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The welfare benefits of stable and efficient payment systems

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Abstract

The Bank of England's second core purpose is to maintain the stability of the financial system. Payment systems, by supporting transactions, are a key aspect of this. In this paper, we examine the importance of smoothly functioning payment systems to the economy by extending a recently developed theoretical model of banks. In the model the risk of theft implies a cost to using cash. This risk can be avoided by depositing cash in banks and transferring money through an interbank payment system. However, agents are then exposed to the risk that the payment system is unreliable. Agents will use a payment system (rather than cash) to make transactions if the system is sufficiently cheap to use and/or it is sufficiently reliable. We show that the introduction of a payment system that buyers and producers choose to use unambiguously increases social welfare if it expands the number of trades occurring in the economy. This is more likely the more reliable is the payment system. When the introduction of a payment system does not increase the number of trades, social welfare may increase or decrease depending on the trade-off between the risk of using cash and the risk that the payment system is unreliable. We again show that the more reliable is the payment system, the more likely welfare is increased by its introduction and we illustrate how this benefit might be quantified.

Summary

The Bank of England's second core purpose is to maintain the stability of the financial system, both domestic and international. A key aspect of financial stability is the ability of consumers, firms and the government to continue making payments to each other in the presence of shocks both external to and emanating from within the systems through which such payments are made. Examples of such shocks could include bankruptcy of payment system participants; liquidity shortages among participants or problems with their operations; events in the wider economy that lead to changes in the profitability or liquidity holdings of participants in the system; or operational problems within the system leading to its temporary closure.

In this paper, we introduce a payment system into a recently developed theoretical model of banks and examine the ability of agents in an economy to make payments to each other in the presence of operational problems within this payment system. In the model agents have a choice between two means of making payments – cash and an alternative – but only one, cash, can be stolen. The safe alternative to cash is referred to as 'cheques' but, in essence, this can be thought of as any reliable interbank payment system. The introduction of a payment system (and banks) enables agents to more easily make payments between each other.

But the payment system in their model is risk free. In the real world this is not the case. In particular, such systems can suffer from operational problems, the focus of this paper. There is a risk that the payment systems temporarily fail to function for some reason and payments cannot be made. In this paper, we model the possibility of shocks to the payment system as a probability that the payment system fails to function when a buyer and producer who meet and agree to trade would like it to. Agents do not know whether the system will function or not when they choose whether to use it rather than using cash. We show that agents have an incentive to use the payment system if it is sufficiently cheap to use and/or sufficiently reliable. We also derive lower bounds on the probability that the payment system functions (given the cost of using it) that are consistent with buyers choosing to use it.

Finally, we compare social welfare with and without the payment system. The presence of a safe and reliable system for transferring money can make people prepared to hold and use money in situations where the presence of thieves would have otherwise stopped this from happening. In such cases, the presence of a payment system unambiguously increases social welfare since it expands the number of trades occurring in the economy. We find that the more reliable the system, the more likely this is to happen. Using our model, we then calculate the welfare gains resulting from an increase in stability. When money is accepted as a medium of exchange in the absence of a payment system, social welfare can increase or decrease with the introduction of a payment system. In this case, the addition of a payment system will not expand the number of trades that occur in our model; so there will be no social benefit arising from this channel. Social welfare will only increase if the reduction in deadweight loss caused by theft in the economy (a cost that thieves incur when they steal successfully) is sufficiently greater than the costs of using the payment system (including both the direct costs of using the system and costs related to

system failures). Again, we show that this is more likely for a more reliable system, and calculate how welfare increases as stability is increased.

1 Introduction

The Bank of England's second core purpose is to maintain the stability of the financial system, both domestic and international. Padoa-Schioppa (2002) defines financial stability as 'a condition where the financial system is able to withstand shocks without giving way to cumulative processes which impairs (*sic*) the allocation of savings to investment opportunities and the processing of payments in the economy'. The purpose of this paper is to concentrate on the latter part of that definition –'the processing of payments in the economy'– since it is clear that a smoothly functioning payment system is a necessary condition for financial stability.

