulative distribution function $F(v_i)$. We assume that $f(v_i)$ is continuous, single-peaked, and lies fully in positive space. Values are also assumed unbounded above. A further desirable property is that payments take low values with high probability, while extreme outturns arise with low probability. A class of distributions exhibiting these characteristics is the Weibull family of distributions, for which the probability density function takes the form:

$$f(v_i; l, b) = \frac{b}{l^b} v_i^{b-1} e^{-(v_i/l)^b} \qquad \textbf{(3)}$$

Examples of the Weibull distribution are presented in Chart 2, below. When $b = 1$ we have the exponential distribution. We adopt the exponential distribution in several illustrations and extensions because of its analytical tractability. We also set $l = 1$, for simplicity.

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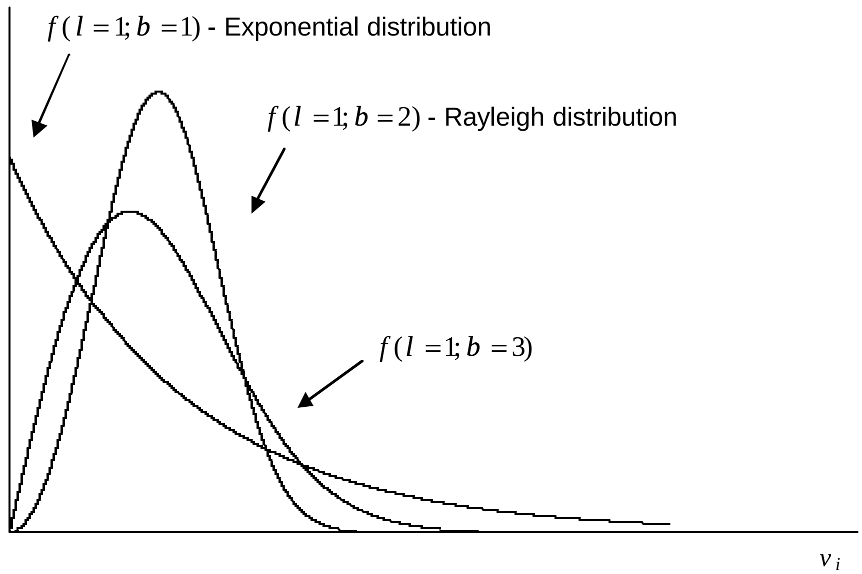
(13) One might model a as an increasing function of the size of the shortfall – ie the larger the shortfall, the more difficult and so expensive it will be to obtain the necessary collateral. For analytical simplicity, however, a is constant in our model.
(14) There may be financial penalties should a bank fail to meet time-critical pay-ins to ancillary systems; a notable example in this regard is the Continuous Linked Settlement (CLS) system for foreign exchange transactions.

Chart 2: Examples of the Weibull distribution with l =1 and alternative values for b



3 Banks' collateral choices

In this section we analyse banks' start-of-day collateral choices in the baseline model for the cases in which (i) there is no cross-border use of collateral; and (ii) cross-border use of collateral is permitted by both central banks. We refer to this second case as symmetric cross-border use of collateral.

3.1 No cross-border use of collateral

We first assume that neither central bank allows banks to use collateral held abroad to support intraday credit requirements. Thus, a bank will always have to obtain further collateral domestically if it is first payer and its start-of-day collateral choice is insufficient to meet its payment obligation. As collateral cannot be used across borders, a bank's conditional expectation of its shortfall in country *i* is independent of its collateral choice in country *j*. Each bank's cost minimisation problem is given by:

$$\min_{c_1, c_2} E[C] = g_1 c_1 + g_2 c_2 + \frac{w_1}{2}\int_{c_1}^{\infty}(v_1 - c_1) f(v_1) dv_1 + \frac{w_2}{2}\int_{c_2}^{\infty}(v_2 - c_2) f(v_2) dv_2 \qquad \textbf{(4)}$$

subject to $c_1, c_2 \geq 0$.

The first two terms in **(4)** are the collateral costs incurred in each country at the start of the day. The remaining terms are the expected shortfall costs incurred if the bank is first payer in each country. The expression for the shortfall captures the expected excess liquidity need, conditional on the value of payments, v_i, being greater than the amount of collateral posted, c_i. Clearly, the expected shortfall is zero for all realisations of payment values below the amount of collateral posted. A bank's optimal choices of collateral to hold at the start of the day are, thus:

$$c_i = \max\left[0, F^{-1}\left(\frac{w_i - 2g_i}{w_i}\right)\right] \quad \textbf{(5)}$$

for i=1,2.

Equation **(5)** implies that a bank's collateral choice in each country will be weakly decreasing in the unit cost of obtaining collateral at the start of the day (g_i), and weakly increasing in the cost of meeting a shortfall later in the day (w_i). A bank will hold less collateral in the country in which it is more expensive to obtain.

3.2 Symmetric cross-border use of collateral

We now turn to the case in which both central banks permit cross-border use of collateral. An immediate implication is that banks will have an incentive to shift their collateral holdings entirely to the cheaper country. For illustrative purposes, we assume that $g_1 < g_2$, implying that banks will find it optimal to hold country 1 collateral only, and hence minimise costs with respect to c_1 only. (The results for $g_1 > g_2$ are analogous.[15]) Given that, with cross-border use of collateral, additional collateral needs will also be met in the cheapest country, the shortfall costs are equivalent in the two countries: $w_2 = w_1$. Therefore, a bank's cost minimisation problem is:

$$\min_{c_1} E[C] = g_1 c_1 + w_1 \int_{c_1}^{\infty} (v_1 - c_1) f(v_1) dv_1 \quad \textbf{(6)}$$

subject to $c_1 \geq 0$.[16] A bank's optimal start-of-the-day collateral choice in each of the countries is then:

$$c_1 = \max\left[0, F^{-1}\left(\frac{w_1 - g_1}{w_1}\right)\right]$$
$$c_2 = 0 \quad \textbf{(7)}$$

As in the case with no cross-border collateral, banks will hold more collateral the lower is the start-of-day cost and the greater is the cost of meeting a collateral shortfall.

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(15) If $g_1 = g_2 = g$, expected costs will be minimised with respect to collateral choices in both countries, and the distribution of holdings across the two countries will be indeterminate.

(16) A non-negativity constraint is still appropriate here in that our focus is on how banks might allocate a dedicated collateral portfolio, the total value of which must necessarily be non-negative. While it would be possible to take a short position in one country's collateral and a (more than) offsetting long position in another, this seems more consistent with a trading portfolio than a dedicated collateral portfolio.

3.3 The implications of cross-border use of collateral

In this subsection we compare the start-of-day collateral choices in the two cases of cross-border use of collateral and consider the implications of cross-border use of collateral for liquidity risk in each country.

