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Abstract

The existence of and uncertainty about structural change in the economy are important features facing policymakers. This paper considers the implications for policy design of uncertainty about structural change, modelling the time variation in parameters of forward-looking models as Markov processes. We extend an algorithm of Backus and Driffill for optimal discretionary policy in rational expectations models to the case with Markov switching in model parameters. As an illustration, we apply our method to determine the optimal monetary policy solution in the presence of structural changes in intrinsic output persistence, within a hybrid New Keynesian model estimated for the euro area. We find that the coefficients of the optimal policy rule are state-dependent, and depend non-linearly on the transition probabilities between states with different values of intrinsic output persistence.

Key words: Macroeconomic policy, optimality, discretion, regime switching.

JEL classification: E52, E58, E61.

Summary

Structural change is an important feature of economies. One aspect of such change is that features of the macroeconomy may vary over time – for example, intrinsic inflation and output persistence, the interest elasticity of demand, or the persistence of shocks. Moreover, uncertainty is an important issue facing policymakers, including uncertainty about structural change, about the best model of the economy, as well as about shocks hitting it. It is therefore interesting to study the implications for policymakers of structural changes that are not known with certainty. This paper considers policy design in the presence of structural change which is not known with certainty, and which may take the form of time variation in the parameters of an economic model. We handle this time variation by assuming there are Markov processes undelying the parameters, so that they can take on several different values and switch between them according to given probabilities. Moreover, structural change may take many different forms, and in particular it may be abrupt, transitory and asymmetric in nature; modelling structural change as Markov processes also enables us to capture these features. By contrast, other work on optimal monetary policy with parameter uncertainty, which assume that policymakers have symmetric uncertainty about parameters, do not capture all of these features.

Optimal policy with Markov switching in model parameters has previously been considered for backward-looking models. This paper extends the analysis to forward-looking models of the economy for the case of discretionary policy, when both the central bank and the private sector face uncertainty about model parameters. Deriving the solution for the case of forward-looking models with rational expectations is useful, since in contrast to purely backward-looking models, such models include forward-looking private sector expectations. This makes the treatment of private sector expectations consistent with the forward-looking behaviour of the policymaker. The macroeconomic models currently used for economic policy analysis mainly incorporate rational expectations, to ensure consistency, and to be able to base them – at least in part – on optimising microeconomic behaviour. In related work at the Bank, Fabrizio Zampolli derives optimal policy for the case of Markov switching of model parameters in backward-looking models, while Andrew Blake and Fabrizio Zampolli consider time-consistent optimal policy in forward-looking models within a semi-structural model representation. In related academic work, Lars Svensson and Noah Williams derive optimal policy with Markov switching in forward-looking models under both commitment and discretion.

As an illustration, we apply our method to study optimal monetary policy in the presence of structural changes in output persistence, within a forward-looking model estimated for the euro area. The main reason for adding this output persistence to the basic forward-looking model is to improve the fit with the data. Output persistence may change, for example, because of changes in the degree to which firms' investment decisions are constrained by cash flow, rather than being purely forward-looking. We assume there is a Markov process driving these changes. We find that the coefficients of the optimal policy rule depend on the state of the economy characterised by different values of output persistence, and the coefficients depend on the transition probabilities of the Markov process governing the structural change. For uncertainty about output persistence, the optimal policy rule is non-linearly related to the transition probabilities. We find that if the probability of moving from a state with low output persistence to a state with high output persistence is high, it is optimal for monetary policy in the former state to respond more aggressively to the lagged output gap, lagged inflation and the two shocks (to output and inflation) we consider, than in the absence of uncertainty about changes in output persistence.

1 Introduction

Structural change is an important feature of economies. One aspect is that parameters of macroeconomic models may vary over time – for example, intrinsic inflation and output persistence, the interest elasticity of demand, the slope of the Phillips curve, or the persistence of shocks. Moreover, uncertainty is an important issue facing policymakers (see for example Issing (2002), King (2004) and Greenspan (2005)), including uncertainty about structural change, about the correct model of the economy, as well as uncertainty about shocks hitting the economy. It is therefore of interest to study the potential implications for policymakers of structural changes which are not known with certainty.

This paper considers policy design in the presence of structural change which is not known with certainty, and which may take the form of time variation in model parameters. We model this time variation as Markov processes, which allows us to keep the analysis tractable. Moreover, structural change may take many different forms, and in particular it may be abrupt, episodic and asymmetric in nature; modelling structural change as Markov processes also enables us to capture these features. By contrast, classic papers on optimal monetary policy with parameter uncertainty (see Brainard (1967), Craine (1979)), which assume that policymakers have a symmetric prior probability distribution of the uncertain parameter, do not capture all of these features.[(1)]

In empirical work, macroeconomic models with Markov switching in parameters have been estimated,[(2)] and in theoretical work Markov switching in parameters has been incorporated in macroeconomic models for modelling regime changes. A number of papers have considered Markov switching in policy regimes described by simple policy rules, incorporated within dynamic stochastic general equilibrium (DSGE) models. This literature considers the optimal behaviour of private agents, assuming switching in the policy regime, which is characterised by simple rules, and not based on optimisation by the policy authority. Davig, Leeper and Chung (2004) consider on-going regime shifts in the feedback parameters of simple monetary and fiscal policy rules, modelling regime shifts as Markov processes in these parameters, within a DSGE model. They review the related literature,[(3)] which includes studies of the fiscal theory of the price

(1) Recent papers on optimal monetary policy with multiplicative uncertainty include for example Söderström (2002), Onatski and Williams (2003) and Levin *et al* (2005).
(2) For an estimation method of models with Markov switching, see Hamilton (1994).
(3) See Davig, Leeper and Chung (2004) for more details.

level incorporating one-time regime shifts in policy rules incorporated into DSGE models,[4] as well as studies considering on-going regime changes for exogenous processes for variables included in the policy rule, such as the inflation target, rather than for feedback parameters.[5]

This paper differs from this literature by considering policy chosen in an optimal manner under discretion in the presence of structural changes in parameters governing the evolution of the economy, for rational expectations equations. It is useful to study optimal policy, rather than simple rules, since it is one representation of how monetary policy might be set.[6]

Optimal policy with Markov switching in model parameters has previously been considered for backward-looking models.[7] This paper contributes to the literature by extending the analysis of optimal policy with Markov switching to forward-looking models of the economy for the case of discretionary policy, when both the central bank and the private sector face uncertainty about model parameters. Deriving the solution for the case of rational expectations models is useful, since in contrast to purely backward-looking models, such models contain forward-looking private sector expectations. This makes the treatment of private sector expectations consistent with the forward-looking behaviour of the policymaker in its optimisation problem (see Sargent (1999)). The macroeconomic models currently used for economic policy analysis are mostly of the rational expectations form, to ensure consistency, and to be able to base them – at least in part – on optimising microeconomic behaviour. In related recent work, Zampolli (2006) derives optimal policy for the case of Markov switching of model parameters in backward-looking models; Blake and Zampolli (2006) consider time-consistent optimal policy in forward-looking models within a semi-structural model representation, and Svensson and Williams (2005) consider optimal policy with Markov switching in forward-looking models under both commitment and discretion.

We extend an algorithm of Backus and Driffill (1986) (see also Oudiz and Sachs (1985)) for optimal discretionary policy in rational expectations models to the case with Markov switching in some of the model parameters governing the feedback of the economy to endogenous variables.[8]

(4) See Sims (1997), Woodford (1998), Loyo (1999), Mackowiak (2002), Weil (2003) and Daniel (2003).

(5) See Erceg and Levin (2003), Andolfatto and Gomme (2003), Davig (2002 and 2003), Leeper and Zha (2003), Schorfheide (2003), and Andolfatto, Hendry and Moran (2002).

(6) This does not imply, however, that optimal policy is an accurate description of how monetary policy is conducted in the United Kingdom, euro area or other countries.

(7) See for example Costa and Fragoso (1995) and Zampolli (2006).

(8) The method presented here also allows us to study the effect of changes in the properties of additive shock processes, such as their persistence.

The algorithm of Backus and Driffill (1986) has been applied for example in Soederlind (1999) and Giordani and Soederlind (2003). For optimal discretionary policy, Svensson and Williams (2005) also derive an algorithm by extending the approach of Oudiz and Sachs (1985); while they consider the case where only the central bank faces uncertainty, we consider the case where both the central bank and the private sector face uncertainty about model parameters.

As well as studying the implications of structural changes for optimal policy within standard rational expectations macroeconomic models, where the Markov switching process is estimated, our method may be used for conducting scenario analysis in such models, where the process governing the uncertainty about structural changes is postulated, for example based on judgement. Our method can be applied to study both one-time and on-going structural changes, to study the optimal response to low as well as high probability events, and to study asymmetric as well as symmetric processes.