He *et al* (2005) present a search theoretic model of a payment system. In their model agents have a choice between two means of making payments: cash and cheques. Using cash carries a cost since it can be stolen. (The cost of using cash resulting from the risk of theft in their model can be interpreted more widely to include all costs of using cash, including the cost of carrying cash.) The alternative to using cash – cheques – involves agents depositing their cash in banks and making payments from their bank account to other agents' bank accounts. But for this to happen, there needs to be an interbank payment system. The authors find that the introduction of a payment system expands the range of parameter values consistent with there being an equilibrium in which money is accepted as a medium of exchange.

But the payment system in He *et al* (2005) is risk free. Lester (2005) extends the He *et al* model to incorporate the potential for banks to go bankrupt as a result of external shocks and default on their payment obligations. When a producer sells a good in exchange for a promise to make a payment he is in effect extending credit to the buyer until the payment is settled. But during this time shocks to the buyer's bank may mean that it does not have enough money to settle the payment on time. If this happens, a producer may find that he does not receive any money for a good that he has sold. Lester recalculates the conditions under which agents use a payment system in the presence of this 'credit' risk and shows that an increase in financial instability – as represented by credit risk – is welfare reducing.

This paper considers shocks to the payment system itself and how these affect agents' incentives to use the system in the He *et al* (2005) model. When a buyer and a producer meet and agree to trade, the payment system only functions with some probability. We will refer to this risk as operational risk. We show that agents have an incentive to use the payment system if it is sufficiently cheap to use and/or it is sufficiently reliable. We derive lower bounds on the probability that the payment system functions (conditional on the cost of using it) that are consistent with buyers choosing to use it. Finally, we compare social welfare with and without the payment system for each of the cases.

The introduction of a payment system that buyers and producers choose to use unambiguously increases social welfare if it expands the number of trades occurring in the economy compared with the situation in which agents use cash. In the model, there is an increase in the number of trades if and only if the introduction of a payment system means that money can be an accepted medium of exchange when it could not be previously. This is more likely the more reliable is the payment system. Indeed, we can calculate – in terms of the parameters of our model – the welfare gains resulting from an increase in reliability. When money is accepted as a medium of exchange in the absence of a payment system, the maximum number of feasible trades already occurs when agents use cash. In this case, the addition of a payment system will not expand the number of trades that occur in our model and so there will be no social benefit arising from this channel. Social welfare will only increase if the reduction in deadweight loss caused by 'theft' in the economy (modelled as a cost that thieves incur when they steal successfully) is sufficiently greater than the costs of using the payment system (including both the direct costs of using the system and costs related to system failures). Again, we can show that this is more likely the more reliable is the system and calculate how welfare increases as stability is increased.

The basic He *et al* (2005) model is introduced in Sections 2 and 3. We analyse a payment system that suffers from operational risk in Section 4. A social welfare comparison is performed in Section 5. Section 6 concludes.

2 The basic model

In this section we will sketch the version of the He *et al* (2005) model where a move by nature in each period determines whether an agent that does not hold money is a producer or a thief. (He *et al* refer to this case as one of 'exogenous robbery'.) Each period is comprised of two subperiods. Centralised trade occurs in one subperiod. Decentralised trade, involving search frictions, takes place in the other subperiod. In other variants of the model the existence of the two subperiods is important. However, this is not true in this paper and we shall proceed without discussing the centralised market.

The economy consists of a unit continuum of agents. We assume that there is a measure, m, of agents holding one unit of money each. We refer to these agents as buyers. A buyer receives utility, u, each time he consumes a good he wants. Money is indivisible. The remaining agents either are thieves or producers, the proportion of thieves being λ. A producer can produce one unit of a specialised, divisible good at a cost c. We assume that trade is worthwhile, ie, $u > c$. One good trades for one unit of money. In a meeting between a buyer and a producer, the buyer wants the producer's good with probability x. We assume that the probability of a double coincidence of wants is zero and, so, barter trade never occurs. In a meeting between a buyer and a thief, the thief attempts to steal the buyer's unit of money. He is successful with probability γ. If he successfully robs a buyer he incurs a cost z. This cost is motivated by a desire to capture in a simple way the idea that theft imposes negative externalities on society as a whole.[1] If there were no cost of theft, then theft would simply represent a transfer of resources and have a zero effect on aggregate welfare.