3.3.1 Collateral holdings

As we have seen, the distribution of collateral holdings across countries will be affected by the shift to cheaper collateral once cross-border use is permitted. The total amount of collateral held will also be affected. When cross-border use of collateral is permitted, there is, for any given collateral choice, a higher expected marginal benefit from each incremental unit of collateral purchased at the start of the day. With no cross-border use of collateral, each unit of collateral in either country will only be used if the bank is first payer in this country: ie with a probability of 0.5. With collateral fungible across the two centres, on the other hand, the probability that each incremental unit of collateral purchased will be used (in either one location or the other) is strictly greater than 0.5.

The higher expected marginal benefit from accumulating collateral that can be used in both countries, and the ability to shift holdings to the cheaper location, will tend to boost total collateral holdings. However, working in the opposite direction, a bank's ability to draw on idle collateral in one country to meet liquidity needs in the other will allow it to economise on its total collateral holdings.

To establish which of these effects will dominate and hence under what conditions a bank's total collateral holdings will either rise or fall, we need to compare the collateral choices shown in equations **(5)** and **(7)**. To compare, assume that payment values are exponentially distributed. For the case of $g_1 < g_2$, banks' total collateral holdings will fall (rise) under cross-border use of collateral if and only if:

$$\ln\left(\frac{w_1}{2g_1}\right) + \ln\left(\frac{w_2}{2g_2}\right) = \ln\left(\frac{w_1 w_2}{4g_1 g_2}\right) > (\leq) \ln\left(\frac{w_1}{g_1}\right) \Leftrightarrow \frac{w_2}{g_2} > (\leq) 4 \qquad \textbf{(8)}$$

That is, for a sufficiently high (low) ratio of shortfall costs to start-of-day collateral costs in country 2, banks will find it optimal to reduce (increase) total collateral holdings. The intuition is that banks will optimally accumulate larger holdings of collateral in each country in the absence of cross-border use of collateral when the costs of meeting a shortfall are higher relative to the costs of obtaining collateral at the start of the day. Hence, there is greater scope to economise on total collateral holdings when symmetric cross-border use is permitted.

3.3.2 Liquidity risk

The expected collateral shortfall can act as a metric for liquidity risk since higher shortfalls mean that a larger portion of payments are delayed. A bank's expected shortfall (in the country in which it is the first payer) will be decreasing in the total amount of collateral that it has available when liquidity needs are realised. Hence, we are able to gauge liquidity risk in each country i, by observing the total amount of collateral that banks can draw upon to satisfy a given obligation.

We find that it is optimal for a bank to hold at least as much collateral in total when symmetric cross-border use of collateral is permitted than in a single country when there is no cross-border use. Hence, a bank is always able to draw on at least as large a pool of collateral, and its shortfall in the event that it experiences a liquidity need is consequently either lower or at least no greater. Indeed, whenever it is optimal for a bank to choose to hold a strictly positive amount of collateral in the symmetric case, the amount available with cross-border use of collateral will be strictly greater. This will be true even though a bank may, under certain scenarios for relative shortfall and start-of-day collateral costs, find it optimal to hold less collateral in total across the two countries at the start of the day. It follows that because the two banks are identical, a fall in each bank's expected shortfall in each country implies a fall in the expected shortfall at the country level also. These results are confirmed in Proposition 1 and Corollary 1.

Proposition 1: Liquidity risk will be the same or will be lower when there is symmetric cross-border use of collateral compared to when there is no cross-border use of collateral.

Proof: Let $g_1 < g_2$. It is clear from the optimal collateral choices in equations **(5)** and **(7)** that the total amount of collateral available to meet a liquidity need in country 1 in the case of symmetric cross-border use of collateral will be at least as large as that with no cross-border use since:

$$\max\left[0, F^{-1}\left(\frac{w_1 - 2g_1}{w_1}\right)\right] \leq \max\left[0, F^{-1}\left(\frac{w_1 - g_1}{w_1}\right)\right] \quad \textbf{(9)}$$

The inequality in equation **(9)**, and equations **(5)** and **(7)**, imply that the total quantum of collateral available to meet a liquidity need in country 2 is also at least as large when there is symmetric cross-border use of collateral, since:

$$\max\left[0, F^{-1}\left(\frac{w_2 - 2g_2}{w_2}\right)\right] \leq \max\left[0, F^{-1}\left(\frac{w_1 - 2g_1}{w_1}\right)\right] \leq \max\left[0, F^{-1}\left(\frac{w_1 - g_1}{w_1}\right)\right] \quad \textbf{(10)}$$

The results are analogous when $g_1 > g_2$. ■

Corollary 1 follows immediately from proposition 1:

Corollary 1: When banks hold positive amounts of collateral at the start of the day, liquidity risk will be strictly lower when there is symmetric cross-border use of collateral.

3.3.3 Cost efficiency

A further implication of symmetric cross-border use of collateral is that, whenever it is optimal to hold positive amounts of collateral at the start of the day, banks' total expected costs will also be reduced. When cross-border use of collateral is permitted, banks could continue to hold the same level of collateral in each country as they would with no cross-border use of collateral. The results above indicate that they do not. Thus, banks' expected costs must fall by revealed preference.

3.3.4 Summary

This concludes our comparison of collateral holdings and liquidity risk in the cases of symmetric and no cross-border use of collateral. We have found that, notwithstanding the potential for a decline in banks' total collateral holdings, cross-border use of collateral will reduce liquidity risk. This is true under all relative cost scenarios in which it is optimal for banks to hold positive quantities of collateral. We also find that banks will naturally shift towards the cheapest collateral.

In order to examine the robustness of these findings to the relaxation of certain assumptions in the baseline model, Section 4 introduces several extensions.

4 Extensions

In this section we examine four extensions to the model. First, recognising that some central banks may accept foreign collateral while others do not, we allow for so-called asymmetric cross-border use of collateral. We then allow for the possibility that operational incidents occur which disrupt the cross-border use of collateral. Third, we relax the assumption that a bank can be first payer in one country only by allowing for the possibility that a bank faces liquidity needs in both countries simultaneously. Finally, we introduce an additional policy option for central banks which is the acceptance of foreign collateral only in times of stress.

4.1 Asymmetric cross-border use of collateral

The case of asymmetric cross-border use of collateral is relevant because central banks typically set collateral policy unilaterally. Indeed, almost all of the examples of existing cross-border collateral arrangements listed in the introduction represent asymmetric cross-border use of collateral. In this section, we again work with the model used in Section 3, but now assume that country 2 collateral can be used to support liquidity needs in country 1, but not *vice versa*.

4.1.1 Banks' collateral choices

The implications of asymmetric cross-border use of collateral for collateral choices, and hence for total collateral held and liquidity risk, depend upon relative collateral costs in the two countries.

We begin with the case of $g_1 \geq g_2$. Here, the collateral that can be used in both countries (country 2 collateral) is also the cheapest and hence the two factors driving collateral choice, its cost and the scope to use it in country 1, are mutually reinforcing. Thus, the implications for both total collateral held and liquidity risk are identical to those in the symmetric case with $g_1 > g_2$.