As an illustration, we apply our method to study optimal monetary policy in the presence of structural changes in intrinsic output persistence, within a hybrid New Keynesian model estimated for the euro area by Smets (2003). The main reason for adding intrinsic output persistence to the basic forward-looking New Keynesian model is empirical, in order to improve the fit with the data. In this paper, we postulate the Markov process for the changes in intrinsic output persistence. Intrinsic output persistence may change for example due to changes in the degree to which firms' investment decisions are constrained by cash flow, rather than being purely forward looking. We find that the coefficients of the optimal policy rule depend on the state of the economy characterised by different values of intrinsic output persistence, and the coefficients depend on the transition probabilities of the Markov process governing the structural change. For uncertainty about intrinsic output persistence, the dependence of the coefficients of the optimal policy rule on the transition probabilities is found to be non-linear. We find that if the probability of moving from a state with low intrinsic output persistence to a state with high intrinsic output persistence is high, it is optimal for monetary policy in the state with low intrinsic output persistence to respond more aggressively to the lagged output gap, lagged inflation, and both shocks, than in the absence of uncertainty about changes in intrinsic output persistence.

The paper is organised as follows. Section 2 presents the algorithm for optimal policy in the presence of regime switching in model parameters. Section 3 describes the hybrid New Keynesian model and the shifts in regime, and determines optimal policy for both one-time and on-going regime changes. Section 4 concludes.

2 Derivation of optimal policy in rational expectations models with regime switching

We consider an optimisation problem where the central bank's loss function is given by

$$L = E_0 \sum_{t=0}^{\infty} \beta^t r(x_t, u_t) \tag{1}$$

where x_t is the vector of state variables, partitioned into the predetermined variables x_{1t} and the jump variables x_{2t}; u_t is the vector of control variables. The function $r(x_t, u_t)$ is assumed to be quadratic, and may be written as

$$r(x_t, u_t) = x_t' Q x_t + x_t' U u_t + u_t' U' x_t + u_t' R u_t \tag{2}$$

The optimisation is subject to linear transition equations for x_t describing the evolution of the economy. In this section, we generalise the formulation and solution of Backus and Driffill (1986) (see also Soederlind (1999)) for optimal discretionary policy without regime switching, to the case with regime switching in model parameters. In the optimisation under discretion without regime switching, the policymaker reoptimises every period, taking x_{1t} and the private agents' expectations as given (with private sector expectations being consistent with actual policy in equilibrium), subject to the equations governing the evolution of the economy (see Backus and Driffill (1986) and Soederlind (1999))

$$\begin{bmatrix} x_{1t+1} \\ E_t x_{2t+1} \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \begin{bmatrix} x_{1t} \\ x_{2t} \end{bmatrix} + \begin{bmatrix} B_1 \\ B_2 \end{bmatrix} u_t + \begin{bmatrix} \varepsilon_{t+1} \\ 0_{n_2 x 1} \end{bmatrix} \tag{3}$$

where the vector $x_t = \binom{x_{1t}}{x_{2t}}$ of state variables has been partitioned into n_1 predetermined state variables x_{1t}, and n_2 jump variables x_{2t}. Here,

$$A \equiv \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix}, \quad B \equiv \begin{bmatrix} B_1 \\ B_2 \end{bmatrix}$$

are constant matrices. The errors ε_{t+1} are assumed to be i.i.d. shocks with zero mean, whose covariance matrix $\Sigma = E_t^{\varepsilon} \left[\varepsilon_{t+1}' \varepsilon_{t+1}\right]$ is time-invariant, and which are uncorrelated with the predetermined variables x_{1t}. $0_{n_2 x 1}$ is a zero matrix of size $n_2 x 1$.

We model regime switching as a Markov process. The probability of moving from one state s^t at time t to another state s^{t+1} at time $t+1$ is modelled as a Markov chain with transition probabilities $p(s^{t+1} | s^t)$, with s^t and s^{t+1} lying in the set S of possible states of the economy. The transition probabilities are summarised in the transition matrix

$$P = (p_{ij}), \quad i, j = 1, ..., N \tag{4}$$

where $p_{ij} \equiv p(s^{t+1} = j | s^t = i)$, and the set of possible states, S, is assumed to contain N different states. At time t, the current state, s^t, is assumed to be observed by all agents in the economy, while the state in the next period, s^{t+1}, is not yet observed.

We generalise the formulation of the optimisation problem under discretion to the case with regime switching in model parameters as follows. The expectation operator of the optimisation problem without regime switching is generalised to include expectation formation over the unknown states $s^{t+1} \in S$ in the next period,

$$E_t = \sum_{s^{t+1} \in S} p(s^{t+1} | s^t) E_t^{\varepsilon} \tag{5}$$

Here and in the following, E_t^{ε} denotes the expectations operator over the additive shocks, ε_{t+1}, for a given state s^{t+1}. E_t^{ε} corresponds to the full expectations operator of equation **(3)** in the optimisation problem without regime switching. However, in the problem with regime switching, E_t^{ε} is only a partial expectations operator, since it does not include the expectations over the unobserved Markov states.

The matrices A and B governing the evolution of the economy from time t to $t+1$ are allowed to be state-dependent, depending on the state in the next period, s^{t+1}, which is not observable at time t by all agents in the economy. Allowing A and B to be state-dependent, and using the full expectations operator E_t of equation **(5)**, yields

$$\begin{bmatrix} x_{1t+1} \\ E_t x_{2t+1} \end{bmatrix} = \begin{bmatrix} A_{11}(s^{t+1}) & A_{12}(s^{t+1}) \\ \bar{A}_{21}(s^t) & \bar{A}_{22}(s^t) \end{bmatrix} \begin{bmatrix} x_{1t} \\ x_{2t} \end{bmatrix} + \begin{bmatrix} B_1(s^{t+1}) \\ \bar{B}_2(s^t) \end{bmatrix} u_t + \begin{bmatrix} \varepsilon_{t+1} \\ 0_{n_2 x1} \end{bmatrix} \tag{6}$$

where the matrices in the second row of the equation are defined as averages over future Markov states,

$$\bar{A}_{21}(s^t) \equiv \sum_{s^{t+1} \in S} p(s^{t+1} | s^t) A_{21}(s^{t+1})$$

$$\bar{A}_{22}(s^t) \equiv \sum_{s^{t+1} \in S} p(s^{t+1} | s^t) A_{22}(s^{t+1})$$

$$\bar{B}_2(s^t) \equiv \sum_{s^{t+1} \in S} p(s^{t+1} | s^t) B_2(s^{t+1})$$

These averages depend on the cuurent Markov state, s^t, which is assumed to be observed by all agents in the economy, *via* the transition probabilities. In the case without regime switching, the

Bellman equation for the value function, $\upsilon(x_t)$, of the optimisation problem is given by (see Backus and Driffill (1986) and Soederlind (1999))

$$\upsilon(x_t) = \min_{u_t} \{r(x_t, u_t) + \beta E_t [\upsilon(x_{t+1})]\} \quad \textbf{(7)}$$

subject to equation **(3)** governing the evolution of the economy, and taking private sector expectations and x_{1t} as given in the optimisation. The value function is the minimum attainable sum of current and expected future losses. The Bellman equation is generalised to the case with regime switching as follows (see Zampolli (2006) for the corresponding Bellman equation and its solution for backward-looking models with regime switching). The value function is allowed to be state-dependent, $\upsilon(x_t, s^t)$, and the expectations operator, which only captures expectations over the additive shocks in the case without regime switching of equation **(7)**, is generalised to the full expectations operator of equation **(5)**, which includes expectations over the Markov states in the next period. This yields a system of N coupled Bellman equations for the value functions $\upsilon(x_t, s^t)$ in each state, which represent the minimum attainable sum of current and expected future losses, given that the economy is in state s^t at time t,

$$\upsilon(x_t, s^t) = \min_{u_t} \left\{ r(x_t, u_t) + \beta \sum_{s^{t+1} \in S} p(s^{t+1}|s^t) E_t^{\varepsilon} \left[\upsilon(x_{t+1}, s^{t+1})\right] \right\}, \; s^t \in S \quad \textbf{(8)}$$

subject to the transition equations **(6)**, with private sector expectations, $E_t x_{2t+1}$, and x_{1t} taken as given in the optimisation.