Let V_m and V_0 be the values of being a buyer and a non-buyer, respectively. Agents have a discount rate r. Their respective Bellman equations are shown in equations **(1)** and **(2)**.

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[1] A part of this cost of theft can also be thought as a proxy for the deadweight cost of using cash. By this, we are thinking of 'cash handling' costs, which can be significant in practice, as well as the actual costs of producing notes and coin.

$$rV_m = x(1-m)(1-\lambda)[u + V_0 - V_m] + \gamma\lambda(1-m)[V_0 - V_m] \qquad \textbf{(1)}$$

$$rV_0 = xm(1-\lambda)[V_m - V_0 - c] + \gamma\lambda m[V_m - V_0 - z] \qquad \textbf{(2)}$$

Trade occurs only in a monetary equilibrium; ie, an equilibrium in which money is an accepted medium of exchange. For this to happen, both buyers and non-buyers must prefer to be active in the economy rather than drop out and live in autarchy. This implies: $V_m \geq 0$ and $V_0 \geq 0$. What is more, when a producer meets a buyer who wants his good, the producer needs to have an incentive to accept a unit of money as payment. This implies $V_m - V_0 \geq c$. Following He *et al* (2005), we assume that a thief steals irrespective of whether his pay-off from successfully robbing an agent, $V_m - V_0$, exceeds his cost from doing so, z. Given these conditions, we need only check that $V_0 \geq 0$ and $V_m - V_0 \geq c$ because when these conditions both hold it must be that $V_m \geq 0$.

Proposition 1

There exists a monetary equilibrium if and only if

$$\frac{x(1-m)(1-\lambda)u + \gamma\lambda zm}{r + x(1-m)(1-\lambda) + \gamma\lambda} \geq c \text{ and}$$

$$\frac{(1-m)[x(1-\lambda) + \gamma\lambda]}{r + (1-m)[x(1-\lambda) + \gamma\lambda]}u - \frac{\gamma\lambda}{x(1-\lambda)}z \geq c$$

Proof

If we subtract rV_0 from rV_m and rearrange, we find

$$V_m - V_0 = \frac{x(1-m)(1-\lambda)u + xm(1-\lambda)c + \gamma\lambda mz}{r + x(1-\lambda) + \gamma\lambda}$$

$$V_m - V_0 \geq c \;\Leftrightarrow\; x(1-m)(1-\lambda)u + xm(1-\lambda)c + \gamma\lambda mz \geq [r + x(1-\lambda) + \gamma\lambda]c$$

$$\frac{x(1-m)(1-\lambda)u + \gamma\lambda mz}{r + x(1-m)(1-\lambda) + \gamma\lambda} \geq c$$

$$V_0 \geq 0 \;\Leftrightarrow\; rV_0 \geq 0$$

$$V_m - V_0 \geq \frac{xm(1-\lambda)c + \gamma\lambda mz}{xm(1-\lambda) + \gamma\lambda m}$$

If we substitute the above expression for $V_m - V_0$ and rearrange, we find

$$V_0 \geq 0 \;\Leftrightarrow\; \frac{(1-m)[x(1-\lambda)+\gamma\lambda]}{r+(1-m)[x(1-\lambda)+\gamma\lambda]}u - \frac{\gamma\lambda}{x(1-\lambda)}z \geq c$$

■

Proposition 1 is the same as proposition 1 in He *et al* (2005). It shows that a producer will accept one unit of money as payment for its good if the utility from consuming, u is sufficiently greater than the production cost, c. We should note that this is not the only equilibrium: autarchy, where no agent carries money and no trade is made, is always an equilibrium in this set-up.

3 Adding a payment system

We now introduce a technology that enables money to flow between buyers and producers without buyers having to carry money around on their persons: that is, a payment system. A buyer deposits his money into a bank and is issued with a cheque book or a debit card. If a buyer carries money on his person we say that he uses cash. The key advantage of using the payment system, rather than cash, is that it is immune from the risk of robbery.(2) Payments flow between banks in the centralised market that we have up until now subsumed. We assume that banks are subject to 100% reserve requirements and, so, do not issue loans. As in He *et al* (2005), we also assume that banks settle their payment obligations without fail. (Lester (2005) relaxes this assumption by introducing bank defaults into the He *et al* model when banks have less than 100% reserve requirements.) Finally, if a buyer uses the payment system, he is charged a price, $p(\geq 0)$. We have assumed, for simplicity, that this cost is unrelated to the proportion of agents using the payment system. In practice, payment systems tend to exhibit economies of scale, implying an inverse relationship between p and the number of users of the system. The likely effect of assuming this within our model is to increase the possibility of multiple equilibria – values of the parameter space within which we could have equilibria with or without payment systems – while not affecting our welfare calculations that simply compare equilibria in which everyone uses the payment system with those in which everyone uses cash.