When $g_1 < g_2$, country 2 collateral remains attractive by virtue of its being eligible in both countries; but banks may have an incentive to hold a proportion of their collateral portfolios in country 1 because of its relative cheapness. Thus, there are two possible outcomes in the asymmetric case with $g_1 < g_2$: either all collateral is held in country 2 because the advantage associated with the eligibility of country 2 collateral in both countries outweighs the cost-disadvantage; or, banks hold some collateral in each country. Which of these outcomes occurs depends on the size of the collateral cost differential.

To see this, we return to the bank's cost minimisation problem. The cost minimisation is now as in equation **(11)**. Throughout this subsection we shall restrict attention to cases in which banks choose to hold strictly positive amounts of collateral when they are not permitted to use collateral across borders. This will ease comparison of outcomes across the various cases.

$$\min_{c_1,c_2} E[C] = g_1 c_1 + g_2 c_2 + \frac{w_1}{2}\int_{c_1+c_2}^{\infty}(v_1 - c_1 - c_2) f(v_1) dv_1 + \frac{w_2}{2}\int_{c_2}^{\infty}(v_1 - c_2) f(v_2) dv_2 \qquad \textbf{(11)}$$

Minimisation of equation **(11)** with respect to c_1 and c_2 leads to the following simultaneous equations:

$$c_1 = F^{-1}\left(\frac{w_1 - 2g_1}{w_1}\right) - c_2 \qquad \textbf{(12)}$$

$$0 = g_2 - \frac{w_1}{2}(1 - F(c_1 + c_2)) - \frac{w_2}{2}(1 - F(c_2)) \qquad \textbf{(13)}$$

The key condition for whether banks hold collateral in both countries is shown in equation **(14)**. This condition is more likely to hold the more expensive is country 2 collateral relative to country 1 collateral.

$$g_2 - g_1 > \frac{g_1 w_2}{w_1} \qquad \textbf{(14)}$$

When the inequality in equation **(14)** holds, a bank's optimal collateral choices are:

$$c_1 = F^{-1}\left(\frac{w_1 - 2g_1}{w_1}\right) - F^{-1}\left(\frac{w_2 - 2(g_2 - g_1)}{w_2}\right) \qquad \textbf{(15)}$$

$$c_2 = F^{-1}\left(\frac{w_2 - 2(g_2 - g_1)}{w_2}\right) \quad \textbf{(16)}$$

In words, when country 2 collateral is sufficiently more expensive than country 1 collateral that the condition in equation **(14)** holds, it will be optimal to hold some collateral in both countries. When country 2 collateral is only modestly more expensive, so that the condition in **(14)** does not hold, it will be optimal for a bank to hold collateral in country 2 only:

$$c_1 = 0$$

$$c_2 = F^{-1}\left(\frac{w_1 + w_2 - 2g_2}{w_1 + w_2}\right) \quad \textbf{(17)}$$

A derivation of these results is available in the appendix.

In the remainder of this subsection, we compare outcomes for total collateral and liquidity risk when there is asymmetric cross-border use of collateral with those in the cases of symmetric or no cross-border use of collateral.

4.1.2 Total collateral

In Table A we compare total collateral holdings under asymmetric cross-border use of collateral with those under symmetric and no cross-border use.

As noted above, when collateral that is eligible in both countries is also cheaper (or at least no more expensive; $g_1 \geq g_2$ in our example), the outcome under asymmetric cross-border use of collateral is identical to that under symmetric use. It follows that total collateral in the asymmetric case may either be higher or lower than that under no cross-border use, as shown in Section 3.3.1. As before, whether total collateral is higher or lower will depend on the costs of obtaining collateral at the start-of-day and the costs of meeting a collateral shortfall during the day.

When the collateral eligible in both countries is more expensive, total collateral holdings will always be lower in the asymmetric case than when there is symmetric cross-border use of collateral. Because both countries' collateral can be used in either country in the symmetric case, a bank is always able to take advantage of the cheaper collateral. In the asymmetric case, by contrast, a bank has to acquire the more expensive collateral if it is to take advantage of its eligibility in both countries. The bank will then optimally acquire less collateral overall than it would if the cheaper collateral was eligible in both countries.

When the collateral eligible in both countries is sufficiently more expensive that it is optimal to hold collateral in both countries, total collateral holdings will be unambiguously lower than when cross-border use is not permitted in either country. Indeed, in this case, the total holding is identical to the optimal holding in country 1 alone under no cross-border use of collateral. When the cost differential is small enough that banks hold only the collateral eligible in both countries, total collateral holdings under asymmetric use could either be higher or lower than when there is no cross-border use.

Table A
Banks' total collateral holdings

<table>
<tr><th></th><th>No cross-border use of collateral</th><th>Symmetric</th><th>Asymmetric †</th><th>Comparison</th></tr>
<tr><td>$g_1 \geq g_2$</td><td rowspan="3">$F^{-1}\left(\frac{w_1 - 2g_1}{w_1}\right) + F^{-1}\left(\frac{w_2 - 2g_2}{w_2}\right)$</td><td>$F^{-1}\left(\frac{w_2 - g_2}{w_2}\right)$</td><td>$F^{-1}\left(\frac{w_2 - g_2}{w_2}\right)$</td><td>No > Sym. = Asym.
or
No ≤ Sym. = Asym.</td></tr>
<tr><td>$g_1 < g_2$
and
$g_2 - g_1 > g_1 \frac{w_2}{w_1}$</td><td rowspan="2">$F^{-1}\left(\frac{w_1 - g_1}{w_1}\right)$</td><td>$F^{-1}\left(\frac{w_1 - 2g_1}{w_1}\right)$</td><td>No > Sym. > Asym.
or
Sym. > No > Asym.</td></tr>
<tr><td>$g_1 < g_2$
and
$g_2 - g_1 \leq g_1 \frac{w_2}{w_1}$</td><td>$F^{-1}\left(\frac{w_1 + w_2 - 2g_2}{w_1 + w_2}\right)$</td><td>No > Sym. > Asym. ††
or
Sym. ≥ No > Asym.
or
Sym.>Asym. ≥ No</td></tr>
</table>

† The results are shown for the case in which country 2 collateral is eligible in both countries but country 1 collateral is not.

†† This result follows by: $g_2 > g_1 \Rightarrow \frac{2g_2}{w_1 + w_2} > \frac{g_2}{w_2} > \frac{g_1}{w_1} \Rightarrow \frac{w_1 - g_1}{w_1} > \frac{w_1 + w_2 - 2g_2}{w_1 + w_2}$

4.1.3 Liquidity risk

As in Section 3.3.2, we can compare outcomes for liquidity risk in each country by comparing the total amounts of collateral a bank can draw upon when it is called upon to meet a payment obligation. Tables B and C compare values of collateral available in each country, for each of the three cross-border collateral cases (no cross-border; asymmetric; and symmetric), under each relative cost scenario.

Relative to the case with no cross-border use of collateral, Table C shows that total collateral available is always higher, and liquidity risk consequently lower, in the country not accepting foreign collateral (country 2 in our example) under asymmetric use. Table B shows that this is also true in the country in which foreign collateral is accepted (country 1 in our example) except when the collateral eligible in both countries is significantly more expensive. In this case, the amount of collateral available when there is asymmetric cross-border use of collateral is the same as that when there is no cross-border use.