In the following, we introduce some simplifying notation. The current state, s^t, is indexed by i, and the state in the next period, s^{t+1}, is indexed by j. Matrices that depend on the current state, s^t, are denoted by superscripts i, and matrices that depend on the state in the next period, s^{t+1}, are denoted by superscripts j. For example, $\bar{A}^i_{21} \equiv \bar{A}_{21}\left(s^t\right)$, $A^j_{11} \equiv A_{11}\left(s^{t+1}\right)$, and similarly for other matrices. Similarly, for the value function, we use the notation $\upsilon(x_t, i) \equiv \upsilon(x_t, s^t)$, and $\upsilon(x_t, j) \equiv \upsilon(x_t, s^{t+1})$. The transition probabilities p_{ij} are defined as above, by $p_{ij} \equiv p(s^{t+1} = j|s^t = i)$. We restrict ourselves to solutions of the following form, which are appropriate for linear-quadratic problems, extending the approach for the case without regime switching (see Backus and Driffill (1986) and Soederlind (1999)) to the case with regime switching, by allowing the matrices in equations **(9)** to **(11)** to be state-dependent,

$$\upsilon(x_t, i) = x'_{1t} V^i_t x_{1t} + \upsilon^i_t \quad \textbf{(9)}$$

$$u_t = -F^i_{1t} x_{1t} \quad \textbf{(10)}$$

$$x_{2t} = C^i_t x_{1t} \quad \textbf{(11)}$$

where private agents form expectations about x_{2t+1} according to equation **(11)**, and C_t^i and V_t^i are assumed not to depend on the additive shocks ε_{t+1}. Using the assumed relationships of **(9)** to **(11)**, the optimisation problem can be written as

$$x_{1t}' V_t^i x_{1t} + \upsilon_t^i = \min_{u_t} \left\{ r(x_t, u_t) + \beta \sum_{j=1}^{N} p_{ij} E_t^{\varepsilon} \left[\begin{array}{c} (A_{11}^j x_{1t} + A_{12}^j x_{2t} + B_1^j u_t + \varepsilon_{t+1})' V_{t+1}^j \\ (A_{11}^j x_{1t} + A_{12}^j x_{2t} + B_1^j u_t + \varepsilon_{t+1}) + \upsilon_{t+1}^j \end{array} \right] \right\}$$

$$s.t.\ E_t x_{2t+1} = \sum_{j=1}^{N} p_{ij} C_{t+1}^j \left[A_{11}^j x_{1t} + A_{12}^j x_{2t} + B_1^j u_t \right]$$

$$E_t x_{2t+1} = \bar{A}_{21}^i x_{1t} + \bar{A}_{22}^i x_{2t} + \bar{B}_2^i u_t$$

$$i = 1, ..., N \qquad \textbf{(12)}$$

where private sector expectations, $E_t x_{2t+1}$, and x_{1t} are taken as given in the optimisation, and $r(x_t, u_t)$ is as defined in equation **(2)**.

Optimisation under discretion implies that at each period of time the policy maker optimises taking the predetermined variables as given, with policy actions chosen as a function of the predetermined state variables, and in the knowledge that the same optimisation procedure will be followed in future periods (see Backus and Driffill (1986)). Under discretion, the central bank cannot manipulate private sector expectations; rather, it takes the expectations formation mechanism of private agents as given. Under the optimal discretionary policy solution, private agents' expectations formation is consistent with that assumed by the policymaker.

Since private sector expectations are taken as given by the policymaker, a transformation can be applied to equation **(12)** which eliminates the jump variables x_{2t} from the specification of the problem, generalising a transformation of Backus and Driffill (1986) for the case without regime switching to the case with regime switching. By eliminating the jump variables, the optimisation problem can be transformed to the form

$$x_{1t}' V_t^i x_{1t} + \upsilon_t^i = \min_{u_t} \left\{ \begin{array}{c} x_{1t}' Q_t^{i*} x_{1t} + x_{1t}' U_t^{i*} u_t + u_t' U_t^{i*\prime} x_{1t} + u_t' R_t^{i*} u_t + \\ \beta \sum_{j=1}^{N} p_{ij} E_t^{\varepsilon} \left[\begin{array}{c} (A_t^{ij*} x_{1t} + B_t^{ij*} u_t + \varepsilon_{t+1})' V_{t+1}^j \\ (A_t^{ij*} x_{1t} + B_t^{ij*} u_t + \varepsilon_{t+1}) + \upsilon_{t+1}^j \end{array} \right] \end{array} \right\}$$

$$i = 1, ..., N \qquad \textbf{(13)}$$

where

$$A_t^{ij*} \equiv A_{11}^j + A_{12}^j D_t^i$$

$$B_t^{ij*} \equiv B_1^j + A_{12}^j G_t^i$$

$$D_t^i = \left[\sum_{j=1}^{N} p_{ij}(A_{22}^j - C_{t+1}^j A_{12}^j) \right]^{-1} \left[\sum_{j=1}^{N} p_{ij}(C_{t+1}^j A_{11}^j - A_{21}^j) \right]$$

$$G_t^i = \left[\sum_{j=1}^{N} p_{ij}(A_{22}^j - C_{t+1}^j A_{12}^j) \right]^{-1} \left[\sum_{j=1}^{N} p_{ij}(C_{t+1}^j B_1^j - B_2^j) \right]$$

and $x_{2t} = D_t^i x_{1t} + G_t^i u_t = C_t^i x_{1t}$, with

$$C_t^i = D_t^i - G_t^i F_{1t}^i \tag{14}$$

Finally,

$$Q_t^{i*} = Q_{11} + Q_{12} D_t^i + D_t^{i\prime} Q_{21} + D_t^{i\prime} Q_{22} D_t^i$$

$$U_t^{i*} = Q_{12} G_t^i + D_t^{i\prime} Q_{22} G_t^i + U_1 + D_t^{i\prime} U_2$$

$$R_t^{i*} = R + G_t^{i\prime} Q_{22} G_t^i + G_t^{i\prime} U_2 + U_2' G_t^i$$

$$i = 1, ..., N$$

where the matrices Q and U have been partitioned conformably with x_{1t} and x_{2t}. While the matrices A_{11}^j, A_{12}^j, B_1^j, B_2^j, Q_{11}, Q_{12}, Q_{21}, U_1, U_2 and R are known from the structure of the model, the matrices C_t^i, F_{1t}^i and V_t^i must be solved for.(9)

Using the assumed property of the shocks ε_{t+1} of being uncorrelated with the predetermined variables x_{1t}, we can write equation **(13)** as

$$x_{1t}' V_t^i x_{1t} + v_t^i = \min_{u_t} \left\{ \begin{array}{c} x_{1t}' Q_t^{i*} x_{1t} + x_{1t}' U_t^{i*} u_t + u_t' U_t^{i*\prime} x_{1t} + u_t' R_t^{i*} u_t + \\ \beta \sum_{j=1}^{N} p_{ij} (A_t^{ij*} x_{1t} + B_t^{ij*} u_t)' V_{t+1}^j (A_t^{ij*} x_{1t} + B_t^{ij*} u_t) + \\ \beta \sum_{j=1}^{N} p_{ij} \left(E_t^{\varepsilon} \left[\varepsilon_{t+1}' V_{t+1}^j \varepsilon_{t+1} \right] + v_{t+1}^j \right) \end{array} \right\}$$

$$i = 1, ..., N \tag{15}$$

Next, we solve for the optimal feedback rule and value function, following Ljungqvist and Sargent (2000). Taking derivatives with respect to u_t in equation **(15)** to obtain the first-order condition, and using the assumed linear form for the optimal feedback rule, equation **(10)**, in the first-order condition, gives

$$F_{1t}^i = \left[R_t^{i*} + \beta \sum_{j=1}^{N} p_{ij} (B_t^{ij*\prime} V_{t+1}^j B_t^{ij*}) \right]^{-1} \left[U_t^{i*\prime} + \beta \sum_{j=1}^{N} p_{ij} (B_t^{ij*\prime} V_{t+1}^j A_t^{ij*}) \right]$$

$$i = 1, ..., N \tag{16}$$

Substituting the optimal feedback rule (see equations **(10)** and **(16)**) back into the Bellman

(9) The matrices D_t^i, G_t^i, Q_t^{i*}, U_t^{i*}, R_t^{i*}, A_t^{ij*} and B_t^{ij*} have been introduced merely to simplify notation; they can be calculated from the other matrices mentioned above.

equation **(15)**, and equating the terms quadratic in x'_{1t} then yields

$$V_t^i = Q_t^{i*} - U_t^{i*} F_{1t}^i - F_{1t}^{i\prime} U_t^{i*\prime} + F_{1t}^{i\prime} R_t^{i*} F_{1t}^i + \beta \sum_{j=1}^{N} p_{ij} (A_t^{ij*} - B_t^{ij*} F_{1t}^i)' V_{t+1}^j (A_t^{ij*} - B_t^{ij*} F_{1t}^i)$$

$$i = 1, ..., N \qquad \textbf{(17)}$$

If the decision problem has an infinite horizon, which is the case considered here, then the matrices may be independent of time t, and we can search for a stationary solution by iterating backwards in time on the set of coupled equations **(14)**, **(16)** and **(17)**.(10) As a criterion for convergence of the matrices in the value function iteration, we choose the maximum of the infinity-norm over the different states of the Markov process,(11)

$$\max_{\{i=1,..,N\}} \left\| V^i \right\|_\infty$$

where the norm for each state is given by

$$\left\| V^i \right\|_\infty = \max_{\{k,l=1,..,n_1\}} \left| \left(V_t^i\right)_{kl} - \left(V_{t+1}^i\right)_{kl} \right|$$

In Section 3, we use this algorithm to determine the optimal monetary policy response to the risk of structural change in the degree of intrinsic output persistence within a hybrid New Keynesian model estimated for the euro area.