Let θ be the measure of buyers who use the payment system, V_{ms} be the value of a buyer who chooses to use the system and V_{mc} be the value of a buyer who chooses to carry cash. Then $V_m = \max\{V_{ms}, V_{mc}\}$ The Bellman equations for when there is a payment system are shown in equations **(3)-(5)**.

$$rV_{ms} = x(1-m)(1-\lambda)(u+V_0-p-V_m) \quad \textbf{(3)}$$

$$rV_{mc} = x(1-m)(1-\lambda)(u+V_0-V_m)+\gamma\lambda(1-m)(V_0-V_m) \quad \textbf{(4)}$$

$$rV_0 = xm(1-\lambda)(V_m-V_0-c)+\gamma\lambda m(1-\theta)(V_m-V_0-z) \quad \textbf{(5)}$$

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(2) To be more precise, the point is not that debit cards, credit cards or cheque books cannot be stolen, rather that they cannot be used by anyone except the particular individual on whose account they draw. This means that there is no incentive for thieves to rob any agent who was not carrying cash.

Now, depending on the values of the parameters, we could identify four possible equilibria:

a) Autarchy: $V_{ms} < 0$, $V_{mc} < 0$ and $V_0 < 0$
b) Cash: $\theta = 0$, $V_{mc} > V_{ms}$, $V_{mc} \geq 0$ and $V_0 \geq 0$
c) Mixture: $0 < \theta < 1$, $V_{ms} = V_{mc}$ and $V_0 \geq 0$
d) Payment system: $\theta = 1$, $V_{ms} > V_{mc}$, $V_{ms} \geq 0$ and $V_0 \geq 0$

Note that, as we said earlier, autarchy will always be an equilibrium. We could find areas of the parameter space in which either autarchy and cash are the only equilibria; areas in which autarchy and payment system are the only equilibria; and areas in which all four types of equilibria are possible. As our goal is to demonstrate the benefits of payment systems we first find those parameter values under which a payment system equilibrium exists. When we carry out our welfare calculations in Section 5, the calculations should be interpreted as telling us which of the possible equilibria society prefers; they do not tell us how society would choose one equilibrium over the other or, indeed, whether a 'payment system' equilibrium would evolve naturally from a 'cash' equilibrium. We leave this for future work.

So, in a 'payment system equilibrium', it must be the case that buyers prefer to use the payment system rather than cash, that is $V_{ms} > V_{mc}$. This implies:

$$-px(1-\lambda) \geq \gamma\lambda\left(V_0 - V_m\right) \quad \textbf{(6)}$$

The participation conditions for buyers and sellers imply

$$rV_m = x(1-m)(1-\lambda)\left(u + V_0 - p - V_m\right) \geq 0 \quad \textbf{(7)}$$

$$rV_0 = xm(1-\lambda)\left(V_m - V_0 - c\right) \geq 0 \quad \textbf{(8)}$$

Finally, we again need producers to have an incentive to accept a unit of money as payment when they meet a buyer who wants their good. This implies the equilibrium condition: $V_m - V_0 \geq c$.

Proposition 2

There exists a payment system equilibrium if and only if

$$p_1 = u - \frac{r + x(1-m)(1-\lambda)}{x(1-m)(1-\lambda)}c \geq p \qquad \text{and}$$

$$p_2 = \frac{(1-m)\gamma\lambda}{r + x(1-\lambda) + \gamma\lambda(1-m)}u - \frac{\gamma\lambda m}{r + x(1-\lambda) + \gamma\lambda(1-m)}c \geq p$$

Proof

First, note from equation **(8)** that $V_m - V_0 \geq c$ implies $V_0 \geq 0$. Hence, $V_m \geq 0$. So, we need only derive the conditions under which $V_m - V_0 \geq c$ and equation **(6)** holds.