Table B
Available collateral in country 1

<table>
<tr><th></th><th>No cross-border use of collateral</th><th>Symmetric</th><th>Asymmetric †</th><th>Comparison</th></tr>
<tr><td>$g_1 \geq g_2$</td><td rowspan="3">$F^{-1}\left(\frac{w_1 - 2g_1}{w_1}\right)$</td><td>$F^{-1}\left(\frac{w_2 - g_2}{w_2}\right)$</td><td>$F^{-1}\left(\frac{w_2 - g_2}{w_2}\right)$</td><td>Sym. = Asym. > No</td></tr>
<tr><td>$g_1 < g_2$
and
$g_2 - g_1 > g_1 \frac{w_2}{w_1}$</td><td rowspan="2">$F^{-1}\left(\frac{w_1 - g_1}{w_1}\right)$</td><td>$F^{-1}\left(\frac{w_1 - 2g_1}{w_1}\right)$</td><td>Sym. > No = Asym.</td></tr>
<tr><td>$g_1 < g_2$
and
$g_2 - g_1 \leq g_1 \frac{w_2}{w_1}$</td><td>$F^{-1}\left(\frac{w_1 + w_2 - 2g_2}{w_1 + w_2}\right)$</td><td>Sym. > Asym. > No</td></tr>
</table>

† The results are shown for the case in which country 2 collateral is eligible in both countries but country 1 collateral is not.

Table C
Available collateral in country 2

<table>
<tr><th></th><th>No cross-border use of collateral</th><th>Symmetric</th><th>Asymmetric †</th><th>Comparison</th></tr>
<tr><td>$g_1 \geq g_2$</td><td rowspan="3">$F^{-1}\left(\frac{w_2 - 2g_2}{w_2}\right)$</td><td>$F^{-1}\left(\frac{w_2 - g_2}{w_2}\right)$</td><td>$F^{-1}\left(\frac{w_2 - g_2}{w_2}\right)$</td><td>Sym. = Asym. > No</td></tr>
<tr><td>$g_1 < g_2$
and
$g_2 - g_1 > g_1 \frac{w_2}{w_1}$</td><td rowspan="2">$F^{-1}\left(\frac{w_1 - g_1}{w_1}\right)$</td><td>$F^{-1}\left(\frac{w_2 - 2(g_2 - g_1)}{w_2}\right)$</td><td>Sym. > Asym. > No</td></tr>
<tr><td>$g_1 < g_2$
and
$g_2 - g_1 \leq g_1 \frac{w_2}{w_1}$</td><td>$F^{-1}\left(\frac{w_1 + w_2 - 2g_2}{w_1 + w_2}\right)$</td><td>Sym. > Asym. > No</td></tr>
</table>

† The results are shown for the case in which country 2 collateral is eligible in both countries but country 1 collateral is not.

It is clear that liquidity risk is generally lowest when there is symmetric cross-border use of collateral. The exception is the case in which the collateral eligible in both countries is cheaper. In this case the implications of symmetric and asymmetric cross-border use of collateral are equivalent because collateral choices are the same.

4.2 Operational risk

So far we have assumed that there are no additional costs or risks associated with the use of foreign collateral relative to domestic collateral, beyond any differences in start-of-day collateral costs. This is why with symmetric cross-border use of collateral, banks will find it optimal to shift their holdings entirely to the cheaper collateral. In this subsection, we relax this assumption by introducing the possibility of operational problems which may prevent the use of foreign collateral. For simplicity, we take each country's payment system and CSD as a single infrastructure. A country's infrastructure is hit by an operational shock with probability r. This shock leaves it out of action for the rest of the day. Importantly, shocks can only hit after banks have made their collateral choices at the start of the day. Assuming that a country's infrastructure is unavailable for the rest of the day implies quite a severe shock, but as an example it suffices to show the qualitative effects that the operational risk might have on banks' collateral choices.

The potential for operational shocks implies that banks face greater risk when they rely on cross-border use of collateral. When a bank only uses domestic collateral, there is a probability, r, that it will be unable to use it. But when a bank uses only foreign collateral, both infrastructures must be fully operational. The probability that at least one of the infrastructures is unavailable is $r^2 + 2r(1-r)$, which is greater than r.

Assume there is symmetric cross-border use of collateral and that $g_1 < g_2$. A bank's cost minimisation problem when there is the potential for operational shocks is

$$\min_{c_1,c_2} E[C] = g_1 c_1 + g_2 c_2 + \frac{r(1-r)w_1}{2}\int_{c_1}^{\infty}(v_1 - c_1) f(v_1)dv_1 + \frac{(1-r)^2 w_1}{2}\int_{c_1+c_2}^{\infty}(v_1 - c_1 - c_2) f(v_1)dv_1$$
$$+ \frac{r(1-r)w_2}{2}\int_{c_2}^{\infty}(v_2 - c_2) f(v_2)dv_2 + \frac{(1-r)^2 w_1}{2}\int_{c_1+c_2}^{\infty}(v_2 - c_1 - c_2) f(v_2)dv_2 \quad \textbf{(18)}$$

$E[C]$ is a bank's expected cost net of the cost of failing to make payments by the end of the day due to infrastructural failure (which is unaffected by a bank's collateral choices). Chart 3, drawn for the case of an exponential distribution with $l = 1$, illustrates the fact that a bank's optimal holding of the cheaper collateral (country 1 collateral) when operational problems can occur is lower than in the absence of operational risk. A greater probability of either infrastructure suffering an operational disruption reduces a bank's incentive to hold country 1 collateral because there is a smaller probability that it will be able to use this collateral in either country.(17)

Chart 3: The effect of operational risk on a bank's collateral choices; for $g_1 = 1$**,** $g_2 = 1.2$**,** $a = 1.1$**,** $d = 1.5$ **and** $l = 1$