Equating the remaining terms in the Bellman equation, which are not quadratic in x_{1t}, yields an expression for the additional term in the value function,

$$v_t^i = \beta \sum_{j=1}^{N} p_{ij} \left(tr \left[V_{t+1}^j \Sigma \right] + v_{t+1}^j \right) \qquad \textbf{(18)}$$

where $\Sigma = E_t^\varepsilon \left[\varepsilon'_{t+1} \varepsilon_{t+1} \right]$ is the covariance matrix of the shocks, as defined above. Given that a stationary solution for $V^i, i = 1, ..., N$, was found, the stationary solutions, v^i, are implicitly given by

$$v^i = \beta \sum_{j=1}^{N} p_{ij} \left(tr \left[V^j \Sigma \right] + v^j \right) \qquad \textbf{(19)}$$

If $(I - \beta P)$ is invertible, equation **(19)** can be written explicitly as

$$\begin{pmatrix} v^1 \\ v^2 \\ . \\ . \\ v^N \end{pmatrix} = \beta (I - \beta P)^{-1} P \begin{pmatrix} tr\, V^1 \Sigma \\ tr\, V^2 \Sigma \\ . \\ . \\ tr\, V^N \Sigma \end{pmatrix} \qquad \textbf{(20)}$$

(10) Initial conditions are chosen as the zero matrices of size $n_2 x n_1$ for the matrices C_t^i in each state, and as 0.01 times the unit matrix of size $n_1 x n_1$ for the matrices V_t^i in each state. These choices apply initial conditions commonly chosen for standard linear quadratic problems to the case with regime switching.

(11) A tolerance level of 10^{-6} is chosen for determining whether convergence has taken place.

According to equations **(10)** and **(16)**, the solution for the optimal feedback rule may be state-dependent. Moreover, in each state, optimal policy may depend on the transition probabilities of the Markov process governing the uncertainty about the regime changes between the different states.

3 Optimal monetary policy in a hybrid New Keynesian model with regime switching

3.1 Hybrid New Keynesian model

In this section we apply the method for determining optimal discretionary monetary policy in the presence of regime switching to a hybrid New Keynesian model. As the benchmark model, we use the following hybrid New Keynesian model based on Clarida, Gali and Gertler (1999),

$$y_t = \mu_y E_t y_{t+1} + (1-\mu_y) y_{t-1} - \gamma (i_t - E_t \pi_{t+1}) + e_{gt} \quad \textbf{(21)}$$

$$\pi_t = \mu_\pi E_t \pi_{t+1} + (1-\mu_\pi)\pi_{t-1} + \alpha y_t + e_{ut} \quad \textbf{(22)}$$

$$e_{gt+1} = \rho_g e_{gt} + \eta_{gt+1} \quad \textbf{(23)}$$

$$e_{ut+1} = \rho_u e_{ut} + \eta_{ut+1} \quad \textbf{(24)}$$

Here, π_t, y_t and i_t denote deviations of the inflation rate, output and the short-term nominal interest rate from their steady-state values. The model consists of an Euler equation and a New Keynesian Phillips curve, augmented to allow for some backward-looking behaviour. There are two shocks, a shock e_{gt} to the output equation, and a shock e_{ut} to the inflation equation, assumed to be serially correlated with autocorrelations of ρ_g and ρ_u; η_{gt+1} and η_{ut+1} are assumed to be i.i.d. shocks with standard deviation σ_g and σ_u, respectively. Without intrinsic persistence, the model may be derived from optimising microeconomic behaviour (see for example Goodfriend and King (1997), Rotemberg and Woodford (1997)). The main reason for adding lagged output and inflation terms, and serially correlated shocks to the basic forward-looking model is to improve the fit with data, which generally exhibit greater persistence than embodied in the basic forward-looking model. While these model equations may be derived from an underlying optimisation problem by private agents,[12] here we consider this model as a rational expectations model in reduced form, rather than being fully micro-founded. The reason for this is that in the presence of a Markov regime switching process, whose parameters are known by all agents, the knowledge about this process would be incorporated in private agents' optimisation problems within a fully micro-founded model, which may affect the form of the structural model equations. As an illustration, we consider regime changes in intrinsic output persistence in Section 3.2. Since the main reason for adding intrinsic output persistence in New Keynesian models is of an empirical nature, and since there is no consensus in the literature about the true mechanism underlying the intrinsic output persistence found empirically, it is useful to consider this example within our framework.

Regarding the parameter values, we choose $\beta = 0.96$, implying an annual steady-state real interest rate of 4%, so that a period in the model is equal to one year. The benchmark values for the parameters μ_y, μ_π, γ, α, σ_g and σ_u are chosen as the values of the parameters estimated for the euro area by Smets (2003), using annual data from 1977 to 1997 (see Table A). In addition, we set the autoregressive parameters of the shocks ρ_g and ρ_u in this model to a very small value of 0.01, following Smets (2000).

Table A: Model parameters; euro-area estimates for 1977-97 (see Smets (2003))

μ_y	μ_π	γ	α	σ_g	σ_u
0.56	0.52	0.06	0.18	0.65	0.70

We consider an optimisation problem where the central bank minimises the loss function

$$L = E_0 \sum_{t=0}^{\infty} \left[\pi_t^2 + \lambda_y y_t^2 + \lambda_R (i_t - i_{t-1})^2 \right] \qquad \textbf{(25)}$$

That is, the central bank cares about the variability of inflation and of the output gap, and it has a concern for interest rate smoothing. A concern for interest rate smoothing when conducting optimal discretionary monetary policy has been shown to be desirable by Woodford (1999). As a benchmark, we choose the parameters of the central bank's loss function as follows, $\lambda_y = 0.5$ to

(12) Including intrinsic output persistence in equation **(21)** may be justified in a micro-founded way for example by the presence of habit persistence in consumption (see Fuhrer (2000)), or of investment adjustment costs (see Clarida, Gali and Gertler (1999)). Including intrinsic inflation persistence in equation **(22)** may be motivated by the presence of a fraction of rule-of-thumb price-setters (see Gali and Gertler (1998)), or by a fraction of firms indexing their prices to past inflation (see Sbordone (2002)).

reflect a concern of the monetary authority for output stabilisation, and $\lambda_R = 0.2$.

The control variable is the nominal interest rate i_t. Predetermined variables are lagged output, inflation and interest rates, y_{t-1}, π_{t-1}, and i_{t-1} , and the two shocks e_{gt} and e_{ut}, $x_{1t} = [i_{t-1}, y_{t-1}, \pi_{t-1}, e_{gt}, e_{ut}]$. Jump variables are output and inflation, y_t and π_t, $x_{2t} = [y_t, \pi_t]$. This optimisation problem can be mapped into the state-space form for which we derived a solution above. In the notation of equation **(3)**, the equations **(21)** to **(24)** of the hybrid New Keynesian model can be written in state-space form with the following matrices,

$$A_{11} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \rho_g & 0 \\ 0 & 0 & 0 & 0 & \rho_u \end{bmatrix}, A_{12} = \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 1 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}, B_1 = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \\ 0 \end{bmatrix} \qquad \textbf{(26)}$$

$$A_{21} = \begin{bmatrix} 0 & \frac{\mu_y - 1}{\mu_y} & \frac{\gamma(1-\mu_\pi)}{\mu_y \mu_\pi} & -\frac{1}{\mu_y} & \frac{\gamma}{\mu_y \mu_\pi} \\ 0 & 0 & \frac{1-\mu_\pi}{\mu_\pi} & 0 & -\frac{1}{\mu_\pi} \end{bmatrix}, A_{22} = \begin{bmatrix} \frac{1}{\mu_y} + \frac{\gamma\alpha}{\mu_y \mu_\pi} & -\frac{\gamma}{\mu_y \mu_\pi} \\ -\frac{\alpha}{\mu_\pi} & \frac{1}{\mu_\pi} \end{bmatrix} \qquad \textbf{(27)}$$

$$B_2 = \begin{bmatrix} \frac{\gamma}{\mu_y} \\ 0 \end{bmatrix} \qquad \textbf{(28)}$$

3.2 Regime changes in intrinsic output persistence

In this section, we consider optimal policy with regime changes in intrinsic output persistence within the model described in Section 3.1. We consider the risk of a change in intrinsic output persistence from the estimated value of $1 - \mu_y^1 = 0.44$ in state 1 to twice the estimated value of $1 - \mu_y^2 = 0.88$ in state 2 (see Table B).