If we subtract rV_0 from rV_m we find

$$V_m - V_0 = \frac{x(1-m)(1-\lambda)(u-p) - xm(1-\lambda)c}{r + x(1-\lambda)} \qquad \textbf{(9)}$$

Hence, $V_m - V_0 \geq c$ if

$$x(1-m)(1-\lambda)(u-p) - xm(1-\lambda)c \geq [r + x(1-\lambda)]c \qquad \textbf{(10)}$$

that is

$$(u-p)\frac{x(1-m)(1-\lambda)}{r + x(1-m)(1-\lambda)} \geq c \qquad \textbf{(11)}$$

Rearranging equation **(6)** implies

$$\frac{\gamma\lambda(V_m - V_0)}{x(1-\lambda)} \geq p \qquad \textbf{(12)}$$

If we substitute the expression for $V_m - V_0$ into equation **(12)** and rearrange we obtain:

$$\frac{(1-m)}{m}u - \frac{r + x(1-\lambda) + \gamma\lambda(1-m)}{\gamma\lambda m}p \geq c \qquad \textbf{(13)}$$

If we rearrange equations **(11)** and **(13)** so that p is on the left-hand side, we get

$$p_1 = u - \frac{r + x(1-m)(1-\lambda)}{x(1-m)(1-\lambda)}c \geq p \qquad \textbf{(14)}$$

$$p_2 = \frac{(1-m)\gamma\lambda}{r + x(1-\lambda) + \gamma\lambda(1-m)}u - \frac{\gamma\lambda m}{r + x(1-\lambda) + \gamma\lambda(1-m)}c \geq p \qquad \textbf{(15)}$$

■

In words, we have shown that a payment system equilibrium exists if it is sufficiently cheap to use. Equation **(14)** shows the condition under which a seller is prepared to accept money through the payment system; equation **(15)** shows the condition under which a buyer is prepared to use the system. Both p_1 and p_2 are increasing in u. That is, the more utility a buyer gets from consumption, the more buyers are prepared to pay for using the payment system and the more sellers are prepared to accept money in the system that they can use at a later date. p_1 and p_2 are decreasing in c since a higher production cost reduces the gains from becoming a producer (following consumption) and hence, both buyers and sellers will only use the payment system if the price is lower. Both p_1 and p_2 are increasing in λ, showing that agents are prepared to pay a higher price for using the payment system the greater is the probability of being robbed.(3) The price that agents are prepared to pay is also increasing in the probability that an attempted robbery is successful, γ, if $p_2 \geq p_1$.(4)

4 Operational risk

So far, we have considered a payment system that functions without fail. In the real world, payment systems are susceptible to operational events that prevent agents from using it. Introducing operational events into the model is relatively straightforward. We simply assume that such events evolve according to a Bernoulli process. In particular, we assume that the probability of a system failure in any given period is $1-\delta$. In practice it is likely that the reliability of a system, δ, is increasing in the cost of using the system, p, since the owners of the system would have to invest money in making the system more reliable.(5) For now, we calculate bounds on the reliability of the system below which a payment system equilibrium does not exist for a given cost of using the system.

The Bellman equations for our extended model are shown in equations **(16)** through **(19)**.

$$rV_{ms} = \delta x(1-m)(1-\lambda)[u+V_0 - p - V_m] \quad \textbf{(16)}$$

$$rV_{mc} = x(1-m)(1-\lambda)[u+V_0 - V_m] + \gamma\lambda(1-m)[V_0 - V_m] \quad \textbf{(17)}$$

$$V_m = \max(V_{ms}, V_{mc}) \quad \textbf{(18)}$$

$$rV_0 = xm(1-\lambda)(1-\theta(1-\delta))(V_m - V_0 - c) + \gamma m\lambda(1-\theta)(V_m - V_0 - z) \quad \textbf{(19)}$$

Again, we restrict ourselves to generating conditions under which there are equilibria in which agents are using the payment system (ie, $\theta = 1$).(6) In these cases, the Bellman equations become:

$$rV_m = \delta x(1-m)(1-\lambda)(u+V_0 - p - V_m) \quad \textbf{(20)}$$

$$rV_0 = \delta xm(1-\lambda)(V_m - V_0 - c) \quad \textbf{(21)}$$

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(3) In fact, $\partial p_2 / \partial \lambda \geq 0$ if and only if $\gamma\lambda(1-m)u \geq \gamma\lambda mc$. However, $p_2 \geq 0$ implies that this inequality will hold; $p_2 < 0$ implies that there can never exist a payment system equilibrium as it would require the system to operate at negative cost.