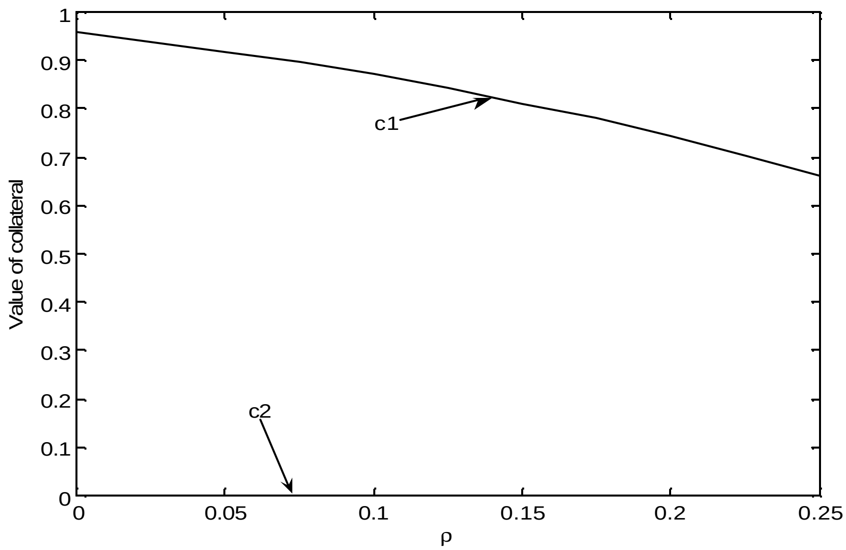
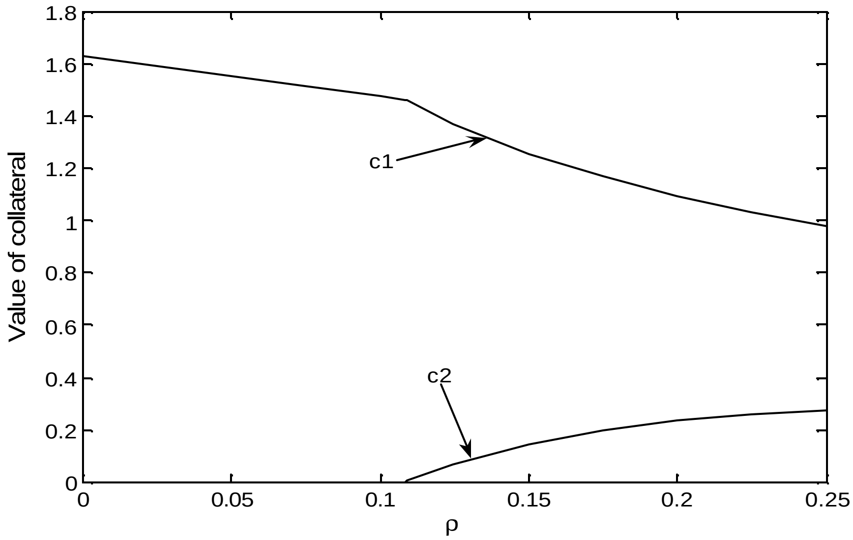


Chart 4 considers the case of a higher delay cost. In this case, not only does the bank reduce its holdings of country 1 collateral, but it will also substitute the more expensive country 2 collateral for country 1 collateral if r is sufficiently high. This reflects the fact that the difference between the cost of obtaining collateral at the start of the day and the cost of suffering a shortfall is greater.

In summary, the possibility of operational disruption tempers banks' incentives to shift collateral holdings towards cheaper collateral and, if the probability of an operational disruption is sufficiently high, they may have an incentive to continue to hold some of the more expensive collateral. In practice, of course, r is likely to be low; operational failure in key infrastructures is an extremely rare event. However, were we to relax the assumption of risk-neutrality in our analysis, banks might place sufficient weight on mitigation of exposure to operational failure that the substitution shown in Charts 3 and 4 might be observed at lower values for r.

(17) It is important to note that the reduction in collateral holdings in country 1 as r increases reflects not only the potential for an operational failure to disrupt cross-border use of collateral, but also the potential for a failure to disrupt usage of domestic collateral to meet domestic needs.

Chart 4: The effect of operational risk on a bank's collateral choices; for $g_1 = 1$, $g_2 = 1.2$, $a = 1.1$, $d = 4$ and $l = 1$



4.3 Simultaneous liquidity needs in the two countries

Each bank is the first payer in only one country in the baseline model. But a bank could face liquidity needs in both countries simultaneously. In this section we extend the baseline model to show that routine cross-border use of collateral may actually increase expected shortfalls (and hence liquidity risk) in states of the world in which simultaneous needs arise. This follows from the fact that, to the extent that banks have an incentive to economise on collateral holdings when cross-border use of collateral is permitted, a smaller pool of collateral will be available when liquidity needs are realised in both countries at the same time.

To address this possibility, we assume that there are four potential states of the world: bank *A* is the first payer in both countries; bank *A* is the first payer in country 1 and bank *B* is the first payer in country 2; bank *A* is the first payer in country 2 and bank *B* is the first payer in country 1; or bank *B* is the first payer in both countries. Each state of the world arises with probability 1/4.

We assume that $g_1 < g_2$, so banks only hold country 1 collateral. A bank's expected shortfall when it realises a liquidity demand in just one of the countries is the same as is in the baseline model with symmetric cross-border use of collateral. A bank's expected shortfalls when it faces liquidity demands in both countries depends on how it distributes collateral between the two countries. Our assumptions that costs are linear in expected shortfalls and that delay costs are the same in both countries mean that a bank's optimal allocation of collateral between the two countries cannot be determined in the model. But suppose that a bank allocates an arbitrary proportion d $(1-d)$ of collateral to country 1(2), and then redistributes any idle collateral between countries as required. The sum of a bank's expected shortfalls when it faces liquidity demands in both countries then equals:

$$\int_{0}^{(1-d)c_1} \int_{dc_1+(1-d)c_1-v_2}^{\infty} (v_1 - dc_1 - (1-d)c_1 + v_2) f(v_1) f(v_2) dv_1 dv_2 + \int_{(1-d)c_1}^{\infty} \int_{dc_1}^{\infty} (v_1 - dc_1) f(v_1) f(v_2) dv_1 dv_2 +$$
$$\int_{0}^{dc_1} \int_{dc_1+(1-d)c_1-v_1}^{\infty} (v_2 - (1-d)c_1 - dc_1 + v_1) f(v_1) f(v_2) dv_1 dv_2 + \int_{dc_1}^{\infty} \int_{(1-d)c_1}^{\infty} (v_2 - (1-d)c_1) f(v_1) f(v_2) dv_1 dv_2 \quad \textbf{(19)}$$

In what follows, we illustrate the implications of this extension for total collateral held. In order to be able to make the relevant comparisons, we again analyse a version of the model in which payment values are exponentially distributed, setting $l = 1$ for convenience. For this case, drawing on the expressions for expected shortfalls from the baseline case and in equation **(19)**, each bank's cost minimisation problem can be expressed as

$$\min_{c_1} E[C] = g_1 c_1 + w_1 \left(1 + \frac{c_1}{4}\right) \exp(-c_1) \quad \textbf{(20)}$$

Minimising expected costs with respect to c_1, subject to $c_1 \geq 0$, yields

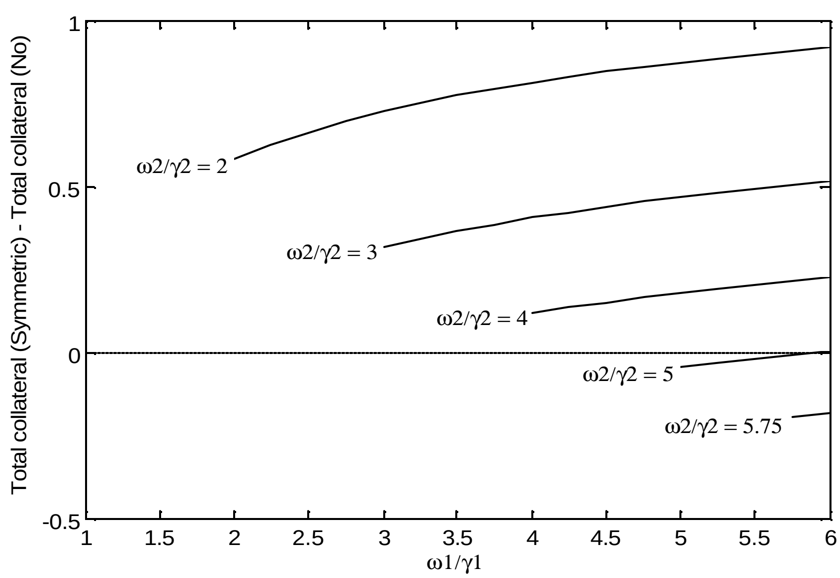
$$c_1 = \max\left[0, \ln\left(\frac{w_1}{4g_1}(3 + c_1)\right)\right] \quad \textbf{(21)}$$

Equation **(21)** confirms that a bank's optimal choice of collateral is again decreasing in the cost of obtaining collateral, g_1, and increasing in the cost of suffering a shortfall, w_1.