Table B: Regimes for degree of intrinsic output persistence

	State 1	State 2
Intrinsic output persistence	$1-\mu_y^1=0.44$	$1-\mu_y^2=0.88$

The other model parameters are as given in Table A. The probability transition matrix of the Markov process used to model the structural change is given by

$$P=\begin{bmatrix} 1-p & p \\ q & 1-q \end{bmatrix} \tag{29}$$

where p is the transition probability in each period (of a year) from state 1 to state 2, and q is the transition probability from state 2 to state 1. The optimal monetary policy response to the uncertainty about structural change in intrinsic output persistence, as a function of the transition probabilities p and q, is calculated according to the algorithm described in Section 2.[(13)]

As mentioned above, the Markov process governing the change in intrinsic output persistence is assumed to be known by all agents in the economy, and at time t the state s^t is assumed to be observed by all agents, while s^{t+1} is unknown at time t. Optimal discretionary policy rules depend on all the predetermined state variables of the economy (see equation **(10)**), in contrast to optimised simple policy rules. As described above, in the presence of regime shifts, the optimal policy rule may be state-dependent. In the hybrid New Keynesian model considered here, the optimal monetary policy rule has the form

$$i_t=f_y^i y_{t-1}+f_\pi^i \pi_{t-1}+f_g^i e_{gt}+f_u^i e_{ut}+f_R^i i_{t-1}\quad ,i=1,2 \tag{30}$$

First, we consider one-time regime changes in intrinsic output persistence. In our first example, where we model a one-time regime change from state 1 to state 2, the structural change in intrinsic output persistence is assumed to be governed by a Markov process with a non-zero transition probability of p from state 1 to state 2, and a transition probability q of zero from state 2 to state 1. This uncertainty about the regime constitutes an asymmetric multiplicative risk, since the reaction of an endogenous variable, the output gap, to the lagged output gap depends multiplicatively on the parameter following a Markov process, and since the transition probability is non-zero in one direction only, while being zero for a transition in the opposite direction. Once the change to the

(13) Using this algorithm presented no problems in terms of convergence to a solution. However, it is not self-evident that the solution to which the algorithm converges is unique. Such a uniqueness result would need to be proven.

state with higher intrinsic output persistence has occurred, the economy is assumed to remain in that state.

The results for the coefficients of the optimal monetary policy rule (see equation **(30)**) in each state are shown in Chart 1 in the appendix, as a function of the transition probabilty, p, from state 1 to state 2. We can see that in state 1 the coefficients of the optimal policy rule depend on the transition probability p. Due to the forward-lookingness of private agents and the existence of intrinsic persistence in the model, it is optimal for monetary policy to take into account possible regime changes in the next period, and the probabilities of their occurrence, when setting policy this period. With intrinsic persistence present in the model, adjustments in current monetary policy affect the future time path of inflation; consequently, optimal monetary policy reacts to forecasts of inflation, as well as to current inflation (see Clarida, Gali and Gertler (1999)). Since the risk of regime change affects the forecasts of inflation, which can be quantified since the Markov process governing the uncertainty is assumed to be known, it is optimal for monetary policy to take the transition probabilities of the Markov regime switching process into account. The effect of the risk of regime change on future inflation depends on its likelihood, ie the transition probability p, so that the optimal policy response in state 1 depends on the transition probability, p, as shown in Chart 1. The effect of the expected regime change is larger for the response coefficients to the lagged output gap than to lagged inflation.

From Chart 1, we can also see that if the probability, p, of moving from state 1 to state 2 with greater intrinsic output persistence is high, it is optimal for policy in state 1 to respond more aggressively to the lagged output gap, lagged inflation, and both shocks, than in the absence of regime switching, since any deviations of the output gap not eliminated in the current period are more likely to persist to a greater extent into the future, potentially requiring more output contraction in future. In this paper we consider the case where both the private sector and the central bank face uncertainty about model parameters. Since under optimal discretionary policy private sector expectations are taken as given by the central bank, the private sector's uncertainty affects the private sector expectations entering the central bank's optimisation problem, by introducing an averaging over future Markov states (see the second row of equation **(6)**). The parameter, μ_y, related to intrinsic output persistence, $(1 - \mu_y)$, enters the state-space matrices inversely, so that it enters such averages in a non-linear way, which in turn affect the central bank's optimisation problem. For $p = 0$, the coefficients of the optimal policy rule in state 1 are equal to the results in the absence of regime switching for $1 - \mu_y = 0.44$, as can be seen by comparing Chart 1 with column 1 of Table C.(14)

Chart 1 shows that the coefficients of the optimal interest rate rule in state 1 depend non-linearly on the transition probability p. This may be due to the non-linear nature of the optimal policy problem with Markov regime switching in a feedback parameter in the dynamic rational expectations model considered here. Chart 1 also illustrates that optimal policy is state-dependent. The coefficients of the optimal policy rule conditional on being in state 2 are different from those conditional on being in state 1. Optimal policy is state-dependent, since the expected values of the model parameters depend on the state. Without Markov switching, optimal policy in each state is identical to the optimal policy calculated under certainty in each state separately. With Markov switching, optimal monetary policy still depends on the state, since it takes into account the expected values of the model parameters, based on the probabilities of remaining in the current state at unchanged parameters, and of switching to another state with different parameters.(15)

Table C: Optimal monetary policy response in states of low and high intrinsic output persistence, assuming no transition between the two states ($p = q = 0$)

	$1 - \mu_y^1 = 0.44$		$1 - \mu_y^2 = 0.88$
f_y^1	0.67	f_y^2	2.86
f_π^1	0.58	f_π^2	0.81
f_g^1	1.55	f_g^2	3.29
f_u^1	1.23	f_u^2	1.71
f_R^1	0.49	f_R^2	0.41

In our second example, we model the case of a one-time regime change in the opposite direction, from state 2 to state 1. Chart 2 shows the coefficients of the optimal policy rule under the assumption that only a transition from state 2 to state 1 can occur, with non-zero probability q, but that there is a zero probability of moving from state 1 to state 2, $p = 0$. We can see that the

(14) Similarly, for $p = 1$, the coefficients of the optimal policy rule in state 1 are equal to the results in the absence of regime switching for $1 - \mu_y = 0.88$ (see the last column of Table C), since the economy will move to state 2 with certainty.
(15) In Chart 1, the coefficients of the optimal policy rule in state 2 are independent of p, since we assumed a zero transition probability q from state 2 to state 1. They are the same as for an economy with high intrinsic output persistence of $1 - \mu_y = 0.88$ in the absence of regime switching, ie, $p = q = 0$ (see the last column of Table C).

optimal policy rule in state 2 depends on the transition probability q, and non-linearly so.

Next, we consider optimal monetary policy with on-going regime changes in intrinsic output persistence. The regime changes are again modelled by a Markov process, which may be asymmetric, with a different transition probability, p, from state 1 to state 2 than from state 2 to state 1, q. This implies that both transition probabilities p and q in the probability transition matrix P of the Markov chain used to model the structural change (see equation **(29)**) are non-zero, and not necessarily equal to each other. Chart 3 shows the coefficients of the optimal policy rules, separately for each state, as a function of the transition probability p from state 1 to state 2, for the case of a transition probability from state 2 to state 1 of $q = 0.2$. Similarly, Chart 4 shows the coefficients of the optimal rule in each state as a function of the transition probability q from state 2 to state 1, assuming a transition probability from state 1 to state 2 of $p = 0.2$. Charts 3 and 4 show that for on-going regime changes, the coefficients of the optimal policy rule in each state, and in particular the aggressiveness of the response to the demand shock and the output gap in each state, depend on both transition probabilities p and q of the Markov process governing the uncertainty about the regime. This is due to the optimal nature of policy setting, the presence of intrinsic persistence in the model, and the forward-lookingness of private agents. The coefficients depend non-linearly on both the transition probabilities p and q.