(4) To get this result we once again appeal to the fact $\gamma\lambda(1-m)u \geq \gamma\lambda mc$.

(5) An analogy to this can be found in the literature on the optimal design of standards, in which it is shown that higher standards are more costly to maintain. See, for example, Immordino and Pagano (2003).

(6) Again, there will always be an 'autarchy' equilibrium and there may exist other equilibria for these parameter values. Our concern here is simply to show that a 'payment system equilibrium' can exist.

As before, we need both buyers and non-buyers to prefer to be active in the economy rather than drop out and live in autarchy and we need producers to have an incentive to accept a unit of money as payment when they meet a buyer who wants their good. This implies the equilibrium conditions: $V_m \geq 0$, $V_0 \geq 0$, and $V_m - V_0 \geq c$. Again, equation **(21)** shows that $V_m - V_0 \geq c$ implies $V_0 \geq 0$ and, hence, $V_m \geq 0$; so, we need only derive the conditions under which $V_m - V_0 \geq c$. Furthermore, for all buyers to use the payment system in equilibrium rather than carry cash we need

$$\delta x(1-m)(1-\lambda)(u+V_0-p-V_m) \geq x(1-m)(1-\lambda)(u+V_0-V_m)+\gamma\lambda(1-m)(V_0-V_m) \quad \textbf{(22)}$$

If we follow the same procedure as before, we find that the payment system will be used if and only if

$$\min\left\{\begin{array}{l} u-\dfrac{r+\delta x(1-m)(1-\lambda)}{\delta x(1-m)(1-\lambda)}c, \\ \dfrac{-r(1-\delta)-\delta xm(1-\lambda)(1-\delta)+\lambda(1-m)}{\delta[r+x(1-\lambda)-xm(1-\lambda)(1-\delta)+\lambda(1-m)]}u+\dfrac{m[x(1-\lambda)(1-\delta)+\lambda]}{r+x(1-\lambda)-xm(1-\lambda)(1-\delta)+\lambda(1-m)}c \end{array}\right\} \geq p \quad \textbf{(23)}$$

To derive lower bounds on the reliability of the payment system consistent with agents having an incentive to use it given the cost of so doing, we first rearrange equation **(23)** to get

$$u-p-\frac{r+\delta x(1-m)(1-\lambda)}{\delta x(1-m)(1-\lambda)}c \geq 0 \geq u-p-\frac{\delta m[x(1-\lambda)(1-\delta)+\lambda]}{r(1-\delta)+\delta xm(1-\lambda)(1-\delta)-\lambda(1-m)}c \quad \textbf{(24)}$$

We will now derive lower bounds on values of δ consistent with equation **(24)**:

$$u-p-\frac{r+\delta x(1-m)(1-\lambda)}{\delta x(1-m)(1-\lambda)}c \geq 0 \quad \Leftrightarrow \quad \delta \geq \frac{rc}{x(1-m)(1-\lambda)(u-p-c)} = \delta_1$$

$$0 \geq u-p-\frac{\delta m\left(x(1-\lambda)(1-\delta)+\lambda\right)}{r(1-\delta)+\delta xm(1-\lambda)(1-\delta)-\lambda(1-m)}c \quad \Leftrightarrow \quad \delta \geq \delta_2$$

where δ_2 is given by the solution to the quadratic equation

$$\delta_2^{\,2}xm(1-\lambda)(u-p-c)+\delta_2[r(u-p)+\lambda mc-xm(1-\lambda)(u-p-c)]-[r-\lambda(1-m)](u-p)=0$$

that lies between zero and unity.