Solving numerically, Chart 5 plots the difference between total collateral held when there is symmetric cross-border use of collateral and that held when there is no cross-border use. As the chart is drawn for the case in which $g_1 < g_2$, w_1 / g_1 is plotted along the horizontal axis; these are the only relevant costs for the symmetric case. Each line on the charts represents a different value for w_2 / g_2 (with the lines starting at different points to reflect the fact that $g_1 < g_2$ implies that $w_1 / g_1 > w_2 / g_2$), which remains relevant in the no cross-border case. The vertical axis plots the difference between total collateral holdings in the cases of symmetric and no cross-border use of collateral. The results indicate that banks may hold either more or less collateral in total when they can use collateral across borders and when they face some probability of simultaneous demands in the two countries. A bank is increasingly likely to reduce its total collateral holdings, the higher is the ratio between shortfall costs and collateral costs in each country. This is consistent with the finding in our baseline model, although the threshold ratio beyond which total collateral holdings will begin to decline will now be higher.

In the baseline model we saw that liquidity risk fell unambiguously when cross-border use of collateral was permitted (as long as it was optimal for banks to hold positive quantities of collateral in the no cross-border case). This was true even when relative collateral costs and costs of suffering a shortfall were such that banks found it optimal to reduce their total collateral holdings. Chart 5 shows that this result no longer necessarily holds when banks can face collateral demands in both countries simultaneously. Here, the implication of a reduction in total collateral holdings is that expected shortfalls (liquidity risk) must be higher in at least one of the countries in the state of the world in which a bank faces simultaneous liquidity needs in the two countries. The effect on liquidity risk in each country depends on how banks choose to allocate collateral once liquidity needs are realised. While, for banks, the distribution of shortfalls between countries is immaterial in our model (given our assumption that delay costs are the same in both countries), the allocation of collateral across countries will have important implications for country-level liquidity risk and hence will be of relevance to central banks. But, importantly, in the states of the world in which liquidity needs arise locally, liquidity risk will still decline with cross-border use of collateral, as in the baseline model.

Chart 5: Difference between total collateral when there is simultaneous cross-border use of collateral and when there is no cross-border use of collateral: $g_1 = 1$**,** $g_2 = 1.2$**,** $a = 1.1$**,** $d = 1.5$ **and** $l = 1$



4.4 Cross-border use of collateral in stress situations only

We have so far restricted attention to cases in which foreign collateral is either always accepted or not accepted at all. An additional option is for a central bank to extend its eligible collateral pool only in situations of stress; ie when banks face very high liquidity demands. In this subsection we will compare banks' collateral choices and country-level liquidity risk when central banks accept foreign collateral only in times of stress with those in which central banks accept foreign collateral routinely. Given banks' decisions, we compare the liquidity risk implications of central banks' permitting cross-border use of collateral routinely with those when central banks only permit cross-border use of collateral in stress situations.

We modify the model in the following way to distinguish between normal and stressed situations. A stressed situation arises in country i when the value of each bank's payment in country i, v_i, exceeds a threshold value, v^*. Central bank i accepts foreign collateral only if $v_i > v^*$. We also introduce three indicator variables I_1, I_2 and H which are defined below:

$$I_i = \begin{cases} 1 \text{ if } c_i \le v^* \\ 0 \text{ if } c_i > v^* \end{cases} \quad \text{for } i = 1,2$$

$$H = \begin{cases} 1 \text{ if } c_1 + c_2 \le v^* \\ 0 \text{ if } c_1 + c_2 > v^* \end{cases}$$

We assume that $g_1 < g_2$. A bank's cost minimisation problem is

$$\begin{aligned}
\min_{c_1,c_2} E[C] &= g_1 c_1 + g_2 c_2 + \frac{w_1}{2} F(v^*) I_1 \int_{c_1}^{v^*} (v_1 - c_1) f(v_1 \mid v_1 \le v^*) dv_1 \\
&+ \frac{w_1}{2}(1 - F(v^*)) H \int_{v^*}^{\infty} (v_1 - c_1 - c_2) f(v_1 \mid v_1 > v^*) dv_1 \\
&+ \frac{w_1}{2}(1 - F(v^*))(1 - H) \int_{c_1+c_2}^{\infty} (v_1 - c_1 - c_2) f(v_1 \mid v_1 > v^*) dv_1 \\
&+ \frac{w_2}{2} F(v^*) I_2 \int_{c_2}^{v^*} (v_2 - c_2) f(v_2 \mid v_2 \le v^*) dv_2 \\
&+ \frac{w_1}{2}(1 - F(v^*)) H \int_{v^*}^{\infty} (v_2 - c_1 - c_2) f(v_2 \mid v_2 > v^*) dv_2 \\
&+ \frac{w_1}{2}(1 - F(v^*))(1 - H) \int_{c_1+c_2}^{\infty} (v_2 - c_1 - c_2) f(v_2 \mid v_2 > v^*) dv_2
\end{aligned} \quad \textbf{(22)}$$

The third and sixth components of the expected cost expression are the costs that a bank faces from a collateral shortfall when the value of the payment does not exceed v^*. These costs equal zero if a bank holds collateral in each country with a value equal to or greater than v^*. The fourth, fifth, seventh and eighth components are a bank's costs from meeting a collateral shortfall when the payment value exceeds v^* and it is permitted to use collateral from either country.