To quantify the impact of uncertain changes in intrinsic output persistence and the optimal policy response to this on macroeconomic variables, we also present impulse responses to cost-push shocks and demand shocks in the presence of regime switching, on average over 1,000 Markov processes (for $p = q = 0.2$), in comparison with the impulse responses in the absence of regime switching (see Charts 5 and 6). We can see that the impulse responses for the output gap, inflation and interest rates lie closer to those in state 2 in the absence of regime switching, which has a higher level of intrinsic output persistence, than to those in the state with low intrinsic output persistence, reflecting the non-linear dependence of the coefficients of the optimal policy rule on the transition probabilities, as discussed above.

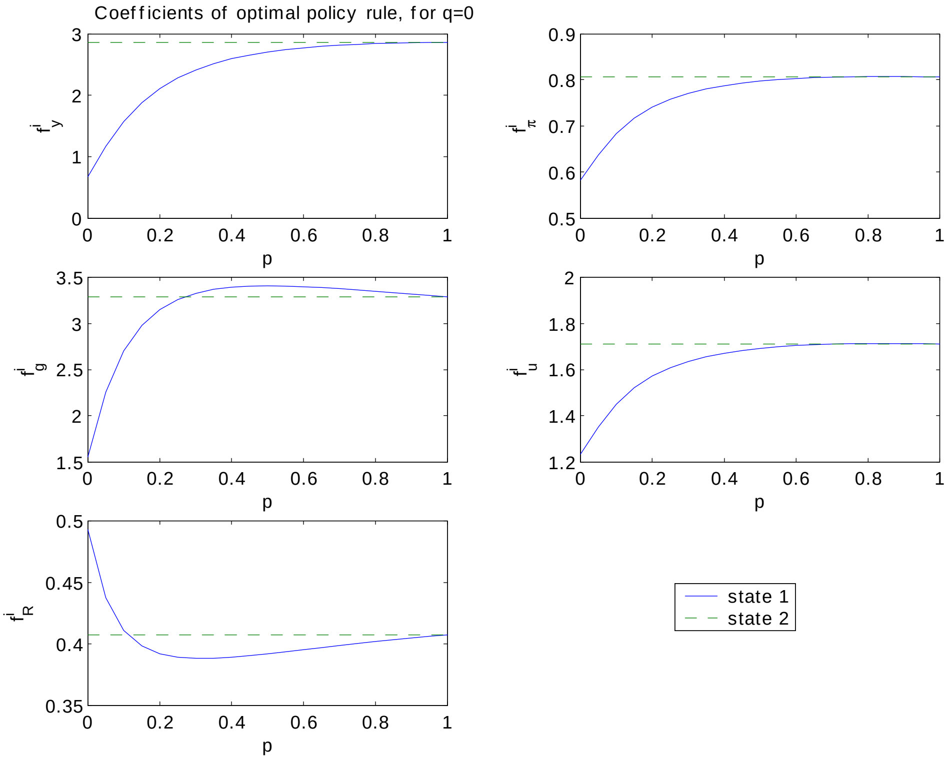
4 Conclusions

In this paper we presented a new method for calculating optimal discretionary policy in rational expectations models with regime changes in parameters governing the feedback of the model to endogenous variables, extending an algorithm of Backus and Driffill (1986) for optimal discretionary policy to the case with regime switching. We assumed that the uncertainty about the regime changes is governed by a Markov process in model parameters. This approach allows us to study the implications of symmetric and asymmetric risks of regime changes for optimal policy within dynamic stochastic rational expectations models, and to study both one-time and on-going regime changes. This paper considered optimal discretionary policy in the presence of regime changes in the evolution of the economy, governed by rational expectations equations. It is useful to study optimal policy, rather than simple rules, since this is one representation of how policy might be set.

The method presented in this paper may be used to determine optimal policy under discretion in the presence of the risk of different kinds of structural changes in rational expectations models. As an illustration, we applied this method to determine optimal policy in the presence of regime changes in intrinsic output persistence, within a hybrid New Keynesian model, a standard model currently used for monetary policy analysis. We found that the coefficients of the optimal policy rule are state-dependent. State-dependent policy arises through the optimising behaviour of the policy authority, rather than by being postulated. We found that if the probability of moving to a state with greater intrinsic output persistence is high, it is optimal for policy in the state with low intrinsic persistence to respond more aggressively to the lagged output gap, lagged inflation, and both shocks, than in the absence of such transitions. Moreover, the coefficients of the optimal policy rule were found to depend non-linearly on the Markov transition probabilities.

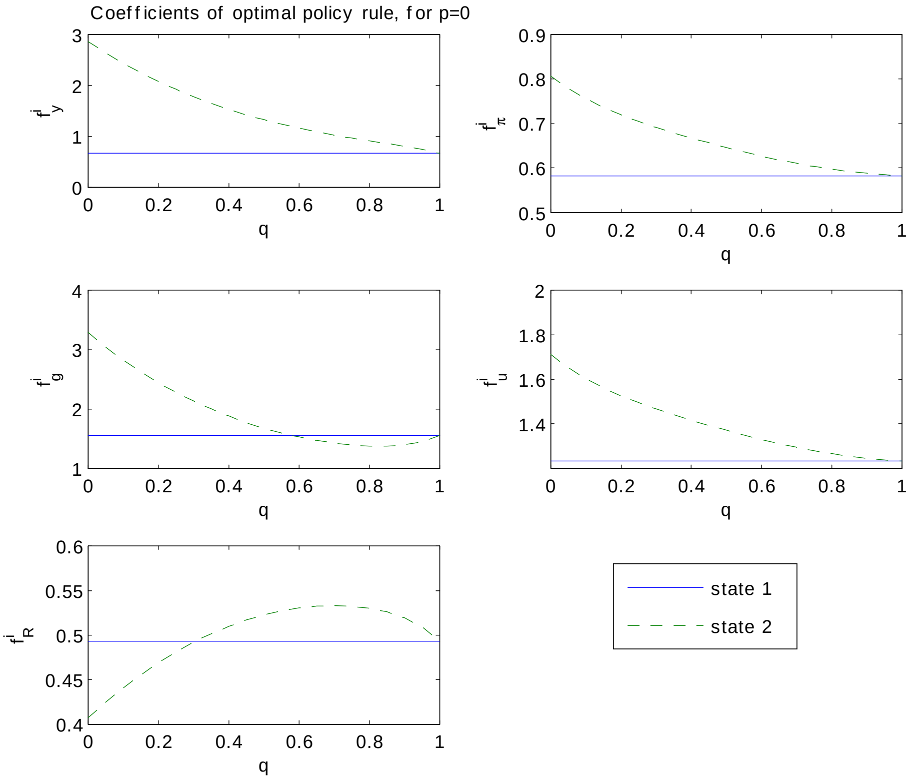
Appendix A: Charts

Chart 1



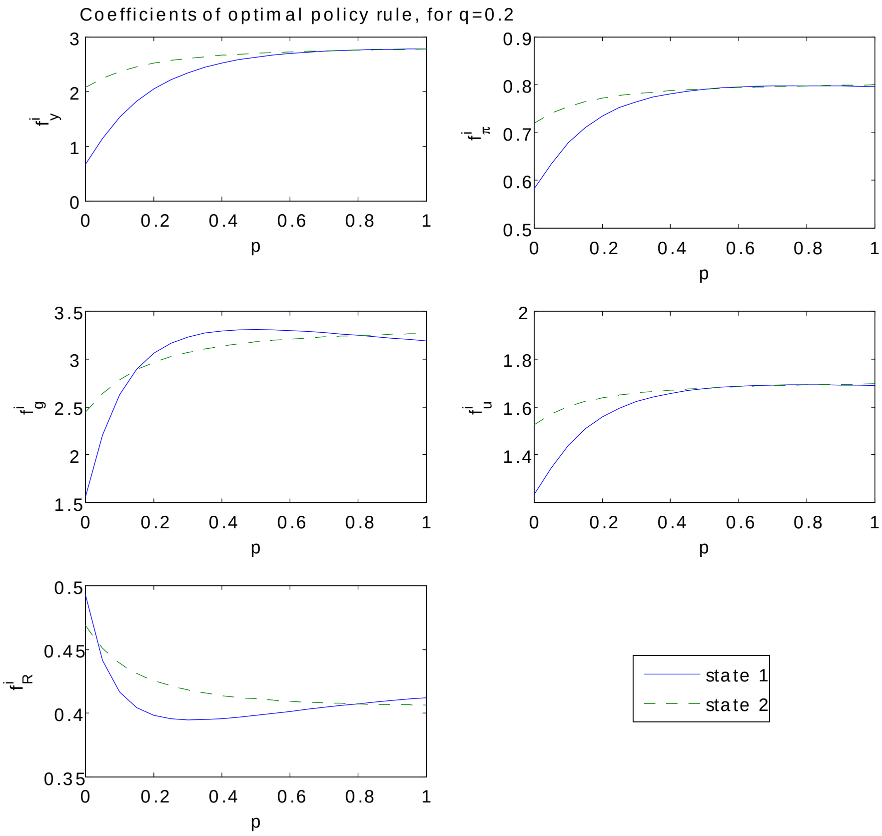
Coefficients of optimal policy rule in states 1 and 2 as a function of the transition probability p from state 1 of low intrinsic output persistence $(1 - \mu_y^1 = 0.44)$ to a state 2 of high intrinsic output persistence $(1 - \mu_y^2 = 0.88)$, assuming zero probability of transition from state 2 to state 1; other parameters are as given in Table A.