Note that both δ_1 and δ_2 are increasing in the cost of using the system, p. Plotting these loci in δ-p space would enable us to find combinations of p and δ that were compatible with the existence of a payment system equilibrium. We could then allow for the fact that δ may depend on p by plotting a 'production possibilities frontier'; that is, those combinations of δ and p that are feasible given the cost of increasing reliability. Having done this, we would be left with the (possibly smaller) area of combinations of δ and p in which a payment system equilibrium exists.

As doing this would depend on the precise calibration of the model – something already left for future work – we leave this extension for future work.

To summarise, we have shown that, for there to be a payment system equilibrium, δ must satisfy:

$$1 \geq \delta \geq \max\{\delta_1, \delta_2\} \quad \textbf{(25)}$$

In words, we have shown that, for a given cost of using the payment system, buyers will choose to use it if it is sufficiently reliable.

5 Welfare

We will now consider whether the introduction of a payment system increases social welfare and to what extent increased reliability and lower costs of using an existing system increase social welfare. Let W be the level of social welfare in a monetary equilibrium when there is no payment system and W_p be the level of social welfare when there is a payment system and it is used. The social welfare functions are given by equations **(27)** through **(29)**. Equation **(27)** is the social welfare function when there is not a monetary equilibrium in the absence of a payment system. It equals zero because there is no trade. Where there is trade, welfare will be given by the appropriately weighted sum of the values of being a buyer and a seller. Mathematically:

$$W = mV_m + (1-m)V_0 \quad \textbf{(26)}$$

Equation **(28)** is the social welfare function when there is a monetary equilibrium even when there is not a payment system.

$$rW = 0 \qquad \text{if } \frac{x(1-m)(1-\lambda)u + \gamma\lambda mz}{r + x(1-m)(1-\lambda) + \gamma\lambda} < c \quad \textbf{(27)}$$

$$rW = xm(1-m)(1-\lambda)(u-c) - \gamma\lambda m(1-m)z \qquad \text{if } \frac{x(1-m)(1-\lambda)u + \gamma\lambda mz}{r + x(1-m)(1-\lambda) + \gamma\lambda} \geq c \quad \textbf{(28)}$$

$$rW_p = \delta xm(1-m)(1-\lambda)(u-p-c) \quad \textbf{(29)}$$

Equation **(29)** shows that in an economy in which a payment system is used, a reduction in the cost of using it, p, leads to an increase in social welfare. An increase in the reliability if the payment system, δ, also leads to an increase in social welfare. In principle, we can use these results to evaluate the benefit of spending resources on making payment systems more reliable. In particular, equation **(28)** suggests that, other things equal, the benefit of increasing payment system reliability from 90% to 95% equals $0.05\frac{xm(1-m)(1-\lambda)(u-p-c)}{r}$. Given values for each of the parameters in our model, we could see if this benefit were greater than the cost of achieving this increase in reliability. However, we would again have to make allowance for the fact that this cost may, in practice, result in the payment system becoming more costly to use and adjust p accordingly. The net benefit would, hence, be lower.

Equations **(27)** and **(28)** show that aggregate activity – that is, the number of trades – is given by $xm(1\text{-}m)(1\text{-}\lambda)$. That is, aggregate activity depends only on the exogenous proportion of buyers who will be interested in purchasing a given seller's good, x, the exogenous proportion of thieves in the population, λ, and the exogenous proportion of agents holding money, m. Since the introduction of a payment system in our model cannot alter any of these proportions, it cannot increase activity. Indeed, activity can only be lowered as a result of the fact that some trades will not happen if the system experiences an operational incident. This suggests that the introduction of a payment system may not necessarily be welfare increasing in our model. Propositions 3 and 4 consider under what conditions the introduction of a payment system increases social welfare.

Proposition 3

The introduction of a payment system into a monetary economy leads to an increase in social welfare if there is not a monetary equilibrium in the absence of a payment system.

Proof

In an equilibrium in which the payment system is used, $V_0 \geq 0$ and $V_m > 0$, and so $W_p > 0 = W$.