For illustrative purposes, we will assume payment values are exponentially distributed in each of the countries. Under the exponential distribution, the probability density functions (pdfs) are

$$f(v_i \mid v_i \le v^*) = \frac{\exp(-v_i)}{1 - \exp(-v^*)}$$

$$f(v_i \mid v_i > v^*) = \frac{\exp(-v_i)}{\exp(-v^*)}$$

Substituting these into equation **(22)**, the cost minimisation problem becomes

$$\min_{c_1,c_2} E(C) = g_1c_1 + g_2c_2 + \frac{w_1}{2} I_1 \int_{c_1}^{v^*} (v_1 - c_1)\exp(-v_1)dv_1$$
$$+ \frac{w_1}{2} H \int_{v^*}^{\infty} (v_1 - c_1 - c_2)\exp(-v_1)dv_1 + \frac{w_1}{2}(1-H) \int_{c_1+c_2}^{\infty} (v_1 - c_1 - c_2)\exp(-v_1)dv_1$$
$$+ \frac{w_2}{2} I_2 \int_{c_2}^{v^*} (v_2 - c_2)\exp(-v_2)dv_2 + \frac{w_1}{2} H \int_{v^*}^{\infty} (v_2 - c_1 - c_2)\exp(-v_2)dv_2$$
$$+ \frac{w_1}{2}(1-H) \int_{c_1+c_2}^{\infty} (v_2 - c_1 - c_2)\exp(-v_2)dv_2 \quad \textbf{(23)}$$

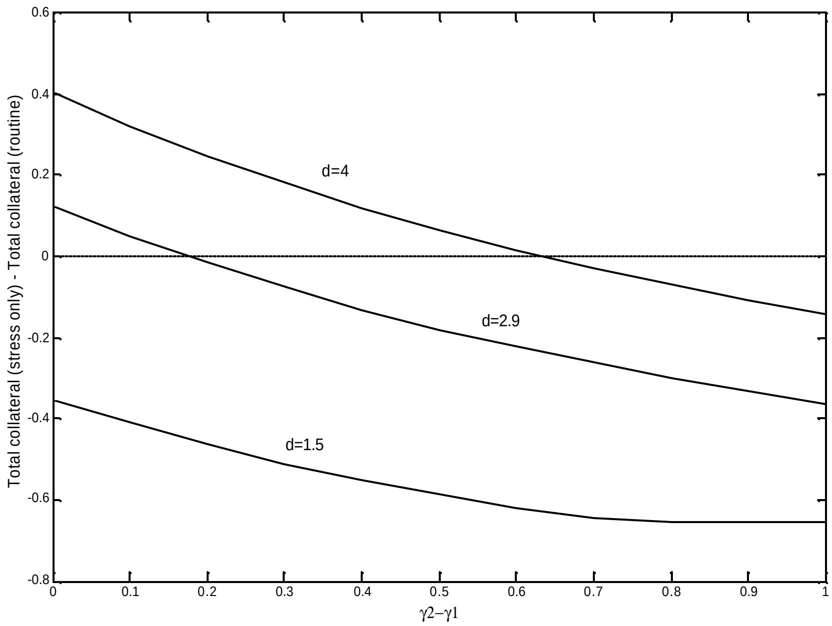
Banks' collateral choices when cross-border use of collateral is only permitted in stress situations could differ in several ways from their choices under routine cross-border use. First, because a bank cannot use country 1 collateral to meet liquidity needs in country 2 when $v_2 \leq v^*$, it may retain an incentive to hold a positive quantity of the more expensive country 2 collateral. Second, the total quantity of collateral held by banks may differ. Whether total collateral is higher or lower when foreign collateral is accepted only in times of stress rather than routinely depends on two opposing factors. The fact that banks cannot always use collateral across borders means that they may have an incentive to hold more collateral in total when cross-border use of collateral is only allowed in stress situations. Countering this effect, the substitution of the more expensive collateral for the cheaper collateral has a negative effect on the amount of collateral held in total.

Both of these effects are illustrated by the simulation results shown in Chart 6, drawn for a case in which each central bank accepts foreign collateral if payment values in its country are at least as high as the 97th percentile of the distribution.

The chart shows that banks may hold more collateral when cross-border use of collateral can only occur in stress situations. This is more likely when delay costs are high, which, as we saw in Section 3.3.1, is when banks are more likely to economise on total collateral holdings under routine cross-border use of collateral. The chart also shows that the difference between total collateral holdings in the two cases, grows smaller, and increasingly negative, as the differential between collateral costs in the two countries increases. The difference in total collateral holdings reaches a lower bound when a bank ceases to hold any country 2 collateral (which is shown in Chart 6 for the case in which delay cost equals 1.5).

The implications for liquidity risk of a central bank's only accepting foreign collateral in times of stress depend on whether banks hold more or less collateral in total. If banks hold less collateral, liquidity risk is higher in both countries, and for all payment values, than when cross-border use of collateral is allowed routinely. Conversely, if banks hold a larger quantum of collateral, liquidity risk may be lower in times of stress than if cross-border collateral is always permitted.

Chart 6: The difference in total collateral held by a bank when cross-border use of collateral is accepted in stress situations only and that when cross-border use of collateral is accepted routinely ($g_1 = 1$, $a = 1.1$, F(v*)=0.97)



5 Conclusion

This paper has introduced a framework for the analysis of cross-border use of collateral. We developed and described a two-country, two-bank model in which risk-neutral banks were members of both countries' payment systems and minimised expected costs with respect to their collateral choices in each country. Decisions were made under uncertainty as to the value of payments to be made and the location in which liquidity (and hence collateral) would be required. In our baseline model a bank could experience a liquidity demand in only one country at any one time. We then considered several extensions: allowing for only one country to accept foreign collateral ('asymmetric' usage); introducing the potential for operational disruptions to the infrastructure; considering the likelihood of simultaneous liquidity needs in the two countries; and, finally, allowing foreign collateral to be delivered in stress situations only.

We showed in our baseline model that expected collateral shortfalls in each country's system, our metric for liquidity risk, were always lower when all collateral could be used. Indeed, even when we allowed for asymmetric cross-border use, expected shortfalls were either lower or unchanged in both countries. This result held despite the possibility that banks would find it optimal to reduce their total holdings of collateral under certain scenarios for start-of-day collateral costs and costs of experiencing a liquidity shortfall. However, when the model was extended to allow for the possibility that liquidity needs arose simultaneously in the two countries, any such economisation on total collateral holdings led to an increase liquidity risk in states of the world in which banks faced liquidity needs in both countries at the same time.

When cross-border use of collateral was allowed only in situations of stress (characterised by high liquidity demands), it was shown that banks could have a lower incentive to reduce collateral holdings compared with the case of routine cross-border use of collateral. If banks did hold more collateral in total this meant that they would have a larger pool on which to draw in times of stress. Hence such a policy could deliver lower expected shortfalls in stressed states of the world than would full routine use. The policy with full routine use continued to deliver lower liquidity risk in normal states of the world. Hence, a central bank's policy choice would rest on the extent to which it placed greater weight on the mitigation of liquidity risk in stress situations.

We leave for future work further development of our model, including the incorporation of costs associated with the mobilisation of assets across borders or the application of haircuts, the modification of shortfall costs to make them a function of the size of the shortfall; the introduction of time-zone frictions.

Appendix

Derivation of optimal collateral choices when there is asymmetric cross-border use of collateral

From the minimisation of a bank's expected cost in equation **(11)** we obtain the simultaneous equations shown in equations **(12)** and **(13)**:

$$c_1 = F^{-1}\left(\frac{w_1 - 2g_1}{w_1}\right) - c_2$$

$$0 = g_2 - \frac{w_1}{2}(1 - F(c_1 + c_2)) - \frac{w_2}{2}(1 - F(c_2))$$

The next step in calculating a bank's optimal collateral choices is to draw these equations in (c_1, c_2) space. To get a sense of what the equations look like we derive the results in Table A1.