Chart 2



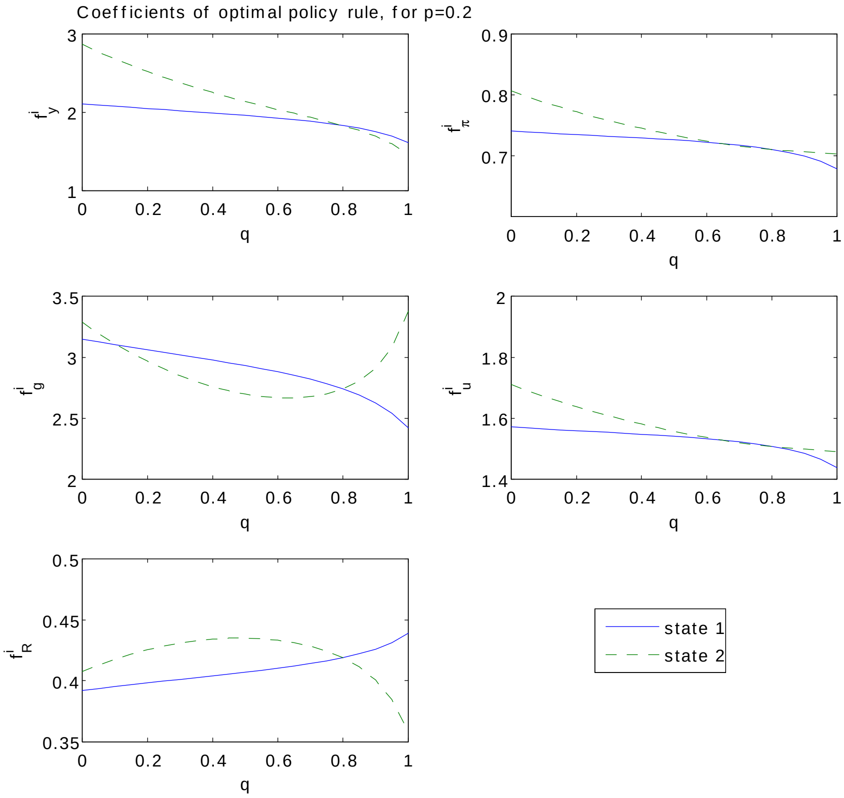
Coefficients of optimal policy rule in states 1 and 2 as a function of the transition probability q from state 2 of high intrinsic output persistence ($1-\mu_y^2 = 0.88$) to a state 1 of low intrinsic output persistence ($1-\mu_y^1 = 0.44$), assuming a zero probability p of transition from state 1 to state 2; other parameters are as given in Table A.

Chart 3



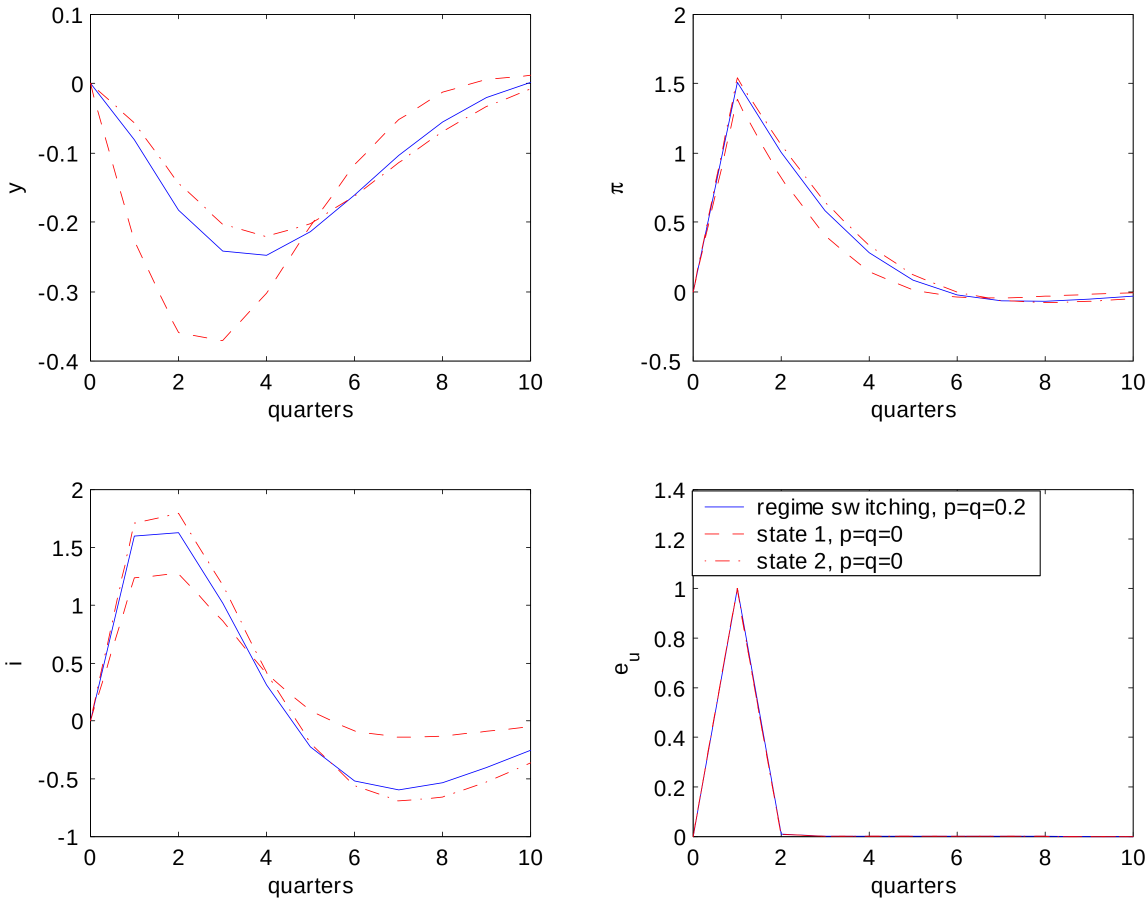
Coefficients of optimal policy rule in states 1 and 2 as a function of the transition probability p from state 1 of low intrinsic output persistence ($1-\mu_y^1=0.44$) to the state 2 of high intrinsic output persistence ($1-\mu_y^2=0.88$), assuming a probability of transition from state 2 to state 1 of $q=0.2$; other parameters are as given in Table A.