■

Proposition 4

The introduction of a payment system into a monetary economy leads to an increase in social welfare if there is a monetary equilibrium in the absence of a payment system if and only if

$$\frac{\gamma\lambda}{\delta x(1-\lambda)} z - \frac{(1-\delta)}{\delta}(u-c) > p$$

Proof

From equations **(27)** and **(28)**

$$r(W_p - W) = m(1-m)[\gamma\lambda z - x(1-\lambda)(1-\delta)(u-c) - \delta x(1-\lambda)p] > 0$$
$$\Leftrightarrow \frac{\gamma\lambda}{\delta x(1-\lambda)} z - \frac{(1-\delta)}{\delta}(u-c) > p$$

■

Robbery reduces social welfare because it can deter agents from trading and because thieves (in practice, society at large) incur the cost z when they are successful. If there is not a monetary equilibrium in the absence of a payment system, it is because the threat of robbery is deterring agents from trading. Thus, the introduction of a payment system that agents use will unambiguously increase social welfare since it leads to trade taking place. When there is a monetary equilibrium without a payment system, the introduction of a payment system will not lead to more trade. In this case, the introduction of a payment system can only increase social welfare through the fact that it results in no thief incurring the cost z anymore. But the expected savings in terms of the cost z must exceed the expected costs from using the payment system.

The expected cost saving from eliminating robbery occurring equals $m(1-m)\gamma\lambda z$. The expected cost of using the payment system has two components. There is the direct cost of using the system, $\delta xm(1-m)(1-\lambda)p$. There is also an indirect cost from trades that cannot take place because the payment system fails to function but that would have been executed if the buyer was carrying cash. This cost is $xm(1-m)(1-\delta)(1-\lambda)(u-c)$.

6 Conclusions

In this paper we have introduced a payment system into a search theoretic model of money and examined its implications for social welfare. The model we use is that of He *et al* (2005). They present a model where money is subject to the threat of theft and then introduce banks into the model. Agents can deposit their money in a bank where they are issued with a cheque book or a debit card that cannot be stolen. They show that cheques/debit cards may be necessary for there to be a monetary equilibrium. If there are banks there must also be an interbank payment system through which banks can make transfers to each other. He *et al* assume that this system is risk free and hence, largely subsume its role in their model. But in the real world payment systems are not risk free. In particular, they can suffer from, *inter alia*, operational risk, the focus of this paper. In payment systems that depend on the sending of electronic messages, there is the risk that the systems temporarily fail to function for some exogenous technological reason and prevent the system from being used.

We introduced operational risk into the He *et al* (2005) model by assuming that operational problems that prevent the payment system from working arrive according to a Bernoulli process. Agents do not know whether the system will function or not when they choose whether to use the payment system rather than carry money on their person. We show that agents have an incentive to use the payment system if it is both sufficiently cheap and sufficiently reliable. Agents have an incentive to use the system whether or not money is an accepted medium of exchange in the absence of a payment system. We then proceeded to consider the implications for social welfare of introducing a payment system. If there is not a monetary equilibrium in the absence of a payment system, introducing a system will unambiguously improve social welfare because it leads to trade occurring. In addition, once a payment system has been introduced, better reliability of (more stability in) the system will increase welfare. When there is a monetary equilibrium in the absence of a payment system, social welfare may increase or decrease. Welfare increases if the reduction in deadweight loss resulting from lower theft is greater than the costs from using the payment system (including these potential lost trades). Again, if this is the case, a more stable system will always increase welfare.

The fact that welfare may not necessarily increase is driven by the result that aggregate activity is determined solely by the exogenous proportions of agents holding money and of agents who are thieves. In our model, using the payment system reduces the total number of trades since, when it goes wrong (operational incidents happen), profitable trades cannot happen. This suggests a number of possible extensions that we leave for future work.[7] In particular, we could endogenise prices as in Shi (1995) and Trejos and Wright (1995) since, in that case, aggregate

[7] We are indebted to an anonymous referee for making these suggestions as to how we might extend this work.

output would also be endogenous. Alternatively, we could introduce divisible money and goods as in Lagos and Wright (2002). Finally, we could endogenise output by endogenising the choice of whether or not to be a thief. The effects of this extension are discussed in He *et al* (2005).

These results, though informative, do not answer the question of the extent to which central banks should devote resources towards improving the operational resilience of the systems they oversee. In order to do that, we would need to weigh up the costs of improving system reliability against the benefits of doing it. Given an appropriate calibration, the model of this paper could be used to quantify the benefits of increased reliability and such benefits could be compared with estimates of the costs. But we leave this to future work.

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