Table A1

	$c_1 = F^{-1}\left(\frac{w_1 - 2g_1}{w_1}\right) - c_2$		$0 = g_2 - \frac{w_1}{2}(1 - F(c_1 + c_2)) - \frac{w_2}{2}(1 - F(c_2))$
If $c_1 = 0$,	$c_2 = F^{-1}\left(\frac{w_1 - 2g_1}{w_1}\right)$	If $c_1 = 0$,	$c_2 = F^{-1}\left(\frac{w_1 + w_2 - 2g_2}{w_1 + w_2}\right)$
If $c_2 = 0$,	$c_1 = F^{-1}\left(\frac{w_1 - 2g_1}{w_1}\right)$	If $c_1 \to \infty$,	$c_2 \to F^{-1}\left(\frac{w_2 - 2g_2}{w_2}\right)$

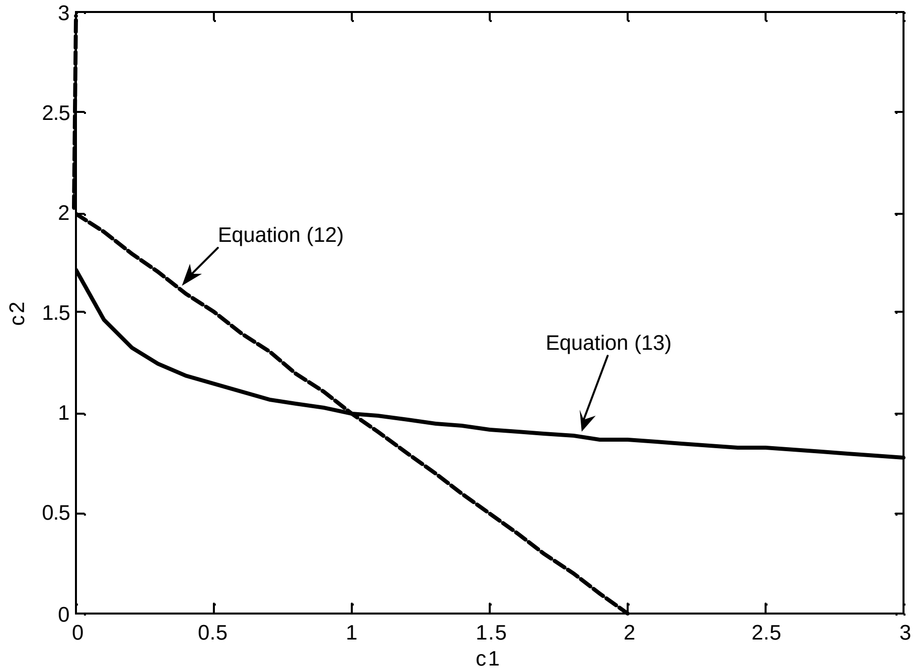
For simplicity, we assume that

$$F^{-1}\left(\frac{w_1 - 2g_1}{w_1}\right), F^{-1}\left(\frac{w_1 + w_2 - 2g_2}{w_1 + w_2}\right) \geq 0$$

This assumption, in combination with the fact that equation **(12)** has a slope of -1 and that equation **(13)** shows c_2 tending to a strictly positive value when c_1 tends to infinity, is sufficient to ensure a unique solution.

Charts A1 and A2 show the two equations subject to the constraints that $c_1, c_2 \geq 0$. A bank's optimal collateral choices are those at which the equations (subject to the constraints) cross. Whether a bank holds positive amounts of both countries' collateral or only country 2 collateral depends on where the two equations meet the c_2 axis. A bank holds a positive amount of either country's collateral if equation **(12)** meets the c_2 axis above the point at which equation **(13)** meets the c_2 axis. This is shown in Chart A1.

Chart A1



Equation **(12)** meets the c_2 axis above where equation **(13)** meets the c_2 axis because

$$F^{-1}\left(\frac{w_1 - 2g_1}{w_1}\right) > F^{-1}\left(\frac{w_1 + w_2 - 2g_2}{w_1 + w_2}\right) \Leftrightarrow g_2 - g_1 > \frac{g_1 w_2}{w_1}$$

This gives us the condition in equation **(14)**. Substituting equation **(12)** into **(13)**, we derive the bank's optimal collateral choices

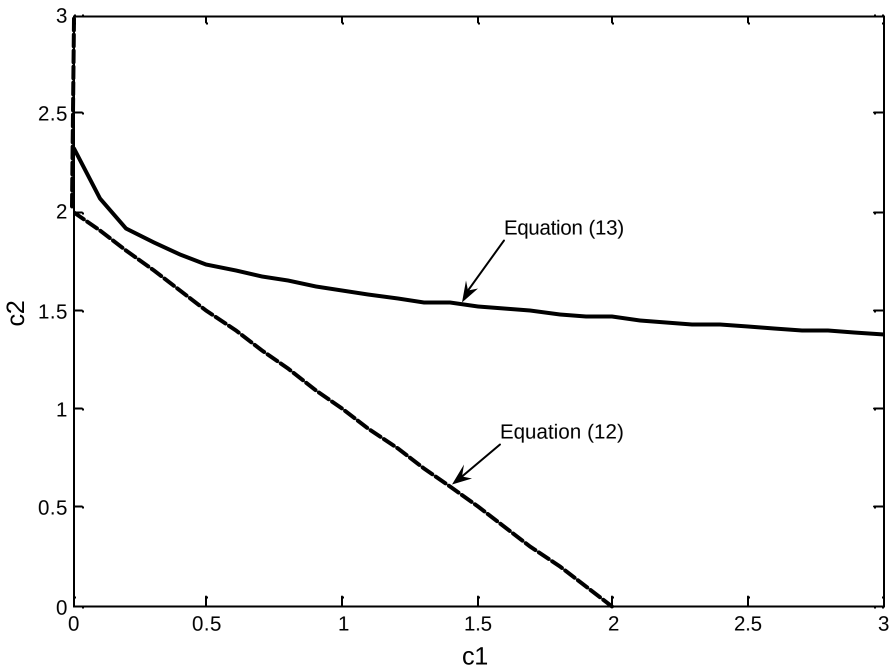
$$c_1 = F^{-1}\left(\frac{w_1 - 2g_1}{w_1}\right) - F^{-1}\left(\frac{w_2 - 2(g_2 - g_1)}{w_2}\right)$$

$$c_2 = F^{-1}\left(\frac{w_2 - 2(g_2 - g_1)}{w_2}\right)$$

When equation **(12)** meets the c_2 axis at the same point or below where equation **(13)** meets the c_2 axis (because equation **(14)** does not hold), a bank holds only country 2 collateral. This is shown in Chart A2. Substituting $c_1 = 0$ into equation **(13)**, we get a bank's optimal holdings of country 2 collateral

$$c_2 = F^{-1}\left(\frac{w_1 + w_2 - 2g_2}{w_1 + w_2}\right)$$

Chart A2



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