Chart 4



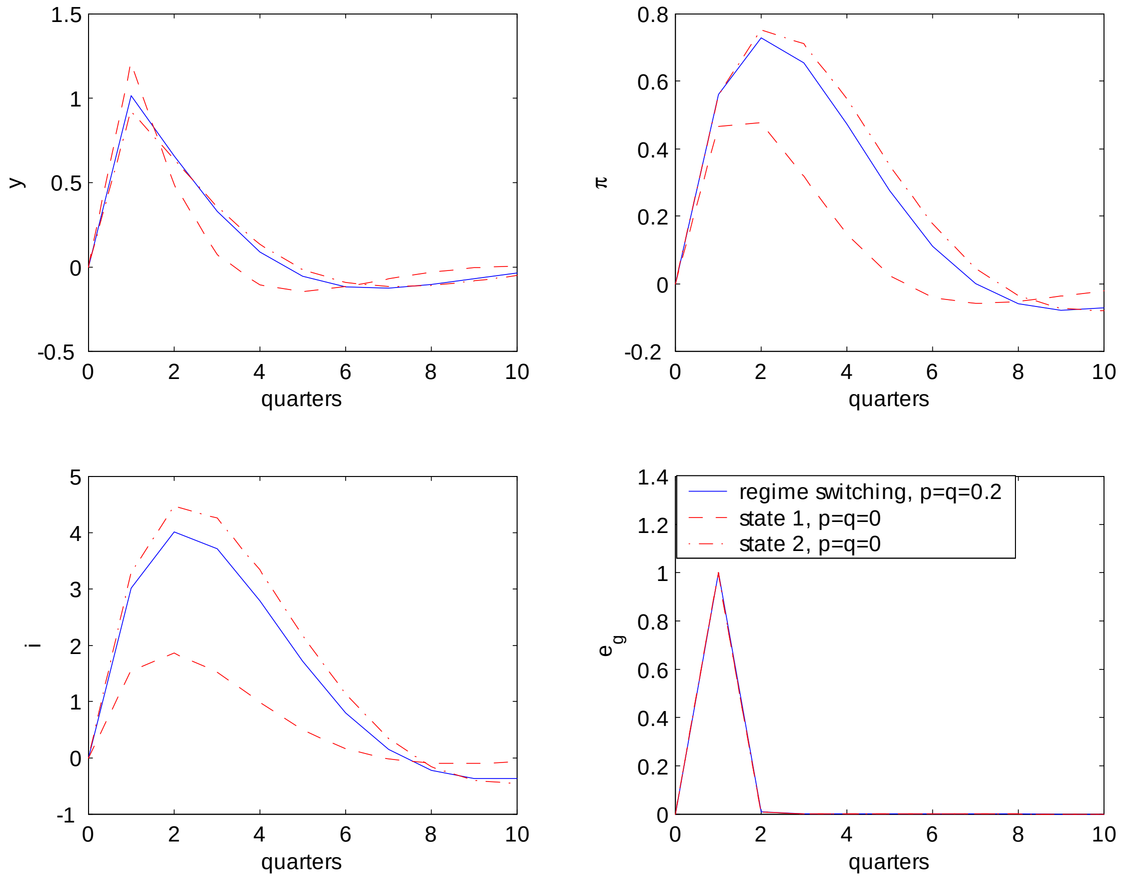
Coefficients of optimal policy rule in states 1 and 2 as a function of the transition probability q from state 2 of high intrinsic output persistence ($1 - \mu_y^2 = 0.88$) to the state 1 of low intrinsic output persistence ($1 - \mu_y^1 = 0.44$), assuming a probability of transition from state 1 to state 2 of $p = 0.2$; other parameters are as given in Table A.

Chart 5



Impulse responses to a cost-push shock for the case with regime switching, on average over Markov processes, with probabilities $p = q = 0.2$ of moving between state 1 of low intrinsic output persistence $(1 - \mu_y^1 = 0.44)$ and state 2 of high intrinsic output persistence $(1 - \mu_y^2 = 0.88)$; the case without regime switching $(p = q = 0)$ is shown for comparison.

Chart 6



Impulse responses to a demand shock for the case with regime switching, on average over Markov processes, with probabilities $p = q = 0.2$ of moving between state 1 of low intrinsic output persistence $(1 - \mu_y^1 = 0.44)$ and state 2 of high intrinsic output persistence $(1 - \mu_y^2 = 0.88)$; the case without regime switching $(p = q = 0)$ is shown for comparison.

References

Andolfatto, D and Gomme, P (2003), 'Monetary policy regimes and beliefs', *International Economic Review*, Vol. 44(1), pages 1-30.

Andolfatto, D, Hendry, S and Moran, K (2002), 'Inflation expectations and learning about monetary policy', *Bank of Canada Working Paper no. 2002-30*.

Backus, D and Driffill, J (1986), 'The consistency of optimal policy in stochastic rational expectations models', *CEPR Discussion Paper no. 124*.

Blake, A and Zampolli, F (2006), 'Optimal monetary policy in Markov-switching models with rational expectations agents ', *Bank of England Working Paper no. 298*.

Brainard, W (1967), 'Uncertainty and the effectiveness of policy', *American Economic Review*, Vol. 57, pages 411-25.

Clarida, R, Gali, J and Gertler, M (1999), 'The science of monetary policy: a New-Keynesian perspective', *Journal of Economic Literature*, Vol. 37, pages 1,661-707.

Costa, O and Fragoso, M (1995), 'Discrete-time LQ-optimal control problems for infinite Markov jump parameter systems', *IEEE Transactions on Automatic Control*, Vol. 40, pages 2,076-88.

Craine, R (1979), 'Optimal monetary policy with uncertainty', *Journal of Economic Dynamics and Control*, Vol. 1, pages 59-83.

Daniel, B (2003), 'Fiscal policy, price surprises and inflation', manuscript, SUNY Albany.

Davig, T (2002), 'Regime-switching debt and taxation', *Journal of Monetary Economics* (forthcoming).

Davig, T (2003), 'Regime-switching fiscal policy in general equlibrium', manuscript, College of William and Mary.

Davig, T, Leeper, E and Chung, H (2004), 'Monetary and fiscal policy switching', manuscript.

Erceg, C and Levin, A (2003), 'Imperfect credibility and inflation persistence', *Journal of Monetary Economics*, Vol. 50, pages 915-44.

Fuhrer, J (2000), 'Habit formation in consumption and its implications for monetary policy models', *American Economic Review*, Vol. 90(3), pages 367-90.

Gali, J and Gertler, M (1998), 'Inflation dynamics: a structural econometric analysis', *Journal of Monetary Economics*, Vol. 44, pages 195-222.

Giordani, P and Soederlind, P (2003), 'Solution to macromodels with Hansen-Sargent robust policies: some extensions', manuscript.

Goodfriend, M and King, R (1997), 'The new neoclassical synthesis and the role of monetary policy', *NBER Macroeconomics Annual*, Vol. 12, pages 231-83.

Greenspan, A (2005), 'Reflections on central banking', speech given at a symposium sponsored by the Federal Reserve Bank of Kansas City, Jackson Hole, Wyoming.

Hamilton, J (1994), *Time series analysis*, Princeton University Press.

Issing, O (2002), 'Monetary policy in a world of uncertainty', *Économie Internationale*, Vol. 92, pages 165-80.

King, M (2004), 'What fates impose: facing up to uncertainty', Eighth British Academy Annual Lecture.

Leeper, E and Zha, M (2003), 'Modest policy interventions', *Journal of Monetary Economics*, Vol. 50(8), pages 1,673-700.

Levin, A, Onatski, A, Williams, J and Williams, N (2005), 'Monetary policy under uncertainty in micro-founded macroeconometric models', *NBER Macroeconomics Annual*, forthcoming.

Ljungqvist, L and Sargent, T (2000), *Recursive macroeconomic theory*, MIT Press.

Loyo, E (1999), 'Tight money paradox on the loose: a fiscalist hyperinflation', manuscript, Harvard University.

Mackowiak, B (2002), 'Macroeconomic regime switching and speculative attacks', manuscript, Humboldt University.

Onatski, A and Williams, N (2003), 'Modeling model uncertainty', *Journal of the European Economics Association*, Vol. 1, pages 1,087-122.

Oudiz, G and Sachs, J (1985), 'International policy coordination in dynamic macroeconomic models', in Buiter, W and Marston, R (eds), *International economic policy coordination*, Cambridge University Press.

Rotemberg, J and Woodford, M (1997), 'An optimisation-based econometric framework for the evaluation of monetary policy', *NBER Macroeconomics Annual*, pages 297-316.

Sargent, T (1999), 'Comment on 'Policy rules for open economies' by Ball, L', in Taylor, J B (ed), *Monetary policy rules*, University of Chicago Press, pages 144-54.

Sbordone, A (2002), 'An optimising model of U.S. wage and price dynamics', manuscript.

Schorfheide, F (2003), 'Learning and monetary policy shifts', manuscript, University of Pennsylvania.

Sims, C (1997), 'Fiscal foundations of price stability in open economies', manuscript, Yale University.

Smets, F (2000), 'What horizon for price stability', *European Central Bank Working Paper no. 24*.

Smets, F (2003), 'Maintaining price stability: how long is the medium term?', *Journal of Monetary Economics*, Vol. 50, pages 1,293-309.

Soederlind, P (1999), 'Solution and estimation of RE macromodels with optimal policy', *European Economic Review*, Vol. 43, pages 813-23.

Söderström, U (2002), 'Monetary policy with uncertain parameters', *Scandinavian Journal of Economics*, Vol. 104, pages 125-45.

Svensson, L and Williams, N (2005), 'Monetary policy with model uncertainty: distribution forecast targeting', manuscript, Princeton University.

Weil, P (2003), 'Reflections on the fiscal theory of the price level', manuscript, ECARES.

Woodford, M (1998), 'Public debt and the price level', manuscript, Princeton University.

Woodford, M (1999), 'Optimal monetary policy inertia', *The Manchester School*, Vol. 67(1), pages 1-35.

Zampolli, F (2006), 'Optimal monetary policy in a regime-switching economy: the response to abrupt shifts in exchange rate dynamics', *Bank of England Working Paper no. 297*.